An Investigation on the Behaviour of a Simple Pendulum

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December 2020

Contents

1 Introduction

The behaviour of simple pendulums was a topic of scientific and practical relevance for centuries. A simple pendulum is a system consisting of a bob attached to a string, who is in turn attached to a pivot. When given an initial push or an increase in the system's gravitational potential, the pendulum will swing, or oscillate, back and forth. Mathematical models have been developed to predict the behaviour of pendulums. However, these models were developed in purely theoretical conditions, so their validities must be tested. I designed and constructed a simple pendulum to compare the actual behaviour of the pendulum, such as the time it takes to oscillate once (the period), to the theoretical behaviour as predicted by various models. In particular, I tested the quality factor, the amplitude-period relationship, the length-period relationship, and the mass-period relationship of my pendulum. First, it is important to note that all pendulums will experience a decrease in amplitude over time due to drag and friction at the pivot. The change in amplitude over time is predicted by the equation

$$\theta\left(t\right) = \theta_0 e^{-\frac{t}{\tau}} \cos\left(2\pi \frac{t}{T} + \varphi_0\right) \tag{1}$$

In 1, $\theta(t)$ is the angular displacement of the pendulum at time t, θ_0 is the initial amplitude, ϕ_0 is the phase shift, T is the period, and τ, the decay constant, represents the rate at which the system's amplitude damps. Although I have control over θ_0 and ϕ_0, the values of T and τ must be experimentally determined. The quantity that measures the quality of my pendulum, the Q factor, depends on both T and τ by the equation $Q = \pi\frac{\tau}{T}$. In general, a pendulum has a high Q factor if its amplitude decreases slowly over time. The Q factor is an important property because it dictates how further experiments on my pendulum should be conducted. Given the initial setup, my pendulum had a Q factor of 230 with an uncertainty of 0.3 due to the statistical uncertainty of the value of τ. Although this value is expected to change while I change the setup of my pendulum, especially the mass, it is a good indicator that my pendulum decays slowly over time. With the knowledge of Q, I went on to examine how changing various parameters of my pendulum will affect the period. The period equation of the pendulum predicts that the period of a pendulum depends only on its effective string length, l, and the strength of gravity, g:

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{2}$$

To test the accuracy of this model, I recorded the period of my pendulum under different amplitudes while keeping all other parameters constant. My experimental results refuted 2 as the period increased with bigger amplitudes. The period as a function of amplitude could be modelled by a quadratic fit within uncertainties. The fit had the equation $T = 1.31 + \ 0.11\theta^2$. However, for angles smaller than 22°, the changes in period over different amplitudes were smaller than the uncertainties involved in measuring the period. This led to the conclusion that period is independent of amplitude for small angles. In other words, 2 may be valid for small amplitudes. Furthermore, I tested whether changing the string length of the pendulum will affect its period, and if yes, whether 2 is an appropriate model for predicting the period. I found that the period change was apparent and I confirmed the simplification of 2 near the surface of the Earth, where given $g = 9.81ms^{-2}$, period is proportional to double the square root of the effective string length. Finally, I changed the mass of the bob of my pendulum and observed its affect on the period. I fitted a constant period curve over changing masses and found the residuals to be within the measurement uncertainties of the period. Therefore, I predict that period does depend on the mass of the bob.

2 Pendulum Design and Setup

Minimizing the uncertainties and maximizing the Q factor were the two goals when designing my pendulum. Specific design choices were motivated by these goals. First, I chose to build the pendulum in a basement storage room because a small, closed room with no one walking around would minimize air currents which may produce uncertainties in the pendulum's oscillation plane and amplitude. Near the ceiling, there was a wooden plank parallel that had a pre-attached industrial staple. The plank was relatively stable as it was previously used as the basement's roof support system. I put a stiff, thin nylon string approximately 1.7mm in diameter through the staple as the string of my pendulum. The industrial staple therefore acted as the pivot of the system. A light and thin string would be ideal as it would lead to angle measurements with greater certainty and minimize the effects of drag forces on the pendulum system. A light string would also simplify the mass and effective string length calculations, as I can assume the string is massless. The string length of my pendulum was easily adjustable by pulling the other end of the string. The bob of the pendulum was a small bag filled with pennies. This was tied to the free end of the pendulum. To accurately measure amplitudes, I glued a protractor in a plane parallel to the pendulum's oscillations and whose origin was at the same height as the point of contact between the pivot point of the pendulum. I recorded the pendulum oscillations with a 60 frame-per-second camera placed in line with the pendulum string and the zero-degree marking of the protractor. This way, I could play back the motion of the pendulum to analyze its behaviour, greatly reducing uncertainties. Finally, I used a rectangular prism-shaped object to ensure the pendulum oscillated in a plane parallel to the wall, and by extension, the protractor. This was important in accurately determining the amplitude. Therefore, all trials where the pendulum string hit the prism after its release were nullified. The prism's width was around 10mm shorter than the distance between the pendulum and the wall. My pendulum setup allowed me to vary one of the key parameters, such as string length or mass, without affecting the others. Therefore, the same setup was used for all subsequent experiments.

