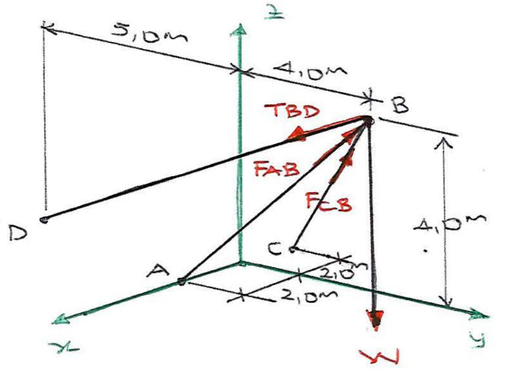
Problem Set 4 (PS4)

Solution

1.



- First, draw a FBD. Logically assume the directions of unknown forces.
- Express all forces in cartesian vector formulation

A(2,0 ; 0,0 ; 0,0)
B(0,0 ; 4,0 ; 4,0)
C(-2,0 ; 0,0 ; 0,0)
D(0,0 ; -5,0 ; 0,0)

- $\underset{\sim}{r}_{AB} = -2{,}0\,\underset{\sim}{i} + 4{,}0\,\underset{\sim}{j} + 4{,}0\,\underset{\sim}{k}$; $r_{AB} = \sqrt{2{,}0^2 + 4{,}0^2 + 4{,}0^2} = 6{,}0\ m$

  $\underset{\sim}{u}_{AB} = \dfrac{\underset{\sim}{r}_{AB}}{r_{AB}} = -0{,}333\,\underset{\sim}{i} + 0{,}667\,\underset{\sim}{j} + 0{,}667\,\underset{\sim}{k}$

  $\underset{\sim}{F}_{AB} = F_{AB} \cdot \underset{\sim}{u}_{AB} = -0{,}333 F_{AB}\,\underset{\sim}{i} + 0{,}667 F_{AB}\,\underset{\sim}{j} + 0{,}667 F_{AB}\,\underset{\sim}{k}$

- $\underset{\sim}{r}_{CB} = +2{,}0\,\underset{\sim}{i} + 4{,}0\,\underset{\sim}{j} + 4{,}0\,\underset{\sim}{k}$; $r_{CB} = 6{,}0\ m$

  $\underset{\sim}{F}_{CB} = 0{,}333 F_{CB}\,\underset{\sim}{i} + 0{,}667 F_{CB}\,\underset{\sim}{j} + 0{,}667 F_{CB}\,\underset{\sim}{k}$

- $\underset{\sim}{r}_{BD} = -9{,}0\,\underset{\sim}{j} - 4{,}0\,\underset{\sim}{k}$; $r_{BD} = 9{,}849\ m$

  $\underset{\sim}{T}_{BD} = -0{,}914\, T_{BD}\,\underset{\sim}{j} - 0{,}406\, T_{BD} \cdot \underset{\sim}{k}$

- $W = -0{,}2 \cdot 9{,}81\,\underset{\sim}{k} = -1{,}962\,\underset{\sim}{k}$

- Apply 3 equilibrium equations:

$\boxed{\Sigma F_x = 0}$ ($\underset{\sim}{i}$ terms) $\quad -0{,}333 F_{AB} + 0{,}333 F_{CB} = 0 \Rightarrow F_{AB} = F_{CB}$

(makes sense due to symm.)

$\boxed{\Sigma F_y = 0}$ ($\underset{\sim}{j}$ terms) $\quad 0{,}667 F_{AB} + 0{,}667 F_{BC} - 0{,}914 T_{BD} = 0 \Rightarrow F_{AB} = 0{,}685\, T_{BD}$

$\boxed{\Sigma F_z = 0}$ ($\underset{\sim}{k}$ terms) $\quad \underbrace{0{,}667 F_{AB} + 0{,}667 F_{BC}} - 0{,}406\, T_{BD} - 1{,}962^{kN} = 0$

