Name: ______________________________

Student number: ______________________

MAT 186
Quiz 9

1. Evaluate

CF16	CF22

$$\int_{-1}^{2} \frac{x^2}{\sqrt{x+2}}\,dx$$

Two good solutions exist. We can substitute for $x+2$ (that is the solution shown here), but it would actually be much better to use $\sqrt{x+2}$. Try that one yourselves.

Let $u = x+2$, so $du = dx$.

$$\begin{aligned}
\int_{-1}^{2} \frac{x^2}{\sqrt{x+2}}\,dx &= \int_{1}^{4} \frac{(u-2)^2}{\sqrt{u}}\,du \\
&= \int_{1}^{4} \frac{u^2-4u+4}{\sqrt{u}}\,du \\
&= \int_{1}^{4} u^{3/2} - 4u^{1/2} + 4u^{-1/2}\,du \\
&= \left[\frac{2}{5}u^{5/2} - \frac{8}{3}u^{3/2} + 8u^{1/2}\right]_1^4 \\
&= \left(\frac{2}{5}2^5 - \frac{8}{3}2^3 + 16\right) - \left(\frac{2}{5} - \frac{8}{3} + 8\right) \\
&= ugh, or\ 26/15
\end{aligned}$$

Continued on back.

AB4	CF16	CF25

2. Find the area of the region between $y = |x|$ and $y = 6 - |2x|$.

We need to take cases:

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases} \qquad 6 - |2x| = \begin{cases} 6 + 2x, & x < 0 \\ 6 - 2x, & x \geq 0 \end{cases}$$

These will intersect at $x = \pm 2$. So, we need to find the area using two integrals (we could use symmetry and just double one of them):

$$\begin{aligned} A &= \int_{-2}^{0} (6 + 2x) - (-x)\ dx + \int_{0}^{2} (6 - 2x) - (x)\, dx \\ &= \int_{-2}^{0} 6 + 3x\, dx + \int_{0}^{2} 6 - 3x\, dx \\ &= \left(6x + \frac{3x^2}{2}\right)_{-2}^{0} + \left(6x - \frac{3x^2}{2}\right)_{0}^{2} \\ &= 12 \end{aligned}$$

3. A spherical snowball melts at a rate that is proportional to its surface area. Show that the rate of change of the radius is constant. CF10

We are given that $\frac{dV}{dt} = k \cdot SA$, for some k. So:

$$\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = k \cdot 4\pi r^2$$

$$4\pi r^2 \cdot \frac{dr}{dt} = k \cdot 4\pi r^2$$

$$\frac{dr}{dt} = k$$

[This one is optional. Solve it if you need better marks in the main attribute tested here. If you try this problem, it will not lower your mark.]

4. Find $\lim_{x\to 2}(x-2)\cos\left(\frac{1}{(x-2)^2}\right)$.

AB6	CS2

This definitely requires the Squeeze Theorem, with positive and negative cases.

Case I: $x-2>0$, or $x>2$.

$$-1 \leq \cos\left(\frac{1}{(x-2)^2}\right) \leq 1$$

$$-(x-2) \leq (x-2)\cos\left(\frac{1}{(x-2)^2}\right) \leq (x-2)$$

$$\lim_{x\to 2} -(x-2) = \lim_{x\to 2}(x-2) = 0$$

Therefore, by the Squeeze Theorem, the limit in case I goes to zero.

Case II is nearly identical and we can get away with saying "Case II is done similarly." Here it is in full:

Case II: $x-2<0$, or $x<2$.

$$-1 \leq \cos\left(\frac{1}{(x-2)^2}\right) \leq 1$$

$$-(x-2) \geq (x-2)\cos\left(\frac{1}{(x-2)^2}\right) \geq (x-2)$$

$$\lim_{x\to 2} -(x-2) = \lim_{x\to 2}(x-2) = 0$$

Therefore, by the Squeeze Theorem, the limit in case II goes to zero.

Therefore, the limit is zero.