UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

ECE 110H1 S -- ELECTRICAL FUNDAMENTALS
FINAL EXAMINATION, APRIL 23, 2018, 6:30 pm

First Year -- Computer, Electrical, Industrial, Mechanical, Materials, and Track One Engineering Programs.
Examiners – S. Aitchison, B. Bardakjian, M. Mojahedi, B. Wang and P. Yoo

($e = 1.6 \times 10^{-19}$ C, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $g = 9.81$ N/kg)

NAME : ______________________________
(PLEASE PRINT) Family (Last) Name Given (First) Name

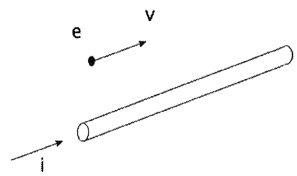
STUDENT NUMBER : ______________________________

EXAMINATION TYPE : D (Students may use a single double sided 8.5" x 11" aid sheet)
CALCULATORS : Casio FX-991 (EX, EX Plus, or MS), Sharp EL-520 (X or W)
DURATION : 2.5 hours
INSTRUCTIONS :
- Answer all five questions. Put the answers in the boxes provided.
- All work is to be done on these pages. Show steps, compute numerical results when requested and state units. Write down any assumption made.
- You may use the back of each page.
- Last blank page may be removed for rough work.

Question	Mark
1	
2	
3	
4	
5	
Total	

Q1 [10 marks] Multiple Choice Questions Section 1
Circle the correct answer for each part.

(a) **[2 marks]** A long wire carries a current, *i*, as shown in the figure below. An electron is launched parallel to the wire with a velocity, *v* as shown. Which of the following statements is true?



a. The electron moves in a straight line parallel to the wire.

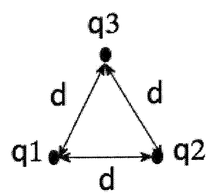
The electron moves in an arc towards the wire.

c. The electron moves in an arc away from the wire.

d. The electron slows down and continues to move parallel to the wire

e. The electron speeds up and continues to move parallel to the wire

(b) **[2 marks]** Three charges are placed on the corners of an equilateral triangle, as shown below. Which expression describes the electric potential at point P, at the center of the triangle?
(Hint: $Cos\ 30 = \frac{\sqrt{3}}{2}$)



a. $V = 0$

b. $V = \frac{\sqrt{3}}{4\pi\varepsilon_o d}(q1 + q2 + q3)$

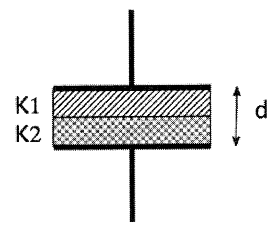
c. $V = \frac{\sqrt{3}}{4\pi\varepsilon_o d^2}(q1 + q2 + q3)$

d. $V = \frac{3}{4\pi\varepsilon_o d^2}(q1 + q2 + q3)$

e. $V = \frac{\sqrt{3}}{4\pi\varepsilon_o d}(q1q2 + q1q3 + q2q3)$

Q1 continued

(c) **[2 marks]** The parallel plate capacitor show in the figure below consists of two plates of area A, separated by a distance d. The capacitor contains two slabs of dielectric material, with equal thickness, with dielectric constants K1 and K2. Which of the following expressions for the capacitance is correct?



a. $C = \frac{\varepsilon_o A}{d}$

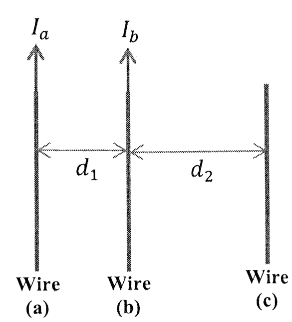
b. $C = \frac{\varepsilon_o A}{d}\left(\frac{K_1+K_2}{K_1 K_2}\right)$

c. $C = \frac{\varepsilon_o A}{d}(K_1 K_2)$

d. $C = \frac{2\varepsilon_o A}{d}\left(\frac{K_1 K_2}{K_1+K_2}\right)$

e. $C = \frac{\varepsilon_o A}{d}(K_1 + K_2)$

(d) **[2 marks]** Figure shows three very long and thin wires carrying currents: I_a, I_b, and I_c (I_c is not shown on the figure). If $d_2 = 2\, d_1$ and wire (b) experiences *no force due to the currents*, what should be the direction of I_c and the ratio I_c/I_a?



