# MIE100 Midterm Examination, 6:15-7:55pm Thursday March 2, 2006

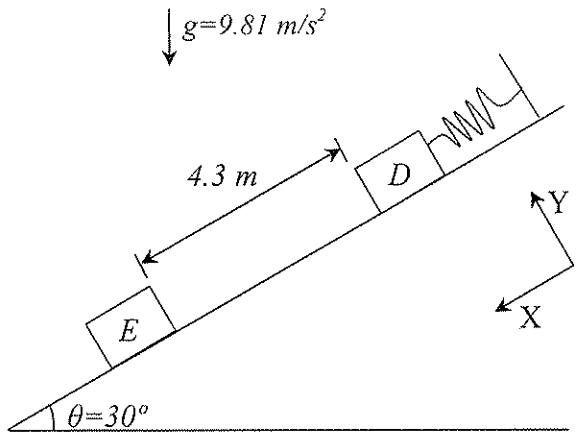
*This is a one hour and 40 minute exam. It is a type C test. Only one aid sheet and an approved calculator (Casio 260, Sharp 520, TI30) are allowed. Answer all questions.*

1. A particle moves along a path given by $r = 5 + \frac{t}{100}$ and $\theta = \pi + \frac{t}{100}$ (units are meters and seconds).

a) Determine the location of the particle and its acceleration at $t=200\ sec$ in polar coordinates. (5 marks)

b) Sketch the location of the particle on a set of standard rectangular axes and at that location, sketch the acceleration vector. (5 marks)

2. At the instant shown in the diagram, block $D$ is moving at $7\ m/s$ down the hill and the spring is compressed $1.8\ m$. The coefficient of dynamic friction between the hill and blocks D and E is $\mu_k=0.45$. The mass of block $D$ is $4\ kg$ and the mass of block $E$ is $3\ kg$. The spring has a stiffness coefficient $k=25\ N/m$. At the instant shown,




a) What is the elastic potential energy of the spring? (5 marks)

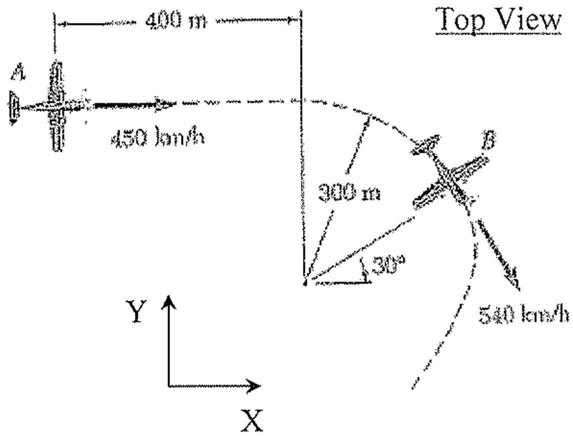
b) What is the acceleration of block $D$? (10 marks) (use the given X-Y coordinates)

After traveling down the slope, block $D$ hits the stationary block $E$ and sticks to it. The two blocks then start sliding together down the slope, stuck together.

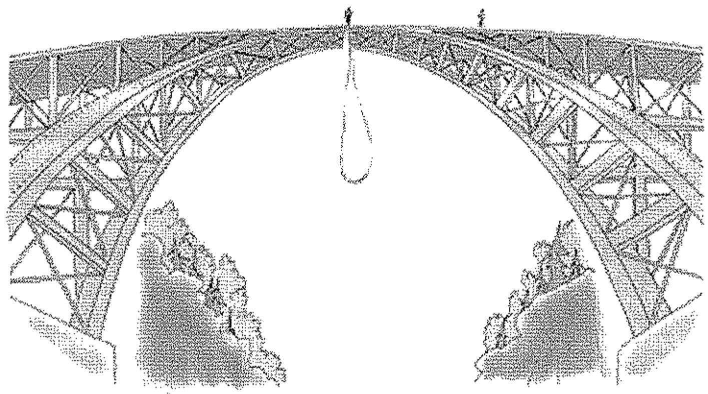
c) What is the speed of block $D$ immediately before the collision with block $E$? (10 marks)

d) By what percentage does the speed of block $D$ change during its collision with block $E$? (10 marks)

3. At a given instant in an airplane race, airplane $A$ is flying horizontally in a straight line, and its speed is increasing at the rate of $8\ m/s^2$. Airplane $B$ is flying at the same altitude as airplane $A$, as it rounds a pylon, is following a circular path of $300\ m$ radius. Knowing that at the given instant the speed of $B$ is decreasing at the rate of $3\ m/s^2$, determine, for the position shown, the acceleration of $B$ relative to $A$. (20 marks)




