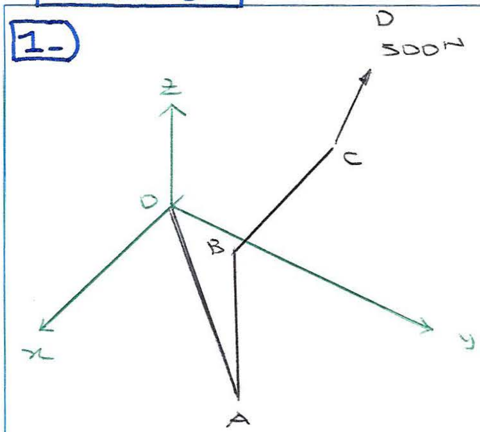
Problem set 5 (PS5)
Solution

1.-



- First, express the force in Cartesian coordinates.

O (0; 0; 0)
A (900; 1200; 0)
B (900; 1200; 1000)
C (100; 1200; 1000)
D (-300; 1400; 2200)

- $\underset{\sim}{r}_{CD} = -400\underset{\sim}{i} + 200\underset{\sim}{j} + 1200\underset{\sim}{k}$; $r_{CD} = 1280,6$ mm

$\underset{\sim}{u}_{CD} = \dfrac{\underset{\sim}{r}_{CD}}{r_{CD}} = -0,3123\underset{\sim}{i} + 0,1562\underset{\sim}{j} + 0,9370\underset{\sim}{k}$

$\underset{\sim}{F}_{CD} = 500\underset{\sim}{u}_{CD} = -156,2\underset{\sim}{i} + 78,1\underset{\sim}{j} + 468,5\underset{\sim}{k}$ N

i.-

- Find the position vector along OA.

$\underset{\sim}{r}_{OA} = 900\underset{\sim}{i} + 1200\underset{\sim}{j}$; $r_{OA} = 1500,0$ mm (we assumed here that M_{OA} is directed from O to A)

$\underset{\sim}{u}_{OA} = \dfrac{\underset{\sim}{r}_{OA}}{r_{OA}} = 0,6\underset{\sim}{i} + 0,6\underset{\sim}{j}$

- Find a moment arm vector from any point on line OA to C. I select AC since its $\underset{\sim}{j}$ component is zero. (easier solution)

$\underset{\sim}{r}_{AC} = -800\underset{\sim}{i} + 1000\underset{\sim}{k}$

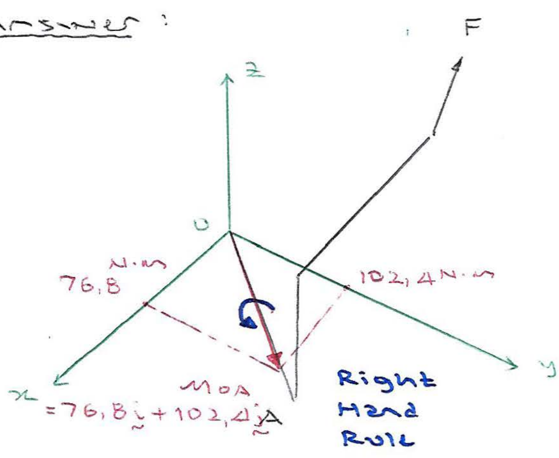
$$M_{OA} = \underset{\sim}{u}_{OA} \overset{\text{dot}}{\cdot} (\underset{\sim}{r}_{AC} \overset{\text{cross}}{\times} \underset{\sim}{F}_{CD}) = \begin{vmatrix} 0,6 & 0,8 & 0 \\ -800 & 0 & 1000 \\ -156,2 & 78,1 & 468,5 \end{vmatrix} = 128\,020 \text{ N·mm}$$

+ve, initial direction assump is correct

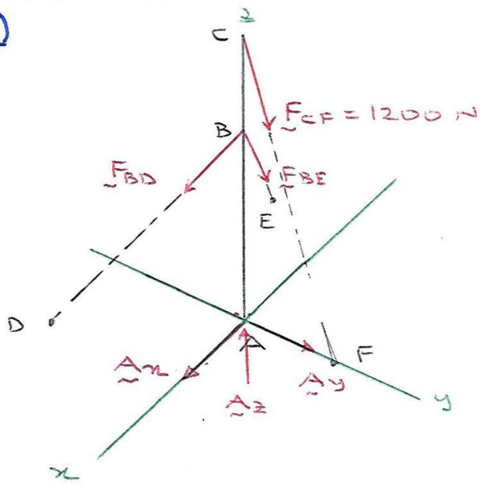
Answer: $M_{OA} = 128$ N·m (3 sig. fig.)

ii.- $\underset{\sim}{M}_{OA} = M_{OA} \cdot \underset{\sim}{u}_{OA} = 128,020 \cdot (0,6\underset{\sim}{i} + 0,8\underset{\sim}{j}) = 76,8\underset{\sim}{i} + 102,4\underset{\sim}{j}$ N·m

Answer:



2.



