ESC103 Group Assignment

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1 Our Contributions

1.1 Caitlin Chee-Kirkpatrick

My role in this group was to help edit/format the document as well as perform calculations on paper to double check the answers.
Caitlin Chee-Kirkpatrick

1.2 Wanda Janaeska

My role in this group assignment is primarily in data analysis. I began as a second eye and verification in the process of forming the 26x23 matrix to ensure consistency in values. From there, I took the matrix we developed and verified as a group and started its analysis question by question with verbal assistance of my teammates, on MATLAB.
Wanda Janaeska

1.3 Kyla Tan

My role was primarily formatting of the report. After the matrices were made in MATLAB I would write the code to display them in LaTeX. I would also be in charge of adding figures (including caption), fixing margins, and troubleshooting code errors (for example the | symbol appearing in the text).
Kyla Tan

1.4 Qingyuan Wu

My contributions to this assignment included constructing the 26x23 matrix A, along with other group members, to ensure that the values are correct. I provided formal responses to most of the questions by writing down our thought processes behind our calculations and explaining our work. Finally, I helped enhance the final section of the assignment asking the group to summarize what we learned from this assignment.

Qingyuan

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2 Introduction

The dashes within all matrices in this report represent entries with a value of 0. The same matrices with 0s (instead of dashes) can be found in the appendix.

3 Problem 1

Presented in the next page is the augmented matrix $[A \mid \overrightarrow{b}]$. Each column in matrix A represents the member force carried in each joint. There are 23 columns in A since there are 23 members in the truss, from AB, BC, BD, to LM. Each row in matrix A represents the component of member force that acts on each of the 13 joints in one of the two translational directions, x or y. There are 13 joints and two directions, resulting in 13x2 = 26 rows. For example, entry $A_{1,1}$ represents that cos60, or 1/2 of the force in member AB acts in the horizontal direction at joint A. Vector $\overrightarrow{b}$ is 26x1 whose entries represent the sum of the reaction forces, in both directions, at each joint. Vector $\overrightarrow{x}$ is a 23x1 vector of unknowns with each entry being the force in each of the 23 members. The product of matrix A and vector $\overrightarrow{x}$ produces the sum of the member forces in each direction at each joint. For example, entry 2,1 of this product represents the total sum of forces in the vertical direction caused by all member forces is equal to -875kN. Solving for vector $\overrightarrow{x}$ in the equation A * $\overrightarrow{x} = \overrightarrow{b}$ yields the forces carried by each member of this truss. The sign convention that we used to construct the system matrix A and vector $\overrightarrow{b}$ was to assign the right and upward directions as positive. We also assumed that all members were subject to tensile forces. Therefore, all negative results in our solution would represent that the member was instead in compression instead of tension.

