University of Toronto

Faculty of Applied Science and Engineering

MIE100 – Dynamics

Final Examination

April 23, 2012, 9:30 a.m. - noon

Instructors: *C. Simmons, A. Sinclair, L. Sinclair and P. Sullivan*

Aids Permitted: One non-programmable calculator
One 8 1/2" by 11" sheet of paper, any colour

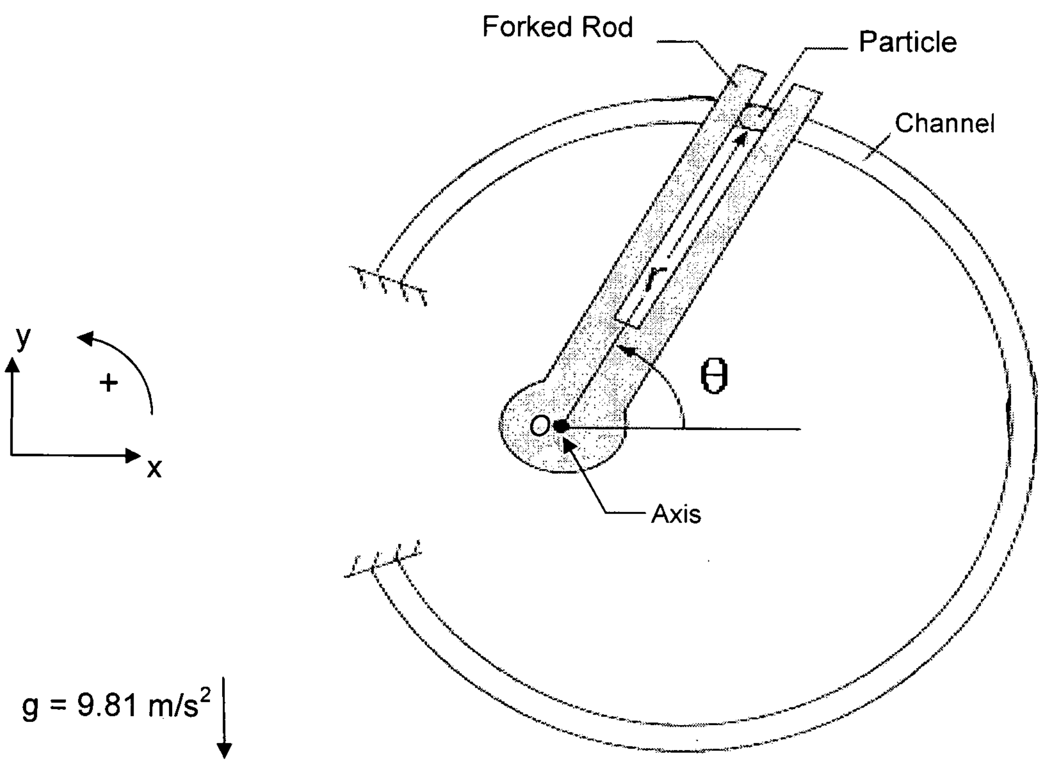
- Answers for each question must be given in the coordinate system specified in the question.
- The correct units must be specified in each final answer
- All rough work must be *neatly* shown to earn full credit for each question.
- This exam has six questions. Answer all six questions.
- Use a very dark pencil or pen.

Total Marks: 100

1. A 1 *kg* particle is pushed by a forked rod along a curved channel that guides its direction of motion. The channel's geometry is given by the equation $r = 0.3(2 + cos\theta)$, where r is the distance in meters from the axis O of the forked rod to the channel. There is no friction. A small motor located at the axis O forces the forked rod to rotate at a constant angular speed of $\dot{\theta} = -0.5\ s^{-1}$.

a) Determine the acceleration $\vec{a}$ of the particle in r-θ coordinates, when $\theta = 90°$. *(10 marks)*

b) Determine the force that the channel exerts on the particle, when $\theta = 0°$. *(5 marks)*

