University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, APRIL, 2011

First Year - CHE, CIV, IND, LME, MEC, MMS

MAT187H1S - CALCULUS II

Exam Type: A
Duration: 150 min.

SURNAME: (as on your T-card) _____

GIVEN NAMES: _____

STUDENT NUMBER: _____

SIGNATURE: _____

Examiners:
D. Burbulla
S. Cohen
A. Moradifam
M. Pugh

Calculators Permitted: Casio 260, Sharp 520 or TI 30. No other aids are permitted.

INSTRUCTIONS: Attempt all questions. Use the backs of the sheets if you need more space. Do not tear any pages from this exam. Make sure your exam contains 10 pages.

Breakdown Of Marks: Questions 1, 2 and 3 consist of short-answer questions. Each part of Questions 1 and 2 is worth 4 marks; each part of Question 3 is worth 5 marks.

Questions 1 and 2 are each worth 12 marks.

Questions 3, 4, 6 and 8 are each worth 10 marks.

Questions 5, 7 and 9 are each worth 12 marks.

Total Marks: 100

PAGE	MARK
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
Q8	
Q9	
TOTAL	

1. Find the general solutions to the three following differential equations:

(a) $\frac{dy}{dx} = \frac{1}{3xy^2}$

(b) $\frac{dy}{dx} - y \tan x = \sin x$

(c) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = 0$

2.(a) What is the interval of convergence of the power series $\sum_{k=0}^{\infty}(-1)^k\frac{(x+5)^k}{k+2}$?

2.(b) Find the first four non-zero terms of the Maclaurin series for $\frac{x^2}{x^2+1}$.

2.(c) What is the sum of the power series $\sum_{k=0}^{\infty}(-1)^k\frac{(2x)^k}{k!}$?

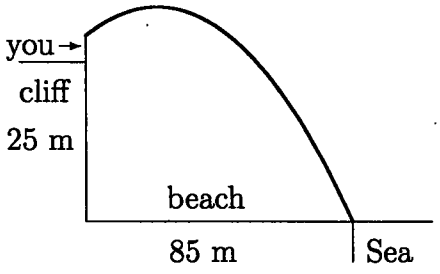
3. Calculate $\dfrac{d^2y}{dx^2}$ at the point $(x, y) = (2, 2)$ for each of the two curves:

(a) the parametric curve with $x = 1 + t^3$, $y = t + t^2$.

(b) the polar curve with $r = \sqrt{8}\, e^{\theta - \pi/4}$.

4. Find the area of the region that is outside the circle with polar equation $r = 1$ but inside the curve with polar equation $r^2 = 2\sin(2\theta)$.

5. You are standing on top of a cliff 25 m above a beach which is 85 m from the sea shore. If you can throw a stone at a maximum speed of 25 m/sec, at least how tall must you be to be able to throw a stone from the top of the cliff into the sea?



For simplicity of calculation, assume the acceleration due to gravity is 10 m/sec^2, and ignore air resistance. Use the diagram at the left to set up your solution. Setting up the problem will be worth half the marks.

6. Find and classify all the critical points of the function $f(x, y) = 4xy - x^4 - y^4$.

7. Approximate

$$\int_0^{\pi/6} \frac{\cos x \, dx}{(1+\sin^4 x)^{2/3}}$$

to within 10^{-5}, and explain why your approximation is correct to within 10^{-5}.

8. The trajectory of a particle in space is given by $\mathbf{r} = (t-5)\,\mathbf{i} + t^2\,\mathbf{j} + 2\sqrt{t}\,\mathbf{k}$. Find the point on the trajectory that is closest to the origin, $(0, 0, 0)$.

9. Consider the cycloid $\mathbf{r} = (t - \sin t)\,\mathbf{i} + (1 - \cos t)\,\mathbf{j}$, for $0 \leq t \leq 2\pi$.

(a) [6 marks] Calculate both $\dfrac{d\mathbf{r}}{dt}$ and $\left\| \dfrac{d\mathbf{r}}{dt} \right\|$.

(b) [6 marks] Find an arc length parameterization of the cycloid, with reference point $(0, 0)$, for which $t = 0$.