a. I_c flows from the **bottom to top** with $\boldsymbol{I_c/I_a = 0.5}$

b. I_c flows from the **top to bottom** with $\boldsymbol{I_c/I_a = 0.5}$

c. I_c flows from the **bottom to top** with $\boldsymbol{I_c/I_a = 2}$

d. I_c flows from the **top to bottom** with $\boldsymbol{I_c/I_a = 2}$

Q1 continued

(e) **[2 marks]** A closed sphere, radius R, is placed close to a long wire, current carrying a current, i. Which expression describes the magnetic flux through the sphere.

a. $\mu_o i$

b. $\frac{\mu_o i}{(4\pi R^2)}$

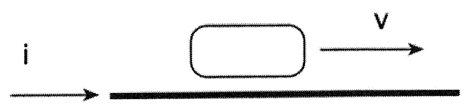
c. $4\pi R^2 \mu_o i$

d. 0

e. $\frac{4}{3}\pi R^2 \mu_0 i$

Q2 [10 marks] Multiple Choice Questions Section 2
Circle the correct answer for each part.

(a) **[2 marks]** A closed loop of wire, with total resistance, R, moves with constant velocity v, parallel to an infinity long wire which carries a current i, as shown below. Which of the following is true?



a. The current induced in the wire is in the clockwise direction.

b) There is no current induced in the wire.

c. The current in the wire oscillates between clockwise and anti-clockwise

d. The current induced in the wire is in the anticlockwise direction.

(b) **[2 marks]** Voltage across an inductor, $L = 2\ [H]$, is given by

For $-\infty < t < 0\ [s]$, $v(t) = 0\ [V]$

For $0 < t < 10\ [s]$, $v(t) = 6\, t^2\ [V]$

For $10 < t < \infty\ [s]$, $v(t) = 0\ [V]$

Circle the correct value of the current at $t = 4\ [s]$

a. 18 [A]

b. 128 [A]

c. 64 [A]

d. 56 [A]

(c) **[2 marks]** Consider the Kirchoff Voltage Law (KVL) and Kirchoff Current Law (KCL). Circle the correct statement below.

a. KVL is a statement of conservation of charges and KCL is a statement of conservation of energy.

b. KVL is a statement of conservation of energy and KCL is a statement of conservation of charges.

c. KVL is a statement of conservation of volume and KCL is a statement of conservation of current.

d. None of the above.

Q2 continued

(d) **[2 marks]** Circle the correct statement.

a. For a capacitor, both voltage and current must be continuous functions of time.

b. For a capacitor, voltage must be a continuous function of time; however, current can be a discontinuous function of time.

c. For a capacitor, current must be a continuous function of time; however, voltage can be a discontinuous function of time.

d. None of the above statements is correct.

(e) **[2 marks]** Considering the voltage across and current through a capacitor in an AC circuit, circle the correct statement.

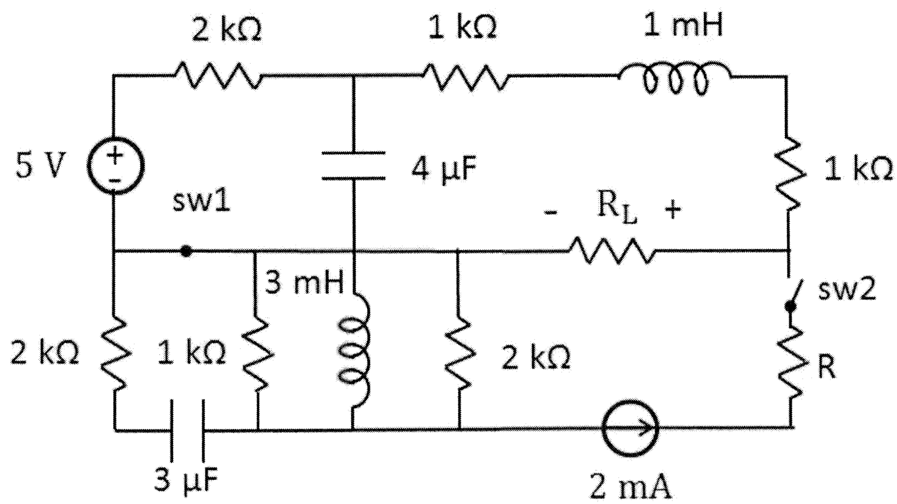
a. Current leads the voltage by 90 degree.

b. Voltage leads the current by 90 degree.

c. Current and voltage are in phase (neither leads or lags the other).

