UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
MIE 100S - Dynamics

FINAL EXAM
April 26, 2006
0930 hrs. to 1200 hrs.
Examiners: R. Ben Mrad, H. Kunov, C. Simmons, A.N. Sinclair, L.A. Sinclair

Allowable Calculators: Casio260, Sharp520 or TI30

Answer all questions

TYPE C

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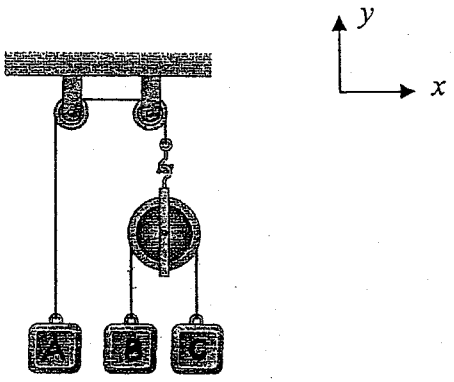
1. (a) The three blocks shown move with constant velocities. The relative velocity of A with respect to C is $+0.3\hat{j}\ m/s$ and the relative velocity of B with respect to A is $-0.2\hat{j}\ m/s$. Find the velocity of B. **(10 marks)**



(b) A particle is located at $(r,\theta)=\left(1,\frac{\pi}{4}\right)$ with r in meters and θ in radians. Its motion can be described as:

$$\dot{r}=1\ m/s$$
$$\dot{\theta}=1\ s^{-1}$$
$$\ddot{r}=1\ m/s^2$$
$$\ddot{\theta}=2\ s^{-2}$$

Using a standard rectangular (x y) co-ordinate system for your answers (centered at $r = 0$, and the x-axis aligned with $\theta = 0$) find:

(i) the direction of the unit vector in the tangential direction. **(3 marks)**

(ii) the acceleration of the particle. **(7 marks)**

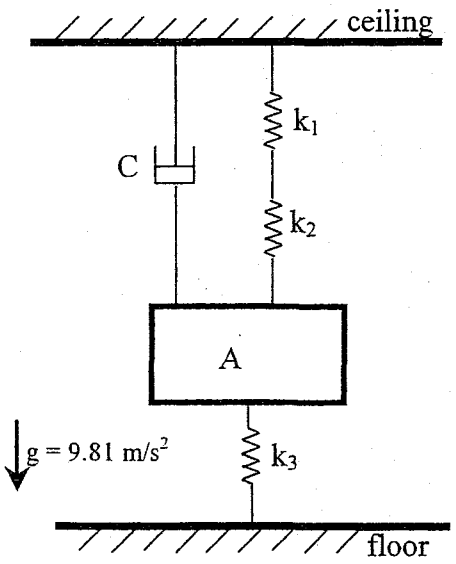
2. The 5-kg block "A" oscillates in the vertical direction. Assume that all springs are stretched at all times. Ignore rotation. $k_1 = 800N/m$; $k_2 = 1000N/m$; $k_3 = 1600N/m$.

(a) Find the undamped natural frequency ω_n of the system, when the damper $c = 0$. **(5 marks)**

(b) Find the damping factor c/c_c such that the damped natural frequency ω_d of the system is 97% of the undamped natural frequency ω_n. **(5 marks)**

(c) Find the damping factor c/c_c such that the damped, unforced oscillations die out to only 1/10 of their original amplitude in 25 seconds. **(5 marks)**

(d) Suppose that an earthquake causes the ceiling and floor to oscillate up and down together at 7 *Hz*, with amplitude of X meters. For what value(s) of X would the mass vibrate with an amplitude of 1 mm? Assume that the damping $c = 0$. **(5 marks)**