3.1 Augmented Matrix

$$
\begin{bmatrix}
cos60 & 1 & - \\
sin60 & - \\
-cos60 & - & cos60 & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
-sin60 & - & -sin60 & - \\
- & -1 & -cos60 & - & cos60 & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & sin60 & - & sin60 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & - & -1 & -cos60 & - & cos60 & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & -sin60 & - & -sin60 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & -1 & -cos60 & - & cos60 & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & sin60 & - & sin60 & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & -1 & -cos60 & - & cos60 & 1 & - & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & -sin60 & - & -sin60 & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & -1 & -cos60 & - & cos60 & 1 & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & sin60 & - & sin60 & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & - & -1 & -cos60 & - & cos60 & 1 & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & -sin60 & - & -sin60 & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & - & -1 & -cos60 & - & cos60 & 1 & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & sin60 & - & sin60 & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & -1 & -cos60 & - & cos60 & 1 & - & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & sin60 & - & -sin60 & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & -1 & -cos60 & - & cos60 & 1 & - \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & sin60 & - & sin60 & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & -1 & -cos60 & - & cos60 \\
- & -sin60 & - & -sin60 \\
- & -1 & -cos60 \\
- & sin60
\end{bmatrix}
\begin{bmatrix}
\overrightarrow{AB} \\ \overrightarrow{AC} \\ \overrightarrow{BC} \\ \overrightarrow{BD} \\ \overrightarrow{CD} \\ \overrightarrow{CE} \\ \overrightarrow{DE} \\ \overrightarrow{DF} \\ \overrightarrow{EF} \\ \overrightarrow{EG} \\ \overrightarrow{FG} \\ \overrightarrow{FH} \\ \overrightarrow{GH} \\ \overrightarrow{GI} \\ \overrightarrow{HI} \\ \overrightarrow{HJ} \\ \overrightarrow{IJ} \\ \overrightarrow{IK} \\ \overrightarrow{JK} \\ \overrightarrow{JL} \\ \overrightarrow{KL} \\ \overrightarrow{KM} \\ \overrightarrow{LM}
