UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION

APRIL 2004
EXAM TYPE: A
First Year Programs: CHE, CIV, IND, LME, MEC, MMS

ECE110H1S: ELECTRICAL FUNDAMENTALS

EXAMINERS: N.P. Kherani, B. Wang

Part A	
Q 1	/5
Q 2	/5
Q 3	/6
Q 4	/10
Part B	
Q 5	/4
Q 6	/6
Q 7	/10
Q 8	/5
Part C	
Q 9	/15
Q 10	/5
Q 11	/4
Part D	
Q 12	/17
Q 13	/8
Total	***/100***

NAME: _____
Last First

STUDENT NO.: _____

INSTRUCTIONS:

- This is a Type A examination; no aids are allowed.
- Only non-programmable calculators are allowed.
- Answer all questions.
- All questions are not of equal weight nor of equal difficulty.
- The weight of each question and its parts are stated in the margins.
- All work is to be done on these pages.
- Place your final answers in the provided boxes unless instructed otherwise.
- When answering the questions include all the steps of your work on these pages. For additional space, you may use the back of the preceding page.
- Do not unstaple this exam.

CONSTANTS:

$e = 1.6 \times 10^{-19}$C $\qquad \varepsilon_o = 8.85 \times 10^{-12}$F/m $\qquad \mu_o = 4\pi \times 10^{-7}$H/m

Part A: Electricity and Magnetism

Question 1: Multiple Choice and True / False [5 marks]

Circle the correct answer.

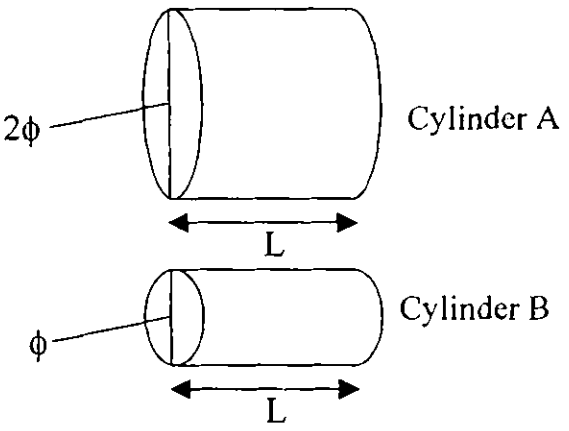
[1] (i) Coulomb's law describes the electric force between point charges.

True / False

[1] (ii) Rank the following in descending order of resistivity (highest resistivity to lowest resistivity): Copper, Rubber, Vacuum.

(a) Copper, Vacuum, Rubber
(b) Vacuum, Rubber, Copper
(c) Rubber, Vacuum, Copper
(d) Vacuum, Copper, Rubber

[1] (iii) Two cylindrical resistors (shown below), fabricated out of copper, are identical except that cylinder A has a diameter, 2ϕ, which is twice that of cylinder B, ϕ. Which of the following statements is true? (NOTE: R_A and R_B represent the resistance of cylinders A and B, respectively.)



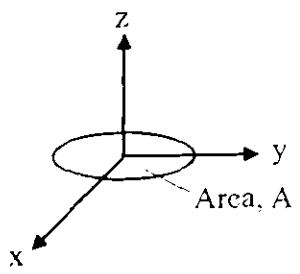
(a) $R_B = 4R_A$
(b) $R_B = 2R_A$
(c) $R_B = R_A$
(d) $2R_B = R_A$
(e) $4R_B = R_A$

[1] (iv) A point charge of 1 mC, released at rest, moves from point A to point B under the influence of an electric field, and undergoes a change in potential of $V_B - V_A = -3V$. The same charge is then forced to traverse from point B to point A along a different path. Assuming that this is an isolated system, the net change in the energy of the system is 0 Joules.

True / False

[1] (v) A loop of copper wire lies in the xy-plane. There is a magnetic field $\vec{B} = B_O \cos(\omega t)\hat{z}$. The induced emf in the loop is $|V| = \omega B_O \sin(\omega t)$.