3 Uncertainties and their Values

Measurement uncertainties are recognized and accounted for in the determination and analysis of all results of this report. The uncertainties can be categorized into four categories: angle-related, time-related, string length-related, and mass uncertainties. I will provide estimates for all sources of uncertainties. However, for any given experiment, only the largest uncertainty was expressed in solutions and analyses.

3.1 Angle Uncertainties

there were three uncertainties that had negligible impact on the amplitude: the plane of oscillation, the unstableness of the plank, and the elasticity of the string. These uncertainties were either inherently insignificant, or a conscious design decision had been made to diminish their influences. As a result, all three uncertainties will contribute to less than 10^{-3} radians to the amplitude uncertainty. A much more significant source of angle uncertainty was my inability to determine the exact amplitude due to the calibration limitations of the protractor and the thickness of the string. The protractor was only accurate to one degree and the string was of significant thickness. As viewed from the recording, the string was almost three times as wide as the protractor graduations and around the same width as the gap between consecutive graduations. Since the tenth digit of degree angle measurements were an estimate and sometimes it was impossible to determine the

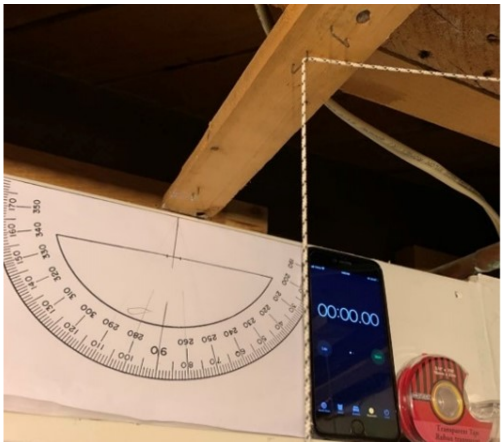

Figure 1: A closeup look at the point of contact between the string and the staple

Figure 2: The box left a 10mm gap of space. It limits the oscillation plane to one that is roughly parallel to the wall

precise position of the string relative to a graduation, I predict that these inaccuracies contributed to up to one degree, or 0.02 radians of angle uncertainty. These uncertainties were represented on vertical and horizontal error bars when plotting amplitude vs. number of oscillations and period vs. amplitude data.

3.2 Time Uncertainties

There were two time-related uncertainties when measuring the period of the pendulum. The first uncertainty was the limitation in the frame rate of the camera, which recorded at a rate of 60 frames per second. Measuring any time interval requires two independent time measurements, the start time and the end time. Therefore, the maximum time uncertainty due to the frame rate was (1/60) x 2 or 1/30 seconds. Another source of time uncertainty, also caused by the frame rate limitation, arose from the behaviour of the bob while the pendulum was at or near its amplitude: for small angles, the pendulum had no observable displacements over two, or sometimes three consecutive frames when it was close to its amplitude. As a result, it was impossible to determine the exact frame at which the pendulum reached its amplitude. For consistency, when the pendulum footage was replayed, the middle of the disputed frames or the first of such frames, depending on how many frames were in question, was used to record the time instant for either the start or end of a time interval. The average time uncertainty for each time measurement was therefore the length of each frame, or 1/60 seconds. Once again, two independent measurements were required for each time interval, so the total uncertainty was 1/30 seconds. To conclude, both the frame rate limitation and the ambiguity on the frame at which the pendulum reached it amplitude produced a time measurement uncertainty of $\pm 1/30$ seconds. When recording period vs. amplitude data, the period was determined for each oscillation. It was therefore equal to the time uncertainty. Vertical error bars of magnitude 0.2s were included on the Period vs. Amplitude plot. In contrast, the process of determining the period uncertainty was not as straightforward for the period vs.

string length and period vs. mass data, mainly because I made a conscious effort to lower the time uncertainty. This will be discussed in greater detail in the second pertaining to period-length relationship. To summarize, I determined the period by measuring the time taken for 10 oscillations and divided the result by 10. The number of oscillations had an uncertainty of zero. According to the uncertainty calculation convention specified in the lab instructions, the uncertainty of the period measurement can be calculated as follows:

$$\left(\frac{t(10) \pm \frac{1}{30}}{10 \pm 0}\right) s = \frac{t(10)}{10}\ s \pm \frac{130}{t(10)} \frac{t(10)}{10}\ s \ = \ \frac{t(10)}{10}\ s \pm 0.003\ s \tag{3}$$

where $t(10)$ is the measured time interval for 10 oscillations. The uncertainty effectively decreased by a factor of 10 by measuring 10 oscillations.

