MIE100 2018 Unofficial Final Solution
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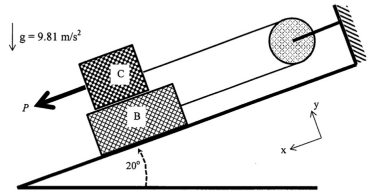
TOPONE Education 2019 Apr 20

1)

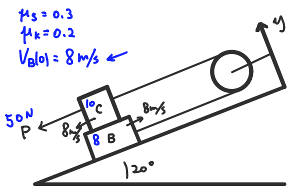
1. Block B and Block C are on a hill sloped at 20 degrees as shown in the diagram. The blocks are linked by a rope in tension that passes around a pulley. The pulley has zero mass, and the center of the pulley is attached to a wall. There is friction between the two blocks, and between the ground and block B; the coefficients of friction between all flat surfaces are $\mu_s = 0.3$ and $\mu_k = 0.2$. The mass of block C is 10 kg, and the mass of block B is 8 kg.

A force $P = 50\ \hat{\imath}$ Newtons is applied to block C as shown. Block C has a velocity of $8\ \hat{\imath}$ m/s at the instant shown in the diagram.



12 Draw separate free-body diagrams of block B and block C, showing clearly the direction in which each of the forces acts. What is the acceleration of block C at the instant shown in the diagram? (Make sure your work is clearly laid out, neat, and easy to understand)

$\mu_s = 0.3$
$\mu_k = 0.2$
$V_B(0) = 8$ m/s



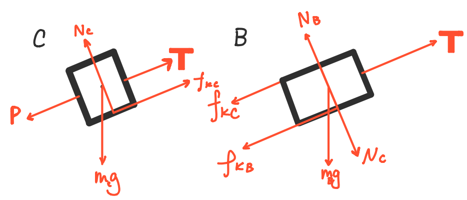
$f_{kC} = N_C \mu_k$; $N_C = mg\cos 20° = 92.1838\,N \Rightarrow f_{kC} = 18.4368\,N$

$f_{kB} = N_B \mu_k$; $N_B = m_B g\cos 20° + N_C = 165.9309\,N \Rightarrow f_{kB} = 33.18618\,N$

$a_C = \frac{\Sigma F_{Cx}}{m_C} = \frac{50N + m_C g\sin 20° - 18.4368N - T}{10} = 6.5115376 - T/10$

$a_B = \frac{\Sigma F_{Bx}}{m_B} = \frac{18.4368N + m_B g\sin 20° + 33.18618N - T}{8} = 9.808 - T/8$

$a_B = -a_C$



$6.5115376 - T/10 = -9.808 + T/8$

$T = 72.53\,N$

$\Rightarrow a_C = -0.74147\ m/s^2$

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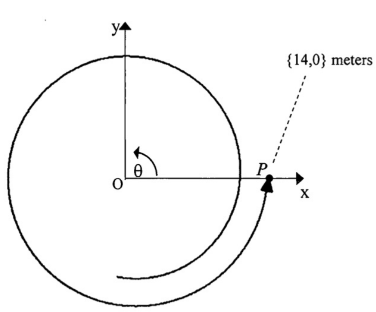
2)

2. A particle P is travelling in a spiral path as shown. At the instant shown in the diagram, the particle's speed is increasing at a constant rate of 2 m/s², and its distance from the origin "O" is increasing at a constant rate of 4 m/s. At the instant shown in the diagram, the particle's position is {x,y} = {14,0} meters, and its speed is 7 m/s.

8 Determine $\frac{d\theta}{dt}$ for the particle P at the instant shown in the diagram, where θ is the angular position of P measured with respect to the x-axis in polar coordinates.

Also determine the x-component of the particle's acceleration a_x at the instant shown in the diagram.



