ENGR 580 Sample Project Report

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I. HANDOUT 2

A. Handout 2 Question 2.1

In the system of two interconnected compartments, the inputs is:

$$\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

states is:

$$\boldsymbol{x} = \begin{bmatrix} V_{s,1} \\ V_{c,1} \\ V_{s,2} \\ V_{c,2} \end{bmatrix}$$

outputs is:

$$\boldsymbol{y} = \begin{bmatrix} y \end{bmatrix}$$

calculate state space function:

$$\begin{aligned}
\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{V_{s,1}} \\ \dot{V_{c,1}} \\ \dot{V_{s,2}} \\ \dot{V_{c,2}} \end{bmatrix} &= \begin{bmatrix} -Q_{r,1} - Q_{u,1} + u_1 \\ Q_{u,1} - Q_{t,1} \\ -Q_{r,2} - Q_{u,2} + 0.25Q_{r,1}u_2 \\ Q_{u,2} - Q_{t,2} \end{bmatrix} \\
&= \begin{bmatrix} -q_r V_{s,1} - q_u V_{s,1} + u_1 \\ q_u V_{s,1} - q_t V_{c,1} \\ -q_r V_{s,2} - q_u V_{s,2} + 0.25 q_r V_{s,1} + u_2 \\ q_u V_{s,2} - q_t V_{c,2} \end{bmatrix} \\
&= \begin{bmatrix} -q_r - q_u & 0 & 0 & 0 \\ q_u & -q_t & 0 & 0 \\ 0.25q_r & 0 & -q_r - q_u & 0 \\ 0 & 0 & q_u & -q_t \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \boldsymbol{u}
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{y} &= \begin{bmatrix} 0.2Q_{r,1} + 0.15Q_{r,2} \end{bmatrix} = \begin{bmatrix} 0.2q_r V_{s,1} + 0.15 q_r V_{s,2} \end{bmatrix} \\
&= \begin{bmatrix} 0.2q_r & 0 & 0.15q_r & 0 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 & 0 \end{bmatrix} \boldsymbol{u}
\end{aligned}$$

then we get $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$ as follows (remain 4 decimals):

$$\begin{aligned}
\boldsymbol{A} &= \begin{bmatrix} -q_r - q_u & 0 & 0 & 0 \\ q_u & -q_t & 0 & 0 \\ 0.25q_r & 0 & -q_r - q_u & 0 \\ 0 & 0 & q_u & -q_t \end{bmatrix} \\
&= \begin{bmatrix} -0.0094 & 0 & 0 & 0 \\ 0.0063 & -0.0042 & 0 & 0 \\ 0.0008 & 0 & -0.0094 & 0 \\ 0 & 0 & 0.0063 & -0.0042 \end{bmatrix} \\
\boldsymbol{B} &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \\
\boldsymbol{C} &= \begin{bmatrix} 0.2q_r & 0 & 0.15q_r & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0.0006 & 0 & 0.0005 & 0 \end{bmatrix} \\
\boldsymbol{D} &= \begin{bmatrix} 0 & 0 \end{bmatrix}
\end{aligned}$$

B. Handout 2 Question 2.2

The limitation of the model in an irrigation system is that it only have one output, which means it can only evaluate the stored water in soil and crops solely by observing a nearby stream. It provides only indirect information about the water content in the soil and the water uptake by the crops. While the stream's flow rate can give some indication of the overall water availability in the area, it does not provide specific and accurate measurements of soil moisture levels or crop water requirements.

To improve the evaluation of stored water in soil and crops in an irrigation system, it is beneficial to implement direct and accurate measurement techniques. Such as soil moisture sensors which can provide real-time data on soil moisture levels, and crop water demand modeling, based on crop type, growth stage, and environmental conditions.

C. Handout 2 Question 2.3

The eigenvalues of the system matrix A is

$$\begin{aligned}
&det(\lambda \boldsymbol{I} - \boldsymbol{A}) \\
&= \begin{bmatrix} \lambda + q_r + q_u & 0 & 0 & 0 \\ -q_u & \lambda + q_t & 0 & 0 \\ -0.25q_r & 0 & \lambda + q_r + q_u & 0 \\ 0 & 0 & -q_u & \lambda + q_t \end{bmatrix} \\
&= (\lambda + q_r + q_u)^2 (\lambda + q_t)^2
\end{aligned}$$

$$\begin{aligned}\lambda_1 = \lambda_2 = -q_r - q_u \quad &= -0.0094 < 0 \\ \lambda_3 = \lambda_4 = -q_t \quad &= -0.0042 < 0\end{aligned}$$

Since all the eigenvalues of $\boldsymbol{A}$ have strictly negative real parts, which means the system is internally and exponentially stable.

