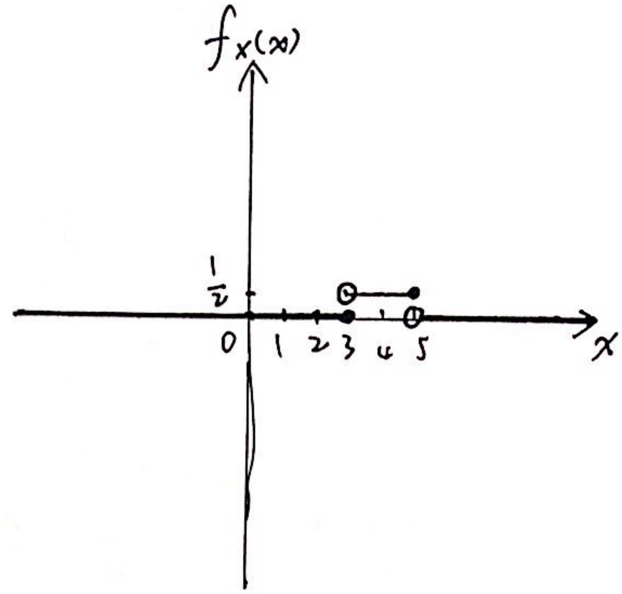
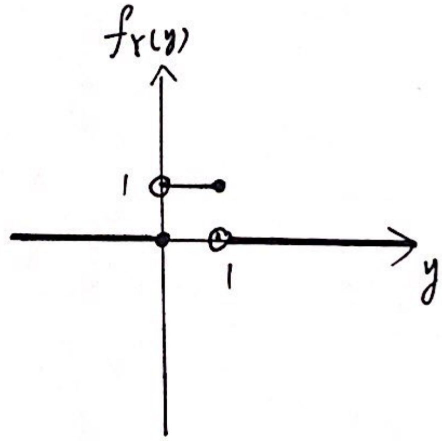
1. (i) $F_X(x) = \begin{cases} 0 & -\infty < x \le 3 \\ \frac{1}{2}(x-3) & 3 < x \le 5 \\ 1 & 5 < x < \infty \end{cases}$



$f_X(x) = F'_X(x) = \begin{cases} 0 & -\infty < x \le 3 \\ \frac{1}{2} & 3 < x \le 5 \\ 0 & 5 < x < \infty \end{cases}$

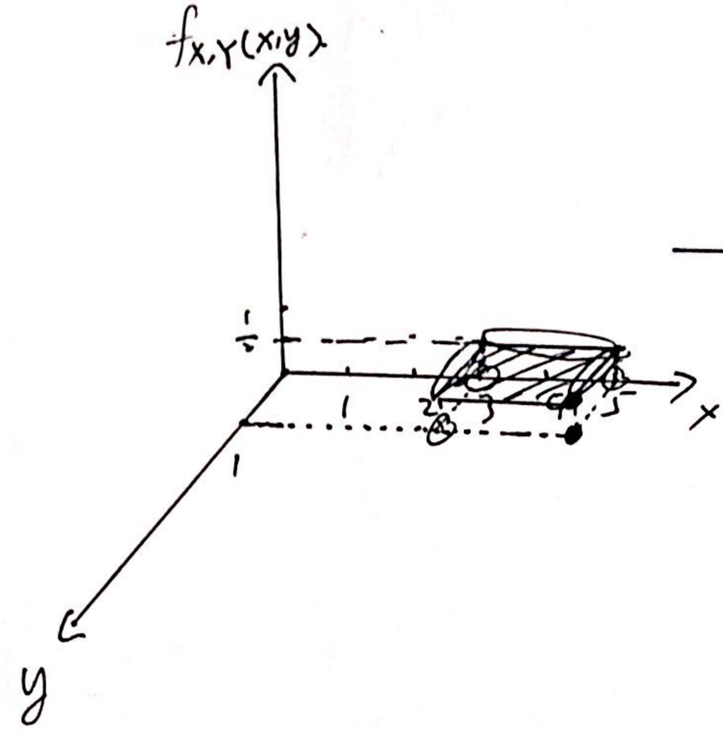
$F_Y(y) = \begin{cases} 0 & -\infty < y \le 0 \\ y & 0 < y \le 1 \\ 1 & 1 < y \le \infty \end{cases}$