$\ddot{r} = 0, \quad \dot{r} = 4$

$a_r = \ddot{r} - r\dot{\theta}^2$

$= -r\dot{\theta}^2$

$v = 4\,\hat{u}_r + r\dot{\theta}\,\hat{u}_\theta$

$v_p = 7\,m/s = \sqrt{16 + (14\dot{\theta})^2}$

$33 = 14^2\dot{\theta}^2 \Rightarrow \dot{\theta} = \frac{\sqrt{33}}{14} = 0.41$ rad/s

$a_x = a_r = -r\dot{\theta}^2 = -14 \times \frac{33}{14^2} = -2.357\ m/s^2$

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3. At the instant shown in the attached diagram, the bar AB is rotating in the positive direction with an angular velocity of $\omega_{AB} = 42.5$ radians/s.

5

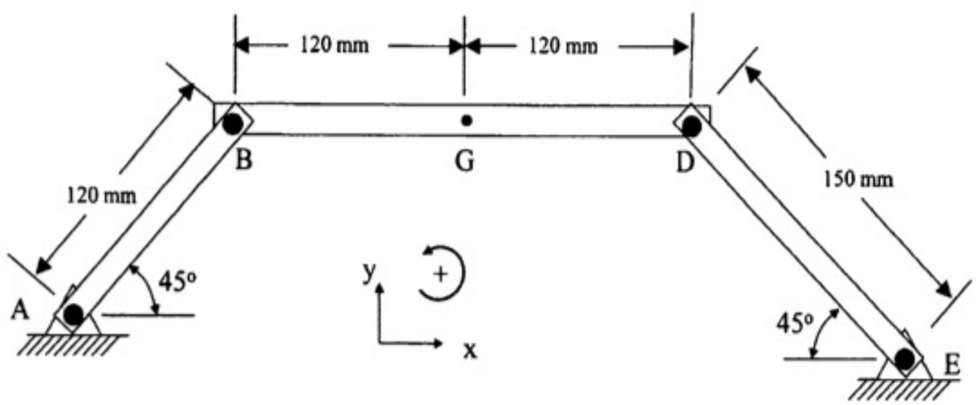
(a) At the instant shown in the diagram, what is the velocity of point B? Use the co-ordinate system provided to express your answer.

10

(b) Using your answer from part (a), find both the velocity of point D and the angular velocity of bar BD at the instant shown in the diagram. Use the co-ordinate system provided to express your answers.

5

(c) Using your answers from part (a) and (b), find the acceleration of point B at the instant shown in the diagram if the speed of B is also increasing by 8m/s^2. Use the co-ordinate system provided to express your answer. (5 marks)



a) $\omega_{AB} = 42.5$ radian/s

$V_B = \omega_{AB} \cdot AB = 42.5 \times 0.12 = 5.1$ m/s

$\vec{V_B} = -3.6062\,\hat{i} + 3.6062\,\hat{j}$

b) $\vec{V_B} = \vec{V_D} + \vec{V_{B|D}}$

$$\Rightarrow \begin{bmatrix} -3.6062 \\ 3.6062 \\ 0 \end{bmatrix} = \begin{bmatrix} V_D' \\ V_D' \\ 0 \end{bmatrix} + \begin{bmatrix} i & j & k \\ 0 & 0 & w \\ -0.24 & 0 & 0 \end{bmatrix}$$

$\Rightarrow -3.6062 = V_D'$

$3.6062 = V_D' - 0.24w$

$\Rightarrow -0.24w = 7.2124$

$\Rightarrow w = -30.05167$ rad/s

$$V_D = \begin{bmatrix} -3.6062 \\ -3.6062 \\ 0 \end{bmatrix}$$

c) $\vec{a_{Bn}} = \vec{w^2 r} = 216.75$ m/s^2 45° $= \begin{bmatrix} -153.27 \\ -153.27 \\ 0 \end{bmatrix}$ m/s^2

$\vec{a_{Bt}} = 8$m/s^2 45° $= \begin{bmatrix} -4\sqrt{2} \\ 4\sqrt{2} \\ 0 \end{bmatrix}$ m/s^2