D. Handout 2 Question 2.4

Jordan Normal Form can be calculated in MATLAB:

$$\begin{aligned}\boldsymbol{J} &= \begin{bmatrix} -q_r - q_u & 1 & 0 & 0 \\ 0 & -q_r - q_u & 0 & 0 \\ 0 & 0 & -q_t & 0 \\ 0 & 0 & 0 & -q_t \end{bmatrix} \\ &= \begin{bmatrix} -0.0094 & 1 & 0 & 0 \\ 0 & -0.0094 & 0 & 0 \\ 0 & 0 & -0.0042 & 0 \\ 0 & 0 & 0 & -0.0042 \end{bmatrix}\end{aligned}$$

And the transfer matrix is

$$\boldsymbol{P} = \begin{bmatrix} 0 & 1.0 & 0 & 0 \\ 0 & -1.2 & 1.2 & 0 \\ 0.0008 & 0 & 0 & 0 \\ -0.0009 & -0.18 & 0.18 & 1.0 \end{bmatrix}$$

We can verify the result as follows:

$$\boldsymbol{PJP}^{-1} = \begin{bmatrix} -0.0094 & 0 & 0 & 0 \\ 0.0063 & -0.0042 & 0 & 0 \\ 0.0008 & 0 & -0.0094 & 0 \\ 0 & 0 & 0.0063 & -0.0042 \end{bmatrix} = \boldsymbol{A}$$

E. Handout 2 Question 2.5

According to Lyapunov's theorem statement 4, we can calculate if P matrix existed in MATLAB and determine whether the eigenvalues of P have strictly positive real parts:

$$P = \begin{bmatrix} 53.0973 & 24.5065 & 2.2124 & 1.0211 \\ 24.5065 & 156.2285 & 2.4349 & 2.5920 \\ 2.2124 & 2.4349 & 53.2817 & 24.6505 \\ 1.0211 & 2.5920 & 24.6505 & 156.4445 \end{bmatrix}$$

$$eig(P) = \begin{bmatrix} 46.1197 \\ 49.0977 \\ 158.5979 \\ 165.2368 \end{bmatrix}$$

Because the eigenvalues of P all have positive real parts, the system is asymptotically and exponentially stable.

F. Handout 2 Question 2.6

Calculate transfer function

$$\begin{aligned}\boldsymbol{g}(s) =& \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D} \\ =& \begin{bmatrix} -\frac{3q_r^2}{80(q_r+q_u+s)^2} - \frac{q_r}{5(q_r+q_u+s)} \\ -\frac{3q_r}{20(q_r+q_u+s)} \end{bmatrix}\end{aligned}$$

The poles of the each entry of the transfer function are:

$$pole = -(q_r + q_u) < 0$$

So the system is BIBO stable.

G. Handout 2 Question 2.7

$$\begin{aligned}\dot{V}_{s,2} =& -Q_{r,2} - Q_{u,2} + 0.25Q_{r,1} + u_2 \\ =& -q_r V_{s,2} - q_u V_{s,2} + 0.25 q_r V_{s,1} + u_2 \\ =& -0.00313889 \times 60000 \\ & -0.00627778 \times 60000 \\ & +0.25 \times 0.00313889 \times 60000 + u_2 \\ =& -517.9169 + u_2 = 0 \\ u_2 =& 517.9169 \text{ L/hr}\end{aligned}$$

H. Handout 2 Question 2.8

We simulate the system response to an impulse in inflow from Sprinkler1 and to an impulse in inflow from Sprinkler 2 in SIMULINK. We set the initial state as half as equilibrium state $x_0 = [30000, 45000, 30000, 45000]^T$ and the impulse in inflow from Sprinkler 1 or 2 are $u_{10} = u_{20} = 30000$.

