

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, APRIL 2004
First Year - CIV, CHE, IND, LME, MEC, MSE

MAT 186H1S - CALCULUS I

Exam Type: A
Examiner: D. Burbulia

INSTRUCTIONS

Non-programmable calculator permitted; no other aids allowed.

Present your solutions to all of the following questions in the exam booklets supplied.
The marks for each question are indicated in parentheses.

TOTAL MARKS: 100.

1. [10 marks; 5 marks for each part] Let the velocity v of a particle at time t be given by

$$v = t^2 - 6t + 8.$$

- (a) What is the average velocity of the particle between time $t = 0$ and time $t = 3$?
(b) What is the total distance travelled by the particle between time $t = 0$ and time $t = 3$?
2. [20 marks; 5 marks for each part] Find the following limits:

(a) $\lim_{x \rightarrow 0} \frac{e^{-x} - 1 + x}{x^2}$

(b) $\lim_{x \rightarrow 1} \left(\frac{1}{\ln x} + \frac{1}{1-x} \right)$

(c) $\lim_{x \rightarrow 0} (1 - \sin x)^{3/x}$

(d) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \left(\frac{i^2}{n^2} + 1 \right)$

3. [10 marks; 5 marks for each part] Find the following:

(a) [5 marks] $\int_0^8 \frac{x^2}{\sqrt{x+1}} dx.$

(b) [5 marks] $F'(2)$, if $F(x) = \int_0^{\sqrt{x}} t^2 \sec^{-1} t dt.$

4. [10 marks] The radial probability density function for the ground state of the hydrogen atom is

$$P(r) = \left(\frac{4r^2}{a^2} \right) e^{-2r/a}, \text{ for } r \geq 0,$$

where $a > 0$ is a constant. Sketch a graph of the function identifying its relative maximum and points of inflection.

5. [10 marks] A circular oil slick of uniform thickness is caused by a spill of $1m^3$ of oil.

The thickness of the oil slick is decreasing at the rate of 0.1 cm/hr . At what rate is the radius of the slick increasing when the radius is 8 m ?

6. [10 marks] Find the area of the region bounded by the graphs of

$$y = 4 - x^2 \text{ and } y = x^2 + 4x - 2.$$

7. [10 marks] Find the volume of the solid obtained by revolving the region bounded by

$$y = \sqrt{x}, y = x, x = 0, \text{ and } x = 1$$

around the line $y = -2$.

8. [10 marks] Find the area of the surface of revolution generated by revolving the curve

$$y = x^2 - \frac{1}{8} \ln x, \text{ for } 1 \leq x \leq 2$$

around the y -axis.

9. [10 marks] A hemispherical tank of radius 3 m is located with its flat side down atop a tower 15 m above the ground. How much work is required to fill this tank with oil of density ρ if the oil is to be pumped into the tank from ground level? (Use acceleration due to gravity $g = 9.8 \text{ m/sec}^2$ and leave your answer in terms of ρ .)