MIE100S Dynamics: Winter 2015

Final Exam – April 29, 2015

9:30 a.m. – noon

General Instructions:

- Answer all questions in the exam booklets provided.
- Print your Last Name and First Name *neatly* on each booklet, as they appear on ROSI.
- Print your student number *neatly* on each booklet.
- All rough work must be *neatly* shown to earn credit for each question.
- You must use a pen or *dark* pencil.
- Answer all five questions. Each question is worth 20%.
- Total marks on this exam = 100.
- Use the given coordinate system and sign conventions to express your final answer to each question.

This is a Type D examination. Permitted Aids:

- Non-communicating/non-programmable calculator: Casio FX-991MS or Sharp EL-520X
- One 8 ½" x 11" aid sheet, any colour. You may write on both sides of the sheet.

Moments of Inertia

Ring: $I_G = mR^2$

Uniform disk: $I_G = \frac{1}{2}mR^2$

Long slender rod: $I_G = \frac{1}{12}mL^2$

Trigonometric Functions

$$\frac{d(\cos\theta)}{d\theta} = -\sin\theta$$

$$\frac{d(\sin\theta)}{d\theta} = \cos\theta$$

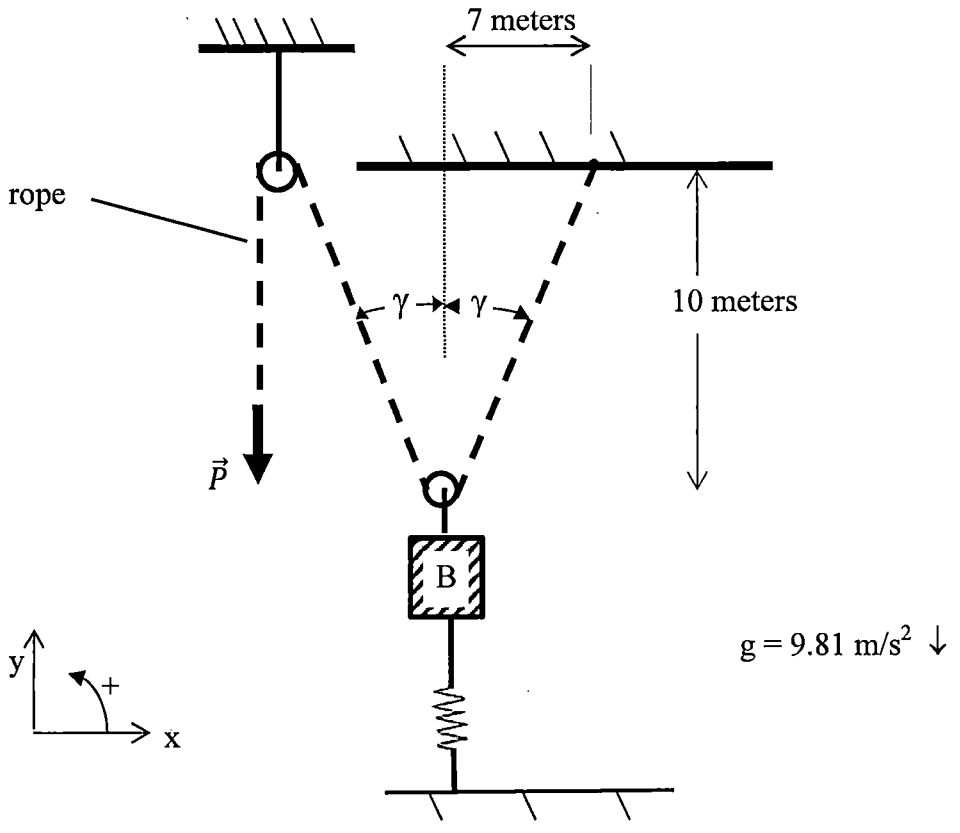
$$\frac{d(tan\theta)}{d\theta} = \frac{1}{cos^2\theta}$$

1. Block "B" has mass 9 kg and can move vertically. It is connected to a spring and a rope passing through two *very small* pulleys as shown. One end of the rope is tied to the ceiling; the other end of the rope is being pulled with a constant force $\vec{P}$ equal to -70 Newtons $\hat{j}$. The spring constant is k = 60 N/m, and the spring is stretched by 0.8 m at the instant shown in the diagram. At the instant shown in the diagram, block "B" has a velocity of $-7\frac{m}{s}\,\vec{j}$, and the angle $\gamma = 35^{\circ}$. Ignore friction and the mass of the pulleys and rope.