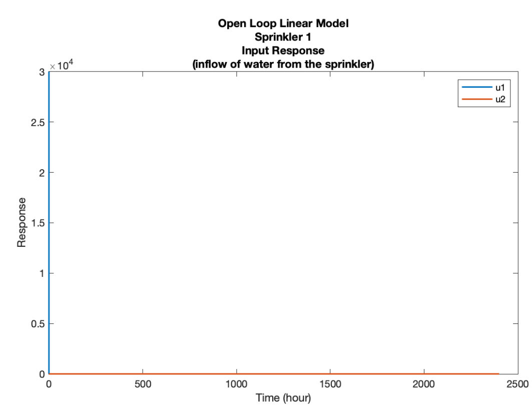

Fig. 1. Input response of the open loop linear model when sprinkler 1 is working

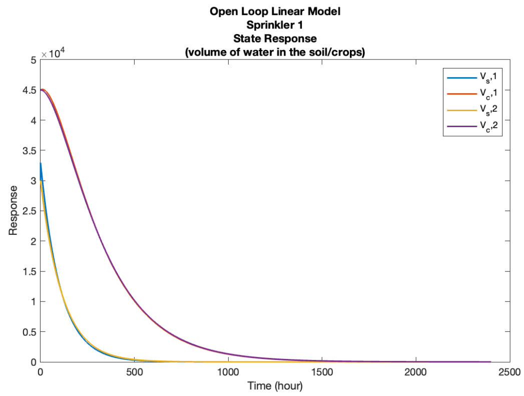

Fig. 2. State response of the open loop linear model when sprinkler 1 is working

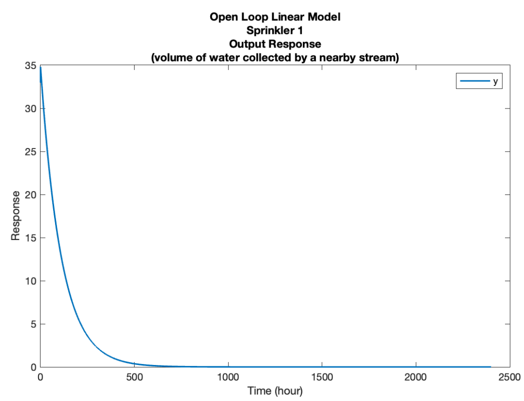

Fig. 3. Output response of the open loop linear model when sprinkler 1 is working

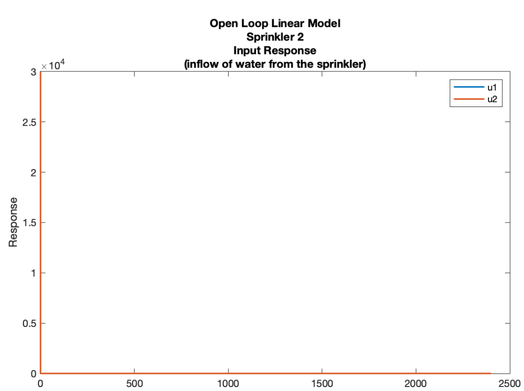

Fig. 4. Input response of the open loop linear model when sprinkler 2 is working

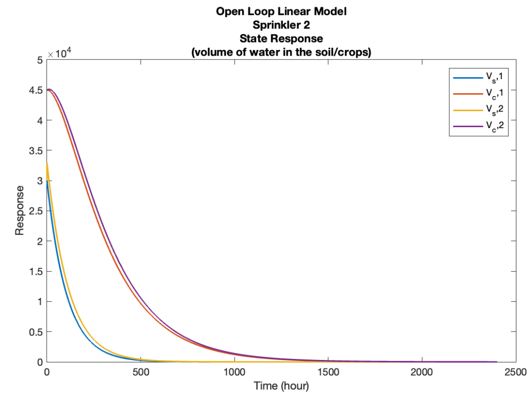

Fig. 5. State response of the open loop linear model when sprinkler 2 is working

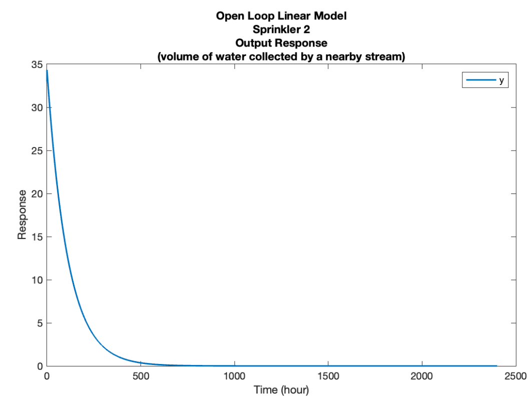

Fig. 6. Output response of the open loop linear model when sprinkler 2 is working

Fig. 1, 2, 3 are the response of the open loop linear model when sprinkler 1 is working. Fig. 4, 5, 6 show the response when sprinkler 2 is working. Since it is impulse input, the responses in the two situations will be different in the initial stage, but will eventually return to 0 over time.

I. Handout 2 Question 2.9

Consider the dry conditions following prolonged hot weather, we assume the initial state is $x_0 = [0; 0; 0; 0]^T$, and the impulse in inflow from both Sprinkler 1 and 2 are $u_{10} = u_{20} = 30000$. Beside, in an irrigation system, hot weather has effect on the flow rate with various factors such as evaporation rates, plant water demand and soil conditions. For example, soils and crops need more water to compensate the increased evaporation. We assume the flow rate are increased as:

$$\begin{cases} q_{r\,hot} = q_r \times 1.5 \\ q_{u\,hot} = q_u \times 1.5 \\ q_{t\,hot} = q_t \times 1.5 \end{cases}$$