- First, express all forces in cartesian coordinate system.

A (0 ; 0 ; 0)
B (0 ; 0 ; 2000)
C (0 ; 0 ; 2800)
D (800 ; −1000 ; 0)
E (−1200 ; −600 ; 0)
F (0 ; 800 ; 0)

- $\underset{\sim}{r}_{BD} = 800\underset{\sim}{i} - 1000\underset{\sim}{j} - 2000\underset{\sim}{k}$; $r_{BD} = 2374,9$ mm
  $\underset{\sim}{F}_{BD} = 0,3369 F_{BD}\underset{\sim}{i} - 0,4211 F_{BD}\underset{\sim}{j} - 0,8422 F_{BD}\underset{\sim}{k}$

- $\underset{\sim}{r}_{BE} = -1200\underset{\sim}{i} - 600\underset{\sim}{j} - 2000\underset{\sim}{k}$; $r_{BE} = 2408,3$ mm
  $\underset{\sim}{F}_{BE} = -0,4983 F_{BE}\underset{\sim}{i} - 0,2491 F_{BE}\underset{\sim}{j} - 0,8305 F_{BE}\underset{\sim}{k}$

- $\underset{\sim}{r}_{CF} = 800\underset{\sim}{j} - 2800\underset{\sim}{k}$; $r_{CF} = 2912,0$ mm
  $\underset{\sim}{F}_{CF} = 329,7\underset{\sim}{j} - 1153,8\underset{\sim}{k}$

- $\underset{\sim}{A}_x = A_x\underset{\sim}{i}$
- $\underset{\sim}{A}_y = A_y\underset{\sim}{j}$
- $\underset{\sim}{A}_z = A_z\underset{\sim}{k}$

(i) $\boxed{\Sigma M_A = 0} \Rightarrow \underset{\sim}{r}_{AB} \times (\underset{\sim}{F}_{BD} + \underset{\sim}{F}_{BE}) + \underset{\sim}{r}_{AC} \times \underset{\sim}{F}_{CF} = 0$; $\underset{\sim}{r}_{AB} = -2000\underset{\sim}{k}$
$\underset{\sim}{r}_{AC} = -2800\underset{\sim}{k}$

$$\begin{vmatrix} i & j & k \\ 0 & 0 & 2000 \\ (F_{BD}+F_{BE})_i & (F_{BD}+F_{BE})_j & (F_{BD}+F_{BE})_k \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 0 & 2800 \\ 0 & 329,7 & -1153,8 \end{vmatrix} = 0$$

$$\begin{matrix} i & j & k & i & j & k \\ 0 & 0 & 2000 & 0 & 0 & 2800 \end{matrix}$$

−ve −ve +ve

- $(0,3369 F_{BD} - 0,4983 F_{BE}) \cdot \underset{\sim}{j} \cdot 2000 - (-0,4211 F_{BD} - 0,2491 F_{BE}) \cdot \underset{\sim}{i} \cdot 2000$
  $- 2800 \cdot 329,7 \cdot \underset{\sim}{i} = 0$

- i-terms : $0,4211 F_{BD} + 0,2491 F_{BE} = \dfrac{2800 \cdot 329,7}{2000} = 461,6$ (I)

- j-terms : $0,3369 F_{BD} - 0,4983 F_{BE} = 0 \Rightarrow F_{BD} = 1,479 F_{BE}$ (II)

- From Eq.(I) and (II): $F_{BE} = 529,4$ N
  $F_{BD} = 783,0$ N
  (4 sig. fig.)

ii. $\boxed{\Sigma F_x = 0}$ (i-terms)

$0,3369 \cdot 783,0^N - 0,4983 \cdot 529,4^N + A_x = 0 \Rightarrow A_x = 0\ N$

$\boxed{\Sigma F_y = 0}$ (j-terms)

$-0,4211 \cdot 783,0^N - 0,2491 \cdot 529,4^N + 329,7^N + A_y = 0 \Rightarrow A_y = 131,9\ N$

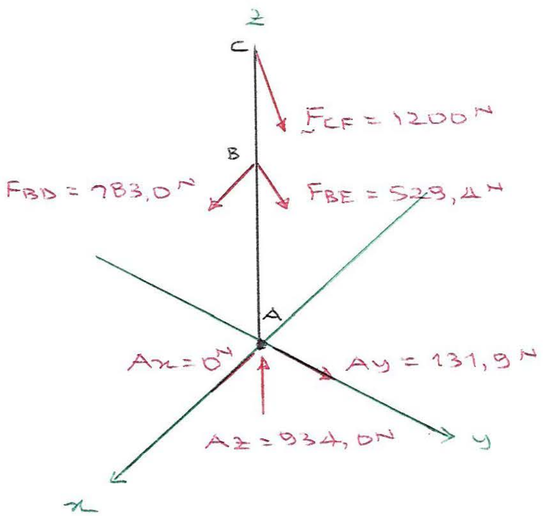
(+ve: assumed direction correct)

