Bidirectional Causality between Energy Consumption and Economic Growth

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1 Introduction

The relationship between energy consumption and economic growth has long been discussed by economists. It has been shown by established studies that bidirectional causality exists between these two factors, and can lead to the endogeneity problem in statistics. A common way to solve this problem is to use instrumental variable. However, in most cases, it is hard to find a satisfying instrumental variable. A new method is suggested by this study to identify the effects on the bidirectional causality between energy consumption and economic growth independently. With moderator variables on both effects, ratio model is established, and proved to be effective in the simulated data. Comparing the results of traditional model and ratio model with the real data from World Bank, this study suggests the possible misrecognition of the effects from economic growth to energy consumption.

2 Literature Review

Various researches have been conducted to explore the relationship between energy consumption and economic growth after the end of the 1970s energy crisis. The main conclusion is that there is a positive relationship between economic growth and energy consumption. A positive impact of energy consumption on economic growth is proved to be existed. [KK78] [EH84] [CWCZ08] [KA15] [Gha12] [LS10]. Similarly, studies on Israel, Kuwait, Oman, Saudi Arabia and Syria also stated that reductions in energy consumption will cause economic recession. [SK16] [MM07] [AB10] [NS10] [NS09] [Sad12a] This bidirectional causality between GDP and energy consumption is also proved to exist in both the short run and long run. Granger test is the most common method used to verify the bidirectional causality. [KK78] [EH84] [CWCZ08] [LS10] [AB10] [Sad12a] [Yan00] [Sad11] [Sad12b] [AP10]

However, the bidirectional causality between energy consumption and economic growth can lead to a statistical problem. Since both of these two factors can influence each other, covariance between error term and independent variable in either direction cannot be zero. This is known as the endogeneity problem in statistics and leads the model estimate to a bias and inconsistent result. In 2010, Antonakis et al. described the situation that the endogeneity was caused by the bidirectional causality as simultaneity. [ABJL10] The most common approach to deal with simultaneity issues is the use of "instruments" to purge the endogenous predictor variable from bias. Instruments are exogenous sources of variance in the explanatory variable that do not correlated with the error term of the outcome. For example, in 2019, to study the car pride and its bidirectional relations with car ownership, Moody et al. identified instrumental variables for both car pride and household car ownership. However, in most of case, a satisfying instrument is hard to find. Even in the study of Moody, he also admitted that the IV he used was imperfect. [MZ19]

3 Approach

1. Model

First, we start with a simple binary model with variables X and Y. To be specific, they are respectively economic growth and energy consumption in this study. Suppose there exists a bidirectional causality between X and Y with a linear form:

$$\begin{cases} Y = \beta_0 + \beta_1 X + e_1 \\ X = \alpha_0 + \alpha_1 Y + e_2 \end{cases} \tag{1}$$

Namely, in the form of first order difference, we have:

$$\begin{cases} \Delta Y_{en} = \beta_1 \Delta X_{ex} \\ \Delta X_{en} = \alpha_1 \Delta Y_{ex} \end{cases} \tag{2}$$

where α_1 and β_1 are respectively the effect from Y to X and from X to Y.

Traditionally, to estimate β_1, we fit a linear regression model in the form of:

$$Y = A + BX + C + e \tag{3}$$

where A and B are respectively the intercept and the coefficient of X. $C = \sum_{i=1}^{n} C_i Z_i$, where Z_i are control variables, and C_i are the corresponding coefficients.

However, as the existed studies shown above, due to the bidirectional causality between X and Y, B is a biased estimate for β_1. To show how B bias estimates β_1, we consider the increment of X and Y.

Suppose within the same period, X has exogenous increment τ_1, and Y has exogenous increment τ_2. We assume that all of the residuals in the model comes from these exogenous increments, so the exogenous increments of X and Y are respectively:

$$\begin{cases} \Delta X_{ex} = \tau_1 + \varepsilon_1' \\ \Delta Y_{ex} = \tau_2 + \varepsilon_2' \end{cases} \tag{4}$$

where $\varepsilon_1' \sim \mathcal{N}(0, \sigma_1'^2)$, $\varepsilon_2' \sim \mathcal{N}(0, \sigma_2'^2)$.

