Coulomb's Law

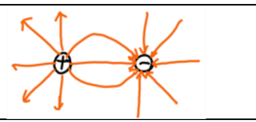
$\boldsymbol{F}_{qQ} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\hat{r}$

Electric Fields

$\mathbf{E}_{Q} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$

$\varepsilon_0 = 8.854 \cdot 10^{-12}\ [C^2/Nm^2]$
$k = 8.99 \cdot 10^{9}\ [Nm^2/C^2]$

Electric Field line

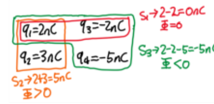


Electric Flux [Nm²/C]

$\phi_{net} = \oint_S \boldsymbol{E} \cdot d\boldsymbol{A}$

$= \oint_S E \cdot dA \cdot cos(\boldsymbol{E}\ d\boldsymbol{A})$

Enclosed charge [C]



$q_{enc} = \varepsilon_0 \Phi_{net}$

$= \varepsilon_0 \oint_S \boldsymbol{E} \cdot d\boldsymbol{A}$

Gaussian surface

Sphere: 1 gaussian surf
Cylinder: 3 gaussian surf
Cube: 6 gaussian surf
$d\boldsymbol{A} = dA\hat{n}\ (\hat{n} \perp A)$

Isolated conductor

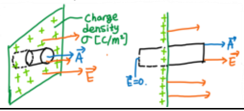
$\mathbf{E} = 0$

Cavity walls, inside isolated conductor, in metal

(charges all reside at surface)

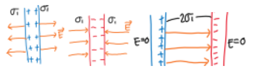
External elec. field

$E = \frac{\sigma}{\varepsilon_0}$ [N/C] or [V/m]



Two conducting planes

$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$



Charge Density (linear, surface, volume)

Linear density	λ	$q_{tot} = \int \lambda(x)\,dx$
Surface density	σ	$q_{tot} = \iint \sigma(x,y)\,dxdy$
Volume density	ρ	$q_{tot} = \iiint \rho(x,y,z)\,dxdydz$

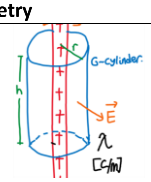
Spherical Symmetry

$E_{ext} = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right)r$

r – radius of gauss surf
R – radius of sphere
q – charge enclosed
$E_{int} = 0$

Cylindrical symmetry

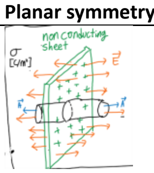
$E = \frac{\lambda}{2\pi\varepsilon_0 r}$



$q_{enc} = \lambda h$

$= \varepsilon_0 \oint_S \boldsymbol{E} \cdot d\boldsymbol{A}$

Planar symmetry

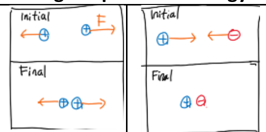


Non-conducting sheet (charge density of σ)

$E = \frac{\sigma}{2\varepsilon_0}$

$\phi = \phi_l + \phi_r = 2EA$

Change in potential energy



$\Delta U > 0$
$\Delta V > 0$
Force applied

$\Delta U < 0$
$\Delta V < 0$
Field does work

Potential Energy and Electric Potential

$\Delta U = U_f - U_i$
$= W_{appl} = -W_{field}\ [J]$
$= q(V_f - V_i) = -qE\Delta S$

$\Delta V = -\int_i^f \boldsymbol{E}\ d\boldsymbol{S}$

$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

$\Delta V = V_f - V_i\ [V]$

$= \frac{\Delta U}{q} = -\int_i^f \boldsymbol{E}\ d\boldsymbol{S} = -E\Delta x$

Equipotential Surfaces

$V_r = \frac{Q}{4\pi\varepsilon_0 r}$

Same potential (+pot E)

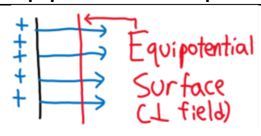
Voltage and Potential Energy

$V_{r>\infty} = \frac{Q}{4\pi\varepsilon_0 r}$

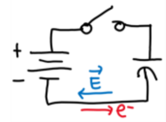
$U = \frac{Qq}{4\pi\varepsilon_0 r}$

Voltages can be added up each other (scalar)

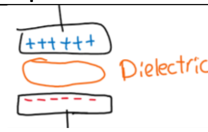
Equipotential surface pic



Basic Circuit with capacitor



Capacitor



*Metal plates > separation of charge
*Dielectric > increase capacitance
*Capacitor is charged

Capacitance

$C = \frac{q}{V}$ [F]

q – charge stored in one plate: not total charge!