3.3 Length Uncertainties

There were multiple uncertainties when measuring the length of the pendulum, L. Most of these uncertainties were extremely small and therefore need not to be considered. These include the inaccuracy of the measuring device and my inability to hold the measuring tape straight. Additionally, the string lengths may be slightly different depending on the angular displacement of the pendulum since the force of tension in the rope was not constant. Due to the relative short length and the small angle condition, these three length uncertainties had negligible contributions to the change in the string length. As a conservative estimate, they each contributed to up to $0.1cm$ of length uncertainty. The most significant length uncertainty was caused by inaccuracies in the estimation of the bob's centre of mass. The dominant component of the bob's mass, which was the collection of pennies, stacked to a height of approximately 6cm. It was difficult to pinpoint the exact position of the centre of mass due to the non-regularity of the group of pennies. A best estimate was made by measuring approximately half way up the stack of pennies. Note that the mass of the bag, which was $8g$, was ignored during the length measurement since the pennies contributed to 95% of the mass. I estimate that the inaccuracies in locating the centre of mass contributed to up to $1cm$ of length uncertainty. Therefore, the largest length uncertainty was $\pm 1cm$. After confirming the model of the relationship between period and string length, uncertainties in the length would also contribute to uncertainties in the period. With a length uncertainty of $\pm$1cm, the period uncertainty would be approximately $0.013s$ according to the relationship $T = 2\sqrt{L}$ a string length of around $50cm$.

3.4 Mass Uncertainties

Mass uncertainties were relevant only when examining the relationship between period and mass. The only source of mass uncertainty was due to the inaccuracy of the kitchen scale, which was accurate to the gram. Therefore, the mass uncertainty was $\pm 1g$ for each mass measurement.

4 The Q Factor

I first investigated how quickly my pendulum decayed over time. This could be quantitatively determined by calculating the quality (Q) factor of the pendulum. I released the bob at an initial amplitude of around 34° or 0.593 radians. I continued the recording until the amplitude damped to clearly below 20% of the initial amplitude. I replayed the footage frame-by-frame to determine the pendulum's amplitude at every oscillation. and stopping when the amplitude damped to less than 20% of the initial. I plotted the data in an amplitude vs. oscillation graph.

4.1 Summary of Data

Figure 2.1 shows the relationship between the number of oscillations and the amplitude at each oscillation. I plotted 154 complete oscillations, which was the number of oscillations it took for the amplitude to damp to definitively less than 20% of the initial amplitude. One point, representing the amplitude, was plotted for every oscillation. Error bars were placed for the amplitude measurements. The magnitude of the error bars was 0.02 radians, determined by the greatest angle measurement uncertainty (see Uncertainties and their Values). There were no horizontal error bars because the number of oscillations was determined with certainty. As an arbitrary sign convention, I measured all amplitudes from the counter clockwise direction and assigned them positive values.

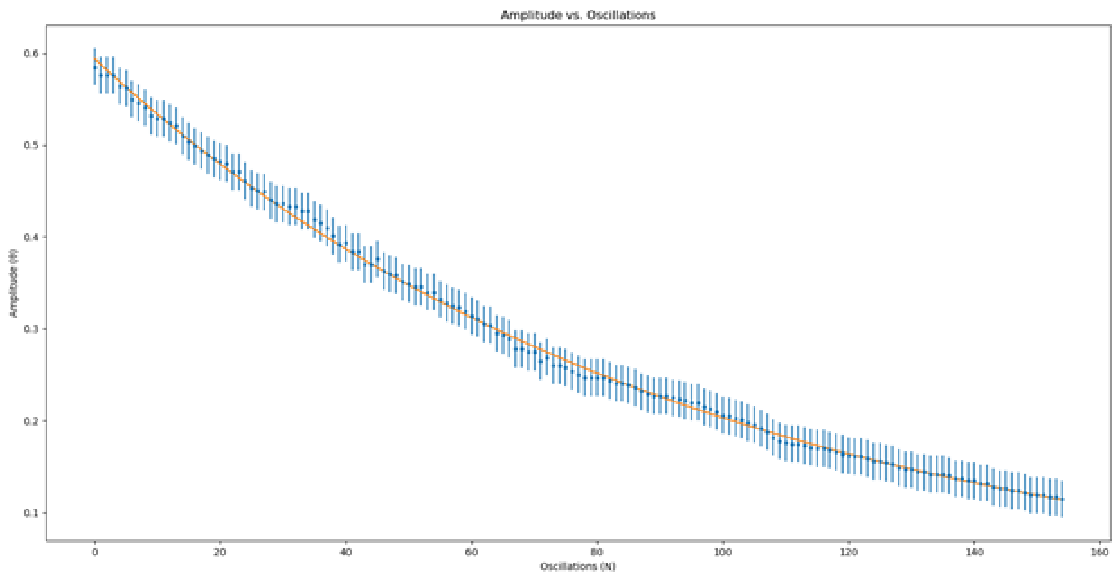

Figure 3: Amplitude vs. Oscillations plot.