\end{bmatrix}
=
\begin{bmatrix}
\Sigma F_{Ax} \\ \Sigma F_{Ay} \\ \Sigma F_{Bx} \\ \Sigma F_{By} \\ \Sigma F_{Cx} \\ \Sigma F_{Cy} \\ \Sigma F_{Dx} \\ \Sigma F_{Dy} \\ \Sigma F_{Ex} \\ \Sigma F_{Ey} \\ \Sigma F_{Fx} \\ \Sigma F_{Fy} \\ \Sigma F_{Gx} \\ \Sigma F_{Gy} \\ \Sigma F_{Hx} \\ \Sigma F_{Hy} \\ \Sigma F_{Ix} \\ \Sigma F_{Iy} \\ \Sigma F_{Jx} \\ \Sigma F_{Jy} \\ \Sigma F_{Kx} \\ \Sigma F_{Ky} \\ \Sigma F_{Lx} \\ \Sigma F_{Ly} \\ \Sigma F_{Mx} \\ \Sigma F_{My}
\end{bmatrix}
=
\begin{bmatrix}
- \\ -875 \\ - \\ - \\ - \\ 350 \\ - \\ - \\ - \\ 350 \\ - \\ - \\ - \\ 350 \\ - \\ - \\ - \\ 350 \\ - \\ - \\ - \\ 350 \\ - \\ - \\ - \\ -875
\end{bmatrix}
$$

3.2 MATLAB Code

Screenshot of MATLAB code in figure 1. The following was the MATLAB code used to construct the augmented matrix [A| $\overrightarrow{d}$]:

```
A = zeros(26,24);
A(1,1) = 1/2;
A(1,2) = 1;
A(2,1) = sqrt(3)/2;
A(3,1) = -1/2;
A(3,3) = 1/2;
A(3,4) = 1;
A(4,1) = -sqrt(3)/2;
A(4,3) = -sqrt(3)/2;
A(5,2) = -1;
A(5,3) = -1/2;
A(5,5) = 1/2;
A(5,6) = 1;
A(6,3) = sqrt(3)/2;
A(6,5) = sqrt(3)/2;
A(7,4) = -1;
A(7,5) = -1/2;
A(7,7) = 1/2;
A(7,8) = 1;
A(8,5) = -sqrt(3)/2;
A(8,7) = -sqrt(3)/2;
A(9,6) = -1;
A(9,7) = -1/2;
A(9,9) = 1/2;
A(9,10) = 1; A(10,7) = sqrt(3)/2;
A(10,9) = sqrt(3)/2;
A(11,8) = -1;
A(11,9) = -1/2;
A(11,11) = 1/2;
A(11,12) = 1;
A(12,9) = -sqrt(3)/2;
A(12,11) = -sqrt(3)/2;
A(13,10) = -1;
A(13,11) = -1/2;
A(13,13) = 1/2;
A(13,14) = 1;
A(14,11) = sqrt(3)/2;
A(14,13) = sqrt(3)/2;
A(15,12) = -1;
A(15,13) = -1/2;
A(15,15) = 1/2;
A(15,16) = 1;
A(16,13) = -sqrt(3)/2;
A(16,15) = -sqrt(3)/2;
A(17,14) = -1;
A(17,15) = -1/2;
A(17,17) = 1/2;
A(17,18) = 1;
A(18,15) = sqrt(3)/2;
A(18,17) = sqrt(3)/2;
A(19,16) = -1;
A(19,17) = -1/2;
A(19,19) = 1/2;
A(19,20) = 1;
A(20,17) = -sqrt(3)/2;
A(20,19) = -sqrt(3)/2;
A(21,18) = -1;
A(21,19) = -1/2;
A(21,21) = 1/2;
A(21,22) = 1;
A(22,19) = sqrt(3)/2;
A(22,21) = sqrt(3)/2;
A(23,20) = -1;
A(23,21) = -1/2;
A(23,23) = 1/2;
A(24,21) = -sqrt(3)/2;
A(24,23) = -sqrt(3)/2;
A(25,22) = -1;
A(25,23) = -1/2;
A(26,23) = sqrt(3)/2;
A(2,24) = -875;
A(6,24) = 350;
A(10,24) = 350;
A(14,24) = 350;
A(18,24) = 350;
A(22,24) = 350;
A(26,24) = -875;
```

