Exam 2006 Solutions.

1(a) using technique #1 for both cables:

Cable #1 $\quad v_A = -v_D \qquad$ D is the pt at the other end of 1st cable from A.

Cable #2 $\quad y_B - y_D + y_C - y_D = cst.$

$$\Rightarrow v_B + v_C = 2v_D = -2v_A. \quad \text{---} (1)$$

but $\quad v_B = v_A + v_{B/A} = v_A - .2 \quad (2)$ (given)

$\quad v_C = v_A + v_{C/A} = v_A - .3 \quad (3)$ (given & $v_{A/C} = -v_{C/A}$)

substitute (2) & (3) in (1)

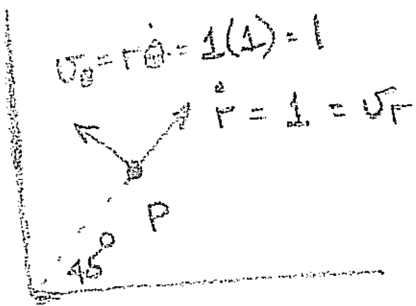
$$\Rightarrow v_A - .2 + v_A - .3 = -2v_A.$$

$$\Rightarrow v_A = \frac{.5}{4} \qquad \Rightarrow v_B = \frac{.5}{4} - .2 = -0.075 \text{ m/s}.$$

$$\Rightarrow \vec{v}_B = -0.075\,\hat{j} \text{ m/s}$$

1(b).

Sketch

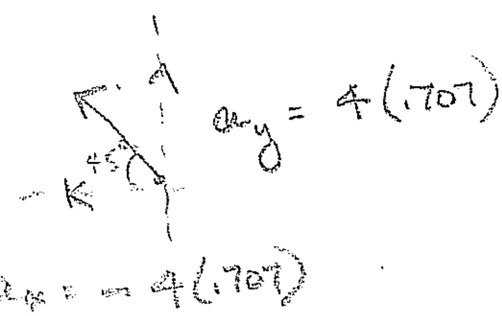


(i) $\hat{u}_t = \hat{j}$

(ii) $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{u}_\theta$

$= \{1 - 1(1)^2\}\hat{u}_r + \{1(2) + 2(1)(1)\}\hat{u}_\theta = 4\hat{u}_\theta$

Sketch



$= -2.83\hat{i} + 2.83\hat{j}$ m/s^2.

2. $m = 5\,kg$

$k_1 = 800\ N/m$

$k_2 = 1000\ N/m$

$k_3 = 1600\ N/m$.

$k_1 + k_2$ are in series: $k_{eff,1} = \dfrac{800(1000)}{1800} = 444$

$\Rightarrow$ k_{eff} for whole system $= 2044\ N/m$.

(a) $\Rightarrow$ $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{2044}{5}} = 20.2\ s^{-1}$

(b) $\omega_d = \omega_n \sqrt{1-\left(\dfrac{c}{c_c}\right)^2} \Rightarrow (.97)^2 = 1 - \left(\dfrac{c}{c_c}\right)^2$

$\Rightarrow$ $c/c_c = 0.243$

(c) $\dfrac{x_a}{x_b} = 0.10 = \exp\left\{-\dfrac{c}{2m}(25)\right\} \Rightarrow +2.30 = +\dfrac{25c}{10}$

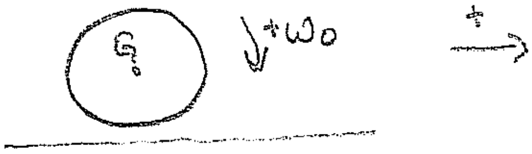
$\Rightarrow c = 0.92$ but $C_c = 2m\omega_n = 2(5)(20.2) = 202$.

$\Rightarrow$ $c/c_c = .00455$

(d) $7\,hz = 7$ cycles/s $= 2 * \pi * 7$ radians/s $= 44.0\ s^{-1}$

$.001 = \left| \dfrac{X}{1-\left(\dfrac{44}{20.2}\right)^2} \right| \Rightarrow X = .0037\ m$

3.



m, R

find v_G @ rolling.

$\Rightarrow \omega_f = v_G/R$

FBD

$F_f = \mu_k mg$

$$\Sigma F_x = m a_{Gx} \Rightarrow \mu_k \cancel{m} g = \cancel{m} a_G \Rightarrow a_G = \mu_k g.$$

$$\Sigma M_G = I\alpha \Rightarrow -\mu_k \cancel{m} g \cancel{R} = \frac{1}{2} \cancel{m} R^{\cancel{2}} \alpha \Rightarrow \alpha = -\frac{2\mu_k g}{R}$$

but $v_G = 0 + at \Rightarrow t = v_G / \mu_k g$

$\omega_f = \omega_0 + \alpha t \qquad t = \dfrac{\omega_f - \omega_0}{\alpha}$

$$\Rightarrow \frac{v_G}{\cancel{\mu_k g}} = \frac{(v_G/R - \omega_0)R}{-2\cancel{\mu_k g}}$$

$$\Rightarrow 2v_G = -v_G + \omega_0 R$$

$$\Rightarrow v_G = \omega_0 R/3.$$

4. (a) this is a straight, $\vec{F} = m\vec{a}$ in rectangular co-ords.

