MIE 100S DYNAMICS: Spring 2011

Midterm Test, Monday, February 28, 2011 6:10 pm- 7:40 pm

Question 1 *[40 marks total]*

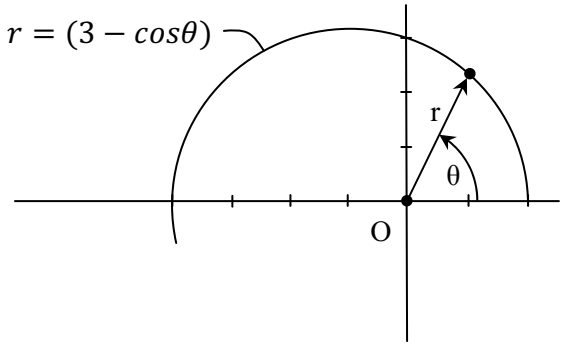


Figure 1

At t = 0, a particle is located at θ = 0°. The particle travels for a short distance on the path $r = (3 - cos\theta)$ meters such that its angular velocity is $\dot{\theta} = 6t$ rad/s, where t is in seconds.

a) Find the time t when the particle is at θ = 60°. *[10 marks]*

b) Find the velocity of the particle at θ = 60°. Express the result in the polar (r-θ) coordinate system shown in the figure. *[10 marks]*

c) Express the velocity of the particle at θ = 60° in normal-tangential (n-t) coordinates. *[10 marks]*

d) Find the radial component of the particle's acceleration, a_r, when the particle is at θ = 60°. *[10 marks]*

Question 2 *[30 marks total]*

A 5 kg block sits on a conical dish that is rotating at a constant rate of 10 rad/s about the vertical axis (Figure 2). The static coefficient of friction between the block and the dish is $\mu_s = 0.28$.

Determine *r*, the distance between the vertical axis and the block, at which the block is on the verge of sliding up the dish.

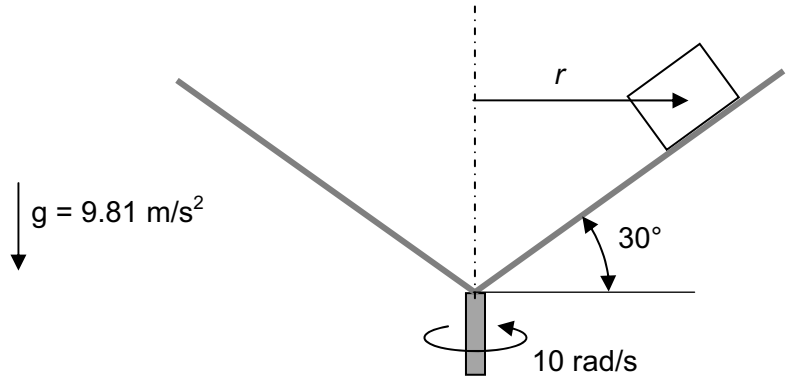


Figure 2

Question 3 *[30 marks total]*

A block "B" of mass 25 kg is connected to a spring with stiffness constant k = 20 N/m. At the position shown in Figure 3, the spring is stretched by 4 meters. The block is on a slope of angle 50° with $\mu_k = 0.15$.

a) Suppose the block is released from the position shown. For what values of μ_s will the block not start to slide down the hill? *[10 marks]*

b) Suppose the block is initially sliding down the slope at 3 m/s at the position shown. What is the magnitude of its acceleration at the position shown? *[10 marks]*

c) Suppose the block is initially moving down the slope at 3 m/s at the position shown. What will be the block's speed when it has traveled 1.5 meters? *[10 marks]*

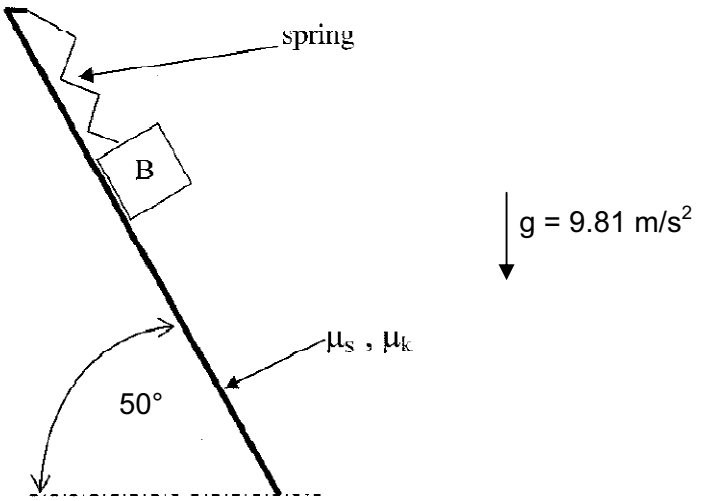
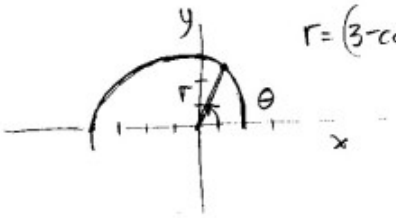