4 Problem 2

The rank of the augmented matrix is r = 23. This was determined by using the MATLAB function rref() and counting the number of rows with leading 1s in the resultant matrix.

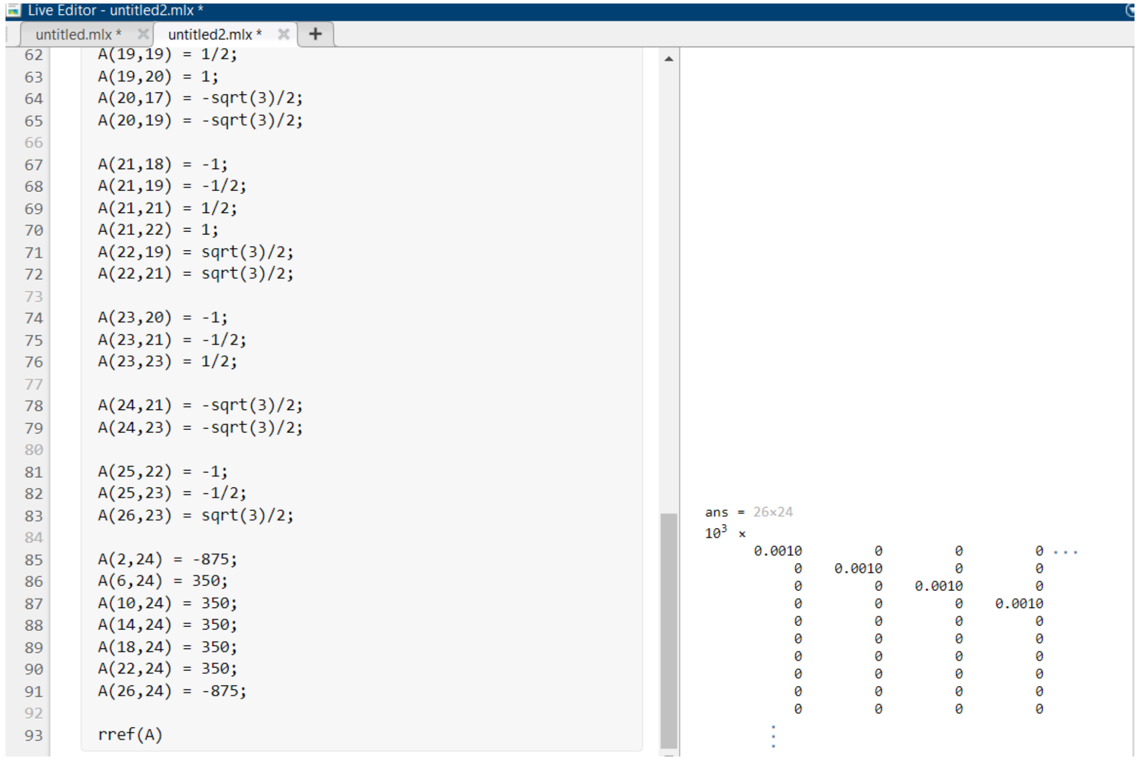

Figure 1: Screenshot of MATLAB code

4.1 RNF

$$
\left[R|\overrightarrow{d}\right] = \left[\begin{array}{ccccccccccccccccccccccc|c}
1 & -1010 \\
- & 1 & - & 505 \\
- & - & 1 & - & 1010 \\
- & - & - & 1 & -1010 \\
- & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 606 \\
- & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1314 \\
- & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 606 \\
- & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & -1617 \\
- & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & -202 \\
- & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & 1718 \\
- & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & 202 \\
- & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & -1819 \\
- & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & 202 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & 1718 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & -202 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & -1617 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & 606 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & 1314 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & -606 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & -1010 \\
- & 1 & - & - & 1010 \\
- & 1 & - & 505 \\
- & 1 & -1010 \\
- & - \\
- & - \\
- & -
\end{array}\right]
$$

5 Problem 3

The solution to this matrix is:

$$\vec{x} = \vec{d} = \begin{bmatrix} AB \\ AC \\ BC \\ BD \\ CD \\ CE \\ DE \\ DF \\ EF \\ EG \\ FG \\ FH \\ GH \\ GI \\ HI \\ HJ \\ IJ \\ IK \\ JK \\ JL \\ KL \\ KM \\ LM \end{bmatrix} = \begin{bmatrix} -1010 \ kN \\ 505 \ kN \\ 1010 \ kN \\ -1010 \ kN \\ 606 \ kN \\ 1314 \ kN \\ 606 \ kN \\ -1617 \ kN \\ -202 \ kN \\ 1718 \ kN \\ 202 \ kN \\ -1819 \ kN \\ 202 \ kN \\ 1718 \ kN \\ -202 \ kN \\ -1617 \ kN \\ 606 \ kN \\ 1314 \ kN \\ -606 \ kN \\ -1010 \ kN \\ 1010 \ kN \\ 505 \ kN \\ -1010 \ kN \end{bmatrix}$$

The vector $\overrightarrow{d}$ of the matrix on the previous page represent the solution to problem 2 of Assignment 5, where each non-zero entry (the first 23 rows of b), represent the member forces in AB, BC, ..., LM, respectively. There are 23 members in the truss, resulting in 23 non-zero rows in vector x. According to the sign convention used to create the augmented matrix $[A|\overrightarrow{d}]$, positive numbers represent that the member is in tension, and negative members represent that the member is in compression. The entries of $\overrightarrow{d}$ are the same as the solutions obtained using traditional truss analysis methods, which is shown in Figure 2.

