University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, DECEMBER 2002
First Year - Programs 1,2,3,4,6,7,8,9

MAT 188H1F
Linear Algebra

Examiners

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SURNAME ______________________________

GIVEN NAME ______________________________

STUDENT NO. ______________________________

SIGNATURE ______________________________

INSTRUCTIONS:

Non-programmable calculators permitted.

Answer all questions.

Present your solutions in the space provided; use the back of the **preceding** page if more space is required.

TOTAL MARKS: 100

The value for each question is shown in parentheses after the question number.

MARKER'S REPORT	
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
TOTAL	

1. [30 marks: 5 marks for each part] Find the following:

(a) the inverse of $\begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \\ 1 & 0 & 2 \end{pmatrix}$

(b) $\det \begin{pmatrix} 1 & 1 & 2 & 3 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 7 \\ 0 & 1 & -1 & 4 \end{pmatrix}$

(c) the eigenvalues of the matrix $\begin{pmatrix} 0 & 0 & 3 \\ 0 & 4 & 0 \\ 3 & 0 & 0 \end{pmatrix}$

(d) the coordinate vector of $\mathbf{p}(x) = -1 + 4x + 5x^2$ with respect to the basis

$$B = \{1 + x, 1 + x^2, x + x^2\}$$

of $\mathbf{P}_2$.

(e) the values of a for which the matrix $\begin{pmatrix} 1 & a & 2+a \\ a & 4 & 4 \\ a & 4 & 6 \end{pmatrix}$ is not invertible.

(f) the point on the plane with equation $x + y + z = 2$ closest to the point $(3, 2, -1)$.

2. [12 marks] Let W be the subspace of $\mathbf{R}^4$ consisting of all vectors of the form

$$(a+c, b+c, a+2b+c, -a-b).$$

Find an orthonormal basis of W. (Use the usual dot product in $\mathbf{R}^4$.)

3. [12 marks] Let W be the set of 3×3 matrices, A, satisfying the condition

$$A^T = -A.$$

(a) [6 marks] Show that W is a subspace of $\mathbf{M}^{3,3}$.

(b) [6 marks] Find a basis for W and its dimension.

4. [12 marks] Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$. Find an orthogonal matrix P and a diagonal matrix D such that $D = P^T AP$.

5. [14 marks] Suppose A is a 3×3 invertible matrix with eigenvalues, 3, 1, and -1. Find the following:

(a) [4 marks] the eigenvalues of A^{-1}.

(b) [5 marks] the eigenvalues of A^T.

(c) [5 marks] the eigenvalues of $\mathrm{Adj}(A)$.

6. [10 marks; 2 marks for each part] Suppose $\mathbf{u}$ and $\mathbf{v}$ are two non-zero vectors in $\mathbf{R}^3$. What does each of the following conditions imply about the linear independence or dependence of the set $\{\mathbf{u}, \mathbf{v}\}$?

(a) $\mathbf{u} = 3\mathbf{v}$

(b) $a\mathbf{u} + b\mathbf{v} = \mathbf{0} \Rightarrow a = b = 0$

(c) $\mathbf{u} \cdot \mathbf{v} = 0$

(d) $\mathbf{u} \times \mathbf{v} = \mathbf{0}$

(e) $\{\mathbf{u}, \mathbf{v}, \mathbf{u} \times \mathbf{v}\}$ spans $\mathbf{R}^3$

7. [10 marks: 5 marks for each part.] Let $B = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_n\}$ be an orthonormal basis of an inner product space V. Prove the following:

(a) For any vectors $\mathbf{u}$ and $\mathbf{v}$ in V,

$$(\mathbf{u}, \mathbf{v}) = \mathbf{x} \cdot \mathbf{y},$$

where $\mathbf{x}$ and $\mathbf{y}$ are the coordinate vectors of $\mathbf{u}$ and $\mathbf{v}$, respectively, with respect to the basis B.

(b) If $\mathbf{x}_i$ is the coordinate vector of $\mathbf{w}_i$ with respect to the basis B, then the set $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\}$ is an orthonormal basis of $\mathbf{R}^n$, with respect to the usual dot product in $\mathbf{R}^n$.