MIE100S Dynamics Midterm Examination

6:10 – 7:55 PM, Thursday March 6th, 2008

This is a 1 hour 45 min exam. It is type C test; only one aid sheet and an approved calculator (Casio 260, Sharp 520, T130) are allowed. Answer all three questions.

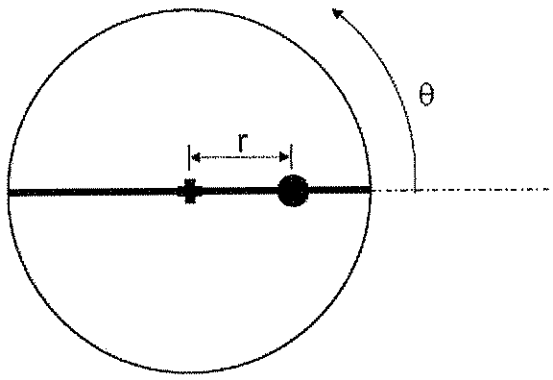
Question (1)

A straight line has been painted along the diameter of the merry-go-round, as shown in the diagram. A woman of mass 60 kg, represented by the bullet in the diagram, is walking along the line towards the center. At the instant shown, she is $r = 4$ meters from the center at $\theta = 0$; her speed relative to the merry-go-round is 0.7 m/s and is decreasing by 0.15 m/s^2 relative to the merry-go-round. The rate of rotation (i.e. angular velocity) of the merry-go-round is 0.1 radians per second at the instant shown, and is increasing by 0.017 **revolutions** per second squared in the positive direction.

a. Determine $\ddot{\theta}$ and $\ddot{r}$ for the woman's position at the instant shown, in S.I. units. (10 marks)

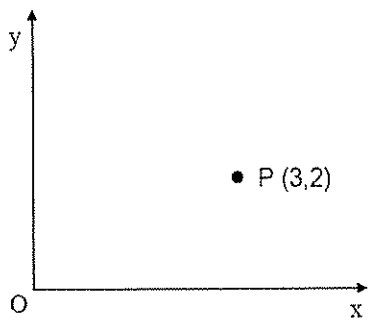
b. Determine the radial component of the friction force exerted by the merry-go-round on her feet, at the instant shown? (15 marks)

c. What will be her position relative to the ground in r-θ coordinates at $t = 5$ seconds? (20 marks)



Question (2)

A particle is located in the x-y plane at x = 3 m and y = 2 m, as shown. The particle has a velocity of $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$ m/s and acceleration $\mathbf{a} = 3\mathbf{j}$ m/s^2.



a. What is the particle's velocity in the n-t system? (5 marks)

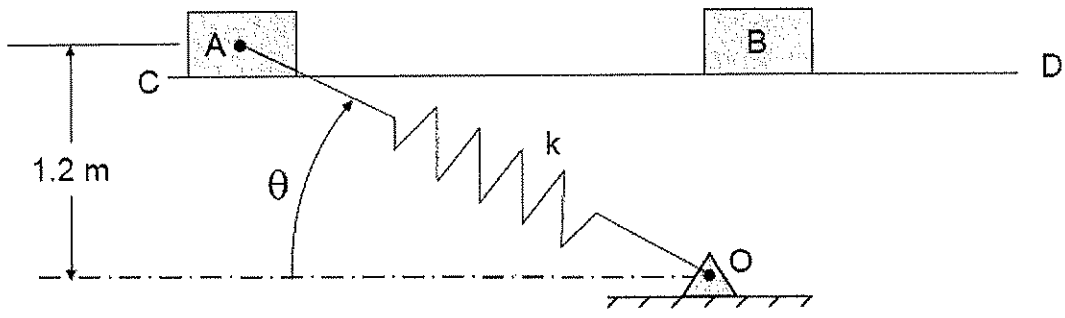
b. What is the particle' acceleration in the r-θ system, with the origin defined as in the figure? (15 marks)

Question (3)

