Q1:

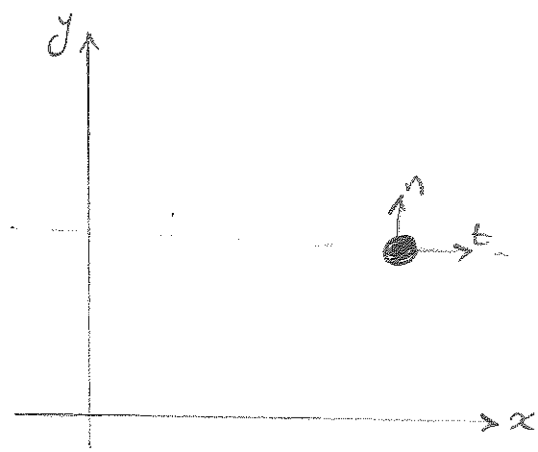
Given: $y = 25\ m$

$x = 4t^3$

a) $V_x = \dot{x} = 12t^2$ $\xrightarrow{t=1s}$ $V_x = 12\ m/s$ $\rightarrow \vec{V} = 12\ \frac{m}{s}\ \hat{i}$

$V_y = \dot{y} = 0$ $\qquad$ $V_y = 0$

b)



$V_x = 12t^2$ $\xrightarrow{t=2s}$

$V_y = 0$

$V_x = 48\ m/s$ $\rightarrow \vec{V} = 48\ \frac{m}{s}\ \hat{i}$

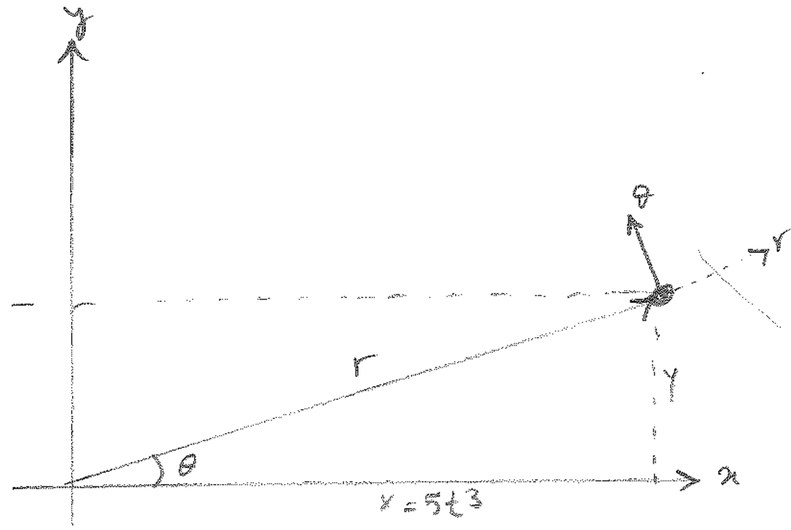
$V_y = 0$

$\rho \rightarrow \infty$

$V_t = 48\ m/s$ $\rightarrow \vec{V} = 48\ \frac{m}{s}\ \hat{e}_t$

$V_n = 0$

c)



$$\begin{cases} y = 25\ m \\ x = 4t^3 \end{cases}$$

$r^2 = x^2 + y^2$

$\theta = \mathrm{Arctan}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{25}{4t^3}\right)$

$$r = \sqrt{x^2 + y^2} = \sqrt{(4t^3)^2 + (25)^2}$$

$$\theta = \tan^{-1}\left(\frac{25}{4t^3}\right)$$

@ $t = 3$

$r = 110.86 \text{ m}$

$$\theta = \tan^{-1}\left(\frac{25}{4 \times 27}\right) = 13.03° = 0.2274 \text{ rad}$$

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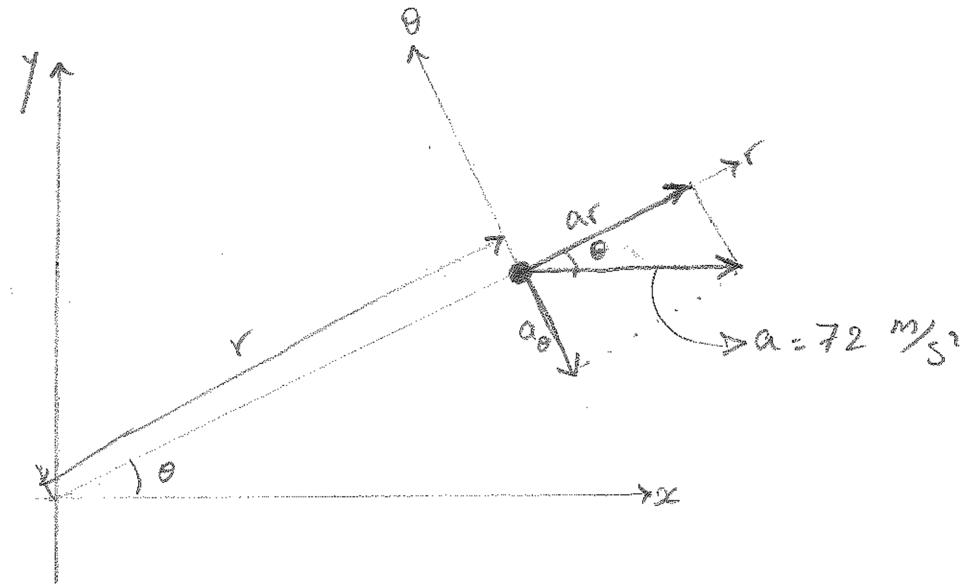
d) <u>Solution 1</u>