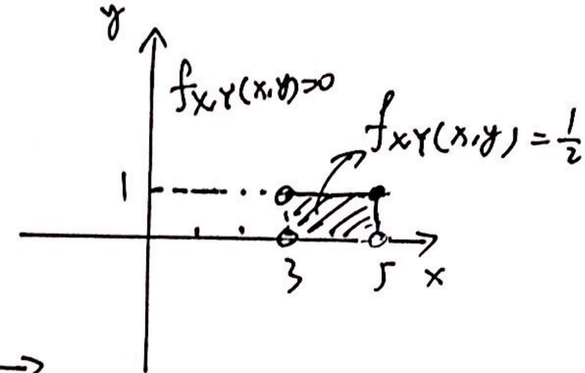


$f_Y(y) = \begin{cases} 0 & -\infty < y \le 0 \\ 1 & 0 < y \le 1 \\ 0 & 1 < y \le \infty \end{cases}$

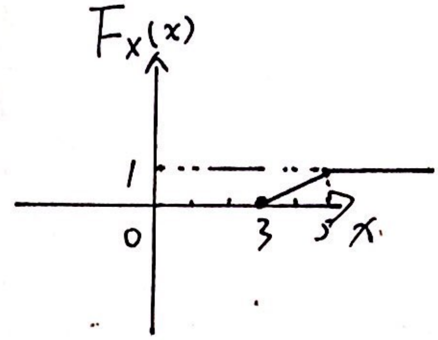
(ii) $F_X(x) F_Y(y) = \begin{cases} 0 & \text{if } -\infty < x \le 3 \text{ or } -\infty < y \le 0 \\ \frac{1}{2}(x-3)y. & \text{if } 3 < x \le 5 \text{ and } 0 < y \le 1. \\ \frac{1}{2}(x-3) & \text{if } 3 < x \le 5 \text{ and } 1 < y \le \infty \\ y & \text{if } 5 < x \le \infty \text{ and } 0 < y \le 1 \\ 1 & \text{if } 5 < x \le \infty \text{ and } 1 < y < \infty \end{cases}$

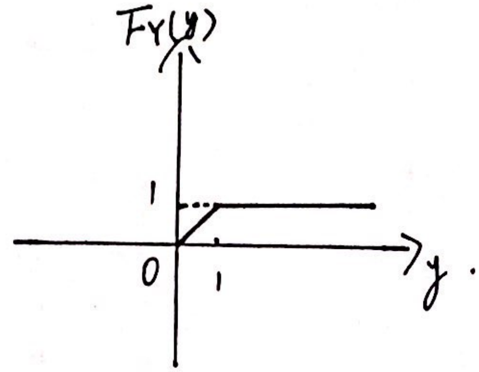
$$f_{X,Y}(x,y) = \frac{\partial F_{X,Y}(x,y)}{\partial x \partial y} = \begin{cases} \frac{1}{2} & \text{if } 3 < x \le 5 \text{ and } 0 < y \le 1. \\ 0 & \text{otherwise} \end{cases}$$





2. $$F_Z(z) = \begin{cases} 0 & -\infty < z \le 0 \\ \frac{1}{2}z & 0 < z \le 1 \\ \frac{1}{2} & 1 < z \le 3 \\ \frac{1}{4}(z-1) & 3 < z \le 5 \\ 1 & 5 < z < \infty \end{cases}$$

