University of Toronto
Faculty of Applied Science and Engineering
Department of Electrical and Computer Engineering

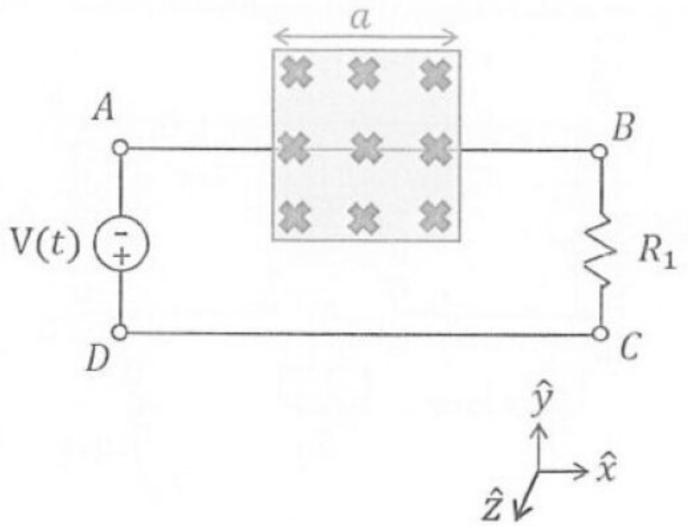
ECE110S – Electrical Fundamentals
Term Test 2 – March 24, 2022, 7:00 – 8:30 p.m.

($e = 1.6 \times 10^{-19}$ C, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $g = 9.81$ N/kg)

ANSWER ALL QUESTIONS ON THESE SHEETS, USING THE BACK SIDE IF NECESSARY.

1. Non-programmable calculators (Casio FX-991MS & Sharp EL-520X) are allowed.
2. You are allowed a one page (8.5" x 11") double-sided aid sheet.
3. For full marks, you must show methods, state UNITS and compute numerical answers (when requested).
4. There is one extra blank page at the end for rough work.

Q1) Consider the following circuit (A-B-C-D) that consists of a voltage source V(t) and a resistor (R_1). The segment of wire connecting nodes A and B overlaps with a uniform magnetic field (B = 2.5 mT) pointing into the x-y plane. The B field is confined within the square area, where a = 30 cm.



a) Determine the resistance (R_1) needed for V(t) = 5 V to exert a force of 7.5 mN on the segment of wire that overlaps with the magnetic field (B). [3 marks]

$F = BIL$

$I = \frac{V}{R_1}$

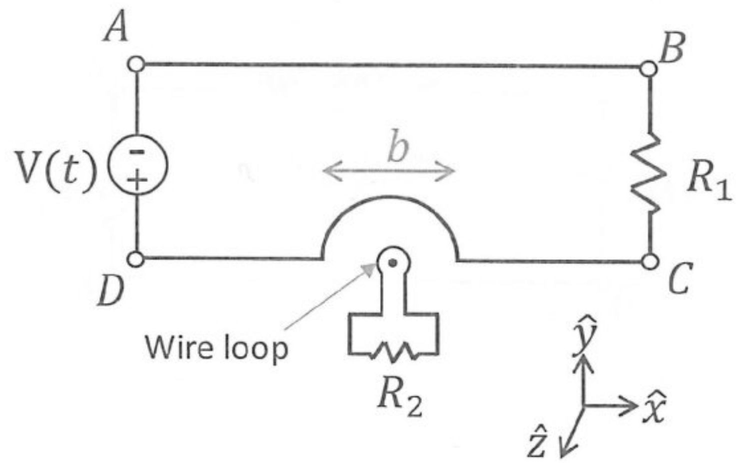
$\therefore\ 7.5\,mN = (2.5\,mT)\left(\frac{5\,V}{R_1}\right)(30\,cm)$

$\boxed{R_1 = 0{,}5\,\Omega}$

b) What is the direction in which the wire is displaced by the magnetic field when V(t) = 2V? [2 marks]

$\boxed{-\hat{y},\ \text{ie,}\ \downarrow}$

Q1 cont.) The circuit is modified such that a segment of the wire between nodes C and D forms a semi-circle in the x-y plane, with a diameter of b = 40 cm. There is a single wire loop (area = 0.01 m^2) placed at the center of the semi-circle and connected to a second resistor (R_2).



c) If V(t) = -5 V, what is the direction of the magnetic field at the center of the loop wire? [2 marks]

