MAT187 - Calculus II - Winter 2019

Term Test 2 - March 19, 2019

MULTIPLE-CHOICE QUESTIONS

AND

FORMULA SHEET

Instructions:

- DO NOT OPEN until instructed to do so.
- **THIS PART WILL NOT BE COLLECTED.**
- This part contains **??** pages.

MULTIPLE-CHOICE PART. **(13 marks)**

ANSWER THESE QUESTIONS ON **PAGE 10** OF THE TEST.

1. (2 marks) What is the solution of the ODE

$$\ln(x)y' + \frac{1}{x}y = x \quad ?$$

(A) $y = \frac{1}{\ln(x)}\left(\frac{x^2}{2} + C\right)$

(B) $y = \frac{1}{x}\left(\frac{x^2}{2} + C\right)$

(C) $y = \frac{1}{\ln(x)}\left(\frac{x^3}{3} + C\right)$

(D) $y = \frac{1}{x}\left(\frac{x^3}{3} + C\right)$

2. (2 marks) Consider the problem:

$$\begin{cases} y'' + 2y' + 2y = 0 \\ y(0) = 0 \end{cases}$$

Which of the following conditions can we add to still have **infinitely many solutions**?

(A) $y'(0) = 0$

(B) $y'\left(\frac{\pi}{2}\right) = 0$

(C) $y\left(\frac{\pi}{2}\right) = 0$

(D) $y(\pi) = 0$

(E) None of the above options allows for infinitely many solutions.

3. (2 marks) Consider the ODE

$$y'' + 2y' + 2y = (t^2 + 1)e^{-t} + t\sin(t)$$

Which of the following terms is **NOT** necessary to form a particular solution?

(A) e^{-t}

(B) te^{-t}

(C) t^2e^{-t}

(D) $\sin(t)$

(E) $t^2\sin(t)$

4. (2 marks) Consider the function

$$f(x) = \sum_{n=0}^{\infty} \frac{n!}{(2n)^n}(x - e)^n.$$

Select the **LARGEST** interval of convergence for this function.

(A) $x \in (-\infty, \infty)$

(B) $x \in (-e, 3e)$

(C) $x \in (e - 1, e + 1)$

(D) $x \in \left(\frac{e}{2}, \frac{3e}{2}\right)$

(E) $x = e$

5. (2 marks) Consider the function

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n-20} n!} (x-5)^n.$$

Select **the one** correct option.

(A) $f^{(23)}(5) = -\dfrac{1}{3^3}$

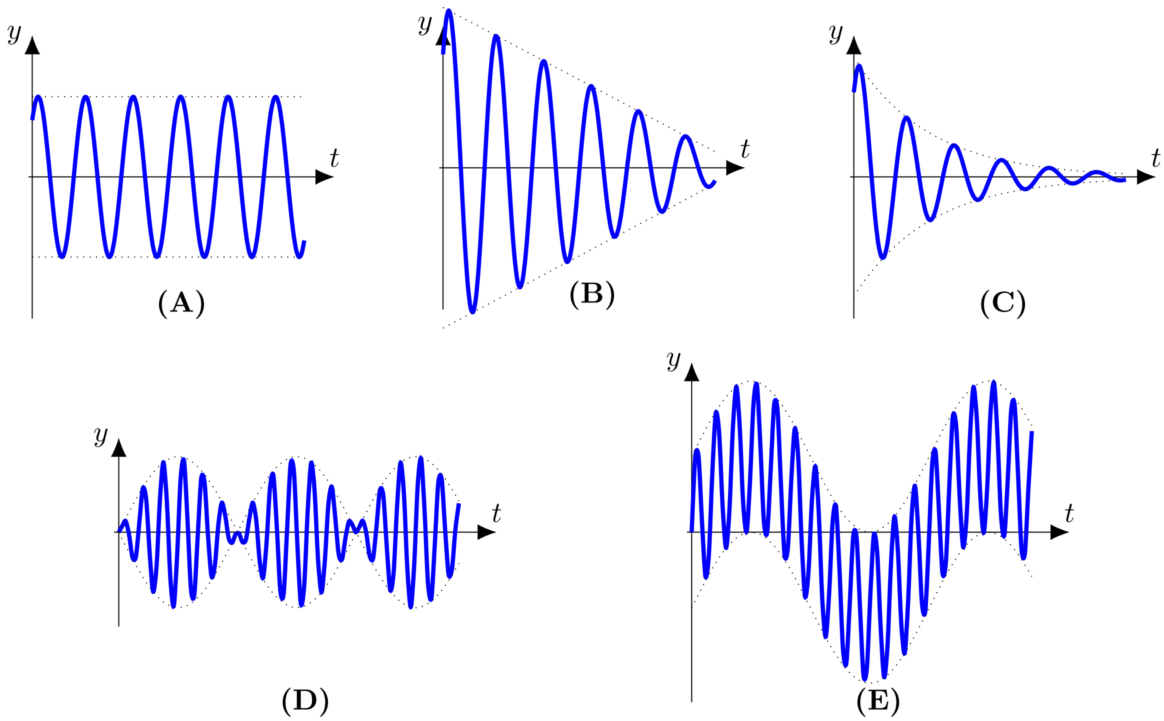
(B) $f^{(23)}(5) = -\dfrac{1}{3^3 \cdot 3!}$

