MAT187 - Calculus II - Winter 2019

Term Test 2 - March 19, 2019

Time allotted: 70 minutes.

Aids permitted: None.

Total marks: 50

Instructions:

- Do not write on the QR code at the top of the pages.
- This test contains 10 pages and a detached booklet for multiple-choice questions and formula sheet. DO NOT DETACH ANY PAGES.
- You can use pages 8–9 to complete questions (**mark clearly** which questions you are answering).
- Calculators, cellphones, or any other electronic devices are not allowed. If you have a cellphone with you, it must be turned off and in a bag underneath your chair.
- DO NOT START the test until instructed to do so.

GOOD LUCK!

LONG ANSWER PART

9. Let $f(x) = \sin(x^3)$. **(12 marks)**

In the following we will consider its Taylor Polynomial and its Taylor Series. You can assume that the Taylor Series converges, no need to prove it.

(a) **(4 marks)** What is the Taylor polynomial of degree 9 centred at 0 for $f(x)$? Justify your answer.

$p_9(x) =$

(b) **(4 marks)** Approximate the integral

$$\int_0^1 \sin(x^3)\, dx$$

using your answer from **(a)**. Justify your answer.

$\int_0^1 \sin(x^3)\, dx \approx$

(c) (4 marks) Calculate

$$\lim_{x\to 0} \frac{\sin(x^3) - x^3}{x^9}.$$

Justify your answer.

$$\lim_{x\to 0} \frac{\sin(x^3) - x^3}{x^9} =$$

10. When you eat something, the food goes into your stomach and then your body starts absorbing it. We will attempt to model the absorption of food in your stomach. **(12 marks)**

Food is absorbed from the surface, so

Rate of change of the volume of food in the stomach is proportional to the food's surface area.

Let's assume that, in the stomach, the food has the shape of a ball (called the bolus) of radius $r(t)$.

(a) **(5 marks)** A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$.

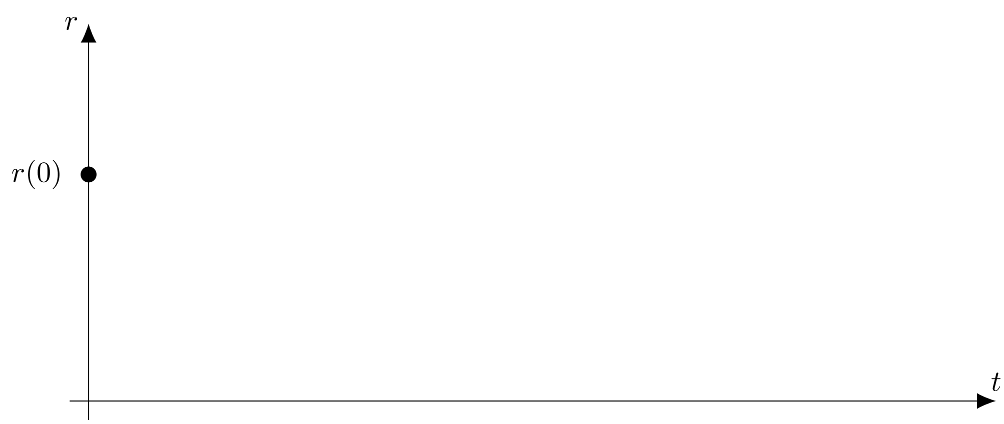
Obtain a differential equation for $r(t)$. Use only positive constants and justify all your steps and define all the constants and variables you use.

(b) **(3 marks)** How is the radius of the ball of food decreasing? Select all the true statements.

- ◯ The radius of the ball of food decreases faster near the beginning than towards the end of digestion.
- ◯ The radius of the ball of food decreases faster towards the end than near the beginning of digestion.
- ◯ The radius of the ball of food decreases at the same rate both near the beginning and towards the end of digestion.

After making a selection, justify your answer.

(c) **(4 marks)** Sketch the graph of $r(t)$. Justify your sketch.



11. It's 1905 and everyone believes that kinetic energy is given by the formula **(13 marks)**

$$K_C(s) = \frac{1}{2} m\, s^2.$$

Here, m is the mass of the particle at rest and s is the speed of the particle.

Albert Einstein just published his paper on Special Relativity. In it he claimed that kinetic energy is actually given by the formula

$$K_R(s) = mc^2 \left[\frac{1}{\sqrt{1 - \frac{s^2}{c^2}}} - 1 \right],$$

where c is the speed of light and s is still the speed of the particle.

The scientific community is very skeptical; after all the new formula $K_R(s)$ looks completely different from the classical one $K_C(s)$.

Your job is to find out how these two formulas are related.

(a) **(7 marks)** Consider the function $f(x) = \dfrac{1}{\sqrt{1-x}}$ and obtain its linear Taylor approximation $p_1(x)$ centred at $a = 0$.

$p_1(x) =$

(b) **(4 marks)** Use part **(a)** to approximate $K_R(s)$.

$$\boxed{K_R(s) \approx \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

(c) **(2 marks)** You just approximated $K_R(s)$. Compare this approximation to the formula for $K_C(s)$. What do you observe?

USE THIS PAGE TO CONTINUE OTHER QUESTIONS.

If you wish to have this page marked, make sure to refer to it in your original solution.

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