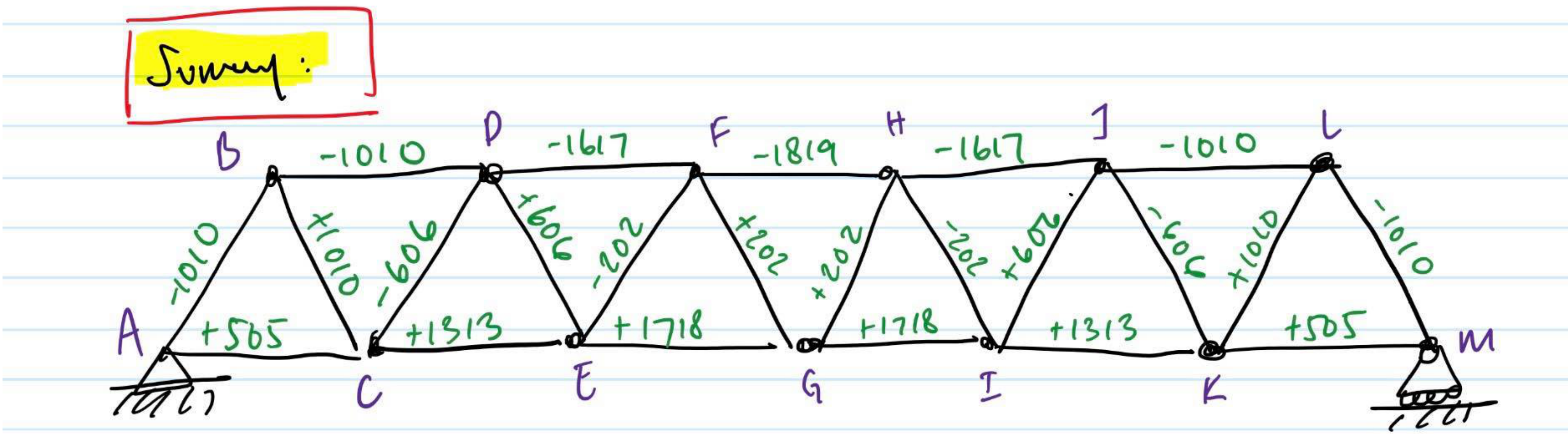

Figure 2: Truss Bridge solved from CIV102 Assignment 5.

6 Problem 4

Three rows can be removed without reducing the determinancy of the system if the three rows do not represent the sum of the forces in the same translational direction. For example, the rows 26, 24, 22 cannot be removed because they all represent reaction forces in the y direction. If they were removed, there will be infinite solutions to the matrix and therefore the truss is statically indeterminate. On the other hand, the rows 1, 2, and 3 can be removed as the solution remains the same because rows 1 and 3 are for x-direction forces, while row 2 is for the y direction. We removed rows 26, 24 and 23 of matrix A, where row 26 was the sum of the y-component of member forces acting on M. Row 24 represented the sum of y-component of member forces acting on L. Row 23 represented the sum of x-component of member forces acting on L. The three corresponding rows in vector x were also removed. The rank and solution to the system were unchanged after the removal of these rows because two of the rows represented forces in the y direction (26 and 24), while one row represented forces in the x direction (23). Intuitively, it is logical that removing three rows that represent forces in the same direction would make the system indeterminate, because there will simply not be enough known member/reaction forces in that direction to solve for all the member forces.

7 Problem 5

7.1 Inverse Matrix

The 26th, 24th, and 23rd rows of matrix A were removed, as well as the 26th, 24th and 23rd elements of vector $\overrightarrow{d}$. After removing the three rows in A. Now A is a 23x23 square matrix $\overrightarrow{d}$ is a 23x1 vector. The modified augmented matrix $[A_r|\overrightarrow{d}_r]$ after removing these rows is:

$$\left[\begin{array}{ccccccccccccccccccccccc|c}
cos60 & 1 & - \\
sin60 & -875 \\
-cos60 & - & cos60 & 1 & - \\
-sin60 & - & -sin60 & - \\
- & -1 & -cos60 & - & cos60 & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & sin60 & - & sin60 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 350 \\
- & - & - & -1 & -cos60 & - & cos60 & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & -sin60 & - & -sin60 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & -1 & -cos60 & - & cos60 & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & sin60 & - & sin60 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 350 \\
- & - & - & - & - & - & - & -1 & -cos60 & - & cos60 & 1 & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & -sin60 & - & -sin60 & - & - & - & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & -1 & -cos60 & - & cos60 & 1 & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & sin60 & - & sin60 & - & - & - & - & - & - & - & - & - & - & 350 \\
- & - & - & - & - & - & - & - & - & - & - & -1 & -cos60 & - & cos60 & 1 & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & -sin60 & - & -sin60 & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & - & -1 & -cos60 & - & cos60 & 1 & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & sin60 & - & sin60 & - & - & - & - & - & - & 350 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & -1 & -cos60 & - & cos60 & 1 & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & sin60 & - & -sin60 & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & -1 & -cos60 & - & cos60 & 1 & - & - \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & sin60 & - & sin60 & - & - & 350 \\
- & -1 & -cos60 & -
\end{array}\right]$$

Originally, A in our MATLAB code represented the augmented matrix $[\mathrm{A}|\vec{d}]$. This augmented matrix had a combined dimension of 26x24. As stated, three rows of this augmented matrix were removed. Additionally, the vector $\vec{d}$ was extracted from this matrix. The resulting matrix A_r was therefore a 23x23 square matrix. The inverse of this matrix was computed using MATLAB's inv(A) function. Below are the modifications on the original augmented matrix (see pg. 7) to arrive at A_r and $\vec{d_r}$. They were multiplied to compute vector $\vec{x}$, which was the solution to the original problem.