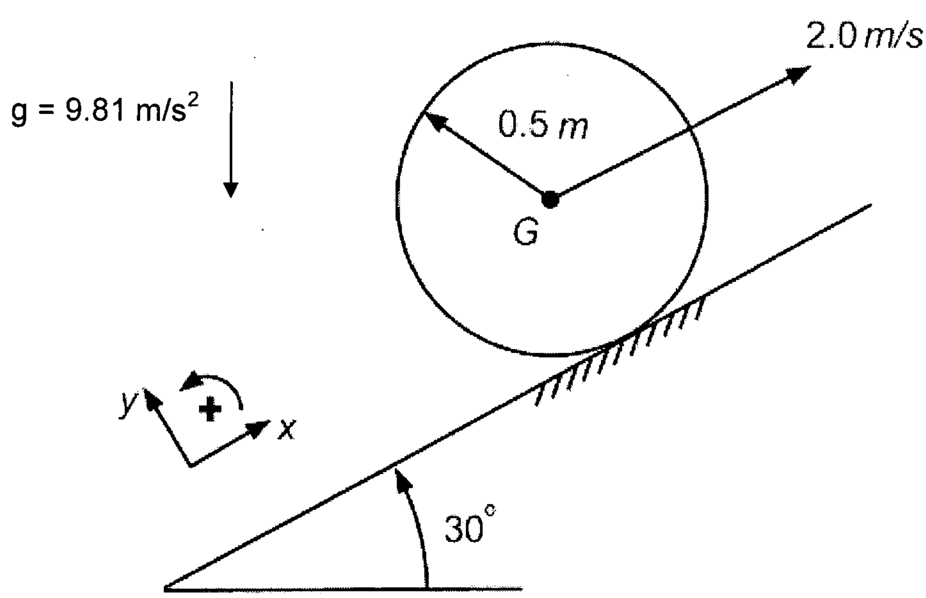


2. A uniform disk with a mass of 5 *kg* and radius of 0.5 *m* is moving up an inclined plane. At the instant shown, the center of the disk *G* has a speed of 2.0 *m*/s.

a) Determine the minimum coefficient of static friction required so that the disk does not start to slip. *(5 marks)*

b) Determine the angular acceleration of the disk at the instant shown, if $\mu_S = 0.85$ and the disk rolls without slipping. *(5 marks)*

c) Determine the kinetic energy of the disk at the instant shown, if $\mu_S = 0.85$ and the disk rolls without slipping. *(5 marks)*

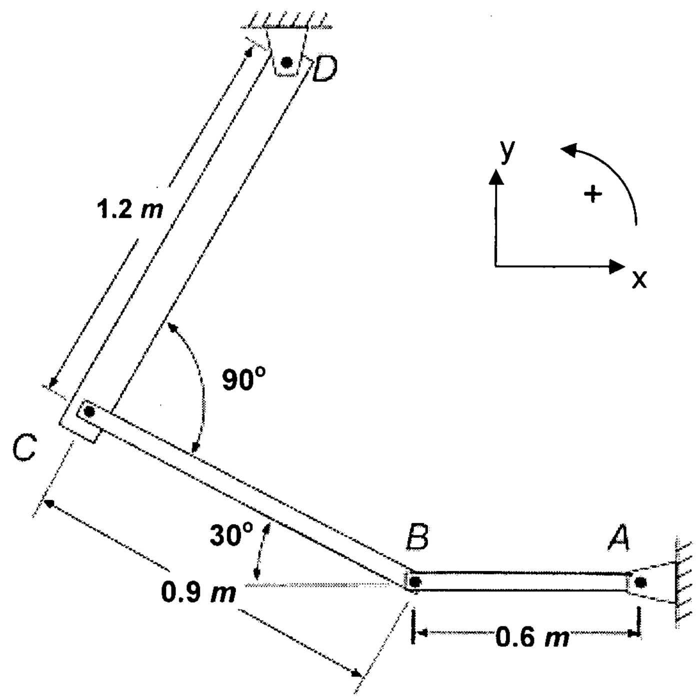


3. Consider the mechanism shown in the figure below. At the instant shown, the link AB has an angular velocity of $\omega_{AB} = -4\ s^{-1}$, and an angular acceleration $\alpha_{AB} = 0$. At the instant shown below, find the following:

a) The acceleration $\vec{a}_B$ of point B in *x-y* coordinates. *(5 marks)*

b) The location of the instantaneous center of zero velocity for link CB. Show your answer clearly in a diagram of link CB. *(5 marks)*

