University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER, 2008

First Year - CHE, CIV, IND, LME, MEC, MMS

MAT186H1F - CALCULUS I

Exam Type: A

SURNAME: (as on your T-card) _____

GIVEN NAMES: _____

STUDENT NUMBER: _____

SIGNATURE: _____

Examiners:
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D. Burbulla
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Calculators Permitted: Casio 260, Sharp 520 or TI 30.

Notation: $\sin^{-1} x, \cos^{-1} x, \tan^{-1} x$ and $\sec^{-1} x$ are inverse trigonometric functions, which can also be represented by $\arcsin x$, $\arccos x$, $\arctan x$ and $\operatorname{arcsec} x$, respectively.

INSTRUCTIONS: Attempt all questions. Present your solutions in the space provided. Use the backs of the sheets if you need more space. Do not tear any pages from this exam. Make sure your exam contains 10 pages.

MARKS: Questions 1 through 6 are Multiple Choice; circle the single correct choice for each question. Each correct choice is worth 4 marks.

Questions 7 through 10 each have two parts and are worth 10 marks; 5 marks for each part.

Questions 11 through 13 are each worth 12 marks.

TOTAL MARKS: 100

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1. Suppose the function $f(x)$ is continuous for all values of x. What is the greatest possible number of asymptotes to the graph of f?

   (a) 1

   (b) 2

   (c) 3

   (d) 4

2. How many vertical asymptotes are there to the graph of $f(x) = \dfrac{\sin(x^2-1)}{x^4-5x^2+4}$?

   (a) 1

   (b) 2

   (c) 3

   (d) 4

3. The arc length of the curve $f(x) = \tan x$ for $0 \leq x \leq 1$ is given by

   (a) $\displaystyle\int_0^1 \sqrt{1+\tan^4 x}\, dx$

   (b) $\displaystyle\int_0^1 \sec x\, dx$

   (c) $\displaystyle\int_0^1 \sqrt{1+\sec^2 x}\, dx$

   (d) $\displaystyle\int_0^1 \sqrt{1+\sec^4 x}\, dx$

4. Which of the following is equal to $\sin(3A)$, for all values of A?

   (a) $3\sin A - 4\sin^3 A$

   (b) $3\sin A\cos A$

   (c) $4\sin A - 3\sin^3 A$

   (d) $3\sin A + 4\sin^3 A$

5. The value of $\int_0^{\ln 2} e^x \ln(e^x+1)\,dx$ is given by

   (a) $\int_0^{\ln 2} \ln u\,du$

   (b) $\int_1^{2} \ln u\,du$

   (c) $\int_2^{3} \ln u\,du$

   (d) $\int_0^{\ln 2} \ln(u+1)\,du$

6. $\int_{-2}^{4} |x|\,dx =$

   (a) 10

   (b) 12

   (c) 8

   (d) 14

7. Find the following limits:

(a) $\lim_{x \to 0} \dfrac{2x - \sin^{-1}(2x)}{x^3}$.

(b) $\lim_{x \to \infty} \left(1 + \dfrac{\pi}{2} - \tan^{-1} x\right)^x$

8(a) Find $F'(4)$ if $F(x) = \int_{\pi}^{\sqrt{x}} e^{-t^2}\, dt$.

8(b) Show that $\int_{-1}^{1} \cos^{-1} x\, dx = \pi$. (Hint: draw a graph.)

9. Suppose the velocity of a particle at time t is given by $v = 3t^2 - 6t$, for $0 \leq t \leq 4$. Find the following:

(a) the net change in position of the particle from $t = 0$ to $t = 4$.

(b) the total distance travelled by the particle for $0 \leq t \leq 4$.

10. The Average Value Theorem, as stated in our text book, says

    If f is continuous on $[a, b]$, then

$$f(\overline{x}) = \frac{1}{b-a} \int_a^b f(x)\, dx$$

    for some number $\overline{x}$ in $[a, b]$.

(a) Illustrate this theorem with a suitable picture, and indicate why, to quote Edwards and Penney, it means:

> Every continuous function on a closed interval attains its average value at some point of the interval.

(b) Use the Intermediate Value Property to prove the Average Value Theorem.

11. A storage tank, full of water with density ρ, is in the shape of a sphere with radius 1 m. How much work is done in emptying the tank by pumping all the water up to a transfer pipe 1 m above the top of the tank?

12. A spherical snowball is melting in such a way that the rate of decrease of its volume is proportional to its surface area. At 9AM its volume is 500 cc and at 10AM its volume is 250 cc. When does the snowball finish melting?

13. Let R be the region in the plane bounded by $x = 1, x = 2, y = 0$ and $y = \dfrac{1}{x^2}$.

(a) [6 marks] Find the volume of the solid of revolution obtained by revolving R about the line $x = -1$.

(b) [6 marks] What is the volume of the solid of revolution obtained by revolving R about the line $y = 2$?