True / False

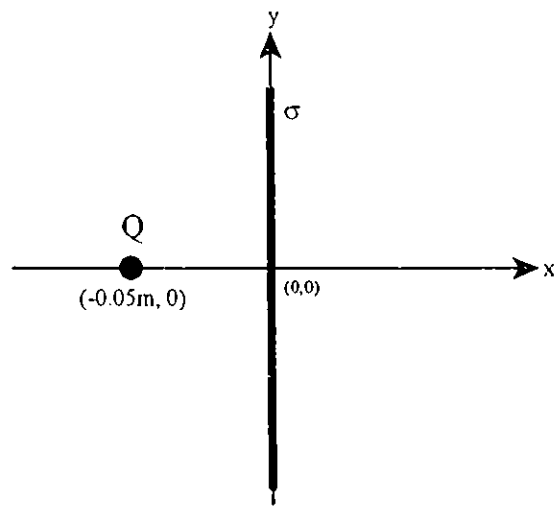


Part A: Electricity and Magnetism

Question 2: Electrostatics [5 marks]

[5] An infinite sheet of charge, havin_g a surface charge density of σ, where $\sigma > 0$, is located at x = 0. A positive point charge Q is located at (x, y) = (-0.05 m, 0). Find the magnitude and direction of the electric field on the x-axis (y=0) in the following regions:

Region I: $x < -0.05$ m
Region II: $-0.05\text{ m} < x < 0$ m
Region III: $x > 0$ m



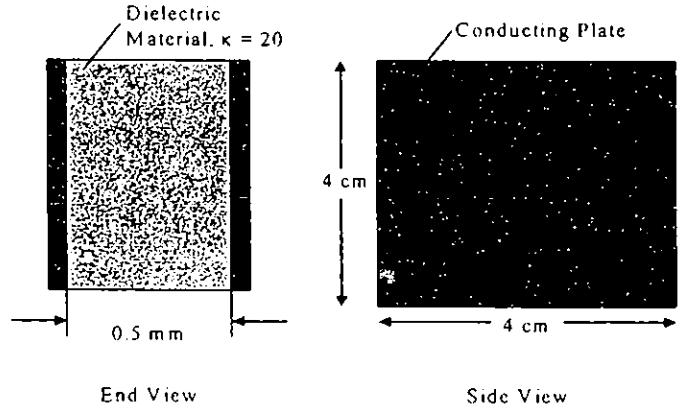
Part A: Electricity and Magnetism

Question 3: Electricity [6 marks]

A parallel plate capacitor is constructed of 2 conducting sheets of metal and a dielectric compound of dielectric constant $\kappa = 20$, sandwiched between the plates. The capacitor, along with its dimensions, is schematically shown below (not to scale).

[2] (i) Calculate the capacitance of the capacitor.

C =

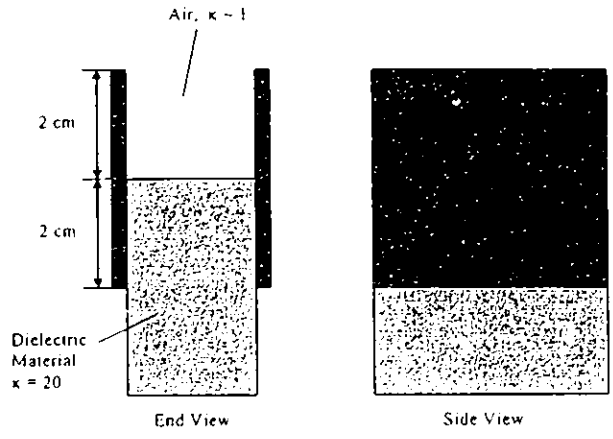


[1] (ii) If, a 12 V battery is connected to the capacitor. What is the magnitude of the charge on each plate?

Q =

[3] (iii) The dielectric material is pulled halfway out of the capacitor as shown below. Treating the new configuration as two capacitors in parallel, calculate the capacitance of this new configuration.

