University of Toronto

Applied Science and Engineering

MIE100 – Dynamics

Final Exam

April 26, 2013: 2:00pm – 4:30pm

Examiners: A. Sinclair, L. Sinclair, P. Sullivan, L. You

Permitted Aids: One $8\frac{1}{2}$" by 11" aid sheet (any colour) and a non-programmable calculator

This exam has 4 questions and 5 pages

Answer all 4 questions – Do all work in your exam answer book

Each question is worth 25 marks

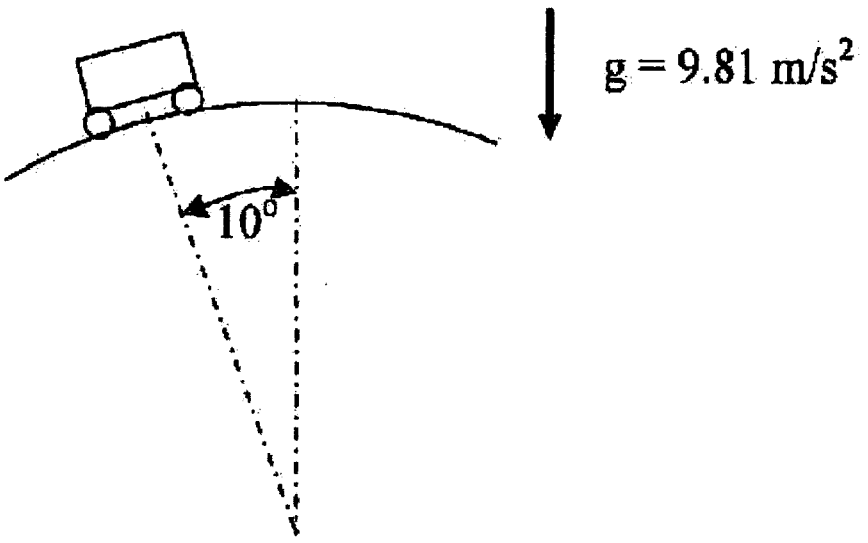
Q.1

A car of mass 1800 kg is going up a hill that is in the shape of a circular arc of radius 175 meters. At the instant shown in the diagram, as it approaches the top of the hill, the car wheels are pointed straight ahead, the car's speed is 25 m/s, and its speed is decreasing by 2 m/s^2. The wheels are not skidding.

(a) At the instant shown in the diagram, find the acceleration of the car. Express your answer in normal-tangential coordinates. (5 marks)

(b) At the instant shown in the diagram, find the total frictional force exerted by the road on the wheels of the car. Express your answer in normal-tangential coordinates. (10 marks)

(c) At the instant shown in the diagram, find the minimum value of the coefficient of static friction (μ_s) required to prevent the wheels from slipping. (10 marks)

