Monopoly Pricing with Social Learning[*]

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Abstract

This paper analyzes optimal dynamic pricing by a monopolist in a market where buyers learn about the quality of the good by observing each other. In the initial phase the monopolist prefers prices that allow more transmission of information from current to future buyers. Eventually the monopolist will stop the learning process, either by exiting or by capturing the entire market. Once an expensive good becomes popular, it is optimal for the monopolist to reduce the price and to sell to all consumers. The expected long-run inefficiency is shown to be generally lower than in the model with fixed prices. Efficiency can be enhanced by pricing below marginal cost. (JEL D83, L12, L15)

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1. Introduction

In many markets information is revealed by observing the behavior of others. Consider a ¯rm deciding whether or not to adopt a new technology; the behavior of other ¯rms with the same opportunity can provide information on the value of the investment. A consumer deciding which good to buy, or which store to patronize, can look at what is popular in an attempt to learn the intrinsic quality of the di®erent alternatives. In addition, in many cases it is di±cult for the decision maker to evaluate the quality of the good, even after usage. For example, it is almost impossible to judge the quality of surgery even after it has been performed, and even by a specialist in the same ¯eld. Patients who would like to choose the best doctor need to rely mostly on the information of others. Similarly, when a doctor decides which drug or treatment to prescribe, the \standard" prescription made by other doctors can weigh more heavily than scienti¯c knowledge and personal experience, because it reveals the information possessed by the profession.

To understand these problems of individual choice in a social context, Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992) have recently proposed a model of rational social learning that provides possible explanations for many social phenomena, like herd behavior and the fragility of fashions. These papers illustrate that imitative behavior can be both individually rational and socially wasteful. However, in these models prices are absent (or ¯xed), even though price °exibility seems to play a crucial role in many economic situations where social learning is at work. The model of dynamic monopoly pricing constructed in this paper is a step in developing an economic theory of social learning with prices.

Consider a situation of dispersed information and bilateral learning: neither the supplier nor any single buyer knows the true value of the product, though the society of buyers as a whole possesses this information. In order to analyze the pricing behavior in markets where consumers learn from each other about product quality, the monopolist is allowed to change the price of the good during the process of social learning. In this way, prices a®ect the decisions of the buyers and the aggregation of information. The main theoretical novelty of this paper is the dynamic use of prices as a screening device to allow information transmission from one consumer to another.

A new good is introduced by a monopolist. On the demand side of the market there is a sequence of potential buyers; one in each period. The consumers have the same preferences, and decide whether or not to purchase one unit of the indivisible good. The quality of the good is either high or low and is unknown both to the buyers and the seller. Each consumer decides whether to buy after observing the price and a private signal about the quality of the good, as well as the decisions of previous consumers and also the prices previously posted. Observation of the behavior of others is used to infer information about the quality of the good. The consumer does not take into account the e®ect of her decision on the information revealed to others. This \social learning externality" concerning the value of information for future consumers leads to an ine±cient aggregation of information.

The monopolist faces a trade o® between myopic pro¯t maximization and acquisition of information. In the ¯rst phase the monopolist posts more informative prices, because learning is valuable. In particular, in the model with binary signals, in this phase prices are set high enough so that consumers with di®erent signals behave di®erently. Because the purchase decision at \high" prices reveals favorable information to others, this pricing strategy allows subsequent consumers to infer the private information of the current buyer. In this phase of active social learning, conditional on the information possessed by the market, the price decreases on average over time. If the monopolist sells, then the price increases, otherwise it decreases. For superior products, however, the price as well as the probability of selling increase on average in the learning phase. Eventually learning will stop, because the monopolist decides either to exit the market, or to capture it entirely by reducing the price. If the price were reduced before the good had become popular, consumers would buy the good regardless of their information, so that their actual information would not be revealed by their decision. This simple observation can help in understanding the strategy for establishing a brand-name product or for becoming a successful professional.

The model presented is stylized, in that a simple sequential structure is imposed and perfect observation of past prices and decisions of all previous decision makers is assumed. Nonetheless, some insights into real-world markets can be gained from the analysis. Social learning may explain why young independent professionals, such as doctors and lawyers, are willing to be underemployed and charge high fees relative to their perceived quality, rather than reduce the price for their services. The observed discounts given on books once they are listed as best sellers are consistent with the prediction that the price should be reduced only after the good becomes popular. In the remainder of the paper the language of a consumption decision is adopted, although the same framework can be applied to many decision problems (see the end of Section 5 for other examples).

In models of social learning the payo® to the consumer does not depend directly on the decisions of other consumers. Learning from others may provide a foundation for the \social in°uence on price" of Becker (1991), who plainly assumes that a consumer's demand for a good depends on the demands by other consumers. Alternative foundations are consumption externalities due to \bandwagon" or \snob" e®ects, as in Pesendorfer (1995), and technological externalities due to network e®ects, as in Bensaid and Lesne (1996) or Cabral, Salant and Woroch (1994).

The paper is organized as follows: Section 2 introduces the model of social learning for a given price sequence. Section 3 characterizes the solution to the discounted dynamic optimization problem of the monopolist and the properties of the optimal price sequence. Section 4 is devoted to welfare analysis: the monopolist solution is compared to the constrained social optimum, and the e®ect of monopoly pricing on the probability that buyers eventually settle on the less e±cient action is studied. In Section 5 the model is extended to consider the possibility that individual consumers delay their purchasing decision, that the monopolist possesses some private information on the value of the good supplied, that past prices are not observed by the consumers, and that the monopolist can control the acquisition of private information by the consumers. Some implications for regulation of competition and applications are discussed at the end of Section 5. Section 6 relates this work to existing literature. Section 7 concludes. The details of the proofs are collected in Appendix A.

2. The Model

A. Supply: Monopoly. In each period the monopolist posts a price for a unit of the good and supplies it at constant marginal cost, set equal to zero for convenience of notation, whenever the buyer demands it. The price quoted by the monopolist in period n is denoted by P^n. The monopolist is risk-neutral and maximizes expected discounted profits. Alternatively, the model can be reinterpreted as one of competition between two varieties of a good, one produced by a monopolist and the other one by a competitive sector. The monopolist has property rights for the "new" variety and can act strategically on prices, while the established variety is sold at marginal cost by a competitive sector with free entry and exit in any period. For more on this, see Section 5.

Prices affect both the decision of the current consumer and the learning process. Both effects are considered by the monopolist. The social learning externality is (partially) internalized by the monopolist, who can appropriate the benefits of learning on the good by acting effectively on prices.

B. Demand: Social Learning of the Consumers. The demand side of the market is based on the simple model of rational social learning of Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992). There is a potential buyer in each period, with a unit demand for the indivisible good supplied by the monopolist. In the basic version of the model the order of buyers is either fixed or random. Individuals, indexed by $n = 1, 2, \ldots$, must decide sequentially which one of the two available actions denoted by a_0 and a_1 to take. Action a_1 corresponds to buying the good and action a_0 to not buying it. There are two states of the world, ω_0 and ω_1, indicating which action is the appropriate one. Let q^1 be the common prior belief that the state is ω_1 at the beginning of time.

All consumers have the same preferences, are risk neutral, and maximize the expected payoff net of the price paid. They update beliefs in a Bayesian fashion. Each consumer would like to buy the good when it is valuable, so that action a_1 is better than action a_0 in state ω_1, while the opposite is true in state ω_0. Assume that action a_0 is a safe action, yielding a payoff normalized to be zero. Without loss of generality, assume that the payoff matrix is symmetric:

$$
\begin{array}{ccc}
\text{PAYOFF} & \text{if state is} \quad \omega_0 & \omega_1 \\
\text{from action} & & \\
a_0 & 0 & 0 \\
a_1 & -1 & 1
\end{array}
\tag{2.1}
$$

Before deciding which good to buy, each consumer n observes one of two possible private signals $\sigma^n \in \{\sigma_0, \sigma_1\}$ and the public history h^n of the decisions (actions and prices) of all preceding individuals. See section 5 for more general signal distributions. The probability distribution of the private signal depends on the state of the world. In this binary signal setting the probability that signal σ_i is realized conditional on the state being ω_j is α if $i = j$ and $1 - \alpha$ if $i \neq j$, with both i and j belonging to $\{0, 1\}$. Assume that the private signal is informative, but not completely revealing, i.e. that $1/2 < \alpha < 1$.[1] For $n \geq 2$ let $H^n \equiv \{\{a_0, a_1\} \times \mathbb{R}\}^{n-1}$ be the space of all possible histories of actions taken by the $n - 1$ predecessors of individual n and prices for the good posted in the past. Let h^n denote an

[1] A signal of quality $\alpha < 1/2$ would be equivalent to one of quality $1 - \alpha > 1/2$ after relabelling the alternatives.

element of H^n.

Let $q^n \equiv \Pr(\omega_1 | h^n)$ be the assessed probability that the state is ω_1 conditional on the publicly observed history h^n. Notice that a higher q corresponds to an higher perceived quality of the monopolist's good. Similarly let $f_i(q^n) \equiv \Pr(\omega_1 | h^n; \sigma_i)$ be the private belief that the state is ω_1 conditional on both the action history and the realization σ_i of the private signal. Bayes' rule yields

$$f_0(q^n) = \frac{(1-\alpha)\,q^n}{\alpha(1-q^n)+(1-\alpha)\,q^n}; \tag{2.2}$$

$$f_1(q^n) = \frac{\alpha q^n}{\alpha q^n + (1-\alpha)(1-q^n)}. \tag{2.3}$$

Clearly $f_1(.) > f_0(.)$, and $f_1(.)$ is a convex function of q^n, while $f_0(.)$ is concave.

The customer observes the signal, updates her private belief, and computes, given the payoff matrix, the expected payoff for the different actions available in monetary terms. The following table summarizes the valuations of the consumer depending on the signal received:

EXPECTED PAYOFF from action	if signal received	$\sigma^n = \sigma_0$	$\sigma^n = \sigma_1$
a_0		0	0
a_1		$2f_0(q^n) - 1$	$2f_1(q^n) - 1$

These gross valuations are used to compute the net expected payoff from buying the good, given that the price of the good is equal to P^n in period n. Clearly, the decision a^n of consumer n is to buy the good if this yields higher net expected payoff than not buying it.

For instance, if the private signal received by individual n is $\sigma^n = \sigma_1$, then action $a^n = a_1$ is taken for all prices P^n that satisfy $2f_1(q^n) - 1 - P^n \geq 0$. The decision rule can be summarized as:

$$\begin{array}{ll} \text{if } \sigma^n = 0 \text{ and } & \left\{ \begin{array}{lll} 2f_0(q^n) - 1 \leq P^n & \Rightarrow & a^n = a_0 \\ 2f_0(q^n) - 1 > P^n & \Rightarrow & a^n = a_1 \end{array} \right. \\ \text{if } \sigma^n = 1 \text{ and } & \left\{ \begin{array}{lll} 2f_1(q^n) - 1 < P^n & \Rightarrow & a^n = a_0 \\ 2f_1(q^n) - 1 \geq P^n & \Rightarrow & a^n = a_1 \end{array} \right. \end{array} \tag{2.4}$$

where the choice in the case of indifference is assumed to be the one which minimizes the possibility of herding.