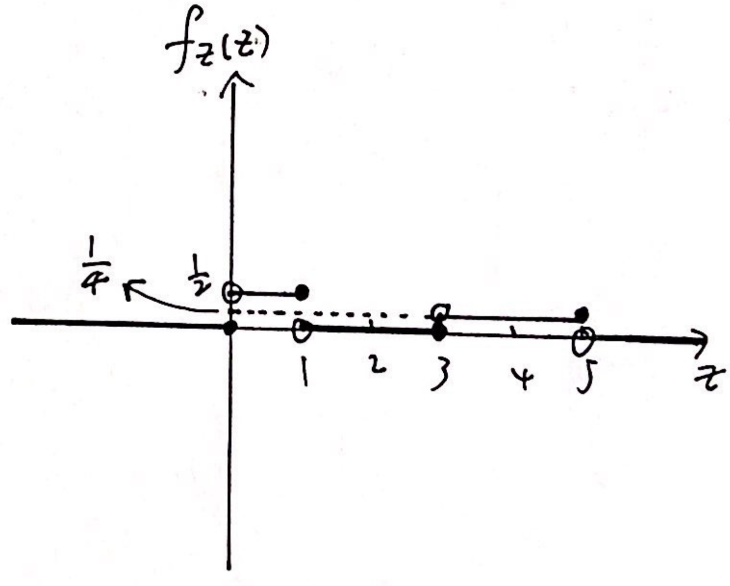




$\uparrow$

$$F_Z(z) = \frac{1}{2}\left[F_X(z) + F_Y(z)\right]$$

$$f_Z(z) = F_Z'(z) = \begin{cases} 0 & -\infty < z \le 0 \\ \frac{1}{2} & 0 < z \le 1 \\ 0 & 1 < z \le 3 \\ \frac{1}{4} & 3 < z \le 5 \\ 0 & 5 < z < \infty \end{cases}$$



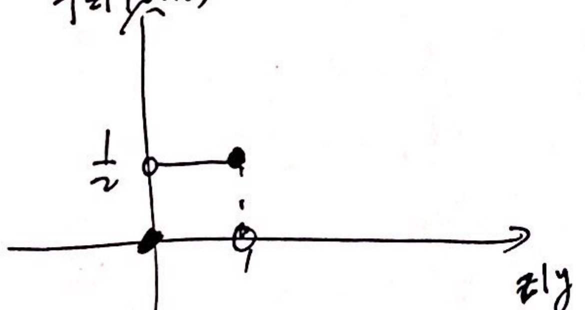
3. $f_{Z|Y}(z|y) = \dfrac{f_{Z,Y}(z,y)}{f_Y(y)}$. $\cancel{\begin{cases} \frac{1}{2} & 0 < z \le 1,\ 0 < y \le 1 \\ 0 & \text{otherwise.} \end{cases}}$

$F_{Z|Y}(z|Y) = \dfrac{F_{Z,Y}(Z,Y)}{F_Y(y)}$.

$$f_{Z,Y}(z,y) = \begin{cases} \frac{1}{2} & 0 < z \le 1,\ 0 < y \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$f_Y(y) = \begin{cases} 0 & -\infty \le y \le 0 \\ 1 & 0 < y \le 1 \\ 0 & 1 < y \le \infty. \end{cases}$$

$$f_{Z|Y}(z|y) = \begin{cases} \frac{1}{2} & 0 < y \le 1. \\ 0 & \cancel{1 \le y \le \infty}\ \text{otherwise} \end{cases}$$



4. We are going to prove that both set $(3.5, 4.0)$ and set $(4.0, 4.5)$ are in the sigma algebra. Call this sigma algebra $\tilde{\mathcal{F}}$.