From Formula (2), there also exist endogenous increments of X and Y at the same time, respectively:

$$\begin{cases} \Delta X_{en} = \alpha_1 \tau_2 + \alpha_1 \varepsilon_2' \\ \Delta Y_{en} = \beta_1 \tau_1 + \beta_1 \varepsilon_1' \end{cases} \tag{5}$$

In summary, the observed total increments of X and Y at the period are respectively:

$$\begin{cases} \Delta X = \alpha_1 \tau_2 + \tau_1 + \varepsilon_1' + \alpha_1 \varepsilon_2' \\ \Delta Y = \beta_1 \tau_1 + \tau_2 + \beta \varepsilon_1' + \varepsilon_2' \end{cases} \tag{6}$$

In the Formula (3), B represents the numerical relationship of total changes on X and Y. Therefore, rather than an unbiased estimate of $\frac{\Delta Y_{en}}{\Delta X_{en}}$ (namely, β_1) as we expect, $\hat{B}$ actually is an unbiased estimate of $\frac{\Delta Y}{\Delta X}$ (namely, B):

$$\begin{aligned} \mathrm{E}(\hat{B} \mid X, Z) &= \frac{\Delta Y}{\Delta X} \\ &= \frac{\beta_1 \tau_1 + \tau_2 + \beta \varepsilon_1' + \varepsilon_2'}{\alpha_1 \tau_2 + \tau_1 + \varepsilon_1' + \alpha_1 \varepsilon_2'} \end{aligned} \tag{7}$$

Therefore, we cannot simply use a linear regression model to estimate β_1. In this study, we use the ratio model to make the estimate. In Formula (7), to isolate the pure effect of β_1, we divide the numerator and the denominator by τ_1, and get:

$$\mathrm{E}(\hat{B} \mid X, Z) = \frac{\beta_1 + \eta + \varepsilon_1}{\alpha_1 \eta + 1 + \varepsilon_2} \tag{8}$$

where $\eta = \frac{\tau_2}{\tau_1}$, namely, the ratio of the exogenous increments of X and Y, and $\varepsilon_1 = \frac{\beta_1}{\tau_1}\varepsilon_1' + \frac{1}{\tau_1}\varepsilon_2'$, $\varepsilon_2 = \frac{\alpha_1}{\tau_1}\varepsilon_2' + \frac{1}{\tau_1}\varepsilon_1'$.

We can see that ε_1 and ε_2 are the linear combinations of ε_1' and ε_2'. Therefore, both of them follow normal distributions, namely, $\varepsilon_1 \sim \mathcal{N}(0, \sigma_1^2)$, $\varepsilon_2 \sim \mathcal{N}(0, \sigma_2^2)$.

We introduce the moderator variables of X and Y, Z_1 and Z_2, so α_1 and β_1 can be expressed as:

$$\begin{cases} \beta_1 = b_1 Z_1 + b_0 \\ \alpha_1 = a_1 Z_2 + a_0 \end{cases} \tag{9}$$

This operation is similar with how we deal with the interactive item of X and Z_1 in the linear regression model:

$$\begin{aligned} Y &= A + B_0 X + B_1 X Z_1 + C + e \\ &= A + (B_1 Z_1 + B_0) X + C + e \end{aligned} \tag{10}$$

Comparing Formula (3) and (10), $B_1 Z_1 + B_0$ plays the role of B. Therefore, with increasing of Z_1, B increases as well. Similarly, we use Formula (9) to replace α_1 and β_1 in Formula (8):

$$\mathrm{E}(\hat{B} \mid X, Z) = \frac{b_1 Z_1 + b_0 + \eta + \varepsilon_1}{\alpha_1 \eta + \alpha_0 \eta + 1 + \varepsilon_2} \tag{11}$$

In Formula (11), the conditional expectation of $\hat{B}$ takes a form of the ratio between linear transforms of Z_1 and Z_2, so we can also write Formula (11) as:

$$\mathrm{E}(\hat{B} \mid X, Z) = \frac{k_1 Z_1 + m_1 + \varepsilon_1}{k_2 Z_2 + m_2 + \varepsilon_2} \tag{12}$$

where $k_1 = b_0$, $k_2 = a_1\eta$, $m_1 = b_0 + \eta$, $m_2 = a_0\eta + 1$.

With the sample information of Z_1, to estimate β_1 is equivalent to estimate b_0 and b_1 in Formula (12). More strongly, we may assume that Z_1 is a complete moderator variable. Namely, b_0 is not significant, and Z_1 can moderate β_1 proportionally. Under this situation, only the estimate of b_1 is needed.