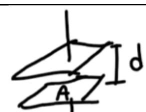
Parallel plate capacitor

$E = \frac{Q}{A\varepsilon_0}$

$V = \frac{Q}{A\varepsilon_0}d$

$C_{pp} = \frac{A\varepsilon_0}{d}$

(Ignore dielectric)



Cylindrical Capacitor

$\mathbf{E} = \frac{Q}{2\pi\varepsilon_0 Lr}\hat{r}$

$V = \frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)$

$C_{cyl} = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$

Spherical capacitor

$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$

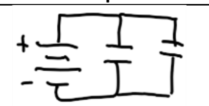
$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$

$C_{sphy} = \frac{4\pi\varepsilon_0 ab}{b-a}$

$C_{single\_sphy} = 4\pi\varepsilon_0 r$

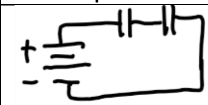
a is smaller radius, b is larger radius, L is height, r is radius of gaussian surface

Parallel Capacitor



$Q_T = Q_1 + Q_2 + \cdots$
$V_T = V_1 = V_2 = \cdots$
$Q_T = C_T V$
$C_T = C_1 + C_2 + \cdots$

Series Capacitor



$Q_T = Q_1 = Q_2 = \cdots$
$V_T = V_1 + V_2 + \cdots$
$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$

Energy in capacitor

$U = \int d\mho = \int q\,dV = \int CV\,dV$

$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C}$

$= \frac{1}{2}qV$

Energy Density

$\mho_{pp} = \frac{U}{Volume} = \frac{U}{Ad} = \frac{CV^2}{2Ad}$

$= \frac{kA\varepsilon_0}{2d}\frac{E^2d^2}{Ad} = \frac{1}{2}k\varepsilon_0 E^2$

Capacitance + dielectric

Increase area + decrease distance + dielectric
= <u>high capacitance</u>

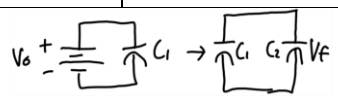
$C_{pp} = \frac{k\varepsilon_0 A}{d}$

$C_{cyl} = \frac{2\pi k\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$

$C_{sphy} = \frac{4\pi k\varepsilon_0 ab}{b-a}$

k: dielectric constant (k>1)

$V_f = \frac{c_1}{c_1+c_2}V_0$



Ohm's Law and current

$R = \frac{V}{I}\ [\Omega]$

$I = \frac{dq}{dt}\ [A]$

$= q_0 NAv_d$

$Q = q_0 NAL\ [C]$

$= q_0 NAv_d t$

Current Density

$I = \iint \boldsymbol{J}\,d\boldsymbol{A} = JA$

$\mathbf{J} = nq_0\boldsymbol{v_d}\ [A/m^2]$

Resistivity/Conductivity

$\rho = \frac{|\boldsymbol{E}|}{|\boldsymbol{J}|}\ [\Omega m]$

$\mathbf{J} = \frac{1}{\rho}\boldsymbol{E} = \sigma\boldsymbol{E}$

$\sigma = \frac{1}{\rho}\ [1/\Omega m]$

$R = \frac{El}{JA} = \rho\frac{l}{A}$

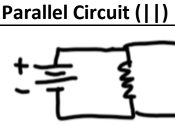
Power in Electric Fields

$U = qRI$

$Power = \frac{dU}{dt}$

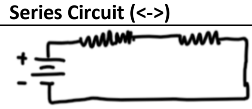
$Power = I^2R = \frac{V^2}{R} = VI\ [W]$

Parallel Circuit (||)



$V_T = V_1 = V_2 = \cdots$
$I_T = I_1 + I_2 + \cdots$
$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$
$\Sigma I = 0$

Series Circuit (<->)



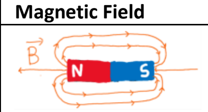
$V_T = V_1 + V_2 + \cdots$
$I_T = I_1 = I_2 = \cdots$
$R_T = R_1 + R_2 + \cdots$
$\Sigma V = 0$

Magnetic Force

$\mathbf{F_B} = q(\mathbf{v} \times \mathbf{B})$
$= Bqv\sin\theta$

Θ is $\angle(\mathbf{v},\mathbf{B})$

Magnetic Field



Measured in [T]

Magnetic field 'inside'?

