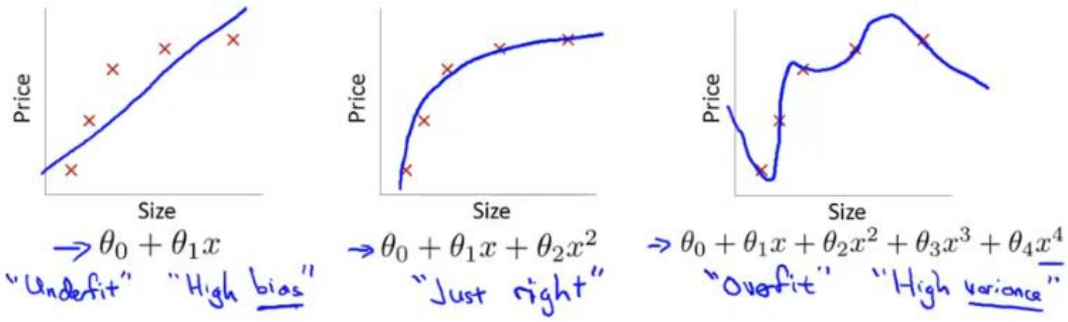
regularization 正规化

过度拟合 overfitting == 高方差 variance
欠拟合 underfitting == 高偏差 bias

Example: Linear regression (housing prices)



Overfitting: If we have too many features, the learned hypothesis may fit the training set very well ($J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 \approx 0$), but fail to generalize to new examples (predict prices on new examples).

过多的特征变量+过少了 training set == over fitting

Addressing overfitting:

Options:

1. Reduce number of features.
   - Manually select which features to keep.
   - Model selection algorithm (later in course).
2. Regularization.
   - Keep all the features, but reduce magnitude/values of parameters θ_j.
   - Works well when we have a lot of features, each of which contributes a bit to predicting y.

lambda 正规化参数

In regularized linear regression, we choose θ to minimize

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^{n}\theta_j^2\right]$$

What if λ is set to an extremely large value (perhaps for too large for our problem, say $\lambda = 10^{10}$)?

$\theta_1, \theta_2, \theta_3, \theta_4$

$\theta_1 \approx 0, \theta_2 \approx 0$

$\theta_3 \approx 0, \theta_4 \approx 0$

$h_\theta(x) = \theta_0$

$h_\theta(x)$

$$\theta_0 + \cancel{\theta_1 x} + \cancel{\theta_2 x^2} + \cancel{\theta_3 x^3} + \cancel{\theta_4 x^4}$$

梯度下降的正规化：：

Gradient descent

θ_0 $\theta_1, \theta_2, \ldots, \theta_n$

Repeat {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m}\boxed{\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_0^{(i)}} \quad \frac{\partial}{\partial\theta_0}J(\theta)$$

$$\theta_j := \theta_j - \boxed{\alpha}\left[\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)} + \frac{\lambda}{m}\theta_j\right]$$

$$(j = \cancel{0}, 1, 2, 3, \ldots, n)$$

}

$$\theta_j := \boxed{\theta_j(1 - \alpha\frac{\lambda}{m})} - \boxed{\alpha\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}}$$

$1 - \alpha\frac{\lambda}{m} < 1$ 0.99 $\theta_j \times 0.99$ θ_j^2

正规方程的正规化：：

Non-invertibility (optional/advanced).

Suppose $m \leq n$,

(#examples) (#features)

$$\theta = \underbrace{(X^T X)}_{\text{non-invertible / singular}}{}^{-1} X^T y$$

pinv inv

If $\lambda > 0$,

$$\theta = \left(X^T X + \lambda \begin{bmatrix} 0 & & & \\ & 1 & & \\ & & 1 & \\ & & & \ddots \\ & & & & 1 \end{bmatrix} \right)^{-1} X^T y$$