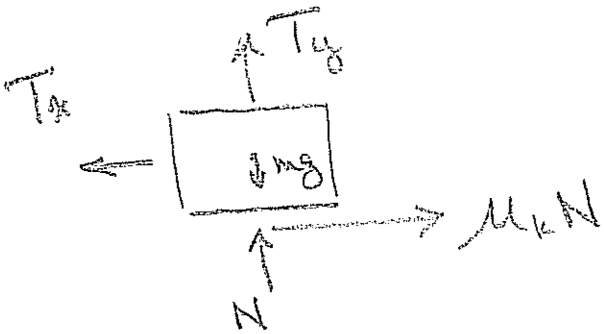
in the x direction:

$T_x = T(.707)$ $\quad$ $T_y = T(.707)$

$a_x = 1.3\ m/s^2$.

$m = 40kg$

$\mu_k = 0.2$



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in the y direction $a_y = 0 \Rightarrow$

$$T(.707) - 40g + N = 0 \Rightarrow N = 40g - T(.707)$$

in the x-direction

$$T(.707) - .2\{40g - T(.707)\} = 40(1.3)$$

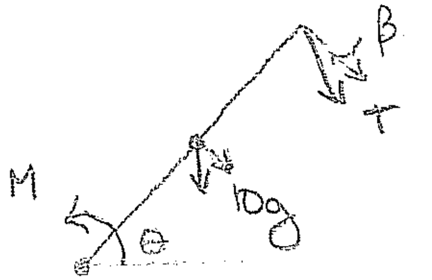
$$T(.707)(1.2) = 40(1.3) + .2(40)(9.81)$$

$$\Rightarrow T = 153.8\ N$$

4(b). $\alpha = 4.5\ s^{-2}$ $\qquad d = \sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = 1.49\,m$

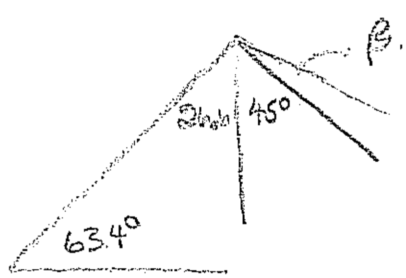
$T = 200N \qquad m = 10kg$

$\Sigma M_O = I_O \alpha \qquad I_O = \frac{1}{3}(10)\,1.49^2 = 7.41\ kg\,m^2$



$\theta = \tan^{-1} \frac{4/3}{2/3} = \tan^{-1} 2 = 63.4^\circ$

geometry to find angle β:



$45^\circ + 26.6^\circ = 71.6^\circ \Rightarrow \beta = 18.4^\circ$

$\Rightarrow$ Tension $(\cos 18.4^\circ)$ causes a moment about O.

moment arm = 1.49 m

$$M - 10g(\cos 63.4^\circ)\left[\frac{1.49}{2}\right] - 200\,(\cos 18.4^\circ)\left[1.49\right] = 7.41(4.5)$$

$$\Rightarrow M = 33.3 + 32.9 + 282.8 = 349.0\ N.m$$

5 (a) energy $T_1 + V_1 = T_2 + V_2$

$m = 1kg$
$l = 10m$

$T_1 = 0$

$T_2 = \frac{1}{2} I_o \omega_f^2 = \frac{1}{2}\left(\frac{1}{3}(1)(10)^2\right)\omega_f^2 = 16.67\,\omega^2$

$V_1 = mgh_G = 1(9.81)(5)$

$V_2 = 0$

$\Rightarrow \quad 49.05 = 16.67\,\omega^2 \Rightarrow \quad \omega_f = 1.72\,s^{-1}$

(b) isolate the stick!

$I_G \cancel{\omega_i}^{0} + M_G \Delta t = I_G \omega_f$

$I_G = \frac{1}{12} m l^2$
$= \frac{1}{12}(.5)(2)^2$
$= .166\ kgm^2$

$F(1)(.01) = .166(6)$

moment arm

$F = 100N.$

(c) angular momentum of the pendulum:

$$I_O \omega_i - F(l)\Delta t = I_O \omega_f$$

$$33.3(1.72) - 100(10)(.01) = 33.3(\omega_f)$$

$$\Rightarrow \omega_f = 1.42 s^{-1}$$