$\Rightarrow a_B = \begin{bmatrix} -158.92 \\ -147.61 \\ 0 \end{bmatrix}$ m/s^2

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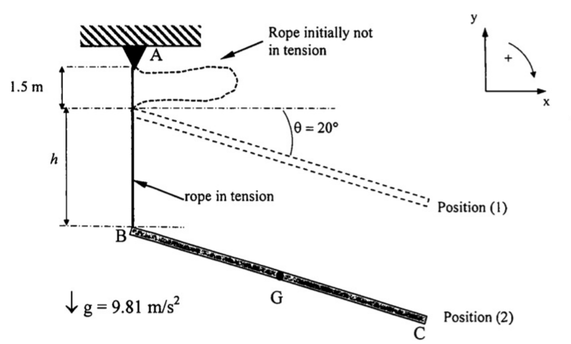
4. A slender rod of mass $m = 1.3$ kg and length $L = 2.0$ m is falling straight down from Position 1, as shown in the diagram (dotted lines). A rope with negligible mass is attached to the fixed point at A on the ceiling, and tied to the end of the rod at B. The rope is initially not in tension. The rope is suddenly in tension when the rod reaches Position 2, and the rope is hanging straight down. The rope stays in tension afterwards, but is allowed to swing.

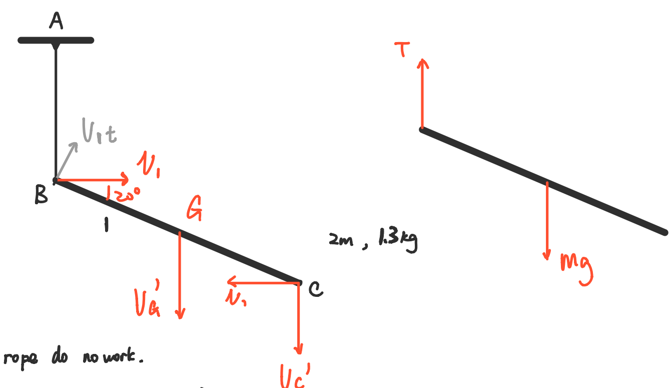
| 2 | (a) Draw the free body diagram of the rod immediately after the rope is in tension.

| 8 | (b) If $\vec{v}_G = -3\hat{j}$ m/s just *before* the rope is in tension, what is the rod's angular velocity, ω, immediately *after* the rope is in tension?

| 6 | (c) If $\vec{v}_G = -2.5\hat{j}$ m/s immediately *after* the rope is in tension, what is the rod's total kinetic energy at that moment?

| 4 | (d) If the rod is initially at rest in Position (1), and $h = 1.5$ m and the rope has a length of 2m, what is velocity of point C, $\vec{v}_C$, just before it reaches Position (2)?





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b) rope do no work.

$U_{total} = \frac{1}{2} \times 1.3 \times 3^2$

↘ transformed to: $U_{\circlearrowright}$ and $U_{\downarrow}$

$U_{\circlearrowright} = \frac{1}{2} I_G w^2 \qquad U_{\downarrow} = \frac{1}{2} m V_G'^2$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{12} \times 1.3 \times 2^2\right) \times w^2 + \frac{1}{2} \times 1.3 \times V_G'^2 = \frac{1}{2} \times 1.3 \times 3^2$$

$$\vec{V_B} = \vec{V_G'} - W \times \vec{BG} \Rightarrow \begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ V_G' \\ 0 \end{bmatrix} - \begin{bmatrix} i & j & k \\ 0 & 0 & w \\ \cos 20^\circ & -\sin 20^\circ & 0 \end{bmatrix}$$

