Name: ______________________________

Student number:______________________

MAT 186
Quiz 6

AB6	CF12	CF13	MW3

1. **Find the local and (if they exist) global extrema of $y = 3x^4 + 8x^3 + \frac{15}{2}x^2 + 3x - \frac{7}{2}$.**

$$\begin{aligned} f'(x) &= 12x^3 + 24x^2 + 15x + 3 \\ &= 3(2x+1)^2(x+1) \end{aligned}$$

So, the critical points are at $x = -1$ and $x = -\frac{1}{2}$.

However, the function is decreasing on $x < 1$ and increasing on $x > 1$, so we end up with $x = -\frac{1}{2}$ being neither a maximum nor a minimum. $x = 1$ gives the only minimum and the function increases without bound on either side of the point. Therefore, $(-1, -4)$ is a global minimum.

AB1	CF11	CF15

2. **The function $y = x^3 + x^2 - 8x - 12$ has one positive $x-$intercept. We build a rectangle with horizontal and vertical sides that has the origin as one vertex and a point on the graph of y for its opposite vertex. If this second point lies between the $y-$ and $x-$ intercepts of the graph, what value(s) would maximize the area of the rectangle?**

Factoring y, we get $y = (x+2)^2(x-3)$, so the second point has $0 \leq x \leq 3$. We should also note that the point will have a *negative* y value, as that will make a difference for the question.

If we write the point on the graph as $(x, x^3 + x^2 - 8x - 12)$, then the area of the rectangle created would be $A(x) = -x(x^3 + x^2 - 8x - 12)$, because of the negative coordinate. You can do the question without the negative for now, but it is a little trickier.

Therefore, we have $A(x) = -x^4 - x^3 + 8x^2 + 12x$ and can get going:

$$A'(x) = -4x^3 - 3x^2 + 16x + 12 = 0$$

$$(x+2)(4x+3)(x-2) = 0$$

$$x = -2, -\frac{4}{3}, 2$$

Of these, the first two points are outside our domain, so they can be dismissed. We are left with $x = 2$ as the only critical point in the interval. We can compare its $y-$ value with that of the other two endpoints, analyze using the first derivative test, or go to the second derivative test, as long as we do

Continued on back.

at least one of them. If you did not realize earlier that the point lies below the axis, you will find that $x = 2$ gives a *minimum*, but that it is still the correct value.

Therefore, the maximum area occurs at $x = 2$.

3. Use <u>two</u> linear approximations (that is, use a different initial point each time) to estimate $\sqrt[3]{4}$ CS12

You will find that the better approximation has a larger value for dx (for comparison $\sqrt[3]{4} \approx 1.59$). How could this happen?

I took a gamble here that everyone would use the function $y = \sqrt[3]{x}$ and attempt to use $a = 1$ and $a = 8$. (Note that $a = 0$ will not work because y is not differentiable there.)

For our function, we have $y' = \frac{1}{3x^{2/3}}$. We have:

a	1	8
$y(a)$	1	2
$dx = x - a$	3	-4
$m = y'(a)$	$1/3$	$1/12$
$dy = m \cdot dx$	1	$-1/3$
$\sqrt[3]{4} \approx y(a) + dy$	2	$5/3$

There are a few ways of thinking about why $a = 8$ gives the more accurate answer. It originates with the fact that the graph of $y = \sqrt[3]{x}$ is 'more flat' the further you go. That is, its second derivative is smaller, so the graph stays close to its tangent lines for longer. We can also think about the tangent lines themselves – since the function is concave down, the lines all lie above the graph, but at $x = 4$, the tangent line that starts at $a = 8$ lies below the one from $a = 1$.

[The following question is strictly to help with older marks – the mark will NOT lower your grade. If you are doing well with the squeeze theorem (CS2) and domains of functions (AB6), this will not help your mark much, but you are welcome to answer it.]

4. Find $\lim_{x\to 2}(x-2)\cos\left(\frac{1}{(x-2)^2}\right)$. AB6 | CS2 | MW2

This definitely requires the Squeeze Theorem, with positive and negative cases.

Case I: $x - 2 > 0$, or $x > 2$.

$$-1 \le \cos\left(\frac{1}{(x-2)^2}\right) \le 1$$

$$-(x-2) \leq (x-2)\cos\left(\frac{1}{(x-2)^2}\right) \leq (x-2)$$

$$\lim_{x\to 2} -(x-2) = \lim_{x\to 2}(x-2) = 0$$

Therefore, by the Squeeze Theorem, the limit in case I goes to zero.

Case II is nearly identical and we can get away with saying "Case II is done similarly." Here it is in full:

Case II: $x - 2 < 0$, or $x < 2$.

$$-1 \leq \cos\left(\frac{1}{(x-2)^2}\right) \leq 1$$

$$-(x-2) \geq (x-2)\cos\left(\frac{1}{(x-2)^2}\right) \geq (x-2)$$

$$\lim_{x\to 2} -(x-2) = \lim_{x\to 2}(x-2) = 0$$

Therefore, by the Squeeze Theorem, the limit in case II goes to zero.

Therefore, the limit is zero.