MIE100S: Dynamics – April 26, 2023

Final Exam – 2.5 hours

E-mail address: _____ **@mail.utoronto.ca**

First name (please write as legibly as possible within the boxes)

Last name

Student ID number

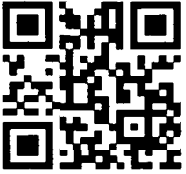|															

- **You must *neatly* show *all* rough work to earn credit for each answer.**
- This answer booklet contains 16 pages, including this cover page, **printed two-sided.**
- If you run out of room answering any of these questions, then *clearly indicate* that your solution is continued on page 15 and 16.
- **Do not** tear any pages from this booklet.
- Avoid folding or tearing any of the sheets to the extent possible.
- **Place your student ID at the front, right-hand corner of your desk.**

Problem	Marks	Page
1a, 1b, 1c	20	2-4
2a, 2b, 2c	20	5-7
3a, 3b, 3c	20	8-10
4a,4b	20	11-12
5a, 5b	20	13-14
Spare Pages		15-16

Permitted Aids:
- Non-communicating/non-programmable calculator: Casio FX-991 (EX recommended; any suffix is acceptable, including ES, PLUS, ES PLUSC, MS, MSPLUS2). Sharp EL-W516 (any suffix is acceptable, including TBSL, XG, XGB-SL)
- One 8 ½" x 11" aid sheet, any colour, brought to the test by the student. You may write on both sides of the sheet.
- Ruler, Protractor

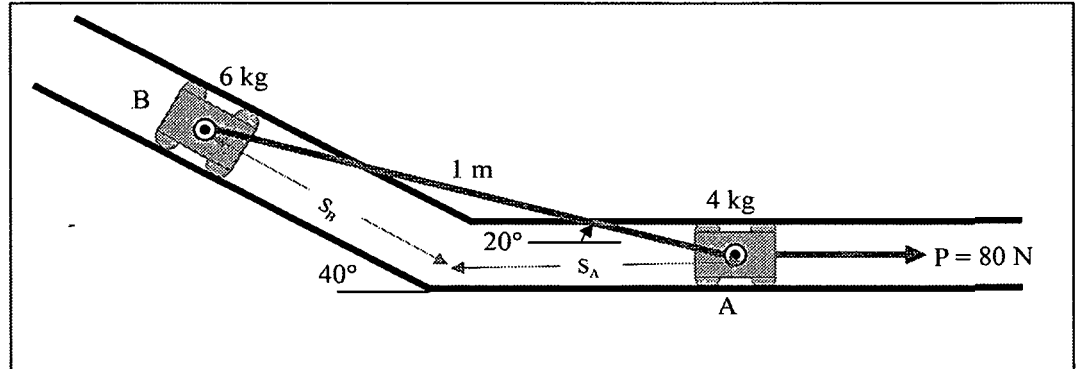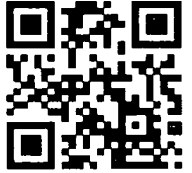**Present your complete solution in the space provided**

Question 1a

The sliders A and B are connected by a light rigid bar and move with negligible friction in the slots, both of which lie in a horizontal plane. For the position shown, the velocity and acceleration of A is 0.8 m/s to the right and 2.94 m/s^2 to the left, respectively.
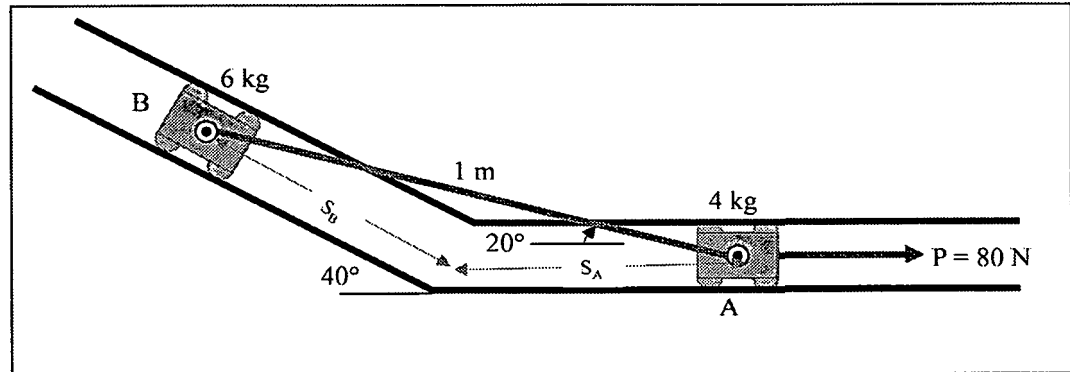
What are the distances s_A and s_B? [4 marks]