$I_{tot} = \pi R^2 J_0$

$I_{enc} = \iint \boldsymbol{J}\,d\boldsymbol{A} = \pi r^2 J_0$

$(dA = rdrd\theta)$

$\oint_C \boldsymbol{B}\,d\boldsymbol{S} = \mu_0 I_{enc} = 2\pi Br$

$B = \frac{\mu_0 I_{tot} r}{2\pi R^2}$

Biot-Savart Law

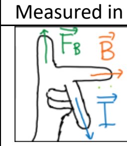
$d\mathbf{B} = \frac{\mu_0 I(d\boldsymbol{S} \times \hat{r})}{4\pi r^2}$

$\mu_0 = 4\pi \cdot 10^{-7}\ [H/m]$

Magnetic force (wire)

$\mathbf{F_B} = I(\mathbf{L} \times \mathbf{B})$
$= BIL\sin\theta$

Θ is $\angle(\mathbf{L},\mathbf{B})$



Ampere's Law

$\oint_C \boldsymbol{B}\,d\boldsymbol{S} = \mu_0 I_{enc} =$

$\oint_C Bcos(\boldsymbol{B}, d\boldsymbol{S})\,dS$

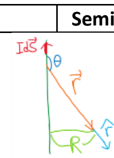
$B = \frac{\mu_0 I}{2\pi R}$

Magnetic field due to current

Infinite wire

$B = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{\sin\theta}{r^2}dx$

$B = \frac{\mu_0 I}{2\pi R}$



Semi-infinite wire

$B = \frac{\mu_0 I}{4\pi}\int_{0}^{\infty}\frac{\sin\theta}{r^2}dx$

$B = \frac{\mu_0 I}{4\pi R}$

Centre of loop

$B = \frac{\mu_0 I\emptyset}{4\pi R}$

$\emptyset$ - angle of loop (partial circular loop)

Force between two parallel currents

$F = \frac{\mu_0\,L\,I_a\,I_b}{2\pi d}$

$\frac{\mu_0}{2\pi} = 2 \cdot 10^{-7}$

Solenoids

$I_{enc} = nhI$

$\mu_0 I_{enc} = \mu_0 nhI = Bh$

$B = \mu_0 nI$

(n=number of loops/m)
(I=current on the loop)

Magnetic flux

$\Phi_B = \iint \boldsymbol{B}\ d\boldsymbol{A}$ [Wb]

$\Phi_B = BA = \mu_0 nIA$

EMF (Faraday/Lenz)

$\varepsilon = -(N)\frac{\partial\Phi_B}{\partial t} =$

$-(N)\frac{\partial}{\partial t}\iint \boldsymbol{B}\ d\boldsymbol{A}$

Faraday's law – $\varepsilon \propto \frac{\partial\Phi_B}{\partial t}$

Lenz' law – Direction of induced I and ε <u>against</u> induced B

EMF/current in loops

$\varepsilon = (-)(N)Bv_x l$

$I = \frac{(N)Bv_x l}{R}$

I is induced by induced B

Power dissipated

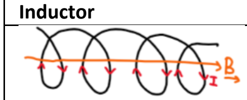
$P = VI = \frac{V^2}{R} = I^2R = \frac{N^2B^2v^2l^2}{R}$

Series Inductor

$V_T = L_T\frac{dI}{dt}$

$L_T = L_1 + L_2 + \cdots$

Inductor



$B = \mu_0 nI$

Solenoid is an inductor.

Inductance

$L = \frac{N\Phi_B}{I}$ [H]

$L = n^2Al\mu_0$

$L = \frac{BAN}{I} = NAn\mu_0$

$(N = nl)$

Self-induction (ε_L)

$\varepsilon_L = -N\frac{\partial\Phi_B}{\partial t}$

$= -L\frac{d}{dt}I(t)$

$V_L = -\varepsilon_L = L\frac{d}{dt}I(t)$

Energy in magnetic field

$U = qV$

$dU = Vdq = qdV$

$Power = \frac{dU}{dt} = IV$

$dU = ILdI$

$U = \frac{1}{2}LI^2$

Ohm's law & power

$V(t) = R\,I(t)$

$P(t) = V(t)I(t) =$

$R(I(t))^2 = \frac{(V(t))^2}{R}$

R=0 > $V(t) = R\,I(t) = 0$

R=∞ > $I(t) = \frac{V(t)}{R} = 0$

Parallel inductor

$V = L\frac{dI}{dt}$

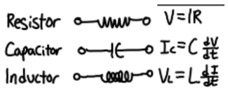
$\frac{V}{L_T} = \frac{V}{L_1} + \frac{V}{L_2} + \cdots$

$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots$

Basic Quantities

$$I(t) = \frac{dq(t)}{dt}$$
$$q(t) = \int_{-\infty}^{t} I(x)\,dx$$
$$V(t) = \frac{W(t)}{q}$$
$$P = IV = I^2R = \frac{V^2}{R}$$

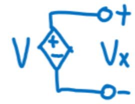
2 terminal devices



Dependent voltage source

Voltage-control

$$V = \alpha V_x$$



Current-control

$$V = rI_x$$

Dependent current source

Voltage-control

$$I = gV_x$$

Current-control

$$I = \beta I_x$$

Independent sources

Indep. Voltage source

Indep. Current source

Node

More than 2 elements join in one point

Branch

Section that contains only one element

Loop

Closed path: start from one node, visit other nodes (only once), return

Conductance

$$G = \frac{1}{R} = \frac{I}{V}\ [S]$$
$$I(t) = GV(t)$$
$$P(t) = \frac{(I(t))^2}{G} = G\big(V(t)\big)^2$$

Kirchhoff's Laws

Current Law (node)

$$\sum I_{enter} = \sum I_{leave}$$

Voltage Law (loop)

$$\sum V_{drop} = \sum V_{rise}$$

Linearity

$$V_{out} = \alpha_1 V_1 + \alpha_2 V_2 + \cdots + \beta_1 I_1 + \beta_2 I_2 + \cdots$$

Linear equations!

Voltage divider

a.k.a series circuit

$$I = \frac{V}{\sum R}$$
$$V_i = V_{source}\frac{R_i}{\sum R}$$

Current divider

a.k.a parallel circuit

$$V = I_s\left(\sum \frac{1}{R}\right)^{-1}$$
$$I_i = I_{source}\frac{1/R_i}{(\sum(1/R))}$$

Open circuit

- No current
- Voltage exists
- Disconnected wire

Short circuit

- No voltage across
- Current goes to R=0
- Connected wire

Nodal Analysis

(#Eq) = (#Node) - 1 - N_v

N_v is independent or dependent voltage sources

- Voltages at nodes
- Kirchhoff's current law
- One ground node set

Mesh Analysis

(#Eq) = (#Loop) – N_I

= Branch – Node + 1 - N_I

N_I is independent or dependent current sources

- Voltages surround loops
- Kirchhoff's voltage law
- Calculate for each loop

Supernode

- Two nodes connected by voltage source
- None of them are reference node

Supermesh

- Used when current source is shared between two loops

Superposition

Deactivate all sources except one source

- Holds for I, V but not power

Deactivate source?

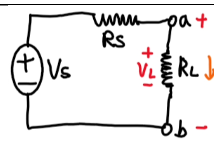
- Voltage source > short
- Current source > open

I_1 – current through R due to V_1 only

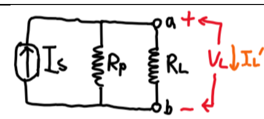
I_2 – current through R due to V_2 only

$$I_{tot} = I_1 + I_2 + \cdots$$
$$P_{tot} = I_{tot}^2 R = R(I_1 + I_2 + \cdots)^2$$

Source transformation



$$V_L = V_s\frac{R_L}{R_L + R_S}$$

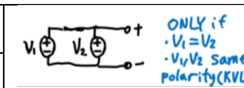


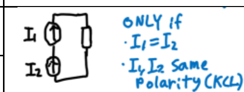
To make this equivalent,

$$V_s = I_s \times R_s$$
$$R_s = R_p$$

Equivalence

$R_1 \leftrightarrow R_2$	$R_1 + R_2$
$R_1 \parallel R_2$	$\frac{R_1R_2}{R_1+R_2}$
$V_1 \leftrightarrow \pm V_2$	$V_1 \pm V_2$
$I_1 \parallel \pm V_2$	$I_1 \pm I_2$





