UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

Department of Mechanical and Industrial Engineering

FINAL EXAMINATION

April 23, 2010 9:30am

Exam Duration: 2.5 hours

First Year

MIE100H1S – DYNAMICS

Calculator Type 2 (non programmable calculator)

Exam Type C (one 8½ x 11 aid sheet)

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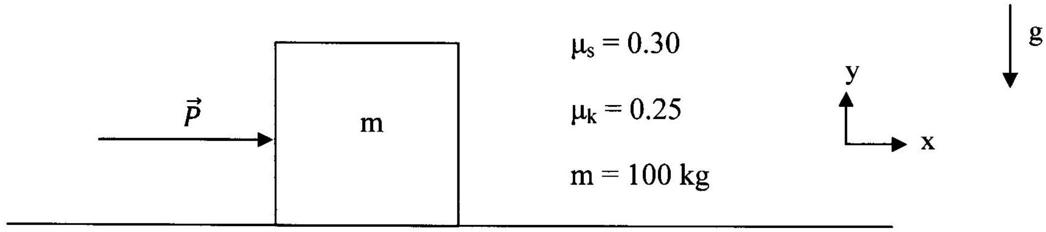
ANSWER ALL FIVE QUESTIONS

(100 marks total)

Question 1

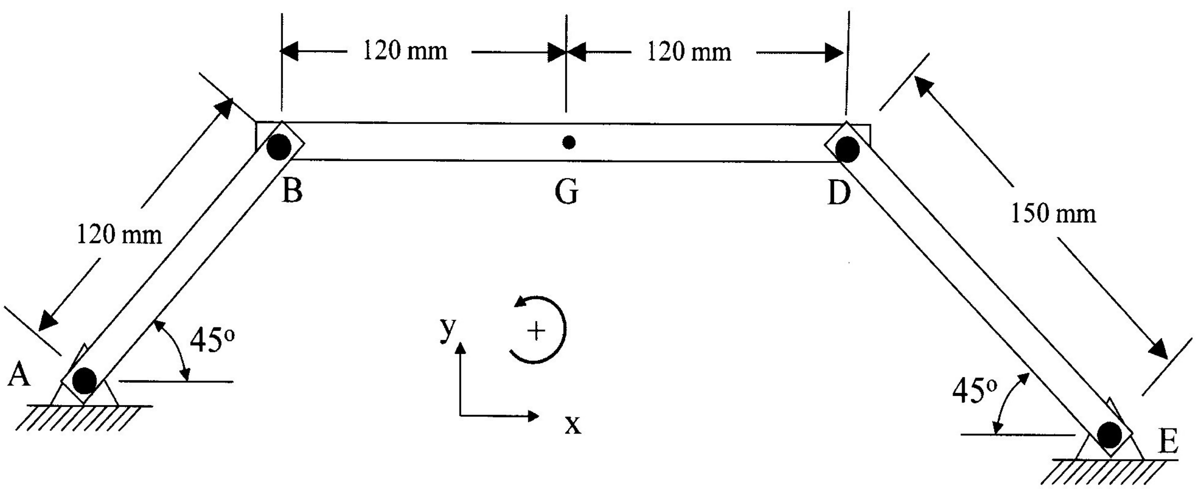
A mass, initially at rest, is subject to a non-constant force, $\vec{P} = 100t\,\hat{\imath}$ Newtons, where t is measured in seconds. You may assume the mass translates and does not rotate.

