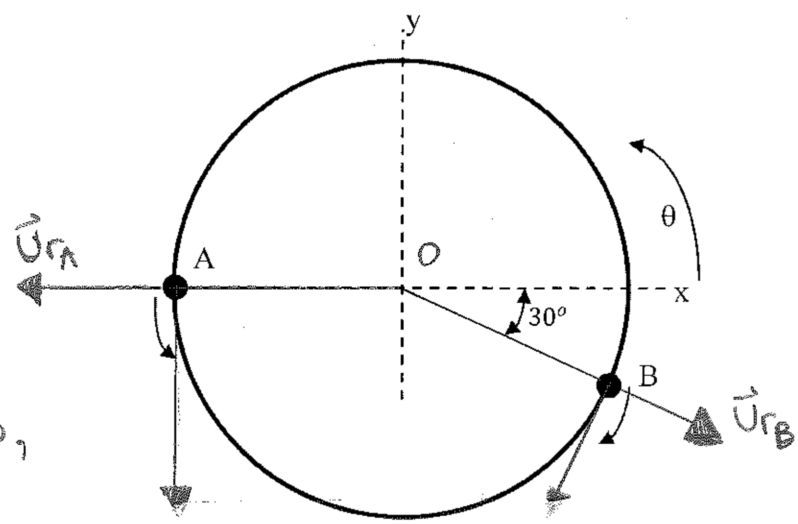
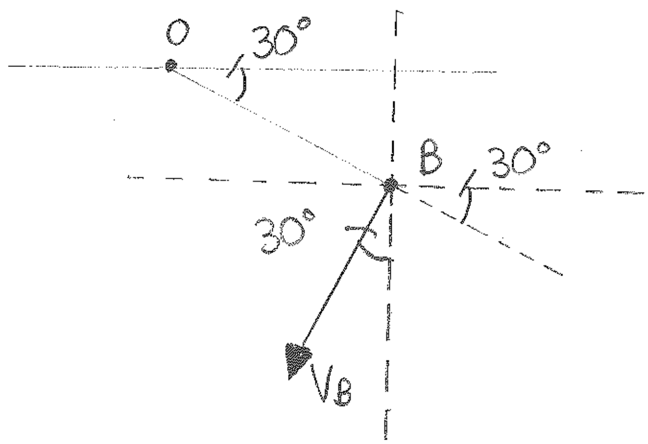
MIE 100 Quiz Solution

$|\vec{V}_A| = 5t$
$|\vec{V}_B| = 2t$



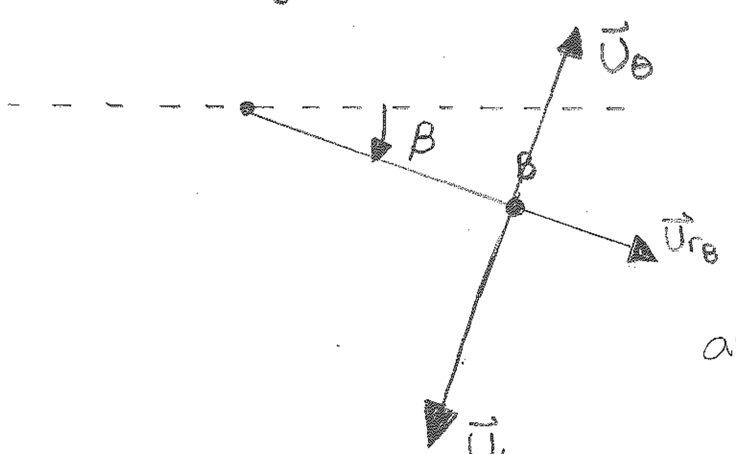
a) determine unit vector for velocity of particle B, initially.



$$\vec{U}_{t_B} = -\sin(30)\vec{i} - \cos(30)\vec{j}$$
$$= \boxed{-\tfrac{1}{2}\vec{i} - \tfrac{\sqrt{3}}{2}\vec{j}}$$

b) determine $\dot{\theta}$ and $\ddot{\theta}$ of B at $t = 3s$.

