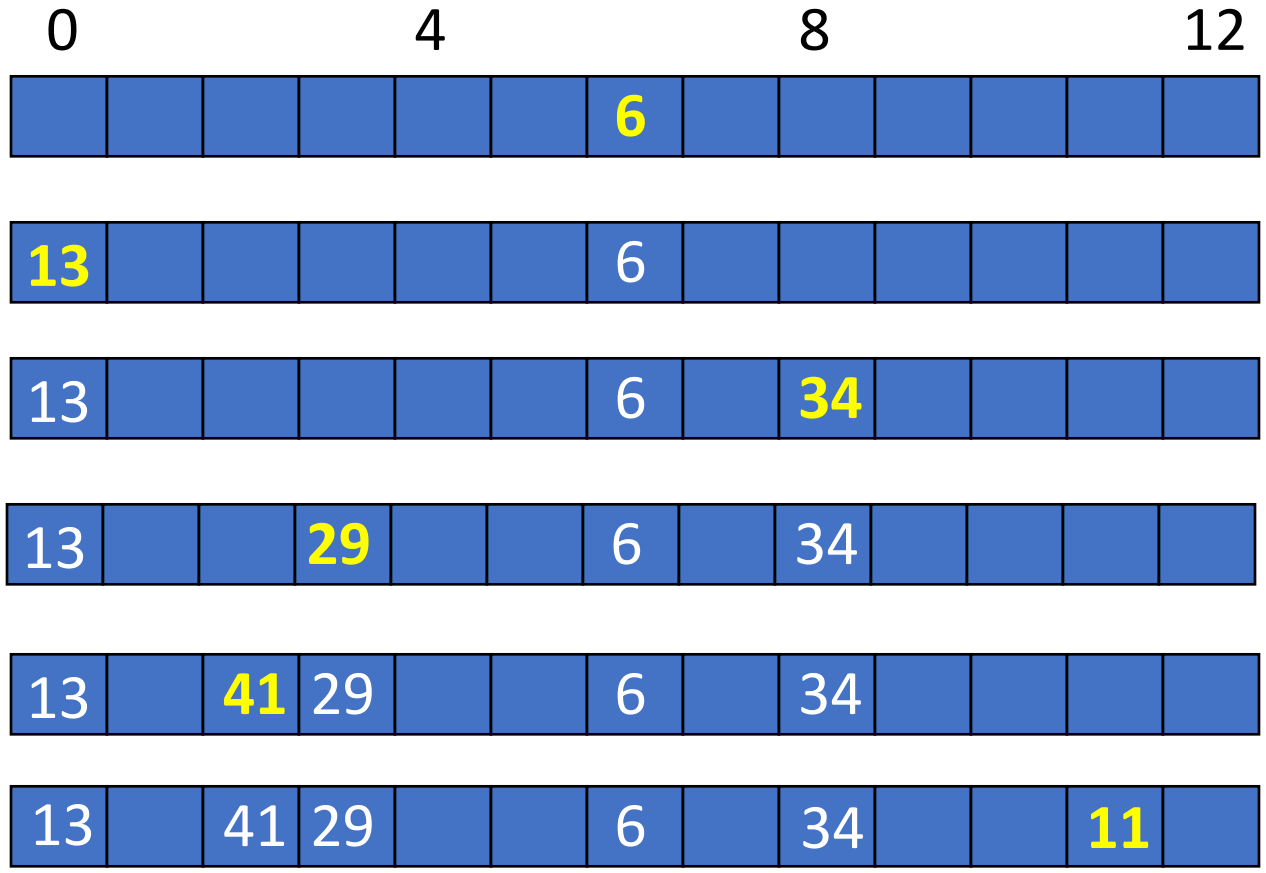
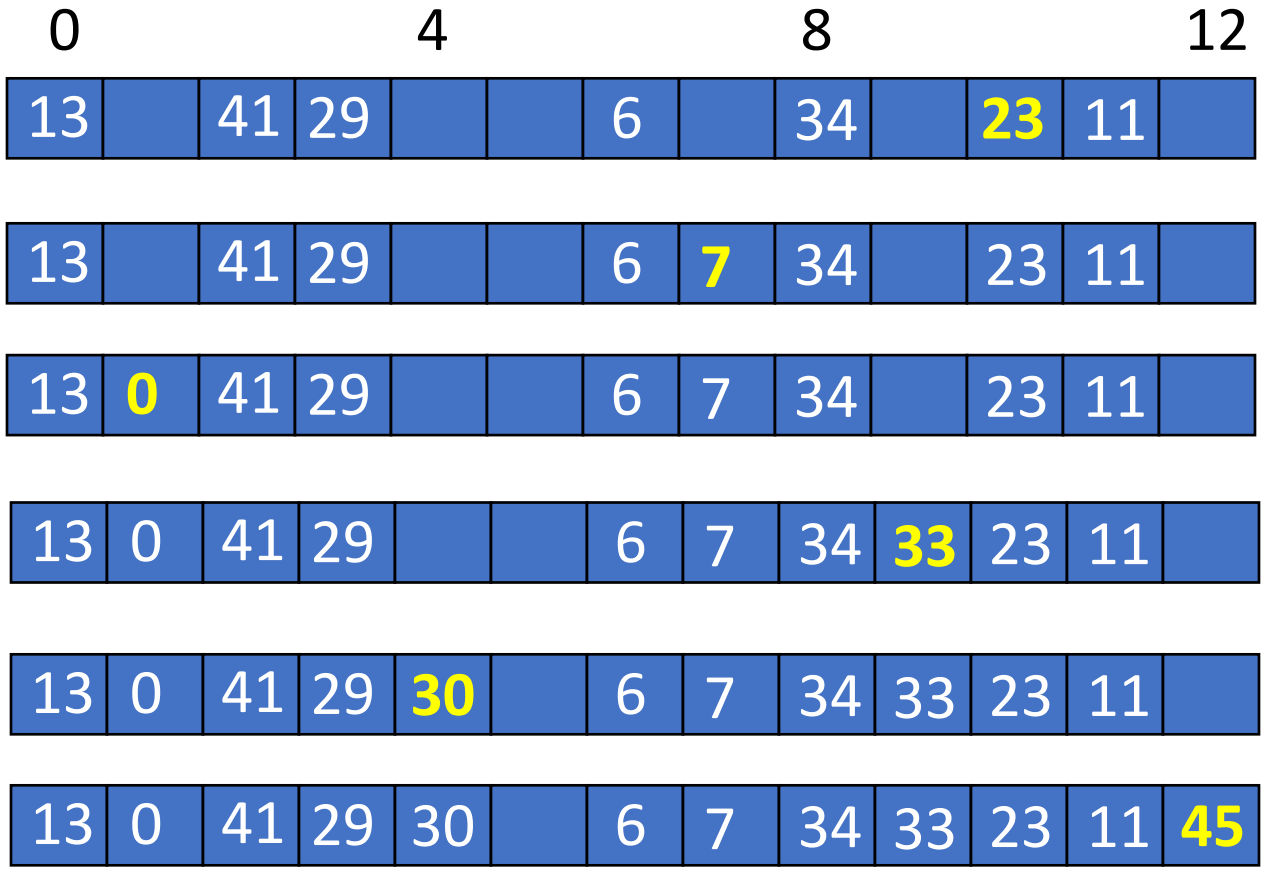
Hashing

