MIE100S Dynamics: Winter 2014

Final Exam – April 25, 2014

9:30 a.m. – noon

General Instructions:

- Answer all questions in the exam booklets provided.
- Print your full ROSI name and student # on each booklet.
- All rough work must be *neatly* shown to earn credit for each question.
- You must use a pen or *dark* pencil.
- Answer all five questions. Each question is worth 20%.
- Total marks on this exam = 100.
- Use the given coordinate system and sign conventions in each question.

Number of Pages:

- 6 (including cover page)

This is a Type D examination. Permitted Aids:

- Non-communicating/non-programmable calculator: Casio FX-991MS or Sharp EL-520X
- One 8 ½" x 11" aid sheet, any colour. You may write on both sides of the sheet.

Moment of Inertia:

Ring: $I_G = mR^2$

Uniform Disk: $I_G = \frac{1}{2}mR^2$

Long slender rod: $I_G = \frac{1}{12}mL^2$

1. A person standing firmly at point P pulls on a rope that is attached to ball A of mass m_A = 20 kg. The ball travels along a curvilinear path parallel to the ground, with initial radius r_0 = 10 m, initial $\theta_0 = 0$, and initial angular rate of rotation $\dot{\theta}_0$ = 0.15 rad/s. Motion is in the horizontal plane, and friction is negligible. Assume that both A and B have extremely small size.

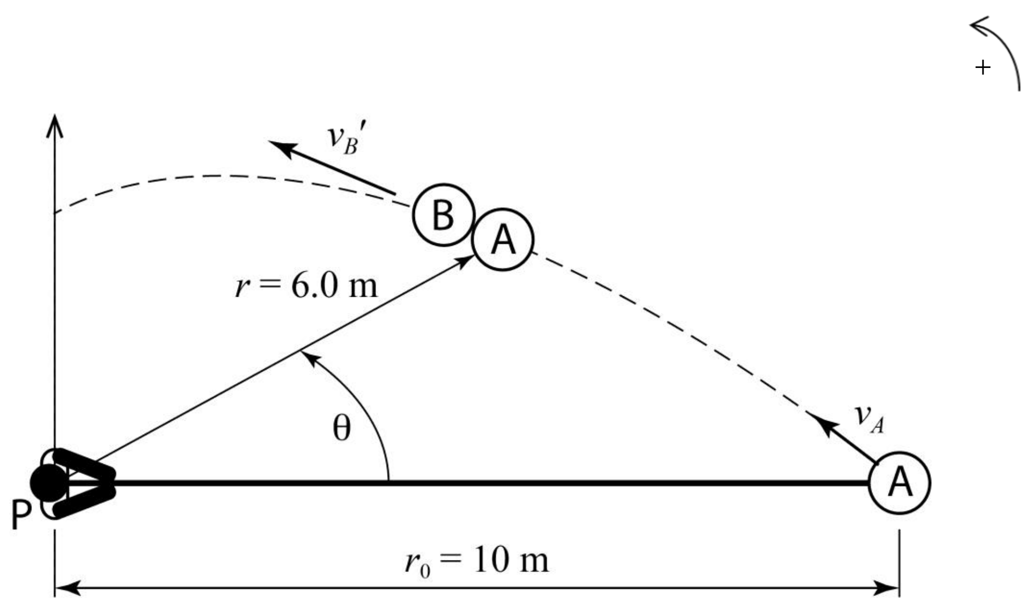
The rope from P to A is pulled inward at a constant speed $\dot{r} = -2.0$ m/s. At r = 6.0 m, ball A collides with ball B (m_B = 25 kg), which is initially at rest.

(a) Find the initial angular momentum of ball A about point P, <u>at r = 10 m</u>.

(b) Determine the angular acceleration a_θ and the angular velocity $\dot{\theta}$ of ball A <u>at r = 8.0 m</u>.

(c) Determine the magnitude of the tension in the rope PA when r = 7.0 m, if the angular velocity is $\dot{\theta}$ = 0.31 rad/s at that time.

(d) If A stops completely after hitting B <u>at r = 6.0 m</u>, what is the speed of B (v_B') after the collision?

