

University of Toronto  
FACULTY OF APPLIED SCIENCE AND ENGINEERING  
**FINAL EXAMINATION, DECEMBER, 2013**

**Duration:** 2 and 1/2 hours

First Year - CHE, CIV, IND, LME, MEC, MMS

**MAT186H1F - CALCULUS I**

Exam Type: A

SURNAME: (as on your T-card) \_\_\_\_\_

YOUR FULL NAME: \_\_\_\_\_

STUDENT NUMBER: \_\_\_\_\_

SIGNATURE: \_\_\_\_\_

**Examiners:**

D. Burbulla

S. Cohen

L. Shorser

K. Tyros

**Calculators Permitted:**

Casio FX-991 or Sharp EL-520.

**INSTRUCTIONS:** Attempt all questions.

Present your solutions in the space provided. You must show your work and give full explanations to get full marks. Partial credit can be obtained for partially correct work, but NO credit may be given if your work is poorly presented, difficult to decipher, uses incorrect mathematical notation, or makes no sense.

Use the backs of the sheets if you need more space. Do not tear any pages from this exam. Make sure your exam contains 9 pages.

**MARKS:** the value of each question is indicated in parentheses beside the question number.

**TOTAL MARKS:** 100

QUESTION	MARK
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
Q8	
TOTAL	

1. [15 marks] Find the following:

(a) [5 marks]  $\int \left( e^x + \frac{1}{1+x^2} + \frac{1}{x} + \sinh x \right) dx$

(b) [5 marks]  $\int_0^{\pi/4} \tan^2 x \sec^2 x \, dx.$

(c) [5 marks] the average speed of a particle over the time period  $t = 0$  to  $t = 3$  if the velocity of the particle at time  $t$  is given by  $v = \sin(\pi t)$ .

2. [10 marks] Find the following:

(a) [4 marks]  $F'(1)$  if  $F(x) = \int_0^{x^4} \sqrt{t^2 + 4t + 11} \, dt$ .

(b) [6 marks] an approximation of the solution to the equation  $\cos x = x$ , correct to 3 decimal places. Note:  $x$  is in radians!

3. [13 marks] Let  $f(x) = \frac{2x^2 - x - 4}{x - 2}$ , for which  $f'(x) = \frac{2(x - 1)(x - 3)}{(x - 2)^2}$ ,  $f''(x) = \frac{4}{(x - 2)^3}$ .

(a) [2 marks] Find the interval(s) on which  $f$  is increasing.

(b) [2 marks] Find the interval(s) on which  $f$  is decreasing.

(c) [1 mark] Find the interval(s) on which  $f$  is concave up.

(d) [1 mark] Find the interval(s) on which  $f$  is concave down.

(e) [2 marks] Find the equations of all asymptotes to the graph of  $f$ .

(f) [5 marks] Sketch the graph of  $f$  labeling all critical points, inflection points and asymptotes, if any.

4. [12 marks] Let  $A$  be the area of the region in the  $xy$ -plane bounded by the curves  $y = \cos^{-1} x$  and  $y = -\cos^{-1} x$  on the interval  $-1 \leq x \leq 1$ . Note:  $\cos^{-1} x = \arccos x$ .

(a) [8 marks] Write down two integrals, one with respect to  $x$  and one with respect to  $y$ , that both give the value of  $A$ .

(b) [4 marks] Find the value of  $A$ .

5. [15 marks] Find the following limits, if they exist:

(a) [4 marks]  $\lim_{x \rightarrow 0} \frac{\sin^{-1}(3x)}{e^x - 1}$

(b) [5 marks]  $\lim_{x \rightarrow +\infty} x \left( \sqrt{1 + \frac{3}{x} + \frac{5}{x^2}} - 1 \right)$

(c) [6 marks]  $\lim_{x \rightarrow 1^-} (2 - x)^{\tan(\pi x/2)}$

6. [10 marks] Sketch the graph of the relation  $x^{2/3} + y^{2/3} = 1$ , for  $-1 \leq x \leq 1, -1 \leq y \leq 1$ , indicating all critical points, inflection points, horizontal tangents and vertical tangents, if any.

7. [10 marks] The lower edge of a painting, 3 m in height, is 1 m above an observer's eye level. Assuming that the best view is obtained when the angle subtended at the observer's eye by the painting is maximum, how far from the wall should the observer stand?



8. [15 marks] The parts of this question are unrelated.

(a) [7 marks] Find the length of the curve  $y = \frac{1}{8} \left( \frac{x^2}{2} - 16 \ln x \right)$  for  $1 \leq x \leq 4$ .

(b) [8 marks] The region bounded by  $x = 1 - y^4$  and  $x = 0$  is rotated about the line  $x = 2$ . Find the volume of the resulting solid.