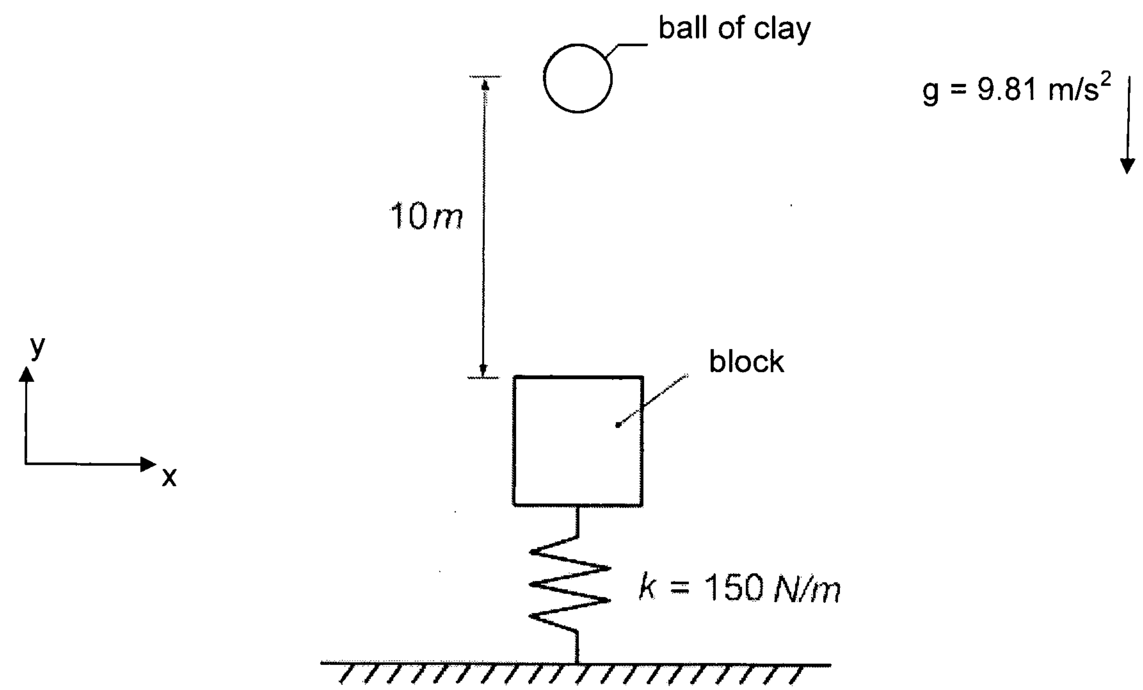
c) The angular velocity ω_{CD} of link CD. *(5 marks)*



4. A 1.0 *kg* block is at rest on top of a spring with a stiffness $k = 150$ N/*m*. A 0.5 *kg* ball of clay is released from rest at a height of 10 *m* above the top surface of the block. The clay hits the 1.0 *kg* block and sticks to it.

a) What is the total amount of kinetic energy of the block with clay stuck onto it, immediately after the collision? *(5 marks)*

b) Determine the force exerted by the spring on the block, at the instant when the block reaches its lowest point. *(10 marks)*



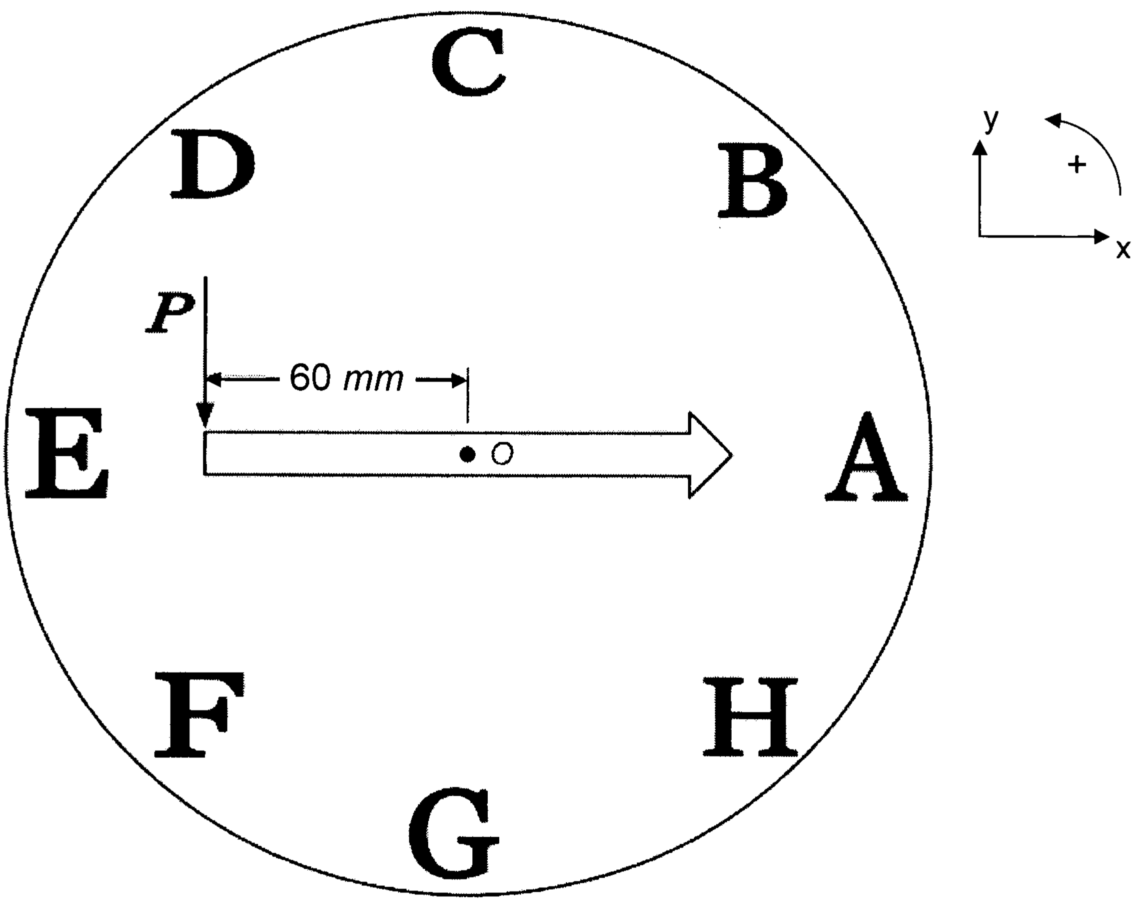
5. You are playing a game where you spin an arrow on a circular game board by applying a force with magnitude $P = 10\ N$ to the arrow with your hand for an extremely short time Δt, as shown below. The game board is fixed and lying flat on a table such that gravity has no effect.