$+\hat{z}$, i.e. ⊙

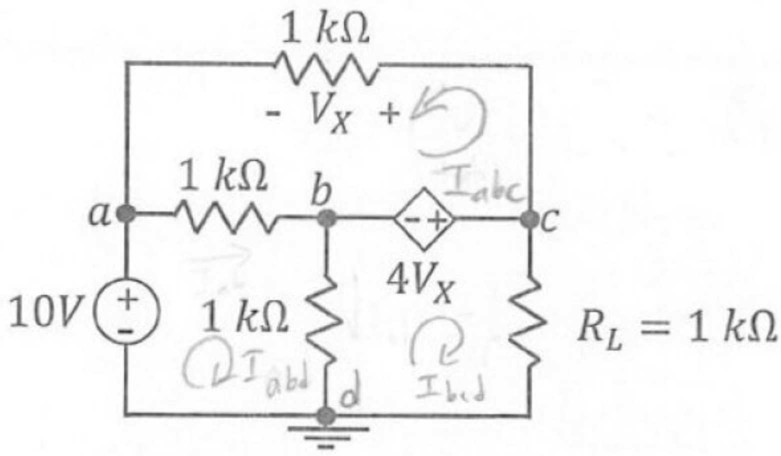
d) If V(t) increases from 0V to +5V over a period of 1 s, what is the direction of the electromotive force generated around the wire loop? [3 marks]

(i) clockwise direction

or

(ii) counter-clockwise direction

Q2) For the circuit shown below, V_x is given in volts.



a) Compute V_x. [5 marks]

Mesh abd:

$10 - (1k)(I_{abd} + I_{abc}) - (1k)(I_{abd} - I_{bcd}) = 0$

$2kI_{abd} + V_x - 1kI_{bcd} = 10$

Mesh bcd:

$4V_x - (1k)(I_{bcd}) - (1k)(I_{bcd} - I_{abd}) = 0$

$4V_x - 2kI_{bcd} + 1kI_{abd} = 0$

Mesh abc:

$4V_x - V_x - (1k)(I_{abc} + I_{abd}) = 0$

$2V_x - 1kI_{abd} = 0$

N.B. $I_{abc} = \frac{V_x}{1k}$

Solving,

$\boxed{V_x = 5\ V}$

$I_{abd} = 10\ mA$

$I_{bcd} = 15\ mA$

b) Compute the power in resistor R_L. Is it absorbed or supplied? [1 mark]

$P = I^2R = I_{bcd}^2 \cdot R_L$

$= (15\ mA) \cdot (1k\Omega)$

$\boxed{P_L = 225\ mW}$

$\boxed{\text{absorbed}}$

since resistors do not supply power

c) Compute the power in the independent voltage source. Is it absorbed or supplied? [2 marks]

$$P = IV = I_{abd} \cdot 10\,V$$
$$= (10\,mA)(10\,V)$$
$$\boxed{P = 100\,mW}$$

$\boxed{\text{supplied}}$, since current runs through it from the -ve to +ve terminals

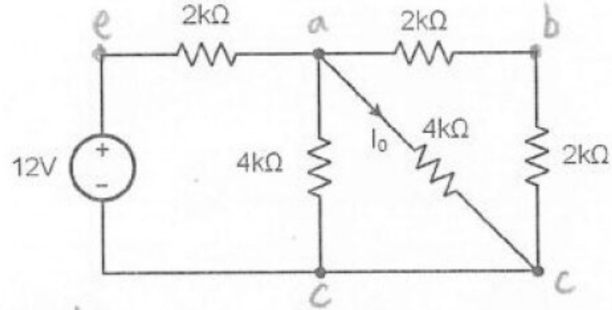
d) Compute the power in the dependent voltage source. Is it absorbed or supplied? [2 marks]

$$P = IV = (I_{abc} + I_{bcd})(4V_x)$$
$$= \left(\frac{5\,V}{1\,k\Omega} + 15\,mA\right)(4 \cdot 5\,V)$$
$$\boxed{P = 400\,mW}$$

$\boxed{\text{Supplied}}$ for similar reasons as (c)

Q3) Solve the following circuits:

a) Find I_0 in the network below using the homogeneity (scaling) property of linear systems. [5 marks]



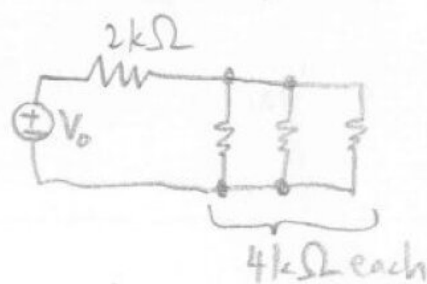
Let $\underline{I_0' = 1\ mA}$, and the voltage of the independent source be V_0.