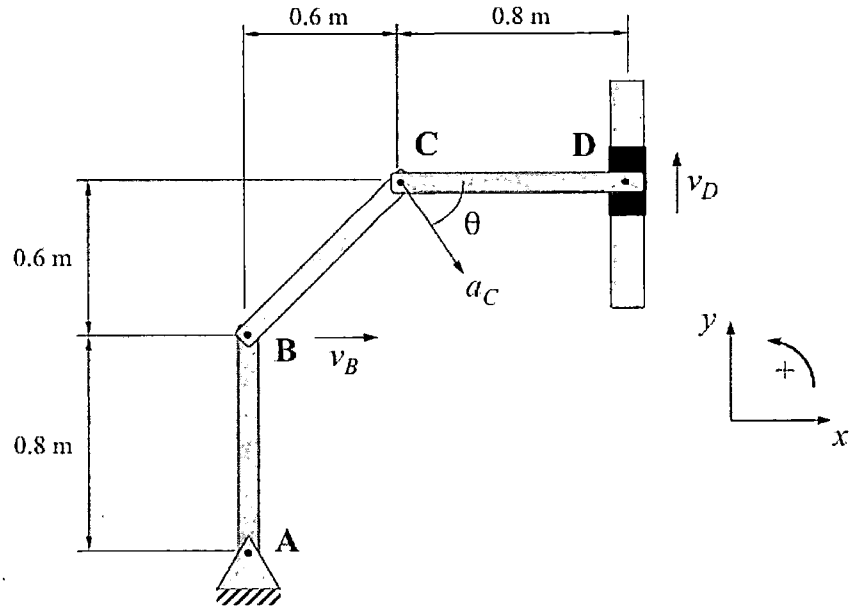
   (a) Let V_g = the gravitational potential energy of block "B". Determine the rate at which the gravitational potential energy of block "B" is changing at the instant shown in the diagram, $\frac{dV_g}{dt}$.

   (b) Find the acceleration of block B at the instant shown in the diagram.

   (c) Determine $\frac{d\gamma}{dt}$ at the instant shown in the diagram, where γ is the magnitude of the angle between the rope and the vertical as shown in the diagram.



2. A three-bar linkage *ABCD* is pinned at point *A* and connected to a collar at point *D* that slides in the vertical *y*-direction only. Express all answers using the sign convention shown.



(a) If $\vec{v}_B = +1.2$ m/s $\hat{\imath}$, and $(\vec{a}_B)_x = +0.4$ m/s^2 $\hat{\imath}$, find ω_{AB} and α_{AB}.

(b) If $\vec{v}_B = +1.2$ m/s $\hat{\imath}$, and $\vec{v}_D = +0.75$ m/s $\hat{\jmath}$, find ω_{BC} and ω_{CD}.

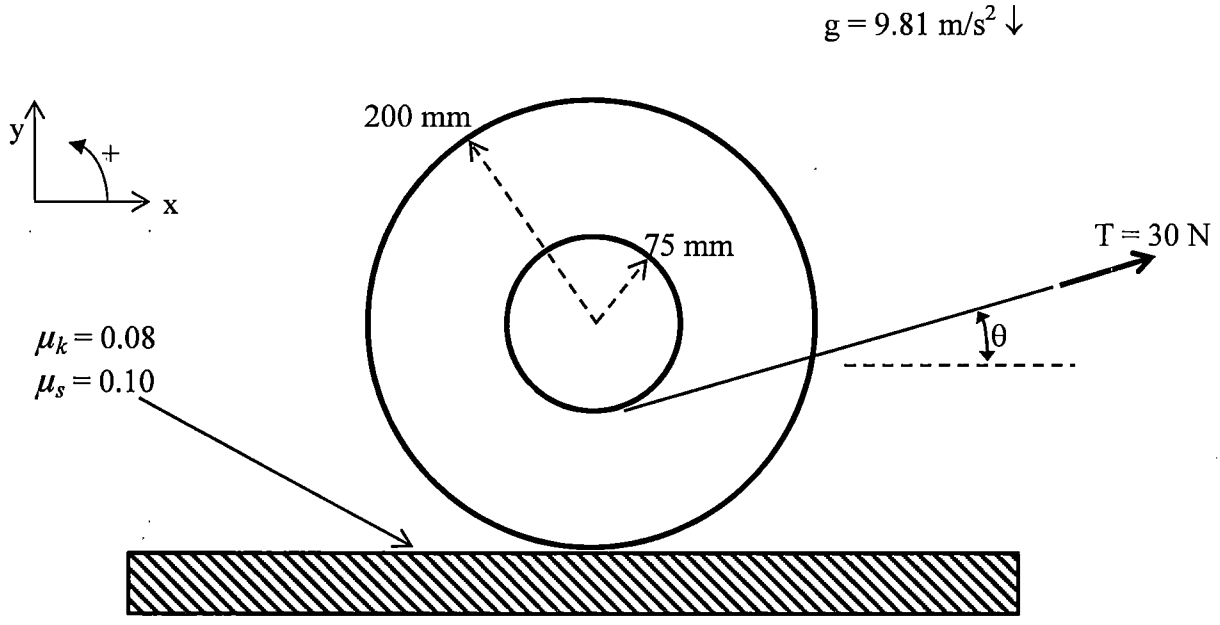
(c) Point C accelerates in the direction shown in the diagram. If $|\vec{a}_C| = 6.46$ m/s^2, $\theta = 88°$, and $\vec{a}_D = 0$, find α_{CD}. Note that ω_{CD} is not required to find α_{CD}.