Blocks A and B slide on the frictionless surface C-D. The mass of block A is 4 kg and the mass of block B is 5 kg. The spring has a stiffness coefficient of 10 N/m and neutral (relaxed, natural) length of 0.9 m. Block A is released from rest when θ = 30°.

a. Determine the speed of block A at the instant when its position is directly above point O. (15 marks)

b. Immediately after block A passes above point O, it collides with and sticks permanently to block B. Determine the maximum distance the pair will travel together to the right. (20 marks)



Question #1

$m = 60\ kg$

$r = 4m \quad \theta = 0$

$\dot{r} = -0.7\ m/s \quad \ddot{r} = 0.15\ m/s^2$

$\dot{\theta} = 0.1\ rad/s \quad \ddot{\theta} = 0.017 \times 2\pi = 0.107\ rad/s^2$

a) at the instant shown:

$\ddot{r} = 0.15\ m/s^2$ & $\ddot{\theta} = 0.107\ rad/s^2$

b) $\sum F_r = m a_r$

$a_r = \ddot{r} - r\dot{\theta}^2$

$= 0.15 - 4 \times 0.1^2 = 0.11\ m/s^2$

$\therefore F_r = 60 \times 0.11 = 6.6\ N$

c) after 5 seconds:

$r = r_0 + \dot{r}t + \frac{1}{2}\ddot{r}t^2$

$= 4 + (-0.7) \times 5 + \frac{1}{2} \times 0.15 \times 25 = 2.375\ m$

$\theta = \theta_0 + \dot{\theta}t + \frac{1}{2}\ddot{\theta}t^2$

$= 0 + 0.1 \times 5 + \frac{1}{2} \times 0.107 \times 25$

$= 1.8375\ rad$

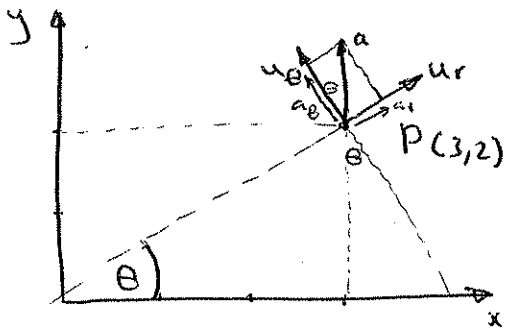
$= 105.3^\circ$

Problem (2)

5 min

$x, y = 3, 2$

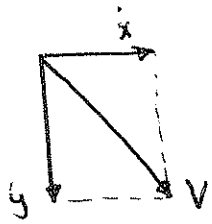
$V = 3i - 4j$ m/s , $a = 3j$ m/s²



a) $V = \sqrt{\dot{x}^2 + \dot{y}^2}$

$= \sqrt{9 + 16} = 5$ m/s

$\therefore \boxed{V = 5\,\hat{u}_t \text{ m/s}}$



b) $\theta = \tan^{-1}\frac{y}{x}$

$= \tan^{-1}\frac{2}{3} = 33.69°$

$\therefore a_r = a \sin\theta$

$= 3 \sin(33.69) = 1.66$ m/s²

$a_\theta = 3 \cos\theta$

$= 3 \cos(33.69) = 2.496$ m/s²

$\therefore \boxed{\bar{a} = 1.66\,\hat{u}_r + 2.496\,\hat{u}_\theta \text{ m/s}}$

