University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING
**FINAL EXAMINATION, DECEMBER, 2011**
**Duration:** 2 and 1/2 hours
First Year - CHE, CIV, IND, LME, MEC, MMS

**MAT186H1F - CALCULUS I**
Exam Type: A

SURNAME: (as on your T-card) _____

YOUR FULL NAME: _____

STUDENT NUMBER: _____

SIGNATURE: _____

**Examiners:**
D. Burbulla
T. Squires
K. Tyros
B. Wang

**Calculators Permitted:** Casio 260, Sharp 520 or TI 30.

**INSTRUCTIONS:** Attempt all questions. Present your solutions in the space provided. Use the backs of the sheets if you need more space. Do not tear any pages from this exam. Make sure your exam contains 10 pages.

**MARKS:** Question 1 is worth 20 marks, 5 marks for each part.

Questions 2 and 4 are each worth 10 marks.

Questions 3, 5 6, 7 and 8 are each worth 12 marks.

**TOTAL MARKS:** 100

| QUESTION | MARK |
|---|---|
| Q1 | |
| Q2 | |
| Q3 | |
| Q4 | |
| Q5 | |
| Q6 | |
| Q7 | |
| Q8 | |
| TOTAL | |

1. Find the following:

(a) $\int \left( \frac{1}{1+x^2} + \tan x + \frac{1}{x} \right) dx$

(b) $\lim_{x \to \infty} \left( 1 - \frac{3}{x} \right)^x$

(c) the linear approximation of $79^{1/4}$, without using your calculator.

(d) an approximation of the solution to the equation $x^3 + x - 1 = 0$, correct to 2 decimal places. (You will need your calculator.)

2. Find the following:

(a) $\int_0^1 x\sqrt{16+9x^2}\,dx.$

(b) $F'(2)$ if $F(x) = \int_0^{x^2} \sqrt{t^2+9}\,dt.$

3. Sketch the graph of $y = 4x^{1/3} - x^{4/3}$, labeling all critical points, inflection points, and vertical tangents, if any. You may assume

$$y' = \frac{4}{3\,x^{2/3}} - \frac{4\,x^{1/3}}{3} \text{ and } y'' = -\frac{8}{9\,x^{5/3}} - \frac{4}{9\,x^{2/3}}.$$

4. The velocity of a particle at time $t$ is given by $v = 3t^2 - 3$. Find

(a) [4 marks] the average velocity of the particle for $0 \leq t \leq 2$.

(b) [6 marks] the average speed of the particle for $0 \leq t \leq 2$.

5. Let $A$ be the area of the region in the $xy$-plane bounded by $x = 1, x = e^2, y = 2$ and $y = \ln x$.

   (a) [8 marks] Write down two integrals, one with respect to $x$ and one with respect to $y$, that both give the value of $A$.

   (b) [4 marks] Find the value of $A$.

6. Sketch the graph of $y = \sec^{-1}\left(\dfrac{x^2+1}{x^2-1}\right)$, labeling all critical points, inflection points, vertical tangents, asymptotes and discontinuities, if any. You may assume

$$y' = -\frac{2x}{(x^2+1)\,|x|} \text{ and } y'' = \frac{4\,|x|}{(x^2+1)^2}.$$

7. If water enters a hemispherical bowl of radius 100 cm at a rate of 10 $cm^3$/sec, how fast will the water level be rising when the depth of the water in the bowl is 50 cm?

8. Let $f(x) = x^{3/2}$.

(a) [6 marks] Find the length of the curve $y = f(x)$ for $0 \leq x \leq 1$.

(b) [6 marks] Find the volume of the solid of revolution obtained by revolving the region in the $xy$-plane bounded by the curves $y = f(x), y = 0, x = 0$ and $x = 1$ about the line $y = -1$.