How much time will it take for the mass to reach a speed of 10 m/s? (20 marks)



Question 2

Bar AB is rotating counterclockwise and, at the instant shown, the speed of point B is 5.1 m/s.



a) What is the velocity of point B in x-y coordinates? (5 marks)

b) What is the angular velocity of bar AB? (5 marks)

c) Determine the speeds of points G and D at this instant. (5 marks)

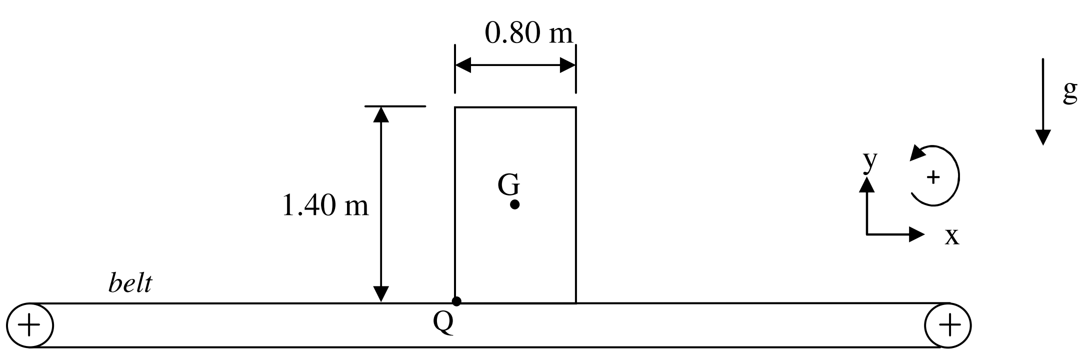
d) Determine the angular velocities of bar DE and bar BD at this instant. (5 marks)

Question 3

A rectangular crate of mass 100 kg rests on a conveyor belt as shown. The crate's height and width are 1.40 metres and 0.80 metres respectively. Assume the crate is uniform such that its centre of mass is located in the middle at G as shown. The coefficients of static and kinetic friction between the crate and the belt are $\mu_s = 0.60$ and $\mu_k = 0.50$.

a) Determine the largest acceleration that the belt can have in the positive x-direction without causing the crate to tip or slip. (10 marks)

b) At t = 0 the belt accelerates from rest with $\vec{a}_{belt}$= 5.70 $\hat{\imath}$ m/s^2 and the crate will begin to tip about point Q. Determine $\vec{a}_G$ and α for the crate at the instant t = 0. The moment of inertia of the crate about G is: $I_G = \frac{1}{12}m(a^2 + b^2)$, where a and b are the height and width. Note that for the crate at this instant, $\omega = 0$, $\alpha \neq 0$, and point Q does not slip relative to the belt. (10 marks)



Question 4

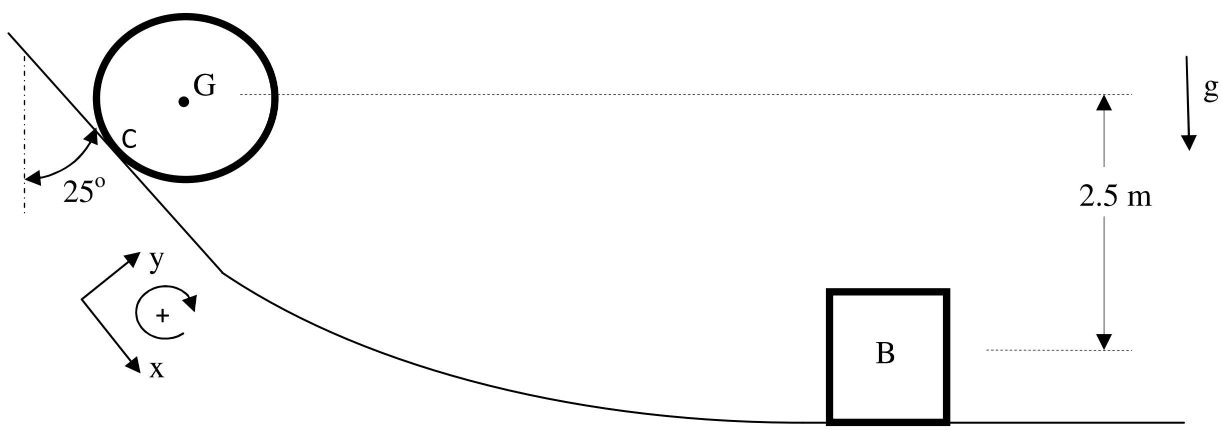
A wheel has mass m = 8.0 kg, radius R = 1.2 metres, and radius of gyration k = 0.70 metres. At the instant shown in the diagram, its center of gravity has a speed of 5.9 m/s, as the wheel rolls without slipping down the hill. At the bottom of the hill, it strikes the stationary block B of mass 5.0 kg; the wheel then stops rolling and sticks to the block B. The wheel and block then both slide to the right together.

