

1. let $x = 4 + 5$ in $(3 * x)$

$\Rightarrow [4+5/x](3*x)$ let reduction

$\Rightarrow (3 * (4+5))$ substitution

$\Rightarrow 3 * 9$ arithmetic

$\Rightarrow 27$ multiplication

2. $(\lambda x \rightarrow 3 * x) (4+5)$

$\Rightarrow [4+5/x](3*x)$ lambda reduction

$\Rightarrow 3 * (4+5)$ substitution

$\Rightarrow 3 * 9$ arithmetic

$\Rightarrow 27$ multiplication

3. $((\lambda x \rightarrow (\lambda y \rightarrow x + (3 * y))) 4) 1$

$\Rightarrow ([4/x](\lambda y \rightarrow x + (3 * y))) 1$ lambda reduction

$\Rightarrow (\lambda y \rightarrow (4 + (3 * y))) 1$ substitution

$\Rightarrow [1/x] (4 + (3 * y))$ lambda reduction

$\Rightarrow 4 + (3 * 1)$ substitution

$\Rightarrow 4 + 3$ multiplication

$\Rightarrow 7$ arithmetic

4. let $x = 4$ in (let $y = 1$ in $(x + (3 * y))$)

$\Rightarrow \text{let } x = 4 \text{ in } ([1/y] x + (3 * y)) \text{ let reduction}$

$\Rightarrow \text{let } x = 4 \text{ in } (x + (3 * 1)) \text{ substitution}$

$\Rightarrow [4/x] x + (3 * 1) \text{ let reduction}$

$\Rightarrow 4 + (3 * 1) \text{ substitution}$

$\Rightarrow 4 + 3 \text{ multiplication}$

$\Rightarrow 7 \text{ arithmetic}$

5. $\text{let } x = 4 \text{ in } (\text{let } y = 1 + x \text{ in } (x + (3 * y)))$

$\Rightarrow \text{let } x = 4 \text{ in } ([1+x/y] (x + (3 * y))) \text{ let reduction}$

$\Rightarrow \text{let } x = 4 \text{ in } ((x + (3 * (1+x)))) \text{ substitution}$

$\Rightarrow [4/x] (x + (3 * (1+x))) \text{ let reduction}$

$\Rightarrow 4 + (3 * (1+4)) \text{ substitution}$

$\Rightarrow 4 + (3 * 5) \text{ arithmetic}$

$\Rightarrow 4 + 15 \text{ multiplication}$

$\Rightarrow 19$

6. $(\lambda x \rightarrow (\lambda y \rightarrow x + (3 * x))) 4) 1$

$\Rightarrow ([4/x] \rightarrow (\lambda y \rightarrow x + (3 * x))) 1 \text{ lambda reduction}$

$\Rightarrow (\lambda y \rightarrow 4 + (3 * 4)) 1 \text{ substitution}$

$\Rightarrow [1/y] (4 + (3 * 4)) 1 \text{ lambda reduction}$

$\Rightarrow 4 + (3 * 4) \text{ substitution}$

$\Rightarrow 4 + 12 \text{ multiplication}$

\Rightarrow 16 arithmetic

7. $((\lambda x \rightarrow (\lambda y \rightarrow y + (3 * y))) 4) 1$

$\Rightarrow ([4/x] \rightarrow (\lambda y \rightarrow y + (3 * y)))$ 1 lambda reduction

$\Rightarrow (\lambda y \rightarrow y + (3 * y))$ 1 substitution

$\Rightarrow [1/y] (y + (3 * y))$ 1 lambda reduction

$\Rightarrow 1 + (3 * 1)$ substitution

$\Rightarrow 1 + 3$ multiplication

$\Rightarrow 4$ arithmetic

8. $(\lambda y \rightarrow y + ((\lambda y \rightarrow 3 * y) 4)) 5$

$\Rightarrow (\lambda y \rightarrow y + ([4/y] 3 * y))$ 5 lambda reduction

$\Rightarrow (\lambda y \rightarrow y + (3 * 4))$ 5 substitution

$\Rightarrow [5/y] y + (3 * 4)$ lambda reduction

$\Rightarrow 5 + (3 * 4)$ substitution

$\Rightarrow 5 + 12$ multiplication

$\Rightarrow 17$ arithmetic

9. $(\lambda y \rightarrow ((\lambda y \rightarrow 3 * y) 4) + y) 5$

$\Rightarrow (\lambda y \rightarrow ([4/y] \rightarrow 3 * y) + y)$ 5 lambda reduction

$\Rightarrow (\lambda y \rightarrow (3 * 4) + y)$ 5 substitution

$\Rightarrow ([5/y] (3 * 4) + y)$ lambda reduction

$\Rightarrow (3 * 4) + 5$ substitution

$\Rightarrow 12 + 5$ multiplication

$\Rightarrow 17$ arithmetic

10. $(\lambda x \rightarrow x * (\text{let } x = 3 * 2 \text{ in } (x+7)) + x) 4$