Figure 4 shows the decay trend of the amplitude over 154 oscillations. The orange line of best fit is modelled by

$$A(N) = A_0 e^{-\frac{NT}{\tau}} \tag{4}$$

, where $A(N)$ is the amplitude at the Nth oscillation, T is the period, and τ is the decay constant. This equation is a special version of 1: $\theta(t) = \theta_0 e^{-\frac{t}{\tau}} \cos\left(2\pi\frac{t}{T} + \varphi_0\right)$ (see 1 for details), where the fundamental relationship $t = NT$ was substituted for t in 1. The term $\cos\left(2\pi\frac{t}{T} + \varphi_0\right)$ is always 1 at any whole number of oscillations, assuming there were no initial phase offset, hence arriving at 4.Vertical error bars were placed for each amplitude measurement to represent the largest measurement uncertainty. The curve of best fit had an initial amplitude, $A_0 = 0.59 \pm 0.0005$ and $\tau = 93.2s \pm 0.1s$.

4.2 Determination of Q

I will use two means to determine Q, 4.2.1 by using the number of oscillations and 4.2.2 by using the Q factor formula.

4.2.1 Method 1

Q/2 is the number of oscillations that is required to decrease the pendulum's amplitude to 20% of the original. The initial amplitude was $0.585rad$. $0.585\dot{2}0\% = 0.117rad$. However, it was impossible to determine with certainty the exact oscillation at which the amplitude was $0.117rad$. However, I am certain that it occurred between the 152nd and 153rd oscillation. Therefore, $N = 152.5 \pm 0.5$. By definition:

$$\frac{Q}{2} \equiv N \tag{5}$$

$$Q_{(1)} = \mathbf{305 \pm 1} \tag{6}$$

4.2.2 Method 2

Use the formula $Q = \pi \frac{\tau}{T}$, where tau is the decay constant and T is the period. According to the Python program, tau was calculated to be $93.2s \pm 0.1s$. I calculated the average period by dividing the total time taken for the 154 oscillations by 154. This calculation assumes that the period either stays constant or changes linearly over the oscillations. Given this assumption, the uncertainty of the period is reduced to effectively zero since the number of oscillations is a large value with no uncertainty (see 3 section for a more detailed explanation).

$$T_{av} = \frac{t_{total}}{N} = \frac{195.85s \pm 0.02s}{154} = 1.27s \pm 0.0s = 1.27s \tag{7}$$

$$Q_{(2)} = \pi \frac{\tau}{T} = \pi \frac{93.2s \pm 0.1s}{1.27s} = \mathbf{230 \pm 0.3} \tag{8}$$

The Q factor calculated using method (1) and method (2) both had a relatively small uncertainty, at 0.33% and 0.13% relative to their magnitudes, respectively. However, the two calculated Q factors differed significantly from each other. This was quite a perplexing result since method (1) was an extension of the definition of the Q factor provided by method (2). However, I assumed that method (2) was a more reliable way of calculating Q since it directly employed its definition. I also have confidence in the τ value since it was used to fit the actual change in amplitude of my pendulum. I expected my Q factor to be relatively large. My calculated Q seemed to confirm my expectations. A large Q factor means that, for any given initial amplitude, I could expect the system to damp at a slow rate. This will have a direct impact on how I will gather period vs. amplitude data: the amplitude change over consecutive oscillations is expected be very small as suggested by the high Q factor. This would allow me to measure the period using one full oscillation instead of half or quarter oscillations. For each period measurement, I will record the amplitude as the angle measurement prior to the oscillation.

5 Period vs. Amplitude

Ideally, one would want to measure all ranges of amplitudes: starting from 90° all the way down to 0° and see how the period varies with all possible amplitudes. However, my pendulum was attached to the ceiling, so its amplitude was limited to around 80°. Therefore, I recorded my largest data at 80° amplitude. My pendulum setup was as described in the Methods section.

5.1 Method of Period vs. Amplitude Data Collection

From the view of the camera, I chose the counter clockwise angular displacements as positive amplitudes. That is, a pendulum swung to the right of its equilibrium position was considered as positive. This assignment was completely arbitrary and whose sole purpose was to distinguish between amplitudes measured at the two different sides, which becomes significant when examining the symmetry of my pendulum. I measured the period at approximately 80°, 67°, 50°, 35°, 10°, and 2.5°. The amplitudes were chosen with the consideration for an expansive range and even spacing of angles, as opposed to amplitudes clustered near one angle. This decision followed that the purpose of this lab was to measure the change in period over all possible amplitudes. I released the pendulum at just over 80°. I recorded the pendulum's amplitude at all six angles. For each amplitude, the period was the time taken for the pendulum to start at a certain height and return to its maximum height. The second height will be slightly lower than the first, but due to the relatively high Q factor, I assumed that they were the same and used the initial amplitude measurement. I repeated this procedure three times and played the videos back, giving me three amplitude and period pairs for each angle. I took the average of these trials and recorded the final data as amplitude and period.

5.2 Asymmetry

To test for the symmetry of my pendulum, I also recorded period and amplitude data from the negative direction using the same method. I recorded the period for roughly the same magnitudes of amplitude, only negative. Like the positive amplitudes, I recorded six period and amplitude data pairs from the negative direction. If my pendulum were symmetric, the period should not depend on the side with which I measured the amplitude. This result seems inevitable for a well-designed pendulum, but needs to be rigorously tested nonetheless. Symmetry will be further discussed in the next section.