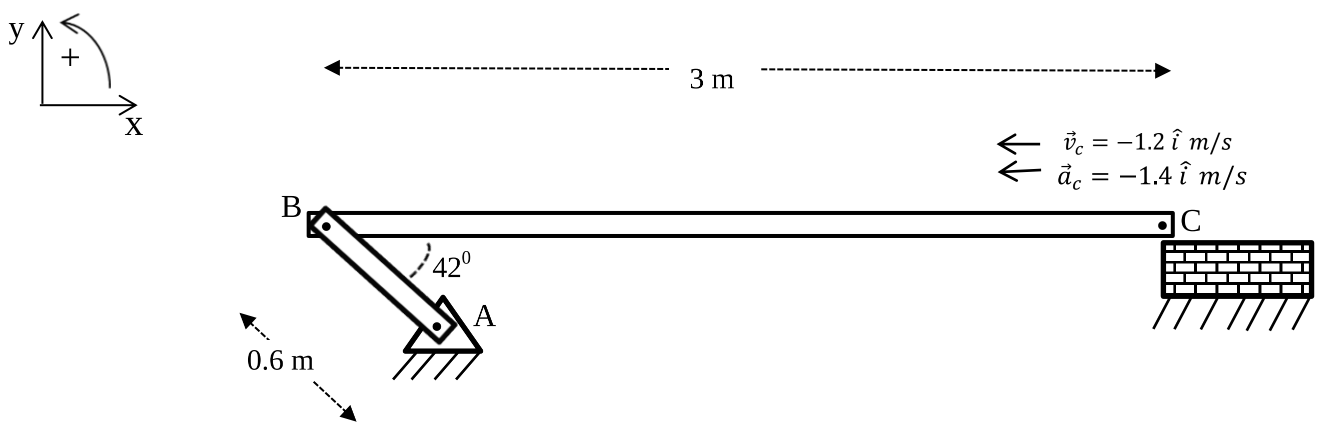


(Note: All of the motion takes place in a horizontal plane as shown in the diagram.)

2. Point C has a velocity of -1.2 $\hat{\imath}$ *m/s*, and an acceleration of -1.4 $\hat{\imath}$ *m/s*^{2} at the instant shown in the diagram. Bar BC is pinned to bar AB at point B, and bar AB is pinned to the ground at point A, as shown in the diagram. One end of bar BC is sliding on a brick wall as shown in the diagram. Use the given coordinate system and sign conventions.

(a) Find ω_{AB} and ω_{BC} at the instant shown in the diagram.

(b) Find α_{AB} and α_{BC} at the instant shown in the diagram.



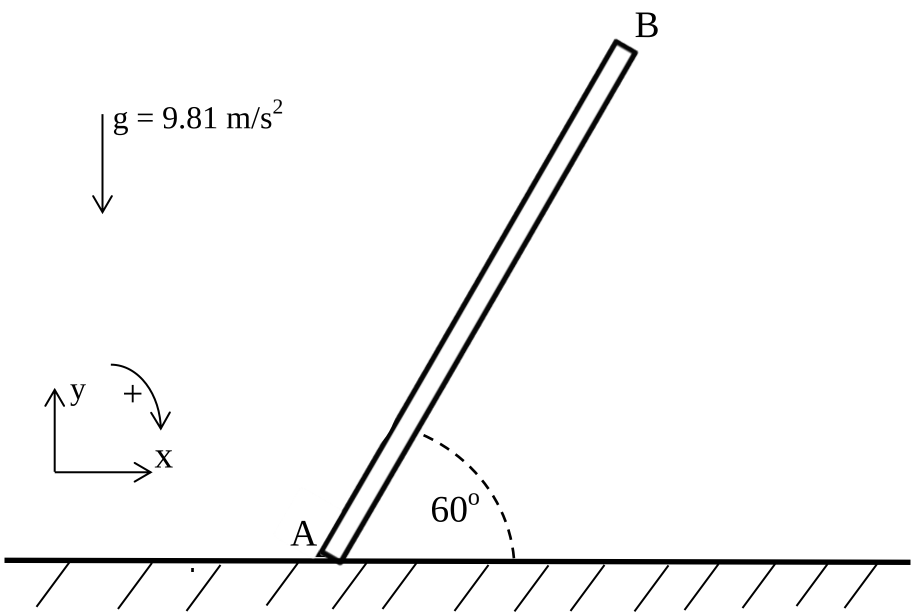
3. A long slender uniform rod AB has a mass 2.0 kg and length 1.5 m. The rod is released from rest on a frictionless surface in the position shown. Use the given coordinate system and sign conventions.