for easy visualization, let us define a curvilinear (polar) coordinate system for particle B.



for the special case of fixed radius circular motion, the $\vec{U}_{t_B}$ tangential and the angular direction, $\hat{U}_\theta$, are on the same axis, tangential to the circular path.

given the positive coordinate directions we have defined

$$\vec{u}_{t_B} = -\vec{u}_\theta$$

and we know that for all time velocity also acts in this direction.

$$\vec{V}_B = \cancel{\dot{r}}^{\,0}\,\hat{u}_{r_B} + (r\dot{\theta})\hat{u}_\theta$$

since r is fixed

$$r = 40$$
$$\dot{r} = 0$$
$$\ddot{r} = 0$$

$$V_B\hat{u}_{t_B} = r\dot{\theta}\hat{u}_\theta$$

but since $\hat{u}_{t_B} = -\hat{u}_\theta$

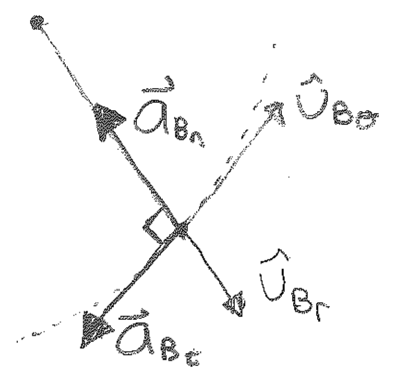
$$V_B = 2t = -r\dot{\theta}$$

$$2(3) = (40)\dot{\theta} \Rightarrow \boxed{\dot{\theta}_B = -0.15\ \text{rad/s}}\ \hat{u}_\theta$$

for $\ddot{\theta}$

$$\vec{a}_{B_t} = \frac{dv_B}{dt} \Rightarrow |a| = \frac{d}{dt}(2t) = 2\ \text{m/s}^2\ \hat{u}_{B_t}$$

$$\vec{a}_{B_n} = \frac{v^2}{\rho}\hat{u}_{B_n} = \frac{(2(3))^2}{40}\hat{u}_{B_n} = 0.9\ \text{m/s}^2\ \hat{u}_{B_n}$$



again with $\hat{u}_{B_r} = -\hat{u}_{B_n}$ and $\hat{u}_{B_t} = -\hat{u}_{B_\theta}$

$$\bar{a}_B = (0.9\hat{u}_{B_n} + 2\hat{u}_{B_t}) = (-0.9\hat{u}_{B_r} - \underline{2\hat{u}_{B_\theta}}) = (\ddot{r} - r\dot{\theta}^2)\hat{u}_{B_r} + \underline{(r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{u}_{B_\theta}}$$

isolating $\hat{u}_{B_\theta}$ direction with $\ddot{\theta}$ term

$$-2\hat{u}_{B_\theta} = (r\ddot{\theta} + 2\cancel{\dot{r}}^{\,0}\dot{\theta})\hat{u}_{B_\theta} \Rightarrow -2 = r\ddot{\theta} = 40\cdot\ddot{\theta}$$

$$\boxed{\ddot{\theta}_B = -0.05\ \text{rad/s}^2}\ \hat{u}_\theta$$

c) for particle A

$$|a_n| = \frac{v^2}{\rho}$$

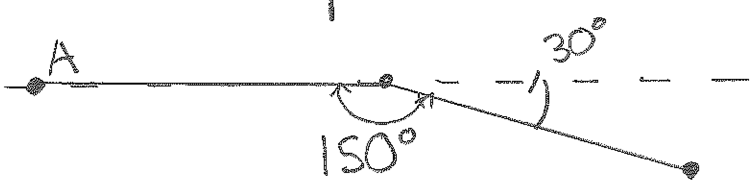
$$|v_A| = 5t$$

must define $|a_t|$ using $a_t = \frac{dv}{dt}$

$$a_t = \frac{d}{dt}(5t) = 5$$

require $5 = \frac{v^2}{\rho} = \frac{(5t)^2}{40} \Rightarrow t = 2\sqrt{2} = \boxed{2.83\text{ s}}$

d) must define position of each particle along the circular path as a function of time.



