FACULTY OF APPLIED SCIENCE AND ENGINEERING

University of Toronto

FINAL EXAM, MONDAY, APRIL 28, 2008

MAT 188S
Linear Algebra

Examiner: S. Cohen
Duration: 2 hours, 30 minutes

Calculators allowed – Casio 260, Sharp 520, or TI 30. **Total: 80 marks**

Family Name: ______________________________

Given Name(s): ______________________________

Please sign here: ______________________________

Student ID number: ______________________________

For Markers Only

Question	Marks
1	/ 10
2	/ 7
3	/ 4
4	/ 8
5	/ 8
6	/ 7
7	/ 7
8	/ 10
9	/ 12
10	/ 7
TOTAL	**/ 80**

1. [10 marks] Let $A = \begin{bmatrix} 1 & -1 & 5 & -2 & 2 \\ 2 & -2 & -2 & 5 & 1 \\ 0 & 0 & -12 & 9 & -3 \\ -1 & 1 & 7 & -7 & 1 \end{bmatrix}$. Determine the rank of A and find bases for col A and null A.

2. [7 marks] Find an orthogonal basis for the subspace $U = \text{span}\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}$.

3. [4 marks] Find all values of k such that $\left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ k \\ 1 \end{pmatrix} \right\}$ spans $\mathbb{R}^3$.

4. a. [4 marks] If $A^3 + A^2 - A - 2I = 0$, show that A is invertible and find A^{-1}.

b. [4 marks] Show that if $A^2 = 0$, then $\operatorname{col} A \subseteq \operatorname{null} A$.

5. [8 marks] Let $A = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$. If B is a 2×2 matrix such that $(BA)^{-1} = A^T$, find B.

6. [7 marks] Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -3 & 0 \\ 1 & 0 & a^2 & a+1 \end{bmatrix}$. Find all values of a, if any exist, for which $\dim(\text{null}\, A) = \dim(\text{col}\, A)$.

7. [7 marks] Find the vector in $\operatorname{span}\left\{\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}\right\}$ that is closest to $\begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$.

8. [10 marks] The augmented matrix of a system $AX = B$ is $\left[\begin{array}{ccc|c} 1 & 1 & 3 & a \\ a & 1 & 5 & 4 \\ 1 & a & 4 & a \end{array}\right]$. For what values of a does the system have infinitely many solutions, one solution,no solutions?

9. Consider the subset $W = \{(a_1, a_2, a_3, a_4) \mid a_1 = a_2, a_3 = a_4 + 2a_2\}$ of $\mathbb{R}^4$.

   a. [5 marks] Find a basis for W.

   b. [7 marks] Extend your answer from (a) to a basis of $\mathbb{R}^4$.

10. [7 marks] If A and B are invertible matrices that commute with each other (i.e., $AB = BA$), show that their inverses also commute.