University of Toronto
Faculty of Applied Sciences and Engineering

MAT187 - Summer 2025

Lecture 1

Instructor: Arman Pannu

We will start at 6:10, use this time to make a new friend

About MAT187

- Continuation of Calculus I:
- Key topics:
  - Integration Techniques
  - Sequences & Series
  - Taylor Approximations
  - ODEs
- Emphasis: Engineering applications & problem-solving.

Course Structure

- **Pre-Class Essentials (PCEs):** short readings + quizzes before class.
- **Lectures:** 6 hours/week, fast-paced.
- **Tutorials:** Weekly, review and preparation for quiz.
- **Self Study:** Double pace course $\rightarrow$ Study twice as much
- **Weekly Quiz:** Demonstrate your knowledge.

Assessments Overview

- **PCEs:** 10% (best 25 of 32).
- **Quizzes:** 50% (6 total; weighted).
- **Final Exam:** 40%.
- **Optional Mini-Project:** Can replace part of quiz weight, bonus marks possible.

Key Online Platforms

- **Quercus:** Course materials & announcements.
- **Gradescope:** Quizzes & grading.
- **Piazza:** Discussion forum.
- **Zoom:** Some office hours/online lectures.

Contact Information and E-mail Policy

arman.pannu@mail.utoronto.ca

- Include "MAT187" in the subject title.
- Ask questions on Piazza whenever possible.
- Refrain from sending math questions over email.
- I will try to respond to all e-mails in 24 hours.

Office Hours

Friday 11-12?

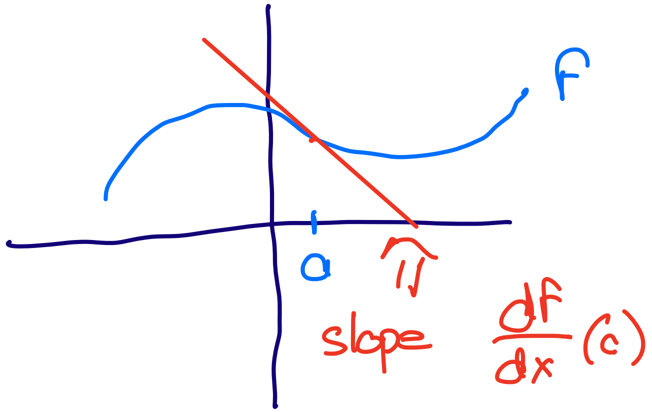
Any questions?

Let's Review Differentiation!

$$\frac{df}{dx}(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

= → slope of tangent

→ rate of change in f with change in input (time)



Techniques for differentiation

→ power rule $\quad \frac{d}{dx}(x^n) = n x^{n-1}$

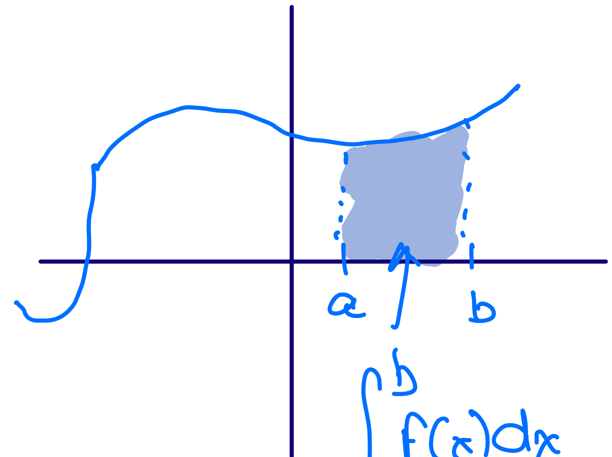
→ Product / quotient rule

→ chain rule

Let's Review Integration!