C. Information and Timing. The quality of the good (i.e. the state ω) is given exogenously, and is unknown both to the seller and to the customers. The assumption that there is no agent with complete knowledge is common in the learning literature.[2] An alternative interpretation is that the seller does not know the preferences of the consumers. The partially informative signal received by each consumer n is her private information. For example, doctors have scientific knowledge on which drug to prescribe, or investors might be informed on the profitability of different opportunities. Consumers might have acquired information through trial periods (See Section 5). The signal is not observed

[2] In Section 5 it is argued that the results of the analysis are valid whenever the state of nature is not perfectly known by the seller.

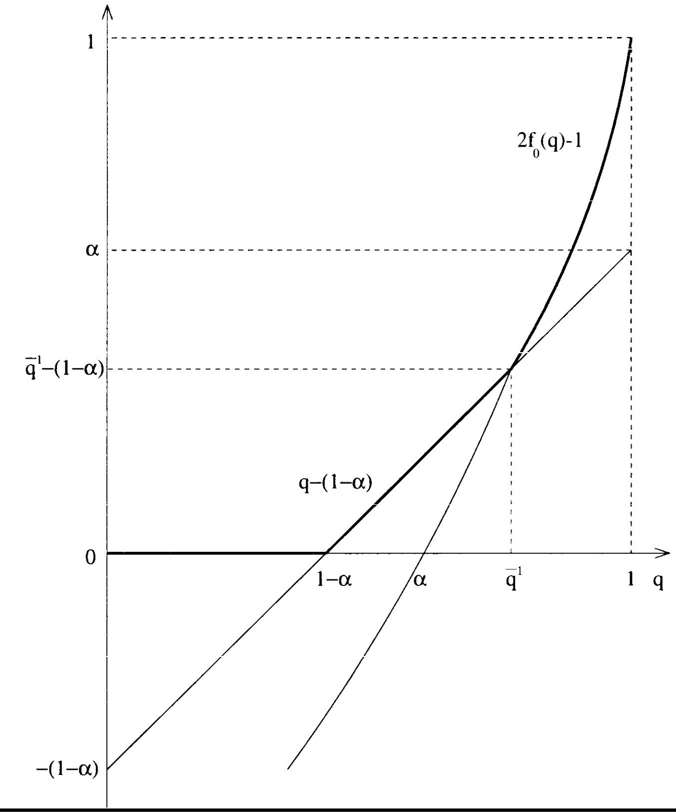
directly either by the sellers or by other consumers, but needs to be inferred from observable behavior. The decision of consumer n (whether to buy and which variety to buy) and the price quoted to consumer n are common knowledge to the current and future consumers.[3] The conditional distribution of the signals and the initial belief q^1 that the good is valuable are also common knowledge.

No further information is communicated to others. There are three possible justi¯-cations for this assumption. First, the payo® realization might not be available to the consumer, even aft

According to the previous definition and (2.4), there is a cascade in period n if $2f_0(q^n) - 1 > P^n$, or $2f_1(q^n) - 1 < P^n$. Notice that anyone can compute these conditional probabilities and determine when a cascade arises. In a cascade on action a_i, the same action $a^n = a_i$ is chosen regardless of the signal σ^n received. The next individual $n + 1$ knows that a^n is uninformative, and computes her prior as

$$q^{n+1} = q^n. \tag{2.7}$$

In summary, the dynamics of the public beliefs prior to the observation of the private signal is determined by (2.6) as long as $2f_0(q^n) - 1 \leq P^n$ and $2f_1(q^n) - 1 \geq P^n$ simultaneously hold (when not in a cascade) and by (2.7) otherwise (during the cascade), with given initial condition q^1.



[redacted]

a payoff of $q-(1-\alpha)$ higher than the pooling price profit $2f_0(q)-1$ for $q<\overline{q}^1$. For $q\in[1-\alpha,\overline{q}^1]$ the monopolist posts the separating price $P_S(q)=2f_1(q)-1$ and sells only to the consumer with a favorable signal. For $q\geq\overline{q}^1$ the monopolist posts the pooling price $P_P(q)=2f_0(q)-1$ and sell to the entire market.

A. Last Period. The solution of the last period problem gives the myopically optimal strategy. The expected payoff of each of the three undominated policies is depicted graphically in Figure 3.1 as a function of the belief q. Not selling at all gives 0, posting the separating price yields $q - (1 - \alpha)$, and posting the pooling price $2f_0(q) - 1$. For $q < 1 - \alpha$ the monopolist decides not to sell since in this region $2f_1(q) - 1 < 0$ and this is the last period. The separating price $P_S(q)$ will be charged if the associated expected payoff $q - (1 - \alpha)$ is higher than $2f_0(q) - 1$ yielded by the pooling price $P_P(q)$. After substitutions it can be easily verified that the pricing strategy in the last stage is

$$P^1(q) = \begin{cases} \text{any} > 2f_1(q) - 1 & \text{for } q \le 1 - \alpha \\ P_S(q) = 2f_1(q) - 1 & \text{for } 1 - \alpha \le q \le \overline{q}^1 \\ P_P(q) = 2f_0(q) - 1 & \text{for } q \ge \overline{q}^1 \end{cases}$$

where $\overline{q}^1 = \frac{-(1-\alpha)^2 + \sqrt{(1-\alpha)^4 + (2\alpha - 1)\alpha^2}}{(2\alpha - 1)}$ is the largest root of the quadratic equation $k^1(q;\alpha) = (2\alpha - 1)q^2 + 2(1-\alpha)^2 q - \alpha^2$.

The probability of selling decreases with the price charged. When the belief is q, if the price $P \in [P_P(q); P_S(q)]$, the probability of selling is $\Pr(\sigma_1|q)$; if instead $P < P_P(q)$, then the good is sold for sure. The demand function has two steps: as q increases the upper step approaches the lower one. Because in the limit as q goes to 1, $f_1(q)$ and $f_0(q)$ both tend to 1, the separating price tends to the pooling one. The demand at the separating price is equal to $\Pr(\sigma_1|q)$ and tends to α as q goes to 1. Therefore it must be optimal to charge the pooling price that sells with probability 1, for q large enough.

The resulting value function is globally convex, being the maximum of convex functions. The last period value function

$$V^1(q) = \begin{cases} 0 & \text{for } q \le 1 - \alpha \\ q - (1 - \alpha) & \text{for } 1 - \alpha \le q \le \overline{q}^1 \\ 2f_0(q) - 1 & \text{for } q \ge \overline{q}^1 \end{cases}$$

is flat at zero for $q < 1 - \alpha$, linearly increasing in q for $q \in [1 - \alpha; \overline{q}^1]$, and strictly convex for $q > \overline{q}^1$. Convexity of the monopolist convex function holds under fairly general conditions on the signal distributions, as discussed in Section 5.

B. Next to Last Period. In the next to last period, for $q < 1 - \alpha$, the monopolist might be willing to bear current losses charging the price $P_S(q) < 0$ in order to gain by selling at a positive price in the next (and last) period in case of a good draw (i.e. if σ_1 is realized). Consider $q < 1 - \alpha$ and the price $P_S(q)$ in the next to last period followed by the optimal policy in the last period. Denoting the discount factor of the firm by β, the expected discounted profit under this policy is $q - (1 - \alpha) + \beta \Pr(\sigma_1|q) V^1(f_1(q))$, because $V^1(f_0(q)) = 0$, as the monopolist exits the market in the last period if unfavorable information (σ_0) is reveled in the next-to-last period, being $f_0(q) < q < 1 - \alpha$. If instead the realization is σ_1, the separating price for the new belief $f_1(q)$ is charged in the last period yielding $V^1(f_1(q)) = f_1(q) - (1 - \alpha)$, so that the total expected discounted payoff from continuing learning is $q - (1 - \alpha) + \beta \Pr(\sigma_1|q)[f_1(q) - (1 - \alpha)]$, which is larger than zero, the payoff obtained by exiting the market right away, when

$$q \ge \frac{(1 - \alpha)(1 + \beta(1 - \alpha))}{1 + \beta[1 - 2\alpha(1 - \alpha)]} \equiv \underline{q}^2. \tag{3.1}$$

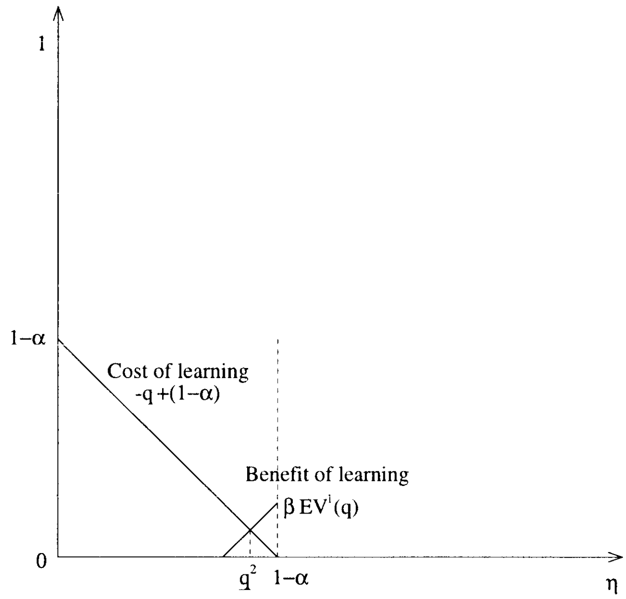


Figure 3.2: For $q < \underline{q}^1$, in the two-period problem the monopolist posts the separating price $P_S(q)$ when the benefit of learning $\beta EV^1(q)$ is larger than the cost of learning $q - (1-\alpha)$.

Notice that $\underline{q}^2 < \underline{q}^1 \equiv 1 - \alpha$. For $q \in \left[\underline{q}^2, \underline{q}^1\right]$ the monopolist is willing to bear immediate losses, that will be more than offset by the expected future profits made if favorable information is revealed to the market through a sale. Figure 3.2 illustrates the trade-off between current losses from selling below marginal cost and future benefits from social learning.

Consider $q > \overline{q}^1$, then there is a trade-off between (a) a higher instantaneous payoff with the pooling price $P_P(q)$ (myopically optimal in this region) than with the separating price $P_S(q)$, and (b) a higher future expected payoff with the separating price $P_S(q)$ that allows for social learning, due to the convexity of $V^1(.)$.