Figure 3

MIE 100S Dynamics - Midterm 2011

Solutions

Question 1



$r = (3 - \cos\theta)$

$t = 0, \theta = 0$ (seconds, radians)

$\dot{\theta} = 6t$

a) $\frac{d\theta}{dt} = 6t \rightarrow \int_0^\theta d\theta = \int_0^t 6t\,dt$

$\theta = 3t^2$

$t_{60^\circ} = \left(\frac{\theta}{3}\right)^{\frac{1}{2}}$

$t_{60^\circ} = \left(\frac{60^\circ \times \frac{\pi}{180^\circ}\,rad}{3}\right)^{\frac{1}{2}}$

$t_{60^\circ} = 0.5908\,s$

b) $\vec{v} = \dot{r}\vec{u}_r + r\dot{\theta}\vec{u}_\theta$

@ 60° $r = 3 - \cos 60^\circ = 2.5\,m$

$\dot{\theta} = 6(0.5908) = 3.545\,rad/s$

$\ddot{\theta} = 6\,rad/s^2$

$\dot{r} = \sin\theta\dot{\theta} = \sin 60^\circ(3.545) = 3.070\,m/s$

$\vec{v} = 3.07\vec{u}_r + 2.5(3.545)\vec{u}_\theta$

$\vec{v} = 3.07\vec{u}_r + 8.86\vec{u}_\theta\ m/s$

$(\ddot{r} = \cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta} = \cos 60^\circ(3.545)^2 + \sin 60^\circ(6) = 11.48\,m/s^2)$

c) $\vec{v} = v\vec{u}_t \qquad v = |\vec{v}| = \sqrt{3.07^2 + 8.86^2} = 9.38\,m/s$

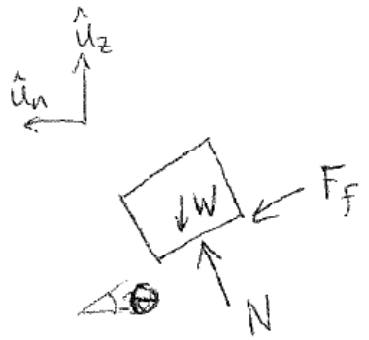
$\vec{v} = 9.38\vec{u}_t\ m/s.$

d) $a_r = \ddot{r} - r\dot{\theta}^2$

$= 11.48 - 2.5(3.545)^2$

$= -19.9\,m/s^2.$

Question 2



$\Sigma F_z = ma_z$

$N\cos\theta - W - F_f \sin\theta = 0$

$F_f = |F_s|_{max} = \mu_s N$

$\therefore N(\cos\theta - \mu_s \sin\theta) = W$

$N = \frac{W}{\cos\theta - \mu_s \sin\theta}$ ~~$= \frac{(5)(9.81)}{\cos 30 - 0.28 \sin 30}$~~

$= \frac{(5)(9.81)}{\cos 30^\circ - 0.28 \sin 30} = 67.6\ N$

$\Sigma F_n = ma_n$

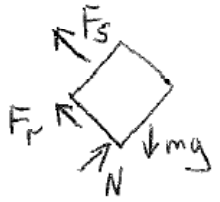
$N\sin\theta + F_f \cos\theta = m\frac{v^2}{\rho}$

$N(\sin\theta + \mu_s \cos\theta) = m\frac{r^2\dot{\theta}^2}{r}$

$r = \frac{N}{m\dot{\theta}^2}(\sin\theta + \mu_s \cos\theta) = \frac{67.6}{(5)(10)^2}(\sin 30 + 0.28\cos 30)$

$= 0.1\ m$

Question 3



(a) $\Sigma F_y = 0 = N - mg\cos 50$

$N = (25)(9.81)(0.643) = 157.6$ Newtons

At critical point, $\Sigma F_x = 0 = -\mu_s N - F_s + mg\sin 50$

$0 = -157.6\,\mu_s - (20)(4) + (25)(9.81)(0.766) \Rightarrow \mu_s = 0.68$

Block does not slide for $\mu_s > 0.68$

(b) $a_x = \Sigma F_x / m = -\mu_k N - F_s + mg\sin 50$

$= [(-0.15)(157.6) - (20)(4) + (25)(9.81)(0.766)]/25$

$a_x = 3.37\ m/s^2$

(c) $\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 - mgah - F_f \Delta s - \frac{1}{2}k(x_2^2 - x_1^2)$

$(\frac{1}{2})(25)(v_2^2) = (\frac{1}{2})(25)(3^2) - (25)(9.81)(-1.5\sin 50) - (15)(158)(1.5^2)$

$-(\frac{1}{2})(20)(5.5^2 - 4^2)$

$\Rightarrow v_2 = 4.16\ m/s$