$$\underbrace{\int_a^b f(x)\,dx}_{\text{continous sum}} = \lim_{\Delta x \to 0} \underbrace{\sum_i f(x_i)\,\Delta x}_{\text{total sum}}$$

density



$\int_a^b f(x)\,dx$

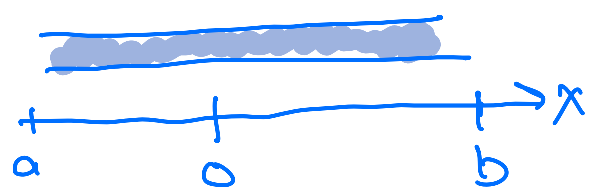
→ integral allows us to take continous sums

density $= f(x)$

ex// m_1 m_2 m_3

total mass $= m_1 + m_2 + m_3$

Kg/per item



total mass $= \int_a^b f(x)\,dx$

Kg/per meter

Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$ where $F'(x) = f(x)$

→ F is anti-derivative

→ motivation to talk about anti-derivatives

$$\int f(x)\,dx$$ ⟸ → indefinite integral

→ anti-derivative

Compute anti-derivatives?

$$\int x^n\,dx = \frac{1}{n+1} x^{n+1} + C$$

$$\int \sin(x)\,dx = -\cos(x) + C \qquad \int \cos(x)\,dx = \sin(x) + C$$

→ Integration by substitution :

$$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du \qquad u = g(x)$$

ex// $\displaystyle\int x \sin(x^2)\,dx = \frac{1}{2}\int \sin(u)\,du$

$= -\dfrac{1}{2}\cos(u) + C$

$= -\dfrac{1}{2}\cos(x^2) + C$

derivative of x^2 x^2

let $u = x^2$

$$\frac{du}{dx} = 2x \Rightarrow du = 2x\,dx$$

Integration by Parts

→ reverse of product rule

$$(fg)' = fg' + gf'$$

$$\int (fg)' = \int fg' + \int gf'$$

$$fg = \int fg' + \int gf'$$

$$\boxed{\int fg' = fg - \int gf'} \Leftarrow \text{integration by parts}$$

start

integrate one part in exchange for differentiating other part

$$\int xe^x\,dx = e^x \cdot x - \int e^x(1)\,dx$$

$f(x) = x$

$g'(x) = e^x$

$g(x) = e^x$

$$\boxed{= e^x x - e^x + C}$$

$$\int x^2 e^x \, dx$$

$g'(x) = e^x$

$f(x) = x^2$

$$= e^x x^2 - \int e^x (2x) \, dx$$

$$= e^x x^2 - 2 \int x e^x \, dx$$

$$= e^x x^2 - 2 \left(e^x x - \int e^x (1) \, dx \right)$$

$$\boxed{= e^x x^2 - 2e^x x + 2e^x + C}$$

some other options:

① $f(x) = e^x$

$\underbrace{g'(x) = x^2}_{\text{complicates}}$

② $f(x) = 1$

$\underbrace{g'(x) = x^2 e^x}_{\text{not helpful}}$

③ $f(x) = x$

$\underbrace{g'(x) = xe^x}_{\text{not bad}}$

option b/c we solved integral of xe^x earlier

→ few more possibilities

$$\int 1 \cdot \ln(x)\,dx$$

$f(x) = \ln(x)$

$g'(x) = 1 \Rightarrow g(x) = x$

$$= x\ln(x) - \int x\frac{1}{x}\,dx$$

$$= x\ln(x) - \int 1\,dx$$

$$\boxed{= x\ln(x) - x + C}$$

$\int sin^2 x dx$

$f(x) = \sin(x)$

$g'(x) = \sin(x) \Rightarrow g(x) = -\cos(x)$

$= \int (\sin x)(\sin x) dx$

$= -\cos(x)\sin(x) - \int (-\cos x)(\cos x) dx$

$= -\cos(x)\sin(x) + \int \cos^2(x) dx$

$= -\cos(x)\sin(x) + \int (1 - \sin^2(x)) dx$

$\int \sin^2 x\, dx = -\cos(x)\sin(x) + x - \int \sin^2(x) dx$

$$\boxed{\int \sin^2 x\, dx = \frac{-\cos(x)\sin(x) + x}{2}}$$