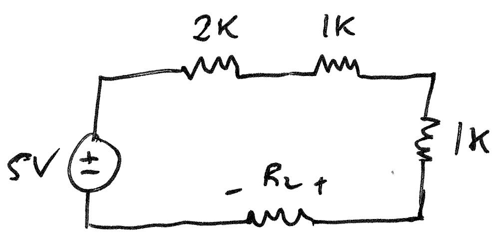
d. None of the above.

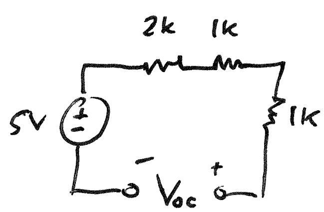
Q3 [10 marks]
Consider the following circuit under DC conditions, where independent voltage (5 volts) and current (2 mA) sources are used to deliver electrical energy to multiple circuit elements. There are 2 switches (sw1 & sw2) that can be independently controlled to be in either the open or closed position.



(a) **[4 marks]** When sw1 is closed and sw2 is open, as shown above. Determine R_L such that maximum power is transferred to the load resistor. Assume R = 10 kΩ.

$R_L = 4k\Omega$

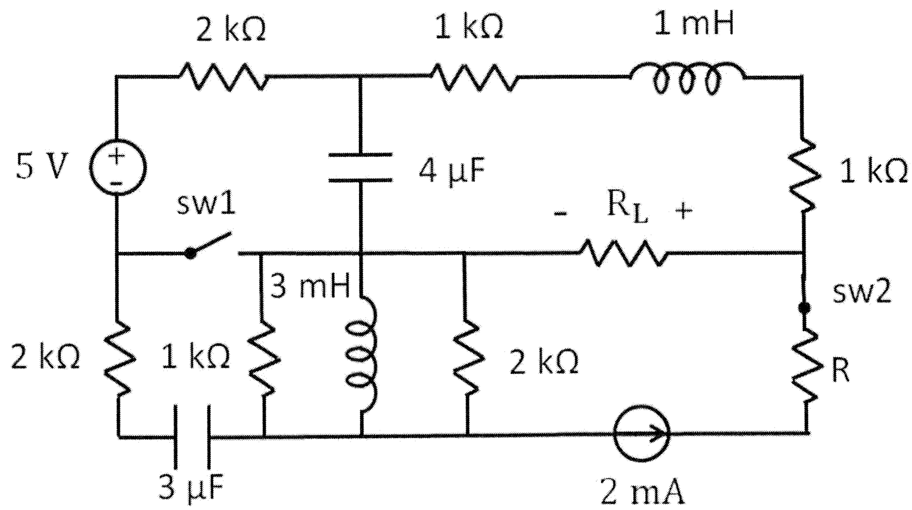




$V_{oc} = 5V$

$R_{th} = 4K\Omega$

Q3 continued.



(b) **[2 marks]** In the case where sw1 is open and sw2 is closed, shown above, determine the value of a replacement resistor, R such that the power dissipated by this resistor is equal to the power dissipated by R_L in part (a). (Use the same value of R_L)

