MIE100S Term Test: March 7, 2017. Place Answers in CrowdMark Booklet

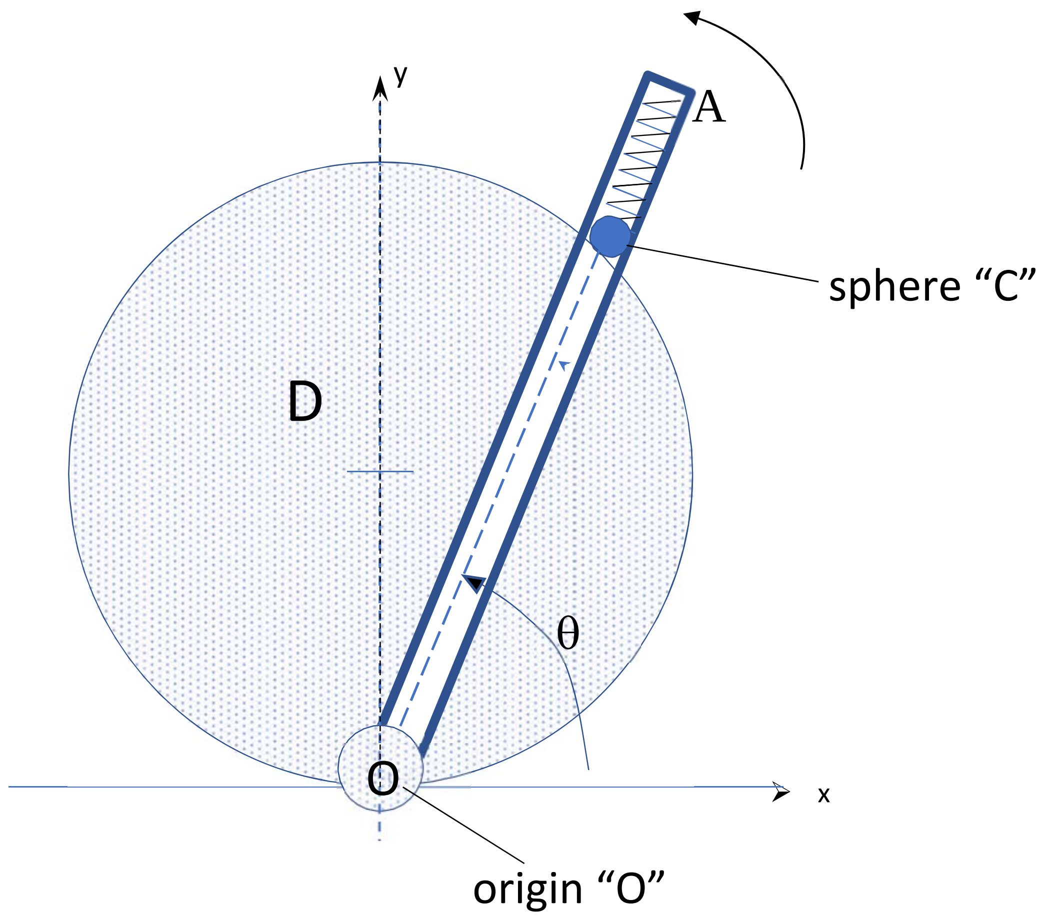
Test instructions are provided on the front page of the Crowdmark booklet.

1. The center of the1.8-kg sphere C travels along the outer surface of a pipe "D" guided by arm OA, on a path described by $r = 0.7\sin\theta$. A small motor at point "O" forces the arm *OA* to rotate counterclockwise with a constant angular velocity of $\dot{\theta} = 2.5\ \frac{radians}{s}$. The spring has a stiffness of k=100 N/m and is compressed by 0.33 m when θ = 63 degrees. The sphere C is in contact with only one edge of the slotted arm. Motion occurs in the *horizontal plane* such that the effects of gravity can be ignored. Assume that there is no friction.

