UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

**FINAL EXAMINATION, DECEMBER 2005**
First Year - CIV, CHE, IND, LME, MEC, MMS

# MAT 186H1F - CALCULUS I

Exam Type: A

SURNAME ______________________

GIVEN NAME ______________________

STUDENT NO. ______________________

SIGNATURE ______________________

**Examiners**

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**Calculators Permitted:** Casio 260, Sharp 520 or Texas Instrument 30

**INSTRUCTIONS:**

Attempt all questions.

Questions 1 through 6 are Multiple Choice; circle the single correct choice for each question. Each correct choice is worth 4 marks.

Question 7 consists of six short parts; each part is worth 2 marks.

Questions 8 through 12 are long questions for which you must show your work.

Question 8 is worth 12 marks; questions 9 through 12 are worth 13 marks each.

TOTAL MARKS: 100

Use the backs of the pages if you need more space.

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1. What is the equation of the tangent line to the graph of $f(x) = \sqrt{x+2}$ at the point $(x, y) = (-1, 1)$?

(a) $y = x + 3$

(b) $2y = x + 3$

(c) $y = 2x + 3$

(d) $2y = x + 1$

2. The area of the region bounded by the two curves $f(x) = x^3$ and $g(x) = 2x^3 + 2x^2 - 3x$ is given by

(a) $\int_{-3}^{1} (f(x) - g(x))\, dx$

(b) $\int_{-3}^{1} (g(x) - f(x))\, dx$

(c) $\int_{-3}^{0} (f(x) - g(x))\, dx + \int_{0}^{1} (g(x) - f(x))\, dx$

(d) $\int_{-3}^{0} (g(x) - f(x))\, dx + \int_{0}^{1} (f(x) - g(x))\, dx$

3. Let $f(x) = x^3 + x - 1$. If Newton's method is used to approximate a solution to the equation $f(x) = 0$, beginning with $x_0 = 0$, then the value of $x_2$ is

(a) 0

(b) 1

(c) $\frac{3}{4}$

(d) $\frac{59}{86}$

4. Suppose $F(x) = \int_{-20}^{x^3} f(t)\,dt$, where $f(t)$ is continuous for all values of $t$.

Then $F'(2) =$

(a) $12f(8)$

(b) $12f(2)$

(c) $f(2)$

(d) $f(8)$

5. $\int_{\pi/2}^{\pi} \sin x \cos^3 x \, dx =$

(a) $\int_{0}^{-1} u^3 \, du$

(b) $\int_{-1}^{0} u^3 \, du$

(c) $\int_{\pi/2}^{\pi} u^3 \, du$

(d) $\int_{\pi}^{\pi/2} u^3 \, du$

6. $\lim_{x \to 0} \dfrac{x - \tan^{-1} x}{x^3} =$

(a) 0

(b) $-\dfrac{1}{3}$

(c) $\dfrac{1}{3}$

(d) $\infty$

7. Suppose $f$ and $g$ are continuous functions such that

$$\int_1^9 f(x)\,dx = 6, \int_1^9 g(x)\,dx = -4 \text{ and } \int_1^{16} f(x)\,dx = 2.$$

Find the values of the following. (Put your answers on the lines at the right of each question.)

(a) $\int_1^9 (3f(x) + 2g(x))\,dx$ ANS: ____________

(b) the average value of $f$ on $[1, 9]$ ANS: ____________

(c) $\int_9^{16} f(x)\,dx$ ANS: ____________

(d) $\int_1^9 f(x)\,dt$ ANS: ____________

(e) $\int_1^3 xf(x^2)\,dx$ ANS: ____________

(f) $\lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{n+15i}{n}\right)\frac{15}{n}$ ANS: ____________

8. Let $f(x) = e^{-e^{x-1}}$. Sketch the graph of $y = f(x)$, labelling all critical points, inflection points and asymptotes, if any.

9. Suppose the velocity of a particle at time $t$ is given by $v = t^2 - 9$, for $0 \leq t \leq 4$. Find the following:

   (a) [3 marks] the average acceleration of the particle for $0 \leq t \leq 4$.

   (b) [4 marks] the average velocity of the particle for $0 \leq t \leq 4$.

   (c) [6 marks] the average speed of the particle for $0 \leq t \leq 4$.

10. A tank is in the shape of a cone with its vertex at the bottom. The radius at the top of the tank is $r$ metres; the vertical height of the tank is $h$ metres. It is full of liquid with density $\rho$, in units of kilograms per cubic metre. Find the work done in pumping out the top $\frac{1}{2}h$ metres of the liquid to the top of the tank.

11.(a) [7 marks] Find the volume of the solid generated by rotating around the $x$-axis the region in the plane bounded by the curves

$$y = \sin x + \cos x; y = 0; x = 0; x = \frac{\pi}{4}.$$

11.(b) [6 marks] Set up an integral that gives the volume of the solid generated by rotating around the $y$-axis the region inside the ellipse with equation $4y^2 + (x - 2)^2 = 1$. (Four BONUS marks if you can evaluate it.)

12. Let $f(x) = \int_1^x \sqrt{t^5 + t^2 - 1}\, dt$, for $1 \leq x \leq 2$.

(a) [4 marks] Show that the arc length differential, $ds$, for the function $f(x)$ is given by

$$ds = \sqrt{x^5 + x^2}\, dx.$$

(b) [9 marks] Find the area of the surface of revolution generated by revolving the curve $y = f(x)$, for $1 \leq x \leq 2$, around the $y$-axis. (You may use the result from part (a).)