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University of Toronto
Faculty of Applied Science and Engineering

FINAL EXAMINATION, December 2006
First Year - CIV, CHE, IND, LME, MEC, MMS

MAT 186H1F, Calculus 1
Exam Type: A

Examiners: S. Cohen, B. Koenig, R. Saghin, B. Stephens

Last Name: ______________________

First Name: ______________________

Student Number: ______________________

Instructions:

- **The use of non-programmable calculators is permitted.**
- Answer all questions. Total marks: 100.
- Please have your student card ready for inspection and turn off all cellular phones.
- This paper has a total of 13 pages, including this cover page. Present your **solutions** (in other words, show your work!) in the space provided. Use the back of the **preceding page** if you need more space. The value of each question is indicated in square brackets beside each question number.
- Do not tear any pages out from this test.

FOR MARKER USE ONLY	
Question	Marks
1	
2	
3	
4	
5	
6	
7	
8	
Total	

1. Evaluate the following limits, or explain why they do not exist.

(i) [5 marks] $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$.

(ii) [5 marks] $\lim_{x\to\infty}(e^{x}+e^{-x})^{1/x}$.

(Question 1 continued)

(iii) [5 marks] $\lim_{x\to 0} x^4 \sin\left(\frac{3}{x} - \frac{8}{x^2}\right)$.

(iv) [5 marks] $\lim_{x\to -\infty} x + \sqrt{x^2+2x}$.

2. The parts are not related.

(i) [3 marks] A particle moves along the curve $y = \sqrt{1+x^3}$. As it reaches the point $(2,3)$, the y-coordinate is increasing at a rate of 4 cm/sec. How fast is the x-coordinate of the point changing at that instant?

(ii) [4 marks] Find a function $f(x)$ such that $f'(x) = x^3$ and the line $x+y=0$ is tangent to the graph of f.

(Question 2 continued)

(iii) [5 marks] Find the equation of the tangent line to the curve $2(x^2+y^2)^2 = 25(x^2-y^2)$ at the point $(3,1)$.

3. Let $f(x) = x\sqrt{9+x^2}$.

(a) [4 marks] Find the average value of $[f(x)]^2$ on the interval $[-1,2]$.

(b) [4 marks] If $g(x) = \int_0^{x^2} f(t)\,dt$, find $g'(2)$.

(c) [6 marks] Find the area of the region in the first quadrant bounded by the curves $y = f(x)$ and $y = 5x$.

4. Consider the function $f(x) = \tan^{-1}\left(\frac{x-1}{x+1}\right)$.

(a) [2 marks] Verify that $f'(x) = 1/(x^2+1)$.

(b) [2 marks] Evaluate $\lim_{x\to -1^+} f(x)$ and $\lim_{x\to -1^-} f(x)$.

(c) [3 marks] Find the horizontal asymptotes and $x-$ and $y-$ intercepts.

(d) [2 marks] Find the intervals for which f is increasing and the intervals for which f is decreasing.

(Question 4 continued) Recall that $f(x) = \tan^{-1}\left(\frac{x-1}{x+1}\right)$ and $f'(x) = 1/(x^2+1)$.

(e) [3 marks] Find the intervals for which f is concave up and the intervals for which f is concave down.

(f) [2 marks] Sketch the graph of f using the information obtained in parts (a) through ~~(d)~~ e.

5. [10 marks] A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into a circle. How should the wire be cut so that the total area enclosed is (a) a maximum? (b) a minimum?

6. [10 marks] Find the surface area of the solid obtained by rotating the curve

$$y = \frac{x^3}{6} + \frac{1}{2x}, \qquad \frac{1}{2} \leq x \leq 1,$$

about the x-axis.

7. [10 marks] Let R be the region bounded by the curves

$$x = 1 - y^4, \qquad x = 0.$$

Find the volume of the solid generated by rotating R about the line $x = 2$.

8. [10 marks] A horizontal cylindrical tank of radius 3 feet and length 8 feet is half full of oil weighing 60 lb/ft^3. Find the work done in pumping out the oil to the top of the tank.

End of examination

(Available for scrap work. Do NOT tear out this page!)