a) At the instant shown in the diagram, what is the magnitude of the total acceleration of point C on the wheel, located at the edge of the wheel where it is touching the ground? (5 marks)

b) At the instant shown in the diagram, what is the acceleration of the centre of the wheel, in n-t coordinates? (5 marks)

c) What will be the speed of the centre of the wheel after the wheel has decreased its elevation by 2.5 metres, just before the wheel strikes the block B? (5 marks)

d) What will be the speed of the block and wheel immediately following the collision, as they slide to the right stuck together? (5 marks)



Question 5

The homogeneous slender rod of mass m = 16 kg and length L = 850 mm is supported by a pin at A and is able to rotate about A. The bar is connected to a spring and damper as shown.

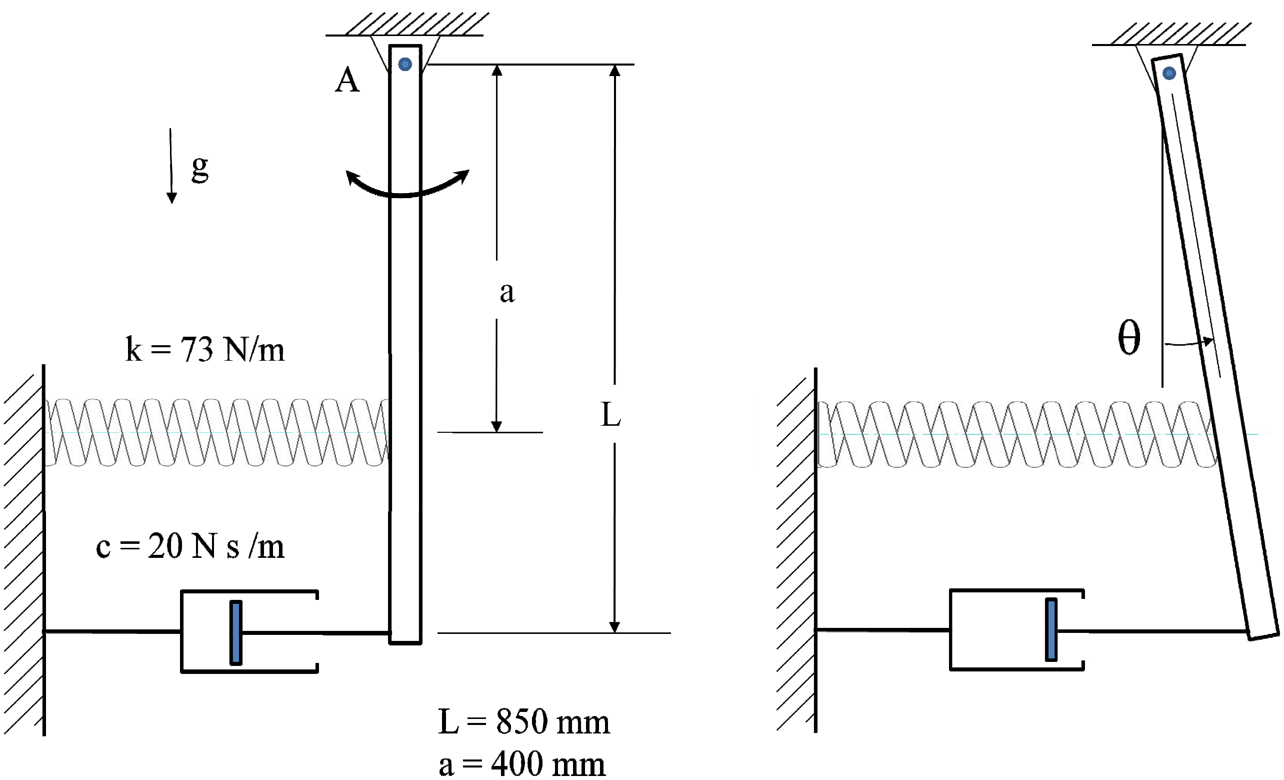
The bar is initially in equilibrium with the spring relaxed at the position shown in the diagram on the left.

