Coulomb's Law

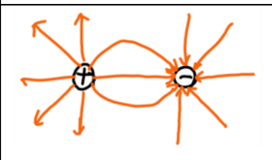
$$\boldsymbol{F}_{qQ} = \frac{kQq}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\hat{r}$$

Electric Fields

$$\mathrm{E}_Q = \frac{\boldsymbol{F}_{qQ}}{q} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

$$\varepsilon_0 = 8.854\cdot 10^{-12}\ [C^2/Nm^2]$$
$$k = 8.99\cdot 10^9\ [Nm^2/C^2]$$

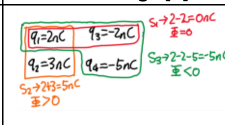
Electric Field line



Electric Flux [Nm^2/C]

$$\phi_{\text{net}} = \oint_S \boldsymbol{E}\cdot d\boldsymbol{A} = \oint_S E\cdot dA\cdot cos(\boldsymbol{E}\ d\boldsymbol{A})$$

Enclosed charge [C]



$$q_{enc} = \varepsilon_0\phi_{\text{net}} = \varepsilon_0\oint_S \boldsymbol{E}\cdot d\boldsymbol{A}$$

Gaussian surface

Sphere – 1 gaussian surf
Cylinder – 3 gaussian surf
Cube – 6 gaussian surf
$\hat{n}$ perpendicular to area
$$d\boldsymbol{A} = dA\hat{n}$$

Isolated conductor

$$\mathbf{E} = 0$$

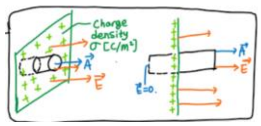
Cavity walls, inside isolated conductor, in metal

(charges all reside at surface)

External elec. field

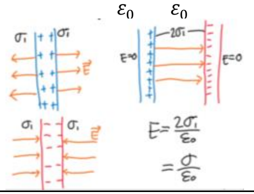
$$\mathrm{E} = \frac{\sigma}{\varepsilon_0}$$

[N/C] or [V/m]



Two conducting planes

$$\mathrm{E} = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$



Charge Density (linear, surface, volume)

Linear density	λ	$\mathrm{q_{tot}} = \int \lambda(\mathrm{x})\,dx$
Surface density	σ	$\mathrm{q_{tot}} = \iint \sigma(\mathrm{x,y})\,dxdy$
Volume density	ρ	$\mathrm{q_{tot}} = \iiint \rho(\mathrm{x,y,z})\,dxdydz$

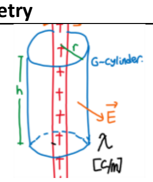
Spherical Symmetry

$$\mathrm{E_{ext}} = \left(\frac{\mathrm{q}}{4\pi\varepsilon_0 R^3}\right)r$$

r – radius of gaussian surface
R – radius of charge sphere
q – charge enclosed
$$\mathrm{E_{int}} = 0$$

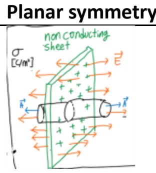
Cylindrical symmetry

$$\mathrm{E} = \frac{\lambda}{2\pi\varepsilon_0 r}$$



$$\mathrm{q_{enc}} = \lambda\mathrm{h} = \varepsilon_0\oint_S \boldsymbol{E}\cdot d\boldsymbol{A}$$

Planar symmetry

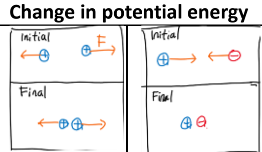


$$\mathrm{E} = \frac{\sigma}{2\varepsilon_0}$$

Non-conducting sheet (charge density of σ)

$$\phi = \phi_l + \phi_r = 2\mathrm{EA}$$

Change in potential energy



$\Delta U > 0$ $\Delta V > 0$ Force applied	$\Delta U < 0$ $\Delta V < 0$ Field does work

Potential Energy and Electric Potential

$$\Delta U = \mathrm{U_f} - U_i = W_{appl} = -W_{field}\ [J] = q(V_f - V_i) = -qE\Delta S$$

$$\Delta V = -\int_i^f \boldsymbol{E}\ d\boldsymbol{S} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$\Delta V = \mathrm{V_f} - V_i\ [V] = \frac{\Delta U}{q} = -\int_i^f \boldsymbol{E}\ d\boldsymbol{S} = -E\Delta x$$

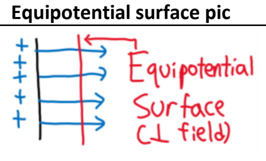
Equipotential Surfaces
$$\mathrm{V_r} = \frac{Q}{4\pi\varepsilon_0 r}$$
Same potential (+pot E)