Question 1b

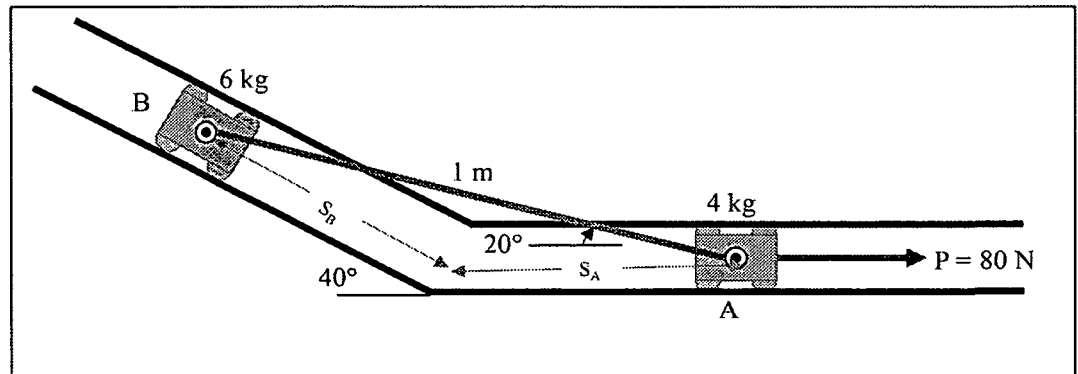What is the magnitude of the velocity of slider B, v_B?

[8 marks]

Question 1c
What is the magnitude of the force T in the bar?
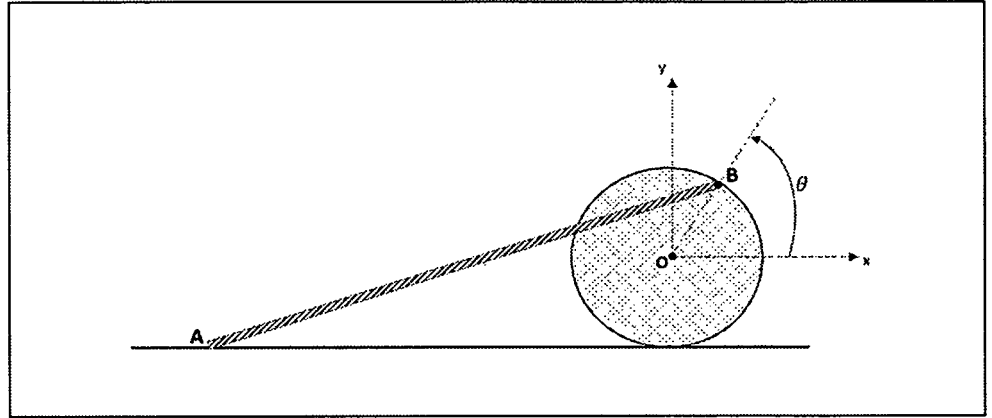
[8 marks]

Question 2a

The wheel of radius $R =$ 0.21 meters rolls without slipping. The bar AB of length 0.85 meters is pinned to the edge of the wheel at point "B". Point "A" of the bar drags horizontally along the ground.

Use the coordinate system specified in the diagram; the line $\overline{OB}$ makes an angle θ with respect to the x-axis, as shown. Watch your plus/minus signs!

The centre of the wheel is at the origin of the coordinate system.