<u>For Θ = 63 degrees determine the following:</u>

8	a) Draw a large, **CLEAR** free body diagram of the sphere. **Include the angles (in degrees) at which forces operate with respect to the x-axis**, but do not calculate the magnitudes of the forces.
5	b) Determine the velocity $\vec{v}$ of the sphere "C" in polar (r-θ) coordinates.
7	c) Determine a_r and a_θ for the sphere "C".
5	d) Determine the "r" component of the force exerted by the pipe on the sphere "C".

25 marks



2. In Figure I shown below, the spring with $k = 100$ N/m is relaxed. The spring is then compressed as shown in Figure II by moving the very small block B of mass 0.5 kg to the left a distance δ. The block is then released so that it can slide to the right and then up the ramp as shown in Figure III. Note that the block is not attached to the spring. Assume no sudden reduction in speed of “B” when it reaches the bottom of the ramp, only a change in direction.

5 a) Determine the amount δ that the spring was compressed in Figure II so that the block reaches point “A” at a speed of 3.5 m/s as shown in Figure III. Assume that there is no friction.

5 b) What is the angular momentum of block “B” about the origin when it reaches “A”? Express your final answer as a vector in x-y-z coordinates.

5 c) What is the linear impulse provided to the block B in the very short time interval when it switches direction to go up the 30-degree slope? Express your final answer using the x-y coordinate system shown.

10 d) Now assume that the coefficient of kinetic friction μ_k is equal to 0.25 on both the sloped and horizontal surfaces. If the spring is initially compressed by an amount $\delta = 35$ cm, what will be the speed of block “B” when it reaches point “A”?

25 marks

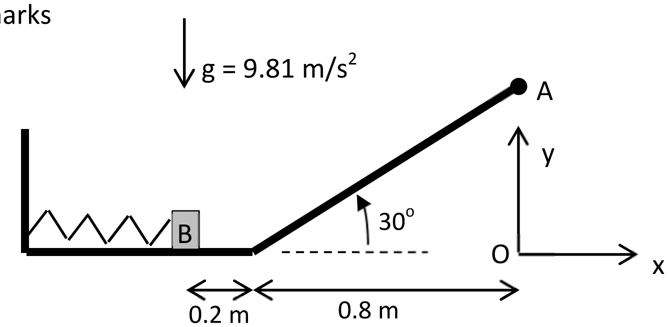


Figure I: The spring is relaxed, with the block B touching the spring.

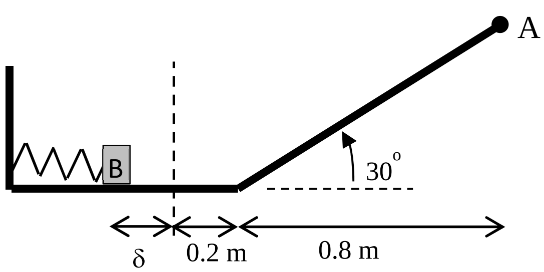


Figure II: The spring has now been compressed a distance δ, with the block B touching the spring.

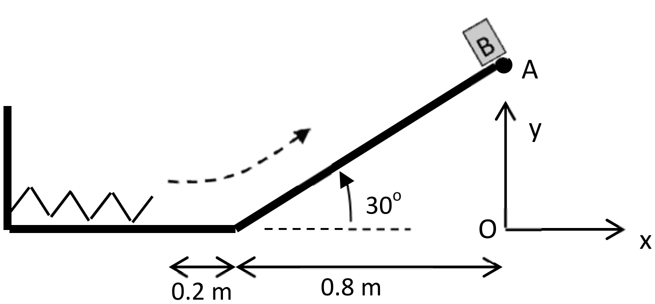
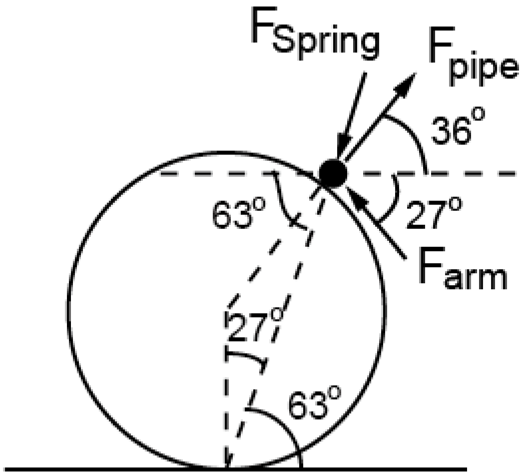


Figure III: The spring has been released, allowing the block B to slide along the ground, and then up the ramp to the point “A”.

