University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, APRIL, 2010
First Year - CHE, CIV, CPE, ELE, ENG, IND, LME, MEC, MSE

MAT188H1S - LINEAR ALGEBRA
Exam Type: A

SURNAME: (as on your T-card) ______________________

YOUR FULL NAME: ______________________

STUDENT NUMBER: ______________________

SIGNATURE: ______________________

Examiner:
D. Burbulla

Calculators Permitted: Casio 260, Sharp 520 or TI 30.

Notation: as in the textbook, 0 represents the zero vector, the zero matrix, or the zero number, depending on context. Also, if A is an $n \times n$ matrix, then $E_\lambda(A) = \{X \in \mathbb{R}^n \mid AX = \lambda X\}$.

INSTRUCTIONS: Attempt all questions. Present your solutions in the space provided. Use the backs of the sheets if you need more space. Do not tear any pages from this exam. Make sure your exam contains 10 pages.

MARKS: Questions 1 through 6 are Multiple Choice; circle the single correct choice for each question. Each correct choice is worth 4 marks.

Question 7 is worth 10 marks; 2 for each part.

Questions 8, 9 and 10 are each worth 10 marks.

Questions 11, 12 and 13 are each worth 12 marks.

TOTAL MARKS: 100

PAGE	MARK
MC	
Q 7	
Q 8	
Q 9	
Q 10	
Q 11	
Q 12	
Q 13	
TOTAL	

1. If U is a subspace of $\mathbb{R}^7$ and $\dim(U) = 4$, then $\dim\left(U^{\perp}\right) =$

(a) 2

(b) 3

(c) 4

(d) 5

2. In the solution to the system of equations $\left\{\begin{array}{rcrcrcr} 3x & + & 4y & - & 3z & = & 6 \\ x & + & 3y & + & z & = & -2 \\ -x & - & 2y & + & 4z & = & -8 \end{array}\right.$ the value of y is

(a) 0

(b) $\frac{6}{19}$

(c) $-\frac{6}{19}$

(d) 2

3. $\dim\left(\operatorname{span}\left\{\begin{bmatrix} 1 \\ 0 \\ -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ -4 \\ 0 \end{bmatrix}\right\}\right) =$

(a) 1

(b) 2

(c) 3

(d) 4

4. If T_1 is a reflection in the x-axis and T_2 is a reflection in the line $y = x$, then

(a) $T_1 \circ T_2$ is a reflection in the line $y = -x$.

(b) $T_1 \circ T_2$ is a reflection in the y-axis.

(c) $T_1 \circ T_2$ is a rotation through $\frac{\pi}{2}$, counterclockwise, around the origin.

(d) $T_1 \circ T_2$ is a rotation through $\frac{\pi}{2}$, clockwise, around the origin.

5. Suppose two lines in space are both parallel to the vector $\vec{d}$, one line passes through the point X_1, and the other line passes through the point X_2. The minimum distance between these two parallel lines is given by

(a) $\left\|\overrightarrow{X_1X_2}\right\|$

(b) $\left\|\operatorname{proj}_{\vec{d}}\left(\overrightarrow{X_1X_2}\right)\right\|$

(c) $\sqrt{\left\|\overrightarrow{X_1X_2}\right\|^2 - \left\|\operatorname{proj}_{\vec{d}}\left(\overrightarrow{X_1X_2}\right)\right\|^2}$

(d) $\sqrt{\left\|\operatorname{proj}_{\vec{d}}\left(\overrightarrow{X_1X_2}\right)\right\|^2 - \left\|\overrightarrow{X_1X_2}\right\|^2}$

6. Suppose $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x + 5y \\ 4x + 6y \end{bmatrix}$. Then the area of the image of the unit square is

(a) -8

(b) 8

(c) -32

(d) 32

7. Decide if the following statements are equivalent or not equivalent to the statement

A is an invertible $n \times n$ matrix.

Give a brief, concise justification for your choice. Circle your choice.

(a) A is diagonalizable. **Equivalent** **Not equivalent**

(b) $A^T A$ is invertible. **Equivalent** **Not equivalent**

(c) $\lambda = 0$ is an eigenvalue of A. **Equivalent** **Not equivalent**

(d) $\operatorname{im}(A) = \mathbb{R}^n$. **Equivalent** **Not equivalent**

(e) $(\operatorname{row}(A))^{\perp} = \mathbb{R}^n$. **Equivalent** **Not equivalent**

8. Given that the reduced row-echelon form of

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & -3 \\ 0 & 0 & 1 & 3 & 11 \\ 1 & 2 & -3 & 4 & 0 \\ 1 & 2 & 4 & 2 & 8 \end{bmatrix} \text{ is } R = \begin{bmatrix} 1 & 2 & 0 & 0 & -6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

state the rank of A, and find a basis for each of the following: the row space of A, the column space of A. and the null space of A.

9. Let A be an $n \times n$ matrix. Let $U = \{X \in \mathbb{R}^n \mid A^T X = AX\}$.

(a) [5 marks] Show that U is a subspace of $\mathbb{R}^n$.

(b) [5 marks] Show that if A is an orthogonal matrix, then $\dim U = \dim E_1(A^2)$.

10. Write $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ as a linear combination of the four vectors $\begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ 3 \\ 1 \end{bmatrix}$.

11. Find an orthogonal matrix P and a diagonal matrix D such that $D = P^T AP$, if

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}.$$

12. Let $U = \operatorname{span}\left\{ \begin{bmatrix} -2 & 1 & 0 & 0 \end{bmatrix}^T, \begin{bmatrix} 1 & 1 & 1 & 2 \end{bmatrix}^T, \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}^T \right\}$;
let $X = \begin{bmatrix} 1 & 1 & 2 & 1 \end{bmatrix}^T$. Find:

(a) [6 marks] an orthogonal basis of U.

(b) [6 marks] $\operatorname{proj}_U(X)$.

13. Find the least squares approximating quadratic for the data points

$$(2, 0), (3, -10), (5, -48), (6, -76).$$