Thevenin Theorem

1) R_L > open ckt > get V_{OC}
2) R_L > short ckt > get I_{SC}

$$R_{Th} = \frac{V_{OC}}{I_{SC}}$$
$$V_{Th} = V_{OC}$$

Norton Theorem

If sources independent

1) Get V_{OC}
2) Deactivate sources
3) Replace R_L into virtual voltage source, find R_{eq}
4) $R_{eq} = R_N$, $I_N = I_{SC}$

Multiple Thevenin usage

1) Divide circuit into two+
2) Get V_{OC},R_{Th} for one side
3) Substitue linear ckt in 2) to $V_{OC} - R_{Th}$ connection
4) Repeat 2) and 3) to get R_{Th} V_{OC} and I_{SC}

Dependent sources in thevenin

1) R_L > open ckt > get V_{OC}
2) R_L > short ckt > get I_{SC}
3) Do not remove dependent sources in 1) and 2)

$$R_{Th} = \frac{V_{OC}}{I_{SC}}$$

Maximum power transfer

$$I_L = \frac{V_{Th}}{R_{Th}+R_L}$$
$$P_L = I_L^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$
$$\frac{dP_L}{dR_L} = V_{Th}(R_L + R_{Th})(R_L - R_{Th})$$
$$P_{Lmax} = P_{outmax} = \frac{V_{Th}^2 R_{Th}}{4R_{Th}^2} = \frac{V_{Th}^2 R_{Th}}{4R_{Th}}$$

Efficiency

$$\text{Efficiency} = \frac{P_{out}}{P_{in}}$$
$$P_{in} = \frac{V_{Th}^2}{R_L + R_{Th}}$$
$$P_{out} = \frac{V_{Th}^2 R_L}{(R_L + R_{Th})^2}$$

Capacitors

$$q(t) = Cv_c(t)$$
$$i_c(t) = C\frac{dv_c(t)}{dt}$$
$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau$$

$$P = \frac{dW}{dt} = v(t)\,i(t)$$
$$W_c(t) = \frac{1}{2}C[v(t)]^2$$

- v_c must be continuous
- DC signal > capacitor looks like open circuit

Inductors

$$L = \frac{N\Phi_B}{i}$$
$$V_L(t) = L\frac{di_L(t)}{dt}$$
$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau$$

$$P = \frac{dW}{dt} = v(t)\,i(t)$$
$$W_L(t) = \frac{1}{2}L[i(t)]^2$$

- i_L must be continuous
- DC signal > inductor looks like short circuit

Series

Inductor	$L_T = \sum_i L_i$
Capacitor	$\frac{1}{C_T} = \sum_i \frac{1}{C_i}$
Resistor	$R_T = \sum_i R_i$

Parallel

Inductor	$\frac{1}{L_T} = \sum_i \frac{1}{L_i}$
Capacitor	$C_T = \sum_i C_i$
Resistor	$\frac{1}{R_T} = \sum_i \frac{1}{R_i}$

Intro to transient circuits

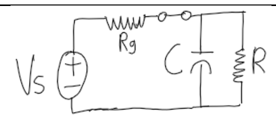
$$C\frac{dv_c(t)}{dt} + \frac{v_c(t)}{R} = 0$$
$$\frac{dv_c(t)}{dt} + \frac{v_c(t)}{RC} = 0$$
$$V_c(t) = V_0 e^{-\frac{t-t_0}{RC}}$$

First-order transient ckts

$$v(t) = v(\infty) + (v(t_0) - v(\infty))e^{-\frac{t-t_0}{\tau}}$$
$$\tau = R_{Th}C = \frac{L}{R_{Th}}$$

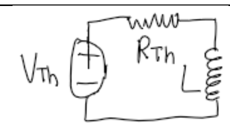
Resistor-Capacitor

$$V_c(t) = V_s\frac{R}{R+R_g}\left(1 - e^{-\frac{t-t_0}{\tau}}\right)$$
$$V_{c-charge} = V_m\left(1 - e^{-\frac{t-t_0}{\tau}}\right)$$
$$V_{c-discharge} = V_m\left(e^{-\frac{t-t_0}{\tau}}\right)$$



Resistor-Inductor

$$V_L + i_L R_{Th} = V_{Th}$$
$$\frac{L}{R_{Th}}\frac{di_L}{dt} + i_L = \frac{V_{Th}}{R_{Th}}$$
$$\tau = \frac{L}{R_{Th}}$$



