ENGR 580: Project handout #2 Open-loop responses and open-loop stability

October 2023

1 Reminder to hand in your system overview for approval

If you have not yet done so, make sure you hand in your system description for approval. Make sure your description includes these discussion points, as indicated in Handout #1.

1.1 Discussion points to include in your overview submitted for approval

1. Provide a description of your chosen system which can include background, motivation, functional explanation, specific technologies (sensors and actuators that you will use for feedback and control). Include picture(s) or diagram(s) to visualize your system.

2. Present a nonlinear state-space model of your system as well as at least one linearization. Make sure the states and parameters are clearly explained in terms of the physical system. Choose realistic numerical values for any parameters in your model. Ideally you will not derive these equations from first principles (this is not a modelling course!) but you should explain how the equations came about.

3. What are the major assumptions or limitations of your model.

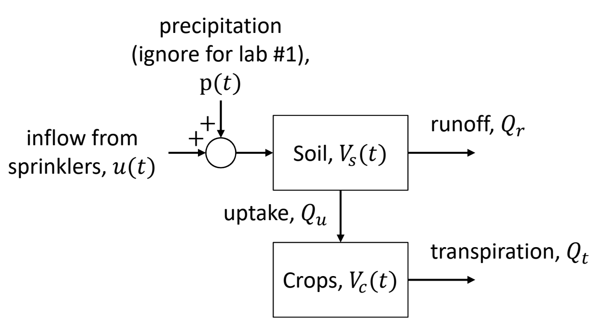
4. What are the references that you used.

Note that this submission can serve as your model description in part B of your design report.

2 Part A: Sample project

In Part A you will work on a sample project of an irrigation system, for which a linearized model is given. An irrigation system is an example of a *compartmental system* [1] in which materials flow between compartments and satisfy conservation laws. Some other examples of compartmental systems include environmental/ecological systems [2], epidemiology/infectious disease propagation [3], and drug/chemical dynamics in biomedical systems [4].

Pretend that you have been contracted by a large-scale farming company to design an automated control system for their sprinklers. The company would like to regulate the amount of water stored by its crops since this variable impacts the quality of the crop. As a first step, a model of the water ecosystem is needed. The soil and the crops are the two relevant compartments that store water in this system. The block diagram is



where u is the inflow of water from the sprinkler, p is the inflow of water from precipitation (which will be ignored for now), V_s is the volume of water in the soil, and V_c is the volume of water present in the crops (water stored in the plant material). The system satisfies the conservation of mass and the mass balance equations are

$$\begin{aligned} \dot{V}_s &= -Q_r - Q_u + u \\ \dot{V}_c &= Q_u - Q_t \end{aligned}$$

where Q_r, Q_u, and Q_t are the flow rates due to runoff, uptake, and transpiration, respectively. The flow rates are linear (flow rate is proportional to the volume of water; more water means more flow!) and satisfy $Q_r = q_r V_s$, $Q_u = q_u V_s$, and $Q_t = q_t V_c$ for flow rate constants $q_r, q_u, q_t > 0$.

There is no way to directly measure the volume of water in the soil or crops, but you estimate that a nearby stream collects 20% of the runoff, nominally. A flow meter in that stream is used as an output measurement and modelled as

$$y = 0.2 Q_r$$

Based on data found in [5], it appears that on average it takes 95,000L of water per week (or 565L/hr on average) to grow crops on one acre. It is difficult to find actual numbers on the volume of water stored in one acre of soil and crops; lets use nominal values of $\tilde{V}_s = 60,000$L of water stored in soil and $\tilde{V}_c = 90,000$L in crops. To maintain an equilibrium,

$$\begin{aligned} (\dot{V}_s =)\ 0 &= -Q_r - Q_u + \tilde{u} \\ &= -q_r \tilde{V}_s - q_u \tilde{V}_s + \tilde{u} \\ &= -60000(q_r + q_u) + 565 \\ q_r + q_u &= 0.0094 \end{aligned}$$

and

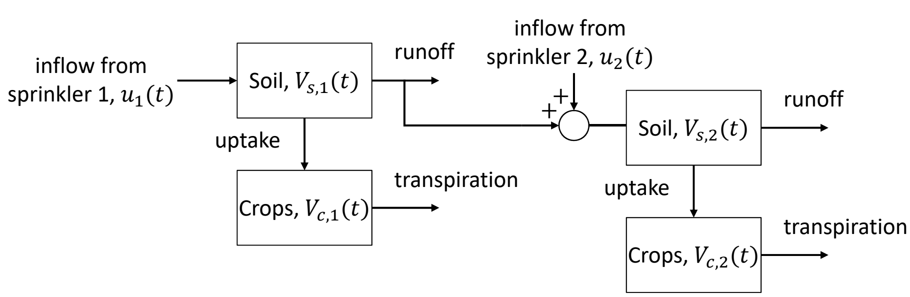
$$\begin{aligned} (\dot{V}_c =)\ 0 &= Q_u - Q_t \\ &= q_u \tilde{V}_s - q_t \tilde{V}_c \\ &= 60000 q_u - 90000 q_t \\ q_u &= 1.5 q_t \end{aligned}$$

