University of Toronto
Faculty of Applied Science and Engineering
Department of Electrical and Computer Engineering

ECE110S – Electrical Fundamentals
Term Test 1 – February 12, 2019, 1:30 – 3:00 p.m.

($e = 1.6 \times 10^{-19}$ C, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $g = 9.81$ N/kg)

ANSWER ALL QUESTIONS ON THESE SHEETS, USING THE BACK SIDE IF NECESSARY.

1. Non-programmable calculators (Casio FX-991MS & Sharp EL-520X) are allowed.
2. You are allowed a one page (8.5" x 11") double-sided aid sheet.
3. For full marks, you must show methods, state UNITS and compute numerical answers (when requested).
4. Write in PEN. Otherwise, no remarking request will be accepted.
5. There is one extra blank page at the end for rough work.

Last Name: ______________________

First Name: Solutions

Student Number: ______________________

Tutorial Section:
(YOU LOSE ONE MARK FOR NOT MARKING YOUR TUTORIAL SECTION CORRECTLY)

- ☐ 01 HS106 Wed. 4-6 p.m.
- ☐ 02 GB404 Wed. 12-2 p.m.
- ☐ 03 SF1101 Wed. 4-6 p.m.
- ☐ 04 SF3202 Fri. 10-12 p.m.
- ☐ 05 HA410 Tues. 4-6 p.m.
- ☐ 06 GB404 Fri. 12-2 p.m.
- ☐ 07 GB248 Tues. 4-6 p.m.
- ☐ 08 MP137 Thurs. 4-6 p.m.
- ☐ 09 SF3201 Wed. 12-2 p.m.
- ☐ 10 MY380 Wed. 4-6 p.m.
- ☐ 11 SF1105 Wed. 4-6 p.m.

Question	Mark
1	
2	
3	
TOTAL	

Q1 [10 marks] Circle the correct answer in each of the following question.

1.1 Which one of the following statements concerning the electric force is true? **(1 mark)**

A) Two charged objects with identical charges will exert an attractive force on one another.

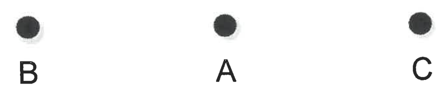
B) It is possible for a small negatively charged particle to float above a negatively charged surface.

C) A positively charged object is attracted toward another positively-charged object.

D) The electric force cannot alter the motion of an object.

E) Newton's third law of motion does not apply to the electrostatic force.

1.2 A charged particle, labeled A, is located at the midpoint between two other charged particles, labeled B and C. The sign of the charges on all three particles is the same. When particle A is released, it starts drifting toward B. What can be determined from this behavior? **(1 mark)**



A) The charge on A is larger than the charge on B.

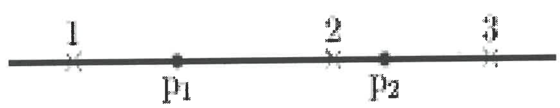
B) The charge on A is larger than the charge on C.

C) The charge on C is larger than the charge on B.

D) The charge on B is larger than the charge on A.

E) The charge on B is larger than the charge on C.

1.3 Two protons (p_1 and p_2) are on the x axis, as shown below. The directions of the electric field at points 1, 2, and 3 respectively, are: **(1 mark)**



A) $\rightarrow, \leftarrow, \rightarrow$

B) $\leftarrow, \rightarrow, \leftarrow$

C) $\leftarrow, \rightarrow, \rightarrow$

D) $\leftarrow, \leftarrow, \leftarrow$

E) $\leftarrow, \leftarrow, \rightarrow$

1.4 Which one of the following statements best describes the concept of the electric field? **(1 mark)**

A) The electric field is a vector quantity that is the reaction force of electrons.

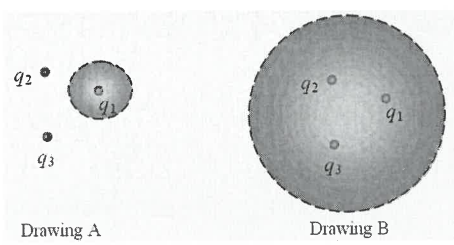
B) The electric field at each point is the potential energy of a test charge divided by the amount of the test charge.

C) The electric field is a distribution of vectors at points due to the presence of one or more charged objects.

D) The electric field is a scalar quantity related to the total amount of charge on one or more charged objects.

E) The electric field is a scalar field, which has a magnitude at each given point, similar to the temperature or pressure field.