The arrow is initially at rest when the force is applied at the instant shown. The arrow spins about its centre of mass at point O, and its rotation is resisted by a constant frictional moment of magnitude $M_o = 0.08\ N{\bullet}m$. The arrow has mass of $0.25\ kg$ and its radius of gyration about point O is $k_o = 0.03\ m$.

(a) Determine the time duration Δt required for application of the force so that the arrow will make exactly two full rotations before coming to rest pointing at the letter "A". *(10 marks)*

(b) What is the angular impulse about O given to the arrow by your hand? *(5 marks)*

(c) How much work is done by the frictional moment M_o? *(5 marks)*



6. A non-uniform bar has a mass m and a moment of inertia I_G about its centre of mass at G. The bar is pinned at point O at a distance L from point G. As the bar rotates about O, the pin joint exerts a resistive moment on the bar with a magnitude $c\dot{\theta}$ that is proportional to its angular speed.

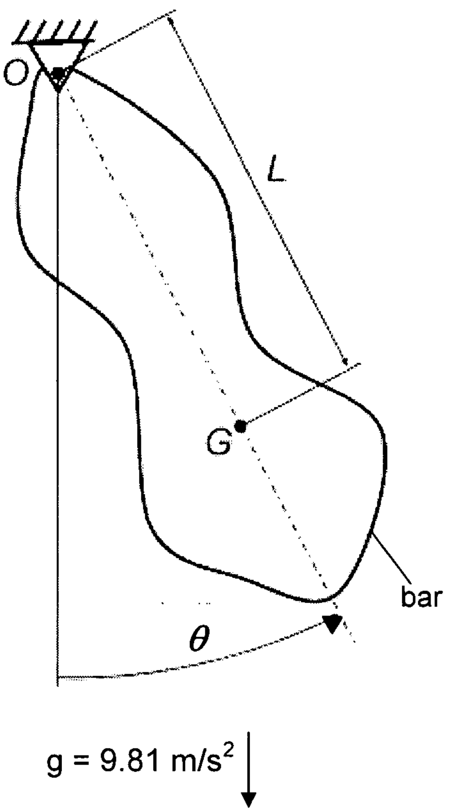
The bar is released from rest at $\theta = 5$ degrees from its equilibrium position at time $t = 0$, resulting in the motion plotted below.