For q close enough to, and strictly larger than $\overline{q}^1$, it is optimal to charge the separating price $P_S(q)$ since the benefit from learning is larger than the reduction in current profits (which is infinitesimal for q close enough to $\overline{q}^1$). Define the "expected value function" with one period to go as $EV^1(q) \equiv \Pr(\sigma_0|q)\, V^1(f_0(q)) + \Pr(\sigma_1|q)\, V^1(f_1(q))$. We need to compare the payoff from the pooling price to the one from the separating price

$$\underbrace{2f_0(q) - 1}_{\substack{\text{Current profit}\\ \text{if pooling}}} + \underbrace{\beta V^1(q)}_{\substack{\text{Continuation value}\\ \text{if pooling}}} \gtrless \underbrace{q - (1-\alpha)}_{\substack{\text{Current profit}\\ \text{if separating}\\ = \Pr(\sigma_1|q)[2f_1(q)-1]}} + \underbrace{\beta EV^1(q)}_{\substack{\text{Continuation value}\\ \text{if separating}}},$$

or, equivalently

$$2f_0(q) - q - \alpha \gtrless \beta \left[EV^1(q) - V^1(q)\right]. \tag{3.2}$$

for $q \geq \overline{q}^1$. In this region the left hand side of (3.2) is increasing in q, while the right hand side is decreasing in q, so that there exists a unique cutoff level of the public belief $\overline{q}^2$

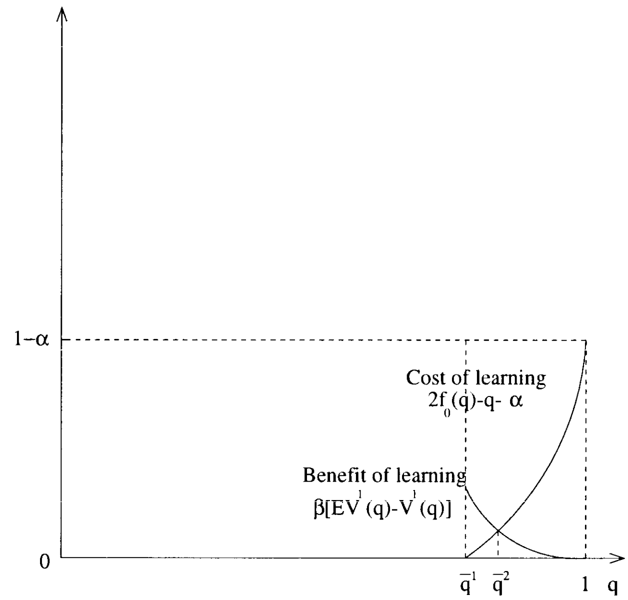


Figure 3.3: In the two-period problem the monopolist posts the separating price $P_S(q)$ when the benefit of learning $\beta\left[EV^1(q) - V^1(q)\right]$ is larger than the cost of learning $2f_0(q) - q - \alpha$.

above which the monopolist strictly prefers to charge the pooling price and below which the separating price is charged. For a graphical illustration see Figure 3.3.

Then the optimal pricing policy in the next to last period (2) is

$$P^2(q) = \begin{cases} \text{any } > 2f_1(q) - 1 & \text{for } q \le \underline{q}^2 \\ P_S(q) = 2f_1(q) - 1 & \text{for } \underline{q}^2 \le q \le \overline{q}^2 \\ P_P(q) = 2f_0(q) - 1 & \text{for } q \ge \overline{q}^2 \end{cases}$$

C. Properties of the Value Function and Solution. The monopolist solves a discounted problem with bounded returns per stage. In this sub-section the properties of the value function of the finite-horizon problem with N periods are characterized. Since this N-stage problem is a truncation of the infinite-horizon one, the properties established in this sub-section will prove useful to characterize the optimal stationary policy of the infinite-horizon problem. Denote by $V^N(q)$ the value function of the N-period problem:

$$V^N(q) = \max\left\{0 + \beta V^{N-1}(q), q - (1-\alpha) + \beta EV^{N-1}(q), 2f_0(q) - 1 + \beta V^{N-1}(q)\right\}, \tag{3.3}$$

where the "expected value function" of the $N-1$-period problem is defined as

$$EV^{N-1}(q) \equiv \Pr(\sigma_0|q)\, V^{N-1}(f_0(q)) + \Pr(\sigma_1|q)\, V^{N-1}(f_1(q)).$$

Clearly $V^N(q)$ is the maximal return that can be obtained starting at state q and proceeding for N stages. The three arguments in the left hand side of (3.3) corresponds to the three non-dominated strategies available to the monopolist. If the monopolist decides not to sell, the discounted continuation payoff is equal to $\beta V^{N-1}(q)$. If the monopolist posts the separating price $P_S(q)$, the expected immediate payoff is equal to $q - (1 - \alpha)$ and the discounted expected continuation payoff $\beta EV^{N-1}(q)$, as the current and continuation payoff depend on the purchase decision of the current consumer, determined by the signal received. Finally the pooling price $P_P(q)$ sells with probability one, and result in a posterior belief equal to the prior, so that the continuation payoff is $\beta V^{N-1}(q)$.

The value function is continuous and non-decreasing in q for any N, as it can be immediately verified.

Lemma 1. (Continuity and monotonicity in q of the value function of the N period problem) The value function $V^N(.)$ of the problem with any horizon N is continuous and, when strictly positive, strictly increasing in q. Similarly, the expected value function $EV^N(.)$ is continuous and, when strictly positive, strictly increasing in q.

Clearly the value function of the problem with N periods left to go is larger than the value function of the problem with $N - 1$ periods left to go. It is also possible to give a lower bound to the increase of the value function due to the addition of one more period. This lower bound will be crucial in proving an important result later in this section.

Lemma 2. (Monotonicity of N-period-to-go value function in N) The value function $V^N(.)$ of the N-period-to-go problem is non-decreasing in the number of periods to go N. For $q \geq \alpha$, it is strictly increasing in N with

$$V^N(q) > V^{N-1}(q) + \beta^{N-1}(2f_0(q) - 1).$$

Proof. See Appendix A.

Corollary 1. $EV^N(.)$ is non-decreasing in the number of periods to go N.

It is now shown that the value function of the finite-horizon optimization problem is convex.

Lemma 3. (Convexity of the value function) The value function $V^N(.)$ is convex.

Proof. See Appendix A.

In the rest of this sub-section the Bellman equation (3.3) is analyzed since $2f_0(q) - 1 > 0$ for $q > \alpha$. By Lemma 2 and the assumption $\beta < 1$, the first argument in the maximum of equation (3.4) can be achieved only for q that satisfy $V^N(q) = V^{N-1}(q) = 0$, so that for $q < \alpha$ the value function satisfies

$$V^N(q) = \max\left\{0; q - (1-\alpha) + \beta E V^{N-1}(q)\right\}:$$

Consider the policy for exiting. It is argued that with N periods left to go, the monopolist does not sell at all to the current consumer whenever the belief is lower than the cutoff level $\underline{q}^N$, and sells for higher beliefs. At the same time, it is shown that the sequence $\{\underline{q}^N\}$ of lower cutoff levels is decreasing in the number of periods left to go N. Since the cutoff levels below which it is optimal not to sell decrease as the horizon of periods becomes longer, the monopolist never reverses the decision to stop selling.

To understand why a cutoff policy for exit is optimal, consider $q \leq 1 - \alpha$. Selling to the current consumer involves an immediate loss of $(1-\alpha) - q$ and a future gain of $\beta\left[EV^{N-1}(q) - V^{N-1}(q)\right]$. The current loss from selling is equal to $(1-\alpha)$ for $q = 0$, linearly decreases in q and is equal to 0 for $q = 1 - \alpha$. The benefit from selling in terms of future expected profits if favorable information on the product is revealed by a current sale is continuous, equal to 0 for $q < f_0\left(\underline{q}^{N-1}\right)$, positive and strictly increasing (by Lemma 1) in q for $q > f_0\left(\underline{q}^{N-1}\right)$. It is clear that there exists a unique belief $\underline{q}^N$ below which it is optimal for the monopolist to exit the market and above which to sell.

The cutoff level below which it is optimal not to sell decreases as the horizon ahead becomes longer. As shown in the previous sub-sections, in the last period it is optimal not to sell at all rather than charging the separating price $P_S(q)$ (with an expected profit of $q - (1-\alpha)$) for $q \leq \underline{q}^1 \equiv 1-\alpha$. In the two-period problem it is optimal for $q \leq \underline{q}^2$ not to sell in both the current and next period rather than quoting the separating price $P_S(q)$ – that sells at loss to the consumer with favorable information – in the current period and following the optimal strategy thereafter. By (3.1): $\underline{q}^2 < \underline{q}^1 = 1 - \alpha$ for $\beta < 1$. The general result on the optimality and monotonicity of the cutoff policy for exit is stated in the next result.

Lemma 4. (Optimality and monotonicity of cutoff levels for exit in N) *In the problem with N periods to go, it is optimal to exit the market when the belief is below the cutoff level $\underline{q}^N$. In addition, the cutoff levels $\underline{q}^N$ is non-increasing in the number of periods to go N, i.e. $\underline{q}^N \leq \underline{q}^{N-1}$.*

Proof. See Appendix A.

When is it optimal to capture the entire market? The decision to continue learning involves a current cost and a future benefit. To keep the learning process going, it is necessary to price high, even though it would be myopically optimal to sell to all the consumers at the pooling price. The "cost of learning" is the loss in current expected profit from learning, equal to $2f_0(q) - q - \alpha$, a continuous strictly increasing function of q that takes the value of 0 for $q = \bar{q}^1$ and the value of $1-\alpha$ for $q = 1$. The benefit in future higher profits from learning is $\beta\left[EV^{N-1}(q) - V^{N-1}(q)\right]$.

It is now proven that a cutoff policy for stopping learning is optimal. In particular, it is optimal to stop the learning process for $q \geq \bar{q}^N$ for the problem with N periods to go.

Lemma 5. (Optimality of cutoff policy for capturing the market) *The optimal policy for the problem with N periods left to go is to capture the whole market when the belief q is above the cutoff level $\overline{q}^N$.*

Proof. See Appendix A.

The sequence of the cutoff levels $\{\overline{q}^N\}$ is increasing in the number of periods to go N. This derives from the fact that experimentation has a larger value with a larger horizon N ahead. In the proof of the n the pooling price strategy that allows the monopolist to stop the learning process of the consumers.

Note that for q small enough it is optimal to post the separating price (even though it is negative), and sell with positive probability at a current loss.

Lemma 7. (Exit) For belief lower than $\underline{q} \in (0; 1 - \alpha)$ the monopolist exits the market.

Proof. See Appendix A.

Learning will eventually be stopped if the threshold $\bar{q}$ above which the monopolist will adopt the low price strategy and sell to all consumer types is strictly smaller than 1. The threshold $\bar{q}$ satisfies

$$\beta [EV(\bar{q}) - V(\bar{q})] = [2f_0(\bar{q}) - \bar{q} - \alpha]: \quad (3.5)$$

Notice that as q tends to 1, $f_1(q)$ converges from above to $f_0(q)$, so that the pooling price converges to the separating price. The separating price sells to a fraction that converges from below to α, while the pooling price sells with probability 1. The value of learning $\beta [EV(q) - V(q)]$ goes to 0 as q tends to 1, so that the myopically optimal pooling price is charged when the belief is larger than the threshold $\bar{q} < 1$.

Lemma 8. (Capture of the entire market) When the belief is larger than $\bar{q} \in (\bar{q}^1; 1)$, learning is stopped by the monopolist.

Proof. See Appendix A.

