UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

ECE 110H1 S -- ELECTRICAL FUNDAMENTALS
FINAL EXAMINATION, APRIL 24, 2017, 2:00 pm

First Year -- Computer, Electrical, Industrial, Mechanical, Materials, and Track One Engineering Programs.

Examiners – S. Aitchison, B. Bardakjian, A. Helmy, M. Mojahedi, K. Truong and P. Yoo

($e = 1.6 \times 10^{-19}$ C, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $g = 9.81$ N/kg)

NAME : ______________________________
(PLEASE PRINT) Family (Last) Name Given (First) Name

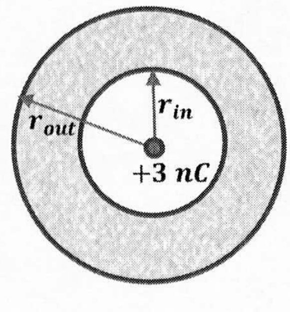
STUDENT NUMBER : ____________________

EXAMINATION TYPE : A
CALCULATORS : Casio FX-991MS & Sharp EL-520X
DURATION : 2.5 hours
INSTRUCTIONS :
- Answer all five questions. Put the answers in the boxes provided.
- For multiple choice questions please circle 1 of the answers
- All work is to be done on these pages. Show steps, compute numerical results when requested and state units. Write down any assumption made.
- You may use the back of each page.
- Last blank page may be removed for rough work.

Question	Mark
1	
2	
3	
4	
5	
Total	

Q1 [10 marks] Electrostatics

(a) **[2 marks]** The figure shows the cross section of a metallic shell with inner radius r_{in} and outer radius r_{out}. The shell is made of perfect metal and is charged with the total charge of -8 nC. A positive charge of +3 nC is placed at the center of the shell. Circle the correct statement regarding the charges appearing at $r = r_{in}$, $r_{in} < r < r_{out}$, and $r = r_{out}$.



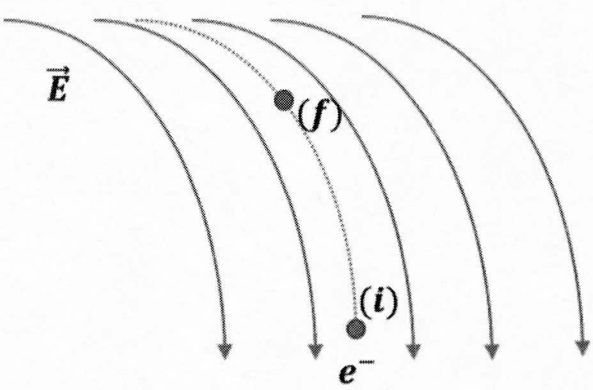
i) Charges at $r = r_{in}$, $r_{in} < r < r_{out}$, and $r = r_{out}$ are respectively, +3 nC, 0, and -5 nC.

ii) Charges at $r = r_{in}$, $r_{in} < r < r_{out}$, and $r = r_{out}$ are respectively, -3 nC, 0, and -5 nC.

iii) Charges at $r = r_{in}$, $r_{in} < r < r_{out}$, and $r = r_{out}$ are respectively, -3 nC, +3, and -5 nC.

iv) Charges at $r = r_{in}$, $r_{in} < r < r_{out}$, and $r = r_{out}$ are respectively, -3 nC, 0 and -8 nC.

(b) **[2 marks]** The figure shows a static electric field, $\vec{E}$. An electron (e^-) moves from the position marked (*i*) to the position marked (*f*). Circle the correct statement regarding the potential energy (*U*) and electrostatic potential (*V*) of the electron.



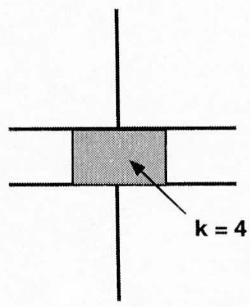
i) Potential energy (*U*) has increased, but electrostatic potential (*V*) has decreased

ii) Both potential energy (*U*) and electrostatic potential (*V*) have decreased

iii) Potential energy (*U*) has decreased, but electrostatic potential (*V*) has increased

iv) Both potential energy (*U*) and electrostatic potential (*V*) have increased

(c) **[2 marks]** A dielectric with k = 4, is inserted into a parallel plate capacitor such that it fills 1/3 of the space as shown in the figure below. If the capacitance of the parallel plate capacitor before the dielectric was inserted was C, what is the new value of capacitance.



