University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, APRIL 2004
First Year - CIV, CHE, IND, LME, MEC, MSE

MAT 187H1S - CALCULUS II

Exam Type: A

SURNAME _____

GIVEN NAME _____

STUDENT NO. _____

SIGNATURE _____

Examiners

D. Burbulla

M. Hamilton

P. Milman

P. Rahimi

INSTRUCTIONS:

Non-programmable calculators permitted.

No other aids permitted.

Answer all questions. Make sure this exam contains 11 pages.

Present your solutions in the space provided; use the back of the **same** page if more space is required.

TOTAL MARKS: 100

The value for each question is shown in parentheses after the question number.

MARKER'S REPORT	
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
Q8	
Q9	
TOTAL	

1. [10 marks] Find and classify the critical points of $f(x, y) = 4xy - 2x^4 - y^2$.

2. [10 marks] For a certain particle travelling in space, the acceleration $\mathbf{a}$ at time t is given by

$$\mathbf{a} = 2\mathbf{j} + 2t\,\mathbf{k}.$$

Given that $\mathbf{r}_0 = \mathbf{0}$ and $\mathbf{v}_0 = 2\mathbf{i}$, find:

(a) (5 marks) $\mathbf{v}$ and $\mathbf{r}$ at time t.

(b) (5 marks) the total distance travelled by the particle for $0 \leq t \leq 3$.

3. [10 marks] Find general solutions, y in terms of x, for each of the following differential equations:

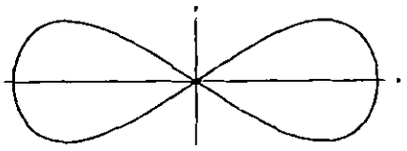
(a) (4 marks) $4\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 9y = 0.$

(b) (6 marks) $x^2\frac{dy}{dx} - 2xy = x^2 + 1.$

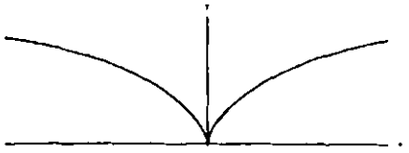
4. [12 marks] Match the following graphs to their parametric equations.

A.

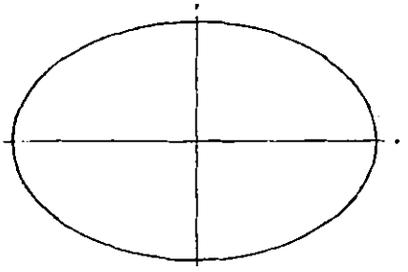
B.



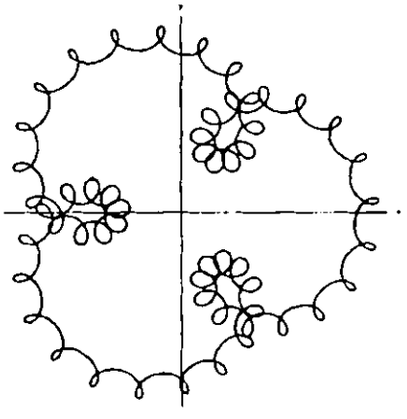
C.



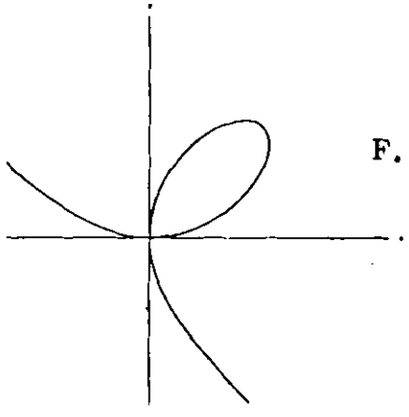
D.



E.



F.



— $\begin{cases} x = 2\cos t \\ y = 3\sin t \end{cases}$ — $\begin{cases} x = t - \sin t \\ y = 1 - \cos t \end{cases}$ — $\begin{cases} x = 3\cos t \\ y = 2\sin t \end{cases}$

— $\begin{cases} x = 3\cos t \\ y = \sin 2t \end{cases}$ — $\begin{cases} x = \cos 2t \\ y = 3\sin t \end{cases}$ — $\begin{cases} x = \dfrac{t^3}{1+t^2} \\ y = \dfrac{t^2}{1+t^2} \end{cases}$

— $\begin{cases} x = \dfrac{t}{1+t^3} \\ y = \dfrac{t^2}{1+t^3} \end{cases}$ — $\begin{cases} x = \dfrac{t^2}{1+t^3} \\ y = \dfrac{t}{1+t^3} \end{cases}$ — $\begin{cases} x = \cos t + \frac{1}{2}\cos 4t + \frac{1}{10}\sin 50t \\ y = \sin t + \frac{1}{2}\sin 4t + \frac{1}{10}\cos 50t \end{cases}$

5. [10 marks] Do the following infinite series converge or diverge? Justify your answer.

(a) (3 marks) $\sum_{n=1}^{\infty} \frac{2+\cos n}{n}$

(b) (3 marks) $\sum_{n=0}^{\infty} \frac{\arctan n}{1+n^2}$

(c) (4 marks) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!}{n^n}$

6.(a) (5 marks) Approximate $\int_0^{1/2} \frac{1}{(1+x^5)^{3/2}}\,dx$ correct to within 10^{-4}.

6.(b) (5 marks) Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x+2)^n}{2^n\sqrt{n+1}}$.

7. [12 marks] Find the exact value of each of the following:

(a) (6 marks) $\int_3^{\infty} \frac{1}{\sqrt{x}(1+x)}\,dx$

(b) (6 marks) $\sum_{n=0}^{\infty} \frac{2^{n+2}}{(n+2)n!}$

8. [10 marks] As the salt KNO_3 dissolves in methanol, the number $x(t)$ of grams of salt in solution after t seconds satisfies the differential equation

$$\frac{dx}{dt} = (0.8)x - (0.004)x^2.$$

(a) (7 marks) If 50 grams of salt are dissolved at $t = 0$, how long will it take for another 50 grams of salt to dissolve?

(b) (3 marks) What is the maximum amount of salt that will ever dissolve in the methanol?

9. [16 marks] The polar graph called a *sunburst* has polar equation

$$r = 1 + \sin^2(n\theta), \text{ for } 0 \leq \theta \leq 2\pi.$$

(a) (8 marks) Plot the sunburst for $n = 1, 2$ and 3.

(b) (8 marks) Show that the area enclosed by the sunburst is independent of n, if n is a positive whole number.