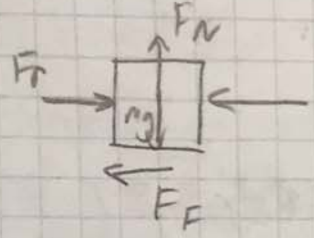
Q1 MIE100 Midterm

a)



/10

$\boxed{3}$

$F_r - F_s - F_F = 0$

$$F_s = K(\Delta x) = 150(0.25 - 0.18) = 150(0.07) = 10.5 \text{ N}$$ $\boxed{2}$

$$F_F = \mu_s mg = \mu_s F_N = 0.3(5)(9.81) = 0.3(49.05) = 14.715 \text{ N}$$ $\boxed{2}$

$$F_r = m(r\dot{\theta}^2) = 5(0.2)\dot{\theta}^2 = mv^2/r$$ $\boxed{2}$

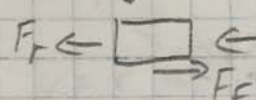
$$\overset{F_r}{5(0.2)\dot{\theta}^2} = \overset{F_s}{10.5} + \overset{F_F}{14.715}$$

$$\dot{\theta}^2 = 25.215$$

$$\dot{\theta}^2 = 5.02 \text{ rad/s}$$

$\boxed{1}$

- wrong F_r / F_F (8)



$F_r = F_F - F_s \rightarrow \dot{\theta}^2 = 2.05$ rad/s

- no Friction (6)

$F_r = F_s \rightarrow \dot{\theta}^2 = 3.24$ rad/s

- no spring

$F_r = F_F \rightarrow \dot{\theta}^2 = 3.836$ rad/s

- kinetic energy conservation (2)

$\frac{1}{2} mv^2 = \frac{1}{2} K\Delta x^2$

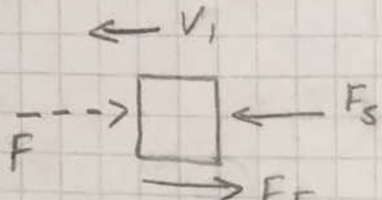
- looking at right components motion and spring
- can still give 5 rad/s

- wrong calc $\boxed{-1}$
  wrong sign
- wrong units $\boxed{-0.5}$

- wrong friction coefficient μ_K $\rightarrow \dot{\theta} = 4.77$ rad/s

$\boxed{-1}$

Q1 b)



$\Sigma F = ma_r$

$F - F_s + F_F = 0$

/5

$\Sigma F = ma_r$, $F = F_s - F_F$ $\boxed{1}$

$F_s = k\Delta x$
$= 10.5 \ N$ $\boxed{1}$

$F_f = \mu_k (5)(9.81)$
$= 0.25 (49.05)$
$= 12.2625 \ N$ $\boxed{1}$

$ma_r = 10.5 - 12.2625$

$a_r = -0.3525 \ m/s^2$

$F = ma_r$ $\boxed{1}$

required to balance, but no balance

$\therefore a_r = 0.3525 \ m/s^2 \ (\rightarrow)$ $\boxed{1}$

$a_r = \ddot{r} - r\dot{\theta}^2$

- solve for $\ddot{r}$ $\quad r = \quad F_c (\leftarrow)$ ③

$F = F_s - F_F + mr\dot{\theta}^2$
$\qquad\qquad 10.5625$
$= 8.8$
$\rightarrow a_r = 1.76 \ m/s^2$ to balance $\therefore (\leftarrow)$ ④

OR

$F = F_s - F_F - F_c$
$= -12.325$
$\rightarrow -2.465 \ m/s^2$ to balance $\therefore (\rightarrow)$ $2.465 \ m/s$ $\ddot{r}$

- assume no slide $\ddot{r} = 0$ ①

$a_r = \cancel{\ddot{r}}^{0} - r\dot{\theta}^2$
$= -0.2(3.25)^2$
$= -2.1125 \ m/s^2 \rightarrow (\leftarrow) \ 2.11 \ m/s^2$

③

- no friction or no springs
$k\Delta x = ma_r \qquad -\mu_k mg = ma_r$
$\hookrightarrow a_r = 2.1 \ m/s^2 \qquad a_r = 2.45 \ m/s^2$

- wrong calc $\boxed{-1}$
wrong direction $\boxed{-0.5}$

wrong units $\boxed{-0.5}$

- wrong friction μ_s $\boxed{-1}$
$\hookrightarrow a_r = 0.843 \ m/s^2$

- sum force $F_s + F_F$ ③
$\hookrightarrow a_r = 4.5 \ m/s^2$

⑤

$-\ddot{r} = a_r = 0.35$

Q1 c) Math doesn't work out

$\frac{}{10}$

$V_{\theta_1} = r\dot{\theta} = 0.2(3.25)$
$= 0.65 m/s$

conservation of momentum $mV_{\theta_1} r_1 = mV_{\theta_2} r_2$

(3) $\boxed{1}$ $\underbrace{0.65(0.2)}_{mV_{\theta_1} r_1} = \underbrace{V_{\theta_2} (0.13)}_{mV_{\theta_2} r_2}$ $\boxed{1}$ $\rightarrow V_{\theta_2} = 1 m/s$
$\dot{\theta}_2 = 7.69$ $\boxed{1}$

conservation of energy

$V_{t_1} = \sqrt{V_{\theta_1}^2 + V_{r_1}^2} = \sqrt{0.65^2 + 0.3^2} = 0.716$ $\boxed{1}$ $V = -0.3 m_r + 0.65\theta_1$

