

University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, DECEMBER 2002
First Year - Programs 1,2,3,4,6,7,8,9

MAT 186H1F
Calculus I

	Examiners
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GIVEN NAME _____	H. Bursztyn
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SIGNATURE _____	H. Li

INSTRUCTIONS:

Non-programmable calculators permitted.

Answer all questions.

Present your solutions in the space provided;
use the back of the preceding page if more
space is required.

TOTAL MARKS: 100

The value for each question is shown in
parentheses after the question number.

MARKER'S REPORT	
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
TOTAL	

1. [30 marks; 5 marks for each part.] Find the following:

$$(a) \lim_{x \rightarrow 0} \frac{x - \sin^{-1} x}{x^3}$$

$$(b) \int_0^{\pi/2} \sin^3 x \cos x \, dx$$

(c) $\int_0^\infty (e^{-2x} + e^{-3x}) dx$

(d) $\frac{dy}{dx}$ at the point $(x, y) = (-2, 2)$ if $y^2 = (4 + \sin(x + y))^{x^2 - 3}$.

$$(e) \lim_{x \rightarrow 0} \frac{\int_0^x \sin(t^2) dt}{x^3}.$$

- (f) the average speed of a particle, with position $x = t^2 - 4t + 2$ at time t , on the interval $0 \leq t \leq 3$.

2. [12 marks] Newton's Law of Cooling states that

$$\frac{dT}{dt} = k(T - A),$$

where T is the temperature of an object at time t , A is the (constant) ambient temperature of the air surrounding the object, and k is a constant.

A hot cup of coffee at 90° C is placed on a table in a room with constant air temperature 20° C . If the coffee cools to 50° C in 5 minutes, what will the temperature of the coffee be 2 minutes later? Sketch a graph of temperature, T , for $t \geq 0$.

3. [12 marks] Two poles are driven into the ground 5 m apart. One pole protrudes 3 m above the ground and the other pole 2 m above the ground. A single piece of rope is attached to the top of one pole, passed through a loop on the ground, pulled taut, and attached to the top of the other pole. Where should the loop be placed in order that the rope be as short as possible?

4. [12 marks; 6 marks for each part] Let $f(x) = \frac{1}{4}x^2$, for $0 \leq x \leq 2$. Find:

- (a) the volume of the solid of revolution obtained by rotating about the line $y = -1$ the region bounded by $y = f(x)$, $y = 0$, $x = 0$ and $x = 2$.

- (b) the area of the surface of revolution obtained by rotating the curve $y = f(x)$ about the y -axis, for $0 \leq x \leq 2$.

5. [12 marks] A funnel in the shape of a right circular cone is 15 cm across the top and 30 cm deep. Water is flowing in at a rate of 80 mL/sec and flowing out the bottom at 15 mL/sec. At what rate is the surface of the liquid rising when the water fills the funnel to a depth of 20 cm? (NOTE: 1 mL = 1 cm³.)

6. [12 marks] Sketch the graph of $f(x) = \tan^{-1} \sqrt{x^2 - 1}$, indicating all critical points, all inflection points, and all asymptotes, if any. (Be careful with domain and range!)

7. [10 marks] A tank filled with water of density $\rho = 1000 \text{ kg/m}^3$ has the shape of a hemisphere, with radius 2 m. Find the work done in pumping all of the water out of the tank and up to a horizontal pipe 1 m above the top of the tank. (Use acceleration due to gravity $g = 9.8 \text{ m/sec}^2$.)