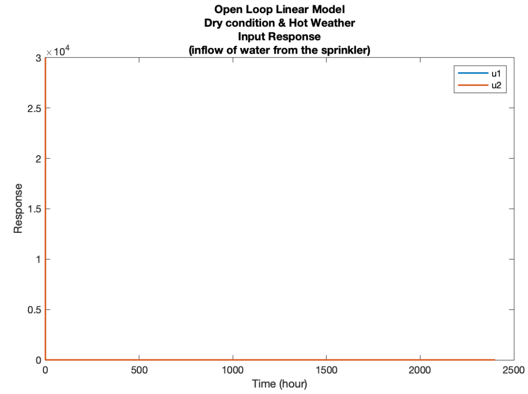

Fig. 7. Input response of the open loop linear model in dry condition and hot weather

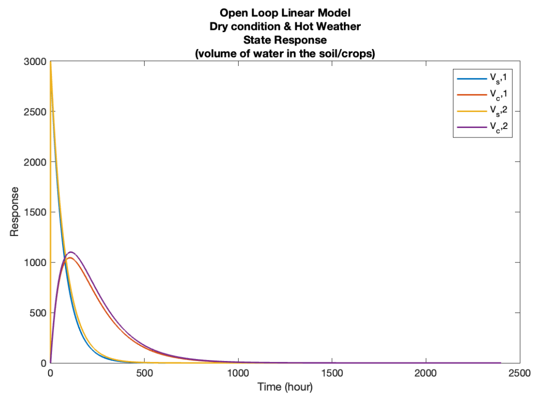

Fig. 8. State response of the open loop linear model in dry condition and hot weather

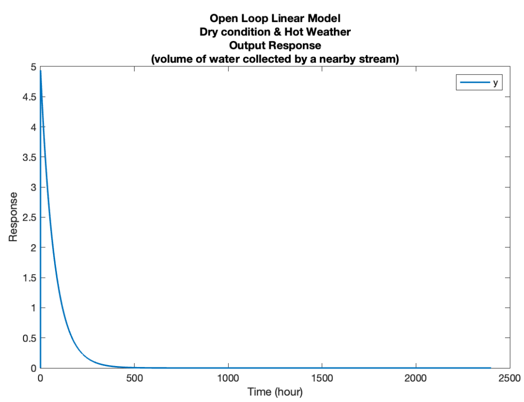

Fig. 9. Output response of the open loop linear model in dry condition and hot weather

Fig. 7, 8, 9 shows the response of the open loop linear model in dry condition and hot weather. Compared to the normal case in section I-H, the water in the soil and crops dries up faster, in other words, the system converges to zero faster.

II. Handout 3

A. Handout 3 Question 1.1

The controllability matrix of the system with two sprinklers is:

$$\mathcal{C} = \begin{bmatrix} 1 & 0 & -0.0094 & 0 & 0.0001 & 0 & 0 & 0 \\ 0 & 0 & 0.0063 & 0 & -0.0001 & 0 & 0 & 0 \\ 0 & 1 & 0.0008 & -0.0094 & 0 & 0.0001 & 0 & 0 \\ 0 & 0 & 0 & 0.0063 & 0 & -0.0001 & 0 & 0 \end{bmatrix}$$

The rank of the controllability matrix is 4, which is full rank, so the system is controllable.

B. Handout 3 Question 1.2

We design a state-feedback controller that stabilizes the system to the equilibrium state and simulate the response to several realistic nonzero initial states $x_0^1 = x_{eq} - 500$, $x_0^2 = x_{eq} - 1000$, $x_0^3 = x_{eq} - 2000$, $x_0^4 = x_{eq} - 5000$, where $x_{eq} = [60000, 90000, 60000, 90000]^T$ and initial input $u_0 = [0, 0]^T$ to check if the system can settle within 48 hours and the response meets the requirements of a real-world irrigation system. In the experiments, we used eigenvalue assignment to finetune parameters.

