

# ENGR 580: Project handout #3 Controllability and state-feedback

November 2023

Use the models you defined in part A and part B of your project following the instructions of handout 1 and 2.

## 1 Part A: Sample project

Given the model you defined for the multiple sprinkler system using the interconnected compartmental model (handout 2):

- 1.1 Is the system with two sprinklers controllable? Explain your answer.
- 1.2 Design a state-feedback controller that stabilizes the system with a settling time of 48hrs or less. Simulate the response to a realistic nonzero initial state  $x_0$  and evaluate whether your response meets the requirements of a real-world irrigation system. Make sure you show the states as well as the inputs.
- 1.3 Design an LQR (optimal) controller for this system. Where are the eigenvalues of the closed-loop system that you designed? Compare the response of this system in simulation with the controller you designed in the question above.
- 1.4 Assume that sprinkler #2 is broken. How can this be reflected in your state-space model?
- 1.5 Is the system with sprinkler #2 out of service controllable? If not, calculate a controllable decomposition. Is this system stabilizable?
- 1.6 How does the controller you designed in 1.2 perform when sprinkler #2 is broken? (Use simulation results to motivate your answer). Can you design a controller that would work better for this case? If so, how?

## 2 Part B: Design project

- 2.1 Is the system you chose in your project controllable?
- 2.2 *If your system is not controllable:* Calculate a controllable decomposition. Is your system stabilizable? Which actuator could you add to make your system controllable? Would that be realistic?
- 2.3 *If your system is controllable:* Is your system still controllable if the first actuator (input) fails? What about if the second actuator fails? If the system with actuator failure is no longer controllable, calculate the controllable decomposition. Is this system stabilizable?
- 2.4 Assume you can measure all states of your system, and that your system is configured with the actuators (inputs) required to make it controllable. Define specifications in terms of a closed-loop controlled system that meets the desired output as defined in Project Handout 2, question 3.10.
- 2.5 Use the following three methods to design (three) state-feedback controllers to achieve the objectives defined above.

- Feedback stabilization based on the Lyapunov test
- Eigenvalue assignment
- LQR (optimal) control

Simulate the homogenous response with a realistic  $x_0$ . Make sure you show the response of the states as well as the inputs. Which method do you prefer? Which controller do you think is the best for your system? Explain why.

2.6 Use your preferred state-feedback controller design to control your linearized system as well as your nonlinear system. Simulate the response to realistic (non-zero) initial conditions  $x_0$ , make sure you show the response of the states as well as the inputs. Simulate the responses to initial conditions  $x(t=0) = 5x_0$ . Explain the differences you see.

2.7 For the designs above, evaluate the homogeneous response with a realistic  $x_0$  in the presence of realistic measurement noise on each state. You can choose to do this for either your linearized system or your nonlinear

system. How do the different designs compare? Make sure you show the states as well as the inputs in your simulation results.