Suppose $\theta = \frac{\pi}{2}; \vec{\omega} = -2\frac{rad}{s}\vec{k}; \vec{\alpha} = +3.5\frac{rad}{s^2}\vec{k}$.

What is the distance to the instantaneous center of zero velocity of bar AB from point A at this instant? **[6 marks]**

Question 2b
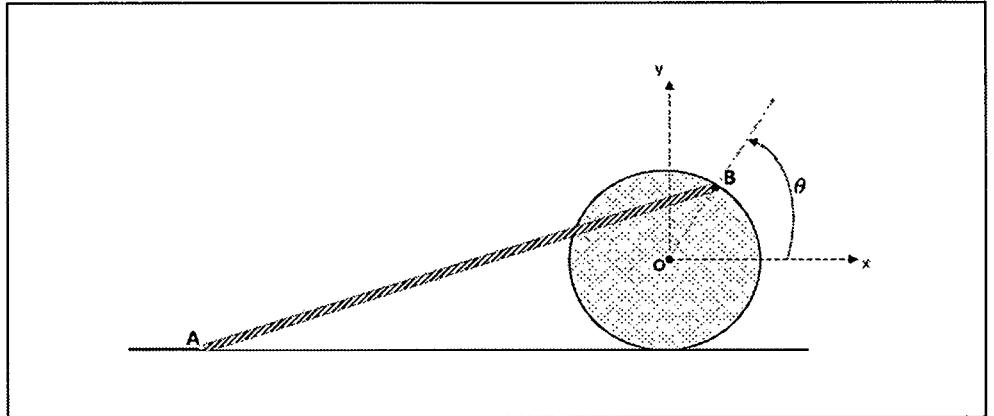
Suppose $\theta = \frac{\pi}{2}$; $\vec{\omega} = -2\frac{rad}{s}\vec{k}$; $\vec{\alpha} = +3.5\frac{rad}{s^2}\vec{k}$.

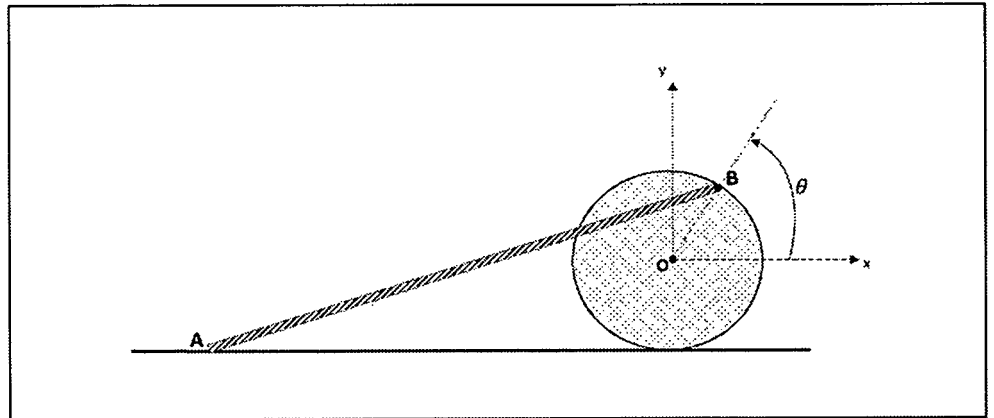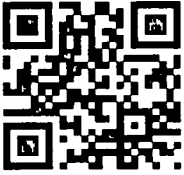Determine the acceleration of point B in x,y coordinates at this instant.

[4 marks]

Question 2c

Suppose $\theta = -\frac{\pi}{2}; \vec{\omega} = +4\frac{rad}{s}\vec{k}; \vec{\alpha} = -5\frac{rad}{s^2}\vec{k}$.

