

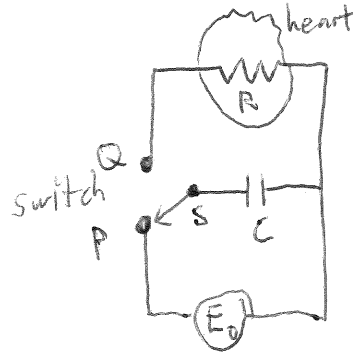
Graded Homework 2

Solutions (not just answers) to the following problems are due on Wednesday, February 13, by the end of the class period.

1. Suppose a tank of salt water with a concentration of 35 g/L has salt water with a concentration of 15 g/L pumped in the rate of 3 L/min, and the well-mixed result is pumped out at 2 L/min. Suppose the tank has an open top and has a total capacity of 400 L.
 - (a) When will the tank overflow?
 - (b) What will be the number of grams of salt in the tank at the instant it overflows?
 - (c) Suppose that salt water continues to be pumped in even while the tank is overflowing. If the inflow occurs at the bottom of the tank, make a conjecture for the amount of salt in the tank as $t \rightarrow \infty$. If the inflow occurs at the top of the tank, does your conjecture change?

2. Suppose an RC -circuit has a resistance at time t given by $\sin(t)$ and an electromotive force at time t given by $\cos(t)$. Find the charge $q(t)$.
3. Two chemical A and B are combined to form a chemical C . The rate of reaction is proportional to the product of the instantaneous amount of A and B not converted to C . Initially there are 40 grams of A and 50 grams of B . For each gram of B , 2 grams of A is used. It is observed that 10 grams of C is formed in 5 minutes.
- (a) How much of C is formed in 20 minutes?
- (b) What is the limiting amount of C ?
4. A heart pacemaker, shown in the pictures, consists of a switch, a battery, a capacitor, and the heart as a resistor. When the switch S is at P , the capacitor charges; when S is at Q , the capacitor discharges, sending an

electrical stimulus to the heart.



- (a) Assume that over the time interval $0 < t < t_1$, the switch S is at position P . When the switch is move to position Q the capacitor discharges, sending an impulse to the heart over the time interval $t_1 \leq t < t_1 + t_2$. Thus

$$\frac{dE}{dt} = \begin{cases} 0 & 0 \leq t < t_1 \\ -\frac{1}{RC}E & t_1 \leq t < t_1 + t_2 \end{cases}$$

By moving S between P and Q , the charging and discharging over the time intervals of lengths t_1 and t_2 is repeated indefinitely. Given $t_1 = 4$ seconds, $t_2 = 2$ seconds, $E_0 = 12$ volts, $E(0) = 0$, $E(4) = 12$, $E(6) = 0$, $E(10) = 12$, $E(12) = 0$, and so on, find $E(t)$ for

$$0 \leq t \leq 24.$$

(b) If $R = C = 1$, plot your solution for $E(t)$ from part (a).