arclength of a circle

$$arc = \theta \cdot r$$

with θ in rads

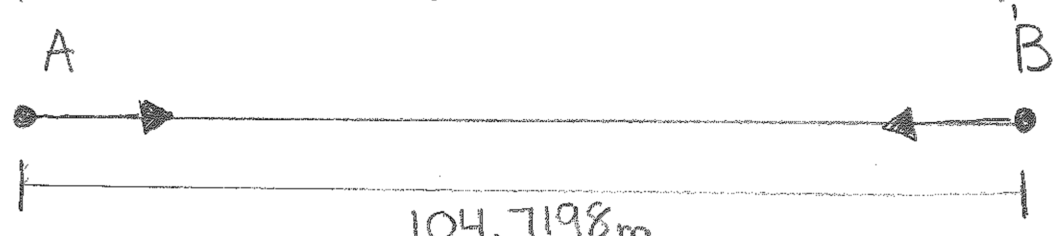
$$arc = (150\,deg)\left(\frac{\pi\ rad}{180\,deg}\right) \cdot 40 = 104.7198\,m$$

can now visualize system as;

and with $v = \frac{ds}{dt}$

$S_{B_0} = 0$ $\quad v\,dt = ds$

$S_{A_0} = 0$



$V_A = 5t$

$$\int_0^t 5t\,dt = \int_0^{S_f} ds_A$$

$$\frac{5}{2}t^2 = S_A$$

$V_B = 2t$

$$\int_0^t 2t\,dt = \int_0^{S_B} ds_B$$

$$t^2 = S_B$$

collision occurs when $S_A + S_B = 104.7198\,m$

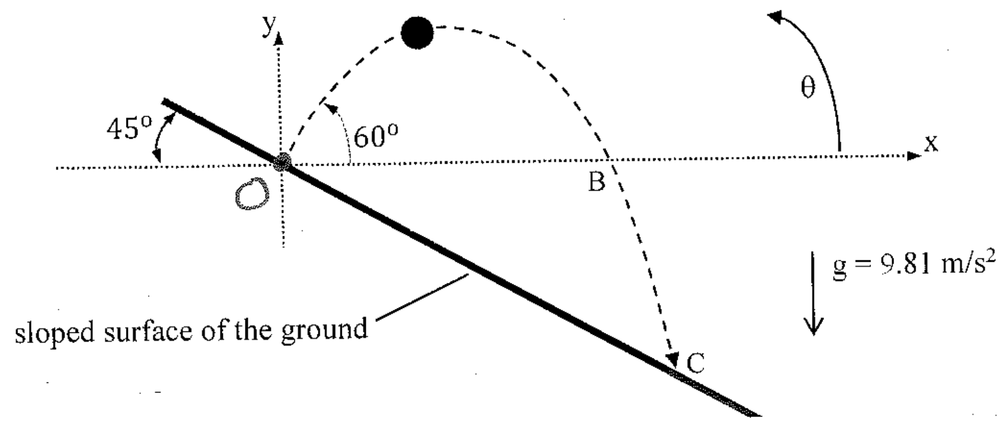
$$\frac{5}{2}t^2 + t^2 = 104.7198$$

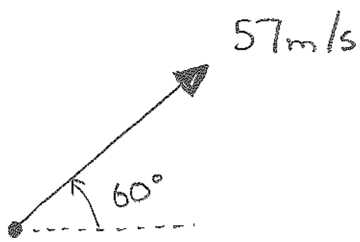
$$\tfrac{7}{2}t^2 = 104.7198$$

$$t = 5.4699\,s$$

$$\boxed{t = 5.47\,s}$$

Question 2





$V_{ox} = 57\cos(60) = 28.5\,m/s\,\vec{i}$

$V_{oy} = 57\sin(60) = 49.36\,m/s\,\vec{j}$

$\vec{r}(t) = S_x(t)\vec{i} + S_y(t)\vec{j}$

a) need to define the equations for position as a function of time with respect to origin $x=0, y=0$.