C =

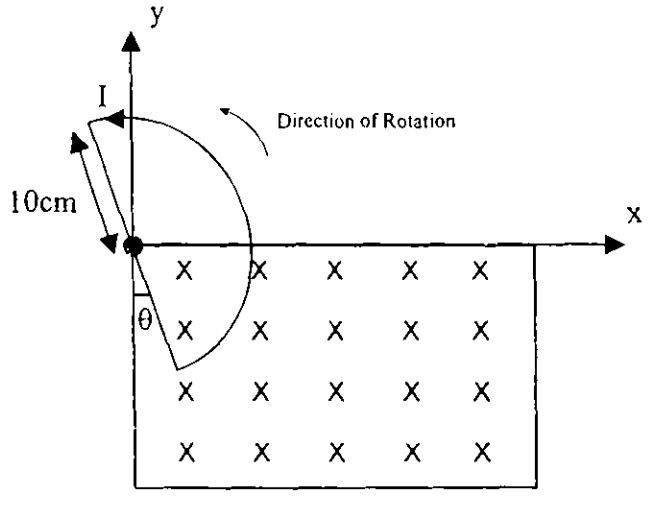


Part A: Electricity and Magnetism

Question 4: Magnetism [10 marks]

A semicircular loop of wire of radius 0.1 m lies in the xy-plane and rotates around the z-axis (pointing out of the page) as shown. A magnetic field of $\vec{B}$ = -20 T $\hat{z}$ exists in the quadrant x > 0, y < 0, as shown.

[4] (i) For $0 < \theta < 2\pi$, calculate the area of the loop that is inside the magnetic field.



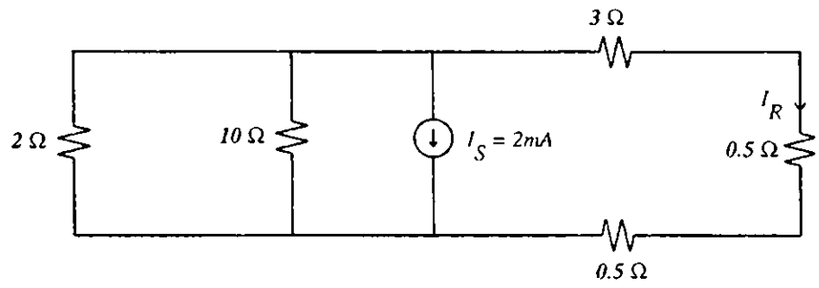
[2] (ii) Calculate the magnetic flux for $0 < \theta < 2\pi$.

[4] (iii) Given that $\theta = \frac{\pi}{2}t$, where θ is in radians and t is in seconds, calculate the induced emf for $0 < \theta < 2\pi$. Also, state the direction of the current (CW: clockwise or CCW: counter-clockwise).

Part B: DC Circuits

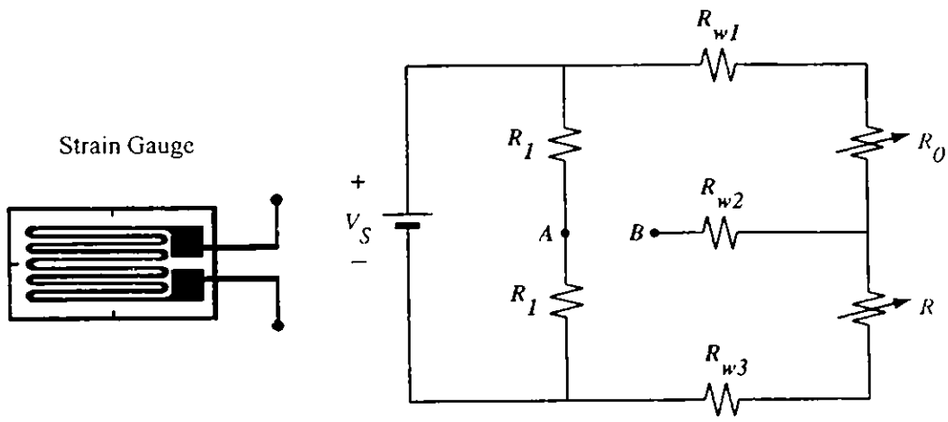
Question 5: Current and Voltage Division [4 marks]

[2] (i) Using current division, find the current I_R.



[2] (ii) A strain gauge is used to measure the amount of mechanical stress on an object. The gauge is composed of a wire such that when stretched the wire's length increases and the width narrows, thus increasing the resistance. The circuit below is used to measure the amount of stress applied to the strain gauge (variable resistor R). The voltage is read using a voltmeter (V) between terminals A and B. Resistances R_{w1}, R_{w2}, and R_{w3} are wire resistances.

