Faculty of Applied Science and Engineering
University of Toronto

MAT 186 H1S - CALCULUS I
FRIDAY, APRIL 20, 2018

FINAL EXAMINATION

FAMILY NAME: _____

GIVEN NAME: _____

STUDENT NUMBER: _____

SIGNATURE: _____

Time allowed: 2 hours, 30 minutes
Total marks: 75
No calculators allowed.
Examiner: S. Cohen
Solutions will be marked for both correctness **and** clarity.
Use the backs of pages when necessary,
indicating clearly where solutions continue.

FOR MARKER'S USE ONLY	
QUESTION	MARK
1	/ 15
2	/ 10
3	/ 20
4	/ 12
5	/ 8
6	/ 10
TOTAL	/ 75

1. Some warm-up questions. Justify your answers fully.

a) Evaluate $\lim_{x\to -2} \frac{x^3+2x+12}{3x^2+4x-4}$.

[3 marks]

b) Evaluate $\frac{d}{dx}(3x^2 + 4x - 1)$ **using the limit definition** of the derivative.

[3 marks]

c) Find the area between the functions $y = x^2 - 2x + 1$ and $y = 5 - x^2$

[3 marks]

d) Find (with justification) the asymptote(s) of $y = \frac{e^x - 1}{x}$.

[3 marks]

e) Evaluate $\int_{-1}^{0} 16t \cdot e^{4t^2-3}\ dt$

[3 marks]

2. Tougher questions, now that you are ready:

a) Approximate the value of $\sqrt[3]{24}$.

[5 marks]

b) Find the equation of the tangent line to $(-xy)^{x+y} = e^{xy}$ at $(1, -1)$.

[5 marks]

3. Graph the function $f(x) = x^3 + 3x^2 - 3x + |6x + 6|$.
   - Organize your solution well.
   - Include all of the important values (one, in particular, requires a detailed analysis) and identify their type (maximum, minimum, etc.).
   - Indicate regions where the function is increasing vs. decreasing, concave up vs. concave down.
   - You have two pages for this problem.

[20 marks]

[Continue Question 3 on this page.]

4. Let $f(x) = e^x$, for $x \leq 0$. For any point on the graph, the tangent line will meet both the x and y axes, creating a triangle. Find the point that maximizes the area of this triangle.

   (**Hint:** Start with a general point, (a, e^a), find its tangent line, and calculate its intersections with the axes.).

[12 marks]

5. A lamp of height 8 feet is standing on one side of a narrow street. On the opposite side, a 6 foot tall person is walking at a rate of 3 feet per second. If the street is 8 feet across, how fast is the person's shadow lengthening two seconds after walking directly across from the lamp?

[8 marks]

6. A trench is created in the shape of a triangle rotated about an axis. If we consider the ground to be the $x-$ axis, then the triangle has coordinates $(-4,0)$, $(-1,0)$, and $(-3,-2)$ and the axis of rotation is $x=2$.
   - How much water is needed to fill the trench?
   - How much work is required to do so, if the water source is a pump three metres below the ground?

[10 marks]

[Extra page for additional work.]

[0 marks]