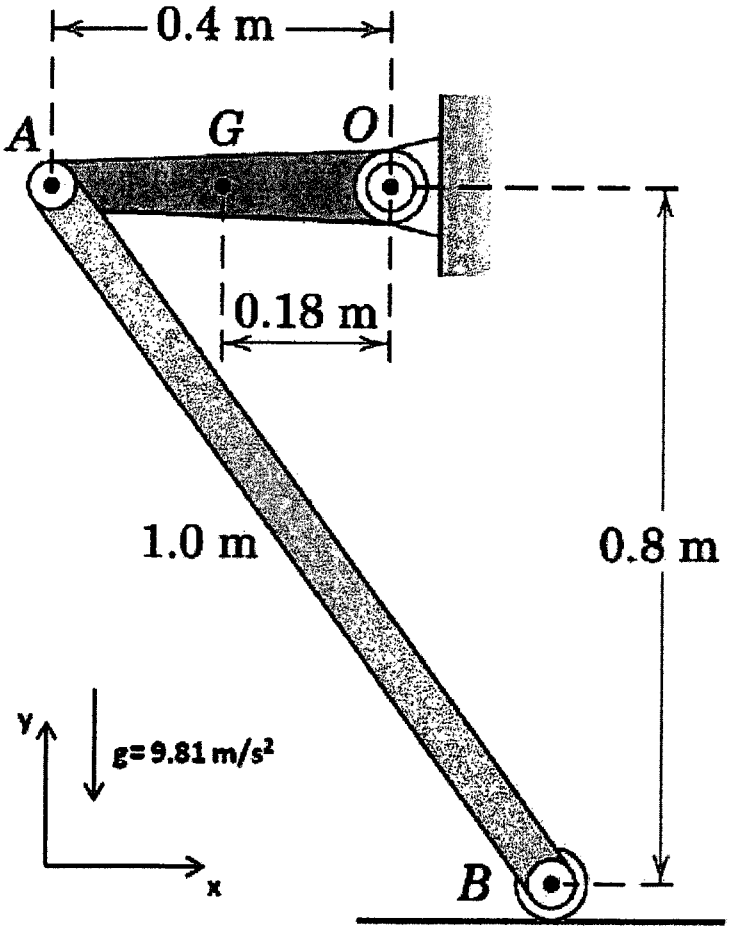


Q.2

The 57 kg crank OA with centre of mass at G has a radius of gyration $k_{G\,crank} = 0.127$ m and is connected to the 9 kg uniform, slender bar AB which has a radius of gyration $k_{G\,bar} = 0.289$ m.
The linkage is released from rest at the position shown; neglect the mass of the wheel at B. Answer both (a) and (b) below for the instant when OA reaches a vertical orientation.

(a) Draw a CLEAR diagram showing the direction of the velocity vectors at the two centres of mass and points A and B. Indicate also the rotational direction of OA as it rotates about O. (10 marks)

(b) Find the velocity of point B. (15 marks)



Q.3

A particle of mass 'm' is embedded at the outer rim (at B) of the otherwise uniform disk of mass M and radius r as shown. The disk starts from rest in the position shown (AD is vertical) and does not slip on the rough incline.

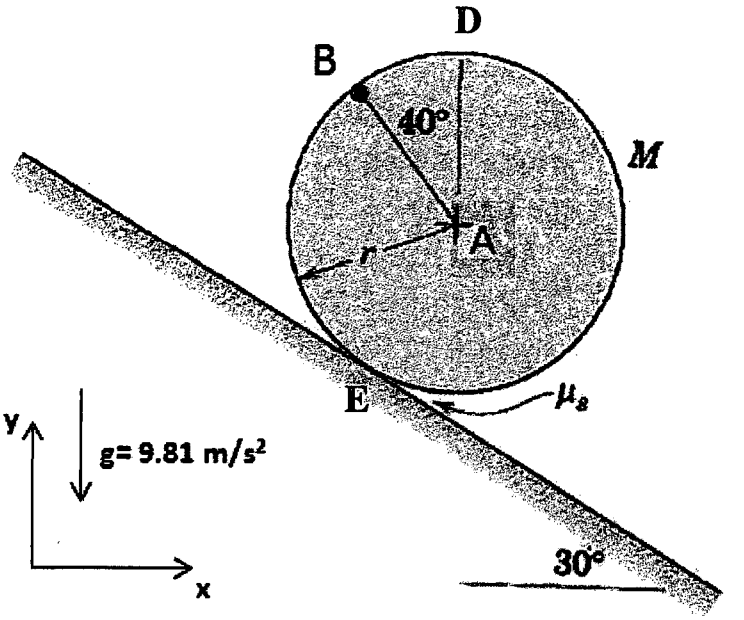
(Hint: for parts (a) and (b) below, treat m and M as separate masses; point A is the center of mass of the uniform disk alone.)

(a) For the position shown, draw a CLEAR free body diagram showing the direction of all external forces exerted on the disk. (5 marks)

(b) For the position shown, find the minimum mass of the particle (in terms of M) so that the disk will begin to roll up the incline. (5 marks)

(c) If m = 4M, find G and I_G of the disk that contains the embedded mass m. Show the location of G on a CLEAR diagram. (5 marks)

(d) If m = 4M, determine the initial angular acceleration of the disk. (10 marks)



Q.4

A block "A" of mass 50 kg is suspended from the ceiling by a spring with stiffness constant k_1 = 900 N/m, and a dashpot of strength C. The mass moves only in the vertical direction, without rotation.

(a) Find the period of natural vibration of "A". Assume that C = 0. (2 marks)

(b) Suppose that "A" is vibrating in the vertical direction, with an amplitude of vibration equal to 5 mm (10mm peak-to-peak). Find the maximum acceleration of "A". Assume that C = 0. (7 marks)

(c) Find the value of C that would be required to reduce the amplitude of vibration of "A" from 0.5 meters to 0.2 meters, in 15 seconds? (8 marks)

(d) Suppose that an earthquake were to make the ceiling vibrate vertically with an amplitude of 3 mm, at a frequency of 5 radians/second. Find the amplitude of vibration of "A"? Assume C = 0. (8 marks)