

MIE 200F - Quiz number 3b - October 5/99
quiz duration = 25 minutes

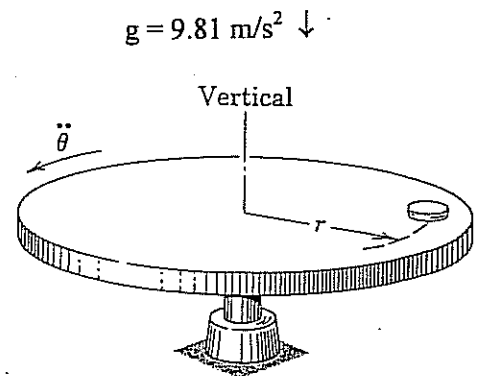
A turntable consists of a disk of radius 18 cm, oriented in the horizontal plane, turning about a vertical axis. It starts to rotate from rest at time $t = 0$, accelerating in a counterclockwise direction with an angular acceleration equal to $\alpha = 0.3 \text{ s}^{-2}$. A coin of mass 10 grams is lying on the surface of the disk, adjacent to the rim.

- (a) What is the magnitude of the total acceleration of the coin at $t = 2$ seconds?
 (b) How much work is done on the coin by the disk in the first 2 seconds?

$$\begin{aligned} (a) \quad \ddot{\theta} &= 0.3 \text{ s}^{-2} \\ \dot{\theta} &= \int \ddot{\theta} dt = (0.3)(2) = 0.6 \text{ s}^{-1} \\ r &= 0.18, \quad \dot{r} = \ddot{r} = 0 \\ a_r &= \ddot{r} - r\dot{\theta}^2 = -(0.18)(0.6)^2 = -0.065 \end{aligned}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.18)(0.3) = 0.054$$

$$|\vec{a}| = \sqrt{a_r^2 + a_\theta^2} = \sqrt{0.065^2 + 0.054^2} = 0.0844 \text{ m/s}^2$$



$$\begin{aligned} (b) \quad \dot{\theta} &= 0.3t \\ \theta &= \int \dot{\theta} = \frac{0.3}{2} t^2 = \frac{0.3}{2} (2^2) = 0.6 \text{ radians.} \\ \Rightarrow \text{distance travelled in } \theta \text{ direction} &= \theta r \\ &= (0.6)(0.18 \text{ m}) = 0.108 \text{ m.} \end{aligned}$$

$$\begin{aligned} \text{Frictional force in } \theta \text{ direction} \\ &= (m)(a_\theta) = (10^{-2} \text{ kg})(0.054) = 5.4 \text{ E-4 Newtons.} \end{aligned}$$

$$\begin{aligned} \text{work} &= \int \vec{F} \cdot d\vec{s} = (5.4 \text{ E-4})(0.108) \\ &= 5.83 \text{ E-5 Joules.} \end{aligned}$$

Note: no displacement in r direction.

$$\text{or } T_{\text{final}} = \frac{1}{2} m v^2 = T_i + U_i \Rightarrow 2$$

$$\left(\frac{1}{2}\right)(0.01)(v^2) = \left(\frac{1}{2}\right)(0.01)(r\dot{\theta})^2 = \left(\frac{1}{2}\right)(0.01)(0.18)^2(0.6)^2 = 5.83 \text{ E-5}$$