17 min

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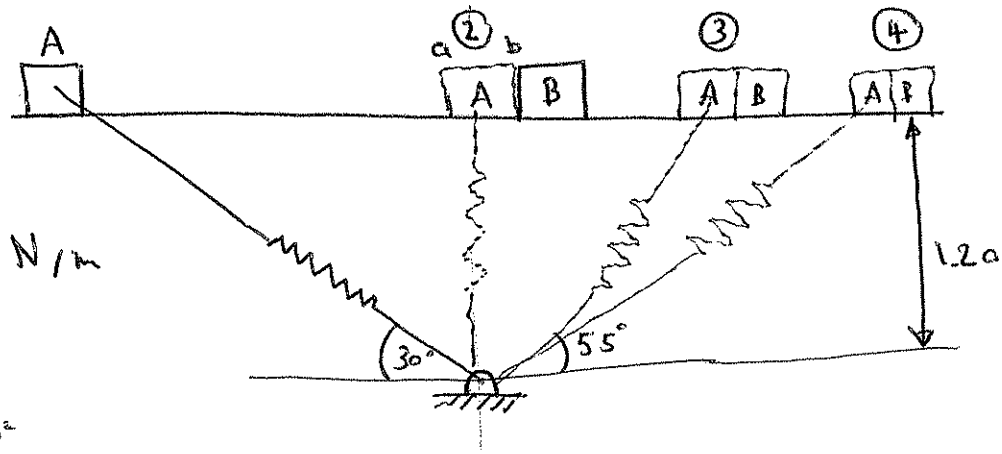
Problem (4)

①

$m_A = 4$ kg

$m_B = 5$ kg

$L_o = 0.9$ m , $k = 10$ N/m



a) $T_{1A} + V_{1A} = T_{A2} + V_{A2}$

$T_{A1} = 0$

$V_{A1} = \frac{1}{2} k (L_1 - L_o)^2$

$= \frac{1}{2} \times 10 (2.4 - 0.9)^2$

$= 11.25$ J

$L_1 = \frac{1.2}{\sin 30} = 2.4$ m

$V_{A2} = \frac{1}{2} k (L_2 - L_o)^2$

$= \frac{1}{2} \times 10 (1.2 - 0.9)^2$

$= 0.45$ J

$\therefore T_{A2} = \frac{1}{2} m_A V_2^2 = T_{A1} + V_{A1} - V_{A2}$

$\therefore \frac{1}{2} \times 4 \times V_2^2 = 0 + 11.25 - 0.45$

$\therefore \boxed{V_A = 2.324 \text{ m/s}}$

b) $m_A V_A + m_B V_B = (m_A + m_B) V_2'$

$\therefore 4 \times 2.324 + 0 = 9 V_2'$

$\therefore \boxed{V_2' = 1.033 \text{ m/s}}$

(b) 4.

$T_2' + V_2' = T_4 + V_4$

$T_2' = \frac{1}{2}(m_A + m_B) V_2'^2$

$= \frac{1}{2} \times 9 \times 1.033^2 = 4.8 \text{ J}$

$V_2' = 0.45 \text{ J}$

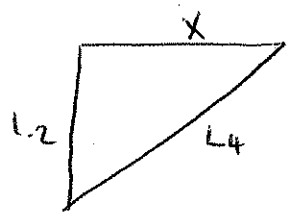
$T_4 = 0$

$V_4 = \frac{1}{2} k (L_4 - L_0)^2$

$= 5 (L_4 - L_0)^2$

$\therefore 5 (L_4 - L_0)^2 = 4.8 + 0.45 - 0$

$\therefore L_4 = 1.925 \text{ m}$



$\therefore$ displacement travelled $(x) = \sqrt{L_4^2 - 1.2^2}$

$= \sqrt{1.925^2 - 1.2^2} = \boxed{1.5 \text{ m}}$

c) $T_2' + V_2' = T_3 + V_3$

$V_3 = \frac{1}{2} k (L_3 - L_0)^2$ $\qquad L_3 = \frac{1.2}{\sin 125} = 1.465 \text{ m}$

$= \frac{1}{2} \times 10 (1.465 - 0.9)^2 = 1.596 \text{ J}$

$\therefore T_3 = \frac{1}{2}(m_A + m_B) V_3^2 = 4.8 + 0.45 - 1.596 = 3.654 \text{ J}$

$\therefore V_3 = \sqrt{\frac{3.654}{0.5 \times 9}} = 0.901 \text{ m/s}$

$H_{B_0} = m_B V_3 r = 5 \times 0.901 \times 1.2 = \boxed{5.4 \text{ J.s}}$