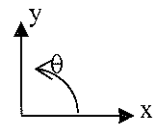
LAST Name: ______________________ First Name: ______________________ Student #: ______________

MIE100S: Dynamics winter 2013: Quiz A
quiz duration – 25 minutes

At time $t = 0$, a particle is moving counterclockwise in a circle of radius 6 meters around the origin, and slowing down at a rate of 0.35 m/s^2. At time $t = 0$, the particle is located at $(r, \theta) = (6 \text{ meters}, 2 \text{ radians})$, and has a speed of 5 m/s.

4 (a) At time $t = 0$, express the velocity in the given x-y rectangular coordinate system.
3 (b) At time $t = 0$, determine $\dot{\theta}$ in S.I. units.
3 (c) At time $t = 0$, determine $\ddot{\theta}$ in S.I. units.
10



Possibly useful equation:

$x = x_o + v_o t + \frac{1}{2} a_o t^2$ $\quad v = v_o + a_o t$ $\quad a\,dx = v\,dv$ $\quad \vec{v} = \dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta$

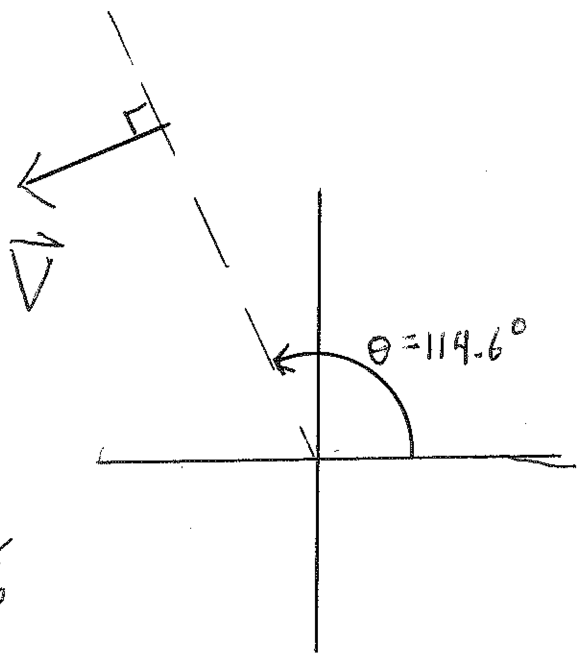
$\vec{a} = \dot{v}\,\hat{u}_t + v\dot{\theta}\hat{u}_n = \dot{v}\,\hat{u}_t + v^2/\rho\hat{u}_n$ $\quad \vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{u}_\theta$

(a) 2 radians $= (2)\left(\frac{180}{\pi}\right) = 114.6^\circ$

$V_x = 5\cos(114.6^\circ + 90^\circ) = -4.54\ m/s$

$V_y = 5\sin(114.6^\circ + 90^\circ) = -2.08\ m/s$

$\vec{V} = -4.54\hat{i} - 2.08\hat{j}\ m/s$



(b) $V_\theta = r\dot{\theta} \Rightarrow \dot{\theta} = \frac{V_\theta}{r} = \frac{V}{r} = 5/6$

$\dot{\theta} = 0.83\ s^{-1}$

(c) For a circle, $a_\theta = r\ddot{\theta} = -0.35\ m/s^2$

$\Rightarrow \ddot{\theta} = \frac{a_\theta}{r} = -\frac{0.35}{6} = -0.0583\ s^{-2}$

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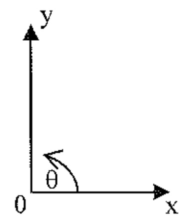
MIE100S: Dynamics winter 2013: Quiz B
quiz duration – 25 minutes

A particle is moving in a circle, and its speed is increasing at a constant rate of 4 m/s^2. At time $t = 0$, the particle is located at $(x,y) = (3,-2)$ meters, and has velocity of $-5\hat{\imath}$ m/s. Use the coordinate system shown. **NOTE: the center of the circular path is not located at the origin.**