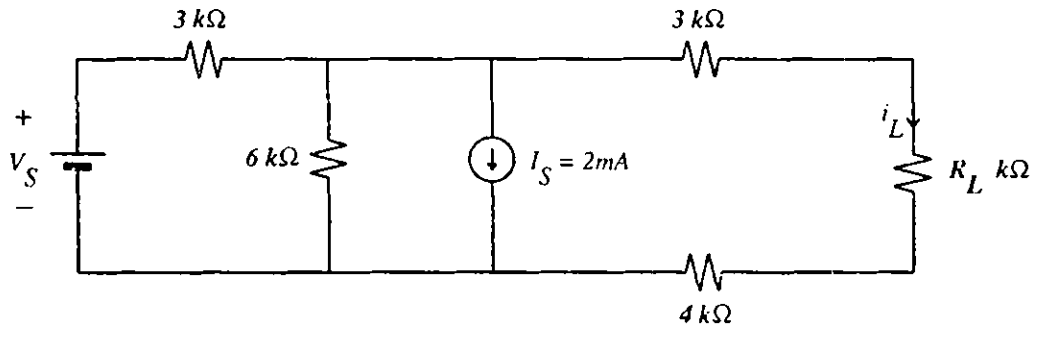
Use voltage division to find the output voltage $V = V_A - V_B$. Express V in terms of the circuit elements (V_S, R_1, R_{w1}, R_{w2}, R_{w3}, R_0, and R).



Part B: DC Circuits

Question 6: Superposition: [6 marks]

Use superposition to find the source voltage V_S (in volts with R_L in $k\Omega$), such that no current flows in the load resistor (that is, $i_L = 0$).

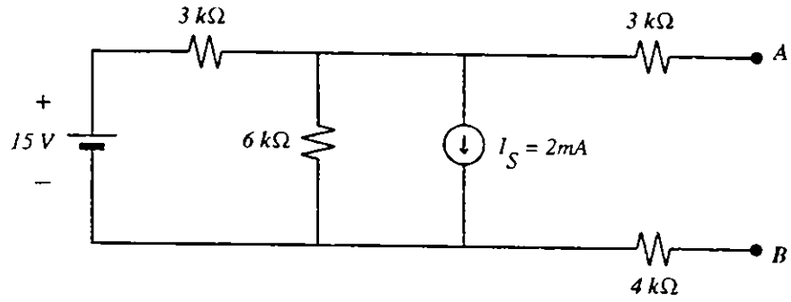


V_S =

Part B: DC Circuits

Question 7: Thevenin and Norton Equivalent [10 marks]

[7] (i) Find the Thevenin and Norton equivalent circuits for the circuit shown below as viewed from nodes A and B. Draw the equivalent circuits in the boxes provided.



Thevenin

Norton

[2] (ii) What is the power associated with the 2 mA current source? Is it absorbing or supplying power?

P= ☐ Absorbing ☐ Supplying

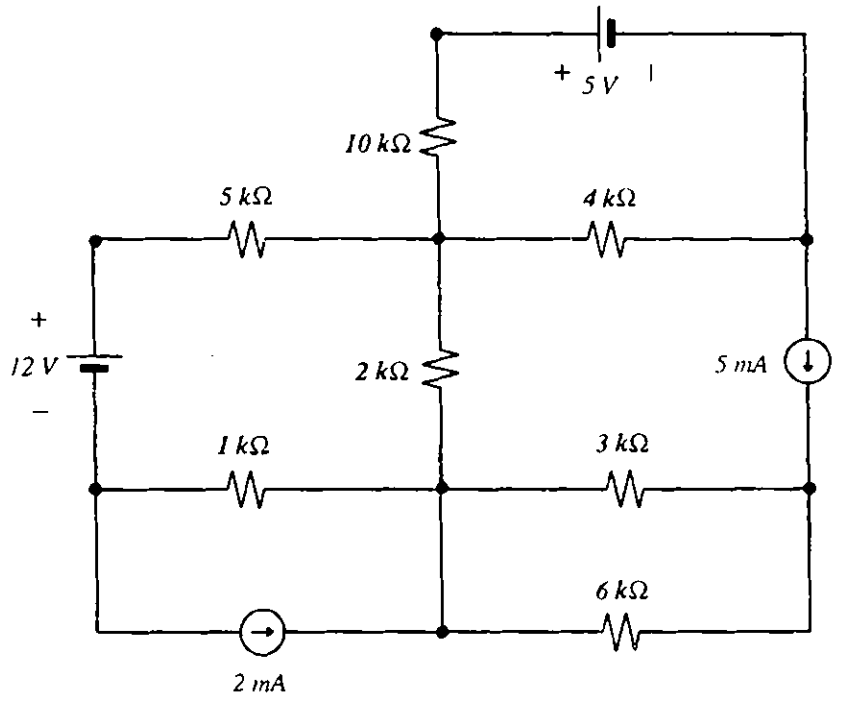
[1] (iii) If a load resistor (R_L) is attached to terminals A and B, for what value of R_L will the power transferred be maximized?

