Faculty of Applied Science and Engineering
University of Toronto

MAT 188H1S – Linear Algebra
WEDNESDAY, APRIL 17, 2013

FINAL EXAMINATION

LAST NAME: ______________________________

FIRST NAME: ______________________________

STUDENT NUMBER: ______________________________

SIGNATURE: ______________________________

Time allowed: 2 hours, 30 minutes
Total marks: 75
Calculators allowed: Casio 260, Sharp 520, or TI 30.
Examiner: S. Cohen

Use the backs of pages when necessary,
indicating clearly where solutions continue.

FOR MARKER'S USE ONLY	
QUESTION	MARK
1	**/ 12**
2	**/ 10**
3	**/ 15**
4	**/ 15**
5	**/ 12**
6	**/ 11**
TOTAL	**/ 75**

$$\det A = \sum_{k=1}^{n} a_{ik} C_{ik}$$

1. [12 marks] Let $A = \begin{bmatrix} 2 & 0 & 4 & 2 & 1 & -1 \\ 2 & 1 & 3 & 3 & 0 & 1 \\ 1 & 2 & 0 & 3 & -1 & 3 \\ -1 & 1 & -3 & 0 & -2 & 3 \end{bmatrix}$.

Find the rank of A and determine bases for its row, column, and null spaces.

2. [10 marks] The augmented matrix of a system is $\left[\begin{array}{ccc|c} 1 & 1 & 3 & a \\ a-1 & 0 & 2 & 4-a \\ 1 & a & 4 & a \end{array}\right]$.

For what values of a does the system have: (i) infinitely many solutions, (ii) one solution, or (iii) no solutions?

3. Let $U = \text{span}\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ 3 \\ -1 \end{pmatrix} \right\}$.

(a) [9 marks] Find an orthogonal basis for U.

(b) [4 marks] Let $X = \begin{pmatrix} 2 \\ 1 \\ 2 \\ 1 \end{pmatrix}$.

Write X as the sum of two vectors, one from U and the other from $U^{\perp}$.

(c) [2 marks] Extend the basis from (a) into an orthogonal basis of $\mathbb{R}^4$.

4. [15 marks] Let $A = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$. Find an **orthogonal matrix** P and a diagonal matrix D such that $A = PDP^{-1}$.

[If you do not remember how to do this, you can simply diagonalize the matrix for part marks]

[More room on the next page]

[Extra room for the previous question]

5. (a) [4 marks] Show that if $A^2 = I$, then $A^5 - 4A^4 + (A^T)^2 - A^{-1} = -3I$.

(b) [8 marks] If U is a subspace, show that $U^{\perp}$ is a subspace.

6. (a) [5 marks] Suppose A is a 2×2 matrix with $A^2 = I$. Show that null $A = \emptyset$.

(b) [6 marks] If $A^2 = A$, show that null $A \cap$ col $A = \emptyset$.