MAT187 - Calculus II

Term Test 2 – Group Part - March 19, 2019

Time allotted: 30 minutes   Aids permitted: None

Total marks: 10

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Instructions   (READ CAREFULLY)

- DO NOT WRITE ON THE QR CODE AT THE TOP OF THE PAGES.

- Read all the instructions carefully.

- This test contains 8 pages and a detached **formula sheet**. Make sure you have all of them.

- You can use page 7 for rough work or to complete a question (**Mark clearly**).
  DO NOT DETACH ANY PAGES.

- No calculators, cellphones, or any other electronic devices are allowed. If you have a cellphone with you, it must be turned off and in a bag underneath your chair.

GOOD LUCK!

MULTIPLE-CHOICE PART. (4 marks)

ANSWER THESE QUESTIONS ON **PAGE 8**.

Only your answer on page 8 will be graded.

1. **(1 mark)** Consider the ODE

$$y'' + 2y' + 2y = (t^2 + 1)e^{-t} + te^{-t}\sin(t)$$

Which of the following terms is **NOT** part of the particular solution?

(A) e^{-t}

(B) te^{-t}

(C) $t^2 e^{-t}$

(D) $e^{-t}\sin(t)$

(E) $t^2 e^{-t}\sin(t)$

2. **(1 mark)** Consider the function

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n-20}n!}(x+2)^{2n}.$$

Select **the one** correct option.

(A) $f^{(23)}(-2) = -\dfrac{1}{3^3}$

(B) $f^{(23)}(-2) = -\dfrac{1}{3^3 \cdot 3!}$

(C) $f^{(23)}(-2) = -\dfrac{1}{3^3 \cdot 23!}$

(D) $f^{(23)}(-2) = 0$

(E) The function f does not have a 23rd derivative at $x = -2$

LONG ANSWER PART

3. It's 1905 and everyone believes that Kinetic energy is given by the formula **(8 marks)**

$$K_C(s) = \frac{1}{2}m\,s^2.$$

Here, m is the mass of the particle at rest and s is the speed of the particle.

Albert Einstein just published his paper on Special Relativity. In it he claimed that kinetic energy is actually given by the formula

$$K_R(s) = mc^2 \left[\frac{1}{\sqrt{1 - \frac{s^2}{c^2}}} - 1 \right],$$

where c is the speed of light and s is still the speed of the particle.

The scientific community is very skeptical; after all the new formula $K_R(s)$ looks completely different from the classical one $K_C(s)$.

Your job is to find out how these two formulas are related.

(a) (2 marks) Consider the function $f(x) = \dfrac{1}{\sqrt{1-x}}$ and obtain its linear Taylor approximation $p_1(x)$ centred at $a = 0$.

$p_1(x) =$

(b) (2 marks) Use part **(a)** to approximate $K_R(s)$.

(c) (1 mark) You just approximated $K_R(s)$. Compare this approximation to the formula for $K_C(s)$. What do you observe?

(d) (1 mark) Assuming $0 < z < x < 1$, show that

$$\left|f''(z)\right| \leqslant \frac{3}{4}(1-x)^{-\frac{5}{2}}$$

(e) (1 mark) Let $R_1(x)$ be the remainder for the approximation found in **(a)**.

Find a formula relating $\left|K_R(s) - K_C(s)\right|$ with $R_1\left(\frac{s^2}{c^2}\right)$.

(f) (1 mark) Estimate $\left|K_R(s) - K_C(s)\right|$ in terms of m, c, and s.

$$\left|K_R - K_C\right| \leqslant$$

USE THIS PAGE TO CONTINUE OTHER QUESTIONS.

If you wish to have this page marked, make sure to refer to it in your original solution.

The end.