University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, DECEMBER, 2012
Duration: 2 and 1/2 hours
First Year - CHE, CIV, IND, LME, MEC, MMS

MAT186H1F - CALCULUS I
Exam Type: A

SURNAME: (as on your T-card) _____

YOUR FULL NAME: _____

STUDENT NUMBER: _____

SIGNATURE: _____

Examiners:
D. Burbulla
S. Cohen
X. Liu
P. Walls
P. Samuelson

Calculators Permitted: Casio 260, Sharp 520 or TI 30.

INSTRUCTIONS: Attempt all questions. Present your solutions in the space provided. Use the backs of the sheets if you need more space. Do not tear any pages from this exam. Make sure your exam contains 10 pages.

MARKS: Questions 1, 2, 3 and 4 are each worth 10 marks.

Questions 5, 6, 7, 8 and 9 are each worth 12 marks.

If a question has multiple parts, the marks for each part are indicated by parentheses beside the question number.

TOTAL MARKS: 100

QUESTION	MARK
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
Q8	
Q9	
TOTAL	

1. Find the following:

(a) [5 marks] $\int \left(e^x + x^{1/3} + \frac{1}{x}\right) dx$

(b) [5 marks] $\int_0^{\pi/2} \left(1 + \sin^3 x\right) \cos x \, dx.$

2. Find the following limits, if they exist:

(a) [2 marks] $\lim_{x \to \infty} e^x \sin x$

(b) [3 marks] $\lim_{x \to -\infty} e^x \sin x$

(c) [5 marks] $\lim_{x \to \infty} \left(1 + \frac{4}{x}\right)^x$

3. Find the following:

(a) [4 marks] $F'(1)$ if $F(x) = \int_0^{x^3} (t^2 + 8)^{3/2}\, dt$.

(b) [6 marks] an approximation of the solution to the equation $e^x + x^3 = 0$, correct to 2 decimal places.

4. The velocity of a particle at time t is given by $v = t^3 - 1$. Find

(a) [4 marks] the average velocity of the particle for $0 \leq t \leq 2$.

(b) [6 marks] the average speed of the particle for $0 \leq t \leq 2$.

5. Sketch the graph of $y = 6x^{1/3} + 3x^{4/3}$. Label all critical points, inflection points, and vertical tangents. You may assume

$$y' = \frac{2}{x^{2/3}} + 4\,x^{1/3} \text{ and } y'' = -\frac{4}{3\,x^{5/3}} + \frac{4}{3\,x^{2/3}}.$$

6. Let A be the area of the region in the xy-plane bounded by $x = 0, x = 1, y = \pi/4$ and $y = \tan^{-1} x$.

   (a) [8 marks] Write down two integrals, one with respect to x and one with respect to y, that both give the value of A.

   (b) [4 marks] Find the value of A.

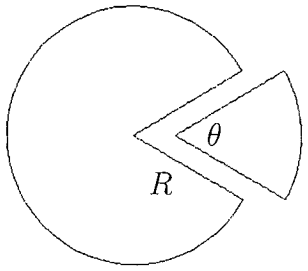
7. Sketch the graph of $y = \dfrac{x^3}{x^2 - 1}$. Label all critical points, inflection points and asymptotes.

You may assume

$$y' = \frac{x^2(x^2 - 3)}{(x-1)^2(x+1)^2} \text{ and } y'' = \frac{2x(x^2 + 3)}{(x-1)^3(x+1)^3}.$$

8. A paper cup in the shape of a cone is made from a circular sheet of paper of radius R by cutting out a sector subtended by an angle θ and then gluing the cut edges of the remaining piece of paper together. What is the maximum volume attainable for the cone? You may assume the volume of a cone with a base of radius r and perpendicular height h is

$$V = \frac{1}{3}\pi r^2 h.$$



9. Let $f(x) = x^{2/3}$.

(a) [6 marks] Find the length of the curve $y = f(x)$ for $1 \leq x \leq 8$.

(b) [6 marks] Find the volume of the solid of revolution obtained by revolving the region in the xy-plane bounded by the curves $y = f(x), y = 1, x = 1$ and $x = 8$ about the x-axis.