MAT197 - Calculus B - Winter 2014

Term Test 2 - February 27, 2014

Time allotted: 90 minutes. Aids permitted: None.

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Last First

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Instructions

- Write only on the front pages (with QR code on top).
- ONLY THE **FRONT PAGES** WILL BE SCANNED. THE BACK PAGES WILL NOT BE SEEN BY THE GRADERS.
- DO NOT WRITE ON THE QR CODE AT THE TOP OF THE PAGES.

- Please have your **student card** ready for inspection, turn off all cellular phones, and read all the instructions carefully.
- DO NOT start the test until instructed to do so.
- This test contains 11 pages (including this title page). Make sure you have all of them.
- You can use the back of pages for rough work.

GOOD LUCK!

PART I No explanation is necessary.

For questions 1–4, please fill in the blanks. **(4 marks)**

1. What is the best substitution to calculate $\displaystyle\int \frac{1}{\sqrt{17+16x+4x^2}}\,dx$?

$u =$ ____

2. What are **two** good substitutions to calculate $\displaystyle\int \frac{x+2}{\sqrt{5+4x+x^2}}\,dx$?

$u =$ ____________________ or $u =$ ____________________

3. Using the Trapezoid Rule with $n = 10$ we obtain an error of 1 when computing the integral $\displaystyle\int_a^b f(x)\,dx$. What is the smallest value for n to obtain an error of at most 10^{-6}?

$n =$ ____

4. During Cinco de Mayo, you shoot a bullet straight up into the sky at the speed of 500 m/s. The altitude of the bullet $y(t)$ at time t seconds after being shot satisfies the differential equation $y'' = -g$. What is the velocity of the bullet when it hits the ground?

The velocity is ____

For questions 5-8, please choose the correct answer. **(4 marks)**

5. Which terms appear in the partial fraction decomposition of $\dfrac{(x-2)^2(x-1)}{(x+1)(x^2+3)^2(x-\pi)^2}$?

(Select **all** that apply)

(a) $\dfrac{A}{x-1}$ **(d)** $\dfrac{D}{x+1}$ **(g)** $\dfrac{Gx+H}{x^2+3}$

(b) $\dfrac{B}{x-2}$ **(e)** $\dfrac{E}{x-\pi}$ **(h)** $\dfrac{Ix+J}{(x^2+3)^2}$

(c) $\dfrac{C}{(x-2)^2}$ **(f)** $\dfrac{F}{(x-\pi)^2}$ **(i)** $Kx+L$

6. Which of the following integrals is convergent?

(a) $\displaystyle\int_{-\infty}^{\infty} x\,dx$ **(c)** $\displaystyle\int_0^1 \frac{1}{x}\,dx$

(b) $\displaystyle\int_1^{\infty} \frac{1}{x}\,dx$ **(d)** $\displaystyle\int_1^{\infty} \frac{1}{\sqrt[5]{x^4}}\,dx$

7. Which of the following integrals is divergent?

(a) $\displaystyle\int_{-\infty}^{-1} \frac{1}{\sqrt{x^4}}\,dx$ **(c)** $\displaystyle\int_0^{\infty} 0\,dx$

(b) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$ **(d)** $\displaystyle\int_0^1 \frac{1}{x^{1+p^2}}\,dx$

8. Which of the following substitutions is best to evaluate $\displaystyle\int \cos^7 x\ \sin^4 x\,dx$?

(a) $u=\sin x$ **(c)** $u=\cos x$

(b) $u=\tan x$ **(d)** $u=\sec x$

PART II **Justify** your answers.

1. Evaluate **(6 Marks)**

$$\int_0^{\frac{3}{5}} \frac{x^2}{\sqrt{9-25x^2}}\,dx.$$

2. Evaluate the integral **(8 Marks)**

$$\int \frac{x^4 + 2x^3 + 9x^2 + 8x + 16}{x(x^2+4)^2}\,dx.$$

 Continued...

3. You are working for a biologist who is studying a new kind of virus that reproduces extremely quickly, with the population following the function **(6 Marks)**

$$p(t) = \int_0^t e^{x^2}\, dx,$$

where t is in days. The size of each virus is $\frac{1}{10}\ \mu m^2$.

The biologist wants to make sure he has enough space for this virus to grow for 4 days.

(a) Using the Midpoint rule with $n = 4$, give an approximation of the necessary area.

(b) Recall the formula of the error E_M we obtain when approximating an integral using the Midpoint Rule:

$$|E_M| \leq \max_{x\in[a,b]} \left|f''(x)\right| \frac{b-a}{24}(\Delta x)^2.$$

What is the smallest value of n that we can use to ensure that the error we are making is at most 1?

4. Gravitational force decreases with the square of the distance, in particular we can write **(6 Marks)**

$$F = \frac{mMG}{y^2},$$

where y is the altitude of the object from the center of a planet of radius R and mass M, m is mass of the object, and G is the gravitational constant.

To find the escape velocity $\boldsymbol{v_e}$, we know that the total kinetic energy of the object is $\frac{1}{2}\, m\, v_e^2$, and that it equals the work necessary to take the object from the surface of the planet ($y = R$) to infinity ($y = \infty$).

(a) Find the escape velocity (in terms of M, G, and R).

(b) Assuming that c is the speed of light, what is the minimum radius R for a black-hole of mass M? (in terms of G)

(**Hint.** A black-hole is a celestial body where the escape velocity is at least the speed of light)

Continued...

5. A bullet of mass m travelling upwards with initial velocity v_0 is slowed by the force of gravity mg and by air resistance kv^2, where k is a positive constant. As the bullet moves upward, its velocity v satisfies **(6 Marks)**

$$m\frac{dv}{dt} = -(kv^2 + mg).$$

(a) Find a formula for $v(t)$.

$\left(\textbf{Hint.}\ \ \text{For simplicity, write } a = \sqrt{\frac{mg}{k}} \text{ and don't forget the initial condition.}\right)$

(b) Assuming $m = k$, and $v_0 = 100\sqrt{g}$, find the maximum height the bullet will reach in the form of an integral. Don't solve the integral.

The end.