University of Toronto

Faculty of Applied Science and Engineering

Department of Mechanical and Industrial Engineering

FINAL EXAMINATION

April 18, 2008 2:00 p.m.

Exam Duration: 2.5 hours

First Year – Mechanical and Industrial Engineering

MIE100 – Dynamics

Calculator Type: 3

Exam Type: C

Examiners:

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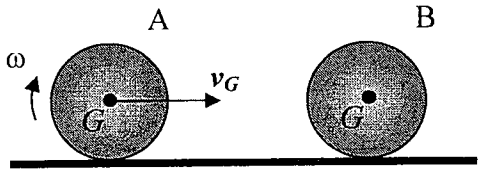
Question 1: 25 marks
Question 2: 25 marks
Question 3: 25 marks
Question 4: 25 marks
Total: 100 marks

ANSWER ALL FOUR QUESTIONS

Question 1

Sphere A and sphere B are identical, each having mass $m = 2.5$ kg and radius $r = 1$ m. Sphere A rolls without slipping with a velocity $v_G = 1$ m/s on a rough horizontal surface until it strikes sphere B, which is initially at rest. Immediately after impact, sphere A stops *translating*. In the questions below you may neglect friction *between sphere A and sphere B*.

Note that I_G for a sphere is $\frac{2}{5}mr^2$.



a) Show that the speed of the centre of mass of sphere B immediately after impact is 1 m/s. (4 marks)

b) Show that the angular velocity of sphere B immediately after impact is zero. State your assumptions. (3 marks)

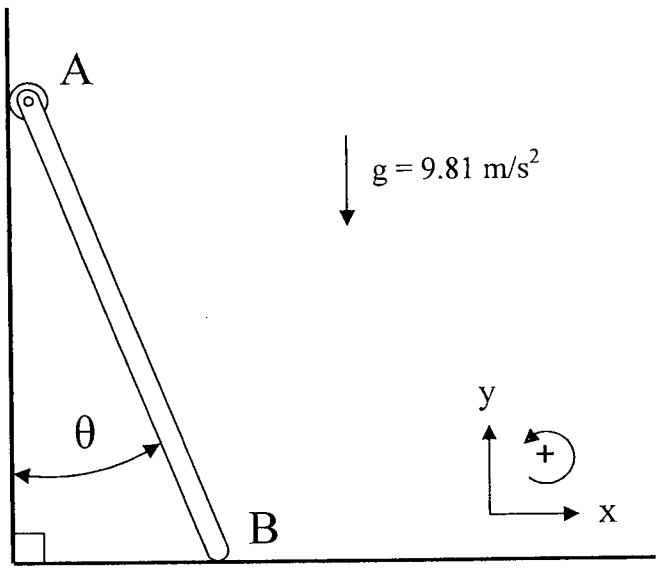
c) Determine the speed of the centre of mass of sphere B once it starts rolling without slipping. (8 marks)

d) Determine the work done by friction on sphere B from the point immediately after impact until the point at which sphere B starts rolling without slipping. (8 marks)

e) How much work is done by friction on sphere B once it starts rolling without slipping? (2 marks)

Question 2

The uniform, slender rod AB is positioned so that $\theta = 30°$ and then is released from rest. The small roller at A, which is frictionless and has negligible mass, will roll down the vertical surface. Point B will slide to the right along the rough horizontal surface. Immediately after release, the initial angular acceleration (α) of rod AB is 0.5 rad/s^2 counterclockwise. The length of rod AB is 1.2 m.



