

MIE 200F - Quiz number 3b - October 5/99
quiz duration = 25 minutes

A turntable consists of a disk of radius 18 cm, oriented in the horizontal plane, turning about a vertical axis. It starts to rotate from rest at time $t = 0$, accelerating in a counterclockwise direction with an angular acceleration equal to $\alpha = 0.3 \text{ s}^{-2}$. A coin of mass 10 grams is lying on the surface of the disk, adjacent to the rim.

- (a) What is the magnitude of the total acceleration of the coin at $t = 2$ seconds?
- (b) How much work is done on the coin by the disk in the first 2 seconds?

$$(a) \ddot{\theta} = 0.3 \text{ s}^{-2}$$

$$\dot{\theta} = \int \ddot{\theta} dt = (0.3)(2) = 0.6 \text{ rad/s}$$

$$r = 0.18 \text{ m}, \dot{r} = \ddot{r} = 0$$

$$a_r = \dot{r}^2 - r \dot{\theta}^2 = -(0.18)(0.6)^2 = -0.065 \text{ m/s}^2$$

$$a_\theta = r \ddot{\theta} + 2\dot{r}\dot{\theta} = (0.18)(0.3) = 0.054 \text{ m/s}^2$$

$$|\vec{a}| = \sqrt{a_r^2 + a_\theta^2} = \sqrt{0.065^2 + 0.054^2} = 0.0844 \text{ m/s}^2$$

$$(b) \dot{\theta} = 0.3t$$

$$\theta = \int \dot{\theta} dt = \frac{0.3}{2} t^2 = \frac{0.3}{2} (2^2) = 0.6 \text{ radians.}$$

⇒ distance travelled in θ direction = θr
 $= (0.6)(0.18 \text{ m}) = 0.108 \text{ m.}$

Frictional force in θ direction
 $= (m)(a_\theta) = (10^{-2} \text{ kg})(0.054) = 5.4 \times 10^{-4} \text{ Newtons.}$

$$\text{work} = \int \vec{F} \cdot d\vec{s} = (5.4 \times 10^{-4})(0.108)$$

$$= 5.83 \times 10^{-5} \text{ Joules.}$$

Note: no displacement in r direction.

Or // $T_{\text{final}} = \frac{1}{2}mv^2 = T_1 + U_1 \rightarrow 2$

$$(\frac{1}{2})(0.01)(v^2) = (\frac{1}{2})(0.01)(r \dot{\theta})^2 = (\frac{1}{2})(0.01)(0.18)^2 (0.6)^2 = 5.83 \times 10^{-5}$$

$$g = 9.81 \text{ m/s}^2 \downarrow$$