$\Rightarrow (\lambda x \rightarrow x * ([3*2/x] x+7) + x) 4$ let reduction

$\Rightarrow (\lambda x \rightarrow x * ((3*2)+7) + x) 4$ substitution

$\Rightarrow [4/x] (x * ((3*2)+7) + x)$ lambda reduction

$\Rightarrow 4 * ((3*2)+7) + 4$ substitution

$\Rightarrow 4 * (6+7) + 4$ multiplication

$\Rightarrow 4 * 13 + 4$ arithmetic

$\Rightarrow 52 + 4$ multiplication

$\Rightarrow 56$ arithmetic

11. $g((\text{let } x = 4 \text{ in } (\lambda y \rightarrow x + y)) 2)$

$\Rightarrow g([4/x](\lambda y \rightarrow x + y) 2)$ let reduction

$\Rightarrow g((\lambda y \rightarrow 4 + y) 2)$ substitution

$\Rightarrow g([2/y] 4 + y)$ lambda reduction

$\Rightarrow g(4 + 2)$ substitution

$\Rightarrow (\lambda z \rightarrow z + 4) 6$ arithmetic

$\Rightarrow [6/z] (z + 4)$ lambda reduction

$\Rightarrow 6 + 4$ arithmetic

$\Rightarrow 10$

12. $\text{let } x = 5 \text{ in } (\lambda z \rightarrow x * z)$

$\Rightarrow [5/x] (\lambda z \rightarrow x * z)$ let reduction

$\Rightarrow \lambda z \rightarrow 5 * z$ substitution

13. $f ((\lambda n \rightarrow \text{fn Rock}) (\lambda x \rightarrow \text{whatItBeats } x))$

$\Rightarrow f ((\lambda x \rightarrow \text{whatItBeats } x) \text{ Rock})$ lambda reduction

$\Rightarrow f ([\text{Rock}/x] \text{ whatItBeats } x)$ lambda reduction

$\Rightarrow f (\text{whatItBeats Rock})$ substitution

$\Rightarrow f ([\text{Rock}/s] \text{ whatItBeats})$ substitution

$\Rightarrow [\text{Scissors}/s]f$ lambda reduction

$\Rightarrow 99$ case reduction

14. $((\lambda f \rightarrow (\lambda x \rightarrow f (f x))) \text{ whatItBeats}) \text{ Paper}$

$\Rightarrow ([\text{whatItBeats}/f](\lambda x \rightarrow f (f x))) \text{ Paper}$ lambda reduction

$\Rightarrow ((\lambda x \rightarrow \text{whatItBeats } (\text{whatItBeats } x))) \text{ Paper}$ substitution

$\Rightarrow [\text{Paper}/x](\text{whatItBeats } (\text{whatItBeats } x))$ lambda reduction

$\Rightarrow \text{whatItBeats } (\text{whatItBeats Paper})$ substitution

$\Rightarrow \text{whatItBeats } ([\text{Paper}/s] \text{ whatItBeats})$ lambda reduction

$\Rightarrow \text{whatItBeats Rock}$ lambda reduction

$\Rightarrow [\text{Rock}/s] (\text{whatItBeats})$ case reduction

\Rightarrow Scissors

15. $\text{whatItBeats} (\text{case Paper of } \{\text{Rock} \rightarrow \text{Paper}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Scissors}\})$

\Rightarrow whatItBeats Rock case reduction

$\Rightarrow [\text{Rock}/s] \text{ whatItBeats}$

\Rightarrow Scissors

16. $(\text{case} (\text{Win Rock}) \text{ of } \{\text{Draw} \rightarrow \text{whatItBeats}; \text{Win } z \rightarrow (\lambda s. \text{Scissors})\}) \text{ Paper}$

$\Rightarrow ([\text{Rock}/z] (\lambda s. \text{Scissors})) \text{ Paper}$ case reduction

$\Rightarrow (\lambda s. \text{Scissors}) \text{ Paper}$ substitution

$\Rightarrow [\text{Paper}/s] \text{ Scissors}$ lambda reduction

\Rightarrow Scissors

17. $\text{case} (\text{Win} (\text{whatItBeats Rock})) \text{ of } \{\text{Draw} \rightarrow n; \text{Win } y \rightarrow (n + f y)\}$

$\Rightarrow \text{case} (\text{Win} ([\text{Rock}/s] \text{ whatItBeats})) \text{ of } \{\text{Draw} \rightarrow 1; \text{Win } y \rightarrow (1 + f y)\}$
lambda + case reduction

$\Rightarrow \text{case} (\text{Win Scissors}) \text{ of } \{\text{Draw} \rightarrow 1; \text{Win Scissors} \rightarrow (1 + f \text{Scissors})\}$
substitution

$\Rightarrow \text{case} (\text{Win Scissors}) \text{ of } \{\text{Draw} \rightarrow 1; \text{Win Scissors} \rightarrow (1 + [\text{Scissors}/s]f)\}$
lambda reduction

$\Rightarrow 1 + 99$ case reduction

$\Rightarrow 100$ arithmetic

18. $\text{let } y = 2 \text{ in } (\text{case} (\text{Win} (\text{whatItBeats Rock})) \text{ of } \{\text{Draw} \rightarrow n; \text{Win } y \rightarrow (n + f y)\} + y)$

\Rightarrow let $y = 2$ in $(([\text{whatItBeats Rock}/y] (n + f y)) + y)$ case reduction

\Rightarrow let $y = 2$ in $((n + f \text{ whatItBeats Rock}) + y)$ substitution

$\Rightarrow [y/2] (n + f \text{ whatItBeats Rock}) + y$ let reduction

$\Rightarrow (n + f \text{ whatItBeats Rock}) + 2$ substitution

$\Rightarrow (1 + f ([\text{Rock}/s] \text{ whatItBeats})) + 2$ lambda reduction

$\Rightarrow (1 + f \text{ Scissors}) + 2$ case reduction

$\Rightarrow (1 + ([\text{Scissors}/s] f)) + 2$ lambda reduction

$\Rightarrow (1 + 99) + 2$ substitution

$\Rightarrow 102$ arithmetic