Fundamental Equations

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

5 Equations of Motion (Constant acceleration, straight line motion)

$$v = v_o + at$$

$$v^2 = v_o^2 + 2a\Delta x$$

$$\Delta x = \frac{v + v_o}{2} t$$

$$\Delta x = vt - \frac{1}{2}at^2$$

$$\Delta x = v_o t + \frac{1}{2}at^2$$

Polar Coordinate Equations

$$\vec{r} = r\widehat{U}_r$$

$$\vec{v} = \dot{r}\widehat{U}_r + r\dot{\theta}\widehat{U}_\theta$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\widehat{U}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\widehat{U}_\theta$$

N-T Coordinate Equations

$$\vec{v} = v\widehat{U}_t$$

$$\vec{a} = \dot{v}\widehat{U}_t + \frac{v^2}{\rho}\widehat{U}_n = a_t\widehat{U}_t + a_n\widehat{U}_n$$

Curvature

$$\rho = \frac{[1 + \left(\frac{dy}{dx}\right)^2]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|}$$

Relative Motion

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$\vec{r}_A$: position A, $\vec{r}_B$: position B, $\vec{r}_{B/A}$: position B – position A
(These are all vectors!!!)

Relative Motion (Pulleys)

$$\vec{r}_c = \frac{\vec{r}_A + \vec{r}_B}{2}$$

$$\vec{v}_c = \frac{\vec{v}_A + \vec{v}_B}{2}$$

$$\vec{a}_c = \frac{\vec{a}_A + \vec{a}_B}{2}$$

For above equations
$\vec{r}_A$ – One point on the circle
$\vec{r}_c$ – Center of the circle
$\vec{r}_B$ – Opposite side of $\vec{r}_A$

Force of a Spring

$$F = -kx$$

k: stiffness [N/m]
x: displacement [m]

Frictional Forces

$$|F_{fsmax}| = \mu_s F_N$$

No slippage, prevents object from moving
μ_s: Coefficient of static friction
F_N: Normal force (mg)

$$|F_{fk}| = \mu_k F_N$$

Relative motion between bodies, tries to stop object from moving
μ_k: Coefficient of kinetic friction
F_N: Normal force (mg)

Kinetic Energy

$$T = \frac{1}{2}mv^2$$

$$T_1 + \sum U_{1\to 2} = T_2$$

$$U_g = -mg\Delta h$$

$$U_f = -F_f \Delta S_T$$

$$U_S = U_e = -\frac{1}{2}k(S_2^2 - S_1^2)$$

Work

$$dU = \vec{F} \cdot d\vec{r}$$

$$dU = |\vec{F}||d\vec{r}|\cos\theta$$

$$U_{1\to 2} = \int_1^2 \vec{F} \cdot d\vec{r}$$

$$\sum F_x = m\ddot{x}$$

$$\sum F_y = m\ddot{y}$$

$$\sum F_t = m\dot{v}$$

$$\sum F_n = m\frac{v^2}{\rho}$$

$$\sum F_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$\sum F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

Impulse + Momentum

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$\int_{t_1}^{t_2} \sum \vec{F} \cdot dt = \int_{\vec{v}_1}^{\vec{v}_2} m \cdot d\vec{v}$$

$$m\vec{v}_1 + \int_{t_1}^{t_2} \sum \vec{F} \cdot dt = m\vec{v}_2$$

$$T_1 + V_{g1} + V_{e1} + U_{1\to 2}^* = T_2 + V_{g2} + V_{e2}$$

The term with the * represents the external energy

Angular Momentum

$$\vec{M}_o = \vec{r}_{F/o} \times \vec{F}$$

(moment)

$$\vec{H}_o = \vec{r} \times m\vec{v}$$

(angular momentum)

Linear Momentum

$$\vec{L} = m\vec{v}$$

Moment

$$M = I\alpha$$

Equations for Constant Angular Acceleration

$$\omega = \omega_o + \alpha_c t$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha_c t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha_c(\theta - \theta_o)$$

Angular Velocity/Acceleration

$$v = \omega r$$

$$a = \alpha r$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

$$\alpha\, d\theta = \omega\, d\omega$$

$a_t = \alpha r$ (Tangential component)