(a) Draw a free body diagram of the rod just after its release.

(b) Find the relationship between the acceleration of the center of mass $\overrightarrow{a_G}$ of the rod and the angular acceleration α of the rod.

(c) Find the force exerted by the floor on the rod just after its release.

(d) Find the acceleration of point A immediately after the release of the bar.

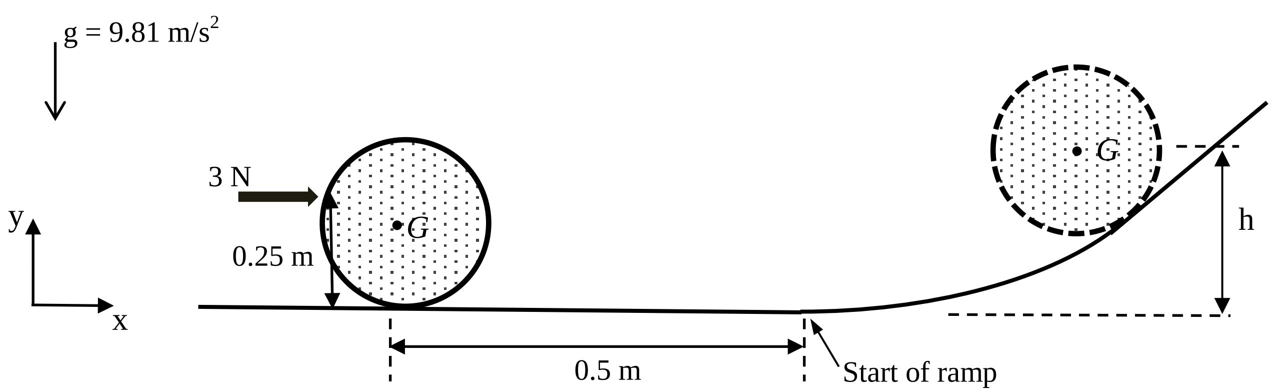


4. A sphere with mass m = 0.1 kg, radius R = 0.2 metres, and radius of gyration k_G = 0.1264 metres sits at rest on the ground. The coefficients of friction between the sphere and the ground are μ_S = 0.4 and μ_K = 0.2. Neglect air friction.

   The sphere is struck with a constant impact force of 3 N at a distance 0.25 metres above the ground as shown in the figure. The impact lasts 0.05 seconds. The sphere then rolls horizontally on the ground for 0.5 m, and then up a ramp, as shown.

   Use the given coordinate system and sign conventions.

a) Prove that the wheel will roll without slip during the impact by the 3 N force.
b) Find the velocity of the sphere's centre of mass immediately after impact.
c) How much work is done by friction on the sphere when it rolls horizontally on the ground for 0.5 m?
d) What is h, the maximum height that the sphere's centre of mass reaches, measured from the ground as shown?



(Figure not to scale)

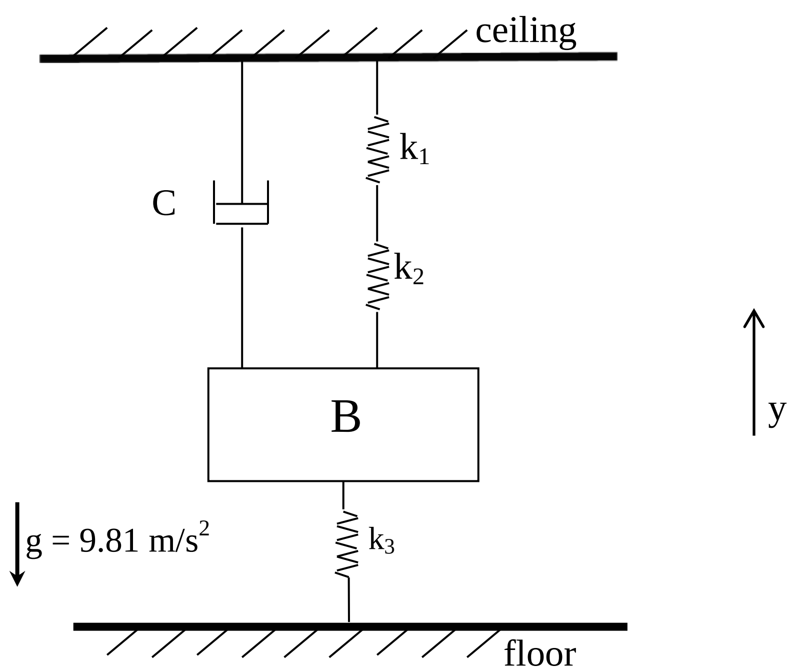
5. The block B has a mass of 4 kg, and is suspended between the floor and ceiling as shown in the diagram. The position of the block is $y = 0$ when the block is at rest.
$k_1 = 10$ N/m; $k_2 = 20$ N/m; $k_3 = 30$ N/m. Ignore any rotation of the block.

