University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, APRIL, 2008

First Year - CHE, CIV, IND, LME, MEC, MMS

MAT 187H1S - CALCULUS II

Exam Type: A

SURNAME: _____

GIVEN NAMES: _____

STUDENT NUMBER: _____

SIGNATURE: _____

Examiners:
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D. Burbulla
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PAGE	MARK
MC	
Q7	
Q8	
Q9	
Q10	
Q11	
Q12	
Q13	
TOTAL	

Calculators Permitted: Casio 260, Sharp 520 or TI 30.

INSTRUCTIONS: Attempt all questions. Use the backs of the sheets if you need more space. Do not tear any pages from this exam. Make sure your exam contains 10 pages.

MARKS: Questions 1 through 6 are Multiple Choice; circle the single correct choice for each question. Each correct choice is worth 4 marks.

Questions 7, 8 and 9 are each worth 12 marks.

Questions 10 through 13 are each worth 10 marks.

TOTAL MARKS: 100

1. What is the third degree Taylor polynomial of the function $f(x) = \dfrac{1}{1+x}$ at $a = 0$?

   (a) $1 + x + x^2 + x^3$

   (b) $1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{6}$

   (c) $1 - x + x^2 - x^3$

   (d) $1 - x + \dfrac{x^2}{2} - \dfrac{x^3}{6}$

2. The length of the polar curve with polar equation $r = e^{-\theta}$ for $\theta \geq 0$ is

   (a) 0

   (b) 1

   (c) $\sqrt{2}$

   (d) ∞

3. What is the radius of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{3^n}(x-1)^n$?

   (a) 1

   (b) $\dfrac{1}{3}$

   (c) $\ln 3$

   (d) 3

4. If the position vector of a particle at time t is given by $\mathbf{r} = t^2\,\mathbf{i} + \ln t\,\mathbf{j} + 4\tan^{-1} t\,\mathbf{k}$, then its speed at time $t = 1$ is

   (a) 3

   (b) $\sqrt{1+\pi^2}$

   (c) $\sqrt{3}$

   (d) $\dfrac{\pi}{3}$

5. The area of the region inside the cardioid with equation $r = 2 + 2\cos\theta$ but outside the circle with equation $r = 2$ is given by

   (a) $\displaystyle\int_0^{\pi/2} [2\cos\theta]^2\,d\theta$

   (b) $\displaystyle\int_0^{\pi/2} [(2+2\cos\theta)^2 - 2^2]\,d\theta$

   (c) $\displaystyle\int_{\pi/2}^{\pi} [2\cos\theta]^2\,d\theta$

   (d) $\displaystyle\int_{\pi/2}^{\pi} [(2+2\cos\theta)^2 - 2^2]\,d\theta$

6. How many inflection points are there on the curve with parametric equations

$$x = t^2 + 4t; y = t^3 - 3t?$$

   Recall: as in Calculus I, an inflection point on a curve is a point where the concavity changes.

   (a) 0

   (b) 1

   (c) 2

   (d) 3

7. [12 marks; 4 for each part.] Decide if the following infinite series converge or diverge. Summarize your work at the right by marking your choice, and by indicating which convergence/divergence test you are using.

(a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{-3n}{4n+1}\right)^n$ ◯ Converges ◯ Diverges

by _____

(b) $\displaystyle\sum_{n=0}^{\infty}\frac{\tan^{-1} n}{n^2+1}$ ◯ Converges ◯ Diverges

by _____

(c) $\displaystyle\sum_{n=1}^{\infty}\frac{n^n}{(n+1)^{n+1}}$ ◯ Converges ◯ Diverges

by _____

8. [12 marks] The displacement, $x(t)$, of an underdamped mass-spring system satisfies

$$x''(t) + 2x'(t) + 65x(t) = 0; x(0) = -2 \text{ and } x'(0) = 4.$$

Solve for x as a function of t and sketch its graph for $0 \leq t \leq \pi$, indicating both its pseudo period and its time-varying amplitude.

9.[12 marks: 6 for each part.]

(a) Write down the first four non-zero terms of the Maclaurin series for each of $f(x) = e^{-x^2}$ and $g(x) = \int_0^x f(t)\,dt$.

(b) Approximate $\int_0^{0.5} \sqrt{1+x^4}\,dx$ to within 10^{-6}, and explain why your approximation is correct to within 10^{-6}.

10. [10 marks] Find $\int_0^\infty \frac{1}{e^{ax}(1+e^{2ax})}\,dx$, if $a > 0$.

11. [10 marks] Given that $-\frac{\pi}{2} < x < \frac{\pi}{2}$, find the general solution to the differential equation

$$\cos x \frac{dy}{dx} + y \sin x = \cos x + \sin x.$$

Is there a solution that passes through the point $(x, y) = (0, 0)$?

12. [10 marks] Torricelli's Law states that

$$A(y)\frac{dy}{dt} = -a\sqrt{2gy},$$

where y is the depth of a fluid in a tank at time t, $A(y)$ is the cross-sectional area of the tank at height y above the exit hole, a is the cross-sectional area of the exit hole, and g is the acceleration due to gravity.

A spherical water tank of radius 1 m is initially full. At 12 noon a plug at the bottom of the tank is removed, and 20 min later the tank is half empty. When will the tank be completely empty?

13. [10 marks] Use power series to find the Taylor series of

$$f(x) = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$$

at $a = 0$. What is its interval of convergence?