(C) $f^{(23)}(5) = -\dfrac{1}{3^3 \cdot 23!}$

(D) $f^{(23)}(5) = -\dfrac{1}{3^{23}}$

(E) $f^{(23)}(5) = -\dfrac{1}{3^{23} \cdot 23!}$

For questions **??–??**, consider the graphs:



6. (1 mark) Which of the graphs **(A)–(E)** represents a solution of

$$y'' + 25y = 0 \quad ?$$

7. (1 mark) Which of the graphs **(A)–(E)** represents a solution of

$$y'' + 25y = \cos(4.8t) \quad ?$$

8. (1 mark) Which of the graphs **(A)–(E)** represents a solution of

$$y'' + 8y' + 17y = 0 \quad ?$$

You can use this page for scratch work.

FORMULA SHEET FOR MAT187

Important Formulas.

- $\cos^2(x) = \dfrac{1+\cos(2x)}{2}$
- $\sin^2(x) = \dfrac{1-\cos(2x)}{2}$
- $\sin(x) = \cos\left(\dfrac{\pi}{2} - x\right)$
- $\lim\limits_{n\to\infty}\left(1+\dfrac{a}{n}\right)^n = e^a$

Trigonometric Integrals.

- $\displaystyle\int \sec(x) = \ln\big(\sec(x)+\tan(x)\big) + C$
- $\displaystyle\int \sec^3\,dx = \int \frac{\cos(x)}{\big(1-\sin^2(x)\big)^2}\,dx$
- $\displaystyle\int \frac{1}{1+x^2}\,dx = \arctan(x) + C$

Applications of integration.

- Arc length for $y = f(x)$ $\qquad \displaystyle\int_a^b \sqrt{1+\big(f'(x)\big)^2}\,dx$
- Area of a surface of revolution $y = f(x)$ revolved around $x-$axis $\qquad \displaystyle\int_a^b 2\pi f(x)\sqrt{1+\big(f'(x)\big)^2}\,dx$

Numerical Integration.

- Left-Hand Rule $\quad \displaystyle\int_a^b f(x)\,dx \approx L_n = \sum_{i=1}^n f(x_{i-1})\Delta x, \qquad |E_L| \leqslant \frac{b-a}{2}(\Delta x)\max_{x\in[a,b]}\left|f'(x)\right|$
- Right-Hand Rule $\quad \displaystyle\int_a^b f(x)\,dx \approx R_n = \sum_{i=1}^n f(x_i)\Delta x, \qquad |E_R| \leqslant \frac{b-a}{2}(\Delta x)\max_{x\in[a,b]}\left|f'(x)\right|$
- Midpoint Rule $\quad \displaystyle\int_a^b f(x)\,dx \approx M_n = \sum_{i=1}^n f\left(\frac{x_{i-1}+x_i}{2}\right)\Delta x, \qquad |E_M| \leqslant \frac{b-a}{24}(\Delta x)^2\max_{x\in[a,b]}\left|f''(x)\right|$
- Trapezoid Rule $\quad \displaystyle\int_a^b f(x)\,dx \approx T_n = \sum_{i=1}^n \frac{f(x_{i-1})+f(x_i)}{2}\Delta x, \qquad |E_T| \leqslant \frac{b-a}{12}(\Delta x)^2\max_{x\in[a,b]}\left|f''(x)\right|$

Differential Equations.

- Linear DE: $y' + p(t)y = g(t)$ $\qquad \mu(t) = e^{\int p(t)\,dt}$

$$y = \frac{1}{\mu(t)}\int \mu(t)g(t)\,dt + \frac{C}{\mu(t)}$$

- Separable DE: $g(y)\dfrac{dy}{dt} = h(t)$ $\qquad \displaystyle\int g(y)\,dy = \int h(t)\,dt$

Power Series.

- Taylor Theorem
$$\begin{cases} f(x) = p_n(x) + R_n(x) \\ p_n(x) = \displaystyle\sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k \\ R_n(x) = \dfrac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1} \end{cases}$$

- Binomial Series
$$(1+x)^p = \sum_{k=0}^{\infty} \frac{p(p-1)\cdots(p-k+1)}{k!}x^k$$

- Sine Series
$$\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}x^{2k+1}$$

- Cosine Series
$$\cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}x^{2k}$$

- Logarithmic Series
$$\ln(1-x) = -\sum_{k=1}^{\infty} \frac{1}{k}x^k$$