Find the angular acceleration α of bar AB in x-y coordinates at this instant. Also find the acceleration of point A in x-y coordinates at this instant. **[10 marks]**

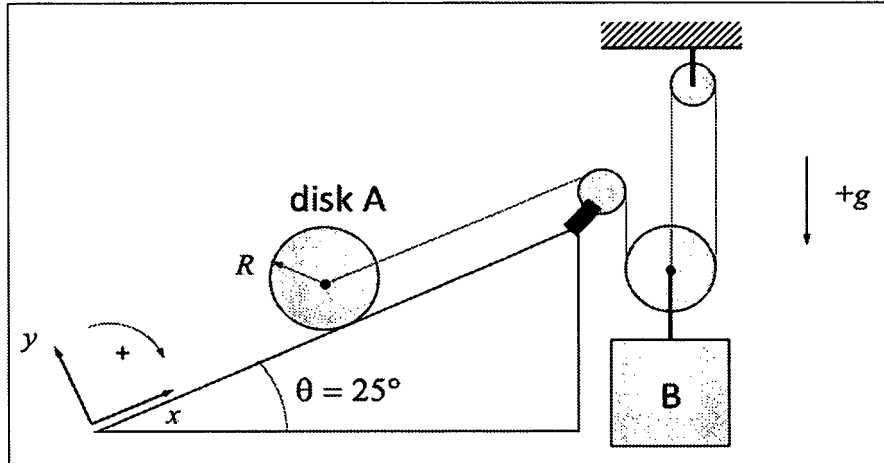## Question 3a
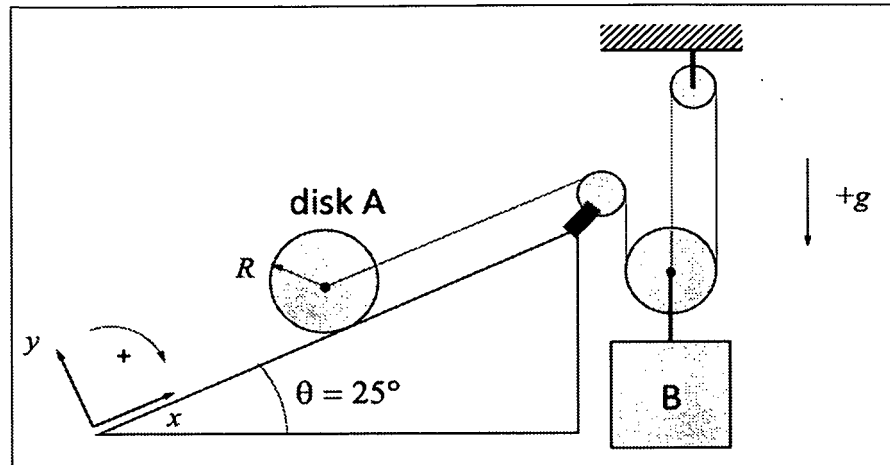

A circular disk of mass $m_A = 16\ kg$ and radius, $R = 0.45m$, is rolling **without slip** on an incline with a slope of $\theta = 25^o$. A rope is attached to the centre of the disk and is wound around a pulley system. At this instant, the disk has an angular acceleration α and angular velocity ω. Block B hangs vertically from the large pulley.
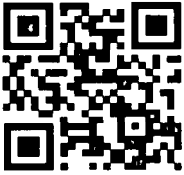
The rope and pulleys all have negligible mass.

Given $\alpha = 7.42\frac{rad}{s^2}$ _clockwise_, find the tension in the rope. (6 marks)