Voltage and Potential Energy

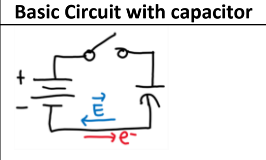
$$\mathrm{V}_{r>\infty} = \frac{Q}{4\pi\varepsilon_0 r} \qquad U = \frac{Qq}{4\pi\varepsilon_0 r}$$

Voltages can be added up each other (scalar)

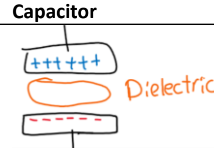
Equipotential surface pic



Basic Circuit with capacitor



Capacitor



*Metal plates > separation of charge
*Dielectric > increase capacitance
*Capacitor is charged

Capacitance

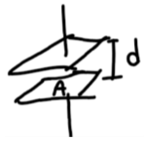
$$\mathrm{C} = \frac{\mathrm{q}}{\mathrm{V}}\ [\mathrm{F}]$$

q – charge stored in one plate: not total charge!

Parallel plate capacitor

$$\mathrm{E} = \frac{Q}{A\varepsilon_0}$$
$$\mathrm{V} = \frac{Q}{A\varepsilon_0}d$$
$$C_{pp} = \frac{A\varepsilon_0}{d}$$

(Ignore dielectric)



Cylindrical Capacitor

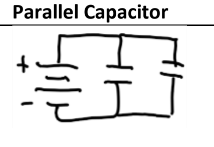
$$\mathbf{E} = \frac{Q}{2\pi\varepsilon_0 Lr}\hat{r}$$
$$\mathrm{V} = \frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)$$
$$\mathrm{C_{cyl}} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

Spherical capacitor

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$
$$\mathrm{V} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$
$$\mathrm{C_{sphy}} = \frac{4\pi\varepsilon_0 ab}{a-b}$$
$$C_{single\_sphy} = 4\pi\varepsilon_0 r$$

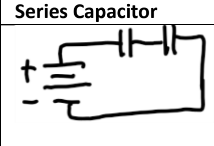
a is smaller radius, b is larger radius, L is height, r is radius of gaussian surface

Parallel Capacitor



$$\mathrm{Q_T} = Q_1 + Q_2 + \cdots$$
$$V_T = V_1 = V_2 = \cdots$$
$$\mathrm{Q_T} = C_T V$$
$$\mathrm{C_T} = C_1 + C_2 + \cdots$$

Series Capacitor



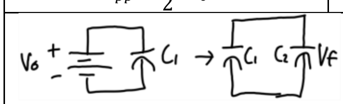
$$\mathrm{Q_T} = Q_1 = Q_2 = \cdots$$
$$\mathrm{V_T} = V_1 + V_2 + \cdots$$
$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

Energy in capacitor

$$\mathrm{U} = \int d\mho = \int q\,dV = \int CV\,dV$$
$$\mathrm{U} = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV$$

Energy Density

$$\mho_{pp} = \frac{U}{Volume} = \frac{U}{Ad} = \frac{CV^2}{2Ad} = \frac{kA\varepsilon_0}{2d}\frac{E^2d^2}{Ad}$$
$$\mho_{pp} = \frac{1}{2}k\varepsilon_0 E^2$$



$$\mathrm{V_f} = \frac{c_1}{c_1+c_2}V_0$$

Capacitance + dielectric

Increase area + decrease distance + dielectric
= <u>high capacitance</u>

$$C_{pp} = \frac{k\varepsilon_0 A}{d}$$
$$\mathrm{C_{cyl}} = \frac{2\pi k\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$
$$\mathrm{C_{sphy}} = \frac{4\pi k\varepsilon_0 ab}{a-b}$$

k: dielectric constant (k>1)

Ohm's Law and current

$$\mathrm{R} = \frac{\mathrm{V}}{I}\ [\Omega]$$
$$\mathrm{I} = \frac{dq}{dt}\ [A] = q_0 NAv_d$$
$$\mathrm{Q} = \mathrm{q_0}NAL\ [C] = q_0 NAv_d t$$

Current Density

$$\mathrm{I} = \iint \boldsymbol{J}\,d\boldsymbol{A} = \mathrm{JA}$$
$$\mathbf{J} = \mathrm{nq_0}\boldsymbol{v_d}\ [\mathrm{A/m^2}]$$

Resistivity/Conductivity

$$\rho = \frac{|\boldsymbol{E}|}{|\boldsymbol{J}|}\ [\Omega m]$$
$$\mathbf{J} = \frac{1}{\rho}\boldsymbol{E} = \sigma\boldsymbol{E}$$
$$\sigma = \frac{1}{\rho}\ [1/\Omega m]$$
$$\mathrm{R} = \frac{El}{JA} = \rho\frac{l}{A}$$