$V_1 = -W \sin 20^\circ$

$V_G' = W \cos 20^\circ$

$$\Rightarrow 0.5 \times \frac{1}{12} \times 1.3 \times 2^2 \times W^2 + \frac{1}{2} \times 1.3 \times W^2 \cos^2 20^\circ = \frac{1}{2} \times 1.3 \times 3^2$$

$\Rightarrow W = 2.72$ rad/s

C) $U_{\downarrow} = \frac{1}{2} m V_G'^2 = \frac{1}{2} \times m \times 2.5^2$

$W = \frac{V'_G}{\cos 20^\circ} = -2.66044$ rad/s²

$U_2 = \frac{1}{2} \times \left(\frac{1}{12} \times 1.3 \times 2^2\right) \times (2.66044)^2 = 1.53355$ J

$\Rightarrow U_{total} = 5.596$ J

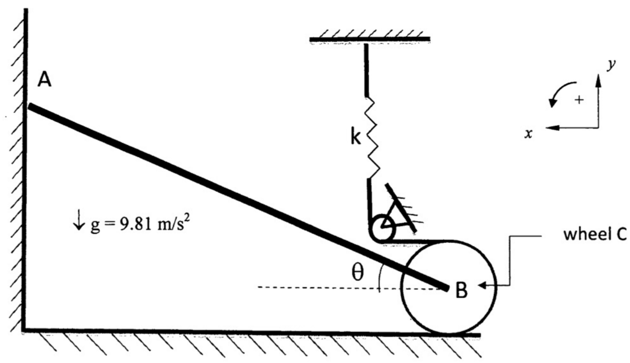
d) $\frac{1}{2} m v^2 = mg \times 1.5$

$\Rightarrow v = \sqrt{3g} = 5.425$ m/s

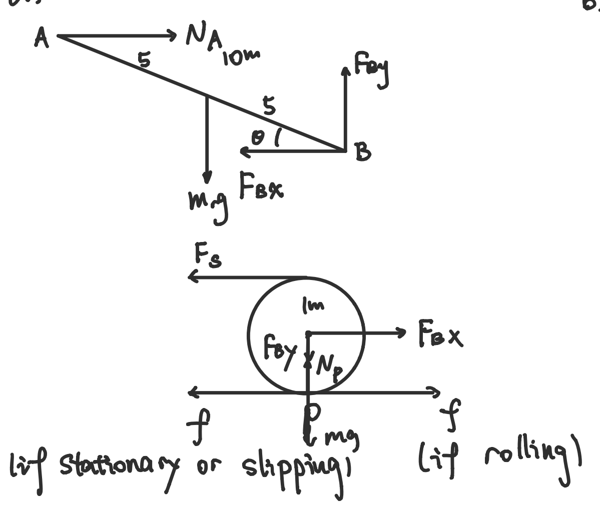
5)

5. The wheel C has a mass = 200 kg, and the bar AB has a mass = 100 kg. One end of the bar AB is connected by a pin to the center of the wheel. The other end of the bar is leaning against a vertical wall and can slide on the wall without friction. Bar AB is 10 meters long, and the radius of wheel C is 1 m. The spring stiffness $k = 200$ N/m. One end of the spring is attached to the ceiling, and the other end is attached to a rope that is wound around a pulley and then wound around wheel C as shown in the diagram. The relaxed length of the spring is 2m. Coefficients of friction between the wheel and ground are $\mu_s = 0.4$, and $\mu_k = 0.3$. The system is initially at rest as shown in the diagram, with the spring length equal to 3m.

2	a) Draw a free body diagram of bar AB and a separate diagram for wheel C at the instant shown in the diagram.
6	b) The system is then released from rest at the position shown in the diagram. Find θ if both the bar AB and the wheel C remain stationary after the system is released.
12	c) If $\theta = 30°$, find α_{AB} and α_C after the system is released from rest.



a)



b) To stay Stationary.

① $\Sigma M_B = 0$ for rod AB.