Let $A = (-\infty, 3.5] \Rightarrow A \in \tilde{\mathcal{F}} . \Rightarrow A^c \in \tilde{\mathcal{F}}$

5. $R_{emp}(\alpha) = \frac{1}{3}\sum_{i=1}^{3}[y_i - (ax_i+b)]^2$

$$= \frac{1}{3}\left((1-(-a+b))^2 + (0-(0+b))^2 + (2-(a+b))^2\right).$$

$$= \frac{1}{3}\left(b^2-2ab+a^2-2(b-a)+1+b^2+a^2+2ab+b^2+4-4(a+b)\right)$$

$$= \frac{1}{3}(2a^2-2a+3b^2-6b+5).$$

$$= \frac{1}{3}\left[2(a-\tfrac{1}{2})^2+3(b-1)^2+\tfrac{3}{2}\right].$$

When choosing $a=\frac{1}{2}, b=1$. $R_{emp}(\alpha)$ is smallest.

6. Suppose all 3 points satisfies $y=ax^2+b+c$

$$\Rightarrow \begin{cases} a-b+c=1 \\ c=0 \\ a+b+c=2 \end{cases}$$ Solve this group of functions.

$$\Rightarrow \begin{cases} a-b=1 \\ a+b=2 \end{cases} \Rightarrow \begin{cases} a=\frac{3}{2} \\ b=\frac{1}{2} \end{cases}$$

So we have $a=\frac{3}{2}, b=\frac{1}{2}, c=0$ makes $R_{emp}(\alpha)=0$ which is smallest.

7. $R_{emp}(a,b) = \dfrac{-\sum_{i=1}^{N} \ln f(x_i;a,b)}{N}$

if $x_i \in [a,b]$. $f(x_i;a,b) = \frac{1}{b-a}$ else $f(x_i;a,b) = 0$.

$R_{emp}(a,b) = -\frac{1}{N} \ln\left(\prod_{i=1}^{N} f(x_i;a,b)\right)$.

$\forall\ 0 < x < y < \infty$. $\ln(x) < \ln(y)$. So we are to maximize.

$\prod_{i=1}^{N} f(x_i;a,b)$. we know that $f(x_i;a,b) \geq 0$.

So making as much as x_is $\in [a,b]$ will make.

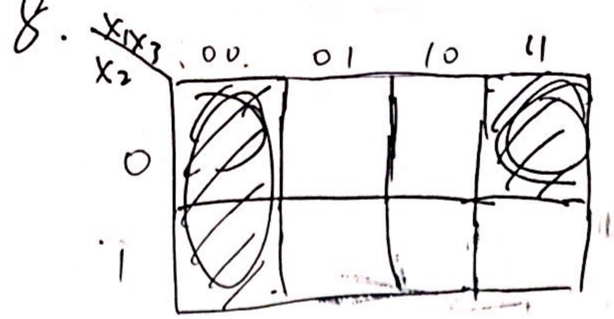
$\prod_{i=1}^{N} f(x_i;a,b)$ largest. if $b-a<1$.

So calculate. $M = \max_i \{x_i\}$ and $\hat{N} = \min_i \{x_i\}$. if $M-N<1$.