The separating price strategy by the monopolist in the learning phase allows the next consumers to infer the private information of the preceding consumers from the observation of their purchase decisions. If the monopolist sells, then the price increases in the next period, otherwise it decreases. In particular, if the monopolist was able to sell at the separating price $2f_1(q) - 1$ in the previous period, the public belief increases from q to $f_1(q)$, and the price increases to $2f_1(f_1(q)) - 1$, if still in the learning phase. If instead the good was not sold, the price decreases from $2f_1(q) - 1$ to $2f_1(f_0(q)) - 1 = 2q - 1$, if still in the learning phase. One of the barriers $\underline{q}$ or $\bar{q}$ will be hit with probability one, since the true unconditional process followed by the belief is a random walk drifted toward the true state. This implies that eventually learning will stop, because the monopolist decides either to exit the market, or to capture it entirely by reducing the price. These results of the analysis of the infinite-horizon model are summarized in:

Proposition 1. In a period t of the first learning phase, the monopolist charges the separating price $P_S(q^t) = 2f_1(q^t) - 1$, high relative to perceived quality q^t. If still in the learning phase, the price in next period increases to $P_S(f_1(q^t)) = 2f_1(f_1(q^t)) - 1$ if the good was sold in the previous period, or decreases to $P_S(f_0(q^t)) = 2q^t - 1$ if the good was not sold. Eventually, the monopolist will almost surely stop the social learning process by either (a) exiting the market the first time ($\underline{t}$) that the belief q^t is below the cutoff level $\underline{q}$, or (b) selling to everyone by reducing deterministically the price to the separating level $P_S(q^t) = 2f_0\left(q^{\bar{t}}\right) - 1$, low relative to perceived quality, from the first time ($\bar{t}$) that the belief crosses the cutoff level $\bar{q}$.

The monopolist is willing to forgo immediate payoff in order to acquire information. Prices act as a screening device that transmits information from current to future consumers. Given the binary structure of signals, the higher separating price is the only informative price. See Section 5 for more general signal structures.

E. Stochastic Properties of the Price Sequence. In this sub-section it is shown that the price decreases on average conditional on the market information. This result will prove useful in Section 5 to consider the incentive for foresighted consumers to wait for lower prices. In the learning phase, the expected next period price if the consumer buys today is $\Pr(\sigma_1|q)\left[2f_1^2(q) - 1\right] + \Pr(\sigma_0|q)\left[2q - 1\right]$. Therefore in the learning phase the expected difference between tomorrow's price and today's price is equal to

$$2\left[\Pr(\sigma_1|q)\, f_1^2(q) + \Pr(\sigma_0|q)\, q - f_1(q)\right]; \tag{3.6}$$

so that the price decreases on average over time.

Proposition 2. The price sequence is a supermartingale.

Proof. Consider first the learning phase, and then the cascade phase. During the learning phase the separating price is posted. The separating price $P_S(:)$ is a convex function of the belief. Jensen inequality and the martingale property of the belief imply that the price is decreasing on average in the learning phase. When instead a cascade on action a_1 is induced the next period if the good is sold today, i.e. for $q \in [f_0(\overline{q}); \overline{q})$, the price decreases deterministically from $2f_1(q) - 1$ to $2q - 1$. This is readily seen, since if the good is sold today, so that the belief is updated to $f_1(q)$, the price tomorrow is set at the "pooling" level $2f_0(f_1(q)) - 1 = 2q - 1$ for the new belief $f_1(q)$. If instead action a_0 is taken today, so that the belief is updated to $f_0(q)$, the price is set at the "separating" level $2f_1(f_0(q)) - 1 = 2q - 1$ for the new belief $f_0(q)$. Finally for $q \in [\overline{q}; 1)$ the price is fixed at the pooling level $2f_0(q) - 1$. ■

The expected reduction in price in the learning region can be rewritten as $-\cdot P_q \equiv (2\alpha - 1)\, q(1-q)\, K(q;\alpha)$, where

$$K(q;\alpha) \equiv \quad 2(2\alpha - 1)^2 q(1-q)$$

Proposition 3. Conditional on the good being valuable, the price sequence increases on average in the learning phase.

Conditional on the good not being valuable (state ω_0), the price decreases on average in the learning phase. The expected decrease in price conditional on state ω_0 and departing from belief q is $-\Delta P_0 = \alpha q K(q;\alpha)$.

Proposition 4. Conditional on the good not being valuable, the price sequence decreases on average.

Clearly the unconditional expected change in price is equal to the average of the conditional expe

B. Do Superior Goods Prevail in the Long Run? This section addresses the question of long-run efficiency. In the herding model of social learning the decisions of initial consumers affect the behavior of those following. This externality is at the origin of the long-run inefficiency occurring when all but a finite number of consumers choose the wrong action. In this paper the property right for the good is given to a monopolist who is allowed to change the price along the social learning path. Intuitively, the monopolist internalizes the externality when it is unfavorable to the good supplied. Therefore it might seem that the allocation of property rights should reduce the inefficiency associated with not selling the good when it is valuable, but increase the inefficiency that arises when the good is sold even if it is inferior. But the monopolist has also the power to extract more rents from the consumers by delaying the capture of the whole market in order to sell at a higher price. This brings in an inefficiency in the short run, because the high price of the good discourages type-0 consumers from buying the good, even if their expected valuation with equal prices would be maximized by buying, but has also the beneficial effect of increasing the region of learning, so that also the long-run inefficiency associated to the externality unfavorable to the monopolist tends to be reduced. In this section it is shown that the expected inefficiency in the monopoly model is lower than in the model without prices.

In the model with fixed (resp. monopoly) prices, if the best action is a_1, the inefficiency arises when the belief hits the lower threshold $1-\alpha$ (resp. $\bar{q}$) before reaching the upper one, α (resp. $\underline{q}$). Similarly, conditional on a_0 being the best action, all but a finite number of consumers take a_1 when the belief hits α (resp. $\bar{q}$) before $1-\alpha$ (resp. $\underline{q}$). We know that $\underline{q} < 1-\alpha$ (Lemma 7) and $\bar{q} > \alpha$ (Lemma 8). Consider the effect of the increase in $\bar{q}$ on the inefficiency favorable to the monopolist – the good is sold when it is not valuable. On the one hand this inefficiency is reduced because it is more difficult that the now higher upper barrier $\bar{q}$ is hit before the lower barrier, on the other hand it is increased because of the reduction in the lower barrier $\underline{q}$. The two effects go in opposite direction, and the total effect is ambiguous for this as well as for the other conditional inefficiency (unfavorable to the monopolist).

Nonetheless, it is possible to sign the change in expected inefficiency due to the enlargement of the learning region. When the upper barrier $\bar{q}$ increases, the reduction of the favorable inefficiency is stronger than the increase of the unfavorable one, because the belief is drifted toward the correct state. Similarly for the effect of a decrease in the lower barrier $\underline{q}$. In this section it is shown that the ex-ante expected inefficiency is reduced by strategic pricing by one firm for initial belief q^1 close enough to $1/2$.

Some preliminary analysis is needed in order to be able to compute conditional inefficiencies. Let $\phi_j(q)$ denote the probability that the public belief starting at $q \in [\underline{q}, \bar{q}]$ reaches $\bar{q}$ before it reaches $\underline{q}$, conditiona

Lemma 9. (Hitting Probability) The probability $\phi_i(q)$ that the public belief starting at $q \in [\underline{q}, \overline{q}]$ reaches $\overline{q}$ before it reaches $\underline{q}$, conditional on the state of the world ω_j is

$$\phi_j(q) = \frac{1 - r_j^{\,\underline{i}(q;\underline{q})}}{1 - r_j^{\,\underline{i}(\overline{q};\underline{q})}}. \tag{4.1}$$

where $\underline{i}\left[q; \underline{q}\right] \equiv \min\left\{m \mid f_0^m(q) < \underline{q}\right\}$ is the minimum number of transition needed to take the belief from q to below the barrier $\underline{q}$.

For example the long-run inefficiency favorable to the monopolist is

$$I_0\left[q^1\right] = \phi_0\left[q^1\right] = \frac{1 - \left[\frac{\alpha}{1-\alpha}\right]^{\underline{i}(q^1;\underline{q})}}{1 - \left[\frac{\alpha}{1-\alpha}\right]^{\underline{i}(\overline{q};\underline{q})}}. \tag{4.2}$$

Lemma 10. The probability that eventually the monopolist captures the market conditional on the good being inferior $\phi_0(q^1)$ is non-increasing in $\overline{q}$ and in $\underline{q}$. Similarly, the unfavorable inefficiency $1 - \phi_1(q^1)$ is non-decreasing in $\overline{q}$ and in $\underline{q}$.

Since $\overline{q} > \alpha$ and $\underline{q} < 1 - \alpha$, the effect of monopoly pricing on both inefficiencies is ambiguous in general, since to quantify them it would be necessary to consider the quantities $[\overline{q} - \alpha]$ and $[(1 - \alpha) - \underline{q}]$ as well. Notice that when the monopolist is completely myopic ($\beta = 0$), $\overline{q} = \overline{q}^1 > \alpha$ and $\underline{q} = 1 - \alpha$, so that by the previous lemma the favorable inefficiency decreases, while the unfavorable inefficiency increases because of profit-maximizing pricing. For $\beta = 0$ the monopolist is a short-run agent and exploits its monopoly power to extract more surplus from the consumers.

When the monopolist is long run ($\beta > 0$) the increase in the upper barrier from $\overline{q}^1$ to $\overline{q}$ has a negative effect on the unfavorable inefficiency and a positive effect on the favorable one. The latter effect is stronger than the former, because of the drift toward higher q when the good is valuable and toward lower q when it is not. Similarly for the effect of the decrease in the lower barrier from $1 - \alpha$ to $\underline{q}$: the reduction of the unfavorable inefficiency is larger than the increase in the favorable one due to the learning effect. For this reason, the ex-ante expected inefficiency $EI\left[q^1; \overline{q}; \underline{q}\right] = (1 - q^1)\, I_0(q^1) + q^1 I_1(q^1)$, is lower in the monopoly case than with perfect competition for intermediate initial beliefs.

Proposition 6. The expected inefficiency decreases by giving the property right of one good to a monopolist for initial belief q^1 close enough to $1/2$.

5. Discussion of Extensions and Applications

A. General Signal Distributions. Convexity of the monopolist value function holds under fairly general conditions on signal distributions. Assume now that, before deciding which good to buy, each consumer n observes one of m possible private signals $\sigma^n \in \{\sigma_0; \sigma_1; \ldots; \sigma_{m-1}\}$ and as well as the public history h^n of the decisions (actions and prices) of all preceding individuals. The probability distribution of the private signal depends on the state of the world. The posterior belief after signal σ_j is

$$f_j(q) \equiv \frac{\Pr(\sigma_j | \omega_1)\, q}{\Pr(\sigma_j | \omega_1)\, q + \Pr(\sigma_j | \omega_0)(1 - q)}.$$

As shown in a companion note and reported in Appendix B, the static profit function of the monopolist is convex if the distributions of signals conditional on the state of nature satisfy the monotone hazard rate condition. If follows immediately that a sufficient condition for convexity is that the distributions of signals conditional on the state of nature satisfy the monotone likelihood ratio property.

When not in a cascade at n, the public belief q^{n+1} for individual $n+1$ after individual n took action a_i at price $P_j = 2f_j(q^n) - 1$ is

$$q^{n+1} = g_{ij}(q^n)$$

with

$$\begin{aligned}
g_{0j}(q) &= \frac{F_1(\sigma_{j-1})\, q}{F_1(\sigma_{j-1})\, q + F_0(\sigma_{j-1})(1 - q)}, \\
g_{1j}(q) &= \frac{(1 - F_1(\sigma_{j-1}))\, q}{(1 - F_1(\sigma_{j-1}))\, q + (1 - F_0(\sigma_{j-1}))(1 - q)},
\end{aligned}$$

where $F_k(.)$ is the c.d.f. of σ conditional on ω_k. If the distributions of signals conditional on the state of nature satisfy the monotone hazard rate condition the value function of the infinite-horizon problem is convex, so that market learning is valuable to the monopolist.