$a_x = \ddot{x} = 24t$

$a_y = \ddot{y} = 0$

$\rightarrow \vec{a} = 24t\,\hat{i}$ @ $t = 3$

$\vec{a} = 72\,\hat{i} \;\left(\frac{m}{s^2}\right)$

$a_r = a\cos\theta = 72\cos 13.03 = 70.146 \text{ m/s}^2$

$a_\theta = a\sin\theta = 72\sin 13.03 = -16.233 \text{ m/s}^2$

$$a = 70.146 \frac{m}{s^2}\,\hat{e}_r - 16.233 \frac{m}{s^2}\,\hat{e}_\theta$$

Solution 2:

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$\dot{r} = \frac{dr}{dt} = \frac{6\times 16 t^5}{2\sqrt{16t^6 + (25)^2}} = \frac{48\, t^5}{\sqrt{625 + 16t^6}}$$

$$\ddot{r} = \frac{d}{dt}\left(\frac{dr}{dt}\right) = \frac{5\times 48 \times t^4 \sqrt{625+16t^6} - (48t^5)\dfrac{48t^5}{\sqrt{625+16t^6}}}{(625 + 16t^2)}$$

$$\dot{\theta} = \frac{d\theta}{dt} = \frac{-3\times\frac{25}{4}\times t^{-4}}{1 + \left(\frac{25}{4t^3}\right)^2} = \frac{-75}{4\left(1 + \frac{625}{16t^6}\right)t^4}$$

$$\ddot{\theta} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{75}{\left(1 + \frac{625}{16t^6}\right)t^5} - \frac{140\,625}{32\left(1 + \frac{625}{16t^6}\right)^2 t^{11}}$$

@ $t = 3\,s$,

$\dot{r} = 105.218\ \frac{m}{s}$, $\ddot{r} = 75.4964\ \frac{m}{s^2}$ and $r = 110.86\ m$

$\dot{\theta} = -0.2197\ \frac{1}{s}$, $\ddot{\theta} = 0.2706\ \frac{1}{s^2}$,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 70.145\ \frac{m}{s^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 110.86\times(0.2706) + 2(105.218)(-0.2197) \rightarrow$$

$$a_\theta = -16.234\ \frac{m}{s^2}$$

$$a = 70.145\,\frac{m}{s^2}\,\hat{e}_r - 16.234\,\frac{m}{s^2}\,\hat{e}_\theta$$

Q2:

Given:

$V_A = 100 \frac{km}{h} = \frac{100}{3.6} \, m/s \quad , \quad \rho_A = 400 \, m$

$V_C = 50 \frac{km}{h} = \frac{50}{3.6} \, m/s \quad , \quad \rho_C = 80 \, m$

$d_{AC} = 200 \, m$

B is inflection point

Uniform acceleration.

a) $V_C^2 - V_A^2 = 2 a_t d_{AC}$

$\left(\frac{50}{3.6}\right)^2 - \left(\frac{100}{3.6}\right)^2 = 2 \, a_t (200)$

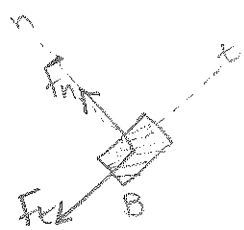
$a_t = -1.447 \, \frac{m}{s^2}$

b) point B is inflection point $\rho_B \longrightarrow \infty$

$a_n = \frac{V_B^2}{\rho_B} = 0$

$\rho \longrightarrow \infty$

c)



$a_n = 0$

$a_t = -1.447 \, \frac{m}{s^2}$

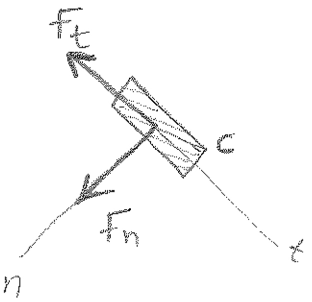
$F_n = m a_n = 0$

$F_t = m a_t = 1500(-1.447) = -2170.5 N$

$\vec{F}_B = -2170 N \, \hat{e}_t$

d) at point "C"

$$a_n = \frac{V_c^2}{\rho_c} = \frac{\left(\frac{50}{3.6}\right)^2}{80} = 2.411 \frac{m}{s^2}$$



$$a_t = -1.447 \frac{m}{s^2}$$

$$F_n = 1500(2.411) = 3616.5 \text{ N}$$

$$F_t = 1500(-1.447) = -2170.5 \text{ N}$$

e) $\vec{F}_c = -2170.5\text{N}\ \hat{e}_t + 3616.5\text{N}\ \hat{e}_n$

Q3:

