University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

# FINAL EXAMINATION, APRIL, 2008

First Year - CHE, CIV, IND, LME, MEC, MMS

## MAT 186H1S - CALCULUS I

Exam Type: A

SURNAME: ______________________

GIVEN NAMES: ______________________

STUDENT NUMBER: ______________________

SIGNATURE: ______________________

**Examiner:**
D. Burbulla

| PAGE | MARK |
|---|---|
| MC | |
| Q7 | |
| Q8 | |
| Q9 | |
| Q10 | |
| Q11 | |
| Q12 | |
| Q13 | |
| TOTAL | |

**Calculators Permitted:** Casio 260, Sharp 520 or TI 30.

**INSTRUCTIONS:** Attempt all questions. Use the backs of the sheets if you need more space. Do not tear any pages from this exam. Make sure your exam contains 10 pages.

MARKS: Questions 1 through 6 are Multiple Choice; circle the single correct choice for each question. Each correct choice is worth 4 marks.

Questions 7, 8 and 9 each have two parts and are worth 12 marks; 6 marks for each part.

Questions 10, 11, 12 and 13 are each worth 10 marks.

TOTAL MARKS: 100

1. What is the equation of the normal line to the graph of $f(x) = e^x$ at the point $(x, y) = (0, 1)$?

   (a) $y = -x$

   (b) $y = -x + 1$

   (c) $y = x + 1$

   (d) $y = -x + e$

2. The arc length of the curve $f(x) = 2\ln x$ for $1 \le x \le e$ is given by

   (a) $\displaystyle\int_1^e \frac{\sqrt{x^2+4}}{x}\,dx$

   (b) $\displaystyle\int_1^e \frac{\sqrt{x^2-4}}{x}\,dx$

   (c) $\displaystyle\int_1^e \frac{\sqrt{2+x^2}}{x}\,dx$

   (d) $\displaystyle\int_1^e \frac{\sqrt{x^2-2}}{x}\,dx$

3. The equation of the slant (or oblique) asymptote to the graph of

$$y = \frac{x^3+1}{x^2+1}$$

   is

   (a) $y = x + 1$

   (b) $y = -x + 1$

   (c) $y = x$

   (d) $y = -x$

4. If $\sin\theta = \dfrac{1}{4}$ and $\cos\theta < 0$, then the exact value of $\sin(2\theta)$ is

(a) $\dfrac{\sqrt{15}}{8}$

(b) $\dfrac{7}{8}$

(c) $-\dfrac{\sqrt{15}}{8}$

(d) $-\dfrac{7}{8}$

5. $\displaystyle\int_0^4 x^3\sqrt{x^2+9}\,dx =$

(a) $\displaystyle\frac{1}{2}\int_0^4 (u+9)\sqrt{u}\,du$

(b) $\displaystyle\frac{1}{2}\int_9^{25} (u+9)\sqrt{u}\,du$

(c) $\displaystyle\frac{1}{2}\int_0^4 (u-9)\sqrt{u}\,du$

(d) $\displaystyle\frac{1}{2}\int_9^{25} (u-9)\sqrt{u}\,du$

6. $\displaystyle\lim_{x\to 0}\frac{x-\sin x}{x^3} =$

(a) $\dfrac{1}{3}$

(b) $\dfrac{1}{6}$

(c) $-\dfrac{1}{3}$

(d) $-\dfrac{1}{6}$

7. [12 marks; 6 for each part] Find the following limits:

(a) $\lim_{x \to 0} \left(5x^2 + e^{-x^2}\right)^{3/x^2}$.

(b) $\lim_{x \to -\infty} x \left(\frac{\pi}{2} + \tan^{-1} x\right)$.

8. [12 marks; 6 for each part] Find the following:

(a) $F'(\pi/4)$, if $F(x) = \displaystyle\int_0^{\tan x} \sqrt{3+t^5}\,dt$.

(b) $\displaystyle\int_0^1 \frac{1}{1+e^{-x}}\,dx$.

9. [12 marks; 6 for each part] Let $R$ be the region in the plane bounded by $y = x$ and $y = \sqrt{x}$. Find the following:

(a) The volume of the solid obtained by revolving the region $R$ about the $x$-axis.

(b) The volume of the solid obtained by revolving the region $R$ about the line $x = 2$.

10. [10 marks; 5 for each part] Suppose the velocity of a particle at time $t$ is given by $v = t^2 - 4t$, for $0 \leq t \leq 5$. Find the following:

(a) the average velocity of the particle for $0 \leq t \leq 5$.

(b) the average speed of the particle for $0 \leq t \leq 5$.

11. [10 marks] A spherical storage tank with radius 1 m is full of water with density $\rho$. How much work is done in emptying the tank by pumping all the water up to a transfer pipe 2 m above the top of the tank?

12. [10 marks] Find the area of the region in the first quadrant bounded by the three curves

$$y = 2, y = \ln x \text{ and } y = 2\sqrt{1-x}.$$

13. [10 marks] Two vertical poles stand 10 m apart on level ground. A wire joining the tops of the two poles is to be attached to a point on the ground between the two poles, so that the wire goes from the top of one pole down to the ground and then up to the top of the second pole. What is the length of the shortest such wire, if one pole is 6 m high and the other pole is 4 m high?