Power in Electric Fields

$$\mathrm{U} = \mathrm{qRI}$$
$$\text{Power} = \frac{\mathrm{dU}}{dt}$$
$$\text{Power} = \mathrm{I^2}R = \frac{\mathrm{V^2}}{R} = VI\ [W]$$

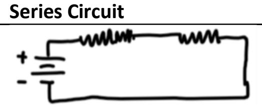
Biot-Savart Law

$$d\mathbf{B} = \frac{\mu_0 I(d\boldsymbol{S}\times\hat{r})}{4\pi r^2}$$
$$\mu_0 = 4\pi\cdot 10^{-7}\ [H/m]$$

Parallel Circuit

$$\mathrm{V_T} = V_1 = V_2 = \cdots$$
$$\mathrm{I_T} = I_1 + I_2 + \cdots$$
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$
$$\Sigma I = 0$$

Series Circuit



$$\mathrm{V_T} = V_1 + V_2 + \cdots$$
$$\mathrm{I_T} = I_1 = I_2 = \cdots$$
$$\mathrm{R_T} = R_1 + R_2 + \cdots$$
$$\Sigma V = 0$$

Magnetic Force

$$\mathbf{F_B} = \mathrm{q}(\mathbf{v}\times\mathbf{B}) = \mathrm{Bqv}\sin\theta$$

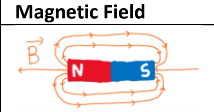
Θ is angle between $(\mathbf{v},\mathbf{B})$

Magnetic force on wire

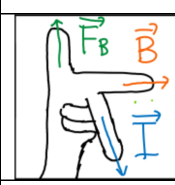
$$\mathbf{F_B} = \mathrm{I}(\mathbf{L}\times\mathbf{B}) = \mathrm{BIL}\sin\theta$$

Θ is angle between $(\mathbf{L},\mathbf{B})$

Magnetic Field



Measured in [T]



Magnetic field 'inside'?

$$\mathrm{I_{tot}} = \pi R^2 J_0$$
$$\mathrm{I_{enc}} = \iint \boldsymbol{J}\,d\boldsymbol{A} = \pi r^2 J_0$$
$$(\mathrm{dA} = \mathrm{rdrd}\emptyset)$$
$$\oint_C \boldsymbol{B}\,d\boldsymbol{S} = \mu_0 I_{enc} = 2\pi Br$$
$$\mathrm{B} = \frac{\mu_0 I_{tot} r}{2\pi R^2}$$

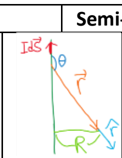
Ampere's Law

$$\oint_C \boldsymbol{B}\,d\boldsymbol{S} = \mu_0 I_{enc} = \oint_C Bcos(\boldsymbol{B},d\boldsymbol{S})\,dS$$
$$\mathrm{B} = \frac{\mu_0 I}{2\pi R}$$

Magnetic field due to current

Infinite wire

$$\mathrm{B} = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{\sin\theta}{r^2}dx$$
$$\mathrm{B} = \frac{\mu_0 I}{2\pi R}$$



Semi-infinite wire

$$\mathrm{B} = \frac{\mu_0 I}{4\pi}\int_0^{\infty}\frac{\sin\theta}{r^2}dx$$
$$\mathrm{B} = \frac{\mu_0 I}{4\pi R}$$

Centre of loop

$$\mathrm{B} = \frac{\mu_0 I\emptyset}{4\pi R}$$

$\emptyset$ - angle of loop (partial circular loop)

Force between two parallel currents

$$\mathrm{F} = \frac{\mu_0\ L\ I_a\ I_b}{2\pi d}$$
$$\frac{\mu_0}{2\pi} = 2\cdot 10^{-7}$$

Solenoids

$$\mathrm{I_{enc}} = nhI$$
$$\mu_0\mathrm{I_{enc}} = \mu_0 nhI = Bh$$
$$\mathrm{B} = \mu_0 nI$$
(n=number of loops)
(I=current on the loop)

Magnetic flux

$\Phi_B = \iint \boldsymbol{B}\ d\boldsymbol{A}$ [Wb]

$\Phi_B = BA = \mu_0 n I A$

EMF (Faraday/Lenz)

$$\varepsilon = -(N)\frac{\partial \Phi_B}{\partial t} = -(N)\frac{\partial}{\partial t}\iint \boldsymbol{B}\ d\boldsymbol{A}$$

Faraday's law – emf is proportional to rate of change of magnetic flux
Lenz' law – Direction of induced current and emf are against induced magnetic field

EMF/current in loops

$\varepsilon = (-)(N)Bv_x l$

$I = \frac{(N)Bv_x l}{R}$

Current is induced by induced magnetic field

Power dissipated

$$P = VI = \frac{V^2}{R} = I^2R = \frac{N^2B^2v^2l^2}{R}$$

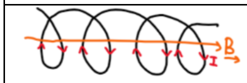
Series Inductor

$V = L\frac{dI}{dt}$

$L_T\frac{dI}{dt} = \frac{dI}{dt}(L_1 + L_2 + \cdots)$

$L_T = L_1 + L_2 + \cdots$

Inductor



$B = \mu_0 n I$
Solenoid is an inductor.

