UNIVERSITY OF TORONTO, FACULTY OF APPLIED SCIENCE AND ENGINEERING

MAT187H1S – Calculus II – Final Exam – Group Part - April 16, 2019

EXAMINERS: G. CHEN, S. COHEN, B. GALVÃO-SOUSA, P. MILGRAM, F. PARSCH, M. PUGH

Time allotted: 45 minutes No aids permitted

Total marks: 8

Name	**UTorID**	**email** @mail.utoronto.ca

Instructions (READ CAREFULLY)

- DO NOT WRITE ON THE QR CODE AT THE TOP OF THE PAGES.
- Read all the instructions carefully.
- This test contains 8 pages and a detached **formula sheet**. Make sure you have all of them.
- You can use pages 6–7 for rough work or to complete a question (**Mark clearly**).

  DO NOT DETACH ANY PAGES.
- No calculators, cellphones, or any other electronic devices are allowed. If you have a cellphone with you, it must be turned off and in a bag underneath your chair.

GOOD LUCK!

MULTIPLE-CHOICE PART. **(2 marks)**

ANSWER THESE QUESTIONS ON **PAGE 8**.

Only your answer on page 8 will be graded.

1. **(1 mark)** The power series $\sum_{n=0}^{\infty} a_n(x-4)^n$ has radius of convergence $R = e$. What is the radius of convergence of

$$\sum_{n=0}^{\infty} a_n \left(\frac{x-2}{e}\right)^n \quad ?$$

(A) $R = 0$

(B) $R = \dfrac{1}{e}$

(C) $R = 1$

(D) $R = e^2$

(E) $R = \infty$

2. **(1 mark)** From the following functions, select **ALL** that satisfy $f^{(2019)}(1) = 2019$.

(A) $f(x) = -(x-1)\cos(x-1)$

(B) $f(x) = (x-1)\cos(x-1)$

(C) $f(x) = \sin(x-1)$

(D) $f(x) = \displaystyle\sum_{\boldsymbol{n=1122}}^{\infty} \frac{(-1)^{n-1}}{(n-1)!}(x-1)^n$

(E) $f(x) = \displaystyle\sum_{\boldsymbol{n=2305}}^{\infty} \frac{(-1)^{n-1}}{(n-1)!}(x-1)^n$

LONG ANSWER PART

3. You work for a roller coaster construction company and Canada's Wonderland **(6 marks)**
is asking you to design a new ride. Your job is to design a specific part of the ride.

The trajectory should follow the curve given by

$$\vec{r}(t) = \left\langle \quad b\int_0^t \cos(u^2)\,du \quad , \quad ct \quad , \quad b\int_0^t \sin(u^2)\,du \quad \right\rangle \qquad \text{in metres,}$$

for $0 \leqslant t \leqslant 10$ seconds.

Consider parts (a) and (b) as independent.

(a) **(1 mark)** Riders should never exceed a speed of 50 m/s. What are the restrictions on b and c? Your answer should not include t. Justify your answer.

(b) **(1 mark)** Taking safety and durability into account, the curvature should not exceed $\frac{1}{10}$ m^{-1}. What are the restrictions on b and c? Your answer should not include t. Justify your answer.

Note. For this question, do not use the restrictions that you found in (a).

(c) **(1 mark)** Give values $b \neq 0$ and $c \neq 0$ that fulfil all the above design requirements. Your answer should not include t. Justify your answer.

(d) **(1 mark)** Your manager asked you to calculate $\vec{r}(1)$.

Approximate using Taylor polynomials of degree 4 centred at 0. Justify your answer.

$$\vec{r}(1) \approx \left\langle \quad\boxed{}\quad , \quad\boxed{}\quad , \quad\boxed{}\quad \right\rangle$$

(e) **(2 marks)** Using your approximation in (d), fill in the boxes below. Justify your answer.

$$\boxed{} \leqslant \int_0^1 \cos(u^2)\,du \leqslant \boxed{}$$

USE THIS PAGE TO CONTINUE OTHER QUESTIONS.

If you wish to have this page marked, make sure to refer to it in your original solution.

USE THIS PAGE TO CONTINUE OTHER QUESTIONS.

If you wish to have this page marked, make sure to refer to it in your original solution.