1.5 Consider the three charges shown in drawings A and B. The values of the charges are $q_1 = +4 \times 10^{-7}$ C; $q_2 = +2 \times 10^{-7}$ C; $q_3 = -2 \times 10^{-7}$ C. In drawing A, a Gaussian surface surrounds charge q_1; and in drawing B, the Gaussian surface surrounds all three charges. Which one of the following statements concerning the electric flux through the two Gaussian surfaces is true? **(1 mark)**



A) The flux through the Gaussian surface in drawing A is greater than the flux through the Gaussian surface in drawing B.

B) The flux through the Gaussian surface in drawing A is the same as the flux through the Gaussian surface in drawing B.

C) The flux through the Gaussian surface in drawing A is less than the flux through the Gaussian surface in drawing B.

1.6 A point charge is placed at the center of a spherical Gaussian surface. The electric flux Φ_E is changed if: **(1 mark)**

A) The sphere is replaced by a cube of the same volume

B) The sphere is replaced by a cube of one-tenth the volume

C) The point charge is moved off center (but still inside the original sphere)

D) The point charge is moved to outside the sphere

E) A second point charge is placed just outside the sphere

1.7 The flux of the electric field (24 N/C)$\hat{\imath}$ + (30 N/C) $\hat{\jmath}$ + (16 N/C) $\hat{k}$ through a 2.0 m^2 portion of the *yz* plane is: **(2 marks)**

A) 32 N · m^2/C

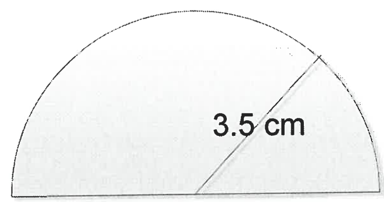
B) 34 N · m^2/C

C) 42 N · m^2/C

D) 48 N · m^2/C

E) 60 N · m^2/C

1.8 A 3.5-cm radius hemisphere contains a total charge of 6.6 × 10^{-7} C. The flux through the rounded portion of the surface is 9.8 × 10^4 N·m^2/C. The flux through the flat base is: **(2 marks)**



A) 0 N·m^2/C

B) +2.3 × 10^4 N·m^2/C

C) −9.8 × 10^4 N·m^2/C

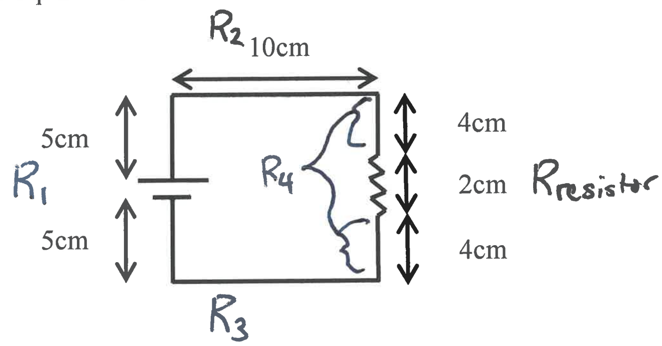
D) +9.8 × 10^4 N·m^2/C

E) −2.3 × 10^4 N·m^2/C

Q2 [10 marks]

a) Consider a circuit (see diagram below) that consists of a battery = 5 V connected in series with a resistor (R). The resistor is a solid cylinder with a given resistivity, $2x10^{-4}\Omega m$, diameter = 1 mm, and length = 2 cm. Each terminal of the battery is connected to the resistor via cylindrical wires that have a radius = 1 mm and resistivity, $1x10^{-5}\Omega m$. Calculate the current density through the resistor (R). [Ignore the length of the battery] **(4 marks)**

Hint: The total resistance equals the sum of resistances connected in series



$$R_1 = R_2 = R_3 = \rho \frac{L}{A} = 0.32\,\Omega$$

$$R_4 = 0.25\,\Omega$$

$$R_{all\ wires} = 1.21\,\Omega$$

$$R_{resistor} = 5.1\,\Omega$$

$$R_{total} = R_{all\ wires} + R_{resistor} = 6.31\,\Omega$$

$$I = \frac{V}{R} = 0.79\,A$$

$$J = \frac{I}{A} = 1 \times 10^{6}\ A/m^2$$

Q2 continued

b) If a capacitor consists of two parallel stainless steel disks (radius = 5 cm) separated by 1mm and with a ceramic material (dielectric constant = 100) inserted between the two disks, calculated the charge stored on this capacitor when a potential difference of 4.75 V is applied across this device. **(2 marks)**

