MAT197 - Calculus B - Winter 2014

Quiz - March 21, 2014

Time allotted: 50 minutes. Aids permitted: None.

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Last First

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Instructions

- Write only on the front pages (with QR code on top).
- ONLY THE **FRONT PAGES** WILL BE SCANNED. THE BACK PAGES WILL NOT BE SEEN BY THE GRADERS.
- DO NOT WRITE ON THE QR CODE AT THE TOP OF THE PAGES.

- Please have your **student card** ready for inspection, turn off all cellular phones, and read all the instructions carefully.
- DO NOT start the test until instructed to do so.
- This test contains 8 pages (including this title page). Make sure you have all of them.
- You can use the back of pages for rough work.
- You can use page 8 for rough work or to complete a question (**Mark clearly**). DO NOT DETACH PAGE 8.

GOOD LUCK!

PART I No explanation is necessary. **(6 marks)**

1. The sequence $\left\{\frac{\sin(n)}{n}\right\}$ is convergent / divergent . (circle the correct option)

2. The integral tests states that if $f(k) = a_k > 0$ where f is a continuous and decreasing function for $x \geqslant 1$, then

$$\sum_{k=1}^{\infty} a_k \text{ converges if and only if } \rule{8cm}{0.4pt}.$$

3. We can write $f(x) = \frac{1}{1+x} = \sum_{k=1}^{\infty} (-1)^k x^k$.

Then $f^{(99)}(0) = $ ____.

4. Consider the series $\sum_{k=1}^{\infty} (-1)^k \frac{k!\, e^k}{k^{2k}}$. Then

(a) The series converges absolutely.

(b) The series converges conditionally.

(c) The series diverges absolutely.

(d) The series diverges.

5. Which of the following is the 3[rd] Taylor polynomial (centered at 1) for the function $f(x) = e^x$:

(a) $1 + (x-1) + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$

(b) $e\left(1 + (x-1) + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}\right)$

(c) $1 + (x-1) + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{6}$

(d) $e\left(1 + (x-1) + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{6}\right)$

6. The series $\sum_{k=42}^{\infty} \frac{k^k x^{2k}}{(k-6)!}$ is convergent in $(-R, R)$. What is the largest possible value for $R > 0$?

(a) $R = \frac{1}{e^2}$

(b) $R = \frac{1}{e}$

(c) $R = \frac{1}{\sqrt{e}}$

(d) $R = \infty$.

PART II **Justify** your answers.

1. A lightbulb is flickering and the time it takes between flickers is $a_k = ke^{-k}$ in years **(7 Marks)** (when it stops flickering, it means that it burned out).

(a) Show that the lightbulb will not last forever.

(b) The lightbulb will last **at least** A years. Find an estimate for $A > 0$.

Continued...

(c) The lightbulb will last **at most** B years. Find an estimate for B.
(**Hint.** Remember the integral test)

2. **Justify** your answers. **(7 Marks)**

(a) Find a formula for the k^{th} derivative of $\ln(1+x)$.

(b) Use part (a) to find the Maclaurin series for $\ln(1+x)$ (starting at $k=1$).

Continued...

(c) Use the geometric series formula to find the Maclaurin series for $\ln(1+x)$ (starting at $k=0$).

(d) In light of the fact that Taylor series are unique, why doesn't part (b) contradict part (c)?

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USE THIS PAGE TO CONTINUE OTHER QUESTIONS.

The end.