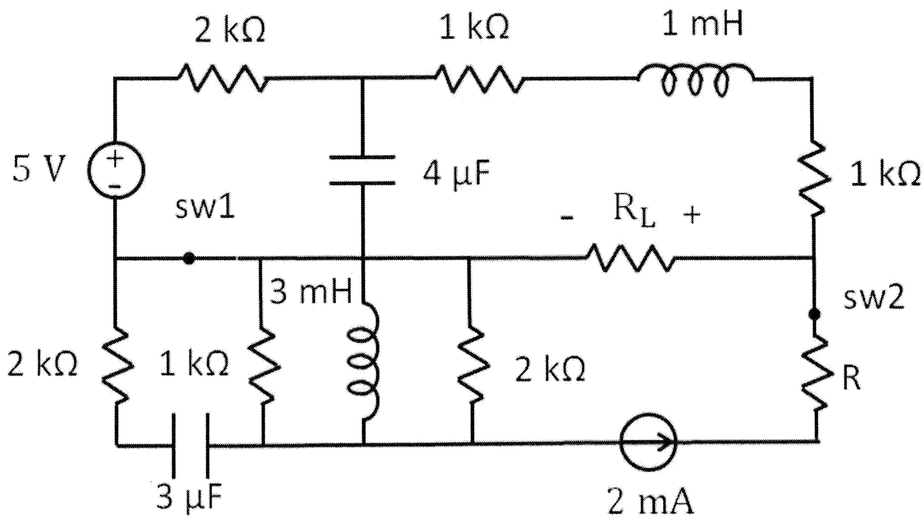
$\boldsymbol{R} =$ 400Ω

$$P = VI = \left(\frac{5}{2}\right) \cdot \left(\frac{5}{8}\right) = 1.56 mW$$

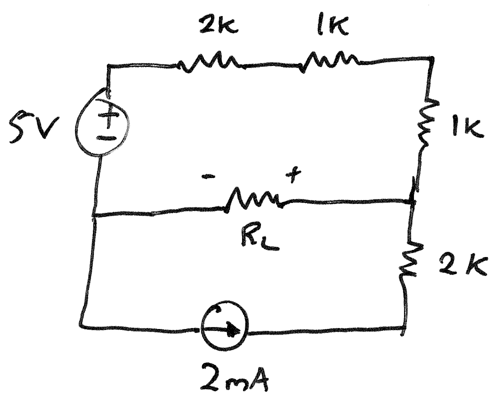
$$P_R = (2mA)^2 R = 1.56 mW$$

$$R = 400\Omega$$

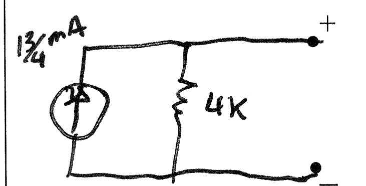
Q3 continued.

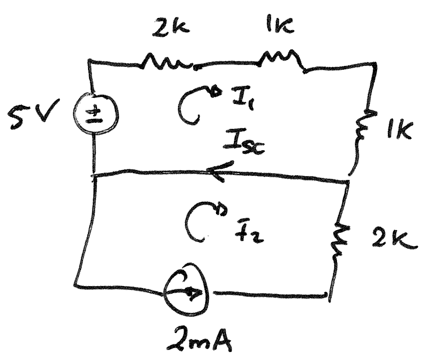


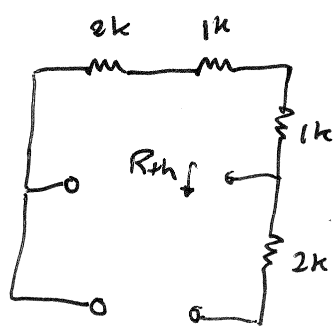
(c) **[4 marks]** If both sw1 & sw2 are closed, determine the Norton equivalent circuit that would allow maximum power transfer to a given R_L. Assume R = 2 kΩ.



Norton Equivalent Circuit:







$I_2 = -2mA$

$I_{sc} = I_1 - I_2$

KVL @ loop 1

$-5 + 4I_1 = 0$

$I_1 = \frac{5}{4}$

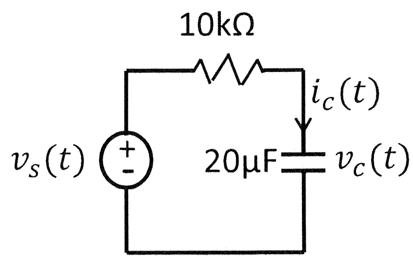
$I_{sc} = 1.25 + 2$

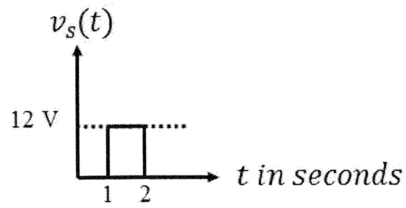
$= 3.25mA$

$R_{Th} = 4K$

Q4 [10 marks]

A pulse voltage source $v_s(t)$ is applied to the series RC circuit shown below.





(a) **[6 marks]** Determine the capacitor voltage $v_C(t)$.