- Q1: Hash function = key % 13, number of buckets = 13. Using linear probing to insert pairs whose keys are 6, 13, 34, 29, 41, 11, 23, 7, 0, 33, 30, 45. Write out the hash table.





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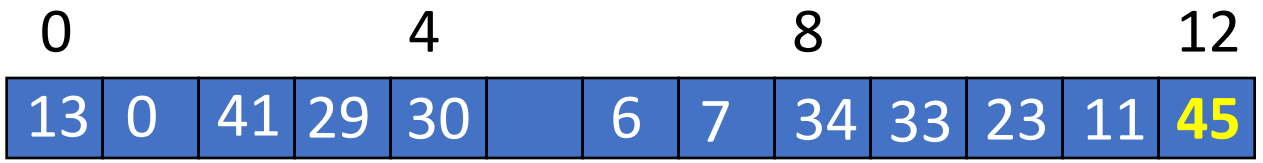
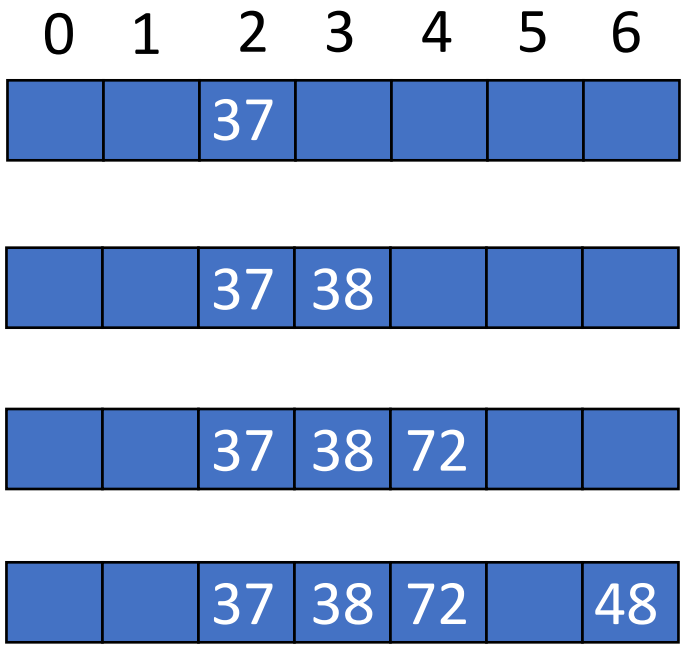