(a) If $C = 14$ N·s/m, find the damped frequency of vibration ω_d of block B.

(b) Let $C = 14$ N·s/m, and the amplitude of oscillation be equal to 0.75 m at time $t = 0$. Find the amplitude of oscillation at $t = 2.5$ seconds.

(c) A force $\vec{F} = 12\,\hat{\jmath}\sin(5t)$ Newtons is applied to block B, where t is time measured in seconds. Find the amplitude of oscillation of block B, **if C = 0 N·s/m**.

(d) For part (c), find the frequency of oscillation in cycles per second.



Solution Q1 :

Given $m_A = 20\ kg$ $\quad r_0 = 10\ m$ $\quad \dot{\theta}_0 = 0.15\ rad/s$

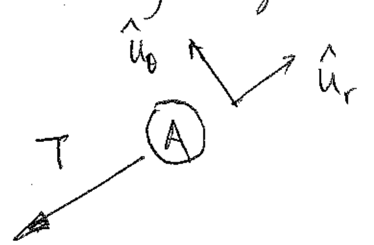
$m_B = 25\ kg$ $\quad \dot{r} = -2\ m/s$

(a) Angular momentum $H_o = \vec{r} \times m\vec{v}$

$$H_o = (10\ m)(20\ kg)(0.15\ rad/s \times 10\ m)$$

$$\boxed{H_o = 300\ kg \cdot m^2/s}$$

(b) Free Body Diagram



$$\Sigma F_r = m a_r = -T \quad (1)$$

$$\Sigma F_\theta = m a_\theta = 0 \quad (2)$$

$\boxed{a_\theta = 0}$ from Eq. (2)

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 = \frac{d}{dt}(r^2\dot{\theta})$$

Thus $r^2\dot{\theta} = \text{constant}$

$$r_0^2\dot{\theta}_0 = (10\ m)^2(0.15) = \underline{15}$$

At $r = 8.0\ m$, $\dot{\theta} = \frac{15}{r^2} = \frac{15}{8^2} = \boxed{0.234\ rad/s}$

Alternatively, angular momentum is conserved

$$(r m v_\theta)_i = (r m v_\theta)_f$$

$$\omega_f = \dot{\theta} = \frac{300 \ kg \cdot m^2/s}{(20 \ kg)(8 \ m)^2} = \boxed{0.234 \ rad/s}$$

(c) $\Sigma F_r = \boxed{-T = ma_r}$ from Eq. (1)

$$T = -m(\cancel{\ddot{r}}_0 - r\dot{\theta}^2)$$

$$T = -20 \ kg \ (0 - 7(0.31)^2)$$

$$\boxed{T = -13.454 \ N}$$

(d) Conservation of momentum.

$$m_A v_A + \cancel{m_B v_B} = \cancel{m_A v_A'} + m_B v_B'$$

$$v_B' = \frac{m_A v_A}{m_B} = \underline{0.8 \ v_A}.$$

$$v_A = \sqrt{v_\theta^2 + v_r^2}$$

$$v_r = \dot{r} = -2.0 \ m/s$$

$$v_\theta = r\dot{\theta} = \frac{15}{6} = 2.5 \ m/s$$

$$v_A = \sqrt{2^2 + 2.5^2} = 3.20 \ m/s$$

$$v_B' = 0.8 \ (3.20 \ m/s) = \boxed{2.56 \ m/s}$$

Marking scheme :

(a) $\boxed{H_0 = 300 \ kg \cdot m^2/s \quad \text{or} \quad 300 \ N \cdot m \cdot s}$ $\boxed{5 \text{ marks}}$

(3 marks) Any form of $H = \vec{r} \times m\vec{v}$, $d\,mv$, $rmv \sin \theta$

(2 marks) Correct choice of velocity component $v_\theta = \omega r$.

