Faculty of Applied Science and Engineering
University of Toronto

MAT 186 H1S - CALCULUS I
WEDNESDAY, APRIL 26, 2017

FINAL EXAMINATION

FAMILY NAME: _____

GIVEN NAME: _____

STUDENT NUMBER: _____

SIGNATURE: _____

Time allowed: 2 hours, 30 minutes
Total marks: 80
No calculators allowed.
Examiner: S. Cohen
Use the backs of pages when necessary,
indicating clearly where solutions continue.

FOR MARKER'S USE ONLY	
QUESTION	MARK
1	/ 15
2	/ 10
3	/ 20
4	/ 12
5	/ 9
6	/ 14
TOTAL	/ 80

1. Some warm-up questions. Justify your answers fully.

a) Evaluate $\lim_{x\to 2} \frac{x^3+2x-12}{3x^2-4x-4}$.

[3 marks]

b) Evaluate $\frac{d}{dx}(4x^2 + 3x - 1)$ **using the limit definition** of the derivative.

[3 marks]

c) Evaluate $\frac{d}{dx}\int_{2x-1}^{x^2-4} 5t^2 - 1\, dt$. [Please don't do this the long way.]

[3 marks]

d) Evaluate $\lim_{x\to 0} \frac{\sin^2(5x)}{e^x - 1}$.

[3 marks]

e) Evaluate $\int_0^2 \mathrm{t} \cdot \cos\left(\pi(t^2 - 1)\right)\, dt$

[3 marks]

2. Tougher questions, now that you are ready:

a) Two cats were standing together when a sudden noise scared them away. The smaller cat ran at 2 km/hr and the larger one ran twice as fast. If the cats ran in straight lines, 45° apart, how fast was the distance between them increasing after two hours?

Note: I know cats won't run in a straight line, much less do anything other than sleep for two hours at a stretch, but we'll pretend these ones do.

[4 marks]

b) Find the equation of the tangent line to $y = (x^2 - 3)^{2x+1}$ at $x = 2$.

[6 marks]

3. Graph the function $f(x) = 3x + \frac{3}{2} + \frac{6}{2x+1}$. Organize your solution well and include all of the important values. You have this page and the next for this problem.

[20 marks]

[Continue Question 3 on this page.]

4. Using calculus (NOT linear algebra), find the point on the function $y = \sqrt{x}$ that is closest to:

   (a) $(2,10)$ (b) $(1,-1)$.

[12 marks]

5. A tall house has a fence 3m tall standing 2m away from it. If we set a ladder on the neighbouring lawn in order to reach the house (that is, the ladder goes from the yard, over the fence, and touches the house), what is the minimum length of ladder needed?

[9 marks]

6. (a) Let R be the region bounded between $y = x^2$ and $y = 2x + 3$. Find the area of R.

[4 marks]

(b) Find the volume of the solid created if R is rotated about $x = 4$.

[5 marks]

(c) The solid created in (b) cannot actually hold any liquid past the point where the line $y = 2x + 3$ creates a boundary. Therefore, there is only a donut-shaped tray that can be filled. How much work is required to fill it with liquid pumped up from the height of the $x -$ axis?

[5 marks]