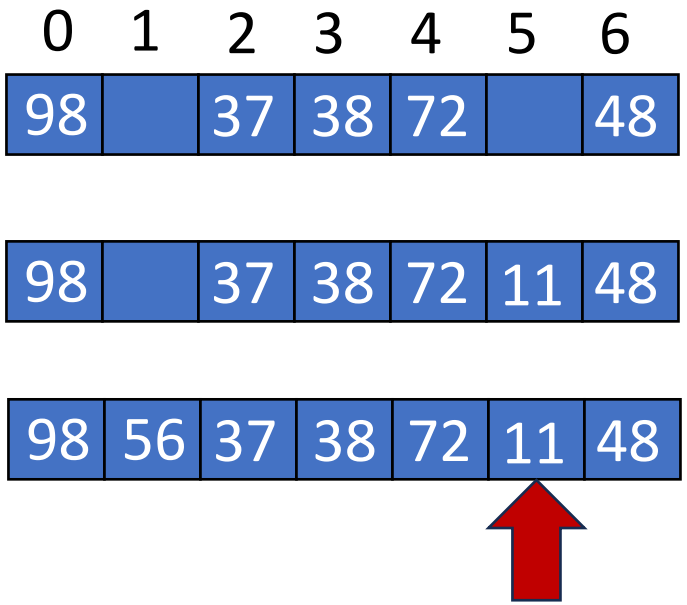
Answers needed to be corrected:

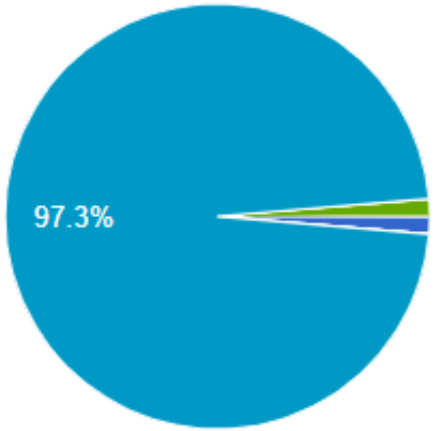
13,0,41,29,45,-,6,7,34,33,23,11,30	The positions of 30 and 45 are incorrect.
13,0,41,29,30,6,7,34,33,23,11,45	There should be a **null** in the bucket[5].

Hashing

- Q2: A hash function h defined as key % 7 with linear probing. Insert the keys 37, 38, 72, 48, 98, 11, and 56 into a table. Where will be 11 in the table?







Exercise

Given m = 13 (size of bit array for the bloom filter BF)

h = 3 (number of hash functions)

- $f_1(k) = (3k) \bmod m$
- $f_2(k) = (2k) \bmod m$
- $f_3(k) = k^2 \bmod m$

Q3: Please write out the bit array after inserting 11.

Q4: (Continue of Q3) Please write out the bit array after inserting 1.

Q5: (Continue of Q4) What is the results of Member(3, BF).