5.3 Representation of Period vs. Amplitude Data

I used a modified version of the fitting.py Python program to generate a graph of my data and its curve of best fit. I fit my graph using a quartic function in the form $T = A + B\theta + C\theta^2 + D\theta^3 + E\theta^4$.

The equation for the quartic fit was $T = 1.31 + 0.0095\theta + 0.1082\theta^2 + 0.0024\theta^3 + 0.0031\theta^4$. The statistical uncertainties for the terms were ± 0.0028, ± 0.0053, ± 0.0085, ± 0.0035, and ± 0.0043, respectively. Horizontal and vertical error bars representing the greatest measurement uncertainties were placed at each data point. Qualitatively, the curve appeared to be a good fit for the data since all period measurements were within one error bar of the fit. Although the first-degree coefficient was not zero, it was deemed to be consistent with zero since its magnitude was less than twice its uncertainty. Additionally, the third-degree coefficient was experimentally zero since its magnitude was less than its uncertainty. From these, I concluded that my pendulum was symmetric within uncertainties. Simply put, the period of the pendulum was not affected by the direction of amplitude measurement. The second-degree term, however, was not consistent with zero. This implies that the period of the pendulum was dependent on its amplitude. As the amplitude decreased, the period of the pendulum decreased in what could be modelled as a quadratic relationship (although I used a quartic model, the coefficient of the highest degree term that was not consistent with zero was the second-degree term). However, for relatively small amplitudes, we can assume that the period is independent of the amplitude. This was confirmed experimentally as measuring the period at 2.5° and 10° both resulted in $T = 1.31s \pm 0.02s$. Now I will determine quantitatively the range of amplitudes for which the period may be approximated as constant and independent of the amplitude. The constant period approximation is accurate for all angles less than one period uncertainty, as in the largest measurement or statistical uncertainty, from the period as amplitude

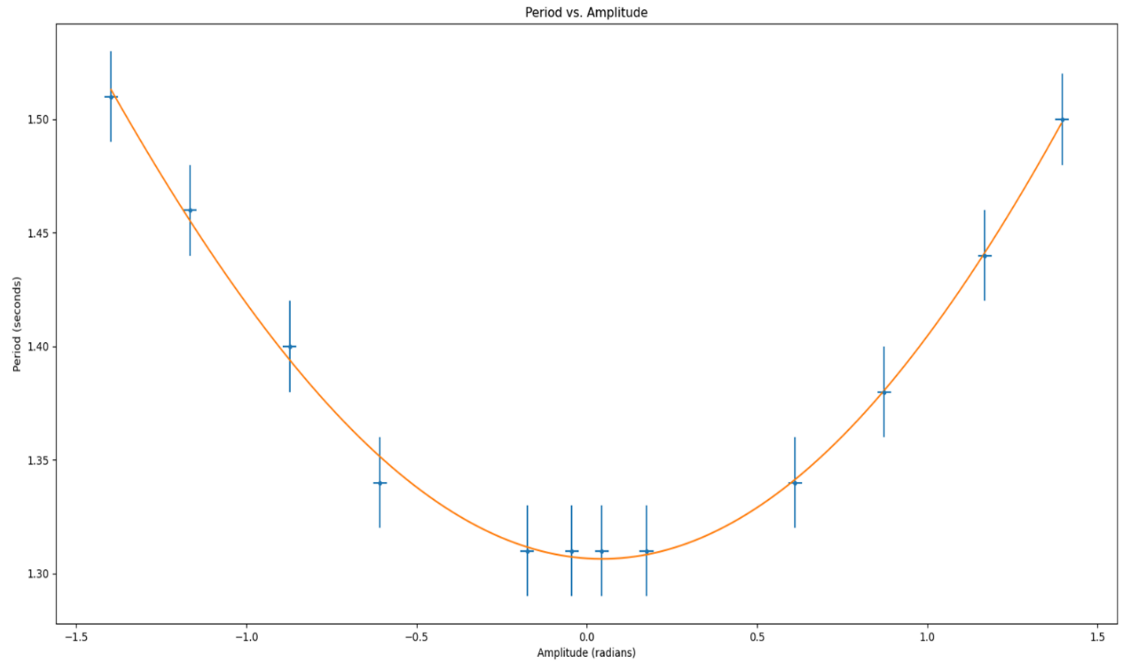

Figure 4: Period vs. Amplitude plot.

approaches 0. We can represent this mathematically as:

$$T_0 + k > T_0 + B\theta + C\theta^2 + D\theta^3 + E\theta^4 \tag{9}$$

where k is the largest period uncertainty and T_0, B, C, D, and E are the coefficients of the fitting function. The value of k was discussed in the Uncertainties and their Values section to be 1/60 seconds. The statistical uncertainty for the period was 0.0028s, which was not considered since it was smaller than the measurement uncertainty.