a) What is the moment of inertia of the rod about A? (5 marks)

b) What is the differential equation for the motion of the rod in terms of θ? Assume the motion only involves small angles of θ. (5 marks)

c) What is ω_d the damped natural frequency of the rod? (5 marks)

d) Is the system underdamped, overdamped or critically-damped? Why? (5 marks)



Q1 Mass doesn't move until after P exceeds $F_{f\,max\ static}$

$F_{f\,max\ static} = \mu_s N$ set $P = \mu_s mg$

$100t = 0.3(100)(9.81)$ $t = 2.94\ s$

Second step: $F_{f\,max\ kinetic} = \mu_k N$

$\sum F_x = ma$

$P - \mu_k mg = ma \longrightarrow a = \dfrac{\cancel{100}t - 0.25(\cancel{100})(9.81)}{1\cancel{00}}$

$a = t - 2.45$

$dv = a\,dt$

$10 = \int_{2.94}^{t} (t - 2.45)\,dt$

$10 = \left(\dfrac{t^2}{2} - 2.45t\right)\Big|_{2.94}^{t}$

$10 = \dfrac{t^2}{2} - 2.45t - 4.322 + 7.210$

$\dfrac{t^2}{2} - 2.45t - 7.111 = 0$

$t = \dfrac{2.45 \pm \sqrt{2.45^2 - 4(\frac{1}{2})(-7.111)}}{2(\frac{1}{2})}$

$t = 6.95\,s.$

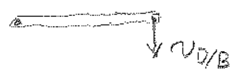
Q2

a) $\vec{v}_B = -5.1\cos45°\,\hat{i} + 5.1\sin45°\,\hat{j}$

$\vec{v}_B = -3.606\,\hat{i} + 3.606\,\hat{j}$ m/s

b) $\omega_{AB} = \dfrac{v_{B/A}}{r_{B/A}} = \dfrac{5.1}{0.120} = 42.5$ rad/s. noting CCW (+), $\omega_{AB} = 42.5$ rad/s.

c) $\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$



$-v_D\cos45°\,\hat{i} - v_D\cos45\,\hat{j} = -3.606\,\hat{i} + 3.606\,\hat{j} - \omega_{BD}(0.240)\,\hat{j}$

$\hat{i}$ $\quad -v_D\cos45° = -3.606 \quad \Rightarrow v_D = 5.1$ m/s

$\hat{j}$ $\quad -v_D\cos45 = 3.606 - \omega_{BD}(0.240) \quad \omega_{BD} = 30.05$ rad/s.

$\vec{v}_G = \vec{v}_B + \vec{v}_{G/B}$

$\vec{v}_G = -3.606\,\hat{i} + 3.606\,\hat{j} - \omega_{BD}(0.120)\,\hat{j}$

$\vec{v}_G = -3.606\,\hat{i}$ m/s $\Rightarrow v_G = 3.606$ m/s.

d) $\omega_{BD} = -30.05$ rad/s. noting that CCW (+)

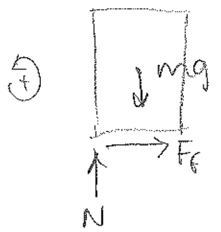
$\omega_{DE} = \dfrac{v_{D/E}}{r_{D/E}} = \dfrac{5.1}{0.150} = +34$ rad/s.