Question 3b

Given $m_B = 25kg$, determine the magnitude and direction of α and the corresponding tension in the rope. (9 marks)
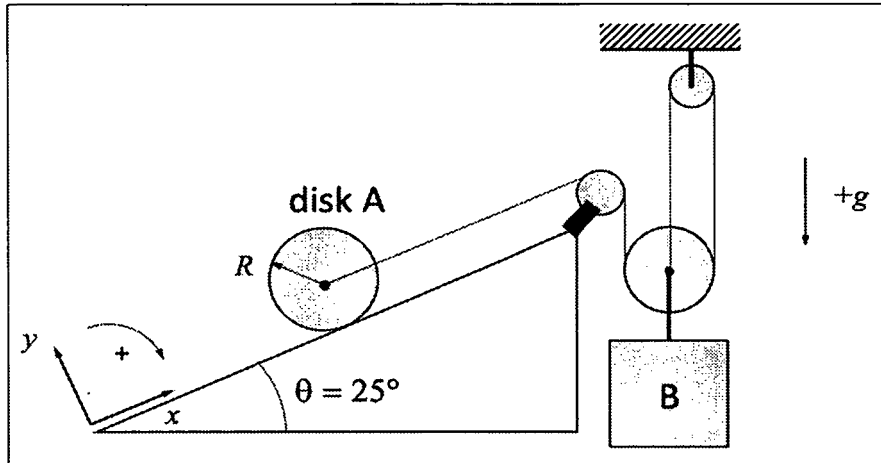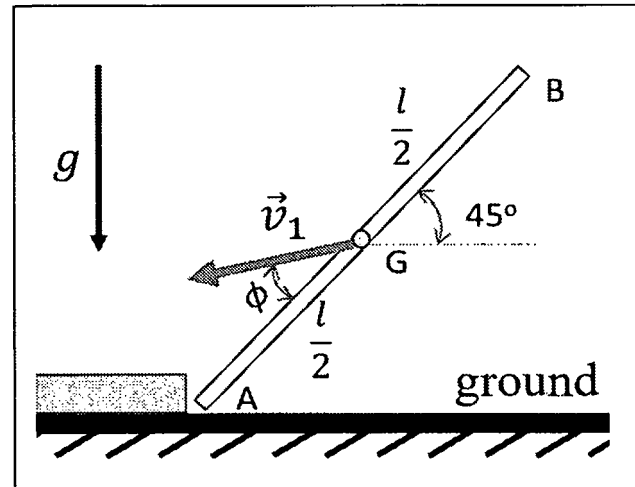
Question 3c

Given $m_B = 25kg$, what must be the smallest possible coefficient of static friction, $\mu_{s\,min}$ that ensures the disk rolls without slip? (5 marks)

Question 4a

The uniform slender bar of mass= 5 kg and length $l = 2\ m$ has no angular velocity as end A strikes the ground against the stop with no rebound. $v_1 = 10\ m/s$ with an angle $\phi = 25^o$ as shown just before the strike.

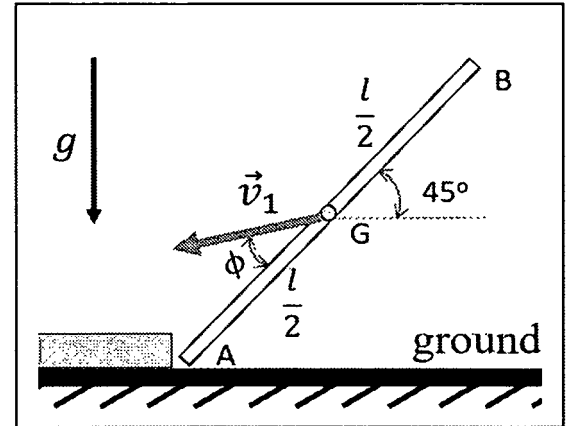What is the magnitude of the angular velocity of the bar right after the strike? **[10 marks]**