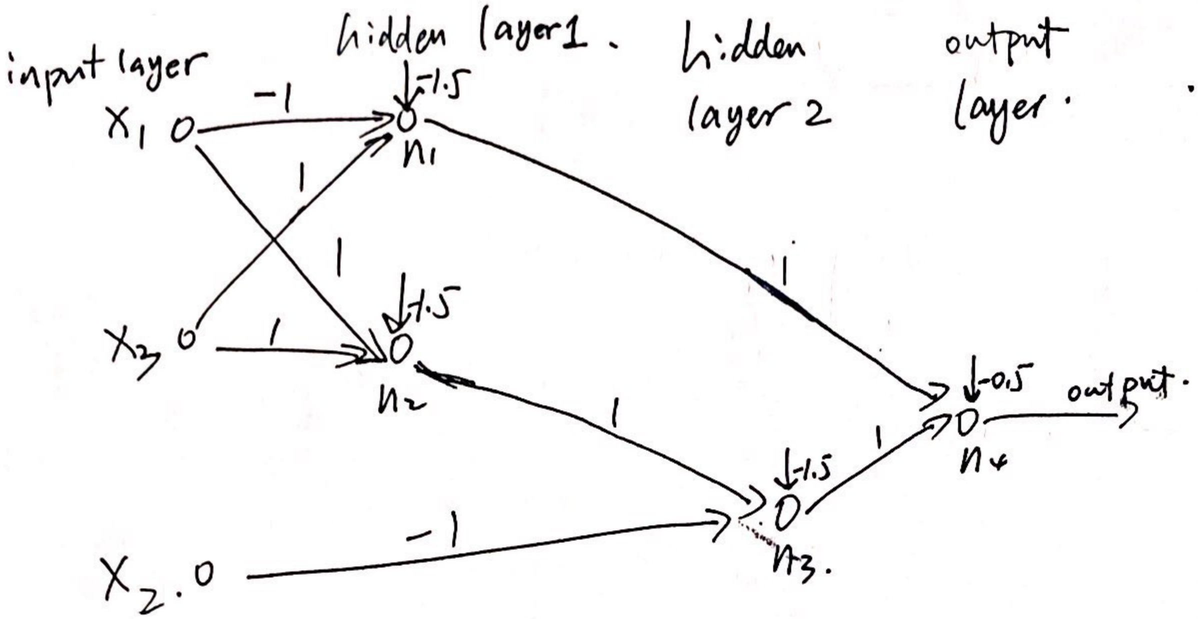
Let $a \leq N$ and $b \geq M$ makes $R_{emp}(\alpha)$ smallest.

Otherwise, in order to making $\ln f(x_i;a,b)$ meaningful. we should keep. $a=N$ and $b=M$. makes $\frac{1}{b-a}$ as large as possible, which makes $R_{emp}(\alpha)$ smallest.

8.



$f(x_1, x_2, x_3) = (x_1 \wedge \neg x_2 \wedge x_3) \vee (\neg x_1 \wedge x_3)$.



for n_1, only with input $x_1 = -1$ and $x_3 = 1$, the output of it will be 1, other wise it's -1.

for n_2, only with input $x_1 = 1$ and $x_3 = 1$, the output is 1, otherwise is -1. for n_3. only if output$(n_2) = 1$ and $x_2 = -1$, the output of n_3 is 1, other wise is -1.

So n_1 is for $(\neg x_1 \wedge x_3)$. n_2 is for $(x_1 \wedge x_3)$. n_3 is for $(x_1 \wedge \neg x_2 \wedge x_3)$. for n_4, when at least one of outputs of n_1 or n_3 is 1. the output of n_4 is 1, otherwise it's -1.

So this MLP is for $f(x_1, x_2, x_3) = (x_1 \wedge \neg x_2 \wedge x_3) \vee (\neg x_1 \wedge x_3)$.

9. Let $g(x) = 2f(x) - 1$. which makes range of $f(x)$ from -1 to 1.

$$g'(x) = \frac{\partial g}{\partial f} \times \frac{\partial f}{\partial x} = 2f(x)(1-f(x)).$$

$$\Rightarrow \frac{\partial o}{\partial net} = 2o(1-o)$$

10. $\frac{\partial o}{\partial w_i} = \begin{cases} x_i & \text{if } w_i x_i = \max_{i=1\cdots N}\{w_1x_1, w_2x_2, \cdots w_Nx_N\}. \\ 0 & \text{otherwise.} \end{cases}$

$\frac{\partial o}{\partial w_i}$ is how output changes as w_i changes.

If $w_i x_i$ is the max, then output $= w_i x_i$.

$\frac{\partial o}{\partial w_i} = x_i$.

If $w_i x_i$ is not the max over all $w_i x_i$ $(i=1 \cdots N)$,

then the small change of w_i won't cause the change of output. $\frac{\partial o}{\partial w_i} = 0$