B = A(:, 24); created b_r by extracting the column 24 of A

A(26, :) = [ ]; removing the rows 26, 24, and 23 in both A and b_r

A(24, :) = [ ];

A(23, :) = [ ];

B(26, :) = [ ];

B(24, :) = [ ];

B(23, :) = [ ];

A(:, 24) = [ ]; removing column 24 of A as this had become b_r

inv(A) * B use the MATLAB inverse function and matrix multiplication to compute $\vec{x}$

The vector $\vec{x}$ that we obtained using this method was in fact the same as $\vec{x}$ in Problem 3 (see pg. 11).

0	1.154701	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	-0.57735	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1.1547	0	-1.1547	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1.154701	1	0.57735	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1.154701	0	1.154701	0	1.154701	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	-1.73205	0	-1.1547	1	-0.57735	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1.1547	0	-1.1547	0	-1.1547	0	-1.1547	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	2.309401	1	1.732051	0	1.154701	1	0.57735	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1.154701	0	1.154701	0	1.154701	0	1.154701	0	1.154701	0	0	0	0	0	0	0	0	0	0	0	0	0
1	-2.88675	0	-2.3094	1	-1.73205	0	-1.1547	1	-0.57735	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1.1547	0	-1.1547	0	-1.1547	0	-1.1547	0	-1.1547	0	-1.1547	0	0	0	0	0	0	0	0	0	0	0
0	3.464102	1	2.886751	0	2.309401	1	1.732051	0	1.154701	1	0.57735	0	0	0	0	0	0	0	0	0	0	0
0	1.154701	0	1.154701	0	1.154701	0	1.154701	0	1.154701	0	1.154701	0	1.154701	0	0	0	0	0	0	0	0	0
1	-4.04145	0	-3.4641	1	-2.88675	0	-2.3094	1	-1.73205	0	-1.1547	1	-0.57735	0	0	0	0	0	0	0	0	0
0	-1.1547	0	-1.1547	0	-1.1547	0	-1.1547	0	-1.1547	0	-1.1547	0	-1.1547	0	-1.1547	0	0	0	0	0	0	0
0	4.618802	1	4.041452	0	3.464102	1	2.886751	0	2.309401	1	1.732051	0	1.154701	1	0.57735	0	0	0	0	0	0	0
0	1.154701	0	1.154701	0	1.154701	0	1.154701	0	1.154701	0	1.154701	0	1.154701	0	1.154701	0	1.154701	0	0	0	0	0
1	-5.19615	0	-4.6188	1	-4.04145	0	-3.4641	1	-2.88675	0	-2.3094	1	-1.73205	0	-1.1547	1	-0.57735	0	0	0	0	0
0	-1.1547	0	-1.1547	0	-1.1547	0	-1.1547	0	-1.1547	0	-1.1547	0	-1.1547	0	-1.1547	0	-1.1547	0	-1.1547	0	0	0
0	5.773503	1	5.196152	0	4.618802	1	4.041452	0	3.464102	1	2.886751	0	2.309401	1	1.732051	0	1.154701	1	0.57735	0	0	0
1	-5.7735	0	-5.19615	1	-4.6188	0	-4.04145	1	-3.4641	0	-2.88675	1	-2.3094	0	-1.73205	1	-1.1547	0	-0.57735	1	-0.57735	1
0.5	-2.88675	0	-2.59808	0.5	-2.3094	0	-2.02073	0.5	-1.73205	0	-1.44338	0.5	-1.1547	0	-0.86603	0.5	-0.57735	0	-0.28868	0.5	0.288675	-0.5
-1	5.773503	0	5.196152	-1	4.618802	0	4.041452	-1	3.464102	0	2.886751	-1	2.309401	0	1.732051	-1	1.154701	0	0.57735	-1	-0.57735	-1