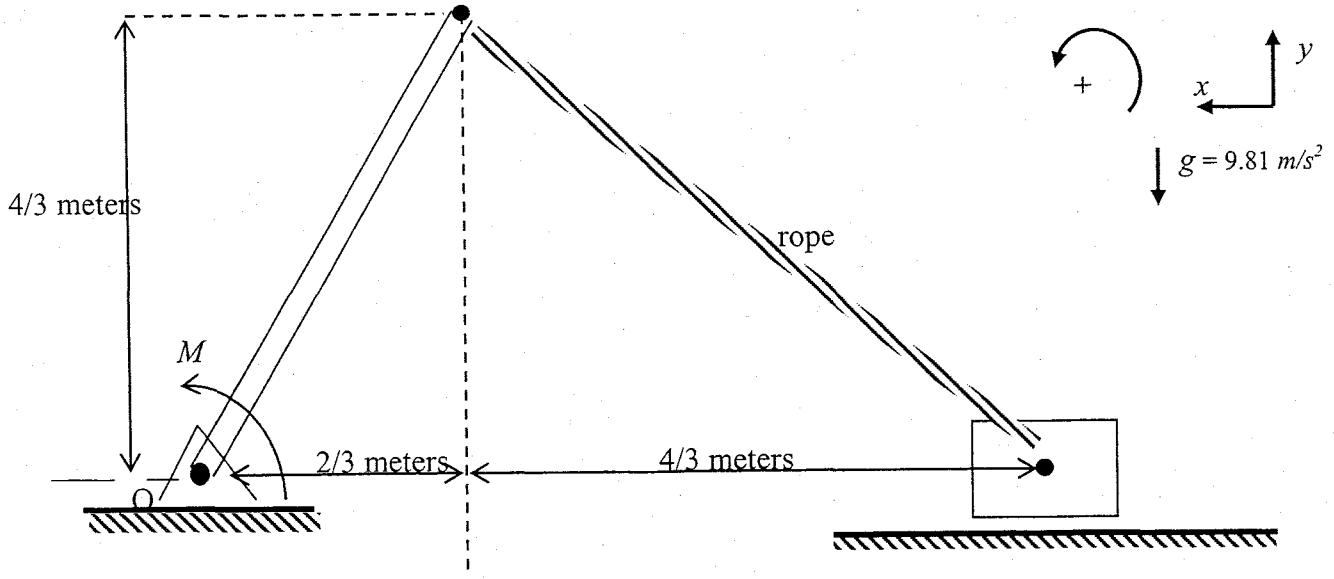
3. A uniform spinning disk drops a very short distance onto a flat horizontal surface. Initially, the disk will slip, and accelerate in a horizontal direction. Eventually, it will roll without slipping, with constant velocity. Determine the final velocity of the center of the disk. Ignore the potential energy associated with this small vertical drop. **(20 marks)**

Radius of the disk = R
Mass of disk = m
Initial angular velocity = ω_0
Kinetic and static friction coefficients = μ_k and μ_s

4. Two different crates are transported by the system shown. In each case, the uniform bar is 10 kg and the coefficient of kinetic friction (μ_k) between the crate and the horizontal surface is 0.2.

(a) The first crate has a mass of 40kg. The crate's linear acceleration is observed to be $+1.3\,\hat{i}\ m/s^2$ when it is in the configuration shown. Find the tension (T) in the rope. **(10 marks)**

(b) When the second crate is transported, the bar's angular acceleration is observed to be $4.5\ s^{-2}$ when it is in the configuration shown. At that point, the tension in the rope is also known to be 200 N. Find the external moment (M) at O that is being applied to the bar. **(10 marks)**



5. A long, slender uniform rod (solid body pendulum) is pinned at its end and is released from rest at the instant shown ($\theta = 0°$). After falling to the vertical position ($\theta = 90°$), the rod strikes the end of a slender, uniform stick that is lying at rest on a frictionless, horizontal table. The duration of impact between the rod and stick is 0.01 seconds and during this time the impact force, F, can be approximated as being constant. Immediately after impact, the stick on the table has an angular speed of 6 s^{-1}.

(a) What is the angular velocity of the pendulum immediately before impact? **(4 marks)**

(b) What is the magnitude of the impact force between the pendulum and stick? **(8 marks)**

(c) What is the angular velocity of the pendulum immediately after impact? **(8 marks)**