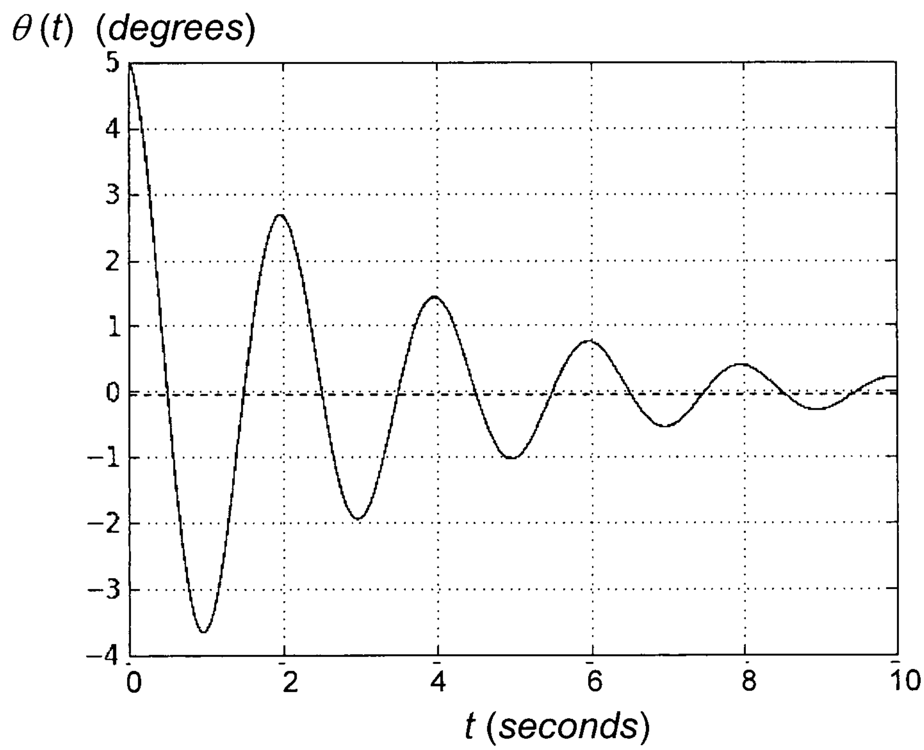
a) Draw a free body diagram of the oscillating bar and derive the second-order differential equation of motion for $\theta(t)$ in terms of the parameters m, L, I_G, and c. Assume that the amplitude of oscillations is small. *(5 marks)*

b) Estimate a value for the *damped* natural frequency ω_d. *(5 marks)*

c) Estimate the damping ratio, c/c_c. *(5 marks)*

d) By how many percent would the period of oscillation τ increase or decrease if the resistive component c were removed from the system? *(5 marks)*





MIE 100 - Final Exam Answers 2012.

1. $r = 0.3(2 + \cos\theta)$ $m = 1\,kg.$

$\dot{\theta} = -0.5\ s^{-1}$

$\ddot{\theta} = 0$

(a) $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{u}_\theta.$

$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = -0.3(\sin\theta)\dot{\theta}$

$\ddot{r} = -0.3(\cos\theta)\dot{\theta}^2 + \cancel{\cdots\ddot{\theta}}^{\,0}$

$\theta = 90^\circ \Rightarrow \theta = \pi/2.$

$r = 0.3(2 + \cos\pi/2) = 0.6.$

$\dot{r} = -0.3(\sin\pi/2)(-0.5) = 0.15$

$\ddot{r} = -0.3(\cos\pi/2)(-.5)^2 = 0.$

$\Rightarrow a_r = 0 - 0.6(-.5)^2 = -0.15.$

$a_\theta = 2(.15)(-.5) = -0.15$

$\Rightarrow \vec{a} = -0.15\hat{u}_r - 0.15u_\theta\ m/s^2$

(b) force that the channel exerts on particle is a contact / surface force $\Rightarrow$ $\perp$ to surface

$\Rightarrow F_r$. see page 1.

$\ddot{r} = -0.3(-.5)^2 = -0.075$ $\qquad r = 0.9$

$a_r = -0.075 - 0.9(-.5)^2 =$

$m = 1\,kg$. $\qquad m a_r = F_r$.

$\Rightarrow F_r = -0.3\ N$

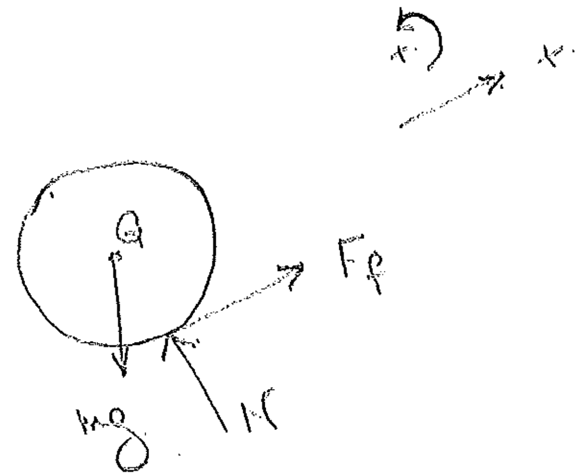
2. $m = 5\,kg$

$\vec{v}_G = +2.0\,\hat{i}\ m/s$

$R = 0.5\,m$

$\mu_s = 0.85$ (for (b))

FBD.



note that the kinematic link using these signs

is $v_G = -R\omega$

$a_G = -R\alpha$

(a) to find μ_s min assume rolling + assume

$F_f = \mu_s N$

$\Sigma F_x = m(-R\alpha)$

$\Sigma M_G = I_G\,\alpha$

$F_f = \mu_s (5)(9.81)(\cos 30°)$ – but don't do arithmetic yet because maybe m goes out

$\mu_s \cancel{m} g \cos\theta - \cancel{m} g \sin\theta = -\cancel{m} R \alpha$. – (1).