in x direction

$a_x = 0 \qquad a = \frac{dv}{dt} \Rightarrow a\,dt = dv$

$\int_0^t a_x\,dt = dv$

$0 = \int_{V_{x0}}^{V_x} dv$

$0 = V_x - V_{x0} \Rightarrow V_x = V_{x0}$

and $v = \frac{ds}{dt} \Rightarrow v\,dt = ds$

$\int_0^t V_{x0}\,dt = \int_0^{S_x} ds$

$V_{x0}t = S_x \Rightarrow \boxed{S_x = V_{x0}t}$

☆ could also work directly from equations of motion for constant acceleration

$a_x = 0\vec{i}$

$a_y = -9.81\vec{j}$

in y direction

$a_y = -9.81\,m/s^2$

$\int_0^t a_y\,dt = \int_{V_{y0}}^{V_y} dv$

$a_y t\Big|_0^t = V_y - V_{y0} \Rightarrow V_y = V_{y0} + a_y t$

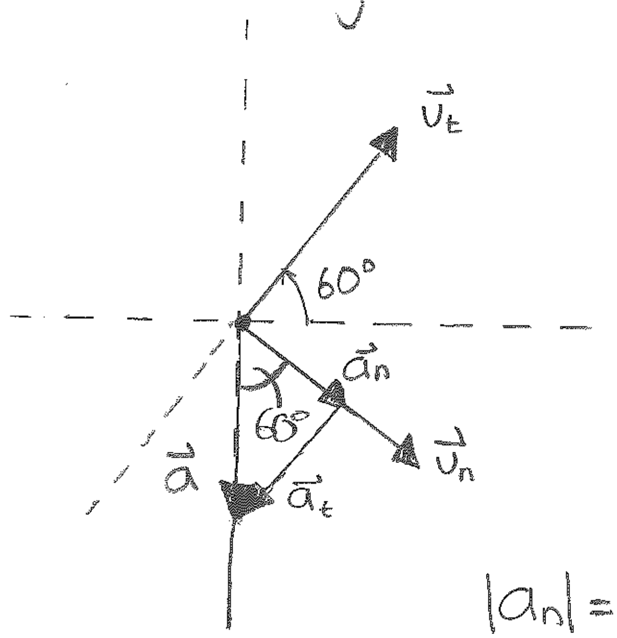
$\int_0^t V_{y0} + a_y t\,dt = \int_0^{S_y} ds$

$V_{y0}t + \frac{1}{2}a_y t^2 = S_y \Rightarrow \boxed{S_y = V_{y0}t + \frac{1}{2}a_y t^2}$

$$\boxed{r(t) = (V_{x0}t)\vec{i} + \left(V_{y0}t + \frac{1}{2}a_y t^2\right)\vec{j}}$$