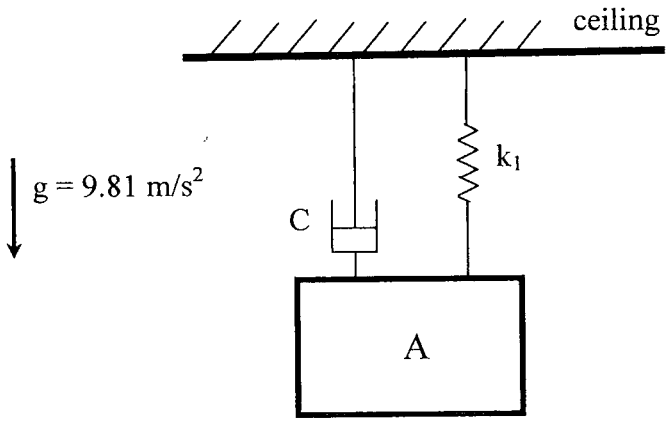
a) Find $\vec{a}_A$, the initial acceleration of point A, expressed in x-y coordinates. (7 marks)

b) Find $\vec{a}_G$, the initial acceleration of the rod's centre of mass, expressed in x-y coordinates. (8 marks)

c) Given that the mass of rod AB is 10 kg, determine the initial friction force that must be present at point B immediately after release. (10 marks)

Question 3

Block A of mass 50 kg is suspended from the ceiling by a spring with stiffness constant $k_1 = 800$ N/m, and a dashpot of strength C. The mass moves only in the vertical direction, without rotation.



a) What is the period (τ) of natural vibration of the mass, if C = 0? (6 marks)

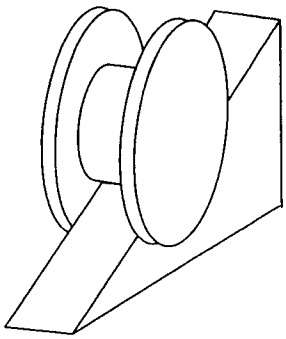
b) Suppose that A is vibrating in the vertical direction, with an amplitude of vibration equal to 0.5 m. What is the maximum acceleration of A? Assume that C = 0. (6 marks)

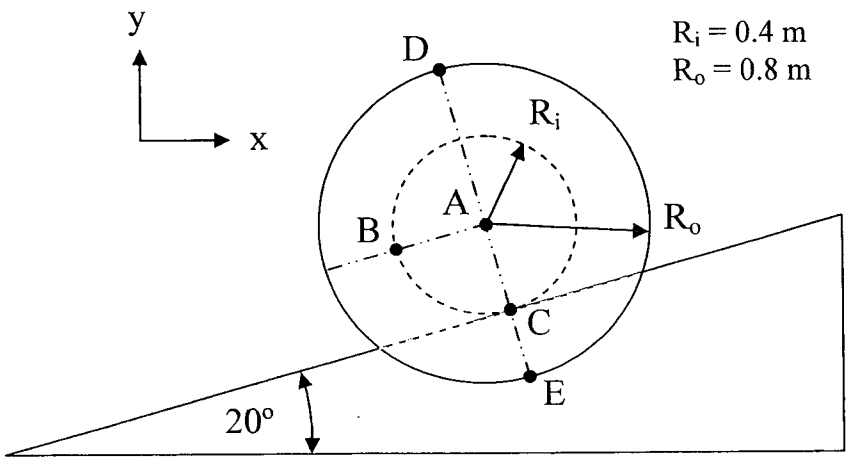
c) What value of C would be required to reduce the amplitude of vibration from 0.5 m to 0.2 m in 15 seconds? (7 marks)

d) Suppose that an earthquake were to make the ceiling vibrate vertically with an amplitude of 3 mm at a frequency of 5 rad/s. What would be the amplitude of vibration of A? Assume C = 0. (6 marks)

Question 4

A spool (a wheel with an inner hub) rolls without slipping down an incline on its inner hub as shown. The radius of the inner hub (R_i) is 0.4 m and the radius of the outer wheel (R_o) is 0.8 m. A, B, C, D and E are points on the wheel.





Point A is the centre of the wheel and is moving with a speed of 2 m/s. The speed of point A is increasing at a rate of 0.25 m/s^2.

a) What is the velocity of point A expressed in x-y coordinates? (4 marks)

b) What is the velocity of point A expressed in n-t coordinates? (4 marks)

c) Where is the instantaneous centre of zero velocity? (3 marks)

d) What is the velocity of point B expressed in x-y coordinates? (6 marks)

e) What is the acceleration of point E expressed in x-y coordinates? (8 marks)