$\Sigma M_G = I_G \alpha$.

$F_f \cancel{R} = \frac{1}{2} m R^{\cancel{2}} \alpha$

$\mu_s \cancel{m} g \cos\theta = \frac{1}{2} \cancel{m} R \alpha$. – (2)

to find μ_s sub for α (2) into (1)

$\mu_s \cancel{g} \cos\theta - \cancel{g} \sin\theta = -\cancel{R}\left(\frac{\mu_s \cancel{g} \cos\theta}{\frac{1}{2} \cancel{R}}\right)$

$\Rightarrow \mu_s \cos\theta - \sin\theta = -2\mu_s \cos\theta$.

$3\mu_s \cos\theta = \sin\theta$.

$\Rightarrow \mu_s = \frac{\tan 30°}{3} = 0.192$

(b) $\sum F_x = m(-R\alpha)$

$\sum M_G = \frac{1}{2} m R^2 \alpha .$

$F_f - mg \sin\theta = -mR\alpha$ (1)

$F_f \cancel{R} = \frac{1}{2} m R \cancel{^2} \alpha$ — (2)

Sub 2 into 1

$\frac{1}{2} mR\alpha - mg \sin\theta = -mR\alpha$

$\frac{3}{2} \cancel{m} R\alpha = + \cancel{m} g \sin\theta$

$\alpha = + \frac{2 g \sin\theta}{3R}$ (units are good).

$= + \frac{2(9.81)\sin 30^\circ}{1.5} = + 6.54\, s^{-2}$

↑

or CCW.

↺

(c) $T = \frac{1}{2} I_{IC} \omega^2 = \frac{1}{2}\left(\frac{3}{2} mR^2\right)\omega^2 = \frac{1}{2}\left(\frac{3}{2} m\cancel{R^2}\right)\left(\frac{v_G}{\cancel{R}}\right)^2$

$= \frac{3}{4} m v_G^2 = 0.75(5)(2)^2 = 15$ joules.

(N.m).

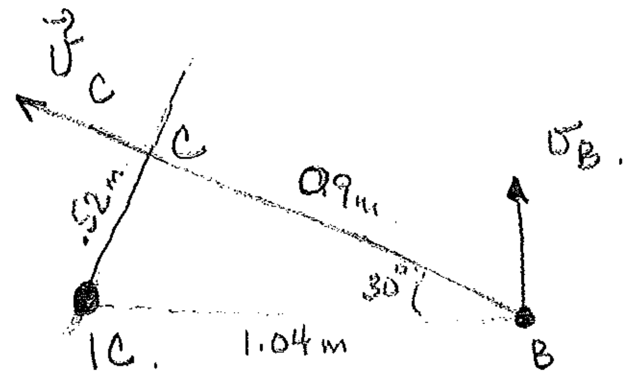
3.

(a) $\longrightarrow r\omega^2$

$0.6\,(4)^2 = 9.6$

$\vec{a}_B = 9.6\,\hat{i}\ \text{m/s}^2$

(b)



(c) to find ω_{CD} I want $|\vec{v}_C|$

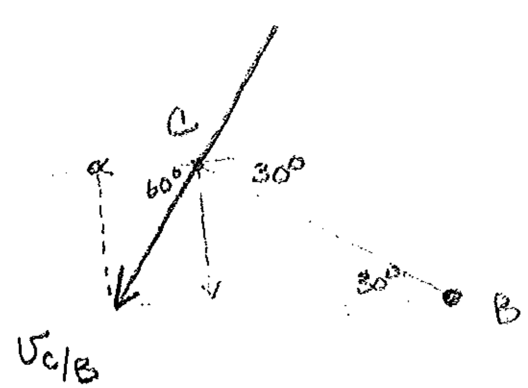
to solve this using the I.C found in (b) (a much shorter calc. - see p.8)

$\vec{v}_C = \vec{v}_B + \vec{v}_{C/B}$

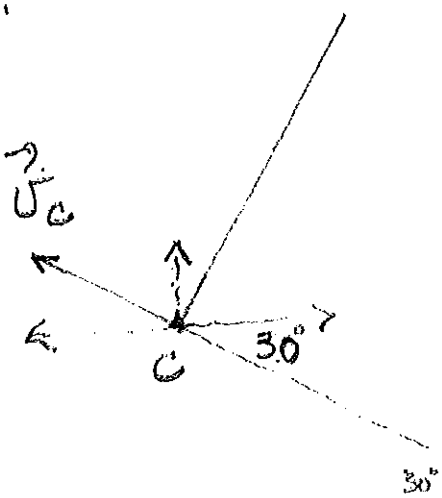
$\vec{v}_B = (.6)(4)\,\hat{j} = 2.4\,\hat{j}$ m/s.

