University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, APRIL 2002
First Year - Programs 1,2,3,4,6,7,8,9

MAT 187H1S
Calculus II

Examiners

D. Burbulla

C. Lun

M. Pugh

B. Wagneur

SURNAME ______________________

GIVEN NAME ______________________

STUDENT NO. ______________________

SIGNATURE ______________________

INSTRUCTIONS:

Non-programmable calculators permitted.

No other aids permitted.

Answer all questions.

Present your solutions in the space provided; use the back of the **same** page if more space is required.

TOTAL MARKS: 100

The value for each question is shown in parentheses after the question number.

MARKER'S REPORT	
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
Q8	
TOTAL	

1. (10 marks; 2 marks for each part) Indicate in the blank to the right, which one of the integral expressions A to O listed below, equals the value of the quantity described on the left:

The length of the spiral with polar equation $r = \theta$, for $0 \leq \theta \leq \pi$.	__________
The distance travelled by a particle along the curve with parametric equations $x = \cos(2\theta); y = \sin(2\theta); z = \theta^2$, for $0 \leq \theta \leq \pi$, where θ represents time.	__________
The length around one petal of the 3-leaved rose with polar equation $r = \sin(3\theta)$	__________
The area within the cardioid with polar equation $r = 1 - \cos\theta$	__________
The area of the region inside the curve with polar equation $r = 2 + \cos\theta$, but outside the circle with polar equation $r = 5\cos\theta$.	__________

Pick your answers from:

A. $\int_0^\pi \sqrt{1+\theta}\, d\theta$	B. $2\int_0^\pi \sqrt{1+\theta}\, d\theta$	C. $\int_0^\pi \sqrt{1+\theta^2}\, d\theta$
D. $2\int_0^\pi \sqrt{1+\theta^2}\, d\theta$	E. $\int_0^\pi (1-\cos\theta)^2\, d\theta$	F. $2\int_0^\pi (1-\cos\theta)^2\, d\theta$
G. $2\int_0^\pi (1-\cos(3\theta))^2\, d\theta$	H. $\int_0^\pi (1-\cos(3\theta))^2\, d\theta$	I. $\int_0^\pi (1+8\cos^2(3\theta))^2\, d\theta$
J. $\int_0^{\pi/3} (1+8\cos^2(3\theta))^2\, d\theta$	K. $\int_0^\pi \sqrt{1+8\cos^2(3\theta)}\, d\theta$	L. $\int_0^{\pi/3} \sqrt{1+8\cos^2(3\theta)}\, d\theta$

M. $\int_{\pi/3}^{\pi} (2+\cos\theta)^2\, d\theta - \int_{\pi/3}^{\pi/2} 25\cos^2\theta\, d\theta$

N. $\int_{\pi/3}^{\pi} \left((2+\cos\theta)^2 - 25\cos^2\theta\right)\, d\theta$

O. $\int_{\pi/3}^{\pi/2} \left((2+\cos\theta)^2 - 25\cos^2\theta\right)\, d\theta$

2. (10 marks) Find the exact sum — not a decimal approximation — of each of the following infinite series. Put your answer in the blank to the right.

(a) (2 marks) $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n}{n!}$ __________

(b) (3 marks) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{5}\right)^n$ __________

(c) (5 marks) $\displaystyle\sum_{n=0}^{\infty} \frac{n+1}{5^n}$ __________

3. (15 marks) Find the critical points of

$$f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$$

and at each critical point determine whether f has a relative maximum point, a relative minimum point, or a saddle point.

4. (10 marks) Plot the curve in the xy-plane with parametric equations

$$x = t^2 \,;\; y = t^3 - 3t, \text{ for } -2 \leq t \leq 2.$$

Be sure to label any maximum or minimum points, and to indicate when the graph is concave up and when it is concave down.

5. (15 marks)

(a) (5 marks) Solve for y as a function of x, if

$$\frac{dy}{dx} + y \tan x = \cos^2 x, \text{ and } y = 5 \text{ when } x = 0.$$

(b) (10 marks) In 1992, ten castaways were stranded on an island. After finding the fresh water, the mango groves, and the man-eating tigers, they settled in for a long stay. Suppose the population, P, of castaways at time t, satisfies the differential equation

$$\frac{dP}{dt} = \frac{1}{1000} P(100 - P),$$

where t is measured in years since 1992.

(i) (5 marks) How many castaways are on the island now, in the year 2002?

(ii) (5 marks) Sketch a graph of P, for $t \geq 0$. What will happen to the population of castaways as more nd more time passes?

6. (15 marks; 5 marks for each part)

(a) Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-2)^{n+1} x^n}{\sqrt{n}}$$

Don't forget to check convergence of the series at the endpoints of the interval!

(b) Write down the first four nonzero terms of the Maclaurin series of $\mathrm{Tan}^{-1}x$.
(Hint: what is $\int \frac{1}{x^2+1}\,dx$?)

(c) What is the maximum possible error if the fifth degree Taylor Polynomial of $\mathrm{Tan}^{-1}x$ about $x = 0$ is used to approximate $\mathrm{Tan}^{-1}x$, for $0 \leq x \leq \frac{1}{2}$?

7. (10 marks) Do the following infinite series converge or diverge? Justify your answer.

(a) (3 marks) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + \ln n}{n^4 - 2n + 3}$

(b) (4 marks) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

(c) (3 marks) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

8. (15 marks)

(a) (10 marks) At what angle to the horizontal should a baseball with initial speed 30 m/sec be thrown from the top of a 100 m high cliff, if the baseball is to hit a target (on the flat ground below) exactly 25 m from the base of the cliff? (Assume the acceleration due to gravity is 9.8m/sec^2; ignore air resistance and any other forces.)

(b) (5 marks) Find $\int_0^\infty \frac{1}{\sqrt{e^{ax} - 1}}\, dx$, for $a > 0$.