UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 2005
First Year - CHE, CIV, IND, LME, MEC, MMS

MAT188H1F – LINEAR ALGEBRA

Exam Type: A

Examiners

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STUDENT NO. ______________________________

SIGNATURE ______________________________

Calculators Permitted: Casio 260, Sharp 520 or Texas Instrument 30

INSTRUCTIONS:

Attempt all questions.

Questions 1 through 6 are Multiple Choice; circle the single correct choice for each question. Each correct choice is worth 4 marks.

Question 7 consists of twelve statements which you must show are True or False; 2 marks each.

Questions 8 through 11 are long questions for which you must show your work. Each long question is worth 13 marks.

TOTAL MARKS: 100.

Use the backs of the pages if you need more space.

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1. If U is a subspace of $\mathbf{R}^5$ and $\dim U = 2$, then $\dim U^{\perp}$ is

(a) 2

(b) 3

(c) 4

(d) 5

2. $\dim\left(\text{span}\left\{\begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -8 \\ 0 \\ 6 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 4 \\ -2 \end{bmatrix}\right\}\right) =$

(a) 2

(b) 3

(c) 4

(d) 5

3. The best approximation $Z = \begin{bmatrix} x \\ y \end{bmatrix}$ to a solution of the inconsistent system of equations

$$\begin{cases} x & & & = & 14 \\ & & y & = & -14 \\ 2x & + & 3y & = & 0 \end{cases}$$

is

(a) $Z = \begin{bmatrix} 14 \\ -14 \end{bmatrix}$

(b) $Z = \begin{bmatrix} 11 \\ -16 \end{bmatrix}$

(c) $Z = \begin{bmatrix} -16 \\ 11 \end{bmatrix}$

(d) $Z = \begin{bmatrix} 16 \\ -11 \end{bmatrix}$

4. What is the matrix of the transformation which is composed of a reflection in the x-axis followed by a rotation through $\frac{\pi}{2}$?

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

5. The eigenspaces of the projection matrix $P_m = \frac{1}{1+m^2}\begin{bmatrix} 1 & m \\ m & m^2 \end{bmatrix}$ are

(a) $E_{-1}(P_m) = \text{span}\left\{\begin{bmatrix} 1 \\ m \end{bmatrix}\right\}$ and $E_1(P_m) = \text{span}\left\{\begin{bmatrix} m \\ 1 \end{bmatrix}\right\}$

(b) $E_0(P_m) = \text{span}\left\{\begin{bmatrix} 1 \\ m \end{bmatrix}\right\}$ and $E_1(P_m) = \text{span}\left\{\begin{bmatrix} m \\ 1 \end{bmatrix}\right\}$

(c) $E_0(P_m) = \text{span}\left\{\begin{bmatrix} -m \\ 1 \end{bmatrix}\right\}$ and $E_1(P_m) = \text{span}\left\{\begin{bmatrix} 1 \\ m \end{bmatrix}\right\}$

(d) $E_1(P_m) = \text{span}\left\{\begin{bmatrix} -m \\ 1 \end{bmatrix}\right\}$ and $E_0(P_m) = \text{span}\left\{\begin{bmatrix} 1 \\ m \end{bmatrix}\right\}$

6. The equation of the plane passing through the point $(x, y, z) = (1, 0, -1)$ and perpendicular to the line $[x\ y\ z]^T = [2\ 3\ 4]^T + t[2\ 1\ 3]^T$ is

(a) $2x + 3y + 4z = -2$

(b) $2x + y + 3z = 1$

(c) $2x + 3y + 4z = 2$

(d) $2x + y + 3z = -1$

7. Suppose A is an $n \times n$ matrix such that $A^2 = O$. Explain clearly and concisely why the following six statements about A are True.

(a) $\det(A) = 0$

(b) $(A^T)^2 = O$

(c) $(I - A)^{-1} = I + A$

(d) If the system $AX = B$ is consistent, then B is in null$A$, where X and B are $n \times 1$ matrices.

(e) The only eigenvalue of A is $\lambda = 0$.

(f) If A is diagonalizable then $A = O$

7. (continued) Suppose A is an $n \times n$ matrix such that $A^2 = O$. Explain clearly and concisely why the following six statements about A are False.

(g) $A = O$

(h) $\text{adj}(A) = O$

(i) A is invertible

(j) $\text{col}A = \text{null}A$

(k) $\text{im}A = \mathbf{R}^n$

(l) $\dim(E_0(A)) = n$

8. Given that

$$A = \begin{pmatrix} 1 & 0 & 1 & -1 & 1 \\ 2 & 0 & 3 & 1 & 1 \\ 1 & 0 & 0 & -4 & 2 \\ 0 & 0 & 1 & 4 & 1 \end{pmatrix} \text{ has reduced row-echelon form } R = \begin{pmatrix} 1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 1 & 0 & -7 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix};$$

state the rank of A, and then find a basis for each of the following: the row space of A, the column space of A, and the null space of A.

9. Let A be an $n \times n$ matrix; let $U = \left\{X \text{ in } \mathbf{R}^n \mid AX = A^T X\right\}$.

(a)[5 marks] Show that U is a subspace of $\mathbf{R}^n$.

(b)[8 marks] Let $A = \begin{bmatrix} 12 & 3 & 4 & 1 & 6 \\ 3 & 4 & 5 & 3 & 7 \\ 3 & 5 & 6 & 4 & 8 \\ 1 & 3 & 5 & 2 & 9 \\ 6 & 7 & 8 & 9 & 1 \end{bmatrix}$. Find a basis for U.

10. Let $U = \text{span}\left\{\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}^T, \begin{bmatrix} 1 & -1 & -1 & 1 \end{bmatrix}^T, \begin{bmatrix} 1 & 2 & -2 & 0 \end{bmatrix}^T\right\}$;
let $X = \begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}^T$. Find $\text{proj}_U(X)$ and $\text{proj}_{U^\perp}(X)$.

11. Find an orthogonal matrix P and a diagonal matrix D such that $D = P^T AP$, if

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$