Exercise

Given m = 13 (size of bit array for the bloom filter BF)

h = 3 (number of hash functions)

$f_1(k) = (3k) \bmod m$ $f_2(k) = (2k) \bmod m$ $f_3(k) = k^2 \bmod m$

Q3: Please write out the bit array after inserting 11.

f1= 33 mod 13=7, f2= 22 mod 13=9, f3 = 121 mod 13=4

0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	1	0	0	1	0	1	0	0	0

Q4: (Continue of Q3) Please write out the bit array after inserting 1.

f1 = 3 mod 13=3, f2 = 2 mod 13=2, f3 = 1 mod 13=1

0	1	2	3	4	5	6	7	8	9	10	11	12
0	1	1	1	1	0	0	1	0	1	0	0	0

Q5: (Continue of Q4) What are the results of Member(3, BF)?

f1 = 9 mod 13=9, f2 = 6 mod 13=6, f3 = 9 mod 13=9

BH[6]=0, BH[9]=1 → NO

Exercise

- Given a bloom filter with m bits of memory size and storing u elements. We set $m = 8u$.
  - Q6: Please compute the optimum number of hash functions that minimizes the false positive probability f. Round the number to have an integer.

$$h = \frac{m}{u}\ln(2) = \frac{8u}{u}\ln(2) = 8\ln(2) \approx 5.545$$

Round(5.545) = 6

- To have an integer value, you can compare 5 and 6 to select the one with better performance.
- Here, we use round(N) to get an answer.

  - Q7: (Continue of Q6) Please compute the false positive probability f.

$$f = (1 - e^{-hu/m})^h$$
$$= (1 - e^{-6/8})^6$$
$$= 0.02157714$$

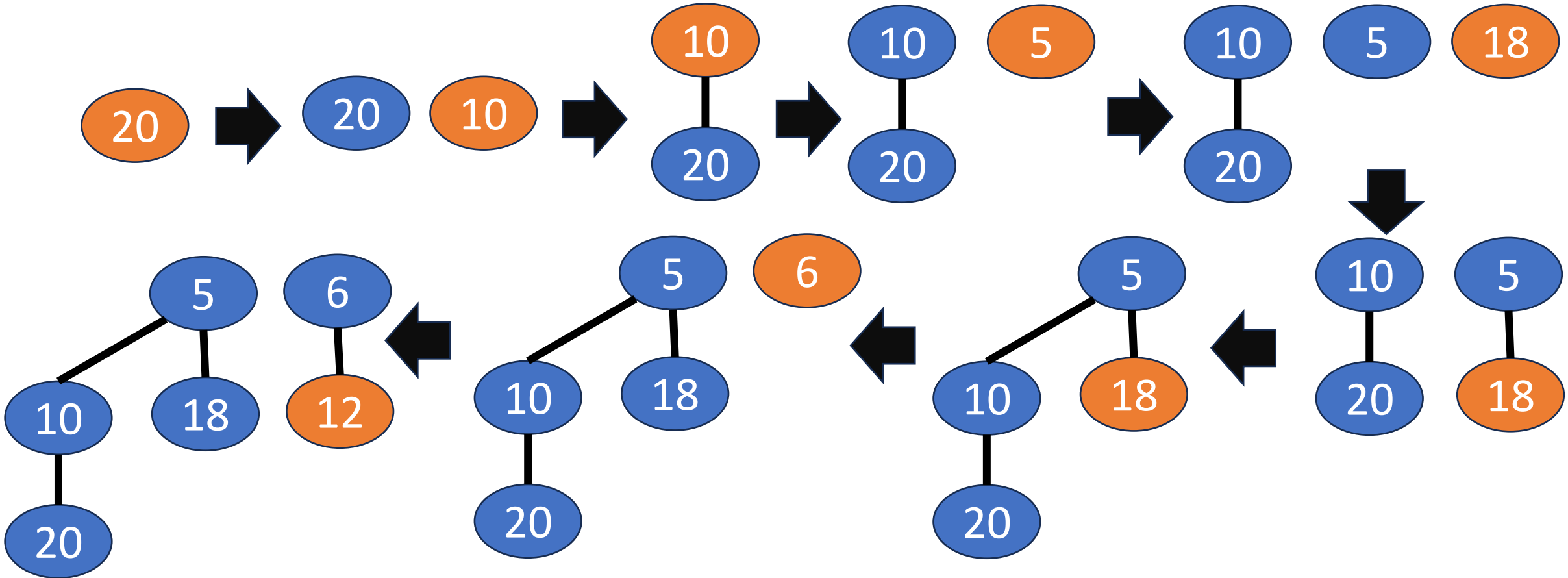
If you use the following two equations, it means that you assume that p is ½. However, the number of hash functions is 6, which is not the optimal value. Thus, the probability p is not ½.