In general, the more informative prices are preferred by the monopolist. If the structure of signals is binary, the higher separating price is the only informative price. Examples of signal structures where higher prices are less informative than lower prices can be constructed, but a full understanding of the problem in this more general setting awaits further research. Nonetheless, it can be concluded in general that for favorable enough beliefs the only way to have some information revelation is to charge a price higher than the myopically optimal pooling price.

B. Reinterpretation of the Model: Competion of a Monopolist vs. Competitive Sector. The model can be reinterpreted as one of competition between two goods, supplied by two different sectors. Good 0 is supplied by a competitive sector, while good 1 is supplied by a monopolist. Action a_i corresponds to buying good i, for $i = 0, 1$. Uncertainty plays a symmetric role, with the payoff matrix:

PAYOFF from action	if state is	ω_0	ω_1
a_0		1	0
a_1		0	1

(5.1)

Each period firms simultaneously quote prices and provide the good at constant marginal cost, set equal to zero for convenience. The price quoted by the monopolist in period n is denoted by P_1^n, and the price quoted by firm j in sector 0 in period n is by P_{0j}^n. The price for buyer n of good 0 is equal to the minimum quoted by any supplier of this good, $P_0^n \equiv \min_j P_{0j}^n$.

The competitive sector is not able to e®ectively act on the price in any perfect Bayesian equilibrium because of a free rider e®ect: the future gains from favorable learning by the consumers will be dissipated by Bertrand competition among the suppliers of the same good. Since information is a public good, the social learning externality cannot be internalized in a competitive sector. Instead, in the other sector the monopolist enjoys the returns from learning and can therefore e®ectively act on prices.

Consider the price of good 0, produced by the competitive sector. Assume that anyone is allowed to enter freely into or exit from this sector at any point in time. Free entry implies that price is not larger than marginal cost in any perfect Bayesian equilibrium, so that it is impossible to appropriate future gains from social learning. If the price were to exceed the marginal cost in at least one period, one ¯rm could have entered in that period, charged a price slightly lower for a unit of good 0 on sale that period and exited immediately afterwards. This ¯rm would have made strictly positive expected pro¯ts. This \deviation" implies that the only possible equilibrium is one in which the price of good 0 is not greater than the marginal cost in any period. By free exit no ¯rm will make negative future expected pro¯ts, that would be generated by a price temporarily lower than the marginal cost, required for more learning to take place. It can be concluded that in any period the price of good 0 is equal to the marginal cost (equal to zero).

Competition in the sector that produces the alternative good 0 has allowed us to reduce the analysis of this model to the optimal decision problem for the monopolist producing good 1 solved in the previous sections of this paper: a monopolist supplies good 1 and competes with good 0, sold at $P_0^n = 0$ in any period n.

C. Foresighted Consumers. In this section the model is extended to foresighted consumers. Suppose that the consumer can wait before exercising her option to purchase. The consumer's cost of waiting derives from the delay of consumption. By waiting the consumer might be able to use the information revealed by others in the meantime, and enjoy reduced future prices (cf. Proposition 2). The consumer has three possibilities: purchase good 1, purchase good 0, or postpone her purchase. Assume that the purchase decision is irreversible, so that it cannot be worthwhile to buy the less desired good in order to manipulate strategically the social learning process and revert later to the more desired good. This corresponds to the assumption that consumers are \small" players and take the social learning process as given, without being able to manipulate it. With the payo® matrix (5.1) the possibility of purchasing the alternative good provides the consumer with a valuable outside option that changes with the perceived relative quality, so that the monopolist cannot fully extract the surplus of the consumer. In contrast, with the payo®s structure (2.1) consumers are left with no surplus in the learning phase.

Consider the incentives for a consumer to postpone her purchase. The timing of private information acquisition and choice of delay are relevant. One possibility is that the consumer can decide whether to purchase or not after the realization of the private signal. Alternatively one could think of situations in which the consumer, once acquired the information, is already committed to purchase. Here the ¯rst case is considered: buyers enter endowed with their information. When is it an equilibrium for individual consumers to buy immediately according to the order in which they are born? Since consumers are born with one of the two signals σ_0 or σ_1, the incentive to deviate for these two types will be considered separately.

As shown below, the solution to the problem with myopic consumers does not violate the additional constraint imposed by foresighted consumers in the learning phase, because the surplus of a consumer with favorable information is a martingale during learning. When instead the monopolist captures the whole market, the price is reduced deterministically, so that consumers need to be impatient enough not to exert the option of delaying the purchase decision. Under this condition, the solution to the original problem is a Markov equilibrium of the problem with foresighted consumers and irreversible decisions.

In general the surplus from consuming today is equal to the difference between the valuation and the price paid. For each of the two types of the consumers there are three regions to be considered. First consider the consumer with signal σ_1. For $q \in [0; \underline{q})$ good 0 is bought at price $P_0 = 0$, so that the surplus of this consumer is $1 - f_1(q)$. If $q \in [\underline{q}; \bar{q})$, then instead good 1 is bought at price $P_1 = 2f_1(q) - 1$, yielding a surplus $S_1(q) = f_1 - [2f_1(q) - 1] = 1 - f_1(q)$ equal to the valuation for the alternative good 0 net of the price $P_0 = 0$, the outside option of type-1 consumer. This is immediate, since in the learning phase the monopolist charges the price $P_1 = 2f_1(q) - 1$ in order to sell (only) to this consumer with favorable information. For $q \in [\bar{q}; 1)$ instead, type-1 consumer buys good 1, valued $f_1(q)$ at price $P_1 = 2f_0(q) - 1$, getting the surplus $S_1(q) = f_1 - [2f_0(q) - 1] = [1 - f_0(q)] + [f_1(q) - f_0(q)]$ equal to the outside option of the type-1 consumer $1 - f_0(q)$ plus the rent $f_1(q) - f_0(q)$ accrued because the monopolist is lowering the price in order to sell also to the type-0 consumer. In summary the surplus of the type-1 consumer is

$$S_1(q) = \begin{cases} 1 - f_1(q) & \text{for } q \in [0; \bar{q}); \\ 1 + f_1(q) - 2f_0(q) & \text{for } q \in [\bar{q}; 1]: \end{cases} \tag{5.2}$$

Now analyze the incentive for deviation. First consider the learning region $[\underline{q}; f_0(\bar{q}))$. As mentioned above, suppose that by delaying one period the purchase of the good, the consumer can wait to use her private (favorable) information on good 1 until the next period. Since the consumer has already received the high signal, the discounted surplus would be

$$\beta\left[\Pr(\sigma_1|f_1(q))\, S_1(f_1(q)) + \Pr(\sigma_0|f_1(q))\, S_1(f_0(q))\right] = \beta(1 - f_1(q)): \tag{5.3}$$

Notice that the probability that the next consumer gets a signal σ_1 is computed with the additional information (signal σ_1) that the consumer who is considering to delay has already acquired.

Lemma 11. For $[\underline{q}; f_0(\bar{q}))$, the payoff of a type-1 consumer from waiting one more period to use her favorable information is $\beta(1 - f_1(q))$.

This implies that the consumer does not have incentive to wait one more period if she has favorable information, even if the price is decreasing on average, and the surplus function in this region is convex in the belief. This is so because the type-1 consumer has more favorable information for the monopolist's product than the market, and believes accordingly that the price is more likely to rise in the future. In other words, both the belief and the price sequence of good 1 are martingales in the learning phase conditional on the posterior belief of the consumer with the private signal σ_1.

Now consider the region $[f_0(\bar{q}); \bar{q})$, where with one more favorable signal the monopolist will decide to reduce the price to capture the entire market. Waiting one more period the surplus would be $S_1(f_1(q)) = 1 + f_1^2(q) - 2q$ if σ_1 is revealed by the following consumer, while if σ_0 is revealed it would be $S_1(f_0(q)) = 1 - q$. In this region the high type consumer will want to postpone the consumption when the discounted expected surplus from waiting $\beta\{\Pr(\sigma_1|f_1(q))[1 + f_1^2(q) - 2q] + \Pr(\sigma_0|f_1(q))(1 - q)\}$ is larger than the surplus from acting immediately $1 - f_1(q)$, so that the no-deviation condition for one period ahead is easily rewritten as $1 - f_1(q) \geq \beta[1 - f_1(q) + R(q; \alpha)]$, where $R(q; \alpha)$ is the deterministic reduction in price (3.7). Notice that this constraint is always satisfied by the solution of the unconstrained problem when consumers are myopic ($\beta = 0$), and never when they are completely impatient ($\beta = 1$). In general there is a threshold $\bar{\beta}(\alpha; q) \equiv (1 - \alpha)/[(1 - \alpha) + 2(2\alpha - 1)q]$ below which the constraint is satisfied. Note that since $\frac{\partial \bar{\beta}(\alpha;q)}{\partial \alpha} < 0$, this constraint is satisfied for more patient consumers for lower α, i.e. when the signal of the single individual is less informative. Similarly it is immediately seen that $\frac{\partial \bar{\beta}(\alpha;q)}{\partial q} < 0$. Clearly if this no-deviation condition is satisfied for the consumer who comes one period before the reduction in price, it is a fortiori satisfied for the consumers who come before and would experience the price reduction farther in the future and only with positive probability.

Consider now the consumer with the signal σ_0. It can be easily shown that the surplus of the type-0 consumer from buying good 0 is equal to $1 - f_0(q)$ for any belief q. Consider the incentives for type-0 consumer to deviate from the proposed equilibrium strategy. Waiting one more period yields, in the case σ_1 is revealed by the next consumer, $\max\{q - [2f_1^2(q) - 1]; 1 - q\} = 1 - q$. If σ_0 is revealed by the next consumer, the surplus to the consumer who has waited one period is $\max\{f_0^2(q) - [2q - 1]; 1 - f_0^2(q)\} = 1 - f_0^2(q)$, so that, as in Lemma 11, the expected discounted surplus from delaying the purchase of good 0 by one period is equal to $\beta(1 - f_0(q))$. This means that a consumer with a signal favorable to good 0 will never have any incentive to delay his consumption, regardless of the discount factor. In conclusion:

Proposition 7. In Markov-perfect equilibrium, no consumer has incentive to wait to buy later in the learning phase. Furthermore, if consumers are patient enough, they do not wish to wait for the price reduction realized when the entire market is captured by the monopolist.

D. Monopolist with Information about Quality: Signalling. Consider what happens when the monopolist has some private information on the quality of the good provided. In many markets it is plausible to assume that the monopolist has this information, and would attempt to give it directly to the potential buyers if it is favorable. This happens, for instance, in the pharmaceutical market where the results of scientific studies on the drugs can be publicized to the medical profession. Similarly, if the monopolist has the ability to make only ex post verifiable statements about quality, full disclosure arises in the equilibrium constructed by Grossman (1981).

When it is difficult to communicate the information directly to the market, some information might be conveyed through signalling. If the equilibrium of the signalling game among monopolists with different information is separating, clearly all information possessed by the monopolist is revealed. The analysis of this model applies for the remaining amount of information that is not possessed by the firm, but present in the market. If instead the equilibrium of the signalling game is pooling, the low quality monopolist imitates the strategy of the high quality one, up to the point where it decides to exit the market, i.e. when the value for continuing to imitate the high quality monopolist, given the additional information possessed by the monopolist, is null. From this point on only the high quality producer is left in the market, and the model applies with this additional information revealed.

