University of Toronto

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, APRIL, 2012

First Year - CHE, CIV, IND, LME, MEC, MMS

MAT187H1S - CALCULUS II

Exam Type: A

Duration: 150 min.

SURNAME: (as on your T-card) ____________________

GIVEN NAMES: ____________________

STUDENT NUMBER: ____________________

SIGNATURE: ____________________

Examiners:
D. Burbulla
P. Milgram
S. Rayan
K. Tyros

Calculators Permitted: Casio 260, Sharp 520 or Texas Instrument 30. No other aids are permitted.

INSTRUCTIONS: Attempt all questions.
Use the backs of the sheets if you need more space.
Do not tear any pages from this exam.
Make sure your exam contains 9 pages.

Part Marks: The value of each question is indicated in parentheses beside the question number.

Total Marks: 100

PAGE	MARK
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
Q8	
TOTAL	

1. [15 marks] Find the general solution, y as a function of x, for each of the following differential equations:

   (a) [4 marks] $\dfrac{dy}{dx} = \dfrac{1}{3xy^2}$

   (b) [6 marks] $\dfrac{dy}{dx} + \dfrac{2y}{x} = \dfrac{1}{x^3+4}$

   (c) [5 marks] $\dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} + 8y = 0$

2. [10 marks] Find the following:

(a) [5 marks] the interval of convergence of the power series $\sum_{k=0}^{\infty}(-1)^k \frac{(x-3)^k}{\sqrt{k+1}}$.

(b) [5 marks] the $8th$ (= $8th$ degree) Maclaurin polynomial of $f(x) = \frac{x^2}{(1+x^2)^{2/3}}$.

3.(a) [8 marks] Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ at the point $(x, y) = (\pi/2, 1)$ if $x = t - \cos t, y = 1 + \sin(2t)$.

3.(b) [7 marks] Approximate $\displaystyle\int_0^1 \sin(x^4)\, dx$ correct to within 10^{-5}, and explain how you know your answer is correct to within 10^{-5}.

4. [10 marks] Plot both the circle with polar equation $r = 1$ and the cardioid with polar equation $r = 1 + \cos\theta$, in the same plane, and then find the area of the region that is inside the circle but outside the cardioid.

5. [13 marks] Find all the critical points of the function $f(x, y) = x^2y^2 - 2xy^2 + 3x^2y - 6xy$, and determine if they are maximum points, minimum points, or saddle points.

6. [12 marks] A small projectile is fired from ground level in an easterly direction with an initial speed of 300 m/s at an angle of $30°$ to the horizontal. A crosswind blows from south to north producing an acceleration of the projectile of 0.4 $\mathrm{m/s^2}$ to the north. Assume the ground is flat, and let $g = 10\ \mathrm{m/s^2}$ (for easier calculations.)

   (a) [6 marks] Where does the projectile land?

   (b) [6 marks] In order to correct for the crosswind and make the projectile land due east of the launch site, at what angle from due east must the projectile be fired? Assume the same initial speed and angle of elevation as in part (a).

7.(a) [6 marks] Use the reduction formula $\int_0^1 x^n(\ln x)^m\,dx = -\frac{m}{n+1}\int_0^1 x^n(\ln x)^{m-1}\,dx$ to show that

$$\int_0^1 x^n(\ln x)^n\,dx = (-1)^n\frac{n!}{(n+1)^{n+1}}.$$

7.(b) [7 marks] Using part (a), or otherwise, show that $\int_0^1 \frac{dx}{x^x} = \frac{1}{1^1} + \frac{1}{2^2} + \frac{1}{3^3} + \frac{1}{4^4} + \frac{1}{5^5}\cdots$

8.[12 marks] Consider the curve $\mathbf{r} = \cos^3 t\,\mathbf{i} + \sin^3 t\,\mathbf{j}$, for $0 \le t \le \pi/2$.

(a) [6 marks] Calculate both $\dfrac{d\mathbf{r}}{dt}$ and $\left\|\dfrac{d\mathbf{r}}{dt}\right\|$.

(b) [6 marks] Find an arc length parameterization of the curve, with reference point $(1, 0)$, for which $t = 0$.