Name: ______________________________

Student number: ______________________

MAT 186
Quiz 3

1. Evaluate:

AB4	CF1

$$\lim_{x\to 3}\frac{|x^2-4x+3|}{|x-3|}$$

Method I:

$$\begin{aligned}\lim_{x\to 3}\frac{|x^2-4x+3|}{|x-3|} &= \lim_{x\to 3}\frac{|(x-1)(x-3)|}{|x-3|}\\ &= \lim_{x\to 3}\frac{|x-1||x-3|}{|x-3|}\\ &= \lim_{x\to 3}|x-1|\\ &= 2\end{aligned}$$

Method II:

$$\lim_{x\to 3}\frac{|x^2-4x+3|}{|x-3|} = \lim_{x\to 3}\frac{|(x-1)(x-3)|}{|x-3|}$$

Draw a number line with the points 1 and 3 on it to get where each term is positive and negative.

$$\begin{aligned}\lim_{x\to 3^-}\frac{|x^2-4x+3|}{|x-3|} &= \lim_{x\to 3^-}\frac{-(x-1)(x-3)}{-(x-3)}\\ &= \lim_{x\to 3^-} x-1\\ &= 2\end{aligned}$$

$$\begin{aligned}\lim_{x\to 3^+}\frac{|x^2-4x+3|}{|x-3|} &= \lim_{x\to 3^+}\frac{(x-1)(x-3)}{(x-3)}\\ &= \lim_{x\to 3^+} x-1\\ &= 2\end{aligned}$$

Therefore, the limit is 2.

2. Find the value(s) of $\boldsymbol{a}$ that makes the following function continuous:

AB6	CS4

$$f(x)=\begin{cases} x^3-4x^2+\dfrac{a}{x}, & x\geq 1\\ 2x^2+3x-a^2x-2, & x<1\end{cases}$$

For continuity, we need the function to have the same limit on either side of $x=1$.

The right-hand limit does not need to be calculated, as the function is continuous and defined up to **and including** $x=1$. So, we only need $f(1)=a-3$.

The left-hand side does require a limit:

Continued on back.

$$\lim_{x \to 1^-} 2x^2 + 3x - a^2x - 2 = 3 - a^2$$

These need to be equal, so we have $a - 3 = 3 - a^2$.

$$a^2 + a - 6 = 0$$

$$(a + 3)(a - 2) = 0$$

Therefore, $a = 2$ or -3 will work.

3. Find the following limit for $\boldsymbol{n = 1, 2, 3}$, and $\boldsymbol{4}$:

CF1	CF2	CF3

$$\lim_{\theta \to 0} \frac{\sin^2 \theta}{\theta^n}$$

Remember to show all work.

$n = 1$:

$$\lim_{\theta \to 0} \frac{\sin^2 \theta}{\theta^1} = \left(\lim_{\theta \to 0} \frac{\sin \theta}{\theta}\right) \cdot \left(\lim_{\theta \to 0} \sin \theta\right)$$
$$= 1 \cdot 0$$
$$= \mathbf{0}$$

$n = 2$:

$$\lim_{\theta \to 0} \frac{\sin^2 \theta}{\theta^2} = \left[\lim_{\theta \to 0} \left(\frac{\sin \theta}{\theta}\right)\right]^2$$
$$= 1^2$$
$$= \mathbf{1}$$

$n = 3$:

$$\lim_{\theta \to 0} \frac{\sin^2 \theta}{\theta^3} = \left[\lim_{\theta \to 0} \left(\frac{\sin \theta}{\theta}\right)\right]^2 \cdot \left(\lim_{\theta \to 0} \frac{1}{\theta}\right)$$

$$= 1 \cdot \lim_{\theta \to 0} \frac{1}{\theta}$$

This requires two one-sided limits, giving us the forms $\left[\frac{1}{0^+}\right]$ on the right and $\left[\frac{1}{0^-}\right]$ on the left. These lead to opposite infinities, so the two-sided limit **does not exist**.

$n = 4$:

$$\lim_{\theta \to 0} \frac{\sin^2 \theta}{\theta^4} = \left[\lim_{\theta \to 0} \left(\frac{\sin \theta}{\theta}\right)\right]^2 \cdot \left(\lim_{\theta \to 0} \frac{1}{\theta^2}\right)$$
$$= 1 \cdot \lim_{\theta \to 0} \frac{1}{\theta^2}$$
$$= \left[\frac{1}{0^+}\right]$$
$$= +\infty$$