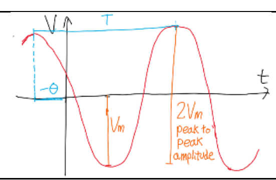
Complex Number Identities ($j = \sqrt{-1} = 1\angle(\pi/2)$)

$$Z = x + jy = r(\cos\theta + j\sin\theta) = re^{j\theta}\ (\theta = \tan^{-1}\frac{y}{x})$$
$$\mathrm{Re}(Z) = x, \mathrm{Im}(Z) = y$$
$$\bar{Z} = x - jy$$

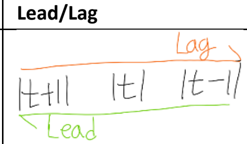
$$Z_1 \pm Z_2 = (x_1 + x_2) \pm j(y_1 + y_2)$$
$$Z_1Z_2 = (x_1x_2 - y_1y_2) + j(x_2y_1 + x_1y_2)$$
$$\frac{Z_1}{Z_2} = \frac{Z_1\bar{Z_2}}{Z_2\bar{Z_2}} = \frac{|Z_1|}{|Z_2|}\angle(\theta_1 - \theta_2)$$

Sinusodials (Fourier)

$$v(t) = V_m\cos(wt + \theta)$$
$$w = 2\pi f = \frac{2\pi}{T}$$
$$T = \frac{1}{f}$$



Lead/Lag



- Same Frequency
- All functions should be cosine or sine
- All amplitudes should be positive or negative
- $0 \le$ separation $< 180°$

Phasor Relationships

$$v(t) = V_m\cos(wt + \theta_v) = Re\left[\underline{V}e^{jwt}\right]$$
$$\underline{V} = V_m e^{j\theta_v} = V_m\angle\theta_v$$

$$i(t) = I_m\cos(wt + \theta_i) = Re\left[\underline{I}e^{jwt}\right]$$
$$\underline{I} = I_m e^{j\theta_i} = V_m\angle\theta_i$$

Impedance - Inductor

$$V_L(t) = L\frac{di_L(t)}{dt}$$
$$Re\left[\underline{V}e^{jwt}\right] = Re\left[L\frac{d}{dt}\underline{I}e^{jwt}\right] = Re\left[Ljw\,\underline{I}e^{jwt}\right]$$

$$Z_L = jwL\ [\Omega]$$
$$I_m = \frac{V_m}{wL}$$
$$i(t) = \frac{V_m}{wL}\cos\left(wt + \theta_v - \frac{\pi}{2}\right)$$

Impedance - Capacitor

$$i_c(t) = C\frac{dv_c(t)}{dt}$$
$$Re\left[\underline{I}e^{jwt}\right] = Re\left[C\frac{d}{dt}\underline{V}e^{jwt}\right] = Re\left[Cjw\,\underline{V}e^{jwt}\right]$$

$$Z_C = \frac{1}{jwC} = -\frac{j}{wC}\ [\Omega]$$
$$I_m = wCV_m$$
$$i(t) = I_m\cos\left(wt + \theta_v + \frac{\pi}{2}\right)$$

Impedance - Resistor

$$Z_R = R\ [\Omega]$$
$$V_R(t) = I_R(t)\,R$$

Impedance and admittance

$$Z = \frac{V}{I} = R + jx$$
$$Y = \frac{1}{Z} = G + jB$$

Z - impedance

Y - Admittance

G - Conductance

B - Susceptance

Parallel/series impedance

series

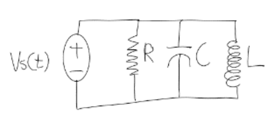
$$Z_{eq} = \sum_i Z_i$$

Parallel

$$\frac{1}{Z_{eq}} = \sum_i \frac{1}{Z_i}$$

Resonance frequency

$$w_0 = \frac{1}{\sqrt{CL}}$$



(When Z_{eq} becomes real)

Trigonometric Identities

$$\sin(wt + \theta) = \cos\left(wt + \theta - \frac{\pi}{2}\right)$$
$$\cos(wt + \theta) = \sin\left(wt + \theta + \frac{\pi}{2}\right)$$

$$-\sin(wt + \theta) = \cos(wt + \theta - \pi)$$
$$-\cos(wt + \theta) = \cos(wt + \theta + \pi)$$