FACULTY OF APPLIED SCIENCE AND ENGINEERING
University of Toronto
FINAL EXAM, MONDAY, APRIL 24, 2012

MAT 188S
Linear Algebra

Examiner: S. Cohen
Duration: 2 hours, 30 minutes

Calculators allowed – Casio 260, Sharp 520, or TI 30. **Total: 80 marks**

Family Name: _____

Given Name(s): _____

Please sign here: _____

Student ID number: _____

For Markers Only

Question	Marks
1	/ 15
2	/ 10
3	/ 12
4	/ 14
5	/ 12
6	/ 12
TOTAL	**/ 75**

1. [15 marks] Let $A = \begin{bmatrix} 2 & 2 & 1 & -1 \\ 0 & 1 & 2 & 1 \\ 4 & 3 & 0 & -3 \\ 2 & 3 & 3 & 0 \\ 1 & 0 & -1 & -2 \\ -1 & 1 & 3 & 3 \end{bmatrix}$.

Find the rank of A and determine bases for its row, column, and null spaces.

2. [10 marks] Find (if possible) conditions on a so that the following system has no solutions, a unique solution, or infinitely many solutions.

$$x + ay = 2$$
$$ax + y = -2$$

3. a) [7 marks] Find an orthogonal basis for the subspace

$$U = \text{span}\left\{\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 1 \\ -3 \end{pmatrix}\right\}.$$

b) [5 marks] Find all values of k such that $\left\{\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} k \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix}\right\}$ spans $\mathbb{R}^3$.

4. Consider the following subspace of $\mathbb{R}^4$:

$$W = \{(a_1, a_2, a_3, a_4) | a_2 - 2a_1 = a_3, a_1 + a_3 = -a_4\}$$

a. [7 marks] Find a basis for W.

b. [7 marks] Extend your answer from (a) to a basis of $\mathbb{R}^4$.

5. [12 marks] Show that the following system is inconsistent and solve for the best approximation(s):

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix} X = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$$

6. a) [8 marks] Let A be a 10×10 matrix, with $A^2 = 0$. Show that $\text{col } A \subseteq \text{null } A$ and that the rank of A is at most 5.

b) [4 marks] Show that if the matrix above was 9×9, then we could not have $\text{col } A = \text{null } A$.