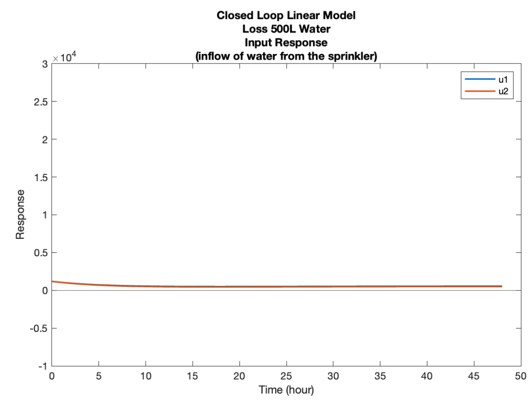

Fig. 10. Input response of the closed loop linear model when losing 500L water

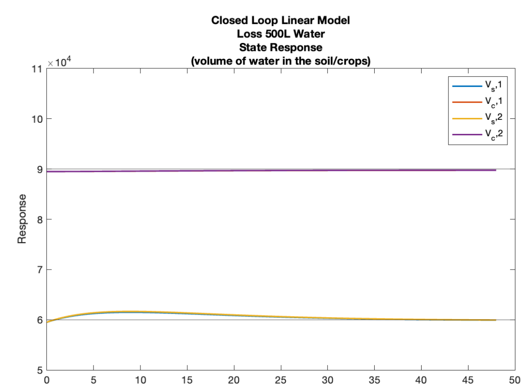

Fig. 11. State response of the closed loop linear model when losing 500L water

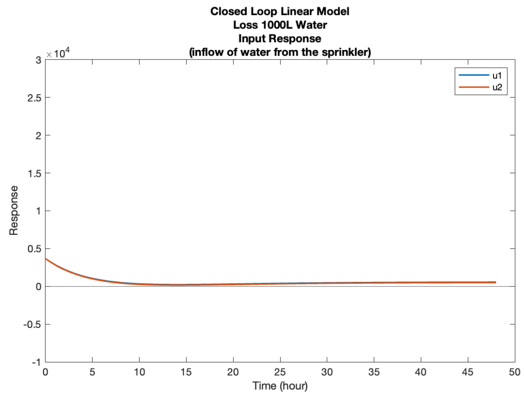

Fig. 12. Input response of the closed loop linear model when losing 1000L water

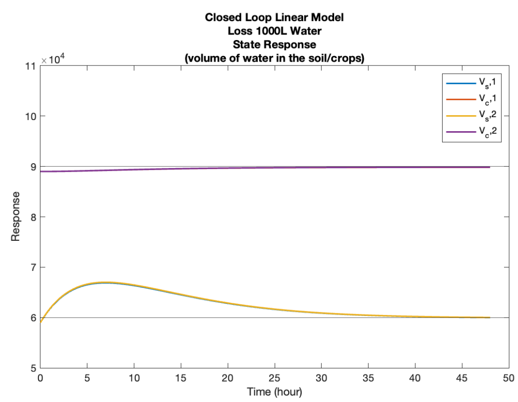

Fig. 13. State response of the closed loop linear model when losing 1000L water

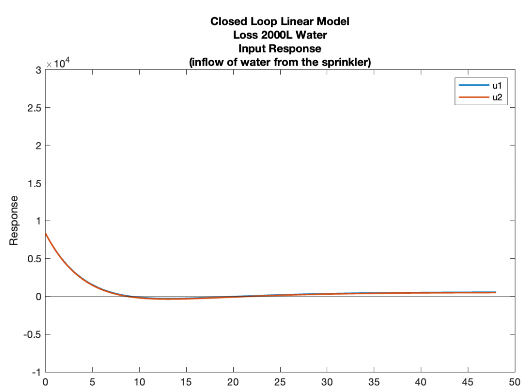

Fig. 14. Input response of the closed loop linear model when losing 2000L water

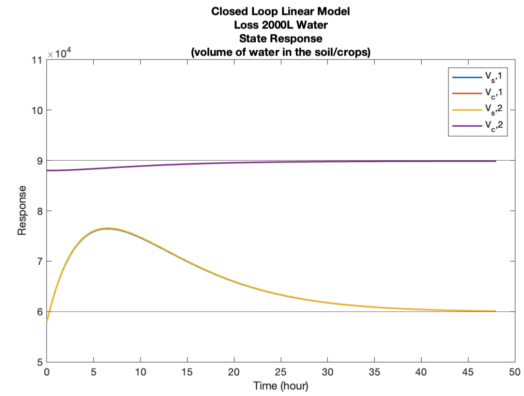

Fig. 15. State response of the closed loop linear model when losing 2000L water

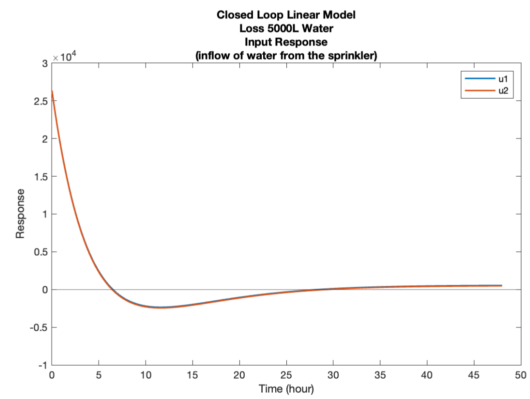

Fig. 16. Input response of the closed loop linear model when losing 5000L water

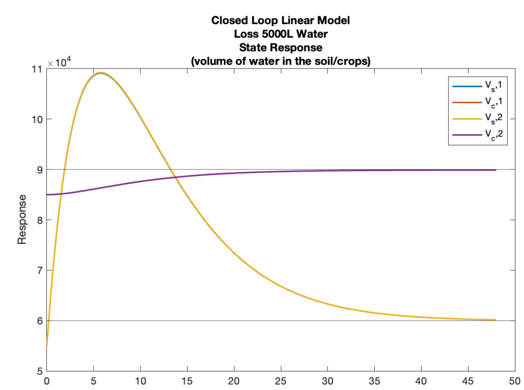

Fig. 17. State response of the closed loop linear model when losing 5000L water

According to the state responses in each initial condition in fig. 11, 13, 15, 17, the system all eventually stabilizes at equilibrium point. But in fig. 17, crops must store twice as much water as the equilibrium state. Crops have limited water storage capacity and are mainly used directly for various physiological processes. Excess water storage within crops can lead to problems such as waterlogging, disease susceptibility, nutrient imbalances and reduced photosynthesis. If 5000L of water is lost in the initial state, it is unrealistic for the system to stabilize within 48 hours.

Analyzing from the perspective of input, we can see from the fig. 14 and 16 that the value of input once became a negative number. For sprinkler, this is unrealistic because sprinkler does not have the function of pumping water.

The conclusion is that if the system does not lose too much water, it can return to equilibrium within 48 hours.

C. Handout 3 Question 1.3