3. The circular disk has a 200 mm radius and mass of 25 kg. The radius of gyration k about the center of gravity is 175mm. The inner radius of the disk is 75 mm. A steady force of T=30 Newtons is applied at an angle $\theta = 17^{o}$ to a cord wrapped around the inner radius.

Find:

a) The angular acceleration of the disk, α
b) The acceleration of the mass centre G
c) The friction force $\vec{F}$ from the ground acting on the disk.

Express your final answers using the coordinate system shown on the left.

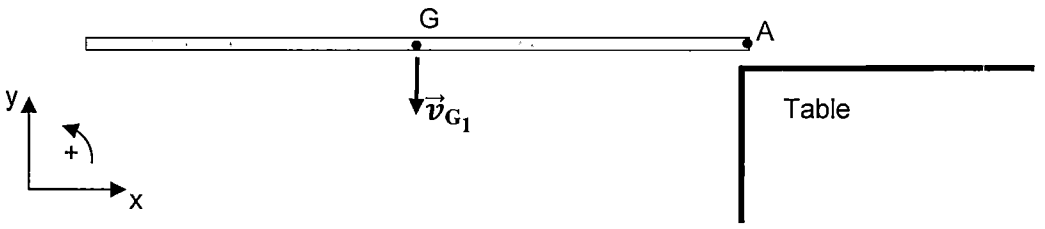


4. A long slender rod has length $L = 2$ meters and mass $m = 1$ kg. Its center of mass G has a velocity $\vec{v}_{G_1} = -2\frac{m}{s}\hat{j}$, and the rod has an initial angular velocity of zero when the end of the rod (point A) strikes a table, as shown in the diagram below. Immediately after impact, the velocity of the centre of mass is $\vec{v}_{G_2}$. Assume that gravity has negligible effect on the rod during the extremely short time period of impact.

   a) Calculate the rod's linear momentum, and its angular momentum about its centre of mass G immediately *before* impact.

   b) Immediately *after* impact, the velocity of the end of the rod at point A is zero. Determine the angular velocity ω of the rod immediately *after* impact.

   c) Write an expression for the energy lost during the impact in terms of the mass of the rod and the velocities $\vec{v}_{G_1}$ and $\vec{v}_{G_2}$. Do not solve the expression or substitute values for the variables – the only variables in your final expression should be m, $\vec{v}_{G_1}$, and $\vec{v}_{G_2}$.



5. The uniform slender bar AH has a mass of 5 kg and length 0.75 meters; it is pinned at point O. The orientation of the bar is horizontal ($\theta = 0$) when the bar is at rest. The spring constant $k = 20$ N/m. $\overline{AB} = \overline{BO} = \overline{OH} = 0.25\ meters$.

(a) Find the natural frequency of vibration ω_n of bar AH if $C = 0$ N·s/m.

(b) Find the damped frequency of vibration ω_d of bar AH if $C = 10$ N·s/m.

(c) A force $\vec{F} = 10\cos(t)$ Newtons $\hat{j}$ is applied to point A on bar AH, where t is time measured in seconds. Find the angular amplitude θ_{max} of oscillation of bar AH when $C = 0$ N·s/m.