

FACULTY OF APPLIED SCIENCE AND ENGINEERING
University of Toronto
FINAL EXAM, WEDNESDAY, APRIL 29, 2009

MAT 188S
Linear Algebra

Examiner: S. Cohen
Duration: 2 hours, 30 minutes

NO AIDS ALLOWED

Total Marks: 80

Family Name:

Given Name(s):

Please sign here:

Student ID number:

For Markers Only

| Question | Marks |
|--------------|-------------|
| 1 | / 12 |
| 2 | / 10 |
| 3 | / 5 |
| 4 | / 8 |
| 5 | / 20 |
| 6 | / 6 |
| 7 | / 8 |
| 8 | / 11 |
| TOTAL | / 80 |

1. [12 marks] Solve the following system of equations using two different methods.
("By observation" does not count.)

$$\begin{aligned}3x + 2y + z &= 1 \\x - y + 2z &= -3 \\2x - y &= 2\end{aligned}$$

[More room on the next page]

[Extra space for question 1]

2. [10 marks] Find the rank of the following matrix and bases for its column and row spaces.

$$A = \begin{bmatrix} 2 & -1 & 0 & 1 & 4 \\ 1 & 2 & 5 & 0 & -1 \\ 0 & -2 & -4 & 3 & 5 \\ 1 & 1 & 3 & 1 & 1 \end{bmatrix}$$

3. [5 marks] Let $W = \{(a_1, a_2, a_3, a_4) \mid 2a_3 = a_1 + a_4 \text{ and } a_2 - 3a_1 = 2a_3\}$.
Find a basis for W .

4. [8 marks] If U is a subspace of \mathbb{R}^n , show that U^\perp is also a subspace.

5. [20 marks] Let $A = \begin{bmatrix} 0 & 2 & 2 \\ 2 & 3 & -1 \\ 2 & -1 & -1 \end{bmatrix}$.

Find an **orthogonal** matrix P and a diagonal matrix D such that $A = PDP^{-1}$

[If you do not remember how to do this, you can simply diagonalize the matrix for part marks]

[More room on the next page]
[Extra space for question 5]

6. [6 marks] Let A and B be $n \times n$ matrices. Show that:

$$\det(A^2 - AB^T) = \det(A^T A - BA)$$

7. [8 marks] Let E be an $n \times n$ matrix and let $A = I_n - 2E$. Prove that $E^2 = E$ if and only if A is its own inverse.

8. a) [8 marks] Let A and B be 8×8 matrices of rank 4, with $AB = 0$. Show that $\text{col}(B) = \text{null}(A)$.

b) [3 marks] Show that $\text{col}(B) \neq \text{null}(A)$ if A and B are 9×9 matrices of equal rank.