When we designed a LQR (optimal) controller for a system, we were trying to see if we could design a controller for a system with an initial loss of 5000L of water that would converge as quickly as possible.

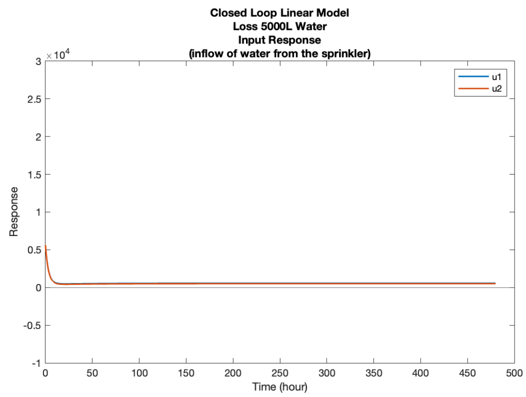

Fig. 18. Input response of the closed loop linear model when losing 5000L water

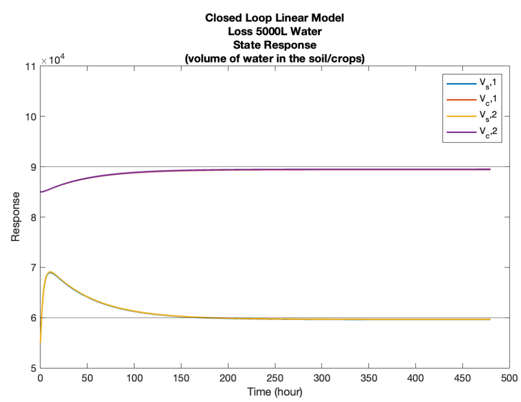

Fig. 19. State response of the closed loop linear model when losing 5000L water

The eigenvalues of the closed-loop system is:

$$\boldsymbol{K} = \begin{bmatrix} 0.3225 & 0.8007 & 0.0004 & 0.0009 \\ 0.0004 & -0.0009 & 0.3225 & 0.8007 \end{bmatrix}$$

The results show that we cannot control the system to converge within 48 hours, but the input values and status values are always within a reasonable range.

D. Handout 3 Question 1.4

If sprinkler 2 is broken, we can remove u_2 and set B and D as:

$$\boldsymbol{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\boldsymbol{D} = \begin{bmatrix} 0 \end{bmatrix}$$

or we can also keep u_2 and set B and D as:

$$\boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$\boldsymbol{D} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

E. Handout 3 Question 1.5

If the system with sprinkler 2 out of service, the controllability matrix is:

$$\mathcal{C} = \begin{bmatrix} 1 & -0.0094 & 0.0001 & 0 \\ 0 & 0.0063 & -0.0001 & 0 \\ 0 & 0.0008 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The rank of the controllability matrix is 3, so the system is uncontrollable.

After controllable decomposition we will get:

$$\bar{\boldsymbol{A}} = \begin{bmatrix} -0.0042 & 0 & 0 & 0 \\ 0.0062 & -0.0093 & 0.0010 & 0 \\ -0.0005 & 0.0004 & -0.0043 & 0.0063 \\ 0 & 0 & 0 & -0.0094 \end{bmatrix}$$
$$\bar{\boldsymbol{B}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$
$$\bar{\boldsymbol{C}} = \begin{bmatrix} 0.0003600 & -0.0002977 & 0.0000584 & 0.0006278 \end{bmatrix}$$

The uncontrollable component of A is:

$$\boldsymbol{A_{uc}} = \begin{bmatrix} -0.0042 \end{bmatrix}$$

Since the eigenvalue of A_{uc} has strictly negative real part, A_{uc} is asymptotically stable, the system is stabilizable.