Common mistakes

(i) wrong units (− 0.5 mark if value correct, no extra penalty if other mistakes found)

(ii) no units (− 1 mark)

(iii) used total v , led to 500 $kg \cdot m^2/s$.
(− 2 marks)

(iv) only calculation error , or missing mass (− 1 mark)

(b) $\boxed{a_\theta = 0}$ (3 marks) $\boxed{5 \text{ marks}}$

$\boxed{\dot{\theta} = 0.234 \text{ or } \frac{15}{64} \ rad/s}$ (2 marks)

(2 marks out of 3.) for $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$ (very generous)

(1 mark out of 2) if tried to use conservation of angular momentum

(c) $\boxed{T = -13.454 \text{ N}}$ $\boxed{5 \text{ marks}}$

Common mistakes :

(i) included gravity (−2 marks)

(ii) calc. error on a_r, if work shown (−1 mark)

(iii) calc. error, no work shown on a_r (−2 marks)

I generally gave part marks for :

(i) F. B. D. (1 mark)

(ii) $\Sigma F_r = m a_r$ (1 mark)

(iii) $a_r = \ddot{r} - r\dot{\theta}^2$ (1 mark)

(d) $\boxed{V_B' = 2.56 \text{ m/s}}$ $\boxed{5 \text{ marks}}$

Conservation of momentum (2 marks)

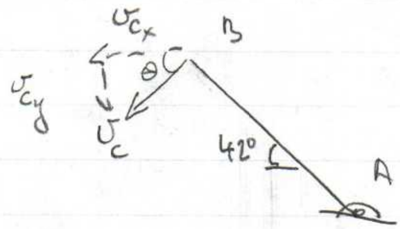
Conservation of energy (0 marks)

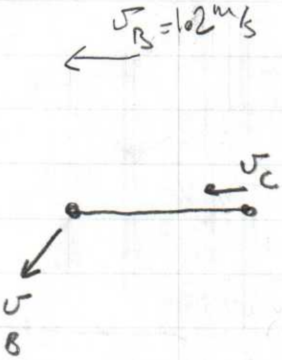
Conservation of momentum + $V_B' = 0.8\, V_A$ (3 marks)

correctly use $V_A = \sqrt{V_\theta^2 + V_r^2}$ (+1 mark)

correct value (+1 mark)

Angular Velocity





$$v_B = v_C + v_{B/C}$$

$$v_B \vec{i} = v_{C_x}\vec{i} + v_{C_y}\vec{j} + v_{B/C}\vec{j}$$

$$v_B = v_{C_x}$$

$$-v_{C_y} = +v_{B/C}$$

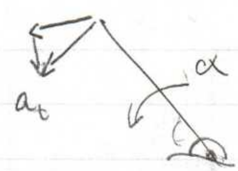
$$v_B = \omega_{AB}\, l_{AB} \sin\theta$$

$$\boxed{\frac{v_B}{l_{AB}\sin\theta} = \omega_{AB}}$$

$$\boxed{\omega_{BC} = -\frac{l_{AB}\cos\theta\, \omega_{AB}}{l_{BC}}}$$

$\underline{\omega_{AB} = 2.99 \frac{RAD}{sec}}$ $\quad$ $\underline{\omega_{BC} = 0.44\ RAD/sec}$

Angular Acceleration



$$a_B = a_C + a_{B/C} = a_C + a_{B/C_t} + a_{B/C_n}$$

$$a_{B_t} = \alpha\, r_{AB}$$

$$a_{B_t} = -l_{AB}\sin\theta\, \alpha\, \vec{i}$$

$$-l_{AB}\cos\theta\, \alpha\, \vec{j}$$

$$a_{B_n} = -\omega_{AB}^2 r_{AB}$$

$$a_{B_n} = \omega_{AB}^2 l_{AB}\cos\theta\, \vec{i} + \omega_{AB}^2 l_{AB}\sin\theta\, \vec{j}$$