$\boxed{\Sigma F_z = 0}$ (k-terms)

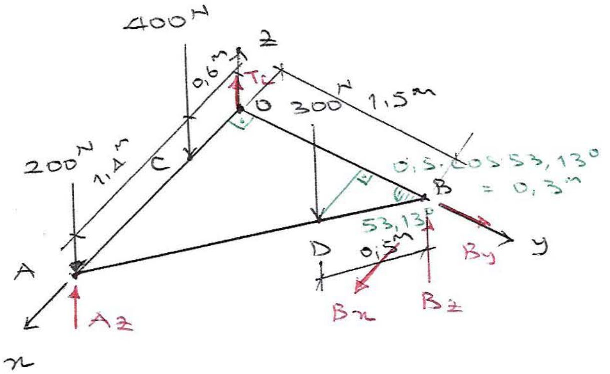
$-0,8422 \cdot 783,0^N - 0,8305 \cdot 529,4 - 1153,8 + A_z = 0 \Rightarrow A_z = 934,0\ N$

(+ve: assumed direction correct)

Answer:



3-) First, draw a FBD. Label all applied forces and unknowns



Then, apply equilibrium equations.

$\Sigma F_x = 0 \Rightarrow B_x = 0$

$\Sigma F_y = 0 \Rightarrow B_y = 0$

$\Sigma F_z = 0 \Rightarrow A_z + B_z + T_C - 200 - 400 - 300 = 0$

$\Rightarrow A_z + B_z + T_C = 900^N$

$\Sigma M_x = 0 \Rightarrow -300 \cdot 1,2 + B_z \cdot 1,5 = 0$

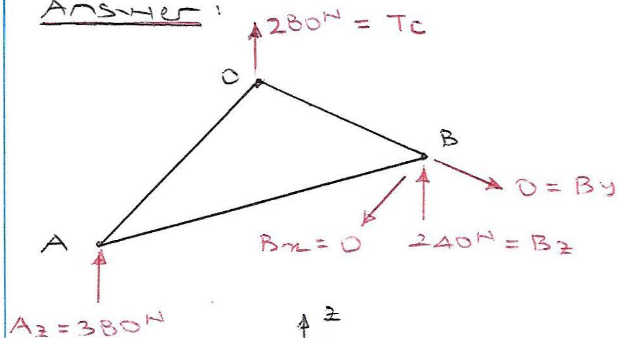
$\Rightarrow B_z = 240\,N$

$\Sigma M_y = 0 \Rightarrow (200 - A_z) \cdot 2,0 + 400 \cdot 0,6 + 300 \cdot 0,4 = 0$

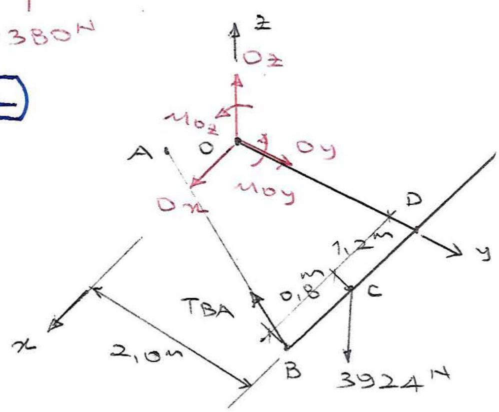
$\Rightarrow A_z = 380\,N$

$\Rightarrow T_C = 280\,N$

Answer:



4-)



First, draw a FBD. Label all applied forces and unknowns.

Then express the cable force as a cartesian vector.

$A(1,0\,;\,0\,;\,0,6)\;;\;B(2,0\,;\,2,0\,;\,0)$

$\underset{\sim}{r}_{BA} = -1,0\underset{\sim}{i} - 2,0\underset{\sim}{j} + 0,6\underset{\sim}{k} \qquad r_{BA} = 2,3152\,m$

$\underset{\sim}{u}_{BA} = \dfrac{\underset{\sim}{r}_{BA}}{r_{BA}} = -0,4319\underset{\sim}{i} - 0,8639\underset{\sim}{j} + 0,2592\underset{\sim}{k}$

$\underset{\sim}{T}_{BA} = T_{BA} \cdot \underset{\sim}{u}_{BA} = -0,4319\,T_{BA}\underset{\sim}{i} - 0,8639\,T_{BA}\underset{\sim}{j} + 0,2592\,T_{BA}\underset{\sim}{k}$

$\underset{\sim}{W} = -3924\underset{\sim}{k}$

Apply equilibrium equations.