$$f \approx 0.6185^{m/u}$$
$$= 0.6185^8$$
$$\approx 0.021414$$

$$f = (1 - p)^h$$
$$= (1 - 1/2)^6$$
$$= 1/64 = 0.015625$$

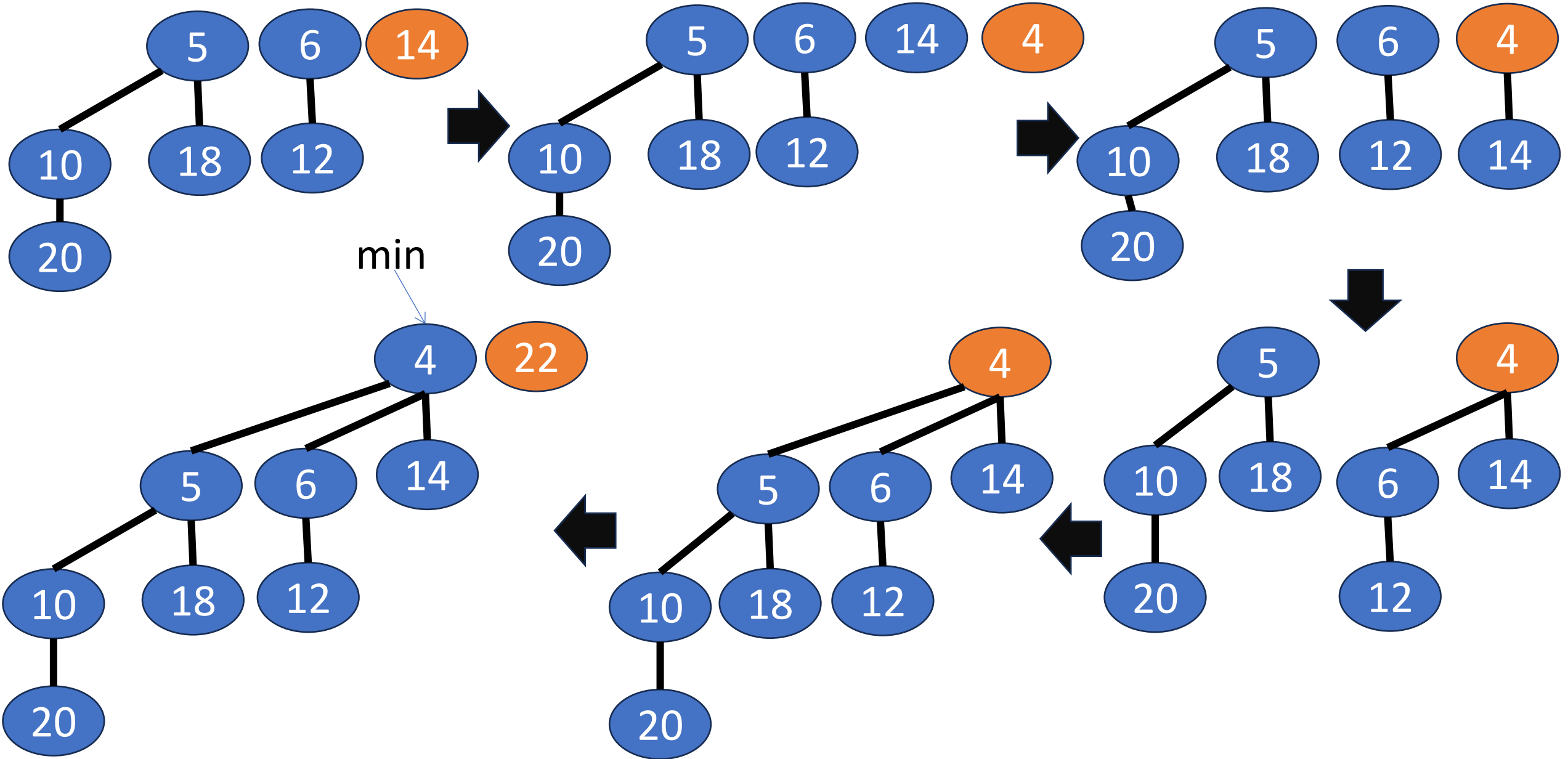
Exercise

- Q8: Into an empty B-heap, insert elements with priorities 20, 10, 5, 18, 6, 12, 14, 4, and 22 (in this order). Each insertion operation includes min-tree joining (pairwise combine). Please write the roots and degrees of min trees in the final B-heap.