4. A "bungee jumper" with a mass of $80\ kg$ jumps from a flat bridge that is $50\ m$ above a river. The person is attached to a "bungee" cord that has an unstretched length of $25\ m$ and has a spring constant of $150\ N/m$. The person is attached to the cord at the ankles and the person's height is $1.8\ m$. An observer watches from $10\ m$ away on the bridge.



a) When the person comes to a complete stop once all oscillations settle down, how far above the river is the top of the head of the jumper? (5 marks)

The same bungee cord is now attached to an $80\ kg$ cannonball, which is pushed from the same spot on the bridge.

b) What is the maximum force the bungee cord exerts on the cannonball? What is the acceleration of the cannonball at that instant? (20 marks)

c) What is the angular momentum of the cannonball (with respect to the observer) after it has fallen $25m$? (10 marks)

Note: please assume that any damping in the cord and also air resistance are negligible in the initial fall of the jumper/cannonball.

**Solution**

# 1.

$$\begin{cases} r = 5 + \frac{t}{100} \\ \theta = \pi + \frac{t}{100} \end{cases} \Rightarrow \begin{cases} \dot{r} = \frac{1}{100} \\ \dot{\theta} = \frac{1}{100} \end{cases} \Rightarrow \begin{cases} \ddot{r} = 0 \\ \ddot{\theta} = 0 \end{cases}$$

(a)

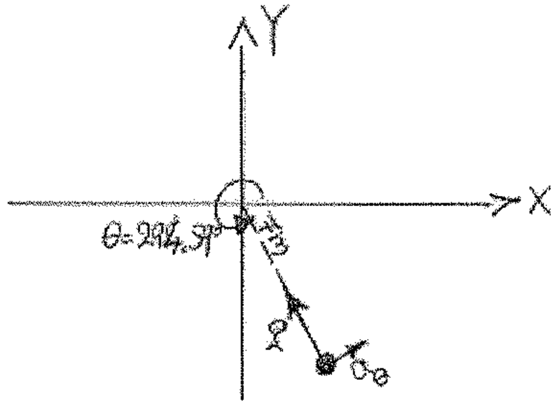
$$t = 200\text{ sec.} \Rightarrow \begin{cases} r = 5 + \frac{200}{100} = 7\text{m} \\ \theta = \pi + \frac{200}{100} = \pi + 2\ (\text{rad}) = 294.59^\circ \end{cases}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 7\left(\frac{1}{100}\right)^2 = -7\times10^{-4}\ \text{m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2\left(\frac{1}{100}\right)\left(\frac{1}{100}\right) = 2\times10^{-4}\ \text{m/s}^2$$

$$\Rightarrow \vec{a} = \frac{1}{10^4}\left(-7\vec{u}_r + 2\vec{u}_\theta\right)\ \frac{m}{s^2}, \quad |\vec{a}| = 7.28\times10^{-4}\ \frac{m}{s^2}$$

(b)

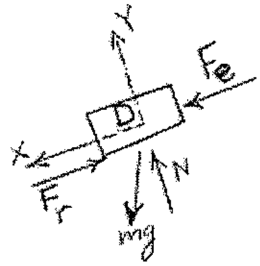


#2.

(a) $V_e = \frac{1}{2}Kx^2 = \frac{1}{2}(25)(-1.8)^2 = 40.5\ J$

(b) $\sum F_y = 0 \Rightarrow N - mg\cos 30 = 0 \Rightarrow$

$N = 4(9.81)(0.866) = 34.0\ N$



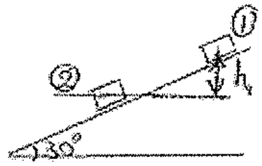

$\sum F_x = m_D a_D \Rightarrow F_e - F_r + m_D g\sin\theta = m_D a_D$

$-Kx - \mu N + m_D g\sin\theta = m_D a_D$

$-25(-1.8) - 0.45(34) + 4(9.81)\sin 30 = 4a_D$

$\Rightarrow a_D = 12.33\ m/s^2$ or $\vec{a_D} = 12.33\,\hat{i}\ (m/s^2)$

(c) $T_1 + V_1 = T_2 + V_2 + U_{F_r}$ ①




$T_1 = \frac{1}{2}m_D V_1^2 = \frac{1}{2}(4)(7)^2 = 98\ J$

$V_1 = V_e + V_g = \frac{1}{2}Kx_1^2 + m_D g h_1 = \frac{1}{2}(25)(-1.8)^2 + 4(9.81)(4.3\sin 30)$

$\Rightarrow V_1 = 124.87\ J$

$V_2 = V_{2e} + V_{2g} = \frac{1}{2}Kx_2^2 + \cancel{m_D g h_2} = \frac{1}{2}(25)(2.5)^2 = 78.13\ J$