Also, the values of B, D, and E were either experimentally zero or consistent with zero. Therefore, we may simplify the equation to obtain $k > C\theta^2$. Solving for θ:

$$\theta < \sqrt{\frac{k}{C}} = \sqrt{\frac{1/60}{0.11}} \tag{10}$$

$$\theta < 0.39\ rad = 22^\circ \tag{11}$$

Therefore, for my pendulum and uncertainties in my experiment, I may approximate the period to be independent of the amplitude for angles less than 22°, because the uncertainty would overcast the inaccuracies of the constant period approximation. This is equivalent to saying that the C value may be ignored for such a range of amplitudes.

6 Period vs. Length (a) and Period vs. Mass (b)

6.1 Measuring Period vs. Length Data

I varied the effective string length of the pendulum and recorded the corresponding period data. The effective string length, or simply length, is defined as the distance between the pivot point of the pendulum to the centre of mass of the bob. The bob was a bag filled with around 80 pennies. Together they weighed 162g ± 1g. When measuring how period varied with different lengths, all the other pendulum properties, especially the mass of the bob, were kept constant. As a result, all changes in the period must be accounted for by the change in length. I recorded period data for string lengths of $25cm, 40cm, 55cm, 70cm, 85cm$, and $100cm$. To determine the period at any given length, I recorded the time it took for the pendulum to undergo 10 oscillations at angles ranging from six to eight degrees. This measured value was then divided by 10 to yield the period of the pendulum for each length. Mathematically, $T = \frac{t(N)}{N} = \frac{t}{10}$, where t(N) is the time taken for N oscillations, in seconds, and T the period, in seconds.

6.2 Discussion and Justification of the Measurement Method

When measuring the period of the pendulum, the primary objective was to minimize the uncertainty. This would help us better understand the direct relationship, if any, between the two variables in question – period and length. The period uncertainty was reduced by calculating the average period through multiple oscillations. This was effective since by recording the time taken for multiple oscillations, the time uncertainty did not change, as two time measurements were required regardless of the length of the time interval. The uncertainty on number of oscillations also did not change; it is always zero. The general form of the period uncertainty is therefore $\frac{t(N)\pm\ 1/30}{N\pm 0}$, where N is the number of oscillations and $t(N)$ is the time required for N oscillations, in seconds. Following the conventions in computing the quotient of two variables each with their own uncertainty, this expression simplifies to $\frac{t(N)}{N} \pm \frac{130}{N}$ or $T \pm \frac{130}{N}$, where T is the period in seconds. It is obvious that the uncertainty decreases if more oscillations, N, were measured at once. However, determining the period through measuring the time interval over multiple oscillations is only justified when period does not change over these oscillations. Since the pendulum had a finite Q factor, this must mean that the period does not change over a range of amplitudes. The major conclusion of the period vs. amplitude investigation (see Lab 3) was that my pendulum's period was independent of its amplitude (within uncertainties) for angles smaller than 22°. However, this value was determined based on the time uncertainty when T = 1.31s, which was ±0.02s. Since the uncertainties had changed for this lab, the angle for which a constant period approximation is valid must be recalculated. The largest period uncertainty for this setup was 0.003s. I will assume the C value remains unchanged to get an estimate of an acceptable starting angle. Using the equation $\theta < \sqrt{\frac{k}{C}}$ (Lab 3), where k is the period uncertainty in seconds and C is the second-degree term of the quartic fit, we get $\theta < 10.1°$. To account for any changes in C from the one calculated in Lab 3, a maximum of 8° was allowed for the period measurements. Additionally, the Q factor of my pendulum was determined to be 290, which was relatively high. This meant that the amplitude would change relatively slowly over multiple oscillations, especially at small angles. This was indeed true as the angle decreased by no more than 2° over 10 oscillations.

To conclude, given initial conditions and uncertainties, the period-amplitude independency is a valid assumption for angles less than or equal to 10°. By dropping the bob at angles smaller than $8° < 10°$, I could measure multiple oscillations and divide by the number of oscillations to obtain the average period. Based on my analyses, the average period should be consistent (within uncertainties) with the actual period of the pendulum and the period during each oscillation. The reason for recording time data over multiple oscillations was to minimize the period uncertainty.

6.3 Analysis Presentation of Data

I acquired six period-length data pairs with lengths of 25cm, 40cm, 55cm, 70cm, 85cm, and 100cm. I fit the six data points to

$$T = k(L_0 + L)^n \tag{12}$$

where T is the period in seconds, L is the effective string length in metres, L_0 is any consistent mismeasurement of L in metres, and k and n are fitting constants. Error bars were placed to represent measurement uncertainties. The horizontal error bars represent the largest length uncertainty, $\pm$1cm. The vertical error bars represent the largest period uncertainty, $\pm$0.003s. My experimental results were consistent with the expected parameter values of equation 12 within uncertainties. Below were the values and their statistical uncertainties:

$$\begin{aligned} k &= 2.0201 \pm 0.02543 \\ L_0 &= -0.0033m \pm 0.0279m \\ n &= 0.5094 \pm 0.0287 \end{aligned}$$

The values of k, L_0, and n were expected to take on values of 2, 0, and 0.5, respectively. All three values specific to my pendulum were within uncertainties from these expected values. Therefore, the data I collected regarding the relationship between period and effective string length agree with equation 12 within uncertainties. There exists a relationship between the period and length of my pendulum. Furthermore, this relationship could be modelled by the equation $T = 2\sqrt{L}$. This is a simplification of 2: $T = 2\pi\sqrt{l/g}$. Figure 5 shows the plot of the period vs. string length data.