i) 0.5 C

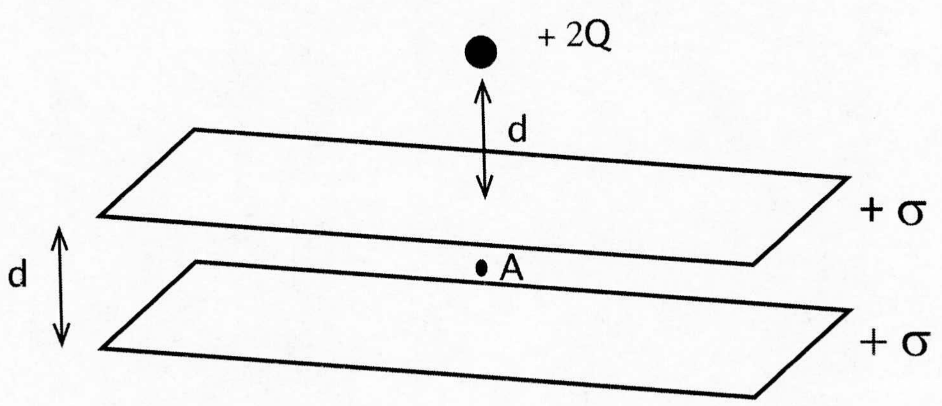
ii) 0.75 C

iii) C

iv) 2 C

v) 6 C

(d) **[2 marks]** Consider the two infinite sheets of charge, separated by a distance d shown below. Each sheet has a charge density of + σ. A point charge of + 2Q is placed at a distance d, above the top sheet. Which expression describes the electric field at point A, midway between the two sheets.



i) $E = \frac{2\sigma}{\varepsilon_0}$

ii) $E = 0$

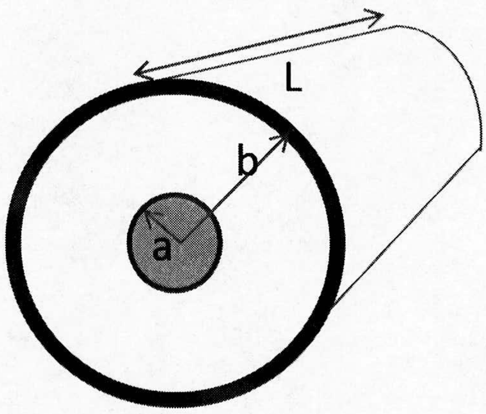
iii) $E = \frac{1}{4\pi\varepsilon_0}\frac{8Q}{9d^2}$

iv) $E = \frac{2\sigma}{\varepsilon_0} + \frac{8Q}{9d^2}$

v) $E = \frac{\sigma}{\varepsilon_0} + \frac{8Q}{9d^2}$

Q1 continued

(e) **[2 marks]** Consider a capacitor of length L that is composed of two concentric conducting cylinders with radii a and b as shown in figure below. Which of the following is the correct expression for the capacitance of these two cylinders?



i) $C = 2\pi\varepsilon_o Ln(\frac{b}{a})$

ii) $C = VQ$

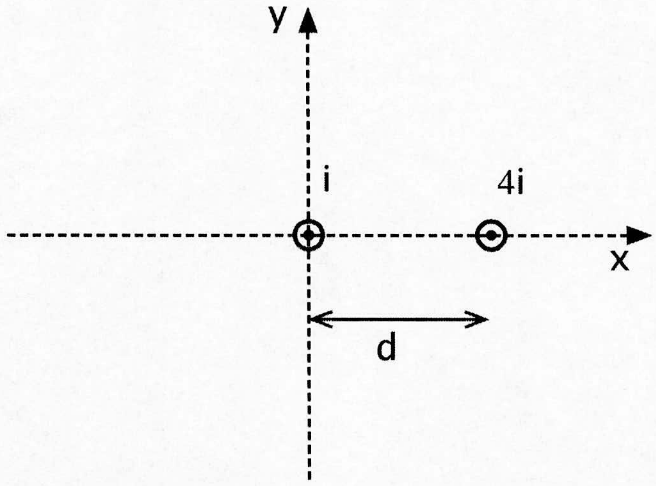
iii) $C = 2\pi\varepsilon_o L\frac{1}{Ln\left(\frac{b}{a}\right)}$

iv) $C = Ln\left(\frac{b}{a}\right)\frac{1}{2\pi\varepsilon_o L}$

v) $C = 2\pi\varepsilon_o L\frac{1}{Ln\left(\frac{a}{b}\right)}$

Q2 [10 marks] Magnetic Fields

(a) **[2 marks]** Consider two wires, carrying currents i and 4i out of the page as shown in the figure below.