1.(a)



Note:

Draw a free body diagram ………………… **1pts**
(Three forces in total with roughly correct force directions)

The angle between F_{pipe} and x axis equals to 36° (The key is that this force points from the center of the pipe)……………………………………………….**3pts**

The angle between F_{string} and x axis equals to -117° ……………………………………………..**2pts**

The angle between F_{arm} and x axis equals to 153° ………………………………………………………………..**2pts**

Either a clear presentation of these angles on the diagram or clear statements of these angles will be considered valid.

1.(b)

As $r = 0.7\sin\theta$

Then, we can obtain $\dot{r} = 0.7\cos\theta \cdot \dot{\theta}$

At $\theta = 36^o$, $\dot{r} = 0.7\cos\theta \cdot \dot{\theta}$ = 0.794 m/s……………………………………………………………………….**1pts**

The velocity of the sphere can be calculated using the following equation,

$\vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$ ………………………………………………………………………………………………………..**2pts**

Then, $\vec{v} = 0.794\vec{e}_r + 0.7 \cdot \sin(\theta)\dot{\theta}\vec{e}_\theta$

$\vec{v} = (0.794\vec{e}_r + 1.56\vec{e}_\theta)m/s$ ………………………………………………………………………………**2pts**

Note: Missing unit in the final presentation will lose **0.5pts.**

1.(c)

$\dot{r} = 0.7\cos\theta \cdot \dot{\theta}$ …………………………………………………………………………………………………….**1pts**

$\ddot{r} = -0.7\sin\theta \cdot \dot{\theta} \cdot \dot{\theta} + 0.7\cos\theta \cdot \ddot{\theta}$ ……………………………………………………………………**1pts**

At $\theta = 36^o$

$$\ddot{r} = -0.7\sin(63^o) \cdot (2.5)^2 + 0.7\cos(63^o) \cdot 0 = -3.9m/s^2$$

$a_r = \ddot{r} - r\dot{\theta}^2$ 1.5pts

$a_r = -3.9 - \left(0.7\sin 63^o\right)\left(2.5\right)^2 = -3.9 - 3.9 = -7.8 m/s^2$ 1pts

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$ 1.5pts

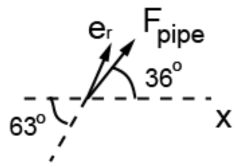
$a_\theta = 0 + 2\times 0.794 \times 2.5 = 3.97\, m/s^2$ 1pts

Note: Missing unit in the final presentation will lose **0.5pts.**

1.(d)

The magnitude of the spring force can be calculated from,

$\left|F_{spring}\right| = \left(100\, N/m\right)\left(0.33m\right) = 33N$ ....



.... **2pts**

The direction of the pipe force F_{pipe} can be shown by

When calculating the r component of the force exerted by the pipe on the sphere, it should satisfy $\sum F_r = ma_r$.

$-F_{spring} + F_{pipe}\big|_r = ma_r$ 2pts

Then,

$F_{pipe}\big|_r = 1.8 \times \left(-7.8\right) + 33 = 18.96N$ 1pts

Note: Missing unit in the final presentation will lose **0.5pts.**

2 (a)

Define initial position with spring compressed as position 1

$T_A = T_1 + U_{1\to A}$

$\frac{1}{2}mv_A^2 = 0 + \frac{1}{2}k\delta^2 - mg\Delta h$

$\frac{1}{2}(0.5kg)(3.5m/s)^2 \quad =$

$\frac{1}{2}\left(\frac{100N}{m}\right)(\delta)^2 - \qquad (0.5kg)(9.81m/s)(0.8tan30°)$

$3.0625 = 50\delta^2 - 2.2655$

$\delta^2 = 0.10656$

$\boldsymbol{\delta = 0.326m}$

2 (b)