R_L =

Part B: DC Circuits

Question 8: Nodal Analysis [5 marks]

Write down a complete set of nodal equations for the circuit shown below. Clearly mark the reference node and all relevant nodes on the circuit diagram.

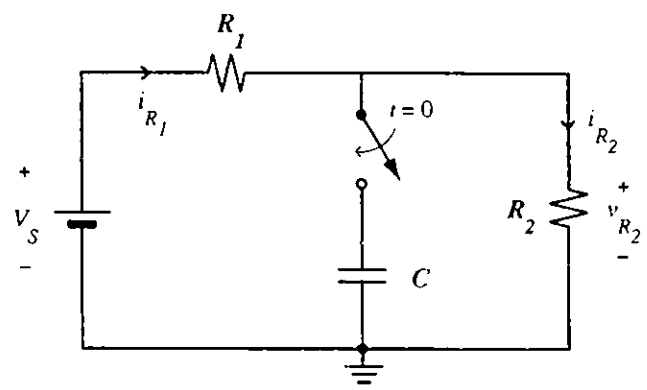


Part C: Transient Analysis

Question 9: RC Circuit [15 marks]

Consider the circuit shown below. Assume that the switch was open for a very long time before time zero and that the capacitor is initially not charged.

[1] (i) Find the voltage $v_{R_2}(0^-)$ across the resistor R_2.



$v_{R_2}(0^-)$ =

[1] (ii) Find the voltage $v_{R_2}(0^+)$ across the resistor R_2.

$v_{R_2}(0^+)$ =

[2] (iii) Find the voltage $v_{R_2}(\infty)$ across the resistor R_2 after the switch is closed.

$v_{R_2}(\infty)$ =

[2] (iv) What is the time constant (τ) for the circuit?

τ =

Part C: Transient Analysis

Question 9 (...continued)

[2] (v) Find $v_{R_2}(t)$ for $t > 0$.

[3] (vi) Sketch $v_{R_2}(t)$ for all t. Clearly mark the key points on the curve (i.e. $v_{R_2}(0^-)$, $v_{R_2}(0^+)$, $v_{R_2}(\infty)$)

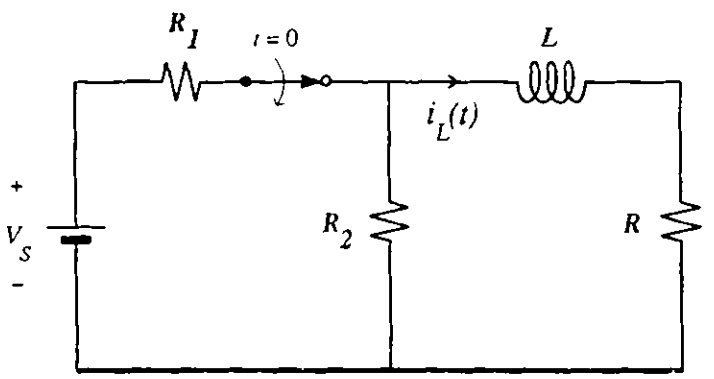
[2] (vii) Find $i_{R_1}(t)$ for $t > 0$.