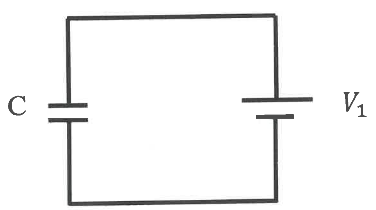
$q = C \cdot V$

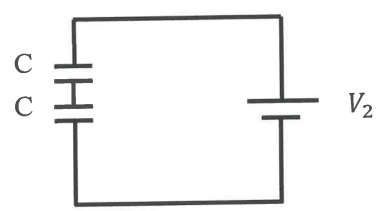
$V = 4.75\ V$

$C = \varepsilon_0 K \frac{A}{d} = 6.9 nF$

$q = 33 nC$

c) Consider the two circuits below, where the capacitance of each individual capacitor is 10 nF. If $V_1 = 4.75\ V$, determine V_2 such that the same amount of capacitive charge (q) is stored in both circuits. Also, determine which circuit requires more energy to store q, and by how much? **(4 marks)**





$q_1 = CV_1$

$q_2 = \frac{1}{2} C V_2$

$CV_1 = \frac{1}{2} CV_2$

$V_2 = 2V_1 = 9.5V$

$U_1 = \frac{1}{2} C V_1^2$

$= 112.8 nJ$

$U_2 = 225.6 nJ$

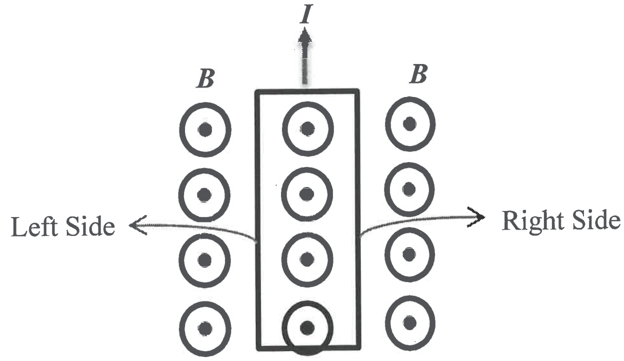
U_2 requires twice as much energy

or

112.8nJ more

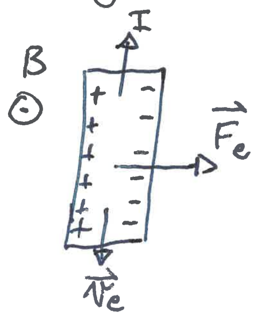
Q3 [10 marks]

a) Figure below shows a copper plate carrying a uniform current ***I*** immersed in a uniform magnetic field ***B***. A voltmeter is used to measure the electrostatic potential (ΔV) between the right and left sides of the plate. Is the voltage at the right side **higher, lower,** or **the same,** when compared to the left side? Justify your answer. **(3 marks)**



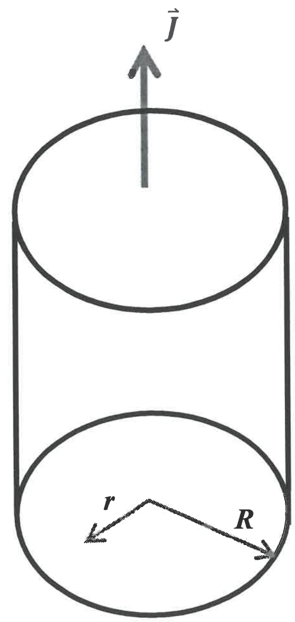
$$\vec{F_e} = -|e|\,\vec{v_e} \times \vec{B}$$

$\vec{F_e}$ points right for electron



$$V_{right} < V_{left}$$

b) A long straight wire of radius R has a non-uniform current density given by $J = \frac{J_0 r}{R}$ that flows as shown in the figure. J_0 is a constant and the wire carries the total current of I. Derive an expression for the magnitude of the magnetic field B, in the region $0 \leq r \leq R$, in terms of I, r, R, μ_0 and π. **(7 marks)**



$$\oint_C B \cdot ds = \mu_0 I_{enc}$$

$$\oint B \cdot ds = B r 2\pi$$

$$I_{enc} = \iint \vec{J} \cdot dA = \frac{J_0 2\pi}{R} \frac{r^3}{3}$$

$$B = \frac{\mu_0 J_0}{3R} r^2$$

but want B in terms of I

$$I = \frac{J_0 2\pi R^2}{3}$$

$$\therefore B = \frac{\mu_0 r^2 I}{2\pi R^3}$$