3 (a) At time $t = 0$, what values of r and θ describe the position of the particle?
2 (b) What will be the tangential comonent of the acceleration at time $t = 2$ seconds?
3 (c) If the magnitude of the total acceleration is 6 m/s^2 at time t =0, find the radius of its circular path.
2 (d) Determine the x-y coordinates of the center of the circular path.
10 Possibly useful equation:

$$v = v_o + a_o t \qquad a\,dx = v\,dv \qquad \vec{v} = \dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta$$

$$\vec{a} = \dot{v}\,\hat{u}_t + v\dot{\theta}\hat{u}_n = \dot{v}\,\hat{u}_t + v^2/\rho\hat{u}_n \qquad \vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{u}_\theta$$

Answer.

(a) $r = \sqrt{3^2 + (-2)^2} = 3.6\ m$

$\theta = \tan^{-1}\left(\frac{-2}{3}\right) = -33.7^\circ$ or 146°

but we must select $\theta = -33.7^\circ$ (or 326.3°)

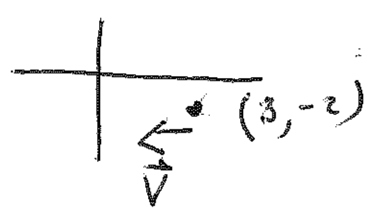
(b) $a_t = \text{constant} = 4\ m/s^2$

(c) $|\vec{a}| = \sqrt{a_n^2 + a_t^2}$

$6 = \sqrt{a_n^2 + 4^2} \Rightarrow a_n = 4.47\ m/s^2$

$4.47 = v^2/\rho = 5^2/\rho \Rightarrow \rho = 5.6\ m$

(d)



From diagram, (3,-2) is the top or bottom of circle of radius 5.6 m.

$\Rightarrow$ center is at (3, -7.6) m or (3, 3.6 m)

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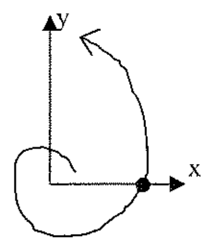
MIE100S: Dynamics winter 2013: Quiz C
quiz duration – 25 minutes

A woman is walking in a spiral path as shown in the diagram. The *magnitude* of her velocity (her speed) is constant and equal to 2 *m/s*. The radial distance between her and the center of the spiral is increasing at a constant rate of 1.1 *m/s*. At the position shown in the diagram, she is 5 meters from the center of the spiral, at the location $(x,y) = (5,0)$ meters.

2 (a) At the position shown, find $\dot{r}$ and $\ddot{r}$. θ
3 (b) At the position shown, find the velocity expressed in the r-θ (polar) coordinate system.
3 (c) At the position shown, find $\dot{\theta}$.
2 (d) At the position shown, find $\ddot{\theta}$.
10



Possibly useful equation:

$$v = v_o + a_o t \qquad a\,dx = v\,dv \qquad \vec{v} = \dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta$$

$$\vec{a} = \dot{v}\,\hat{u}_t + v\dot{\theta}\hat{u}_n = \dot{v}\,\hat{u}_t + v^2/\rho\hat{u}_n \qquad \vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{u}_\theta$$

Answer

(a) From the wording of the question, $\dot{r} = 1.1$ m/s , $\ddot{r} = 0$

(b) $V_r = 1.1$, $V = 2$, $V_r^2 + V_\theta^2 = 2^2$

$1.1^2 + V_\theta^2 = 4 \Rightarrow V_\theta = 1.67$ m/s

(c) $V_\theta = 1.67 = r\dot{\theta} = 5\dot{\theta} \Rightarrow \dot{\theta} = 0.334\, s^{-1}$

(d) $V^2 = V_r^2 + V_\theta^2$

but V is constant, V_r is constant $\Rightarrow V_\theta = \text{constant}$

$V_\theta = r\dot{\theta} = \text{constant}$

$$\Rightarrow \frac{d}{dt} V_\theta = r\ddot{\theta} + \dot{r}\dot{\theta} = 0$$

$$\ddot{\theta} = -\dot{r}\dot{\theta}/r$$

$$= -(1.1)(0.334)/5 = -0.0735\, s^{-2}$$