$0{,}914\, T_{BD} - 0{,}406\, T_{BD} = 1{,}962\ kN \Rightarrow T_{BD} = 1{,}486^{kN}$

$\Rightarrow F_{AB} = 1{,}018^{kN}$

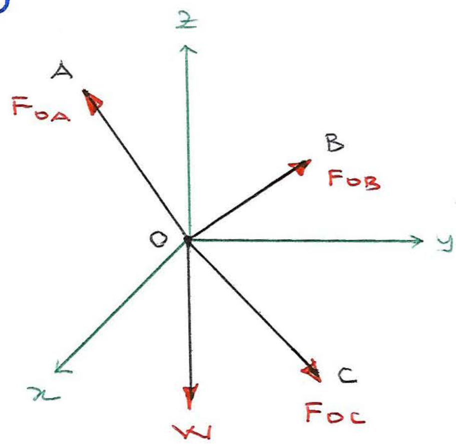
$\Rightarrow F_{CB} = 1{,}018^{kN}$

Answer: Leg AB : $F_{AB} = 1{,}02\ kN$ compression

Leg CB : $F_{CB} = 1{,}02\ kN$ compression

Cable BD : $T_{BD} = 1{,}49\ kN$ tension

2-



- First, draw a FBD. Logically assume directions of unknowns. All cables → tension.
- Express all forces in cartesian vector formulation.

A(−1,0 ; −3,0 ; 4,2)
B(−1,0 ; 2,1 ; 1,6)
C(6,0 ; 5,5 ; 0,0)

- $\underset{\sim}{r}_{OA} = -1{,}0\,\underset{\sim}{i} - 3{,}0\,\underset{\sim}{j} + 4{,}2\,\underset{\sim}{k}$; $r_{OA} = 5{,}257$ m

$\underset{\sim}{T}_{OA} = \dfrac{\underset{\sim}{r}_{OA}}{r_{OA}} = -0{,}190\,T_{OA}\,\underset{\sim}{i} - 0{,}571\,T_{OA}\,\underset{\sim}{j} + 0{,}799\,T_{OA}\,\underset{\sim}{k}$

- $\underset{\sim}{r}_{OB} = -1{,}0\,\underset{\sim}{i} + 2{,}1\,\underset{\sim}{j} + 1{,}6\,\underset{\sim}{k}$; $r_{OB} = 2{,}823$ m

$\underset{\sim}{T}_{OB} = (-0{,}354\,\underset{\sim}{i} + 0{,}744\,\underset{\sim}{j} + 0{,}567\,\underset{\sim}{k}) \cdot T_{OB}$

- $\underset{\sim}{r}_{OC} = 6{,}0\,\underset{\sim}{i} + 5{,}5\,\underset{\sim}{j}$; $r_{OC} = 8{,}139$ m

$\underset{\sim}{T}_{OC} = 0{,}737\,T_{OC}\,\underset{\sim}{i} + 0{,}676\,T_{OC}\,\underset{\sim}{j}$

- $\underset{\sim}{W} = -W\,\underset{\sim}{k}$

- Apply 3 equilibrium equations.

$\boxed{\Sigma F_x = 0}$ $\quad -0{,}190\,T_{OA} - 0{,}354\,T_{OB} + 0{,}737\,T_{OC} = 0 \qquad (I)$
$\underset{\sim}{i}$ terms

$\boxed{\Sigma F_y = 0}$ $\quad -0{,}571\,T_{OA} + 0{,}744\,T_{OB} + 0{,}676\,T_{OC} = 0 \qquad (II)$
$\underset{\sim}{j}$ terms