Inductance

$L = \frac{N\Phi_B}{I}$ [H]

$L = n^2 A l \mu_0$

$L = \frac{BAN}{I} = NAl\mu_0$

$(N = nl)$

Self-induction (ε_L)

$\varepsilon_L = -N\frac{\partial\Phi_B}{\partial t} = -L\frac{d}{dt}I(t)$

$V_L = -\varepsilon_L = L\frac{d}{dt}I(t)$

Energy in magnetic field

$U = qV$

$dU = Vdq = qdV$

Power $= \frac{dU}{dt} = IV$

$dU = ILdI$

$U = \frac{1}{2}LI^2$

Ohm's law & power

$V(t) = R\,I(t)$

$P(t) = V(t)I(t) = R(I(t))^2 = \frac{(V(t))^2}{R}$

R=0 > $V(t) = R\,I(t) = 0$

R=∞ > $I(t) = \frac{V(t)}{R} = 0$

Parallel inductor

$V = L\frac{dI}{dt}$

$\frac{V}{L_T} = \frac{V}{L_1} + \frac{V}{L_2} + \cdots$

$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots$

Basic Quantities

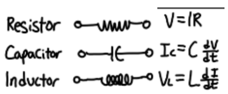
$I(t) = \frac{dq(t)}{dt}$

$q(t) = \int_{-\infty}^{t} I(x)\,dx$

$V(t) = \frac{W(t)}{q}$

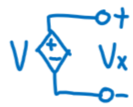
$P = IV = I^2R = \frac{V^2}{R}$

2 terminal devices



Dependent voltage source

Voltage-control
$V = \alpha V_x$



Current-control
$V = rI_x$

Dependent current source

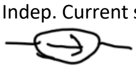
Voltage-control
$I = gV_x$

Current-control
$I = \beta I_x$

Independent sources

Indep. Voltage source

Indep. Current source



Node

More than 2 elements join in one point

Branch

Section that contains only one element

Loop

Closed path: start from one node, visit other nodes (only once), return

Conductance

$G = \frac{1}{R} = \frac{I}{V}$ [S]

$I(t) = GV(t)$

$P(t) = \frac{(I(t))^2}{G} = G(V(t))^2$

Kirchhoff's Laws

Current Law (node)
$\sum I_{enter} = \sum I_{leave}$
Voltage Law (loop)
$\sum V_{drop} = \sum V_{rise}$

Linearity

$V_{out} = \alpha_1 V_1 + \alpha_2 V_2 + \cdots + \beta_1 I_1 + \beta_2 I_2 + \cdots$

Linear equations!

Voltage divider

a.k.a series circuit

$I = \frac{V}{\sum R}$

$V_i = V_{source}\frac{R_i}{\sum R}$

Current divider

a.k.a parallel circuit

$V = I_s\left(\sum\frac{1}{R}\right)^{-1}$

$I_i = I_{source}\frac{1/R_i}{(\sum(1/R))}$

Open circuit

- No current
- Voltage exists
- Disconnected wire

Short circuit

- No voltage across
- Current goes to R=0
- Connected wire

Nodal Analysis

(#Eq) = (#Node) - 1 - N_v
N_v is independent or dependent voltage sources

- Voltages at nodes
- Kirchhoff's current law
- One ground node set

Mesh Analysis

(#Eq) = (#Loop) – N_I
= Branch – Node + 1 - N_I
N_I is independent or dependent current sources

- Voltages surround loops
- Kirchhoff's voltage law
- Calculate for each loop

Supernode

- Two nodes connected by voltage source
- None of them are reference node

Supermesh

- Used when current source is shared between two loops

Superposition

Deactivate all sources except one source
- Holds for I, V but not power

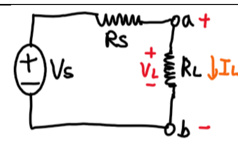
Deactivate source?
- Voltage source > short
- Current source > open

I_1 – current through R due to V_1 only
I_2 – current through R due to V_2 only

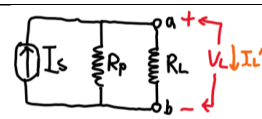
$I_{tot} = I_1 + I_2 + \cdots$

$P_{tot} = I_{tot}^2 R = R(I_1 + I_2 + \cdots)^2$

Source transformation



$V_L = V_S\frac{R_L}{R_L + R_S}$



To make this equivalent,
$V_s = I_s \times R_s$
$R_s = R_p$

Equivalence