$v_C(t) =$
$12(1-e^{-\frac{t-1}{0.2}})$, $1<t<2$
$11.92e^{-\frac{t-2}{0.2}}$, $t>2$

$$v_C(t) = K_1 + K_2 e^{-\frac{t-t_0}{\tau}}$$

$$\tau = R_{th}C = (10k\Omega)(20\mu F) = 0.2s$$

For $1<t<2$, $t_0 = 1$

$$v_C(t) = K_1' + K_2' e^{-\frac{t-1}{0.2}}$$

$$v_C(\infty) = K_1' = 12V$$

$$v_C(1^+) = v_C(1^-) = 0 = K_1' + K_2'$$

$$K_2' = -K_1' = -12$$

$$v_C(t) = 12(1 - e^{-\frac{t-1}{0.2}})$$

For $t>2$, $t_0 = 2s$

$$v_C(t) = K_1'' + K_2'' e^{-\frac{t-2}{0.2}}$$

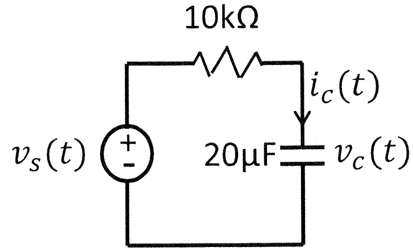
$$v_C(\infty) = K_1'' = 0$$

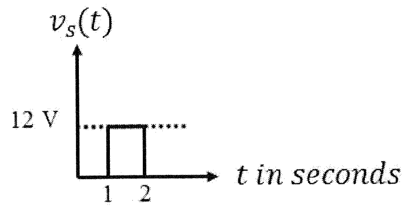
$$v_C(2^+) = v_C(2^-) = 12(1-e^{-1/0.2}) = 11.92V = K_1'' + K_2''$$

$$K_2'' = 11.92V$$

$$v_C(t) = 11.92 e^{-\frac{t-2}{0.2}}$$

Q4 continued. (Circuit diagram and pulse diagram have been duplicated for your convenience)





(b) **[2 marks]** Determine the capacitor current $i_c(t)$.

$i_c(t) = \frac{d}{dt} v_c(t)$

for $1 < t < 2$

$i_c(t) = \frac{d}{dt} 12(1 - e^{-\frac{t-1}{0.2}})$

$= -12 e^{1/0.2} \cdot -1/0.2\, e^{-t/0.2}$

$= 60 e^{-\frac{t-1}{0.2}}$

$t > 2$

$i_c(t) = -59.6 e^{-\frac{t-2}{0.2}}$

$\boxed{i_c(t) = 60 e^{-\frac{t-1}{0.2}},\ 1 < t < 2 \quad -59.6 e^{-\frac{t-2}{0.2}},\ t > 2}$

(c) **[2 marks]** Determine the energy stored in the capacitor at $t_1 = 1 + \tau$ and $t_2 = 2 + \tau$, where t_1, & t_2 are time in seconds, and τ is the time constant of the circuit.

$\mathcal{E}(t) = \frac{1}{2} C v_c^2(t)$

$= \frac{1}{2} \cdot 20 \times 10^{-6} \cdot v_c^2(t)$

$Energy_{t_1} =$	571.5 µJ
$Energy_{t_2} =$	192.7 µJ

$\mathcal{E}(t_1) = 10 \times 10^{-6} \cdot v_c^2(1.2) = 571.5\,\mu J$

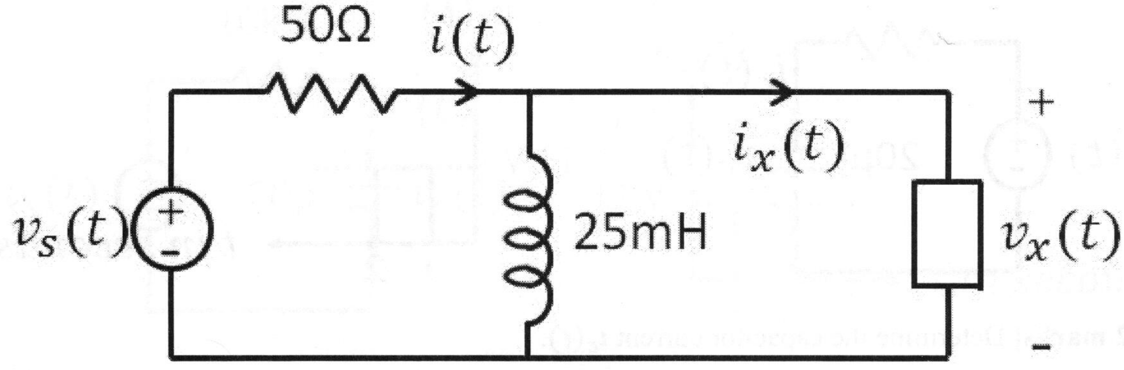
$\mathcal{E}(t_2) = 10 \times 10^{-6} \cdot v_c^2(2.2) = 192.7\,\mu J$