$\vec{v}_{C/B} = -0.9\,\omega_{BC}\cos 60^\circ\,\hat{i}$

$-\ 0.9\,\omega_{BC}\sin 60^\circ\,\hat{j}$



& v_C :



$\vec{v}_C = -v_C \cos 30° \,\hat{i}$
$+ v_C \sin 30° \,\hat{j}$

all together :

$$-v_C \cos 30° \,\hat{i} + v_C \sin 30° \,\hat{j} = 2.4 \hat{j}$$
$$- 0.9\,\omega_{BC} \cos 60° \,\hat{i} - 0.9\,\omega_{BC} \sin 60° \,\hat{j}$$

i direction :

$$-v_C \cos 30° = -0.9\,\omega_{BC} \cos 60°$$

$$\Rightarrow \quad v_C = +0.52\,\omega_{BC}. \quad \Rightarrow \quad \omega_{BC} = +1.92\, v_C$$

j direction

$$+ v_C \sin 30° = 2.4 - 0.9(+1.92 v_C) \sin 60°$$

$$+ v_C = \frac{2.4}{\sin 30°} - (0.9)(1.92)\, v_C \frac{\sin 60°}{\sin 30°}$$

$$+ v_C = 4.8 - 3 v_C \quad \Rightarrow \quad v_C = +1.2 \text{ m/s}$$

3 (a) to find ω_{CD} find v_C.

$v_B = 2.4\,\hat{j}$ m/s.

$|IC|:B = 1.04$ m

$IC:C = 0.52$ m

$\Rightarrow v_C = 1.2$ m/s.

$\Rightarrow \omega_{CD} = \dfrac{1.2}{1.2} = 1\ s^{-1}$

by observation ω_{CD} is $-ve$
or CW.

final answer: $\omega_{CD} = -1$ radian/s

4. (a) momentum in the y direction is conserved during impact

$$m_c v_c = (m_c + m_b)\, v_{b+c}$$

but $v_c = \sqrt{2gh} = \sqrt{2(9.81)(10)} = 14\ \text{m/s}.$

$$\Rightarrow v_{b+c} = 14\left(\frac{.5}{1.5}\right) = 4.67\ \text{m/s}.$$

$$\Rightarrow T = \frac{1}{2}(1.5)(4.67)^2 = 16.35\ \text{joules}.$$

or

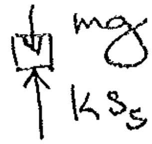
16.4 N·m.

(b) use energy to find how far down it goes.

$$T_1 + V_1 = T_2 + V_2.$$

$$T_1 + V_{1e} + V_{1g} = \cancel{T_2}^{\,0} + V_{2e} + \cancel{V_{2g}}^{\,0}$$

to find V_{1e}: statics.



$$9.81 = 150(s) \Rightarrow s = .0654\,m.$$

$$\Rightarrow V_{e,1} = \frac{1}{2}(150)(.0654)^2 = 0.32 \text{ joules.}$$

let h be <u>extra</u> depression in spring:

$$16.4 + 0.32 + 1.5g\,h = \frac{1}{2}(150)(h + .0654)^2$$

$$16.72 + 14.72h = 75(h^2 + .13h + .0043)$$

$$16.72 + 14.72h = 75h^2 + 9.75h + .32$$

$$\Rightarrow 75h^2 - 4.97h - 16.4 = 0$$

$$h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4.97 \pm \sqrt{24.7 + 4920}}{150}$$

$\Rightarrow$ = 0.50 m (using + sign) $\Rightarrow F_s = + 75.0$
extra h. $+ mg = 84.8\,N$

5. $M_2 = -0.08$ Nm
$m = 0.25$ kg
$k_G = 0.03$

$$\left.\right\} I_o = .25\,(.03)^2 = 2.25 \times 10^{-4}\ kg\,m^2$$

a simpler answer than the following was also accepted see 11A + 12A

divide up the $4\pi = \Theta_1 + \Theta_2$

Θ_1: P is applied

Θ_2: the rest of the 4π rotation.

[illegible]: use energy from start $(T_1 = 0)$ to end $(T_2 = 0)$.

$$\Rightarrow U_{M_1+M_2}\,\Delta\Theta_1 + U_{M_2}\Theta_2 = 0.$$

$$\Rightarrow \left(10\,(.06) - .08\right)\Theta_1 - .08\,(\Theta_2) = 0$$

$$\& \quad \Theta_1 + \Theta_2 = 4\pi$$

2 equations; 2 unknowns.