The wires are separated by a distance of d = 5 cm. At what position is the magnetic field zero.

i) (-1 cm, 0)

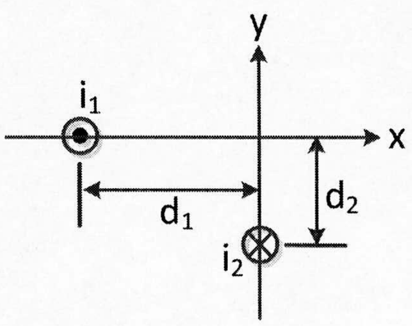
ii) (1 cm, 0)

iii) (4 cm, 0)

iv) (2 cm, 0)

v) (2.5 cm, 0)

(b) **[2 marks]** For the two long straight wires carrying currents i_1 and i_2 respectively as shown below, which one of the following represents the expression for the magnetic field at the origin.



i) $\frac{\mu_o i_2}{2\pi d_2}\,\hat{\imath}$

ii) $\frac{\mu_o i_1}{2\pi d_1}\,\hat{\jmath}$

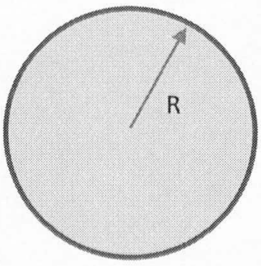
iii) $\frac{\mu_o i_2}{2\pi d_2}\,\hat{\imath} - \frac{\mu_o i_1}{2\pi d_1}\,\hat{\jmath}$

iv) $\frac{\mu_o i_1}{2\pi d_2}\,\hat{\imath} + \frac{\mu_o i_2}{2\pi d_1}\,\hat{\jmath}$

v) $\frac{\mu_o i_2}{2\pi d_2}\,\hat{\imath} + \frac{\mu_o i_1}{2\pi d_1}\,\hat{\jmath}$

Q2 continued.

(c) **[2 marks]** The schematic in the figure below shows the cross-section of a metallic wire with a radius R, which carries a uniform current *I*, out of the page. Which of the following is an expression for the Magnetic Field, *B* at a radius, r < R ?



i) $B = \frac{\mu_0 i}{2\pi r}$

ii) $B = r\left(\frac{\mu_0 i}{2\pi R^2}\right)$

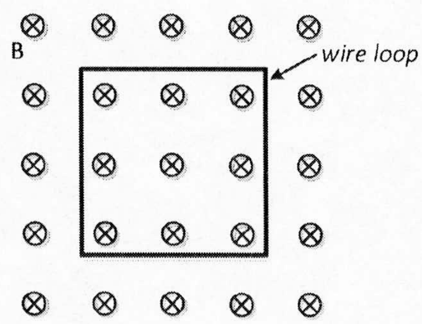
iii) $B = \frac{\mu_0 i}{2\pi r^2}$

iv) $B = \frac{2\pi i r}{R}$

v) $B = \frac{2\pi i R^2}{r}$

Q2 continued.

(d) **[2 marks]** The uniform magnetic field $\vec{B}$ as shown below, changes over time according to the following equation: $B(t) = \frac{1}{200\pi}\sin(2\pi t)$ where B is in Tesla and t is in second. Which of the following is the magnetic flux (Φ_B) as a function of time, which passes through a 1 cm × 1 cm square-shaped wire loop.



i) $\Phi_B = \left(\frac{5}{\pi}\right) 10^{-5} \sin(2\pi t)$

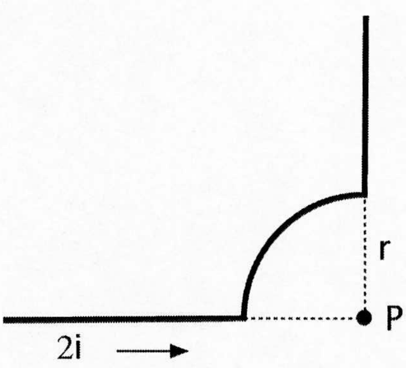
ii) $\Phi_B = \left(\frac{1}{200\pi}\right) cos\,(2\pi t)$

iii) $\Phi_B = \left(\frac{5}{\pi}\right) 10^{-5} \cos(2\pi t)$

iv) $\Phi_B = -\left(\frac{1}{200\pi}\right) sin\,(2\pi t)$

v) $\Phi_B = \left(\frac{1}{200\pi}\right) 10^{-5} cos\,(2\pi t)$

(e) **[2 marks]** Consider the wire, which carries a current of 2i as shown in the figure below. What is the magnitude of the magnetic field at point P?



i) $B = \frac{\mu_o i}{2r}$

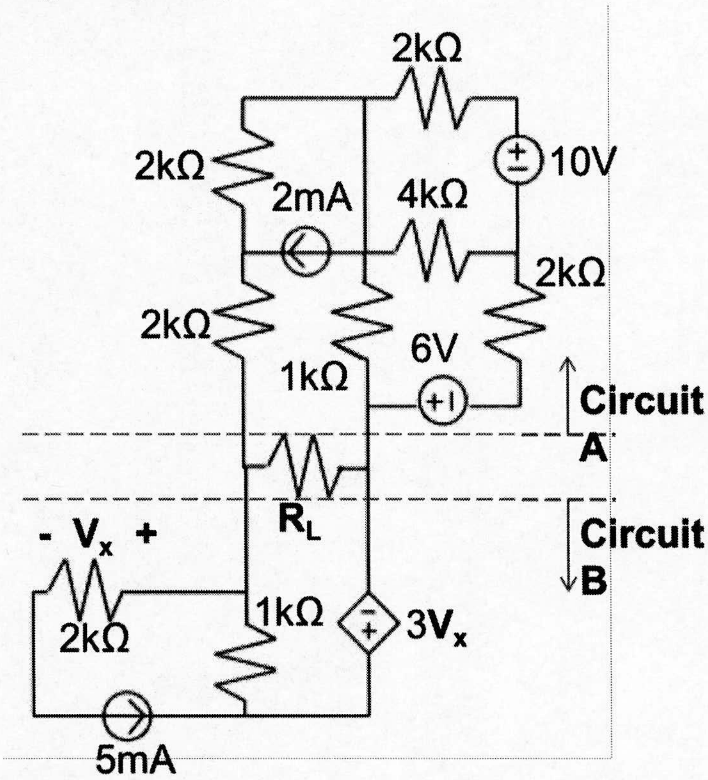
ii) $B = \frac{\mu_o i}{8r}$

iii) $B = \frac{\mu_o i}{2\pi r}$

iv) $B = \frac{\mu_o i}{4\pi r}$

v) $B = 0$

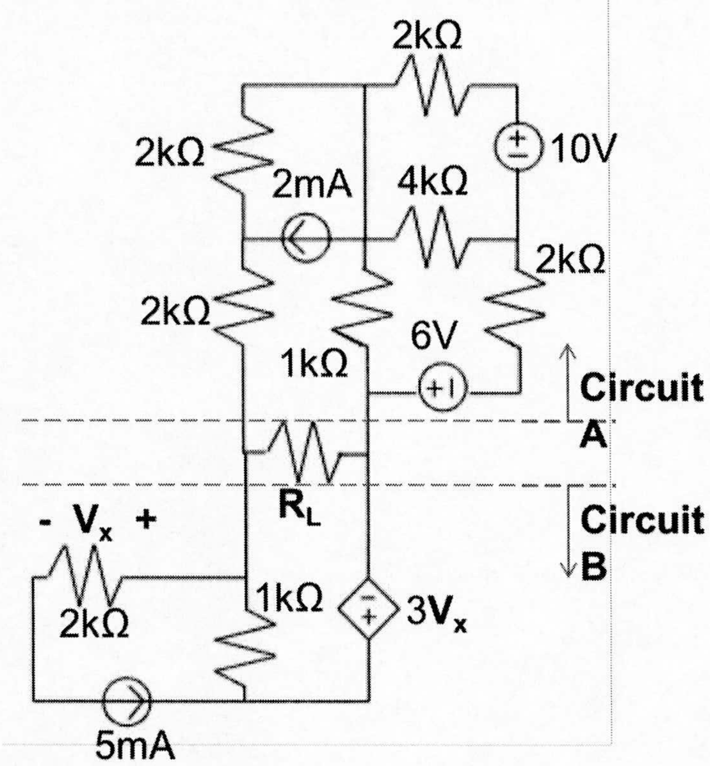
Q3 [10 marks]
Consider the following circuit below,



(a) **[4 marks]** Find the Thévenin equivalent of circuit A

Thévenin Equivalent Circuit:

Q3 continued. (Circuit diagram has been duplicated for your convenience)



(b) **[4 marks]** Find the Thévenin equivalent of circuit B

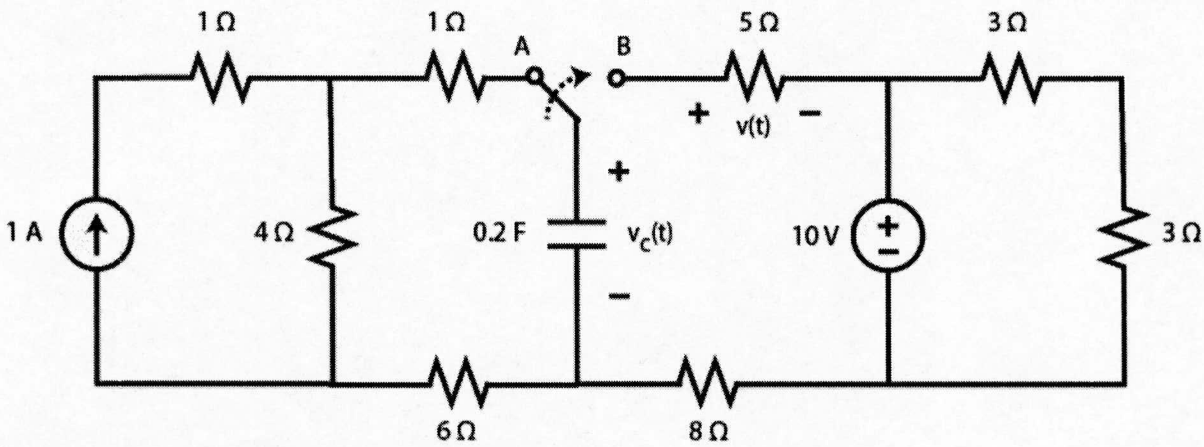
Thévenin Equivalent Circuit:

(c) **[2 marks]** Determine R_L that yields maximum power transfer from all sources in circuits A & B

$R_L =$

Q4 [10 marks]

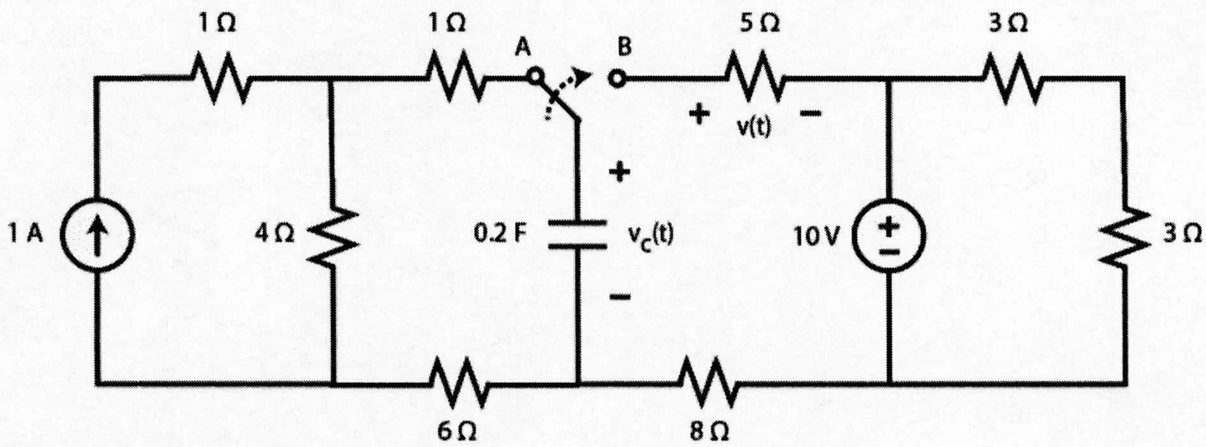
Consider the following circuit below, the switch is in position A for a long time. At t = 0, the switch is moved to position B.



(a) **[6 marks]** Derive the function $v_c(t)$ for $t \geq 0$.

$v_c(t) =$

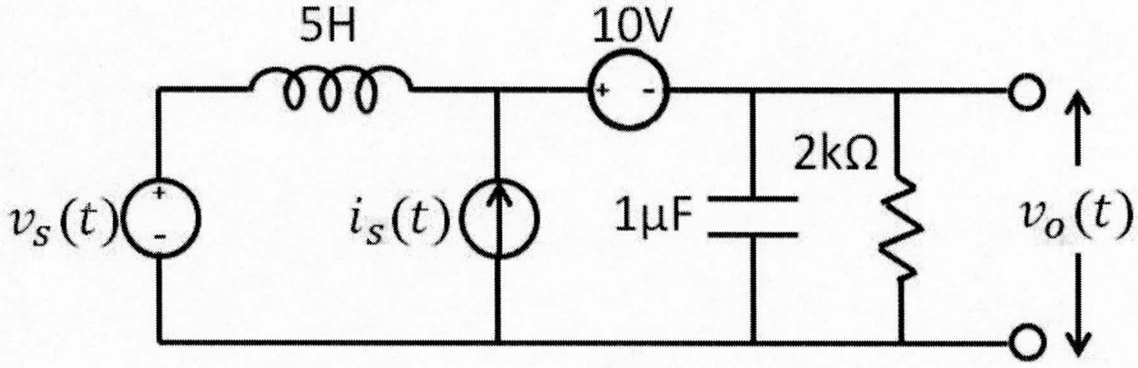
Q4 continued. (Circuit diagram has been duplicated for your convenience)



(b) **[4 marks]** Derive the function v(t) for $t \geq 0$.

$v(t) =$

Q5 [10 marks]
Consider the following circuit below,



Where $v_s(t) = 10\sin(200t)$ in V, and $i_s(t) = 5\cos(200t)$ in mA.

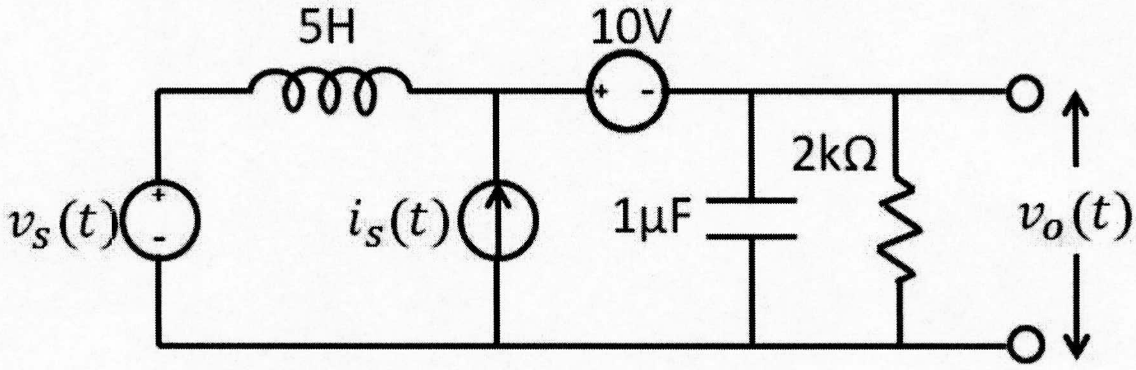
(a) **[2 marks]** Compute the contribution of the DC voltage source to the output voltage $v_o(t)$.
HINT: Use source superposition to compute $v_o(t)$

$v_o^{DC}(t) =$

(b) **[2 marks]** Compute the impedances.

$Z_{cap} =$
$Z_{ind} =$

Q5 continued. (Circuit diagram has been duplicated for your convenience)



(c) **[6 marks]** Compute the contribution of the AC sources to the output voltage $v_o(t)$.

$v_o^{AC}(t) =$