Exercise

- Inserting 20, 10, 5, 18, 6, 12, 14, 4, and 22 (in this order).



Exercise

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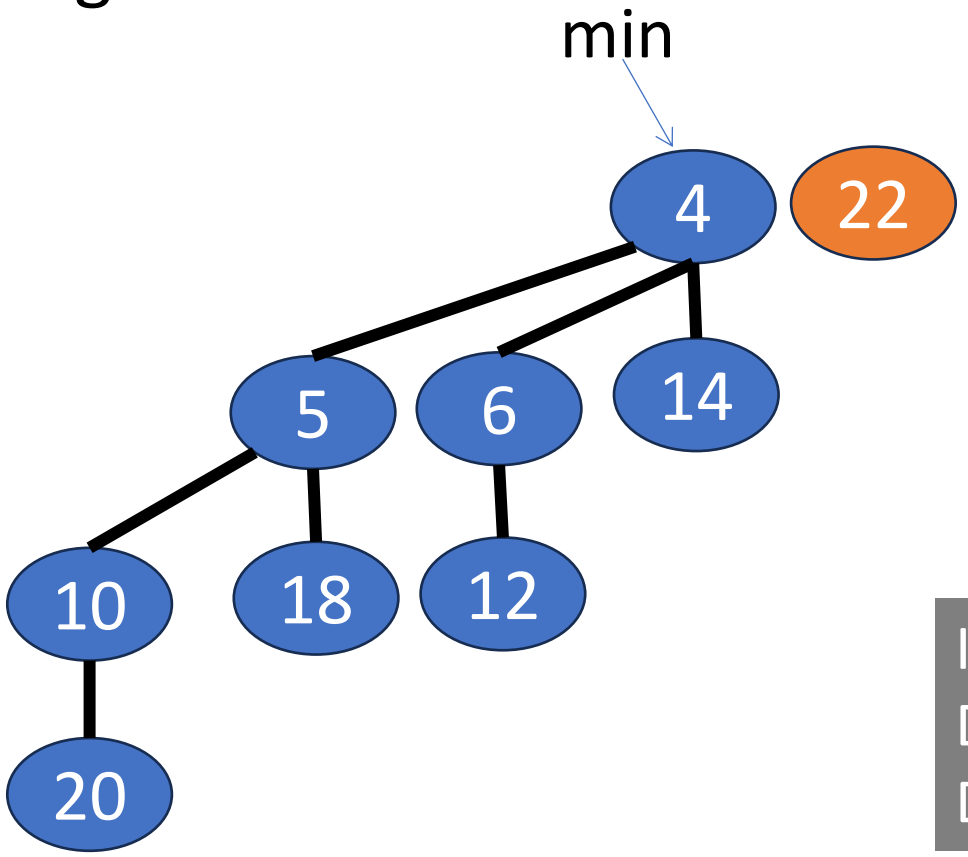


