UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 2017

DURATION: 2 AND 1/2 HRS

FIRST YEAR - CHE, CIV, CPE, ELE, ENG, IND, LME, MEC, MMS

MAT186H1F - Calculus I

EXAMINERS: S. AMELOTTE, F. BISCHOFF, D. BURBULLA, S. COHEN, J. ENNS, M. MATVIICHUK, M. PALASCIANO, K. PHAM

Exam Type: A. Aids permitted: Casio FX-991 or Sharp EL-520 calculator.

Full Name: _____

Student Number: _____

UTor email: _____ @mail.utoronto.ca

Signature: _____

Instructions:

- DO NOT WRITE ON THE QR CODE AT THE TOP OF THE PAGES.
- This exam contains 12 pages, including this cover page, printed two-sided. Make sure you have all of them. Do not tear any pages from this exam.
- This exam consists of eight questions, some with many parts. Attempt all of them. Each question is worth 10 marks. Marks for parts of a question are indicated in the question. **Total Marks: 80**
- Notation: $\sin^{-1} x, \cos^{-1} x, \tan^{-1} x$ and $\sec^{-1} x$ are all inverse functions, not reciprocals.
- PRESENT YOUR SOLUTIONS IN THE SPACE PROVIDED. You can use pages 10, 11 and 12 for rough work. If you want anything on pages 10, 11 or 12 to be marked you must indicate in the relevant previous question that the solution continues on page 10, 11 or 12.

1. Find the following:

(a) [3 marks] $\lim_{x \to 0} \dfrac{\sin(2x)}{3x}$

(b) [4 marks] the value of $\dfrac{dy}{dx}$ at the point $(x, y) = (1, 2)$ if $x^2 + xy^3 = x + 8$.

(c) [3 marks] the equation of the vertical tangent line to the graph of $y = x^{7/3} - 7\,x^{1/3}$

Continued...

2. Let R be the region bounded by the curves $y = 5\,x$ and $y = x^2$ for $0 \leq x \leq 5$. Find the following:

   (a) [5 marks] the volume of the solid generated by revolving R about the x-axis.

   (b) [5 marks] the volume of the solid generated by revolving R about the line $x = -3$.

3. Let $v = -t^2 + 5t - 4$ be the velocity of a particle at time t, for $0 \leq t \leq 4$. Find:

(a) [4 marks] the average velocity of the particle.

(b) [6 marks] the average speed of the particle.

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4. Find the exact values of the following:

(a) [3 marks] $\int_{-3}^{3} (x^7 \cos x - x^2)\, dx$

(b) [3 marks] $\int_{0}^{4} \sqrt{16 - x^2}\, dx$ (Hint: interpret the integral as an area.)

(c) [4 marks] $\int_{0}^{4} x\sqrt{16 - x^2}\, dx$

Continued...

5. Let A be the area of the region bounded by $y = \tan^{-1} x$ and $y = \dfrac{\pi x}{3\sqrt{3}}$. (Draw a diagram.)

(a) [3 marks] Express the value of A in terms of one or more integrals with respect to x.

(b) [3 marks] Express the value of A in terms of one or more integrals with respect to y.

(c) [4 marks] Find the value of A.

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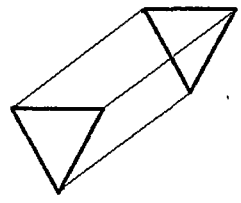
6. A water trough has a horizontal length of 2 m, and vertical cross sections which are equilateral triangles, with each side of length 0.5 m . (For the shape of the trough, see the diagram to the right.) If the trough is full, how much work is required to pump all the water up to an exit pipe 1 m above the top of the trough? (Assume the density of water is $\rho = 1000$ kg/m^3 and that $g = 9.8$ m/sec^2.)



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7. Consider the curve with equation $y = \dfrac{e^x + e^{-x}}{2}$ for $-\ln 2 \leq x \leq \ln 2$.

(a) [5 marks] Find the length of the curve.

(b) [5 marks] Find the area of the surface generated by revolving the curve about the x-axis.

8. Find the following:

(a) [5 marks] the equations of *two* continuous and differentiable functions defined on the interval $[2, 6]$ with graphs that pass through the point $(3, 5)$ and have arc length given by $\int_2^6 \sqrt{1 + 9\,x^{-4}}\,dx$.

(b) [5 marks] the critical point(s) of $C(x) = \int_0^x \cos(t^2)\,dt$ on the interval $[0, \sqrt{2\pi}]$, and determine if C has a maximum or minimum value at each critical point.

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