E. Past Prices Not Observed by Consumers: Signal Jamming. Consider the possibility that the consumer can observe only the decision of others, but not the prices at which these decisions were taken. By reducing the price the monopolist can affect the social learning process of the consumers. In equilibrium the consumers anticipate the incentive of the monopolist to "signal jam" their inference.[5]

The following two-period example illustrates the effect of non-observability of past prices on social learning dynamics and equilibrium prices. Clearly the analysis of the last period is unaffected. Consider the next-to-last period. There are three cases to be considered.

Pure strategy separating equilibrium. The separating price $P_S(q)$ is charged in period 1, and the following consumer (2) updates the belief according to the separating dynamics (2.6):

$$q^2 = \begin{cases} f_0(q^1) & \text{if } a^1 = a_0 \\ f_1(q^1) & \text{if } a^1 = a_1. \end{cases}$$

This gives a payoff of $q - (1-\alpha) + \beta E V^1(q)$ to the monopolist. By deviating to the pooling price the payoff would be $2f_0(q) - 1 + \beta V^1(f_1(q))$, because the sure sale would be erroneously taken as a good signal by the market. The no-deviation condition for such a pure strategy equilibrium is

$$q - (1-\alpha) + \beta E V^1(q) \geq 2f_0(q) - 1 + \beta V^1(f_1(q)). \tag{5.4}$$

Pure strategy pooling equilibrium. The pooling price $P_P(q)$ is charged in period 1, and the following consumer (2) updates the belief according to the cascade dynamics (2.7): $q^2 = q^1$, for any $a^1 = a_i$. This strategy yields $2f_0(q) - 1 + \beta V^1(q)$ to the monopolist. Since by deviating to the separating price the monopolist would instead obtain $q - (1-\alpha) + \beta V^1(q)$, the no-deviation condition for such a pure strategy equilibrium is

$$2f_0(q) - 1 \geq q - (1-\alpha). \tag{5.5}$$

or $q \geq \bar{q}^1$.

Mixed strategy Hybrid equilibrium. When neither (5.4) nor (5.5) are satisfied, there is no pure strategy equilibrium. Consider the following hybrid equilibrium: the pooling price $P_P(q)$ is charged with probability y, and the separating price $P_S(q)$ with probability in period $1 - y$, and the following consumer (2) updates the belief according to:

$$q^2 = \begin{cases} f_0(q^1) \equiv \frac{(1-\alpha)q^1}{\alpha - (2\alpha - 1)q^1} & \text{if } a^1 = a_0 \\ \tilde{f}_1(q^1) \equiv \frac{yq^1 + (1-y)\alpha q^1}{y + (1-y)[(1-\alpha) + (2\alpha - 1)q^1]} & \text{if } a^1 = a_1 \end{cases}$$

[5] For other models with signal jamming see Holmström (1982) and Caminal and Vives (1994).

For this to be an equilibrium it is necessary that the payoff from the separating price be equal to the one from the pooling price:

$$q - (1 - \alpha) + \beta E V^1(q) = 2 f_0(q) - 1 + \beta V^1\left(\tilde{f}_1(q)\right); \tag{5.6}$$

where $E V^1(q) \equiv \Pr(\sigma_0 | q) V^1(f_0(q)) + \Pr(\sigma_1 | q) V^1\left(\tilde{f}_1(q)\right)$, and $y(q) \in (0; 1)$.

Concluding, if only decisions of others are observed, but no information on past prices is available, in equilibrium prices tend to be lower than otherwise at the beginning of learning, and the learning phase is shortened. The monopolist has all the incentive to publicize the information on past separating prices and benefits from committing credibly to disclose honestly past prices, both in the case they resulted in a sale and they did not.

F. Control on Information and Trial Periods. What happens if the monopolist is allowed to control the quality of private information (α) received by the consumers? In particular consider the option of the monopolist to stop the consumers from acquiring private information. This option dominates statically the strategy of giving information and capturing the whole market and has the same dynamic effects. Still, when the belief is below a certain level the monopolist will prefer to commit to allow information acquisition and charge the separating price to not giving information at all. This strategy allows the monopolist to "gamble for resurrection". The optimality of this policy derives from the convexity of payoff function of the monopolist around the belief at which the marginal revenue is equal to the marginal cost. For this belief the monopolist is indifferent between selling and not selling, for a belief strictly below this level the monopolist prefers not to sell and has a payoff of zero, while for beliefs strictly above this level the monopolist sells at a positive profit linearly increasing in the belief.

Giving information is equivalent to having a period of trial, and thereby facilitating the acquisition of information (good or bad) on the product by the consumer. Since periods of trial are associated with separating prices, the good is likely to be returned if bad information is acquired by the consumer. The monopolist of a successful products prefers, instead, to stop consumers from acquiring information, and can achieve this by not allowing them to return the product.

G. Implications for Regulation of Competition. Some interesting implications for industrial policy can also be drawn from this model. An argument for why pricing below marginal cost might increase efficiency is provided. The monopolist achieves the objective of learning by pricing below marginal cost when its good is perceived as inferior (for $q < 1 - \alpha$). Learning improves efficiency, so that long-run inefficiencies can be reduced by entitling a monopolist to the property right of one good.

Some of the conclusions on the effect of competition on diffusion of information and market learning are not specific to social learning, but they also arise in any market where the acquisition of information by the market requires a costly investment, as is the case, for example, in the pharmaceutical industry. The appreciation of the product is a public good to the sector that provides it: once information has been given to the market by the incumbent, a new entrant who can provide the same product and compete à la Bertrand on prices will destroy completely the profits deriving from the investment of the incumbent. In general, competition erodes, at least partially, the return from the initial investment required to give information to the market. In more competitive markets, therefore, information is less likely to be provided directly by the suppliers, but would need to be acquired through time-consuming experimentation by the consumers.

H. Examples. This model provides a possible explanation of why the price of books and compact disks is reduced only after they become best sellers, but not before then. If, instead, the price were reduced sooner, the book would sell more so that it would be more likely to become a best seller. Popularity at separating prices (high relative to the perceived quality) lets the information °ow from past consumers to future customers. This model also predicts introductory discounts, reduced if the good is successful and increased if it is not. On average, prices are predicted to decrease, even though superior products are predicted to become more expensive during the learning phase.

Social learning may be the reason why high prices are crucial to establish a new brand-name product, a new restaurant, a fashionable cafe or a vacation resort. Similarly, social learning provides an explanation for why young independent professionals (doctors and lawyers, for instance) tend to charge fees that are high relative to their perceived quality and are willing to be underemployed, but not to reduce the price for their services. It might be argued that fees of older professionals are higher than those of younger ones, even when their more extensive experience is taken into account. But this implication is in line with this model, since not everyone survives in this market, and higher quality professionals are more likely to succeed. According to this model, a professional conditional on being successful, it is much more likely to be good, and good professionals are predicted to charge increasing prices during the learning phase.

Social learning can be important in the pharmaceutical market. Information on drugs is crucial in determining the prescription choice, and companies spend fortunes to detail drugs to doctors. Some otherwise puzzling empirical observations in the anti-ulcer drug market (see Berndt, Bui, Reiley and Urban (1994)) are consistent with the prediction possible. If, on the other hand, there are multiple dimensions of uncertainty, herd behavior can result.

Welch (1992) studies the choice of optimal price by an issuer who sells a new security to a sequence of partially informed investors. He considers, however, ¯xed-price sales, since the Securities and Exchange Commission has banned variable-price sales in initial public o®erings of stocks. He also argues that a highly risk-averse issuer would prefer a ¯xed low price that induces an immediate informational cascade to changing prices in response to past sales. In this paper, instead, the optimal °exible-price strategy for a risk-neutral issuer is characterized.

Simons and Bhattacharya (1994) study quality and price choice by two sellers when consumers learn from observing the purchases of other consumers. They di®er from this paper in that they constrain each seller to supply the good at the same price (and quality) during the play of the game.

Vettas (1995a) studies the joint evolution of demand and supply in new markets where consumers learn about quality and suppliers learn about quantity demanded. Information about the quality of the good di®uses among consumers; whether or not previously informed consumers buy again is observable by other consumers. The equilibrium entry rate of ¯rms into the market is determined by a zero-expected pro¯t condition. The entry decision by ¯rms causes another externality; entry reveals information about the pro¯tability of the good, so that it a®ects pro¯tability of subsequent entry. The model builds on Rob's (1991) analysis of the pattern of entry in new markets, and shows that by combining learning on both sides of the market it is possible to explain some commonly observed phenomena of product life cycle (such as S-shaped di®usion paths).

In Caminal and Vives (1996) consumers learn by observing market shares, but past prices are assumed not to be observed. They construct a duopoly model where consumers are uncertain about the quality di®erential of the varieties. A higher current market share is interpreted by consumers as a signal of higher relative quality and tends to increase future demand. If the ¯rms are uninformed as well about the quality di®erential, prices do not signal quality and tend to be set below the short-run pro¯t maximizing level in the introductory phase. When, instead, ¯rms are informed about the quality di®erential, prices as well as quantity convey relevant information about quality.

Finally, Bergemann and Valimaki (1996b) analyze a problem of social experimentation with strategic pricing. They build on the continuous-time model of strategic experimentation of Bolton and Harris (1993), where the outcome of the experiment of any buyer is freely available to everyone. Experimentation produces a public good, so that too little investment in information acquisition results in a model without sellers. In contrast to this, Bergemann and Valimaki (1996b) ¯nd that, once price-setting sellers are introduced, there is overinvestment in social experimentation. Sellers induce learning by consumers because it results in di®erentiation, that reduces co sales levels in markets where consumers receive information from others. Scarpa (1990) studies the optimal pricing behavior for a monopolist informed about quality when future demand is an increasing function of past sales (a proxy for word-of-mouth communication). Both Glaister (1974) and Scarpa (1990) predict a low introductory price, gradually increased over time. In Scarpa (1990) the informational role of prices is suppressed: although goods of di®erent qualities are sold at di®erent prices, consumers do not invert the pricing decision to learn the true quality.

Vettas (1995b) considers the e®ect of word-of-mouth communication for the dynamic pricing strategy of a monopolist who knows the quality of the durable good. At the beginning of each period consumers who have not yet purchased the good learn the true quality whenever they communicate with someone who knows it by having already consumed. The probability of communication is equal to the proportion of those who have already purchased the good. Past and current prices are publicly observable, but consumers do not know the aggregate quantities sold. Consumers can learn from other consumers, from prices, and from the fact that they were not able to purchase and did not receive information from others.

Third, this work relates to the industrial organization literature on pricing of \experience" goods. According to the de¯nition of Nelson (1970) the quality of experience goods can be veri¯ed only with usage, after purchase. Shapiro (1983), for instance, shows that a monopolist charges a high and declining price if consumers optimistically overestimate product quality and charges a low introductory price otherwise. These results depend on the assumption that consumers have adaptive expectations on product quality with no possibility of price signalling.

Fourth, the problem of experimentation by a long-lived agent has been extensively studied by probabilists in the armed bandit literature and introduced in economics by Rothschild (1974). Recently, Bergemann and Valimaki (1996a) have considered an experimentation problem by an individual consumer where the cost of the di®erent \arms" are determined by competition between sellers. The timing of their model and the assumption of bilateral learning are common to my paper, but their problem, individual optimal experimentation, is intrinsically di®erent from that of learning from the behavior of others. The supply sector is modeled in a similar way to the duopoly problem studied in Moscarini and Ottaviani (1994) in the social learning context.