$a_C = -1.2\ m/s^2$

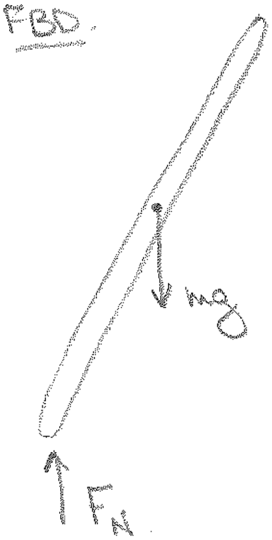
$$a_{B/C} = -l_{BC}\,\alpha_{BC}\,\vec{j} - \omega_{BC}^2\, l_{BC}\,\vec{i}$$

Summing up components

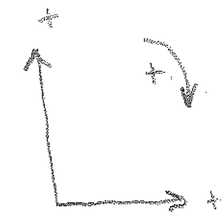
$$\boxed{\alpha_{AB} = 6.933\ \frac{RAD}{s^2}\quad,\quad \alpha_{BC} = 2.1\ RAD/s^2}$$

question 3: MIE final 2014

(a) FBD.

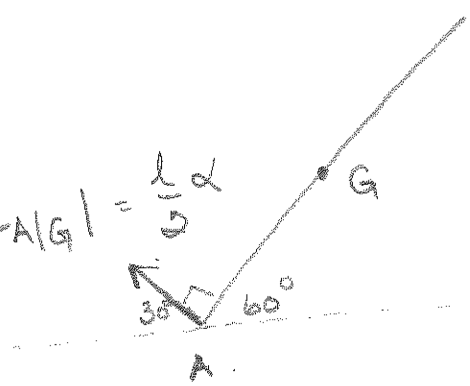


5 marks.



(b) Kinematics:

$|a_{A|G}| = \frac{l}{2}\alpha$



since $\omega = 0$ $\vec{a}_{A|G} = (\vec{a}_{A|G})_t$ only

$\vec{a}_A = a_A \hat{i}$

+ since $\sum \vec{F}_x = \vec{0}$

$\vec{a}_G = a_G \hat{j}$

$\Rightarrow \vec{a}_A = \vec{a}_G + \vec{a}_{A|G} \Rightarrow$

$$a_A \hat{i} = a_G \hat{j} - \frac{l}{2}\alpha \cos 30^\circ \hat{i} + \frac{l}{2}\alpha \sin 30^\circ \hat{j}$$

$\Rightarrow$ to find relationship between a_G and α use the j component:

$$0 = a_G + \frac{l}{2}\alpha \sin 30^\circ$$

$$\Rightarrow \quad a_G = -.375\,\alpha \qquad \text{5 marks.}$$

(c) <u>kinetics</u> ($\Sigma F_x = 0$ does not enter into it)

$$\Sigma F_y = m a_{Gy}$$
$$\Sigma M_G = I_G \alpha \qquad I_G = \frac{1}{12} m l^2 = \frac{1}{12}(2)(1.5)^2 = .375\,\text{kgm}^2$$

$$F_N - mg = m(-.375\,\alpha)$$
$$F_N(.75\cos 60^\circ) = .375\,\alpha \quad \Rightarrow \quad F_N = \alpha \quad \text{substitute}$$

$$F_N = 2(-.375 F_N) + 2(9.81)$$
$$1.75 F_N = 19.6 \quad \Rightarrow \quad F_N = 11.2 \text{ Newtons} \qquad \text{5 marks}$$

$$(\alpha = 11.2\ s^{-2})$$

(d) to find $\vec{a}_A$ use the $\hat{\imath}$ component from (b).

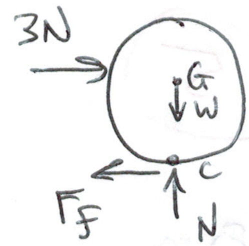
$$a_A = -\frac{l}{2}\alpha\cos 30^\circ$$

$$= -.75(11.2)\cos 30^\circ = -7.27$$

$$\Rightarrow \vec{a}_A = -7.27\,\hat{\imath}\ \ m/s^2$$

5 marks.