(6) $\boxed{1}$ $\frac{1}{2} m (V_{t_2})^2 + \frac{1}{2} k (\Delta x_2)^2 = \frac{1}{2} m (V_{t_1})^2 + \frac{1}{2} k (\Delta x_1)^2 - \mu_k m g \Delta x$ $\boxed{1}$ $\boxed{1}$ $\boxed{1}$ $\boxed{1}$

$\frac{1}{2}(5) V_{t_2}^2 = \frac{1}{2}(5)(0.716)^2 + \frac{1}{2}(150)(0.07)^2 - 0.25(5)(9.81)(0.07)$
$= 1.28 + 0.3675 - 0.8584$
$= 0.789$ $\boxed{1}$

(1) $V_{t_2}^2 = 0.3156$ $\qquad V_t = 0.5618 m/s$

$V_{\theta_2} > V_t$ $\therefore$ not enough energy to move color to 0.13

- no friction, only V_r in conservation of energy
$\frac{1}{2} m (V_{r_2})^2 = \frac{1}{2} m (V_{r_1})^2 + \frac{1}{2} k (\Delta x_1)^2$
$\hookrightarrow V_{r_2} = 0.4868 m/s$ (4)

- no friction, total V_t in conservation of energy (or V_θ)
$\frac{1}{2} m (V_{t_2})^2 = \frac{1}{2} m (V_{t_1})^2 + \frac{1}{2} k (\Delta x_1)^2$
$\hookrightarrow V_{t_2} = 0.812 m/s$ (5)

- V_t in conservation of momentum
$mV_2 r_2 = mV_1 r_1$ (3)
$\hookrightarrow V_2 = 1.1 m/s$

- same V_r and $\dot{\theta}$ at end (2)
$= -0.3 m/s \hookrightarrow V_\theta = 4.225 m/s$

- wrong number in equation (no penalty except for answer)

$\boxed{-0}$

2 (a)

The current length of the spring equals to

$$l1=\sqrt{(\frac{L}{2})^2+\left(\frac{3}{4}L\right)^2}=\sqrt{(\frac{2.8}{2})^2+\left(\frac{3}{4}\times 2.8\right)^2}=2.52m$$

$$l2=\sqrt{\left(\frac{L}{2}\right)^2+\left(\frac{L}{4}\right)^2}=\frac{\sqrt{5}}{4}L=1.565m$$

The unstreched length of the spring equals to

$$l1_0=l2_0=l_0=\frac{\sqrt{2}}{2}L=1.97m \qquad (1)$$

The spring forces will be

$$F_1=k\left(l1-l1_0\right)=4000\times\left(2.52-1.97\right)=2200N$$
$$F_2=k\left(l2-l2_0\right)=4000\times\left(1.565-1.97\right)=-1620N \qquad (2)$$

The angle between the pole and the spring will be θ_1 and θ_2

$$\theta_1=\tan^{-1}\left(\frac{\frac{L}{2}}{\frac{3}{4}L}\right)=\tan^{-1}\left(\frac{\frac{2.8}{2}}{\frac{3}{4}\times 2.8}\right)=33.69^o=0.5877rad$$
$$\theta_2=\tan^{-1}\left(\frac{\frac{L}{2}}{\frac{1}{4}L}\right)=\tan^{-1}\left(\frac{\frac{2.8}{2}}{\frac{1}{4}\times 2.8}\right)=63.43^o=1.1rad \qquad (2)$$

Or

$$F_x=F_1\times\sin\left(\theta_1\right)+F_2\times\sin\left(\theta_2\right)=1220-1448=-228N$$
$$F_y=F_1\times\cos\left(\theta_1\right)-F_2\times\cos\left(\theta_2\right)=1830.5+724.6=2555.1N$$

$$|F|=\sqrt{228^2+2555.1^2}=2565.25N \quad \theta=\arctan\left(\frac{F_x}{F_y}\right)=5.099^o$$

2 (b)

Using the energy conservation law.

$$T_1+U_1=T_2+U_2 \qquad (2)$$

At the point shown in diagram, the kinetic energy and the potential energy can be expressed in the following equations.

