ECE110 - Quiz #2

Only non-programmable calculators are allowed.

First Name: _____ *Last Name:* _____

*Student #:*_____ *Tutorial Location:* _____

A capacitor consisting of two concentric cylindrical shells with radii ***a*** and ***b*** (***b>a***) and a length of ***L (L>>b)*** is considered. The inner shell has a total charge of ***+q*** and the outer shell has a total charge of ***-q***.

a) Draw a diagram and clearly show the above configuration.

Develop the following expressions in terms of ***a***, ***b***, and ***q*** for:

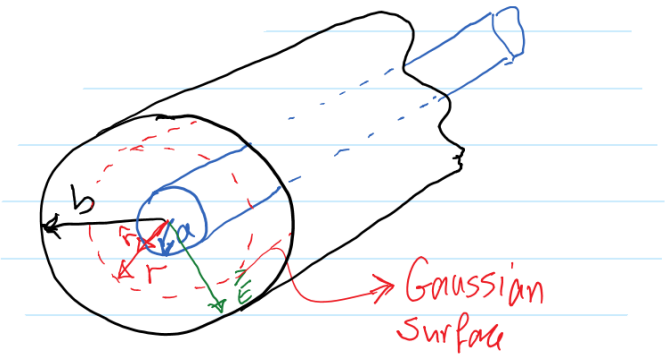
b) Electric Field in space between two cylindrical shells.

c) Electric Potential (Voltage) between two cylindrical shells.

d) Capacitance of this device.

Solution:

Part a)



Part b)

Apply Gauss' Law

Place cylindrical shell Gaussian surface (radius r)

$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$

$\varepsilon_0 E 2\pi r L = q \Rightarrow \vec{E} = \frac{q}{2\pi\varepsilon_0 L} \cdot \frac{1}{r} \hat{r}$

or E radially outward with magnitude of

$E = \frac{q}{2\pi\varepsilon_0 L} \frac{1}{r}$ $\left(\frac{V}{m}\right)$ or $\left(\frac{N}{C}\right)$

Part c)

$dW = \vec{F} \cdot d\vec{r}$ and $\vec{F} = q_0 \vec{E}$

$W = q_0 \int_a^b \vec{E} \cdot d\vec{r}$, $\vec{E} \cdot d\vec{r} = E\,dr$ since $\theta = 0$

$V_f - V_i = V_+ - V_- = V_a - V_b = V_{ab} = V = \frac{-W}{q_0}$

$\frac{-W}{q_0} = \int_a^b \vec{E} \cdot d\vec{r} \Rightarrow V = \int_a^b E\,dr = \int_a^b \frac{q}{2\pi\epsilon_0 L} \frac{dr}{r}$

$V = \frac{q}{2\pi\epsilon_0 L} \left(\ln r \Big|_a^b \right) = \frac{q}{2\pi\epsilon_0 L} (\ln b - \ln a)$

$$\boxed{V = \frac{q}{2\pi\epsilon_0 L} \ln \frac{b}{a}} \quad (\text{volt})$$

Part d)

$q = CV \Rightarrow C = \frac{q}{V}$

$$C = \frac{q}{\frac{q}{2\pi\epsilon_0 L} \ln \frac{b}{a}} = \frac{2\pi\epsilon_0 L}{\ln \frac{b}{a}}$$