To create a sample set for Formula (12), we perform boostrap on the original sample set, and get a group of estimate $\hat{B}_i$. In each boostrap set, we approximately substitute Z_1 with $\bar{Z}_{1i}$ [1], and get:

$$\begin{aligned} \mathrm{E}(\hat{B}_i \mid \{X_j, Z_j\}_{j \in set_i}) = B_i& \\ &= \frac{k_1 \bar{Z}_{1i} + m_1 + \varepsilon_1}{k_2 \bar{Z}_{2i} + m_2 + \varepsilon_2} \end{aligned} \tag{13}$$

To perform MLE, the distribution of B_i is needed. As shown above, ε_1 and ε_2 follow normal distributions. Therefore, condition on $\{Z_{1j}, Z_{2j}\}_{j \in set_i}$, B_i is the ratio of two variables following uncorrelated non-central normal distributions. In 1969, Hinkley proved that the probability density function of an uncorrelated non-central normal ratio took the following form:

$$p_Z(z) = \frac{b(z) \cdot d(z)}{a^3(z)} \frac{1}{\sqrt{2\pi}\sigma_x \sigma_y} \left[\Phi\left(\frac{b(z)}{a(z)}\right) - \Phi\left(-\frac{b(z)}{a(z)}\right) \right] + \frac{1}{a^2(z) \cdot \pi \sigma_x \sigma_y} e^{-\frac{c}{2}} \tag{14}$$

where $a(z) = \sqrt{\frac{1}{\sigma_x^2} z^2 + \frac{1}{\sigma_y^2}}$, $b(z) = \frac{\mu_x}{\sigma_x^2} z + \frac{\mu_y}{\sigma_y^2}$, $c = \frac{\mu_x^2}{\sigma_x^2} + \frac{\mu_y^2}{\sigma_y^2}$, $d(z) = e^{\frac{b^2(z) - ca^2(z)}{2a^2(z)}}$, and $\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\, du$. [Hin69]

With Formula (14), we can make an estimate on b_1, and therefore, β_1.

2.Data

The data of this study consists with two parts, simulated data set and real data set. From the model above, we estimate b_1 and β_1. However, neither b_1 nor β_1 are unable to be observed in real world, so we cannot make a crossing validation within a real data set. To examine the performance of the model, we need to create a simulated data set, and see to what degree the estimate result of the model is close to our preset parameters. The real data set comes from the World Bank. For the main variables, we use GDP per capita (current international dollars, measured by PPP) to measure

[1] In a specific boostrap set Set_i, $\hat{B}_i$ takes the form of $f(\{Z_1 j, Z_2 j\}_{j \in set_i})$. Therefore, the operation here is an approximate substitute, but as we can see blow, this substitute is satisfying under situations.

the economic growth, and CO2 emissions per capita (metric tons) to measure energy consumption. In the bidirectional causality between these two variables, the effect of energy consumption on economic growth is considered happening on production field, while the opposite effect happening on consumption field. We use the employment rate percentage of of male labor force, modeled by ILO estimate) to moderate the effect on production field [2] , and GNI per capita (current international dollars, measured by PPP) for the effect on consumption field, population urbanization rate (percentage of total population) are used as control variables.

4 Result

1. Simulated data

With Formula (2) and (4), we generate a group of ΔX and ΔY, with $\tau_1 \sim \mathcal{N}(\bar{\tau}, \sigma_\tau^2)$, $\tau_2 = \eta\tau_1$, $\varepsilon_1' \sim \mathcal{N}(0, \sigma_1'^2)$, $\varepsilon_2' \sim \mathcal{N}(0, \sigma_2'^2)$, $Z_1 \sim \mathcal{U}(0,1)$, $Z_2 \sim \mathcal{U}(0,1)$, $\bar{\tau} = 500, \sigma_\tau^2 = 20, \eta = 1, \sigma_1' = \sigma_2' = 1.b_1 = 0.35, a_1 = 0.5, b_0 = a_0 = 0$. With Formula (14), we construct the log-likelihood as the function of k_1, m_1, k_2, m_2, σ_1^2, σ_2^2 :

$$L(\hat{k_1}, \hat{m_1}, \hat{k_2}, \hat{m_2}, \hat{\sigma_1^2}, \hat{\sigma_2^2}) = \sum_{i=1}^{t} logp(\bar{Z}_{1i}, \bar{Z}_{2i}) \tag{15}$$

It is noticed that the optimal solution of Formula (15) is depend on σ_1^2, σ_2^2. This is different from the common situation of MLE. In this way, we would like to see the influence of σ_1^2, σ_2^2 on the log-likelihood.