F. Handout 3 Question 1.6

In order to compare the performance of the system before and after sprinkler 2 is broken, we choose the initial loss of 500L water for comparison.

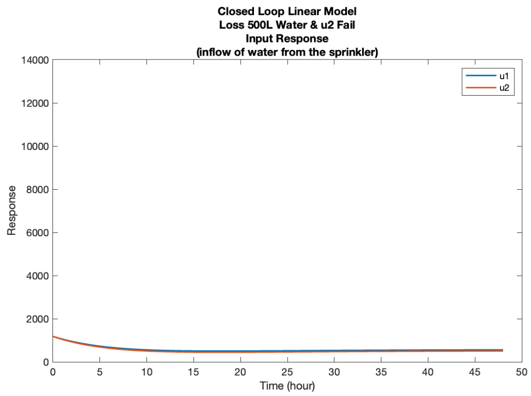

Fig. 20. Input response of the closed loop linear model when sprinkler 2 work

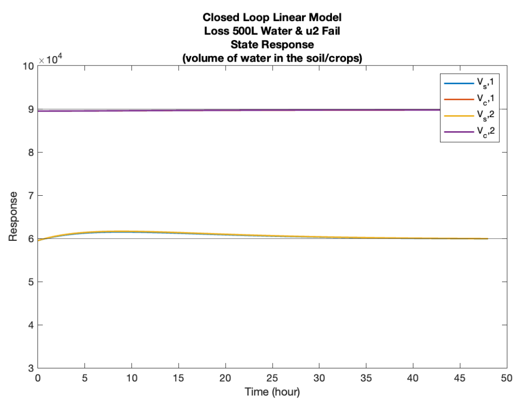

Fig. 21. State response of the closed loop linear model when sprinkler 2 work

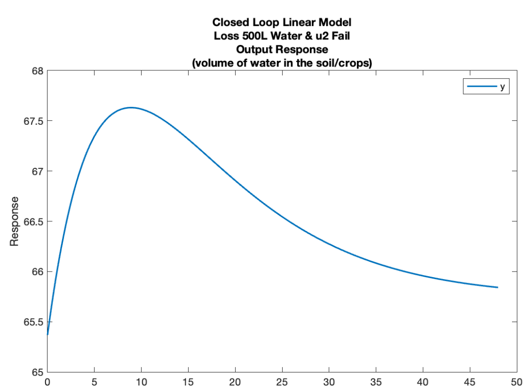

Fig. 22. Output response of the closed loop linear model when sprinkler 2 work

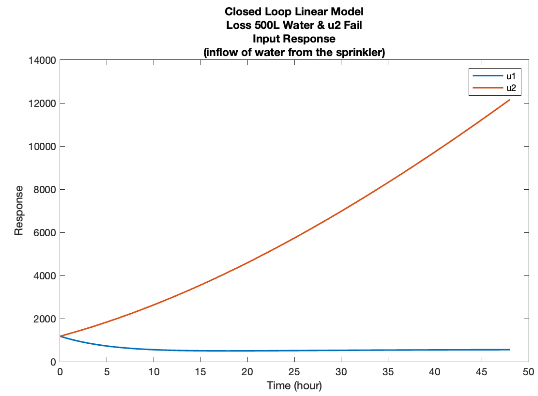

Fig. 23. Input response of the closed loop linear model when sprinkler 2 fail

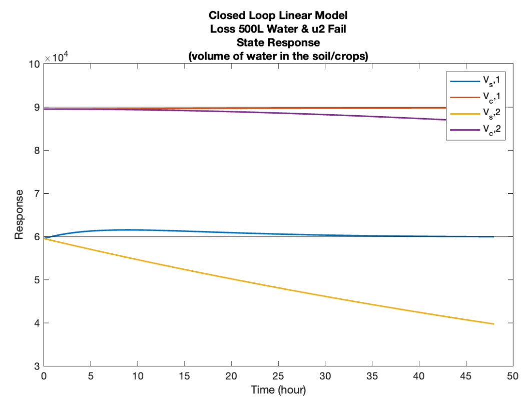

Fig. 24. State response of the closed loop linear model when sprinkler 2 fail

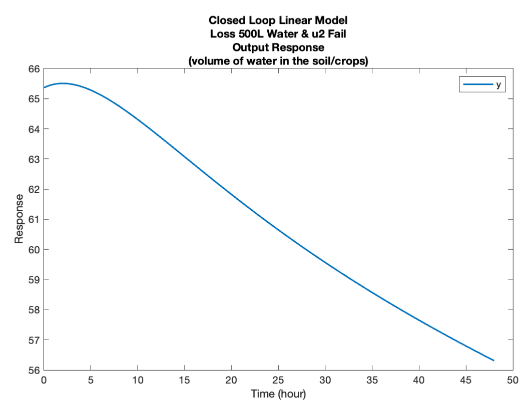

Fig. 25. Output response of the closed loop linear model when sprinkler 2 fail

Compare fig. 20 and 23, we can see that when sprinkler 2 is broken, the feedback control system requires more and more water from sprinkler 2. As a result, from fig. 24, we can see that the water stored in soil 2 and crop 2 gradually decreases.