$\Rightarrow$ $5m \cdot g \cos\theta = 10 N \sin\theta$

$N = F_{Bx}$

② $\Sigma M_P = 0$, for wheel C

F_s initial $= 1m \times 200 N/m = 200$

$200 \times 2 = F_{Bx} \times 1 \Rightarrow F_{Bx} = 400$ Newton

③ At this time, $f = 200$ N

$f_{max} = (F_{By} + mg)\mu_s$

$= (300) g \mu_s = 1177.2 > 200$

$\Rightarrow \theta = \arctan \dfrac{5 \times 100kg \times 9.81}{4000 \text{ Newton}} = 50.8°$

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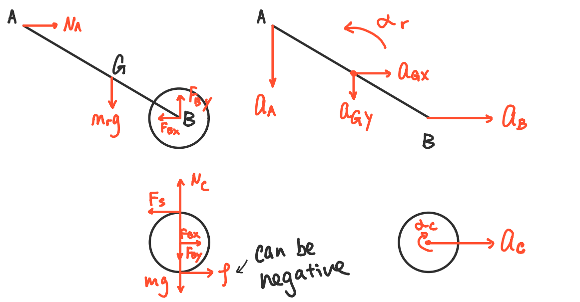
C



for rod, $v_A = \omega_r = 0$

$\Sigma(F_G)_x = N_A - F_{Bx} = m(a_G)_x$ ①

$\Sigma(F_G)_y = F_{By} - m_r g = m(a_G)_y$ ②

$\circlearrowleft \Sigma M_G = I_G \alpha_r$

$\Leftrightarrow -N_A \times 2.5m - F_{Bx} \times 2.5m + F_{By} \times 2.5\sqrt{3}\,m = \frac{1}{12} \times 100 \times (10m)^2 \alpha_r$

$\Leftrightarrow -2.5(N_A + F_{Bx}) + 2.5\sqrt{3}\,F_{By} = \frac{1}{12} \times 10^4 \alpha_r$ ③

$a_A = 2(a_G)_y = 2 \times \frac{F_{By} - m_r g}{m}$

$a_B = 2(a_G)_x = 2 \times \frac{N_A - F_{Bx}}{m}$

by kinematics

$$\begin{bmatrix} 0 \\ a_A \\ 0 \end{bmatrix} = \begin{bmatrix} a_B \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} i & j & k \\ 0 & 0 & \alpha_r \\ -5\sqrt{3} & 5 & 0 \end{bmatrix}$$

$\Rightarrow a_B = 5\alpha_r$

$a_A = -5\sqrt{3}\,\alpha_r$

this section yields

$-2.5(N_A + F_{Bx}) + 2.5\sqrt{3}\,F_{By} = \frac{1}{12} \times 10^4 \alpha_r$ ③

$-5\sqrt{3}\,\alpha_r = 2\,\frac{F_{By} - m_r g}{m_r}$ ②

$5\alpha_r = 2\,\frac{N_A - F_{Bx}}{m_r}$ ①

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for the wheel. $v_c = \omega_c = 0$, $F_s = 200N$

Assume no slipping:

$$\Sigma(F_B)_x = m(a_c)_x = F_{Bx} + f - 200N$$

$$\Sigma(F_B)_y = 0 \Rightarrow N_C = F_{By} + mg$$

$$\circlearrowright \Sigma M_B = I_B \alpha_c$$

$$\Rightarrow 200N \cdot 1m + f \cdot 1m = \frac{1}{2} \times 200 \times 1^2 \alpha_c$$

$$\Rightarrow 200 + f = 100 \alpha_c$$

Also. by kinematic, $a_B = a_C = -\alpha_c$

$$F_{Bx} + f - 200N = 200 a_B$$

$$200 + f = -100 a_B$$

$$\Rightarrow F_{Bx} - 400 = 300 a_B$$

$$\Rightarrow F_{Bx} = 300 a_B + 400. \quad (4)$$

Solve (1)(2)(3)(4)