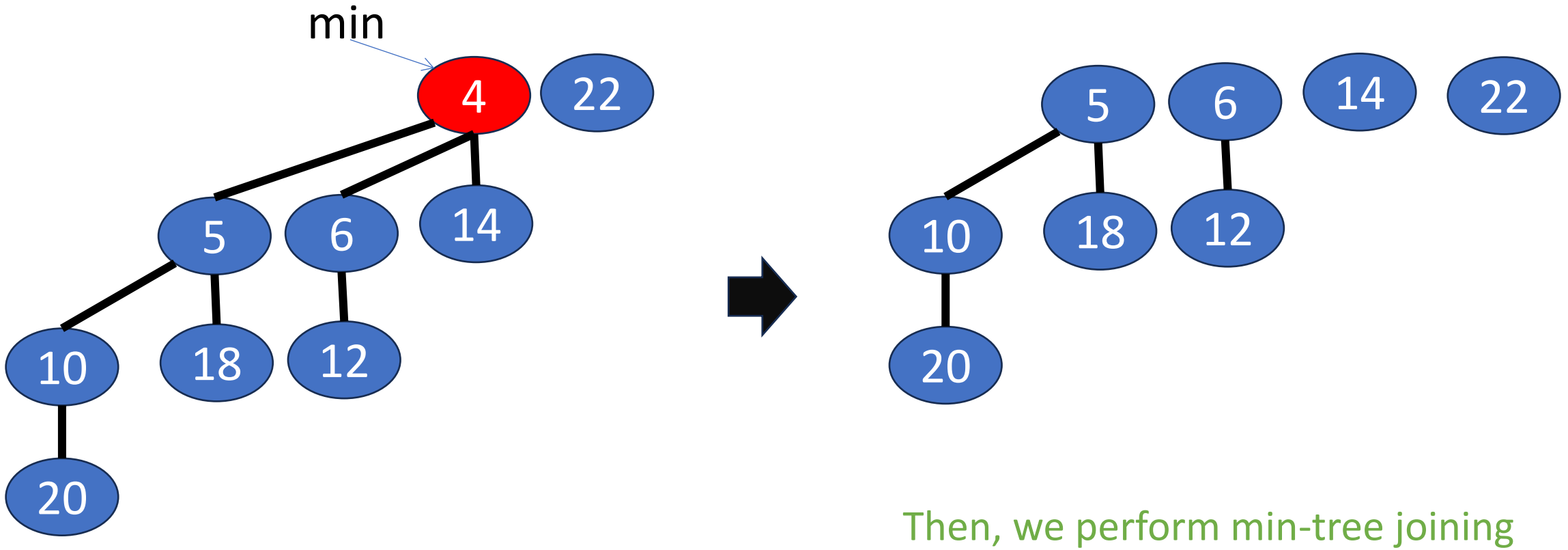
Roots: 4, 22

Degree of min-trees: 3, 0

In B heap:
Degree of a node = number of its children
Degree of tree = root's degree

Exercise

- Q9: Delete the min element from the final B-heap of Q8. Please write the roots and degrees of min trees in the resulting B-heap.



Then, we perform min-tree joining (pairwise combine).

Exercise

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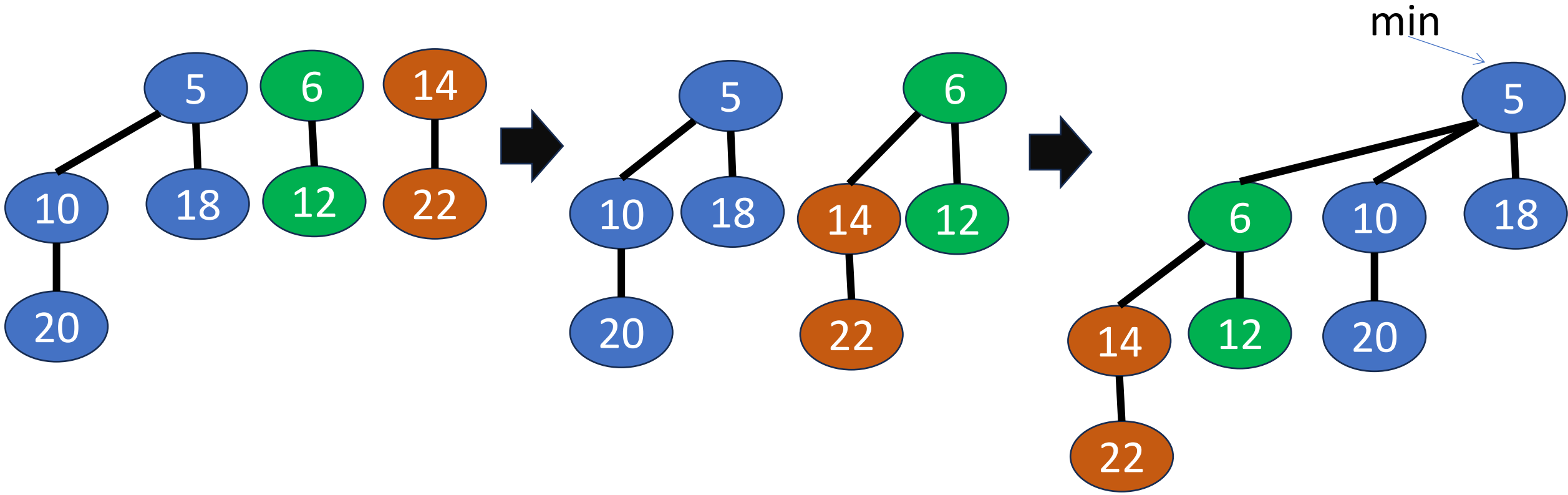

Roots: 5

Degree of min-trees: 3