Figure 3: Screenshot of the inverse matrix from MATLAB

```
B = A(:,24)

A(:,24) = []

inv(A) * B
```

Figure 4: MATLAB code used to obtain the inverse

7.2 Matrix in Reduced Normal Form 23 x 23

$$
[R_r|\overrightarrow{d}_r] = \left[\begin{array}{ccccccccccccccccccccccc|c}
1 & -1010 \\
- & 1 & - & 505 \\
- & - & 1 & - & 1010 \\
- & - & - & 1 & -1010 \\
- & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 606 \\
- & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1314 \\
- & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 606 \\
- & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & -1617 \\
- & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & -202 \\
- & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & 1718 \\
- & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & 202 \\
- & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & -1819 \\
- & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & 202 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & 1718 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & -202 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & -1617 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & 606 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & 1314 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & -606 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & -1010 \\
- & 1 & - & - & 1010 \\
- & 1 & - & 505 \\
- & 1 & -1010
\end{array}\right]
$$

8 Problem 6

$$
\left[\begin{array}{ccccccccccccccccccccccc|c}
1 & -1299 \\
- & 1 & - & 650 \\
- & - & 1 & - & 1299 \\
- & - & - & 1 & -1299 \\
- & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & -779 \\
- & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1689 \\
- & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 779 \\
- & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & -2078 \\
- & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & - & -260 \\
- & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & - & 2208 \\
- & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & - & 260 \\
- & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & - & -2338 \\
- & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & - & 260 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & - & 2208 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & - & -260 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & - & -2078 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & - & 779 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & - & 1689 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & - & -779 \\
- & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & - & 1 & - & - & - & -1299 \\
- & 1 & - & - & 1299 \\
- & 1 & - & 650 \\
- & 1 & -1299
\end{array}\right]
$$

Each entry in the b_r vector of the augmented matrix $[\mathrm{A}_r|\overrightarrow{b_r}]$ is scaled by a factor of $\frac{90}{70}$ or $\frac{9}{7}$. This is because all the reaction forces at joints A and M and the load are all proportional to the magnitude of the uniformly distributed load. To solve the new augmented matrix $[\mathrm{A}_r|\overrightarrow{b_r}]$ without applying Gaussian Elimination again, multiply each element of vector $\overrightarrow{x}$, i.e., each member force, as obtained from Problems 3 and 5, by $\frac{9}{7}$. In other words, the forces carried by each member of the truss is increased by a factor of $\frac{9}{7}$.

The forces in each member of the truss for a uniformly distributed load of 90 $\frac{kN}{m}$ is:

$$\begin{bmatrix} AB \\ AC \\ BC \\ BD \\ CD \\ CE \\ DE \\ DF \\ EF \\ EG \\ FG \\ FH \\ GH \\ GI \\ HI \\ HJ \\ IJ \\ IK \\ JK \\ JL \\ KL \\ KM \\ LM \end{bmatrix} = \begin{bmatrix} -1299 \ kN \\ 650 \ kN \\ 1299 \ kN \\ -1299 \ kN \\ -779 \ kN \\ 1689 \ kN \\ 779 \ kN \\ -2078 \ kN \\ -260 \ kN \\ 2208 \ kN \\ 260 \ kN \\ -2338 \ kN \\ 260 \ kN \\ 2208 \ kN \\ -260 \ kN \\ -2078 \ kN \\ 779 \ kN \\ 1689 \ kN \\ -779 \ kN \\ -1299 \ kN \\ 1299 \ kN \\ 650 \ kN \\ -1299 \ kN \end{bmatrix}$$

9 Problem 7

This problem can be solved by setting up the ratio as stated by equation (1) below. In (1), x is the amount of loading, in Newtons per metre, required to cause member $\overrightarrow{FH}$ to carry a load of 2000kN and 1818kN is the magnitude of the force in member $\overrightarrow{FH}$ under 70kN of uniformly distributed load. Recall that the force in member FH under the current loading was determined in Problem 3. Solving for x in equation (1) yields the magnitude of the uniformly distributed load that is required to create a member force in $\overrightarrow{FH}$ of 2000kN. Any load that exceeds x will therefore cause the force in $\overrightarrow{FH}$ to exceed 2000kN. The solution to this equation is x $= 77.0\frac{kN}{m}$.