Given:

$m = 2kg$

$k = 30\ N/m$

l_o = Unstretched length of Spring = 1.5 m

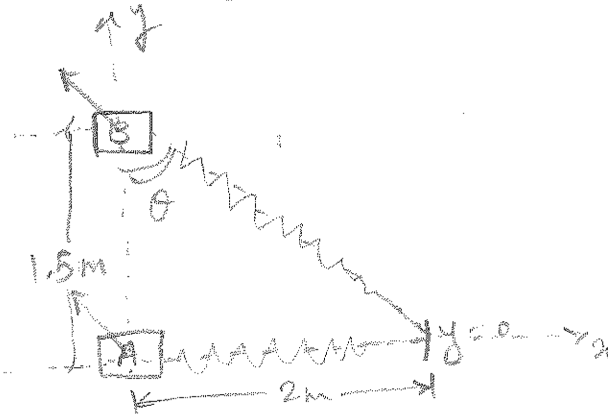
$V_A = 0\ m/s$ @ $t = 0$

a) A $t = 0$

$l_A = 2\ m$

$S_A = l_A - l_o = 2 - 1.5 = 0.5 m$

$V_e = \frac{1}{2} k S^2 = \frac{1}{2} \times 30 \times (0.5)^2 = 3.75\ J$



b) $T_A + V_A + \left(\sum U_{1-2}\right)_{Uncon} = T_B + V_B$

$T_A = \frac{1}{2} m V_A^2 = 0$

$V_A = mgh + \frac{1}{2} k S_A^2 = 0 + 3.75 J = 3.75 J$

$T_B = \frac{1}{2} m V_B^2 = V_B^2$

$S_B = l_B - l_o$

$l_B = \sqrt{2^2 + 1.5^2} = 2.5\ m$

$S_B = 2.5 - 1.5 = 1\ m$

$$V_B = mgh_B + \frac{1}{2} k x_B^2 =$$

$$2 \times 9.81 \times 1.5 + \frac{1}{2} \times 30 \times 1^2 = 44.43 \text{ J}$$

$$\Sigma U_{1-2} = F \cos 30 (y_B - y_A) = 50 \cos 30 (1.5) = 64.95 \text{ J}$$

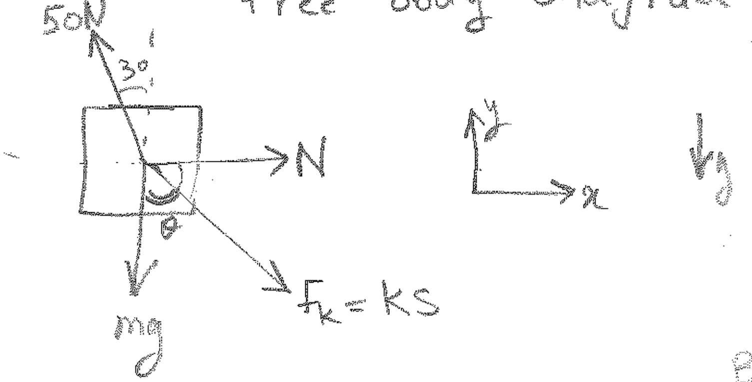
$$T_A + V_A + \Sigma U_{1-2} = T_B + V_B$$

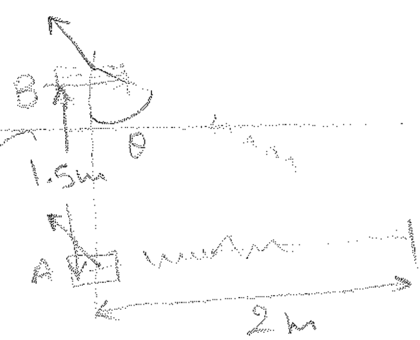
$$0 + 3.75 + 64.95 = V_B^2 + 44.43$$

$$V_B^2 = 24.27 \; \left(\frac{m^2}{s^2}\right)$$

$$V_B = 4.92 \; m/s$$

c) Free body diagram in position B





d) $\Sigma F_x = ma_x \rightarrow \Sigma F_x = 0$

$\Sigma F_y = ma_y$

$$50 \cos 30 - mg - F_k \cos\theta = m a_y$$

$$\theta = tg^{-1}\left(\frac{2}{1.5}\right) = 53.13°$$

$$F_k = k S_B = 30 (1) = 30 \; N$$

$$50\cos 30 - 2\times 9.81 - 30\cos(53.13) = 2a_y$$

$$5.68 = 2a_y \rightarrow$$

$$a_y = 2.84 \frac{m}{s^2} \uparrow$$