$T_2 = \frac{1}{2}m_D V_2^2 = 2V_2^2$

$U_{F_r} = F_r \cdot d = 15.29 \times 4.3 = 65.75\ J$

① $\Rightarrow 98 + 124.87 = 2V_2^2 + 78.13 + 65.75 \Rightarrow V_2 = 6.28\ m/s$

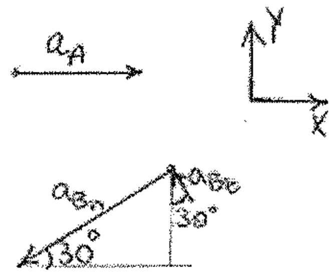
(d) $$\left.\begin{array}{l} m_D V_D + m_E V_E = m_D V_D' + m_E V_E' \\ \text{but } V_E' = V_D' \end{array}\right\} \Rightarrow V_D' = \frac{m_D}{m_D + m_E} V_D = \frac{4}{7} V_D$$

$\Rightarrow$ Speed of D decreases by $\frac{3}{7}$, or $\sim 43\%$.

# #3.

$a_A = 8\hat{i}\, m/s^2$

$\vec{a}_B = \vec{a}_{B_n} + \vec{a}_{B_t}$ , $|a_{B_n}| = \frac{v_B^2}{\rho} = 75\, m/s^2$




$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$-75\cos 30\,\hat{i} - 75\sin 30\,\hat{j} - 3\sin 30\,\hat{i} + 3\cos 30\,\hat{j} = 8\hat{i} + \vec{a}_{B/A}$

$-64.95\,\hat{i} - 37.5\,\hat{j} - 1.5\,\hat{i} + 2.60\,\hat{j} = 8\,\hat{i} + \vec{a}_{B/A}$

$\Rightarrow \vec{a}_{B/A} = -74.45\,\hat{i} - 34.9\,\hat{j}\ (m/s^2)$ , $|\vec{a}_{B/A}| = 82.22\,(m/s^2)$

# 4.

(a) When the jumper comes to a complete stop, the force in the cord is exactly equal to the weight of the jumper, i.e.:

$mg = Kx$ ($x$ is the expansion of the cord)

$$\Rightarrow x = \frac{mg}{K} = \frac{80 \times 9.81}{150} = 5.23\ m$$

Distance from river $= 50 - (25 + 5.23 + 1.80) = 17.96\ m$

(b) At the beginning of the fall, the ball is only subject to the gravitational acceleration. Once the ball reaches 25m below the bridge, then the cord will also start slowing it down.

During the initial 25m: $v_1^2 = \cancel{v_0^2} + 2g(s - \cancel{s_0}) \Rightarrow v_1 = \sqrt{2gs}$

at 25m $\Rightarrow v_1 = 22.1\ m/s$

Once the spring force kicks in, the ball is subject to conservative forces only. The maximum force is exerted when the ball is at the maximum extended position. The position is determined by applying the principle of mechanical energy conservation between position 1 (moment when spring

Kicks in) and position 2 (maximum cord extension position):

$$T_1 + V_{g_1} + \cancel{V_{e_1}}^{\,0} = \cancel{T_2}^{\,0} + \cancel{V_{g_2}}^{\,0} + V_{e_2}$$

$$\frac{1}{2} m v_1^2 + mgh = \frac{1}{2} K h^2 \Rightarrow$$

$$\frac{1}{2}(80)(22.1)^2 + 80(9.81)h = \frac{1}{2}(150)h^2 \Rightarrow$$

$$75h^2 - 784.8h - 19714 = 0 \Rightarrow$$

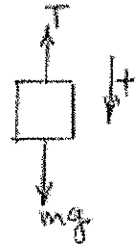
$$h^2 - 10.5h - 263 = 0 \Rightarrow \begin{cases} h_1 = 22.3\,m \\ h_2 = -11.75\,m \text{ (rejected)} \end{cases}$$

$$\Rightarrow h = 22.3\ m$$

The maximum force: $T_{max} = Kh = 150 \times 22.3$

$$T_{max} = 3345\ N$$

Applying Newton's law to the Cannonball:




$$mg - T = ma \Rightarrow a = \frac{mg - T}{m} = \frac{80(9.81) - 3345}{80}$$

$$a = -32\ m/s^2$$

(C) Angular momentum of the ball w.r.t. the point O:


$\theta = 21.8°$

$$m(V\sin\theta)r = 80(22.1 \sin 21.8°)(26.93)$$

$$= 17679\ Kg\,\frac{m^2}{s}$$