$\boxed{\Sigma F = 0} \Rightarrow (-0,4319\,T_{BA} + O_x)\underset{\sim}{i} + (-0,8639\,T_{BA} + O_y)\underset{\sim}{j} + (0,2592\,T_{BA} - 3924 + O_z)\underset{\sim}{k} = 0$

$\Rightarrow \boxed{O_x = 0,4313\,T_{BA}\;;\; O_y = 0,8639\,T_{BA} \quad O_z = -0,2592\,T_{BA} + 3924}\;(*)$

$\boxed{\Sigma M_{OD}} = 0 \Rightarrow$ Selecting simple r's
or ΣM_y

$$\Sigma M_{OD} = \underset{\sim}{u}_{OD} \cdot (\underset{\sim}{r}_{OB} \times \underset{\sim}{T}_{BA} + \underset{\sim}{r}_{OC} \times \underset{\sim}{W}) + M_{Oy} = 0$$

B(2,0 ; 2,0 ; 0) ; $\underset{\sim}{r}_{OB} = 2{,}0\underset{\sim}{i} + 2{,}0\underset{\sim}{j}$; $r_{OB} = 2{,}8284$ m

C (1,2 ; 2,0 ; 0) ; $\underset{\sim}{r}_{OC} = 1{,}2\underset{\sim}{i} + 2{,}0\underset{\sim}{j}$; $r_{OC} = 2{,}3324$ m

D (0 ; 2,0 ; 0) ; $\underset{\sim}{r}_{OD} = 2{,}0\underset{\sim}{j}$; $r_{OD} = 2{,}0$; $\underset{\sim}{u}_{OD} = \underset{\sim}{j}$

$$\Sigma M_{OD} = \begin{vmatrix} 0 & 1 & 0 \\ 2{,}0 & 2{,}0 & 0 \\ -0{,}4313\,T_{BA} & -0{,}8639\,T_{BA} & 0{,}2592\,T_{BA} \end{vmatrix}$$

$$+ \begin{vmatrix} 0 & 1 & 0 \\ 1{,}2 & 2{,}0 & 0 \\ 0 & 0 & -3924 \end{vmatrix} = 0 \Rightarrow \boxed{-0{,}5184\,T_{BA} + 4708{,}8 + M_{Oy} = 0}$$

(**)

$\boxed{\Sigma M_x = 0} \Rightarrow \Sigma M_x = \underset{\sim}{u}_x \cdot (\underset{\sim}{r}_{OB} \times \underset{\sim}{T}_{BA} + \underset{\sim}{r}_{OC} \times \underset{\sim}{W}) = 0$; $\underset{\sim}{u}_x = \underset{\sim}{i}$

$$\Sigma M_x = \begin{vmatrix} 1 & 0 & 0 \\ 2{,}0 & 2{,}0 & 0 \\ -0{,}4313\,T_{BA} & -0{,}8639\,T_{BA} & 0{,}2592\,T_{BA} \end{vmatrix}$$

$$+ \begin{vmatrix} 1 & 0 & 0 \\ 1{,}2 & 2{,}0 & 0 \\ 0 & 0 & -3924 \end{vmatrix} = 0 \Rightarrow 0{,}5184\,T_{BA} - 7848 = 0$$

$$\Rightarrow T_{BA} = 15139 \text{ N}$$

Sub. TBA into (*) : $O_x = 6530$ N
$O_y = 13079$ N
$O_z = 0$ N

Sub. TBA into (**) : $M_{Oy} = 3139$ Nm

$\boxed{\Sigma M_z = 0} \Rightarrow \Sigma M_z = \underset{\sim}{u}_z \cdot (\underset{\sim}{r}_{OB} \times \underset{\sim}{T}_{BA} + \underset{\sim}{r}_{OC} \times \underset{\sim}{W}) + M_{Oz} = 0$; $\underset{\sim}{u}_z = \underset{\sim}{k}$

$$\Sigma M_z = \begin{vmatrix} 0 & 0 & 1 \\ 2{,}0 & 2{,}0 & 0 \\ -0{,}4313\,T_{BA} & -0{,}8639\,T_{BA} & 0{,}2592\,T_{BA} \end{vmatrix} \ldots$$

See the next page

$$+\begin{vmatrix} 0 & 0 & 1 \\ 1{,}2 & 2{,}0 & 0 \\ 0 & 0 & -392A \end{vmatrix} + M_{Oz} = 0 \Rightarrow -0{,}8652\,T_{BA} + M_{Oz} = 0$$

$$\Rightarrow M_{Oz} = 13098\ \text{Nm}$$

Moment directions: Right Hand Rule

Answer: