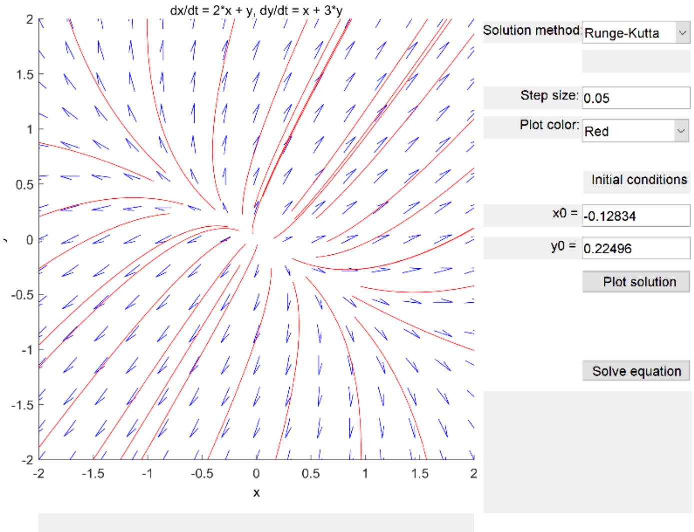
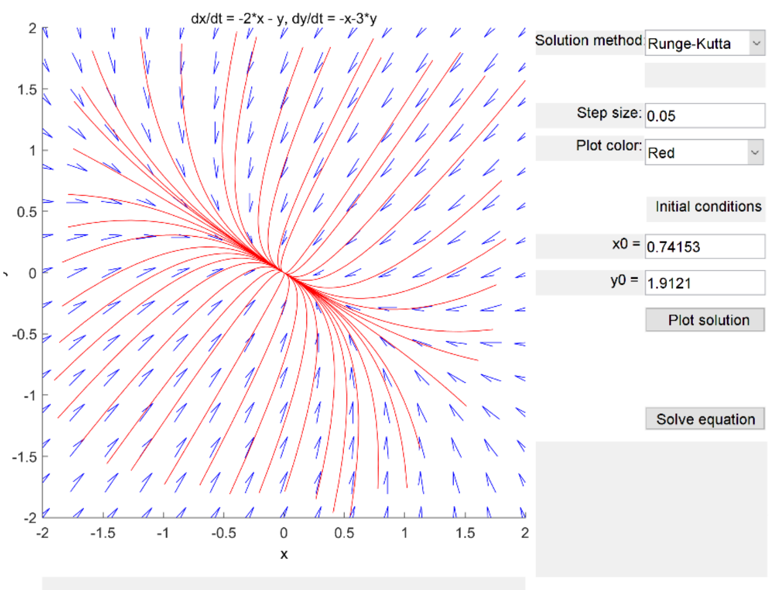
4.1 dx/dt = [2 1; 1 3] x



Equilibrium: Unstable, nodal source.

Eigenvalues: $\frac{5+\sqrt{5}}{2}$, $\frac{5-\sqrt{5}}{2}$. We have two real, distinct, positive eigenvalues. Since $\lambda_1, \lambda_2 > 0$, the solutions are unstable and diverge to infinity – this is of type nodal source.

4.2 dx/dt = [-2 -1; -1 -3] x



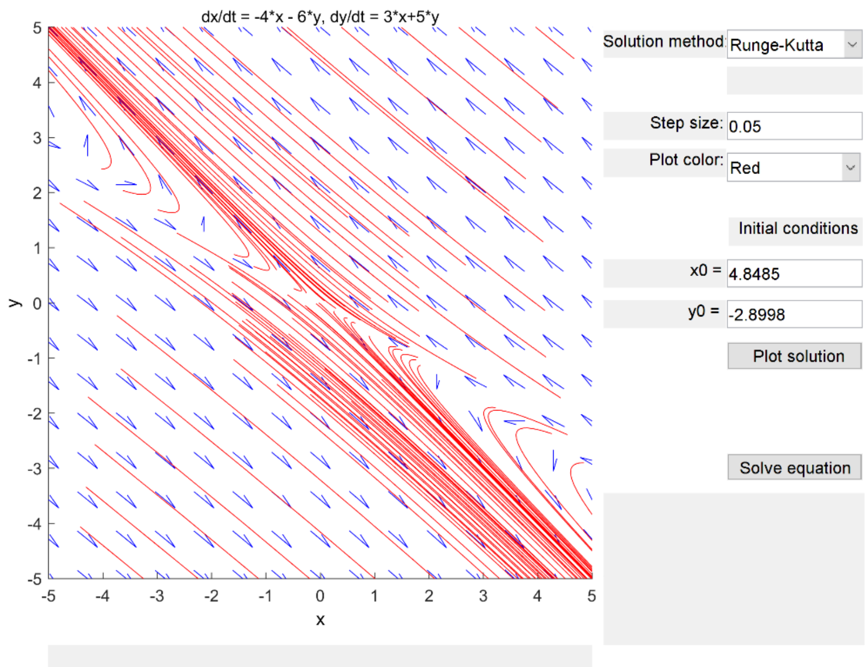
Equilibrium: Stable, nodal sink

Eigenvalues: $\frac{-5+\sqrt{5}}{2}$, $\frac{-5-\sqrt{5}}{2}$. We have two real, distinct, negative eigenvalues. Since $\lambda_1, \lambda_2 < 0$, the solutions are stable and a nodal sink – all solutions converge to (0,0).

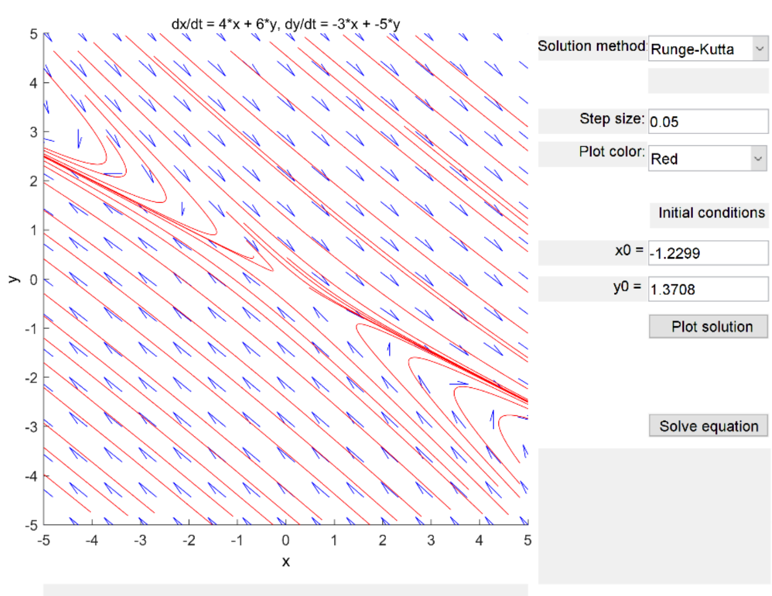
4.3 dx/dt = [-4 -6; 3 5] x

Equilibrium classification: Saddle point, unstable

Eigenvalues: 2, -1 We have two real, distinct eigenvalues. Since $\lambda_1 > 0$ and $\lambda_2 < 0$, the solutions are unstable since the $exp(2t)$ term approaches infinity. However, solutions starting on the line where the constant for the eigenvector of the negative eigenvalue is 0 converges to zero. This is due to the $exp(-t)$ term present (from the negative eigenvalue). The solution is a saddle point because the two eigenvalues have different signs.



4.4 dx/dt = [4 6; -3 -5] x



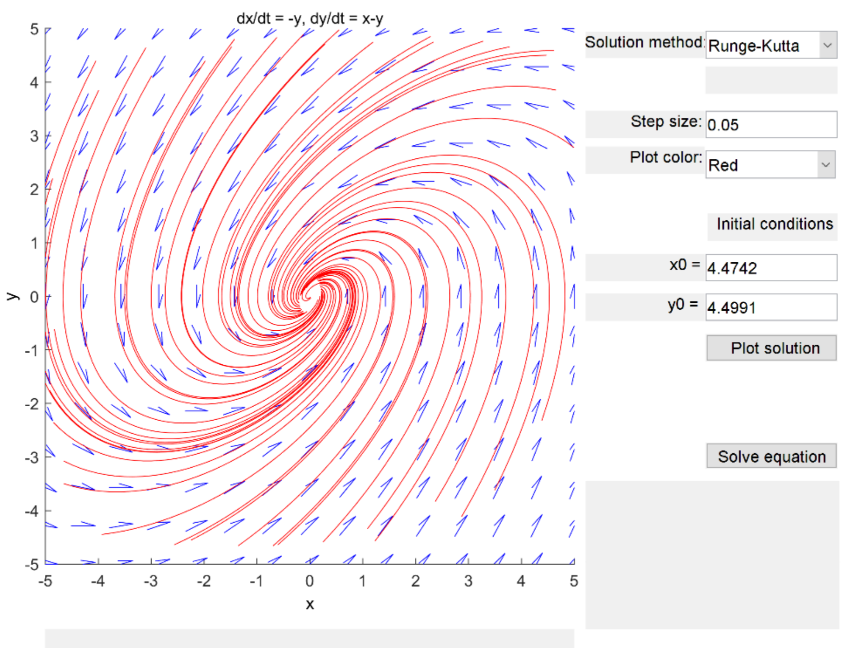
Equilibrium: Saddle point, unstable

Eigenvalues: 1, -2. We have two real, distinct eigenvalues. Since we have the $exp(t)$ term from the positive eigenvalue, the solution is unstable. The eigenvalues have opposing signs, therefore the solutions are a saddle point.

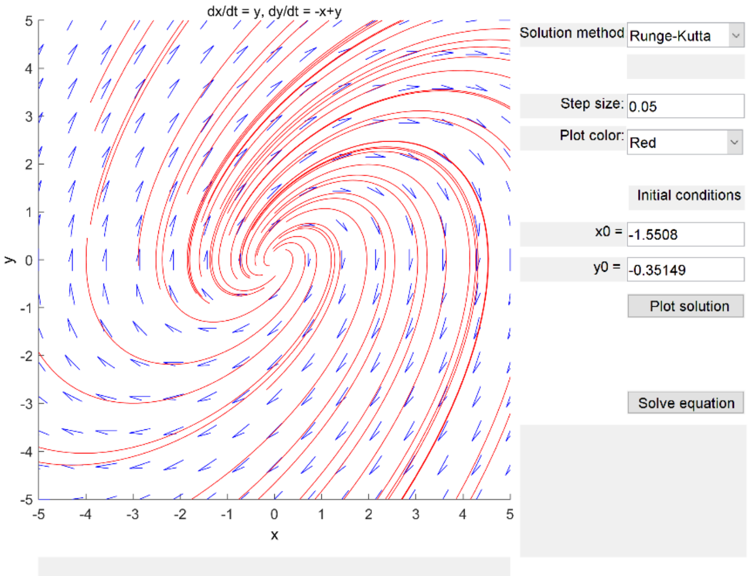
4.5 dx/dt = [0 -1; 1 -1] x

Equilibrium: spiral sink, stable, counter-clockwise

Eigenvalues: $\frac{-1+i\sqrt{3}}{2}$, $\frac{-1\text{-}i\sqrt{3}}{2}$. Complex eigenvalues with negative real part mean that the solution is stable (converges to (0,0)).



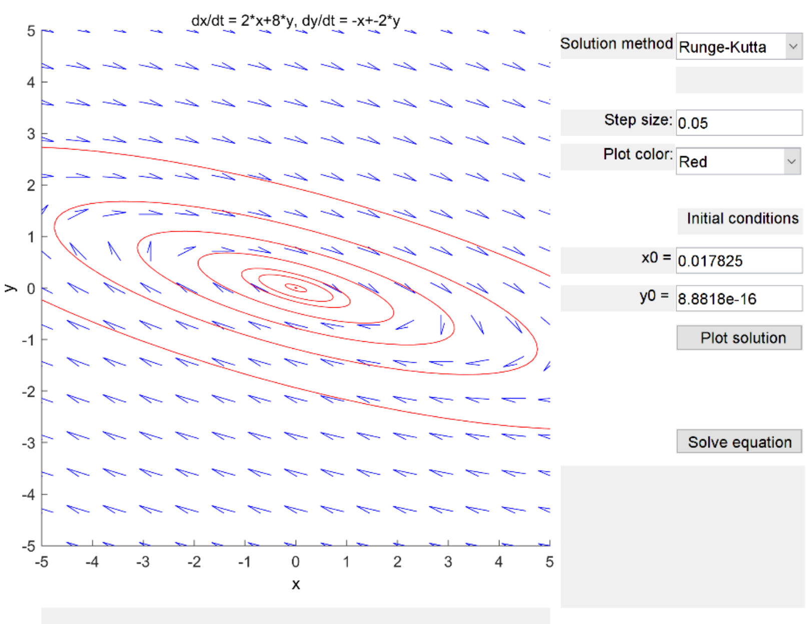
4.6 dx/dt = [0 1; -1 1] x



Equilibrium: spiral source, unstable, clockwise

Eigenvalues: $\frac{1+i\sqrt{3}}{2}$, $\frac{1-i\sqrt{3}}{2}$. Complex eigenvalues with positive real part mean that the solution is unstable.

4.7 dx/dt = [2 8; -1 -2] x



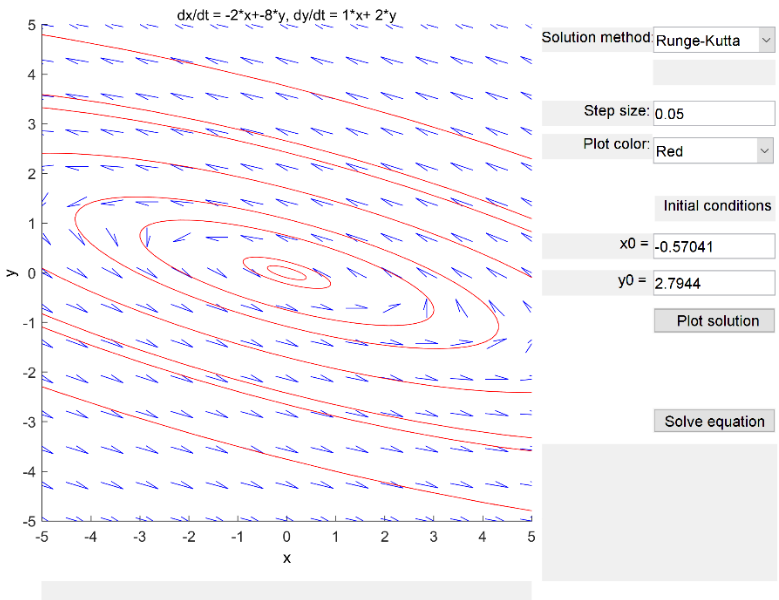
Equilibrium: centre, stable, clockwise

Eigenvalues: $2i, -2i$. The real part of the eigenvalues is zero therefore the solutions are stable and circle in ellipses around (0,0).

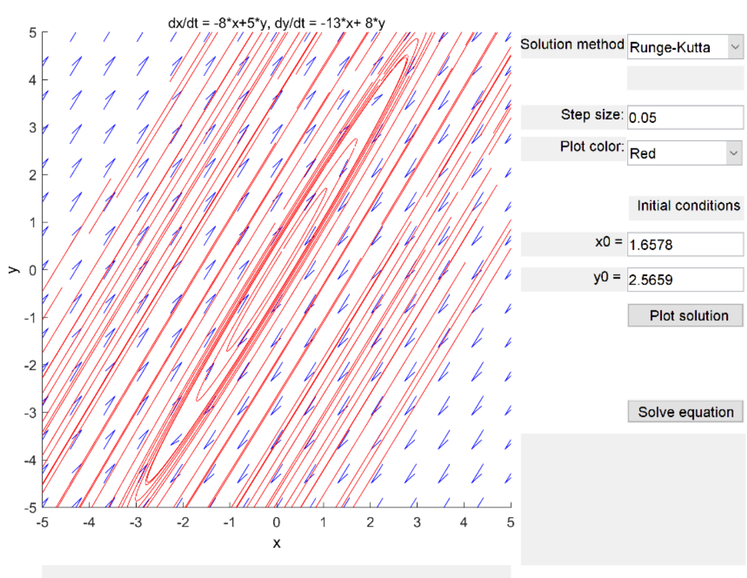
4.8 dx/dt = [-2 -8; 1 2] x

Equilibrium: centre, stable, counter-clockwise

Eigenvalues: $2i, -2i$. The real part of the eigenvalues is zero therefore the solutions are stable and circle in ellipses around (0,0).



4.9 dx/dt = [-8 5; -13 8] x



Equilibrium: centre, stable, clockwise

Eigenvalues: $i, -i$. The real part of the eigenvalues is zero. Therefore, the solutions do not diverge to infinity or converge to (0,0).

4.10 dx/dt = [8 -5; 13 -8] x

Equilibrium: centre, stable, counter-clockwise.

Eigenvalues: $i, -i$. The real part of the eigenvalues is zero. Therefore, the solutions do not diverge to infinity or converge to (0,0).