Name: ______________________________

Student number:____________________

MAT 186
Quiz 4

1. Use the limit definition of the derivative to find $f'(x)$ for

CF4	CS1

$$f(x) = \frac{3}{2x-1}$$

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{1}{h}\left(\frac{3}{2(x+h)-1}-\frac{3}{2x-1}\right)\\
&= \lim_{h\to 0}\frac{3}{h}\left(\frac{1}{2(x+h)-1}-\frac{1}{2x-1}\right)\\
&= \lim_{h\to 0}\frac{3}{h}\left(\frac{2x-1}{(2(x+h)-1)(2x-1)}-\frac{2(x+h)-1}{(2x-1)(2(x+h)-1)}\right)\\
&= \lim_{h\to 0}\frac{3}{h}\left(\frac{-2h}{(2(x+h)-1)(2x-1)}\right)\\
&= \lim_{h\to 0}\left(\frac{-6}{(2(x+h)-1)(2x-1)}\right)\\
&= -\frac{6}{(2x-1)^2}
\end{aligned}$$

2. Find $\lim\limits_{x\to-\infty}\frac{3x^3-4x^2-x\sin(x^2)}{3x^3+7x^2-7x-3}$ (Make sure to write this up properly.)

CF2	CS1	CS2	MW1	MW2

The first step is the obvious one:

$$\lim_{x\to-\infty}\frac{3x^3-4x^2-x\sin(x^2)}{3x^3+7x^2-7x-3}\cdot\frac{\left(\frac{1}{x^3}\right)}{\left(\frac{1}{x^3}\right)}$$

$$=\lim_{x\to-\infty}\frac{\left(3-\frac{4}{x}-\frac{\sin(x^2)}{x^2}\right)}{\left(3+\frac{7}{x}-\frac{7}{x^2}-\frac{3}{x^3}\right)}$$

The last term in the numerator needs work, using the Squeeze Theorem:

$$-1\le\sin(x^2)\le 1$$

$$-\frac{1}{x^2}\le\frac{\sin(x^2)}{x^2}\le\frac{1}{x^2}$$

$$\lim_{x\to-\infty}-\frac{1}{x^2}=\lim_{x\to-\infty}\frac{1}{x^2}=0$$

$$\therefore\lim_{x\to-\infty}\frac{\sin(x^2)}{x^2}=0$$

So, our limit becomes $\frac{3-0-0}{3+0-0-0}=1$.

Continued on back.

3. Find all asymptotes and identify all discontinuities (both their place and kind) for

$$\boldsymbol{y = \frac{x^2 - 5x + 6}{|2x^2 - 3x - 2|}}$$

AB1	AB4	CS3	CS4

The only possible discontinuities (and vertical asymptotes) can occur when the denominator is zero. We have $2x^2 - 3x - 2 = (2x+1)(x-2)$, so we need to examine $x = -1/2$ and $x = 2$.

Note that since these are zeroes for the function inside an absolute value, we have to be careful, although one part is actually made easier. We can take a two-sided limit, although it is not necessary to do so.

$\lim_{x \to -1/2} \frac{x^2-5x+6}{|2x^2-3x-2|}$ is of the form $\left[\frac{\left(-\frac{33}{4}\right)}{0^+}\right]$, so the two-sided limit is $-\infty$. This means there is a vertical asymptote.

For $x = 2$, we need one-sided limits.

$$\begin{aligned}
&\lim_{x \to 2^+} \frac{x^2 - 5x + 6}{|2x^2 - 3x - 2|} \\
&= \lim_{x \to 2^+} \frac{(x-3)(x-2)}{(2x+1)(x-2)} \\
&= \lim_{x \to 2^+} \frac{x-3}{2x+1} \\
&= -1/5
\end{aligned}$$

Meanwhile,

$$\begin{aligned}
&\lim_{x \to 2^-} \frac{x^2 - 5x + 6}{|2x^2 - 3x - 2|} \\
&= \lim_{x \to 2^-} \frac{(x-3)(x-2)}{-(2x+1)(x-2)} \\
&= \lim_{x \to 2^-} \frac{x-3}{-2x-1} \\
&= 1/5
\end{aligned}$$

Which gives a jump discontinuity.

Finally, for large values of x in both directions, the function inside the absolute value is positive, so:

$$\begin{aligned}
&\lim_{x \to \pm\infty} \frac{x^2 - 5x + 6}{|2x^2 - 3x - 2|} \\
&= \lim_{x \to \pm\infty} \frac{x^2 - 5x + 6}{2x^2 - 3x - 2} \\
&= \lim_{x \to \pm\infty} \frac{1 - \frac{5}{x} + 6/x^2}{2 - 3/x - 2/x^2} \\
&= \frac{1}{2}
\end{aligned}$$

So, $y = \frac{1}{2}$ is the horizontal asymptote in both directions.