University of Toronto

Faculty of Applied Science and Engineering

Final Examination, December 2016

Duration: 2 and 1/2 hrs

First Year - CHE, CIV, CPE, ELE, ENG, IND, LME, MEC, MMS

MAT186H1F - Calculus I

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Exam Type: A. Aids permitted: Casio FX-991 or Sharp EL-520 calculator.

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Instructions:

- DO NOT WRITE ON THE QR CODE AT THE TOP OF THE PAGES.
- This exam contains 12 pages, including this cover page, printed two-sided. Make sure you have all of them. Do not tear any pages from this exam.
- This exam consists of nine questions, some with two parts. Attempt all of them. Each question is worth 10 marks. Marks for parts of a question are indicated in the question. **Total Marks: 90**
- Notation: $\sin^{-1} x, \cos^{-1} x, \tan^{-1} x$ and $\sec^{-1} x$ are all inverse functions, not reciprocals.
- PRESENT YOUR SOLUTIONS IN THE SPACE PROVIDED. You can use pages 11 and 12 for rough work. If you want anything on pages 11 or 12 to be marked you must indicate in the relevant previous question that the solution continues on page 11 or 12.

1. Find the following:

   (a) [3 marks] the equation(s) of the vertical asymptote(s) to the graph of $y = \dfrac{x^2 - 1}{x^2 - 4x + 3}$.

   (b) [3 marks] $\lim\limits_{x \to 0} \dfrac{\sin^{-1}(2x)}{\tan x}$

   (c) [4 marks] the maximum and minimum values of $f(x) = x^3 - 3x$ on the interval $[0, 2]$.

2. Let R be the region bounded by the curves $y = 3x$ and $y = x^2$ for $0 \leq x \leq 3$. Set up (**but do not calculate**) integrals with respect to x that give the following volumes:

   (a) [3 marks] the volume of the solid generated by revolving R about the x-axis.

   (b) [3 marks] the volume of the solid generated by revolving R about the y-axis.

   (c) [2 marks] the volume of the solid generated by revolving R about the line with equation $x = -2$.

   (d) [2 marks] the volume of the solid generated by revolving R about the line with equation $y = 9$.

3. Let $v = 10\sin(2t)$ be the velocity of a particle at time t, for $0 \leq t \leq 3\pi/2$. Find:

   (a) [4 marks] the average velocity of the particle.

   (b) [6 marks] the average speed of the particle.

4. Find and simplify the following:

(a) [4 marks] $F'(3)$, if $F(x) = \int_0^{\sqrt{x}} \tan^{-1} t \, dt$

(b) [6 marks] $\int_0^4 x^3 \sqrt{16 - x^2} \, dx$

5. Let A be the area of the region bounded by $y = \ln x, y = 2, y = 0$ and $x = 0$. (Draw a picture!)

   (a) [3 marks] Express the value of A in terms of one or more integrals with respect to x.

   (b) [3 marks] Express the value of A in terms of one or more integrals with respect to y.

   (c) [4 marks] Find A.

6. A water tank is shaped like a sphere with radius 3 m. If the tank is full, how much work is required to pump all the water to an exit pipe 1 m above the top of the tank? (Assume the density of water is $\rho = 1000$ kg/m^3 and that $g = 9.8$ m/sec^2.)

7. Consider the curve $y = x^3 + \dfrac{1}{12x}$, for $1 \le x \le 2$.

(a) [5 marks] Find the length of the curve.

(b) [5 marks] Find the area of the surface generated by revolving the curve about the y-axis.

8. Let $f(x) = x^{6/5} + 6\,x^{1/5}$, for which $f'(x) = \dfrac{6}{5}x^{1/5} + \dfrac{6}{5}x^{-4/5}$ and $f''(x) = \dfrac{6}{25}x^{-4/5} - \dfrac{24}{25}x^{-9/5}$.

(a) [1 mark] On which interval(s) is f increasing?

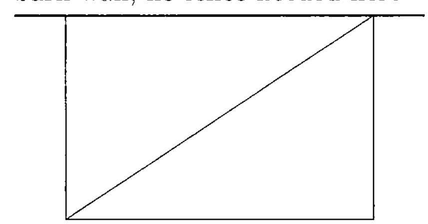
(b) [1 mark] On which interval(s) is f decreasing?

(c) [2 marks] On which interval(s) is f concave up?

(d) [1 mark] On which interval(s) is f concave down?

(e) [5 marks] Sketch the graph of f indicating critical points and inflection points, if any.

9. Consider the problem:



Two triangular pens (areas for animals) are built against a barn. Two hundred meters of fencing are to be used for the three sides and the diagonal dividing fence. See figure to the right. The total area of the two pens is to be maximized.

(a) [3 marks] Set this problem up in terms of the dimensions of the rectangle in the figure. That is: which function is to be maximized, subject to which constraint(s)?

(b) [3 marks] Now set this problem up in terms of an acute angle in one of the triangles, and the length of the diagonal dividing fence.

(c) [4 marks] Express the total area of the pens in terms of one variable. What equation would you have to solve to find the critical point(s) of your total area function? (You do not actually have to find the critical point(s).)

This page is for rough work or for extra space to finish a previous problem. It will not be marked unless you have indicated in a previous question to look at this page.

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