University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, APRIL 2005
First Year - CIV, CHE, IND, LME, MEC, MMS

MAT 187H1S - CALCULUS II

Exam Type: A

Examiners

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SURNAME ______________________________

GIVEN NAME ______________________________

STUDENT NO. ______________________________

SIGNATURE ______________________________

INSTRUCTIONS:

Non-programmable calculators permitted.

No other aids permitted.

Answer all questions. Make sure this exam contains 11 pages.

Present your solutions in the space provided; use the back of the **same** page if more space is required.

TOTAL MARKS: 100.
Each question is worth 10 marks.

The value for each part of a question is shown in parentheses after the question number.

MARKER'S REPORT	
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
Q8	
Q9	
Q10	
TOTAL	

1.(a) (5 marks) Find the length of the curve with parametric equations

$$x = \sin(4t); y = \cos(4t); z = 2t^{3/2}, \text{ for } 0 \leq t \leq 1.$$

1(b) (5 marks) Find the first three non-zero terms in the Maclaurin series of $f(x) = \dfrac{3}{5 - 2x^2}$.

2. For a certain vibrating mass-spring system, the displacement, $x(t)$, of the spring from equilibrium at time t, satisfies the differential equation

$$2x''(t) + 12x'(t) + 50x(t) = 0.$$

Solve for $x(t)$ if $x(0) = 2$ and $x'(0) = 6$; then plot a graph of $x(t)$ for $t \geq 0$, indicating the pseudoperiod and the time-varying amplitude.

3.(a) (5 marks) Approximate $\int_0^{1/2} x^2\sqrt{1+x^4}\,dx$ correct to within 10^{-4}, and explain why your approximation *is* correct to within 10^{-4}.

3.(b) (5 marks) Find the interval of convergence of the power series $\sum_{n=2}^{\infty} \frac{1}{\ln n}\frac{x^n}{3^n}$.

4. Do the following infinite series converge or diverge? Justify your answer.

(a) (3 marks) $\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n!}$

(b) (3 marks) $\sum_{n=0}^{\infty} \frac{1}{(1+n^2)^{3/2}}$

(c) (4 marks) $\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n^2}\right)$

5. Find and classify all the critical points of

$$f(x, y) = 2x^2y + 4xy^2 + xy.$$

6. An archer fires an arrow from the top of a 20-m high castle wall. At what angle to the horizontal should the arrow be aimed if it leaves the bow with a speed of 50 m/sec and it is to hit a target located 150 m from the base of the wall? (The acceleration due to gravity is 9.8 m/sec^2.)

7. Newton's Law of Cooling states that

$$\frac{dT}{dt} = k(T - A),$$

where T is the temperature of a body placed in a surrounding medium of constant temperture A, t is time, and k is a constant.

A body is placed in a room with constant air temperature 70F. At noon the temperature of the body is 80F, and at 1 PM its temperature is 75F. At what time was the temperature of the body 110F?

8. Consider the curve with parametric equations

$$x = e^{-2t}; y = 4 - t^2, \text{ for } -2 \le t \le 2.$$

Plot the curve, and find the area of the region between the curve and the x-axis.

9. Consider the curve with polar equation $r = \cos\theta + \sin\theta$.

(a) (5 marks) Plot the curve and find the area of the region enclosed within it.

(b) (5 marks) Find the length (or perimeter) of the curve.

10. The human body metabolizes the drug Zilinium (i.e. removes it from the bloodstream) so that the rate at which the amount of Zilinium in the bloodstream decays is equal to five percent of the amount of Zilinium not yet decayed, where time is measured in hours.

(a) (3 marks) Suppose that 1 gram of Zilinium enters the bloodstream at 9 AM on Monday morning. Show that, at 9 AM on Tuesday morning approximately 0.301 grams remain.

(b) (3 marks) Suppose that 1 gram of Zilinium enters the bloodstream at 9 AM on Monday morning, and a further 1 gram enters the bloodstream at 9 AM on Tuesday morning. Write down an expression for the amount of Zilinium in the bloodstream at 9 AM on Wednesday morning.

(c) (4 marks) Starting at 9 AM on Monday morning, and repeating daily at 9 AM for each day in the forseeable future, a patient must receive an injection of one gram of Zilinium. Using an infinite geometric series, determine how much Zilinium remains in the patient's bloodstream in the long run.