To satisfy the equilibrium conditions and based on intuition of the system, the following parameter values are estimated:

parameter	value	units
q_r	0.00313889	1/hr
q_u	0.00627778	1/hr
q_t	0.00418519	1/hr

Interconnected compartmental systems

The farm is very large and there are multiple sprinklers. Assume that the topology and type of crop is the same from acre to acre and thus the system parameters are also the same. The multiple sprinkler system can be modelled as two interconnected compartment systems according to the following block diagram



where the subscripts 1 and 2 differentiate states between the two systems. It is estimated that 25% of the runoff from field 1 ends up as an input to field 2

$$\dot{V}_{s,2} = -Q_{r,2} - Q_{u,2} + 0.25Q_{r,1} + u_2$$

where $Q_{r,2} = q_r V_{s,2}$, and so on. It is easy to see how large complex networks can be modelled by interconnecting smaller systems.

With this new model, assume that 20% of the runoff from field 1 and 15% of the runoff from field 2 both enter the nearby stream, so the measurement model is now

$$y = 0.2Q_{r,1} + 0.15Q_{r,2}$$

2.1 System model, open-loop response and stability

Note that you are free to use Matlab in this work. If you use Matlab to calculate an answer, make sure you report the complete answer in your report and that you perform tests to verify that your answer is correct if possible.

2.1 Write the system of two interconnected compartments in state-space form:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

What does the state x physically represent? How many inputs, outputs and states does this model have? How does this show in your model and in the matrices A, B, C and D?

Hint: Write the state-space form of one compartment and then interconnect the two systems.

2.2 Discuss any limitations of this model and possible improvements.

2.3 What are the eigenvalues of the system matrix A? Is this system internally stable? Exponentially stable?

2.4 Find the Jordan form of this system. Note that you can use Matlab to answer this question. Verify that your answer is correct.

2.5 Use Lyapunov's theorem statement 4 or 5 to check for stability of this open-loop system.

2.6 Is this system BIBO stable?

2.7 How can you show that $\tilde{u}_2 = 517.92$L/hr is the equilibrium flow rate for sprinkler 2?

2.8 Simulate the system response to an impulse in inflow from Sprinkler 1 and to an impulse in inflow from Sprinkler 2. Make sure you show the input signals, states and outputs in your report.

2.9 Simulate the system response to an initial condition that corresponds to dry conditions following prolonged hot weather.

3 Part B: Design project

Note that you are free to use Matlab in this work. If you use Matlab to calculate an answer, make sure that you perform tests to verify that your answer is correct if possible. Report sufficient detail to allow for evaluation of your work. For simulations, make sure you show the input signals, states and outputs in your report as appropriate.

3.1 How many inputs, states and outputs does your system have?

3.2 Calculate the Jordan form corresponding to your linearized system. What are the eigenvalues of your system and what is their multiplicity?

3.3 Is your linearized system internally stable? Is it asymptotically stable? What about exponentially stable?

3.4 Is your nonlinear system locally exponentially stable at the equilibrium point?

3.5 Is your system BIBO stable?

3.6 Is it possible to perform an open-loop experiment on your (physical) system? If so, what experiment would be practically and physically meaningful, while at the same time give you insight into the behaviour of your system?

3.7 Implement your non-linear state-space model as well as your linearization in Simulink. Simulate the homogeneous response of your system (both linear and nonlinear) to practically meaningful initial conditions.

3.8 Simulate the forced response of your system (both linear and nonlinear) to step changes or impulses in each input, where the input type is chosen such that the experiment can be performed on the real system. If a step is chosen, the size of the step must be within the physical and practically meaningful range.

3.9 How does the response of your nonlinear system compare to the response of the linearized model?
3.10 As your system is a "control system", what is the desired output of the system and how does that compare to the open-loop response?

References

[1] F. Bullo, *Lectures on Network Systems.* CreateSpace, 1 ed., 2018. With contributions by J. Cortes, F. Dorfler, and S. Martinez.

[2] I. Noy-Meir, "Desert ecosystems I: Environment and producers," *Annual Review of Ecology and Systematics*, vol. 4, pp. 25–51, 1973.

[3] C. Nowzari, V. M. Preciado, and G. Pappas, "Analysis and control of epidemics: A survey of spreading processes on complex networks," *IEEE Control Systems Magazine*, vol. 36, no. 1, pp. 26–46, 2016.

[4] N. D. Charkes, P. T. Maklar, and C. Philips, "Studies of skeletal tracer kinetics I: Digital-computer solution of a five-compartment model of 18 fluoride kinetics in humans," *Journal of Nuclear Medicine*, vol. 19, no. 12, pp. 1301–1309, 1978.

[5] B. Byelich, J. Cook, and C. Rowley, "Small acreage irrigation guide," tech. rep., U.S. Department of Agriculture (USDA), June 2013.