Question 4b

Given an initial velocity $v_1 = 10\ m/s$ just before striking the corner, will the bar rotate about A to the vertical position? Show all your work to prove either "yes" or "no" to the above question. **[10 marks]**
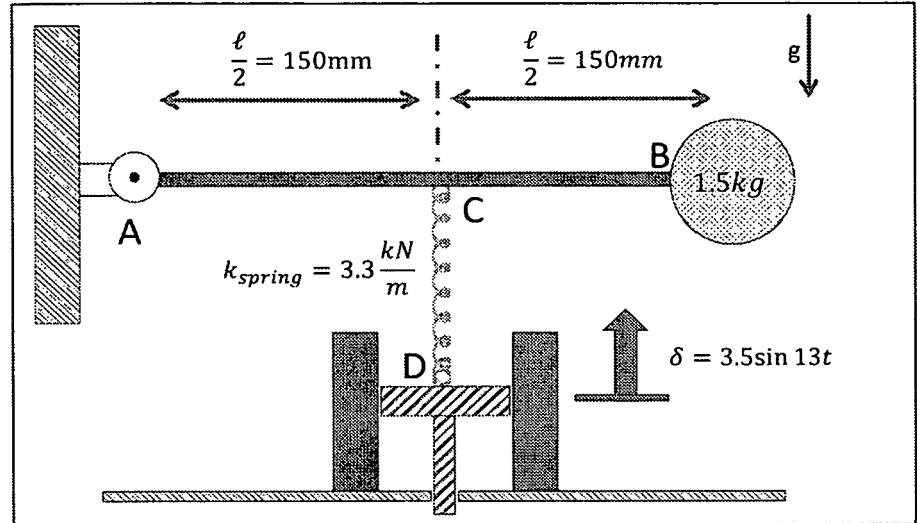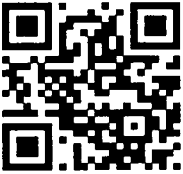
Question 5a and 5b

A 1.5-kg sphere B is attached to bar AB. Bar AB is pinned at A, and there is a spring ($k_{spring} = 3.3\ kN/m$) connecting the bar at point C bar to the moving support D. The bar has zero mass and is horizontal when the system is at rest.

Support D experiences a vertical displacement $\delta = \delta_m sin\ \omega_f t$ where $\delta_m = 3.5\ mm$ and $\omega_f = 13\ rad/s$,

- Show the Free Body Diagram [6 marks]
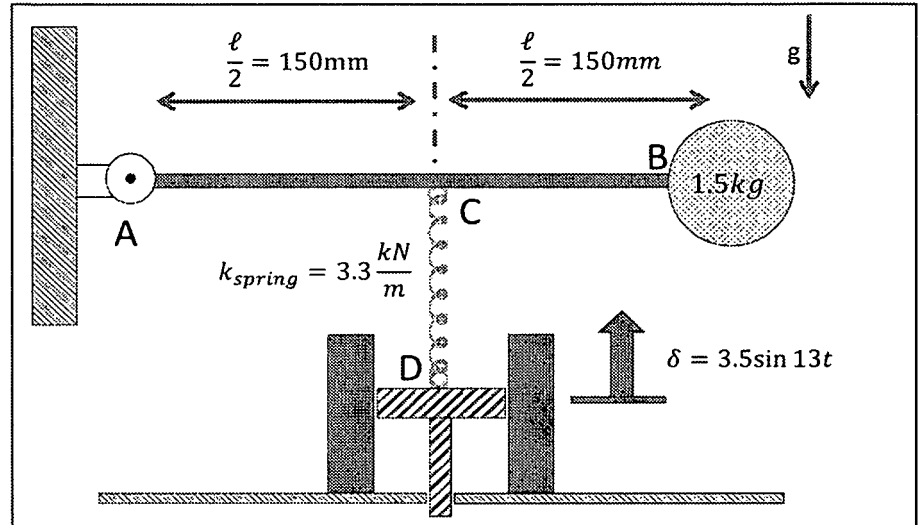- Determine the natural frequency of the system [8 marks]

$\frac{\ell}{2} = 150mm$

$\frac{\ell}{2} = 150mm$

g

B

1.5kg

A

$k_{spring} = 3.3\frac{kN}{m}$

C

D

$\delta = 3.5sin\ 13t$

Question 5c

Determine the magnitude of the maximum acceleration of sphere B **once the system is at steady-state. [6 marks]**

$$\frac{\ell}{2} = 150\text{mm} \qquad \frac{\ell}{2} = 150mm$$

$$k_{spring} = 3.3 \frac{kN}{m}$$

$$\delta = 3.5\sin 13t$$

Spare Page 1

Spare page in case **you ran out of room on one of the questions – Indicate clearly which question you are writing.**

Spare Page 2

Spare page in case **you ran out of room on one of the questions – Indicate clearly which question you are writing**