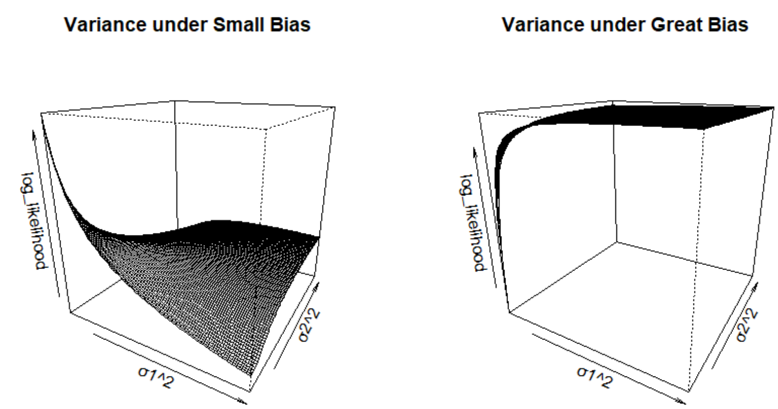

Figure 1: Influence of Variance on Log-Likelihood

Figure 1 shows the performance of σ_1^2, σ_2^2 under the situations of small estimate bias and great estimate bias. Under the situation of small bias, we set $\hat{k_1} = 0.38$, $\hat{m_1} = 0.05$, $\hat{k_2} = 0.55$, and $\hat{m_2} = 0.05$. σ_1^2, σ_2^2 trend to 0. Under great bias, we set $\hat{k_1} = 1.35$, $\hat{m_1} = -1$, $\hat{k_2} = -0.5$, and $\hat{m_2} = 1$. σ_1^2, σ_2^2 trend to positive infinity. Since with small estimate bias, small variances are helpful to improve the log-likelihood. However, with great estimate bias, the estimate bias can be attributed to the great variances.

Since an estimate with small bias is preferred, σ_1^2, σ_2^2 are very likely to be underestimated. To prevent the tendency of underestimating σ_1^2, σ_2^2 we add a loss function into the log-likelihood function:

$$L(\hat{k_1}, \hat{m_1}, \hat{k_2}, \hat{m_2}, \hat{\sigma_1^2}, \hat{\sigma_2^2}) = \sum_{i=1}^{t} logp(\bar{Z}_{1i}, \bar{Z}_{2i}) + log(\sigma_1) + log(\sigma_2) \tag{16}$$

where $log(\hat{\sigma_1})$ and $log(\hat{\sigma_1})$ can punish the result with small estimates of σ_1^2, σ_2^2.

[2] Considering that in some countries, females are forbidden to be employed, it may lead to a national level gap if the employment rate of the total population is used. Therefore, we use the employment rate of male as the moderator variables.

	Interaction Model
(Intercept)	44.88 t=40.92
X	0.9141 t=445.87
Z1	155.5 t=122.23
Z2	-232.4 t=-489.79
Z1 on X	0.03099 t=15.31
N	6000

Table 1: Results of Interaction Model in Simulated Data

First, we start from the result of the traditional method, linear regression model with interactive item in Table 1. In the simulated data set, moderate effect of Z_1 on X, namely, b_1 is set as 0.35. However, the estimate of interaction model is 0.03099, significantly smaller than b_0. Since the direct effect of Z_1 and Z_2 are highly overestimated.

	Ratio Model	Ratio Model (Adjusted)
σ1^2	0.0001299 t=0.05767	0.003147 t=1.639
σ2^2	0.002993 t=1.429	0.00009968 t=0.05198
Z1 on X	0.3490 t=1.598	0.3588 t=1.278
X (+η)	2.446 t=17.39	2.443 t=11.20
Z2 on Y (×η)	1.620 t=10.40	1.701 t=7.870
Y (×η+1)	2.010 t=20.86	1.977 t=9.673
N	6000	6000
Boostrap (coefficient)	2000	2000
Boostrap (t statistic)	100	100

Table 2: Results of Interaction Model in Simulated Data

Second, as shown in Table 2, we use the ratio model with and without loss function to make estimate. The moderate effect of Z_1 on X are correctly recognized and estimated. However, the moderate effect of Z_2 on Y, direct effect of X and Y, (namely, a_1, b_0 and a_0) are mixed with η and remained unrecognizable. Compared with model without loss function, the adjusted ratio model successfully prevent the estimate of σ_1^2 being too small, and make it significant, Although with a little large bias on the estimate of b_1, the result is still reliable.

