University of Toronto
Faculty of Applied Science and Engineering

FINAL EXAMINATION, December 2006
First Year - CIV, CHE, IND, LME, MEC, MMS

MAT 188H1F, Linear Algebra
Exam Type: A

Examiners: S. Chulkov, Y.-H. Kim, B. Stephens, S. Uppal

Last Name: ______________________

First Name: ______________________

Student Number: ______________________

Instructions:

- **The use of non-programmable calculators is permitted.**
- Answer all questions. Total marks: 100.
- Please have your student card ready for inspection and turn off all cellular phones.
- This paper has a total of 13 pages, including this cover page. Present your **solutions** (in other words, show your work!) in the space provided. Use the back of the **preceding page** if you need more space. The value of each question is indicated in square brackets beside each question number.
- Do not tear any pages out from this test.

FOR MARKER USE ONLY	
Question	Marks
1	
2	
3	
4	
5	
6	
7	
Total	

1. (a) [5 marks] Given the point $P(5,3,2)$, find the orthogonal projection onto the plane $x+2y+3z=0$.

   (b) [5 marks] Find the scalar equations of the line of intersection of the planes $x+2y+3z=0$ and $2x-y+z=0$.

(Question 1 continued)

(c) [5 marks] Find the plane perpendicular to the line obtained in part (b) that contains the point $(0,0,2)$.

2. Let $A = \begin{bmatrix} 2 & -2 & 1 & 4 & 7 & 5 \\ 1 & -1 & 0 & 3 & 2 & 1 \\ 1 & -1 & 0 & 3 & 3 & 2 \\ -2 & 2 & 0 & -6 & -6 & -4 \end{bmatrix}$.

(a) [6 marks] Find a basis for row A.

(b) [6 marks] Find a basis for col A.

(c) [3 marks] Find rank A.

3. Let $U = \text{span}\left\{ \begin{bmatrix} 2 \\ 4 \\ -2 \\ 6 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 0 \\ 10 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ -1 \\ 14 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ -3 \\ 10 \\ -3 \end{bmatrix} \right\}$.

(a) [4 marks] Find $\dim U$ and $\dim U^{\perp}$.

(b) [5 marks] Find a basis for $U^{\perp}$.

(Question 3 continued)

(c) [6 marks] Given $X = \begin{bmatrix} 2 \\ -1 \\ 0 \\ -1 \\ 3 \end{bmatrix}$, find the orthogonal projection $\text{proj}_{U^{\perp}} X$.

4. [15 marks] Find the least squares approximating line $y = mx + b$ for the points $(2,4)$, $(4,3)$, $(7,2)$, $(8,1)$.

5. [15 marks] Given

$$A = \begin{bmatrix} 8 & 2 & -2 \\ 2 & 5 & 4 \\ -2 & 4 & 5 \end{bmatrix},$$

find an *orthonormal* basis of $\mathbb{R}^3$ consisting of the eigenvectors of A.

6. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the orthogonal projection on the line $y = x$.

(a) [4 marks] Find $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$ and $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$.

(b) [2 marks] Using the result from part (a), find the matrix of T.

(c) [4 marks] If A is the matrix of T found in part (b), find A^{2007}.

7. For each of the following five statements determine whether the statement is true or false (by putting a checkmark next to the word "<u>True</u>" or "<u>False</u>") and then justify your answer with an appropriate explanation or an example showing that it is false. For each question, failing to check either True or False will result in no credit (and no part marks).

(a) [3 marks] The matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ is diagonalizable. <u>True ☐ or False ☐</u>

(b) [3 marks] The matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is diagonalizable. <u>True ☐ or False ☐</u>

(Question 7 continued)

(c) [3 marks] It is impossible to have a $n \times n$ matrix with linearly independent rows but linearly dependent columns. True ☐ or False ☐

(d) [3 marks] If a square matrix A is diagonalizable, then A^{2007} is also diagonalizable. True ☐ or False ☐

(Question 7 continued)

(e) [3 marks] If A is a 2×2 matrix and A^2 is diagonalizable, then A is also diagonalizable. True ☐ or False ☐

End of examination

(Available for scrap work. Do NOT tear out this page!)