Fifth, the informational content of prices is emphasized in signalling models

7. Conclusion

This paper studies how a monopolist exploits the process of sequential information aggregation in a market with dispersed information, as a part of a broader research program aimed at analyzing the effect of property rights and competition on the outcome of social learning. The motivation of this program is twofold. First, it provides a stylized model of real markets where buyers learn from each other, and suppliers compete in an attempt to exploit this learning process. The focus of the earlier work on social learning is on the demand side of the market, with no consideration for the supply side. Both demand and supply sides are considered in order to study the effects of herd behavior in different markets. Suppliers can only infer - as can every other consumer - information revealed by the decisions of consumers. The second motivation is to explore the effect of flexible prices on the inefficiency created by the informational externality associated with social learning. An important question concerning the dynamics of product introduction is whether superior products are eventually adopted. Do long-run inefficiencies decrease when long-run players - the firms supplying the good - are brought into the picture?

Different specifications of the supply side of the market are possible. Consider the following three cases with two varieties of the good, each one produced by a separate sector. First, the "standard" model of Bikhchandani, Hirshleifer and Welch (1992) corresponds to the case of perfect competition with free entry and free exit in both sectors, and can be applied to study the adoption of a new standardized technology. Second, Moscarini and Ottaviani (1994) study a duopoly, in which the supplier of each good tries to influence the learning process of the consumers by acting on the price of its own good. The duopoly model may illustrate competition among firms in the aircraft industry, where the purchase decisions of the airline companies are easily observable by the competing producer and by the other airline companies. Finally, the monopoly problem analyzed in this paper has been interpreted as the case of a monopolist competing against a perfectly competitive sector that produces an alternative good.

To establish a brand name the monopolist decides to be "expensive", because if something were popular but cheap, nothing would be indicated about quality. On the other hand, the popularity of an expensive good conveys the favorable information possessed by the buyers. Eventually the monopolist stops the social learning process, either by quitting the market or by capturing it entirely with a reduction in the price. Once stopped, social learning will not be reactivated again. From then on, all private information is lost, because nothing is revealed by the purchase decisions of the consumers.

With dynamic monopoly pricing, the expected long-run inefficiency is shown to decrease for two reasons. First, the social learning externality is internalized by the monopolist, so that the good is less likely to disappear when it is valuable. Second, the monopoly position allows the firm to delay the capture of the whole market in order to sell at a higher price. The latter factor works to decrease the inefficiency associated with selling the good when it is not valuable. As a result, the expected inefficiency is generally lower when compared with a model with fixed prices. The remaining inefficiency associated with incomplete learning is common in learning models (cf. for example Aghion, Bolton, Harris and Jullien (1991)) where the long-run player is impatient. Compared to the constrained social optimal level of learning, monopoly pricing results in over-experimentation for favorable beliefs and under-experimentation for unfavorable beliefs.

This model provides an insight into why and how market shares matter. Since selling can be valuable even at a loss because it might convey valuable information to the market, learning provides an argument for why pricing below marginal cost might increase e±ciency. Long-run ine±ciency can be reduced by entitling a monopolist to the property right of one good, while more competition can reduce the return from the investment required to let the market acquire information on the good.

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A. Appendix: Omitted Proofs

Lemma 2 (Monotonicity of N-period-to-go value function in N) The value function $\{V^N(\cdot)\}$ of the N-period-to-go problem is non-decreasing in the number of periods to go N. For $q \geq \alpha$, it is strictly increasing in N with

$$V^N(q) > V^{N-1}(q) + \beta^{N-1}(2f_0(q) - 1). \tag{A.1}$$

Proof. The value function of the $N-$period problem is trivially non-decreasing in the number of periods left to go, since it is always possible to adopt the strategy optimal for the problem with $N-1$ periods, and not sell at all in the last period. Furthermore, in the N-periods problem it is always possible to adopt in the first $N-1$ periods the policy optimal for the $N-1$-period problem and in the last period to sell at the price $2f_0\left(q^N\right) - 1$, whatever the belief in the last period q^N will turn out to be. This feasible policy yields the expected discounted payoff of $V^{N-1}(q) + \beta^{N-1}\left(2E\left[f_0\left(q^N\right)|q\right] - 1\right)$. Jensen inequality implies that $E\left[f_0\left(q^N\right)|q\right] > f_0\left(E\left[q^N|q\right]\right)$ because $f_0(\cdot)$ is a strictly convex function of q. Finally notice that $E\left[q^N|q\right] = q$, because the public belief is a martingale. This completes the proof of (A.1), because the optimal policy in the N-periods problem cannot give a lower expected discounted payoff than this feasible policy and $2f_0(q) - 1 > 0$ for $q > \alpha$. ■

Lemma 3 (Convexity of the value function) The value function $V^N(\cdot)$ is convex.

Proof. The property of convexity is clearly verified for $N = 1$, as $V^1(q) = \max\langle 0; q - (1-\alpha); 2f_0(q) - 1\rangle$ is constructed as the maximum of 0, a linear function of q, and a strictly convex function of q. $V^1(\cdot)$ is strictly convex for $q \geq \bar{q}^1$. It is immediately seen that $EV^1(\cdot)$ is convex (strictly for $q \geq f_0(\bar{q}^1)$) in q. Now consider in general

$$V^N(q) = \max\left\{\beta V^{N-1}(q); q - (1-\alpha) + \beta EV^{N-1}(q); 2f_0(q) - 1 + \beta V^{N-1}(q)\right\}$$

and suppose by induction that $V^{N-1}(q)$ is convex, and strictly convex for $q \geq \min\langle f_0^{N-2}(\bar{q}^1); \underline{q}^{N-1}\rangle$. Then by the linearity of probability, clearly $EV^{N-1}(q)$ is convex, and strictly convex for $q \geq \min\langle f_0^{N-1}(\bar{q}^1); f_0\left(\underline{q}^{N-1}\right)\rangle$. Then $V^N(q)$ is convex (strictly for $q \geq \min\langle f_0^{N-1}(\bar{q}^1); \underline{q}^N\rangle$) being the maximum of convex functions. ■

Lemma 4 (Optimality and monotonicity of cutoff levels for exit in N) In the problem with N periods to go, it is optimal to exit the market when the belief is below the cutoff level $\underline{q}^N$. In addition, the cutoff levels $\underline{q}^N$ is non-increasing in the number of periods to go N, i.e. $\underline{q}^N \leq \underline{q}^{N-1}$.

Proof. By induction. It has been shown in the text that a cutoff policy for exit is optimal in the one-period problem and in the two-period problem, with $\underline{q}^1 < \underline{q}^2 = \frac{(1-\alpha)(1+\beta(1-\alpha))}{1+\beta[1-2\alpha(1-\alpha)]}$. Suppose that it is optimal to follow a cutoff policy for exit for the $N-1$-period problem, and consider the payoff from selling at the separating price to the left of $\underline{q}^{N-1}$:

$$q - (1-\alpha) + \beta \Pr(\sigma_1|q) V^{N-1}(f_1(q)); \tag{A.2}$$

a continuous function of q, by Lemma 1. From the definition of $\underline{q}^{N-1}$

$$\underline{q}^{N-1} - (1-\alpha) + \beta \Pr(\sigma_1|q) V^{N-2}\left(f_1\left(\underline{q}^{N-1}\right)\right) = 0 \tag{A.3}$$

since $V^{N-2}\left(f_1\left(\underline{q}^{N-1}\right)\right) = 0$, as $f_1\left(\underline{q}^{N-1}\right) < \underline{q}^{N-1} \le \underline{q}^{N-2}$, with the last inequality following from the inductive hypothesis. Clearly

$$V^{N-1}(f_1(q)) \ge V^{N-2}(f_1(q)); \tag{A.4}$$

by Lemma 2. Note that (A.2) is a continuous function of q by Lemma 1, is strictly increasing in q for $q \in \left[f_0\left(\underline{q}^{N-1}\right); \underline{q}^{N-1}\right]$ by Lemma 1, and takes a value no smaller than 0 (the alternative payoff from exiting the market) at $\underline{q}^{N-1}$ by combining (A.4) for $q = \underline{q}^{N-1}$ with (A.3), so that there exist a cutoff level $\underline{q}^{N} \in \left[f_0\left(\underline{q}^{N-1}\right); \underline{q}^{N-1}\right]$ such that it is optimal to sell at the separating price for beliefs $q \ge \underline{q}^{N}$ and exit the market for beliefs $q \le \underline{q}^{N}$. Clearly the inequality is strict, $\underline{q}^{N} < \underline{q}^{N-1}$, if $V^{N-1}\left(f_1\left(\underline{q}^{N-1}\right)\right) > V^{N-2}\left(f_1\left(\underline{q}^{N-1}\right)\right)$. ■

Lemma 5 (Optimality of cutoff policy for capturing the market) The optimal policy for the problem with N periods left to go is to sell to the entire market when the belief q is above the cutoff level $\overline{q}^{N}$.

Proof. Define the average value function as

$$\overline{V}^{n}(q) = \frac{1-\beta}{1-\beta^{n}} V^{n}(q):$$

Then for $q \ge \overline{q}^{n-1}$, the Bellman equation can be rewritten in terms of average payoff as

$$\overline{V}^{n}(q) = \max\left\{0; \frac{1-\beta}{1-\beta^{n}}[q-(1-\alpha)] + \frac{\beta-\beta^{n}}{1-\beta^{n}} E\overline{V}^{n-1}(q); 2f_0(q)-1\right\}:$$

The cost of learning in terms of average payoff $\frac{1-\beta}{1-\beta^{n}}[2f_0(q)-q-\alpha]$ is increasing in q. The benefit of learning in terms of average payoff is

$$\frac{\beta-\beta^{n}}{1-\beta^{n}}\Big[E\overline{}$$

Proof. The property $\bar{q}^2 \ge \bar{q}^1$ has been verified in the text. We will now show that $\bar{q}^{N+1} \ge \bar{q}^N$. As discussed above, the current cost of learning is $2f_0(q) - q - \alpha$, a continuous strictly increasing function of q that takes the value of 0 for $q = \bar{q}^1$ and the value of $1 - \alpha$ for $q = 1$, and the benefit in future higher profits from learning is $\beta \left[EV^N(q) - V^N(q) \right]$.