$$r(t) = (28.5t)\vec{i} + \left(49.36t - 4.905t^2\right)\vec{j}$$

b) immediately after launch



recognize $\vec{a} = 0\hat{i} - 9.81\hat{j}$

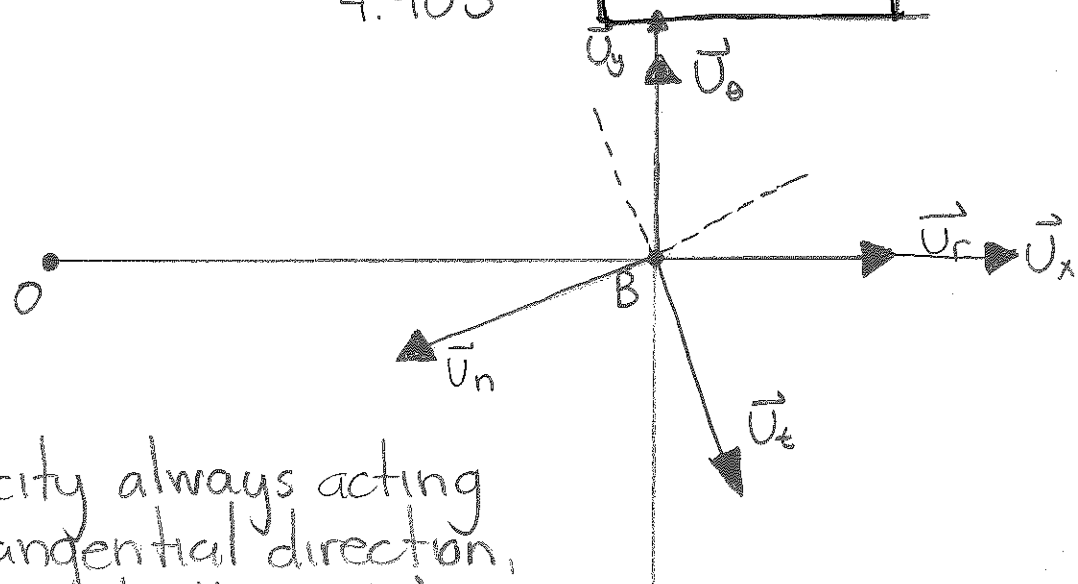
still true that $|a_n| = \frac{v^2}{\rho}$

need component of $\vec{a}$ in the normal direction.

$$|a_n| = |\vec{a}|\cos(60)$$
$$= |9.81|(\tfrac{1}{2}) = 4.905 \; m/s^2$$

$$\rho = \frac{(57)^2}{4.905} = \boxed{662.4\,m}$$

c)



velocity always acting in tangential direction, (tangent to the path)

find velocity at point B to define tangential direction

in y-direction

$$S_y = V_{y0}t + \tfrac{1}{2}a_y t^2 = 0 \quad \text{at point B}$$

$$(49.36)(t) - 4.905t^2 = 0 \Rightarrow t(49.36 - 4.905t) = 0$$

$\therefore\ t = 0\,s$

or $t = \dfrac{49.36}{4.905} = 10.063\,s$

at this time

$V_x = V_{xo} = 28.5\,\vec{i}$

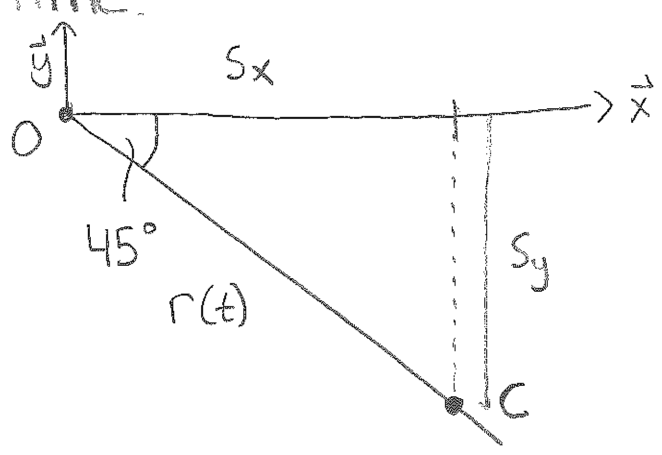
$V_y = V_{yo} + a_y t = 49.36 - 9.81(10.063) = -49.36\ m/s\ \vec{j}$

$V_B = 28.5\vec{i} - 49.36\vec{j}$

at point B the x-y coordinate system is aligned with the r-θ coordinate system

$$\boxed{\vec{V}_B = 28.5\hat{u}_r - 49.36\hat{u}_\theta}$$

d) again use vertical position to determine the time.



at point C

$S_y = -|r(t)|\sin(45)$

or $S_y = -S_x$

$49.36t - 4.905t^2 = -28.5t$

$\Rightarrow 4.905t^2 - 77.86t = 0$

$t(4.905t - 77.86) = 0$

$t = 0$ or $t = 15.874$

$$\boxed{t = 15.8\,s}$$