$$\begin{cases} -2.5(N_A + F_{Bx}) + 2.5\sqrt{3} F_{By} = \frac{1}{12} \times 10^4 \alpha_r & (3) \\ -5\sqrt{3}\alpha_r = 2 \dfrac{F_{By} - m_r g}{m_r} & (2) \\ 5\alpha_r = 2 \dfrac{N_A - F_{Bx}}{m_r} & (1) \\ F_{Bx} = 300 a_B + 400 = 1500\alpha_r + 400 & (4) \end{cases}$$

$$250\alpha_r = N_A - 1500\alpha_r - 400$$

$$\Rightarrow N_A = 1750\alpha_r + 400$$

$$-250\sqrt{3}\alpha_r = F_{By} - 981$$

$$\Rightarrow 981 - 250\sqrt{3}\alpha_r = F_{By}$$

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$$-2.5\left(1750\alpha_r + 400 + 1500\alpha_r + 400\right) + 2.5\sqrt{3}\left(981 - 250\sqrt{3}\alpha_r\right) = \frac{1}{12} \times 10^4 \alpha_r$$

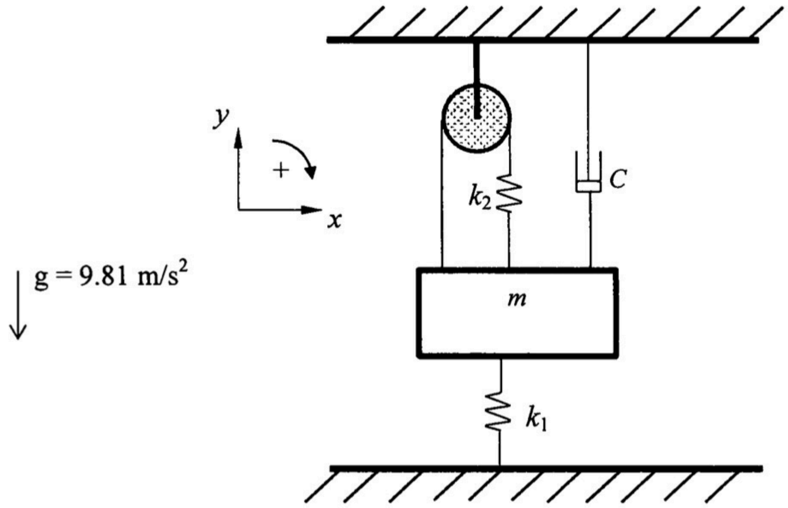
$$\Rightarrow \alpha_r = 0.2075 \text{ rad/s}^2$$

6. The mass moves only in the vertical direction. Ignore any rotation of the mass. Assume that the rope wound around the pulley remains in tension at all times. At time t=0, the mass is lifted a small distance from its equilibrium position, and then released.

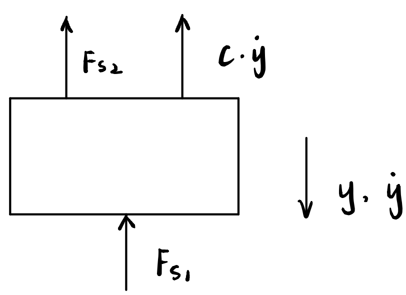
$m = 15$ kg $\quad k_1 = 100$ N/m $\quad k_2 = 35$ N/m $\quad C = 40\ \frac{N}{m/s}$

[6] (a) Find the period τ_d for damped, unforced vibrations of this system.

[2] (b) How long will it take for the vibration amplitude to diminish to 3% of its initial value?