$$T_1 = \frac{1}{2}mv^2 = \frac{1}{2}\times 21.4\times 13^2 = 1808.3J$$

$$U_1 = \frac{1}{2}k\left(\sqrt{\left(\frac{3}{4}L\right)^2+\left(\frac{L}{2}\right)^2}-\frac{\sqrt{2}}{2}L\right)^2+\frac{1}{2}k\left(\sqrt{\left(\frac{L}{2}\right)^2+\left(\frac{L}{4}\right)^2}-\frac{\sqrt{2}}{2}L\right)^2$$

$$=\frac{1}{2}k\left(\sqrt{\left(\frac{3}{4}\times 2.8\right)^2+\left(\frac{2.8}{2}\right)^2}-\frac{\sqrt{2}}{2}\times 2.8\right)^2+\frac{1}{2}k\left(\sqrt{\left(\frac{2.8}{2}\right)^2+\left(\frac{2.8}{4}\right)^2}-\frac{\sqrt{2}}{2}\times 2.8\right)^2 \quad (3)$$

$$=\frac{1}{2}k(2.52-1.98)^2+\frac{1}{2}k(1.565-1.98)^2 = 0.1458k+0.0861125k = 0.232k$$

At the top of the pole,

$$U_2 = mg\frac{3}{4}L+\frac{1}{2}k\left(\sqrt{\left(\frac{L}{2}\right)^2}-\frac{\sqrt{2}}{2}L\right)^2+\frac{1}{2}k\left(\sqrt{\left(\frac{L}{2}\right)^2+L^2}-\frac{\sqrt{2}}{2}L\right)^2$$

$$=21.4\times 9.81\times\frac{3}{4}\times 2.8+\frac{1}{2}k(1.4-1.98)^2+\frac{1}{2}k\left(\sqrt{1.4^2+2.8^2}-1.98\right)^2 \quad (3)$$

$$=440.86+0.1682k+0.662k = 0.8302k+440.86$$

$$T_2 = 0$$

Then we can have,

$$0.232k+1808.3 = 0.8302k+440.86 \quad (2)$$

Then, $k = 2285.92N/m$

2 (c)

The acceleration will be applied towards the positive direction of y axis,

$$\cos(\theta_1) = \cos(33.69^o) = 0.832$$

$$\cos(\theta_2) = \cos(63.43^o) = 0.4473 \quad (2)$$

$$F_1 = k(l1-l1_0) = 2500\times(2.52-1.97) = 1375N$$

$$F_2 = k(l2-l2_0) = 2500\times(1.565-1.97) = -1012.5N \quad (2)$$

$$F_y = F_1\times\cos(\theta_1)-F_2\times\cos(\theta_2) = 1375\times 0.832+1012.5\times 0.4473 = 1596.89125N \quad (2)$$

$$a = \frac{F_y}{m}-g = \frac{1596.8912}{21.4}-9.81 = 64.81m/s^2 \quad (4)$$

Solution and grading scheme

Question 3

(a) angular momentum $(H_o) = r \cdot mv$ -- (1)

either calculate the velocities or directly input them

$$\left.\begin{aligned} V_{\theta A} &= r_a \dot{\theta} = (3)(3.5) = 10.5 \text{ m/s} \\ V_{\theta B} &= r_B \dot{\theta} = (4)(3.5) = 14 \text{ m/s} \end{aligned}\right\} \text{ -- (1)}$$

Total angular momentum $= H_a + H_b$

$$= m_a V_a r_a + m_b V_b r_b \quad \text{-- (1)}$$

$$= (6)(10.5)(3) + (4)(3)(14)$$

$$= 357 \text{ N.S or kg.m}^2\text{/s} \quad (2)$$

(1) final answer (1) unit

(b) $\Delta H_o = \int_0^t M\, dt$ -- (3)

$$\cancel{H_f}^{\,0} - H_i = \int_0^t kt\, dt \quad \text{-- (1)}$$

$$0 - (6)(3)(3.5)(3) = \frac{kt^2}{2}\Big]_0^5 \quad \text{-- (2)}$$

(1) for integration

(1) for the angular momentum calcution

$$\frac{-(189)(2)}{25} = k \quad \text{-- (1) for calcution of } k$$

$k = -15.12$ N.m/s or kg.m²/s² (3)

(1) final answer (1) unit (1) direction (−,+)

(c) $\Delta L = \int f \, dt$ — (2)

in — y direction — (2) → (1) for calculation → (1) for $L_{y_f} = 0$

$\Delta L_y = \int f_y \, dt$

$\cancel{L_{y_f}}^{\,0} - L_{y_i} = f_y \Delta t$

$$-\frac{m_b V_{bi}}{\Delta t} = f_y$$

$$f_y = \frac{(-3)(5.4)}{0.2}$$

$f_y = -81 \text{ N}$ — 0.5

in — x direction — (2) → (1) for calculation → (1) for $L_{x_i} = 0$

$L_{x_f} - \cancel{L_{x_i}}^{\,0} = f_x \Delta t$

$$f_x = \frac{L_{x_f}}{\Delta t}$$

$$= \frac{(3)(4.2)}{0.2}$$

$f_x = 63 \text{ N}$ — 0.5

$f_t = \sqrt{f_x^2 + f_y^2}$ — (2) → (1) for final answer → (1) unit

$= 102.61 \text{N}$

$\theta = \tan^{-1}\left(\frac{f_x}{f_y}\right)$

$= 51.7^\circ$ — (1) for direction or vector

or $f = 63\hat{i} - 81\hat{j}$