(12)

$.52\,\theta_1 = .08\,\theta_2$. $\qquad \theta_1 = \frac{.08}{.52}\theta_2 = .154\,\theta_2$

$.154\,\theta_2 + \theta_2 = 4\pi \Rightarrow \theta_2 = 10.89$ radians

$\theta_1 = 1.68$ "

can find ω_1 (at the end of the push).

again use energy

$\cancel{T_1}^{0} + U_{1\to 2} = T_2$

$(.6 - .08)(1.68) = \frac{1}{2}(2.25\times 10^{-4})\,\omega_1^2$

$\Rightarrow \omega_1 = \sqrt{\frac{1.747}{2.25\times 10^{-4}}} = 88.1\ s^{-1}$

can use angular momentum to get Δt.

$\cancel{I_0\omega_1}^{0} + M\Delta t = I_0\omega_2$

$.52(\Delta t) = 2.25\times 10^{-4}(88.1)$

$\Delta t = .038$ s.

(b) $10(.06)(.038) = .023$ N.m.s.

(c) $-.08 * 4\pi = -1$ N.m.

5. $M = -0.08\ N\cdot m$

$m = 0.25\ kg$

$k = 0.03$

$$\left.\begin{array}{l} \\ \\ \end{array}\right\} \quad I_o = .25(.03)^2 = 2.25 \times 10^{-4}\ kg\,m^2$$

$$I_o \omega_i + \Sigma M \Delta t = I_o \omega_f.$$

$\uparrow$?

this is a two step problem : after the arrow has started we have an energy problem ; we need to find ω_i so we can work backward with angular momentum.

$$T_1 + V_1 + U_{1\to 2} = T_2 + V_2.$$

no springs no gravity $\Rightarrow$ only, negative work done by frictional moment

$$\frac{1}{2}\left(2.25\times10^{-4}\right)\omega^2 = .08\,(4\pi)$$

$$\omega = 94.5\ s^{-1}$$

now go back & use angular momentum.

$$-.08\,\Delta t + 10(.06)\,\Delta t = \left(2.25\times10^{-4}\right)(94.5)$$

$$\Rightarrow \Delta t = .041\ s.$$

(b) $10 * .06 * .041 = \cancel{.0025}$ $.025$ N·m·s.

$$N\cdot ms = \frac{kg\,m}{s^{\cancel{2}}}\cdot m\cdot\cancel{s} = \frac{kg\,m^2}{s}$$

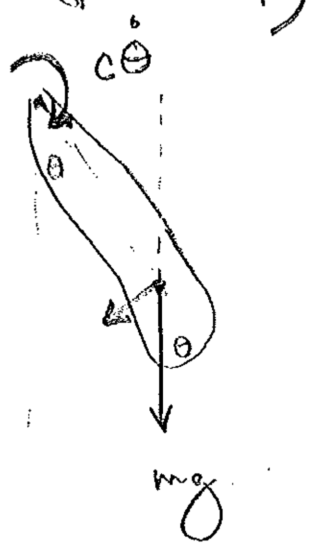
(c) $-0.08 * 4\pi = -1$ N·m

Check : initial $T = \frac{1}{2} I_o \omega^2$ ← all of this is dissipated by the -ve work.

(see top of page 12)

$U = -1$ joule

6. careful with the sign of this. $+$



$$\Sigma M_O = I_O \ddot{\theta}$$

$$-mg\sin\theta(L) - c\dot{\theta} = (I_G + mL^2)\ddot{\theta}$$

+ve $\dot{\theta}$ perturbation

Signs: make a +ve perturbation

$$(I_G + mL^2)\ddot{\theta} + c\dot{\theta} + mgL\theta = 0$$

(a) $\Rightarrow$

$$\Rightarrow -mg\theta(L) - c\dot{\theta} = (I_G + mL^2)\ddot{\theta}$$

(c)

from notes: $\dfrac{x_2}{x_1} = e^{-2\pi(c/c_c)}$

$$\ln\frac{x_2}{x_1} = -2\pi\frac{c}{c_c}$$

$$\ln\left(\frac{2.6}{1.4}\right)^{-1} = -2\pi\frac{c}{c_c} \Rightarrow \frac{c}{c_c} \simeq 0.10$$

(b) $\omega_d = \omega_n \sqrt{1-(.1)^2} = \omega_n(.994)$

$\Rightarrow \omega_d \approx \omega_n$. but we are just looking on the graph for that.

recall ω_d is radians /s.

but this bar is going 1 cycle / 2 seconds.

$\Rightarrow 2\pi$ rad / 2 s $= \pi\ s^{-1} = 3.14\ s^{-1}$

(d) Δ period $= \Delta\omega \Rightarrow$ 0.5 % – see above.