(a)



$$m\cdot\ddot{y} = -c\cdot\dot{y} - y\cdot k_1 - 2y\cdot k_2$$

$$m\cdot\ddot{y} + c\dot{y} + (k_1 + 2k_2)y = 0$$

$$\frac{\sqrt{c^2 - 4m\cdot(k_1+2k_2)}}{2m} = \frac{\sqrt{86}}{3}\ i$$

$$\therefore \tau = \frac{2\pi}{\omega} = \frac{6\pi}{\sqrt{86}}$$

(b) $\frac{-c}{2m} = -\frac{4}{3}$ $\quad$ Amplitude : $A\cdot e^{-\frac{4}{3}t}$

$$\frac{e^{-(4/3)\cdot t}}{e^{-(4/3)\cdot 0}} = 0.03 \qquad t = 2.63\ s$$

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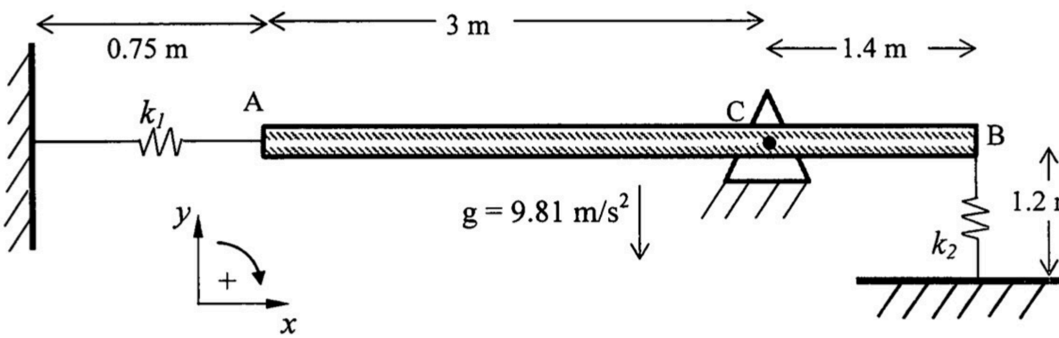
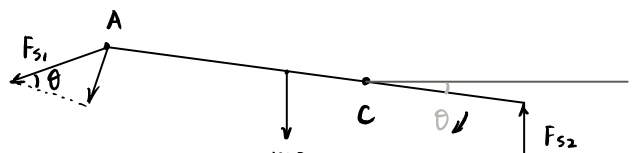
7. A uniform thin rod AB of mass 10 kg and length 4.4 meters is pinned at point C. The rod is in equilibrium in the horizontal position as shown. Spring $k_1 = 120$ N/m, and $k_2 = 180$ N/m.

6 (a) Determine the natural frequency of vibration ω_n for very low-amplitude oscillations.

6 (b) A moment $M = 4\sin(\omega_o t)$ Newton-meters is then applied to make the rod oscillate with a very small amplitude about point C. Determine the steady-state vertical amplitude of oscillation of point B, if $\omega_o = 5\ s^{-1}$.





Be balanced.

(a) At equilibrium: $k_2 \cdot l \cdot 1.4 = 0.8 \cdot mg$

Moment caused by s1: $F_{s1} \cdot \sin\theta \cdot r_{AC}$

$$F_{s1} = \left(\sqrt{(0.75-3+3\cdot\cos\theta)^2 + (3\sin\theta)^2} - 0.75\right)\cdot k_1$$

For small θ: $\sin\theta = \theta$, $F_{s1} = 0$, $M_{s1} = 0$.

$22.53\ddot{\theta} + 352.8\theta = 0 \Rightarrow \omega_n = 3.957$ rad/s

$\Rightarrow I_c \cdot \alpha = -\cancel{0.8mg} - 1.4\cdot(k_2(1.4\cdot\theta - \cancel{l}))$

(b) $22.53\ddot{\theta} + 352.8\theta = 4\sin(\omega_o t)$

$\theta_p = \theta_{p\,max} \cdot \sin(\omega_o t)$

$(-5^2\cdot 22.53 + 352.8)\cdot\theta_{p\,max} = 4 \quad \theta_{p\,max} = -0.018$ rad

Amplitude at B: $A_{max} = r_{CB} \cdot \sin\theta_{p\,max} = 0.0266$ m

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