In order to design a controller that would work better, we try to combine the feedback control signal together and send it into sprinkler 1, which means, each time the sprinkler 1 will produce more water.

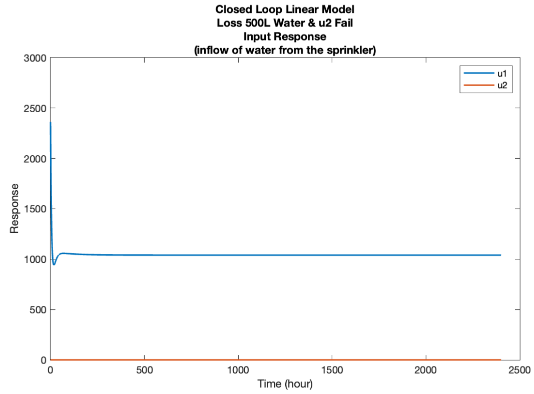

Fig. 26. Input response of the closed loop linear model when sprinkler 2 fail (redesign)

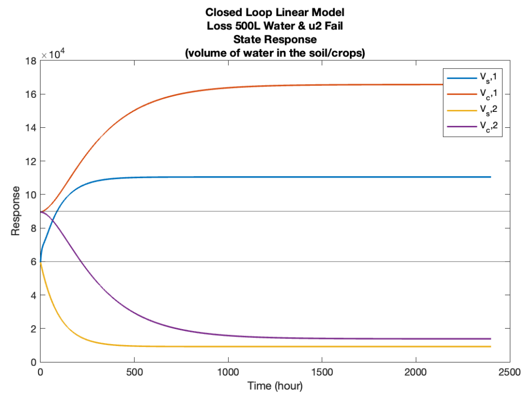

Fig. 27. State response of the closed loop linear model when sprinkler 2 fail (redesign)

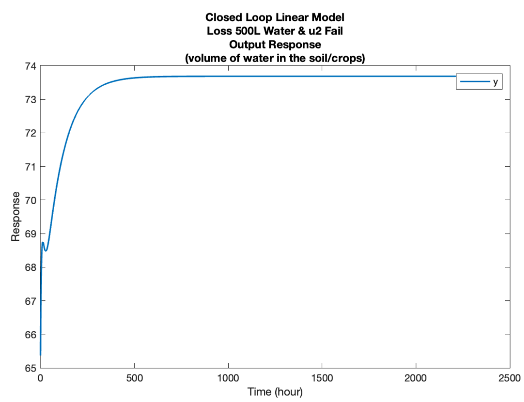

Fig. 28. Output response of the closed loop linear model when sprinkler 2 fail (redesign)

As we can see in fig. 27, although the water stored in crop 2 and soil 2 will not completely dry out over time, which is more ideal than the original system, the corresponding crop 1 and soil 1 will store 1.5 times more water, which has It may cause a series of problems such as waterlogging, so it is still not the most perfect solution.

III. Handout 4

A. Handout 4 Question 1.1

The observability matrix of the system with two sprinklers is:

$$\mathcal{O} = \begin{bmatrix} 0.0006278 & 0 & 0.0004708 & 0 \\ -0.0000055 & 0 & -0.0000044 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The rank of the observability matrix is 2, so the system is unobservable.

B. Handout 4 Question 1.2

Since the number of states is 4 and the rank of the observability matrix is 2, the minimum number of additional sensors required to make the multi-sprinkler system observable is 2, in addition to the flow meters already used to calculate runoff in nearby streams.

C. Handout 4 Question 1.3

We calculate Kalman decomposition in MATLAB with function ”minreal”, and we get

$$\bar{\boldsymbol{A}} = \begin{bmatrix} -0.0094 & 0 & 0 & 0 \\ 0.0008 & -0.0094 & 0 & 0 \\ 0.0063 & 0 & -0.0042 & 0 \\ 0 & 0.0063 & 0 & -0.0042 \end{bmatrix}$$

$$\bar{\boldsymbol{B}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\bar{\boldsymbol{C}} = \begin{bmatrix} 0.0006278 & 0.0004708 & 0 & 0 \end{bmatrix}$$

The controllable but unobservable component of A is:

$$\boldsymbol{A_{c\bar{o}}} = \begin{bmatrix} -0.0042 & 0 \\ 0 & -0.0042 \end{bmatrix}$$