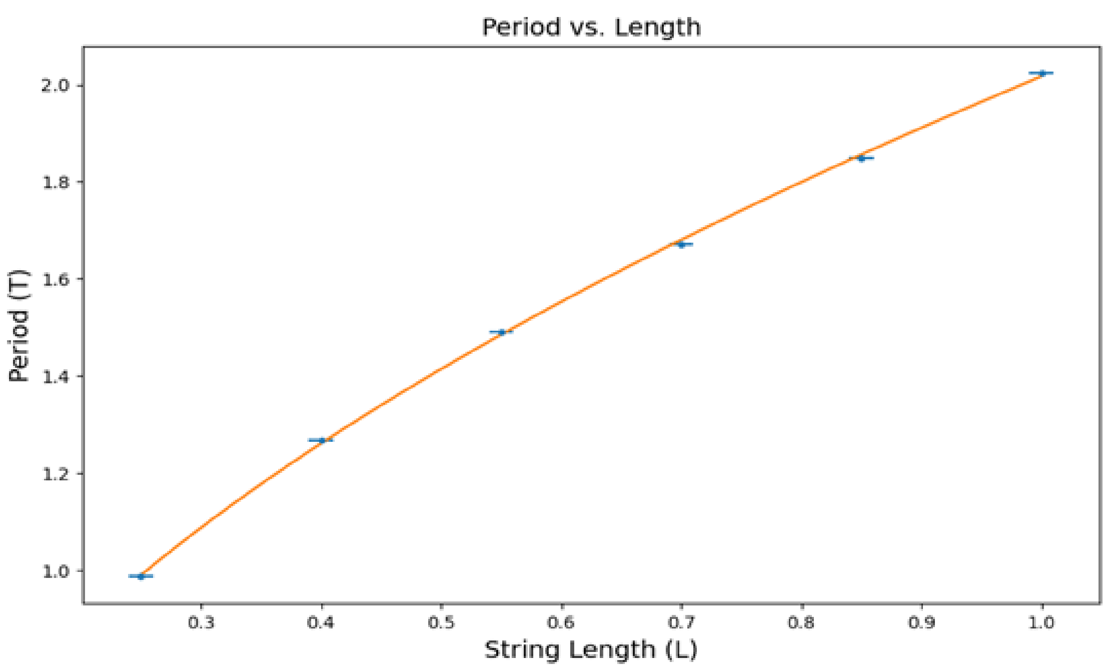

Figure 5: Period vs. String Length plot. Period is proportional to the square root of the length by the proportionality factor of 2 within uncertainties. The string length was measured in metres.

Measuring Period vs. Mass Data For the second part of the investigation, the period was measured over a wide range of masses to observe the relationship, if any, between the pendulum's period and the mass of the bob. To prevent variables other than the mass from affecting the period, all other properties of the pendulum, especially its effective string length L, were kept constant. For each period measurement, the length was taken to be $55cm$. The bob of the pendulum consisted of a bag filled with various number of pennies. A kitchen scale was used to weigh each mass. Initially, four pennies were placed in the bag and, along with the bag, were weighed to give a total mass of $24g$. Pennies were added to the bag such that each subsequent bob mass was approximately double the previous mass. Finally, 160 pennies were placed in the bag which resulted in a bob mass of $384g$. I chose an exponential growth of mass to efficiently analyze a wide range of masses while taking few data points. This would help me quickly observe any relationship between period and mass. Due to the changing of the centre of mass and stretching of the string from increasing the mass, the length of the string was adjusted and re-measured for each mass to ensure the length stayed constant at $55cm$. The period was measured using the same technique as in the period vs. length investigation.

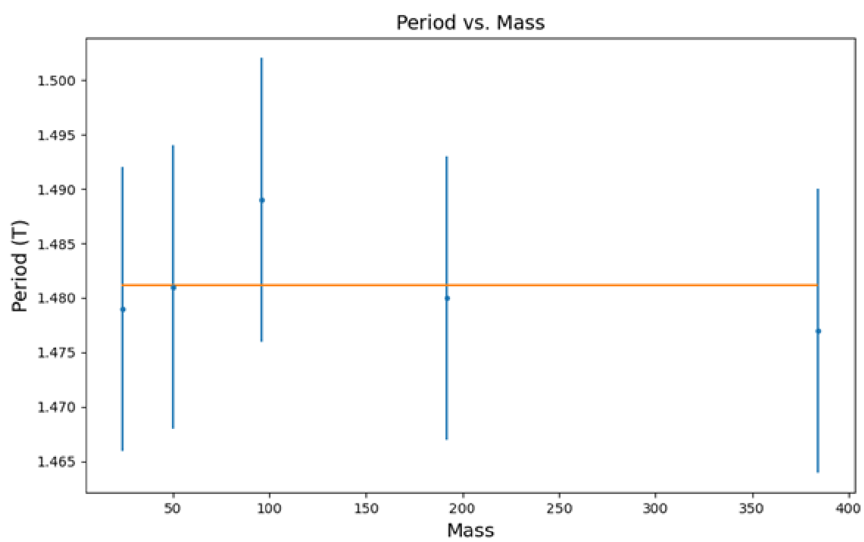

Figure 6: Period vs. mass data was fitted to a constant, base period. This period was $1.481s \pm 0.002s$. Period was measured in seconds and mass was measured in grams.

Since there were no definitive pattern of how the period varied with mass and almost all such variations were explained by the measurement uncertainty of the period, it could be concluded that the period of my pendulum is constant over changing masses. In other words, period is independent of mass.

7 Discussion on Means of Improving the Experiment

A source that may have contributed to the inaccuracy for the period vs. amplitude plot was the amplitude measurements for large angles. As predicted by the exponential decay model and confirmed by empirical evidence, the amplitude decreased quickly for large angles. Empirical evidence suggested that, for angles near 80°, the amplitude damped by up to 2° per oscillation. However, this was not reflected in my amplitude data – I used the simple approximation of consistently recording the amplitude prior to each oscillation. I made this decision because I solely considered the Q fac-

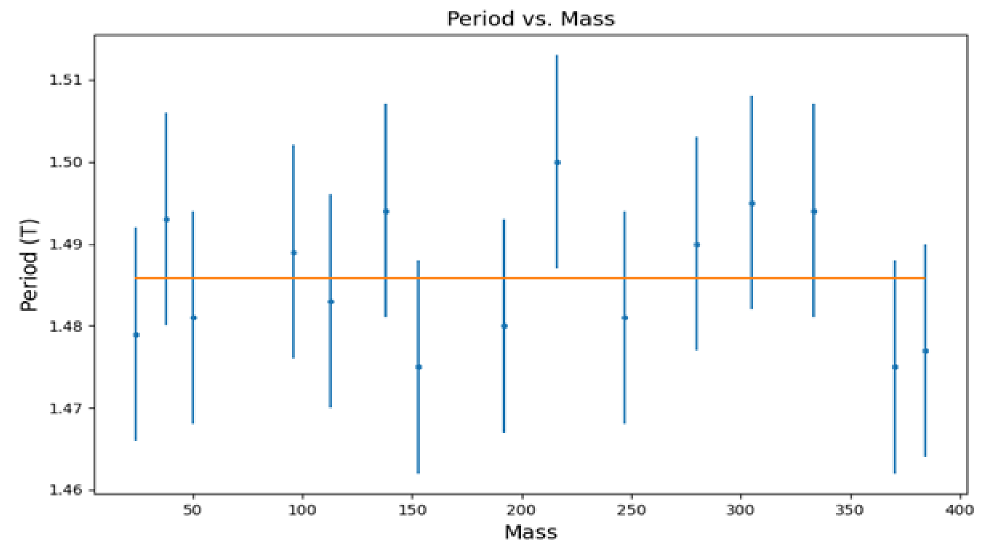

Figure 7: Period vs. mass data for 15 masses. The line of best fit was $T = T_0$, where T_0 is the period of the pendulum over small angles at $L = 55cm$. Here, $T_0 = 1.486s \pm 0.002s$. The predicted period is $T = 2\sqrt{L} = 2\sqrt{0.55} = 1.483s$. The two values are consistent within the largest period uncertainty, which was $0.013s$.

tor, which was relatively large, when I made the decision of the method of measurement. To reflect a more accurate period measurement, I should have instead measured the time for a half oscillation and multiplied the resulting value by two. This would have been a more accurate representation of the period at the recorded amplitude. Additionally, for future investigations, a step forward would be to develop equations to model the pendulum's period under varying amplitudes. Results from this experiment merely refuted their independency and recommended a quadratic relationship, but more could be done to determine the precise relationship between period and amplitude.

Period is independent of mass within uncertainties. All the variance in period could be explained by the uncertainties in measuring the string length. Consequently, if the length uncertainty was decreased by employing a better measurement technique, the period is expected to be constant over changing masses. One such technique could be using a software to accurately determine the centre of mass of the bob. This would be very helpful since this was the greatest length uncertainty, which also contributed to significant period uncertainty.

The range of masses recorded could be increased to observe the affects, if any, of extremely large or small masses. This was not done for my pendulum because I anticipated significant air resistance if I had removed all the pennies in the bag, and my pendulum was not designed to withstand high masses. However, I predict that there would still not be any relationship between the period and the mass.

8 Conclusion

Pendulums exhibit fascinating behaviours, many of which may seem unintuitive to the average person. This report concluded that the period of a pendulum is independent of its amplitude for small angles, confirmed the square root proportionality between the period and the effective string length of a pendulum, and confirmed the period-mass independency model of a simple pendulum.