The property that $\bar{q}^{N+1} \ge \bar{q}^N$ would then follow from the fact that the expected future gain from experimentation is increasing in the number of periods left to go, i.e.

$$EV^N(q) - V^N(q) \ge EV^{N-1}(q) - V^{N-1}(q). \qquad \text{(A.8)}$$

Suppose, by induction that $\bar{q}^N \ge \bar{q}^{N-1}$. Notice that for $q \ge \bar{q}^N$, $V^N(q) - V^{N-1}(q) = \beta^{N-1}(2f_0(q) - 1)$ since

$$V^n(q) = \frac{1-\beta^n}{1-\beta}(2f_0(q) - 1) \qquad \text{(A.9)}$$

for any $n \le N$ (by $q \ge \bar{q}^n$). Then (A.8) becomes $EV^N(q) - EV^{N-1}(q) \ge \beta^{N-1}(2f_0(q) - 1)$. From the definition of $EV^N(q)$ and (A.9) it is easily seen that

$$EV^N(q) - EV^{N-1}(q) = \Pr(\sigma_0|q)\left[V^N(f_0(q)) - V^{N-1}(f_0(q))\right] + \beta^{N-1}\Pr(\sigma_1|q)(2q-1):$$

so that it is left to show that

$$\Pr(\sigma_0|q)\left[V^N(f_0(q)) - V^{N-1}(f_0(q))\right] + \beta^{N-1}\Pr(\sigma_1|q)(2q-1) \ge \beta^{N-1}(2f_0(q) - 1). \qquad \text{(A.10)}$$

Applying inequality (A.1) proven in Lemma 2, to $f_0(q)$, we can conclude that

$$V^N(f_0(q)) - V^{N-1}(f_0(q)) > \beta^{N-1}(2f_0(f_0(q)) - 1). \qquad \text{(A.11)}$$

For any $q \in (0, 1)$

$$\Pr(\sigma_1|q)\,q + \Pr(\sigma_0|q)\,f_0(f_0(q)) - f_0(q) > 0 \qquad \text{(A.12)}$$

being equal to the positive fraction (A.7). Combining (A.12) and (A.11) it is found that necessarily (A.10) holds for any $q \ge \bar{q}^N$, so that $\bar{q}^{N+1} > \bar{q}^N$. ■

Lemma 7 (Exit) For belief lower than $\underline{q} \in (0, 1-\alpha)$ the monopolist exits the market.

Proof. Define $\underline{q}$ as the largest belief at which the value function of firm 1 is equal to zero, $\underline{q} - (1-\alpha) + \beta EV\left(\underline{q}\right) = 0$. Such a value uniquely exists and is in the interval $(0, 1-\alpha)$, because $q - (1-\alpha) + \beta EV(q)$ is a continuous function of q, $V(0) = 0$ and $\beta EV(1-\alpha) > 0$. Since for $q < \underline{q}$ the firm has a strictly negative value from operating, it will prefer to quit this market. ■

Lemma 8 (Capture of the entire market) When the belief is larger than $\bar{q} \in (\bar{q}^1, 1)$, learning is stopped by the monopolist.

Proof. The proof follows combining the analysis of the finite-horizon problem with the fact that $\bar{q} < 1$. Suppose by contradiction that $\bar{q} = 1$. Notice that by continuity of $V(\cdot)$, $f_0(\cdot)$ and $f_1(\cdot)$, $\lim_{q\to 1} V(q) = V(1)$, $\lim_{q\to 1} f_0(q) = \lim_{q\to 1} f_1(q) = 1$, $\lim_{q\to 1}\Pr(\sigma_1|q) = 1 - \lim_{q\to 1}\Pr(\sigma_0|q) = \alpha$, $\lim_{q\to 1} EV(\bar{q}) = V(1)$, so that (3.5) implies that $0 = 1 - \alpha$ contradicting the hypothesis that $\alpha < 1$. ■

Lemma 9 (Hitting Probability) *The probability $\psi_i(q)$ that the public belief starting at $q \in [\underline{q}; \overline{q}]$ reaches $\overline{q}$ before it reaches $\underline{q}$, conditional on the state of the world ω_j is*

$$\psi_j(q) = \frac{1 - r_j^{\iota(q;\underline{q})}}{1 - r_j^{\iota(\overline{q};\underline{q})}}. \tag{A.13}$$

where $\iota(q;\underline{q}) \equiv \min\{m \mid f_0^m(q) < \underline{q}\}$ is the minimum number of transition needed to take the belief from q to below the barrier $\underline{q}$.

Proof. The probability that the threshold $\overline{q}$ is reached before than $\underline{q}$, starting with a belief of q and conditional on the state of the world ω_j satisfies the following recursive equation

$$\psi_j(q) = \Pr(\sigma_1 \mid \omega_j)\,\psi_j(f_1(q)) + \Pr(\sigma_0 \mid \omega_j)\,\psi_j(f_0(q));$$

provided that $f_0(q) \geq \underline{q}$ and $f_1(q) \leq \overline{q}$. This last equation can be rewritten as

$$\psi_j(f_1(q)) - \psi_j(q) = r_j\,[\psi_j(q) - \psi_j(f_0(q))]; \tag{A.14}$$

where $r_j = \Pr(\sigma_0 \mid \omega_j) / \Pr(\sigma_1 \mid \omega_j)$. The boundary conditions are

$$\psi_j(\underline{q}) = 0; \tag{A.15}$$

and

$$\psi_j(\overline{q}) = 1. \tag{A.16}$$

By making use of (A.15) it is immediate to verify the identity:

$$\psi_j(q) = \sum_{k=1}^{\iota(q;\underline{q})} \left[\psi_j\left(f_1^k(\underline{q})\right) - \psi_j\left(f_1^{k-1}(\underline{q})\right)\right]. \tag{A.17}$$

where $f_j^k(.)$ is the k-th iterate of the function $f_j(.)$, and $\iota(q;\underline{q}) \equiv \min\{m \mid f_0^m(q) < \underline{q}\}$ is the smallest number of iterations of the map $f_0(.)$ that brings the posterior belief below the level $\underline{q}$. Substitute from (A.14) to get

$$\sum_{k=1}^{\iota(q;\underline{q})} \left[\psi_j\left(f_1^k(\underline{q})\right) - \psi_j\left(f_1^{k-1}(\underline{q})\right)\right] = \sum_{k=1}^{\iota(q;\underline{q})} r_j^{\,j-1}\left[\psi_j\left(f_1(\underline{q})\right) - \psi_j(\underline{q})\right]$$

so that by (A.17) and (A.15)

$$\psi_j(q) = \frac{1 - r_j^{\iota(q;\underline{q})}}{1 - r_j}\psi_j\left(f_1(\underline{q})\right). \tag{A.18}$$

By definition of $\iota(q;\underline{q})$ it follows that $\overline{q} = f_1^{\iota(q;\underline{q})}(q)$. Computing $\psi_j(.)$ at $\overline{q}$, and making use of (A.16), (A.18) can be rewritten as (4.1). ■

Lemma 10 *The probability that eventually the monopolist captures the market conditional on the good being inferior $\psi_0(q^1)$ is non-increasing in $\overline{q}$ and in $\underline{q}$. Similarly, the unfavorable inefficiency $1 - \psi_1(q^1)$ is non-decreasing in $\overline{q}$ and in $\underline{q}$.*

Proof. $\psi_0(q^1)$ is non-increasing in $\overline{q}$, since it is decreasing in $\underline{i}\left(\overline{q};\underline{q}\right)$, and $\underline{i}\left(\overline{q};\underline{q}\right)$ is non-decreasing in $\overline{q}$. A change in $\underline{q}$ effects $\psi_0(q^1)$ through $\underline{i}\left(q^1;\underline{q}\right)$ and $\underline{i}\left(\overline{q};\underline{q}\right)$: $\psi_0(q^1)$ is increasing in $\underline{i}\left(q^1;\underline{q}\right)$, and $\underline{i}\left(q^1;\underline{q}\right)$ is non-increasing in $\underline{q}$, and $\psi_0(q^1)$ is decreasing in $\underline{i}\left(\overline{q};\underline{q}\right)$, and $\underline{i}\left(\overline{q};\underline{q}\right)$ is non-increasing in $\underline{q}$. The first effect is stronger than the second one due to the drift toward $q = 0$ in the dynamics of the belief conditional on the state ω_0. In conclusion $\psi_0(q^1)$ is non-increasing in $\underline{q}$. ■

Proposition 6 The expected inefficiency decreases by giving the property right of one good to a monopolist for initial belief q^1 close enough to $1/2$.

Proof. By making use of (4.1), $EI\left(1/2;\overline{q};\underline{q}\right)$ is easily seen to be non-increasing in $\overline{q}$ and non-decreasing in $\underline{q}$. Furthermore $EI\left(q;\overline{q};\underline{q}\right)$ is non-increasing in $\overline{q}$ for $q \le \overline{\overline{q}}$ (with $\overline{\overline{q}} \ge 1/2$) and non-decreasing in $\underline{q}$ for $q \ge \underline{\underline{q}}$ (with $\underline{\underline{q}} \le 1/2$). These two facts combined with $\overline{q} > \alpha$ and $\underline{q} < 1 - \alpha$ give the result that the expected inefficiency decreases by giving the property right of the good to a monopolist for initial belief q^1 close enough to $1/2$. ■

B. Appendix: Convexity with General Signal Distributions

Lemma 12. (Convexity of the profit function) The profit function is convex if the distributions of signals conditional on the state of nature satisfy the monotone hazard rate condition:

$$\sum_{i=j}^{m-1} \left[\Pr(\sigma_i \mid \omega_1)\Pr(\sigma_j \mid \omega_0) - \Pr(\sigma_i \mid \omega_0)\Pr(\sigma_j \mid \omega_1)\right] \geq 0$$

for all j.

Proof. Consider a finite number of signals $\sigma_1; \sigma_2; \ldots; \sigma_j; \ldots; \sigma_{m-1}$. When price $P_j(q) \equiv 2f_j(q) - 1$ is char and $E_jV(q) = F(\sigma_{j-1}|q)\,V(g_{0j}(q)) + [1 - F(\sigma_{j-1}|q)]\,V(g_{1j}(q))$ where $g_{0j}(q)$ (resp. $g_{1j}(q)$) are the posterior beliefs after observation of no purchase (resp. purchase) at price $P_j(q)$:

$$
\begin{aligned}
g_{0j}(q) &= \frac{F_1(\sigma_{j-1})\,q}{F_1(\sigma_{j-1})\,q + F_0(\sigma_{j-1})\,(1-q)}, \\
g_{1j}(q) &= \frac{(1-F_1(\sigma_{j-1}))\,q}{(1-F_1(\sigma_{j-1}))\,q + (1-F_0(\sigma_{j-1}))\,(1-q)}.
\end{aligned}
$$

Since the Bellman operator $(TV)(q) = \max_j \langle \pi_j(q) + \beta E_jV(q) \rangle$ satisfies the Blackwell sufficient conditions for a contraction, to show convexity of the value function by the Contraction Mapping Theorem (see page 52 of Stokey and Lucas (1989)) it is enough to show that the operator T preserves convexity. For each j, $\pi_j(q)$ is convex under our assumption. By making use of $F_{qq}(\sigma_{j-1}|q) = 0$, $2 \cdot F_q(\sigma_{j-1}|q)\,g'_{0j}(q) = -F(\sigma_{j-1}|q)\,g''_{0j}(q)$ and $2F_q(\sigma_{j-1}|q)\,g'_{1j}(q) = F(\sigma_{j-1}|q)\,g''_{1j}(q)$, we get

$$
E''_jV(q) = F(\sigma_{j-1}|q)\,V''(g_{0j}(q))\left[g'_{0j}(q)\right]^2 + [1 - F(\sigma_{j-1}|q)]\,V''(g_{1j}(q))\left[g'_{1j}(q)\right]^2 \geq 0
$$

when $V(.)$ is a convex function of q. Then it follows that for any j $\pi_j(q) + \beta E_jV(q)$ is a convex function of q. Since TV is the upper envelope of a family of convex mappings, it is also convex. ■