$\overrightarrow{H_o} = \vec{r} \times m\vec{v}$

$\quad = (0.8tan30°\hat{\jmath}m) \times (cos30°\hat{\imath} + sin30°\hat{\jmath})(3.5m/s)(0.5kg)$

$\quad = 0.70(-\hat{k})kg.m^2/s$

$\boldsymbol{\overrightarrow{H_o} = -0.70\,\hat{k}\ kg.m^2/s}$

2 (c)

Find speed of B at the bend (v_o), using energy balance:

$E_{initial,Fig\ II} = E_{bend}$

$\frac{1}{2}k\delta^2 = \frac{1}{2}mv_o^2$

$v_o^2 = \frac{k}{m}\delta^2$

$v_o = \sqrt{\frac{k}{m}}\delta = \sqrt{\frac{100N/m}{0.5kg}}0.326m$

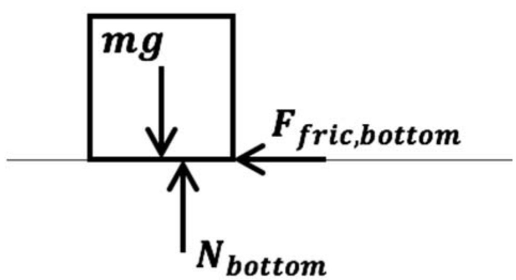
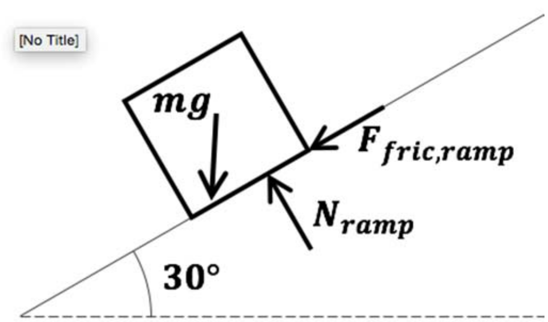
$v_o = 4.616m/s$

Note that the *speed* immediately before ($v_{o,-}$) and after ($v_{o,+}$) entering the ramp are equal (but their velocities have different directions)

$$Linear\ impulse = \Delta(m\vec{v}) = m(\overrightarrow{v_{o,+}} - \overrightarrow{v_{o,-}})$$
$$= (0.5kg)(v_o(cos30°\hat{\imath} + sin30°\hat{\jmath}) - (v_o\hat{\imath}))$$
$$\boldsymbol{Linear\ impulse} = (\mathbf{-0.309}\hat{\boldsymbol{\imath}} + \mathbf{1.154}\hat{\boldsymbol{\jmath}})\boldsymbol{kg.m/s}$$

2 (d)

Calculate normal force for the bottom and the ramp

Free body diagram (bottom)	Free body diagram (ramp)
	
$+\uparrow\ \sum F_y = 0$ $N_{bottom} - mg = 0$ $N_{bottom} = (0.5kg)(9.81\frac{m}{s})$ $N_{bottom} = 4.905N$	$+\nwarrow\ \sum F_{Normal} = 0$ $N_{ramp} - mgcos30° = 0$ $N_{ramp} =$ $(0.5kg)(9.81\frac{m}{s})(0.8660)$ $N_{ramp} = 4.248N$

Energy balance between initial position (Figure II) and position A (Figure III)

$$T_A = T_{initial,Fig\ II} + U_{grav} + U_{spring} + U_{friction}$$

$$\frac{1}{2}mv_A^2 = 0 + (-mg\Delta h) + \frac{1}{2}k\delta^2 - \mu_k(N_{bottom}\Delta s_{bottom} + N_{ramp}\Delta s_{ramp})$$

$$\frac{1}{2}(0.5kg)(v_A)^2 = (-0.5kg)(9.81m/s)(0.8tan30°m) + \frac{1}{2}(100N/m)(0.35m)^2 - 0.25\left(4.905N(0.55m) + 4.248N\left(\frac{0.8m}{cos30°}\right)\right)$$

$$0.25v_A^2 = -2.2655 + 6.125 - 0.6744 - 0.9815$$

$$v_A^2 = 2.2036/0.25$$

$$\boldsymbol{v_A = 2.97m/s}$$