$a_n = \omega^2 r$ (Normal component)

$$\vec{a} = \vec{a}_t + \vec{a}_n = (\alpha \times r)\widehat{U}_t - (\omega^2 r)\widehat{U}_n$$

Relative Velocity/Acceleration

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega^2\vec{r}_{B/A}$$

$$\omega_{AB} = \frac{|\vec{v}_{B/A}|}{\vec{r}_{B/A}} = \frac{|\vec{v}_A - \vec{v}_A|}{\vec{r}_{B/A}}$$

$$\alpha_{AB} = \frac{|\vec{a}_{B/A}|}{\vec{r}_{B/A}} = \frac{|\vec{a}_A - \vec{a}_A|}{\vec{r}_{B/A}}$$

Impulse and Momentum for shit that's spinning:

$$\boldsymbol{I_G\omega_1 = I_G\omega_2}$$

Vibrations

$$\ddot{x} + \omega_n^2 x = 0$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$x(t) = A\sin(\omega_n t) + B\cos(\omega_n t)$$

$$x(t) = C\sin(\omega_n t + \emptyset)$$

$$\tau_n = \frac{2\pi}{\omega_n}$$

τ_n – Natural period [s]

$$f_n = \frac{1}{\tau_n}$$

f_n – Frequency [Hz]

$$\omega_n = \frac{2\pi}{\tau_n}$$

ω_n – Natural frequency [rad/s]

$$A = C\cos(\emptyset)$$

$$B = C\sin(\emptyset)$$

$$k_{eq} = k_1 + k_2 + \cdots$$

Springs in parallel

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots$$

Springs in series

Second Order Differential Equations

$$x(t) = x_c + x_p$$

$$x_c = A\sin(\omega_n t) + B\cos(\omega_n t)$$

$$x_p = C\sin(\omega_o t)$$

ω_o – Forcing frequency

$$M = \frac{C}{F_o/_k} = \frac{1}{1 - (\frac{\omega_o}{\omega_n})^2}$$

Energy of a Rigid Body

$$T_1 + V_{g1} + V_{e1} + U^*_{1\to 2} = T_2 + V_{g2} + V_{e2}$$

Kinetic Energy

$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2 = \frac{1}{2}I_{IC}\omega^2$$

Moment of Inertia for Different Shapes

Rod about center:

$$I = \frac{1}{12}ml^2$$

Rod about end:

$$I = \frac{1}{3}ml^2$$

Solid cylinder, symmetry axis:

$$I = \frac{1}{2}mr^2$$

Solid cylinder, central diameter:

$$I = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$$

Parallel axis theorem:

$$I_A = I_G + mr^2$$

Radius of Gyration

$$K = \sqrt{\frac{I}{m}}$$

$$I = mk^2$$

Damped Vibration

$$F_d = -c\dot{x}$$

$$\mathrm{m}\ddot{x} + c\dot{x} + kx = 0$$

$$\mathrm{m}\lambda^2 + c\lambda + ke = 0$$

$$\lambda_{1,2} = -\frac{c}{2m} \pm \sqrt{(\frac{c}{2m})^2 + \frac{k}{m}}$$

$$(\frac{c_c}{2m})^2 = \frac{k}{m}$$

$$c_c = \sqrt{\frac{k}{m}}2m = 2m\omega_n$$

When $\lambda_{1,2}$ are real and negative (overdamped, $\frac{c}{c_c} > 1$):

$$x(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$

When $\lambda_{1,2}$ are equal (critically damped, $\frac{c}{c_c} = 1$):

$$x(t) = (A + Bt)e^{-\omega_n t}$$

When $\lambda_{1,2}$ are complex (underdamped, $\frac{c}{c_c} < 1$):

$$x(t) = De^{-\frac{c}{2m}t}\sin(\omega_d t + \emptyset)$$

$$\omega_d = \omega_n\sqrt{1 - (\frac{c}{c_c})^2}$$

$$\omega_d = \frac{\sqrt{c^2 - 4mk}}{2m}$$