Finally, we compare the result of these models. All of these three models present a significant estimate of the moderate effect, with the result of adjusted ratio slightly weak. However, if we make a t-test between the expected effect and the model estimates, there is a significant bias existing in the

	Expected Effect	Interaction Model	Ratio Model	Ratio Model
Z_1 on X	0.35	0.04021	0.349	0.3588
		t=15.31	t=1.598	t=1.277
Difference with Expected Effect		-0.3098	-0.0009651	0.008808
		t=-153.05	t=-0.004420	t=0.03137
N		6000	6000	6000
Boostrap (coefficient)			2000	2000
Boostrap (t statistic)			100	100

Table 3: Results of Interaction Model in Simulated Data

interaction model, but no in the ratio model and the adjusted ratio model. Hence, we can see the improvement of ratio model compared with the linear regression model.

2. real data

	Interaction Model
(Intercept)	-0.002906
	t=-0.387
GDPPer	-0.009983
	t=-0.583
GNI	0.702497
	t=63.202
GNI on GDPPer	0.012471
	t=2.764
Employment	-0.001375
	t=-0.174
Capital	0.00899
	t=0.797
Labor	0.03151
	t=2.811
Trade	-0.038707
	t=-4.707
Urbanlization	0.186362
	t=17.334
N	5618

Table 4: Results of Interaction Model in Simulated Data

With the real data set from the World Bank, three models above are performed. First, in the interaction model, significant effect is observed in moderating of GNI on GDP per capita, but not in the direct effect of GDP. This suggests that GNI is the complete moderator variable as we hope. However, no matter with or without loss function, ratio model shows that the moderate effect is not significant. Moreover, although mixed with η, the direct effect of GDP per capita is significantly negative. Therefore, it can be expected that the whole effect of GDP per capita is actually not so significantly positive as we generally observed in the linear regression model. This is possibly due to the significantly positive effect in the opposite direction. Namely, in the linear regression model, the effect of energy consumption on economic growth may be partially recognized as the effect of economic growth on energy consumption oppositely for a long term. Similar operation as above can also be

	Ratio Model	Ratio Model (Adjusted)
σ1^2	0.9595 t=0.5737	0.8678 t=0.3137
σ2^2	1.455 t=0.8359	2.009 t=0.6305
GNI on GDPPer	0.00001510 t=0.00004812	0.00003471 t=0.00004812
GDPPer (+η)	-0.100209226 t=-1.603	-0.05970 t=-1.603
Employment on CO2Per (×η)	0.07533 t=0.04568	0.8650 t=0.04568
CO2Per (×η+1)	4.644 t=2.371	4.441 t=2.371
N	6000	6000
Boostrap (coefficient)	2000	2000
Boostrap (t statistic)	100	100

Table 5: Results of Interaction Model in Simulated Data

performed in the opposite direction.

5 Conclusion

The aim of this study is to solve the problem of recognizing and estimating the effect on a single direction in a bidirectional causality. We use the MLE to fit the model with the form of ratio distribution, and moreover, control the estimate of variance with the loss function. In simulated data set, it is proved that, compared with the traditional linear regression model, the ratio model considerably improves the estimate result. Further, the ratio model is performed on the real data set from World Bank, and suggests the possible misrecognition of the opposite direction effect in the linear regression model. There still exists some possible ways to improve the ratio model. The main problem of the ratio model is that the estimate of variance is not robust enough, and can be very sensitive to the unbalanced data. Thus, only under the data satisfying the restrict assumption, can the ratio model perform well. To make an improvement, a better form of loss function, or some Bayes estimate methods can be take into consideration. For example, it is observed that, with the increase of k, the estimate of variance would also have a rapid increase. To control this tendency, loss function in the form of $w^{\frac{1}{k}} log(\hat{\sigma})$ can be used, where w is a flexible parameter can be used to control the degree that estimate of variance is close to 0. $w^{\frac{1}{k}}$ means that, with larger k, small estimate of variance is punished less, and thus, encouraged.

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