UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, APRIL 2004
MAT 188 S – LINEAR ALGEBRA. FIRST YEAR: T-PROGRAM
EXAMINER: FELIX J. RECIO

INSTRUCTIONS:

1. ATTEMPT ALL QUESTIONS.
2. SHOW AND EXPLAIN YOUR WORK IN ALL QUESTIONS.
3. GIVE YOUR ANSWERS IN THE SPACE PROVIDED. USE BOTH SIDES OF PAPER, IF NECESSARY.
4. DO NOT TEAR OUT ANY PAGES.
5. USE OF NON-PROGRAMMABLE POCKET CALCULATORS, BUT NO OTHER AIDS ARE PERMITTED.
6. THIS EXAM CONSISTS OF SEVEN QUESTIONS. THE VALUE OF EACH QUESTION IS INDICATED (IN BRACKETS) BY THE QUESTION NUMBER.
7. THIS EXAM IS WORTH 50% OF YOUR FINAL GRADE.
8. TIME ALLOWED: 2 ½ HOURS.
9. PLEASE WRITE YOUR NAME, YOUR STUDENT NUMBER, AND YOUR SIGNATURE IN THE SPACE PROVIDED AT THE BOTTOM OF THIS PAGE.

PLEASE DO NOT WRITE HERE

QUESTION NUMBER	QUESTION VALUE	GRADE
1	15	
2	20	
3	15	
4	10	
5	15	
6	15	
7	10	
TOTAL:	100	

NAME: ______________________ , ______________________

(FAMILY NAME. PLEASE PRINT.) (GIVEN NAME.)

STUDENT No.: ______________ SIGNATURE: ______________________

1. a) (5 marks) Find all vectors $\mathbf{v}$ in R^3, if any, such that $\|\mathbf{v}\| = \|\mathbf{v}+\mathbf{i}\| = \|\mathbf{v}+2\mathbf{j}\| = \|\mathbf{v}+3\mathbf{k}\|$, where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ denote the standard unit vectors in R^3.

   b) (5 marks) Let L be the line with parametric equations $x = 2 - t$, $y = 1 + 3t$ and $z = -1 + 2t$. Find the coordinates of the point on the line L closest to the point $(1, -4, -1)$.

   c) (5 marks) Let W be the plane that passes through the points $(2, 0, -1)$, $(1, -1, 1)$ and $(0, 1, 2)$. Find the coordinates of the point at which the plane W intersects the z-axis.

2. a) (10 marks) Solve the linear system $\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 1 \\ x_1 + 2x_2 + 2x_3 + x_4 + 2x_5 = 1 \\ -2x_1 - x_4 + x_5 = 1 \end{cases}$.

b) (10 marks) Consider the linear system $\begin{cases} x + y + z = 1 \\ x + y - z = b \\ -x + y + z = 3 \\ 2y - z = 1+b \end{cases}$.

Find all the possible values of the parameter b, if any, for which this linear system has a unique solution and find this solution.

3. Let $A = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 2 & 1 \\ 0 & 3 & 1 \end{pmatrix}$ and let I be the 3×3 identity matrix.

a) (5 marks) Compute $(2I - A^T)^2$.
b) (5 marks) Compute $(2I - A^T)^{-1}$.
c) (5 marks) Find all matrices M, if any, for which $A + MA^T = 2M$.

4) (10 marks) Find the value of k, if any, for which $\det \begin{bmatrix} 2 & 1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 0 & 2 & -1 & 1 \\ -1 & 1 & 1 & k \end{bmatrix} = 3$.

5. (15 marks) Let $\mathbf{S}$ be the set consisting of all vectors $\mathbf{v} = (x_1, x_2, x_3, x_4, x_5)$ of $\mathbf{R}^5$, such that $x_1 - x_2 = x_3 - x_4 = x_5 - x_1$. Show that $\mathbf{S}$ is a subspace of $\mathbf{R}^5$, find its dimension and give a basis for this subspace.

6. Given the matrix $A = \begin{pmatrix} 3 & 3 & -1 \\ -1 & -1 & 1 \\ 2 & 6 & 0 \end{pmatrix}$.

a) (6 marks) Find all the eigenvalues of the matrix A.
b) (5 marks) Find a basis for each of the eigenspaces of the matrix A.
c) (4 marks) Find an invertible matrix P and a diagonal matrix D such that $P^{-1} A P = D$.

7. (10 marks) Solve the initial value problem: $\begin{cases} y_2' = y_1 + 2y_2 \\ y_1' = 4y_1 + 3y_2 \end{cases}$, $y_1(0) = 2$, $y_2(0) = -1$.