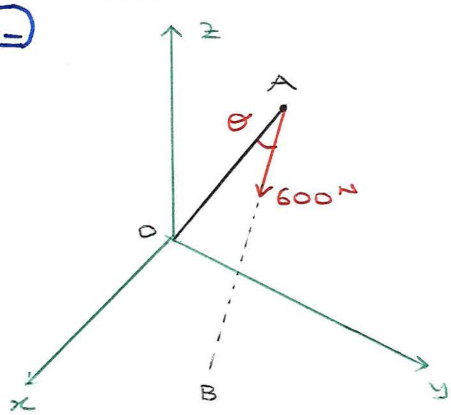
$\boxed{\Sigma F_z = 0}$ $\quad 0{,}799\,T_{OA} + 0{,}567\,T_{OB} - W = 0 \qquad (III)$
$\underset{\sim}{k}$ terms

- 3 equations with 4 unknowns. cannot solve. must make an assumption. Assume $T_{OA} = 1200$ N and solve. If $T_{OB} \leq 1200$N and $T_{OC} \leq 1200$N, the assumption is correct.

- Solving eq. (I) and (II) · $T_{OB} = 445{,}5$ N < 1200N $\boxed{OK}$ ✓
$T_{OC} = 523{,}3$ N < 1200N $\boxed{OK}$ ✓

- From eq. (3) : $W = 1211{,}4$ N <u>Answer</u> :
$W = m \cdot g \Rightarrow m = 123$ N

3.



- First, draw a FBD. Show all forces.
- Express all forces in cartesian formulation.

A(0,0 ; 3,2 ; 5,0)

B(2,6 ; 4,0 ; 0,0)

$\underset{\sim}{r}_{AB} = 2{,}6\underset{\sim}{i} + 0{,}8\underset{\sim}{j} - 5{,}0\underset{\sim}{k}$; $r_{AB} = 5{,}692$ m

$\underset{\sim}{F}_{AB} = 274{,}1\underset{\sim}{i} + 84{,}3\underset{\sim}{j} - 527{,}1\underset{\sim}{k}$

check : $F = \sqrt{274{,}1^2 + 84{,}3^2 + 527{,}1^2} = 600{,}1$ N $\boxed{\text{ok}}$ ✓

- Then, find the unit vector in requested direction (i.e, AO)

$\underset{\sim}{r}_{AO} = -3{,}2\underset{\sim}{j} - 5{,}0\underset{\sim}{k}$; $r_{AO} = 5{,}936$ m

$\underset{\sim}{u}_{AO} = \underset{\sim}{r}_{AO} / r_{AO} = -0{,}539\underset{\sim}{j} - 0{,}842\underset{\sim}{k}$

i. dot product

$F_{AO\parallel} = \underset{\sim}{F}_{AB} \bullet \underset{\sim}{u}_{AO} = (274{,}1)(0) + (84{,}3)(-0{,}539) + (-527{,}1)(-0{,}842)$

$= +398{,}4 \text{ N} \cong 399 \text{ N}$

$\underset{\sim}{F}_{OA\parallel} = F_{OA\parallel} \cdot \underset{\sim}{u}_{AO} = -214{,}7\underset{\sim}{j} - 335{,}5\underset{\sim}{k}$

check : $\cos\theta = \dfrac{\underset{\sim}{r}_{AO} \bullet \underset{\sim}{r}_{AB}}{r_{AO} \cdot r_{AB}} = \dfrac{(-3{,}2)(0{,}8) + (-5{,}0)(-5{,}0)}{5{,}692 \cdot 5{,}936} = 0{,}664$

$\Rightarrow \theta = 48{,}4°$

$F_{OA\parallel} = 600 \cdot \cos 48{,}4° = 398{,}4$ N $\boxed{\text{ok}}$ ✓

ii. $\underset{\sim}{F}_{OA\perp} = \underset{\sim}{F}_{AB} - \underset{\sim}{F}_{OA\parallel} = 274{,}1\underset{\sim}{i} + (84{,}3 + 214{,}7)\underset{\sim}{j} + (-527{,}1 + 335{,}5)\underset{\sim}{k}$

$= 274{,}1\underset{\sim}{i} + 299{,}0\underset{\sim}{j} - 191{,}6\underset{\sim}{k}$