Q3

a) on verge of slip $F_f = \mu_s mg$ $\quad \Sigma F_x = ma_B$

$$\mu_s \cancel{m} g = \cancel{m} a_B \quad a_B = \mu_s g = \underline{0.6g} = 5.886 \text{ m/s}^2$$

on verge of tip



$\Sigma M_G = 0$ $\qquad \Sigma F_y = 0$ $\qquad \Sigma F_x = ma_B$

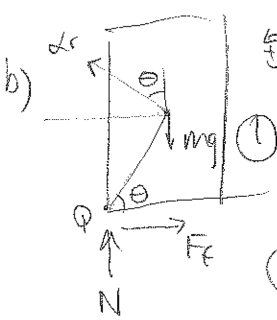
$\underbrace{0.7F_f - 0.4N = 0 \qquad N = mg}$ $\qquad F_f = ma_B$

$F_f = \dfrac{0.4(mg)}{0.7}$ $\qquad \dfrac{0.4}{0.7}\cancel{m}g = \cancel{m}a_B$

$a_B = \dfrac{4}{7}g = \underline{\underline{0.57g}} = 5.606 \text{ m/s}^2$ lower

$\therefore a_{B\,max} = 0.57g = 5.606 \text{ m/s}^2$

b)



$\circlearrowleft+$ $\Sigma M_G = I_G \alpha$ $\qquad I_G = \frac{1}{12}(100)(0.8^2 + 1.4^2) = 21.6 \text{ kgm}^2$

① $0.7F_f - 0.4N = 21.6\alpha$

$\Sigma F_x = ma_{Gx}$

② $F_f = 100 a_{Gx}$

$\Sigma F_y = ma_{Gy}$

③ $N - mg = 100 a_{Gy}$

3 eqn, 5 unknown, But

$$\vec{a}_G = \vec{a}_Q + \overset{\alpha r}{(\vec{a}_{G/Q})_t} + \underset{0}{\overset{\omega^2 r}{(\vec{a}_{G/Q})_n}}$$

$$\vec{a}_G = 5.7\hat{\imath} - \alpha r \sin\theta\,\hat{\imath} + \alpha r \cos\theta\,\hat{\jmath}$$

$$\vec{a}_G = 5.7\hat{\imath} - \alpha \cancel{r}\left(\frac{0.7}{\cancel{r}}\right)\hat{\imath} + \alpha\cancel{r}\left(\frac{0.4}{\cancel{r}}\right)\hat{\jmath}$$

$a_{Gx} = 5.7 - 0.7\alpha \qquad a_{Gy} = 0.4\alpha$

continued:

②′ $F_f = 100(5.7 - 0.7\alpha)$
$F_f = 570 - 70\alpha$

③′ $N - 100(9.81) = 100(0.4\alpha)$
$N - 981 = 40\alpha$

① ②′ ③′ 3 eqn 3 unknown, by calculator

$F_f = 564.7 \text{ N}$

$N = 984.0 \text{ N}$

$\alpha = 0.076 \text{ rad/s}^2$

$\vec{a}_G = 5.7\hat{\imath} - 0.076(0.7)\hat{\imath} + 0.076(0.4)\hat{\jmath}$

$\vec{a}_G = 5.65\hat{\imath} + 0.0305\hat{\jmath} \text{ m/s}^2$

Q4 a) $a_c = \omega^2 R$

$\omega = \frac{v}{R} = \frac{5.9}{1.2} = 4.91\dot{6}$ rad/s $\quad a_c = (4.91\dot{6})^2(1.2)$