[2] (viii) Sketch $i_{R_1}(t)$. Clearly mark $i_{R_1}(0^+)$ and $i_{R_1}(\infty)$ in the curve.

Part C: Transient Analysis

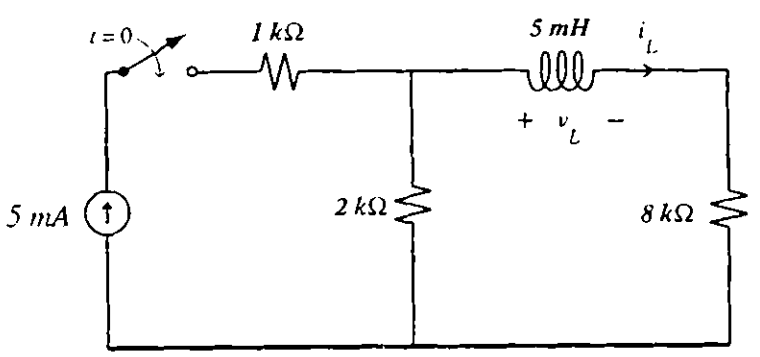
Question 10: Differential Equation [5 marks]

Derive the first order differential equation in terms of $i_L(t)$ for $t > 0$. Note, at $t=0$ the switch is opened from the position shown in the circuit given below.



Question 11: RL Circuit [4 marks]

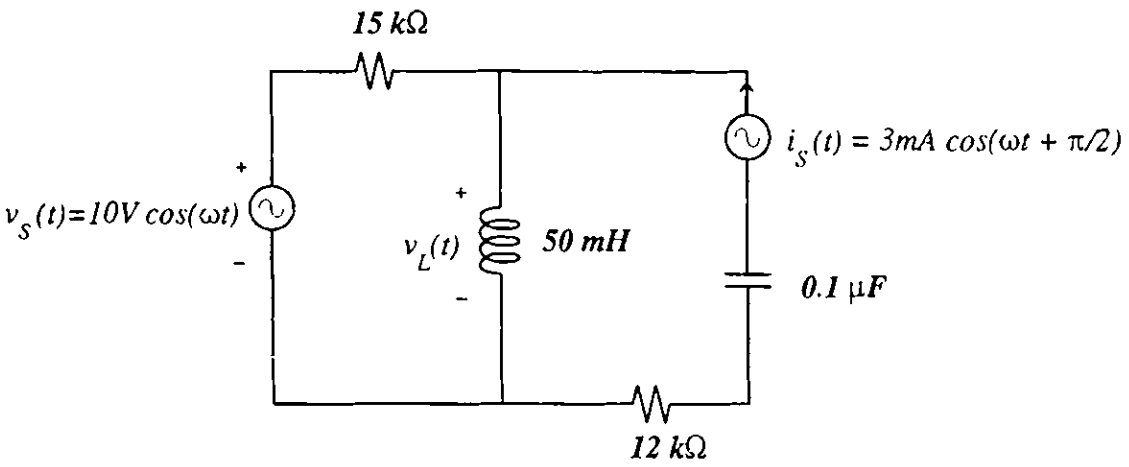
Find the voltage $v_L(\infty)$ and current $i_L(\infty)$.



Part D: AC Circuits

Question 12: AC Circuit [17 marks]

The following circuit is powered by both a current source and a voltage source, both operating at the same angular frequency, $\omega = 1000$ rad/s.



[4] (i) Draw the circuit in the frequency domain, clearly stating the source phasors and the impedances.

[6] (ii) Using superposition, find the voltage phasor across the inductor.

[2] (iii) Calculate the inductor current phasor.

[2] (iv) State the inductor voltage and inductor current in the time domain.

[3] (v) Calculate the real power, reactive power and apparent power absorbed by the inductor.

Part D: AC Circuits

Question 13: AC Circuit [8 marks]

An electric motor rated at 1200 W is powered by a 240 V_{rms}, 60 Hz source. The motor is operating at a lagging power factor of 0.9.

[4] (i) Assuming that the motor can be represented by a resistor and an inductor in series, calculate the corresponding resistance and inductance.

[4] (ii) What component would you introduce in the circuit to operate the motor at a power factor of unity? What is the magnitude of this component and how is it connected to the circuit?