By homogeneity, $\frac{12\ V}{V_0} = \frac{I_0}{I_0'} \Rightarrow I_0 = \frac{12\times10^{-3}}{V_0}$

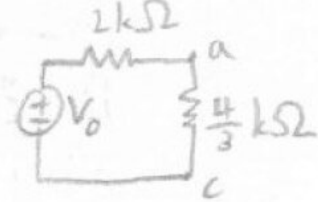
$V_{ac} = I_0' \cdot (4k\Omega)$

$= 4V$

Combine resistors;



$\Rightarrow$



Voltage divider:

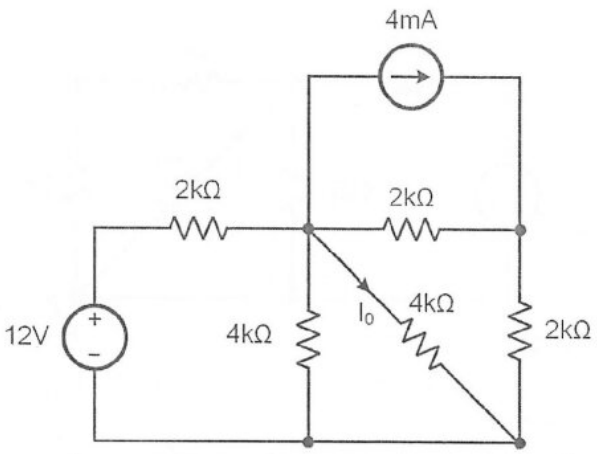
$V_{ac} = \frac{\frac{4}{3}k}{\frac{4}{3}k + 2k} V_0$

$V_0 = 10\ V$

$\rightarrow I_0 = \frac{12\times10^{-3}}{10}$

$\therefore \boxed{I_0 = 1.2\ mA}$

(b) Find I_0 in the network below using the superposition (additivity) property of linear systems. [5 marks]



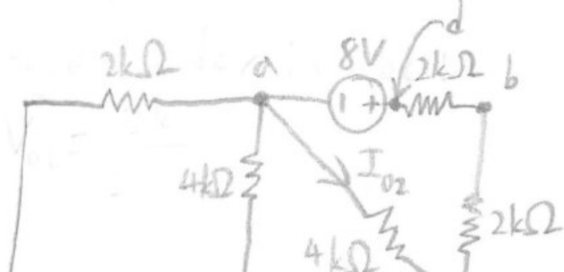
Set the current source to zero: it becomes an open circuit.

This is what we did in part (a)! $\therefore$ $\boxed{I_{01} = 1.2\,mA}$

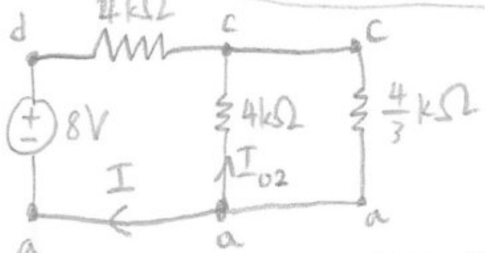
Set the voltage source to zero: it becomes a short circuit.

$\Rightarrow$

Source transformation:



Combine resistors:



$\Rightarrow$

$$\therefore I = \frac{8V}{5k\Omega} = 1.6\,mA$$

Current division:

$$-I_{02} = \frac{\frac{4}{3}k}{\frac{4}{3}k + 4k} I$$

(cont'd on next page)

For Extra work

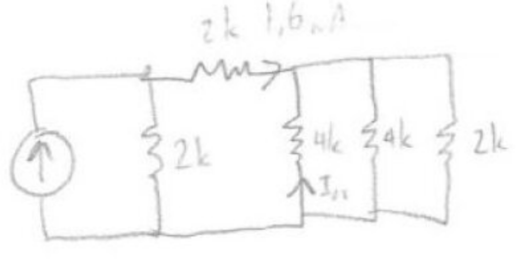
$\boxed{\text{(cont'd from Q3(b))}}$

$\boxed{I_{02} = -0.4\,mA}$

By the additive property of linear systems,

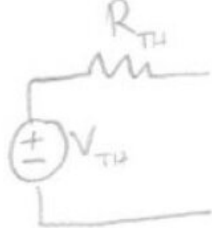
$I_0 = I_{01} + I_{02} = 1.2\,mA - 0.4\,mA$

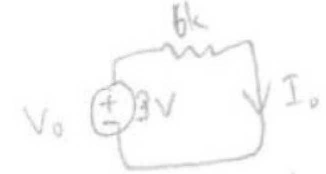
$\boxed{I_0 = 0.8\,mA}$



$V_{TH} =$

$I_{TH} = \frac{V_{TH}}{R_{TH}}$





$I_0 = \frac{3V}{6k} = 0.5mA$ $\quad I_0' = 1m$

$= \frac{3V}{V_0} \cdot I_0'$ $\quad V_0 = 6k \cdot I_n = 6V$