$a_c = 29.01$ m/s²

$a_x = \alpha R$

b) $\Sigma F_x = ma_x \Rightarrow mg\cos\theta - F_f = m\alpha R$ ①

$\Sigma M_G = I_G\alpha \Rightarrow F_f R = k^2 m\alpha$

$F_f = \frac{k^2 m\alpha}{R}$ sub into ①

$\cancel{m}g\cos\theta - \frac{k^2\cancel{m}\alpha}{R} = \cancel{m}\alpha R$

$\alpha = \frac{g\cos\theta}{R + \frac{k^2}{R}} = \frac{9.81(\cos 25°)}{1.2 + \frac{0.9^2}{1.2}}$

$\alpha = 5.528$ rad/s²

$a_x = \alpha R$

$= 5.528(1.2)$

$a_x = 6.63$ m/s² $\quad \vec{a}_G = 6.63\,\hat{u}_t$ m/s²

c) $T_1 + V_1 = T_2 + \cancel{V_2}^{\,0}$

$\frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 + mgh = \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2$

$\frac{1}{2}v_1^2 + \frac{1}{2}k^2\omega_1^2 + gh = \frac{1}{2}v_2^2 + \frac{1}{2}k^2\omega_2^2 \quad \left(\omega_2 = \frac{v_2}{R}\right)$

$5.9^2 + 0.9^2(4.9167)^2 + 2(9.81)(2.5) = v_2^2\left[1 + 0.9^2/1.2^2\right]$

(May also use: $T = \frac{1}{2}I_K\omega$) $\quad v_2 = 8.45$ m/s.

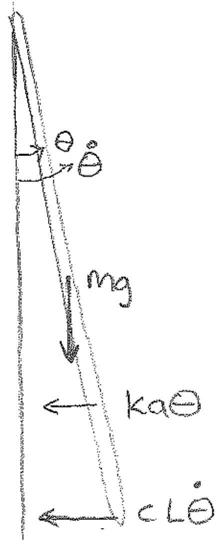
d) $m_w v_{w1} = v_2(m_w + m_B)$

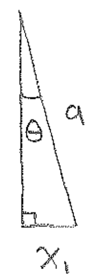
$v_2 = \frac{8(8.45)}{(8+5)} = 5.2$ m/s.

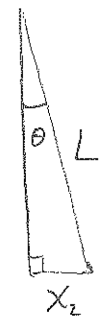
Q5

a) $I_A = \frac{1}{3}mL^2 = \frac{1}{3}(16)(0.85)^2 = 3.853 \ \text{kgm}^2$

b).







$x_1 = a\sin\theta$
$x_1 \approx a\theta$

$x_2 = L\sin\theta$
$\dot{x}_2 = L\cos\theta\,\dot{\theta}$
$\dot{x}_2 \approx L\dot{\theta}$

$$\Sigma M_A = I_A\ddot{\theta}$$

$$-mg\frac{L}{2}\theta - ka\theta(a) - cL\dot{\theta}(L) = I_A\ddot{\theta}$$

$$I_A\ddot{\theta} + cL^2\dot{\theta} + \left[\frac{mgL}{2} + ka^2\right]\theta = 0$$

$$3.853\ddot{\theta} + 20(0.85)^2\dot{\theta} + \left[\frac{16(9.81)(0.85)}{2} + 73(0.4)^2\right]\theta = 0$$

$$3.853\ddot{\theta} + 14.45\dot{\theta} + 78.388\theta = 0$$

c) $\omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2}$

$$\ddot{\theta} + \frac{14.45\dot{\theta}}{3.853} + \frac{78.388}{3.853}\theta = 0$$

$$\omega_n = \sqrt{\frac{78.388}{3.853}} = 4.5103 \ \text{rad/s}$$

Critical damping when the characteristic equation has two equal roots, λ.
ie when "$b^2 - 4ac = 0$"

$$\left(c_c(0.85)^2\right)^2 = 4(3.853)(78.388)$$

$$c_c = 48.11 \ \text{N/m/s}$$

$$\omega_d = 4.5103\sqrt{1 - \left(\frac{20}{48.11}\right)^2} = 4.102 \ \text{rad/s}$$