Quiz 9

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In order to prove the maximum flow is equal to the minimum cut capacity, I first prove that minimum cut capacity is greater and equal to the maximum flow capacity. Considered any cut of the basic graph, any flow must pass through the edge of cut. So we have the total flow which equals the sum of capacity of all forward edges minus sum of all capacity of the backward edges. So the total flow must less and equal to the sum capacity of forward edges, which is the capacity of any cut (forward edge means flow from source to target and backward capacity means from target to source). Since this property is true for every flow and every cut, the maximum possible flow must be less and equal to capacity of minimum cut.

We have already prove that maximum flow is less and equal to minimum cut capacity. In order to prove the equality, what we have to do is to show that there exists a cut C such that capacity of C equals to the maximum flow. We first find the maximum flow in the graph network. We construct a cut with set X which is vertices that can be reached from source S through flow augmenting path and set Y which is all the other vertices. In order to prove this cut is equal to the maximum flow, we have to prove the following 2 properties.

First of all we have to prove that what we have define is a cut, which means source and target are in different sets. By the way we have defined the set X, source is always included in X. Suppose target is also included in X. Based on the definition of set X, there exists a flow augmenting path from source to target which mean our flow is not maximum. As a result, if our flow is maximum flow, target will not be included in X.

Afterward, we have to show the total flow over cut is equal to the capacity of the cut. Since capacity of the cut equals to the sum of the capacity of all forward edges. We want to show that each forward edge e in cut C is equal to forward edge e in maximum flow and the backward edges equal to 0. We first select an arbitrary forward edge which connects set X and Y. Since it connects the set X and Y, there is a flow augmenting path from S to edge's source vertex. If the flow of edge in cut is not equal to the flow of same edge in maximum flow. Then, there must be a flow augment path from source to the target vertex, which contradict to the set we have defined above. So the flow of cut equals to the forward edges of maximum flow.

Then we show that each backward edge e in cut is equal to 0. We select an arbitrary backward edge which connected two set we defined above. The target of edge must be in set X based on what we defined. If the flow of this edge is not equal to 0, then there will be a flow augmenting path from source to the source of this edge. This will also contradict what we defined above.

Based on the two fact we have proved, the capacity of the cut equals to the sum of the capacity of maximum flow. Since the capacity of the cut is always greater and equal to the maximum flow, the capacity of minimum cut is equal to the capacity of maximum flow.