$v_c(1.2) = 7.56$

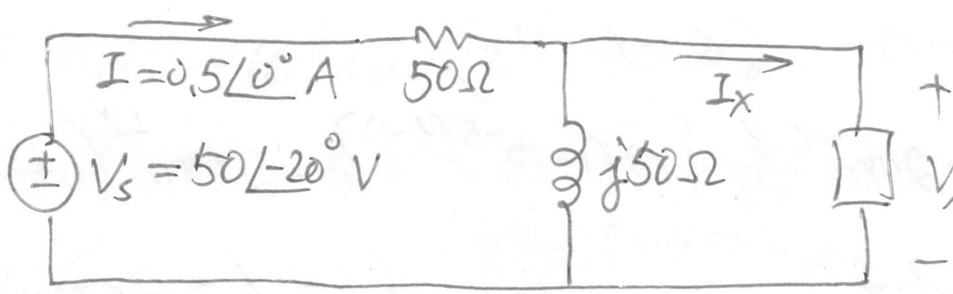
$v_c(2.2) = 4.39$

Q5 [10 marks]

The circuit shown below is operating in the sinusoidal steady state with
$v_s(t) = 50\cos(2000t - 20^\circ)$ V and $i(t) = 0.5\cos(2000t)$ A.



(a) **[1 mark]** Draw the circuit in the frequency domain, indicating all phasors and impedances.



(b) **[4 marks]** Find the phasor voltage, $\boldsymbol{V_x}$ and the phasor current, $\boldsymbol{I_x}$.

$\boldsymbol{V_x} = 27.9\angle -37.9^\circ$ V	$\boldsymbol{I_x} = 0.95\angle 27.6^\circ$ A

1

$$V_x = V_s - (50\Omega)I = 50\angle -20^\circ - 50\times 0.5\angle 0^\circ$$

$$= 27.9\angle -37.9^\circ \text{ V} \quad (22 - j17.1) \quad 1$$

1

$$I_x = I - \frac{V_x}{j50} = 0.5\angle 0^\circ - \frac{27.9\angle -37.9^\circ}{j50}$$

$$= 0.95\angle 27.6^\circ \text{ A} \quad (0.84 + j0.44) \quad 1$$

(c) **[1 mark]** Determine the impedance of the elements in the rectangular box.

$\boxed{Z_x = 12.2 - j26.7\ \Omega}$

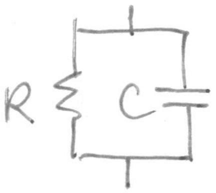
0.5

$$Z_X = \frac{V_X}{I_X} = \frac{27.9\angle -37.9^\circ\ \text{V}}{0.95\angle 27.6^\circ\ \text{A}} = 29.4\angle -65.5^\circ\ \Omega$$

$$(12.2 - j26.7\ \Omega)\quad 0.5$$

(d) **[4 marks]** If the elements in the rectangular box can be modeled as a resistor connected in parallel with a capacitor, find the values of the resistance and the capacitance.

R = $70.9\ \Omega$	**C** = $15.5\ \mu F$



$$Y_{total} = Y_R + Y_C = \frac{1}{R} + j\omega C \quad 2$$

$$Y_{total} = \frac{1}{Z_X} = \frac{1}{29.4\angle -65.5^\circ}$$

$$= 0.0141 + j0.03095$$

$$\frac{1}{R} = 0.0141 \rightarrow R = 70.9\ \Omega \quad 1$$

$$\omega C = 0.03095 \rightarrow C = 15.5\ \mu F \quad 1$$

Alternative Solution

$$Z_{total} = \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega RC} = \frac{R - j\omega R^2 C}{1 + (\omega RC)^2}$$

$$\frac{R}{1+(\omega RC)^2} = 12.2 \text{ and } \frac{\omega R^2 C}{1+(\omega RC)^2} = 26.7$$

Solve for R and C