$F_{OA\perp} = \sqrt{274{,}1^2 + 299{,}0^2 + 191{,}6^2} = 448{,}6 \text{ N} \cong 449 \text{ N}$

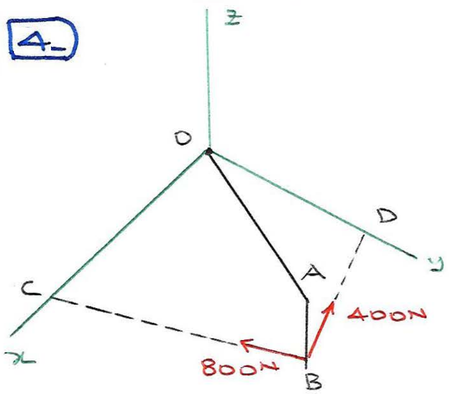
check : $F_{OA\perp} = \sqrt{600^2 - 398{,}4^2} = 448{,}6$ N $\boxed{\text{ok}}$ ✓

$F_{OA\perp} = 600 \cdot \sin 48{,}4° = 448{,}7$ N $\boxed{\text{ok}}$ ✓

iii

$\theta = 48{,}4°$ (see above solution)

$\boxed{4.}$



- First, draw a FBD. Show all forces.
- Then express all forces in cartesian formulation.

B(500 ; 1600 ; −400) mm
C(2000 ; 0 ; 0)
D(0 ; 1600 ; 0)

- $\underset{\sim}{r}_{BC} = 1500\underset{\sim}{i} - 1600\underset{\sim}{j} + 400\underset{\sim}{k}$; $r_{BC} = 2229,3$ mm
  $\underset{\sim}{F}_{BC} = 538,3\underset{\sim}{i} - 574,2\underset{\sim}{j} + 143,5\underset{\sim}{k}$

- $\underset{\sim}{r}_{BD} = -500\underset{\sim}{i} + 0\underset{\sim}{j} + 400\underset{\sim}{k}$; $r_{BD} = 640,3$ mm
  $\underset{\sim}{F}_{BD} = -312,4\underset{\sim}{i} + 0\underset{\sim}{j} + 249,9\underset{\sim}{k}$

- Then, select moment arms with as many zero components as possible (to simplify the math.)

  For $\underset{\sim}{F}_{BC}$, select $\underset{\sim}{r}_{OC} = 2000\underset{\sim}{i} + 0\underset{\sim}{j} + 0\underset{\sim}{k}$
  For $\underset{\sim}{F}_{BD}$, select $\underset{\sim}{r}_{OD} = 0\underset{\sim}{i} + 1600\underset{\sim}{j} + 0\underset{\sim}{k}$

- Finally, apply the moment equation :
  $\underset{\sim}{M}_O = \underset{\sim}{r}_{OC} \times \underset{\sim}{F}_{BC} + \underset{\sim}{r}_{OD} \times \underset{\sim}{F}_{BD}$ cross-product

$$M_O = \begin{vmatrix} i & j & k \\ 2000 & 0 & 0 \\ 538,3 & -574,2 & 143,5 \\ i & j & k \\ 2000 & 0 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 1600 & 0 \\ -312,4 & 0 & 249,9 \\ i & j & k \\ 0 & 1600 & 0 \end{vmatrix}$$

0, 0, $-287\,000\underset{\sim}{j}$; 0, $-1148400\underset{\sim}{k}$, 0 ; $-499840\underset{\sim}{k}$, 0, 0 ; $399840\underset{\sim}{i}$, 0, 0

$= [-1148400k - (-287\,000j)] + [399840i - (-499840\underset{\sim}{k})]$

$M_O = 399840\underset{\sim}{i} + 287\,000\underset{\sim}{j} - 648560\underset{\sim}{k}$ N.mm

- Need 4 significant figures.

<u>Answer</u> :
$M_O = 399,8\underset{\sim}{i} + 287,0\underset{\sim}{j} - 648,6\underset{\sim}{k}$ N.m