$$\frac{x}{70\frac{kN}{m}} = \frac{2000kN}{1818kN} \tag{1}$$

10 What did we Learn through this Assignment?

In this assignment, we used a new approach to solve a common problem in structural engineering - finding the member forces in a truss. Our biggest takeaway from this assignment was realizing that the courses we are taking this term as Engineering Science students are closely linked. From our studies in CIV102, we were already familiar with how to approach and solve problems involving finding the forces in truss bridges. At around the same time, In ESC103, we learned the Gaussian Elimination (GE) algorithm. This tool seemed fairly abstract and did not have any obvious applications. However, after going through the process of constructing an augmented matrix that summarized a truss analysis problem in the CIV102 assignment, two seemingly unrelated problems instantly had a clear link. From going through the calculations in MATLAB, we started to get a hint of the usefulness of GE and matrices in general. It was especially satisfying to see that through transforming the augmented matrix $[\mathrm{A}|\vec{d}]$ to its reduced normal form, we got the same answer for the force in each member as the one we had obtained in solving the truss by conventional physics methods. The variedness in the approaches to the same problem led us to appreciate that there is never a "linear approach" to problem-solving, despite being in a linear algebra course! This assignment gave us many opportunities to work with matrices in MATLAB. As a result, we are now more familiar with and have a better understanding of MATLAB's built-in functions such as rref() and inv(), and methods of slicing columns and rows. Finally, we learned the importance of teamwork and communication when working in a group. In particular, we must ensure the data that we analyze are correct, consistent, and can be verified and reproduced by others.

To conclude, the four main things we took away from this assignment were the interconnectedness of all disciplines of math and sciences, the practical usefulness of GE, some important MATLAB syntax, and the importance of proper communication.

11 Appendix

The following matrices are identical to the matrices shown in its corresponding questions, only presented differently as the zeroes are represented using 0 instead of dashes. This conventional representation was not used in the sparse matrices in the body of the assignment since it was visually confusing and hard to read.

11.1 Problem 1 Original Matrix

$$\begin{bmatrix}
cos60 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
sin60 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-cos60 & 0 & cos60 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-sin60 & 0 & -sin60 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & -cos60 & 0 & cos60 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & sin60 & 0 & sin60 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & -cos60 & 0 & cos60 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -sin60 & 0 & -sin60 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & -cos60 & 0 & cos60 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & sin60 & 0 & sin60 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -cos60 & 0 & cos60 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -sin60 & 0 & -sin60 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -cos60 & 0 & cos60 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & sin60 & 0 & sin60 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -cos60 & 0 & cos60 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -sin60 & 0 & -sin60 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -cos60 & 0 & cos60 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & sin60 & 0 & sin60 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -cos60 & 0 & cos60 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & sin60 & 0 & -sin60 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -cos60 & 0 & cos60 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & sin60 & 0 & sin60 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -cos60 & 0 & cos60 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -sin60 & 0 & -sin60 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -cos60 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & sin60
\end{bmatrix}$$

11.2 Problem 2 RNF

$$
\left[\begin{array}{ccccccccccccccccccccccc|c}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1010\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&505\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1010\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1010\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&606\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1314\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&606\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1617\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-202\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&1718\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&202\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&-1819\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&202\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1718\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&-202\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&-1617\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&606\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&1314\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&-606\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&-1010\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1010\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&505\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&-1010\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{array}\right]
$$

11.3 Problem 6 RNF

$$
\left[\begin{array}{ccccccccccccccccccccccc|c}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1299\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&650\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1299\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1299\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-779\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1689\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&779\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-2078\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-260\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&2208\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&260\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&-2338\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&260\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&2208\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&-260\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&-2078\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&779\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&1689\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&-779\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&-1299\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1299\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&650\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&-1299
\end{array}\right]
$$