

University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, APRIL 2005
First Year – CIV, CHE, IND, LME, MEC, MMS

MAT 186H1S - Calculus I

Exam Type: A

Examiner: D. Burbulla

INSTRUCTIONS

Aids permitted: a Casio 260, Sharp 520, or Texas Instrument 30 calculator.

Present your solutions to all of the following questions in the exam booklets supplied.
The marks for each question are indicated in parentheses.

TOTAL MARKS: 100.

1. [20 marks; each part is worth 5 marks] Find the following:

(a) $\int_0^3 x\sqrt{16+x^2} dx$

(b) $\int \frac{e^{-1/x}}{x^2} dx$

(c) $\lim_{x \rightarrow 0} \frac{x - \sin^{-1} x}{x^3}$

(d) $\lim_{x \rightarrow \infty} (e^{2x} + x)^{1/x}$

2. [10 marks; each part is worth 5 marks] Find the following:

(a) $\frac{dy}{dx}$ at the point $(x, y) = (0, 0)$, if $\ln(e + y) = e^{\sin(x+y)}$.

(b) $F'(-1)$ if $F(x) = \int_0^{x^2} \frac{1}{\sqrt{1+t^2}} dt$.

3. [10 marks; 5 marks for each part] Suppose that the position of a particle at time t is given by $x = t^3 - 3t + 4$. Find the following:
- the average velocity of the particle for $0 \leq t \leq 3$.
 - the average speed of the particle for $0 \leq t \leq 3$.
4. [10 marks; 5 marks for each part] Let $f(x) = x^3 + 2x - 4$. Explain why the equation

$$f(x) = 0$$

has *exactly* one solution in the interval $[1, 2]$ and then approximate the solution correctly to four decimal places by Newton's method.

5. [10 marks] Let R be the region bounded by the curves $x = 0$, $x = 2$, $y = 2x$ and $y = x^2$. Find the volume of the solid of revolution obtained by revolving the region R about the line $x = -1$.
6. [10 marks] Find the area of the region bounded by the curves with equations

$$y = x^3 + x \text{ and } y = 4x + 2x^2.$$

7. [10 marks] Find the length of the curve with equation

$$y = (1 - x^{2/3})^{3/2}$$

for $-1 \leq x \leq 1$.

8. [10 marks] A tank filled with water of density $\rho = 1000 \text{ kg/m}^3$ has the shape of a sphere of radius 2 m. Find the work done in pumping all of the water out of the tank and up to a horizontal pipe 1 m above the top of the tank. (Use acceleration due to gravity $g = 9.8 \text{ m/sec}^2$.)
9. [10 marks] Let $f(x) = e^{-1/x} + x$. Find the intervals on which f is increasing, decreasing, concave up, concave down. Plot the graph of $y = f(x)$, labelling all critical points, inflection points and asymptotes, if any.