Q4 SOLUTION

a) 

For no slip, $F_f < F_{s_{max}}$

$$F_{s_{max}} = \mu_s N = \mu_s mg \quad (\because \Sigma F_y = 0)$$

$$= (0.4)(0.1)(9.81)$$

$$= \underline{0.39\,N}$$

To get F_f:

Kinetics $\quad \Sigma F_x = m a_{Gx} \qquad \Sigma M_G = I_G \alpha$

$3 - F_f = m a_G \qquad 3(0.05) + F_f(0.2) = (0.1)(0.1264)^2 \alpha$

$0.2 F_f - 1.598 \times 10^{-3} \alpha = -0.15$ ①

typically gave ~4 marks if general approach was appropriate, but errors if F_f, calculations, signs, etc.

Assume rolls w/o slip $\Rightarrow a_G = R\alpha = 0.2\alpha$

$\therefore \; F_f + (0.1)(0.2\alpha) = 3$ —②

Solve ① + ② $\Rightarrow \underline{F_f = 0.32\,N} \quad (\alpha = 134\ s^{-2})$

+8

$\therefore \; F_f < F_{s_{max}}$

b) ~~Ang~~ Momentum $\quad \cancel{I_c \omega_1}^{\,0} + \int_{t_1}^{t_2} M_c\,dt = I_c \omega_2$

$$(3)(0.25)(0.05) = (\underbrace{(0.1)(0.1264)^2}_{m\,K_G^2} + \underbrace{(0.1)(0.2)^2}_{m\,R^2})\omega_2$$

$$\omega_2 = 6.7\ s^{-1}$$

$$v_G = R\omega_2 = \underline{1.34\ m/s}$$

or linear momentum

$$\cancel{m v_{G_1}}^{\,0} + \Sigma F_x \Delta t = m v_{G2}$$

$$(3 - 0.32)(0.05) = (0.1) v_{G2}$$

$$v_G = \underline{1.34\ m/s}$$

+5

c) No work done by friction +2 if correct / 0 if not

d) $T_1 + \cancel{V_1}^{0} = \cancel{T_2}^{0} + V_2$ +5

$\frac{1}{2} I_C \omega_1^2 = m g h_G$

$h_G = \frac{I_C \omega_1^2}{2mg} = \underline{\underline{0.128\ m}}$ ← accepted either answer for full marks

From ground $h = h_G + R = \underline{\underline{0.328\ m}}$

- typical problems in part d included:
  - incl. friction −1
  - exclude rotational K.E. −1

~~3 marks if~~

Alternative sol'n to (a)

$m\cancel{v_{G1}}^{0} + \int F_x\, dt = m v_{G2}$

$(3 - F_f)(0.05) = (0.1)\cancel{v_{G2}}(1.34)$ ← solved by ang momentum as in (b)

$\cancel{v_{G2}} =$ $F_f = 0.32\,N$

5. (a) $k_{effective} = 30 + \left(\frac{1}{10} + \frac{1}{20}\right)^{-1}$

$= 30 + 6.67 = 36.67$ N/m.

$\omega_n = \sqrt{k_{eff}/m} = \sqrt{36.67/4} = 3.03\ s^{-1}$

$C_c = 2m\omega_n = (2)(4)(3.03) = 24.22$ N·s/m

10

$\omega_d = \omega_n\sqrt{1 - \left(\frac{c}{C_c}\right)^2} = 3.03\sqrt{1 - \left(\frac{14}{24.22}\right)^2}$

$= 2.47\ s^{-1}$

(or $\omega_d = \sqrt{k/m - \left(\frac{c}{2m}\right)^2}$)

(b) Amplitude $= (0.75\ m)\left(\exp\left(\frac{-c}{2m}t\right)\right)$

$= 0.75\exp\left(\frac{-14}{(2)(4)}(2.5)\right)$

4

$= 0.0094$ m.

(c) Amplitude $= \dfrac{F_0/k_{eff}}{1 - (\omega_0/\omega_n)^2}$

$= \dfrac{(12)/36.67}{1 - (5/3.02)^2}$

$= 0.1895$ meters

(d) frequency $= \frac{\omega_0}{2\pi} = \frac{5}{2\pi} = 0.796$ Hz.