University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, DECEMBER, 2009
First Year - CHE, CIV, IND, LME, MEC, MMS

MAT186H1F - CALCULUS I
Exam Type: A

SURNAME: (as on your T-card) _____

GIVEN NAMES: _____

STUDENT NUMBER: _____

SIGNATURE: _____

Examiners:
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Calculators Permitted: Casio 260, Sharp 520 or TI 30.

Notation: $\sin^{-1} x, \cos^{-1} x, \tan^{-1} x$ and $\sec^{-1} x$ are inverse trigonometric functions, which can also be represented by $\arcsin x, \arccos x$, $\arctan x$ and $\operatorname{arcsec} x$, respectively.

INSTRUCTIONS: Attempt all questions. Present your solutions in the space provided. Use the backs of the sheets if you need more space. Do not tear any pages from this exam. Make sure your exam contains 11 pages.

MARKS: Questions 1 through 6 are Multiple Choice; circle the single correct choice for each question. Each correct choice is worth 4 marks.

Questions 7 through 10 are worth 10 marks.

Questions 11 through 13 are each worth 12 marks.

TOTAL MARKS: 100

PAGE	MARK
MC	
Q7	
Q8	
Q9	
Q10	
Q11	
Q12	
Q13	
TOTAL	

1. Let $f(x) = x^2$. What is the average value of f on the interval $[-1, 1]$?

   (a) $\frac{1}{4}$

   (b) $\frac{1}{3}$

   (c) $\frac{1}{2}$

   (d) $\frac{2}{3}$

2. If Newton's method is used to approximate a solution to the equation $x^3 - x - 7 = 0$ and x_0 is chosen to be 2, then the value of x_2 is

   (a) 2

   (b) 2.09090909...

   (c) 2.08675431...

   (d) 2.45454545...

3. Let $f(x) = \frac{x^2 + 5}{x + 2}$. Which of the following statements is a true statement about the function f on the interval $[-8, -2)$?

   (a) The absolute maximum value of f on the interval $[-8, -2)$ is -10 and the absolute minimum value of f on the interval $[-8, -2)$ is -11.5

   (b) The absolute maximum value of f on the interval $[-8, -2)$ is -5 and the absolute minimum value of f on the interval $[-8, -2)$ is -8.

   (c) The absolute maximum value of f on the interval $[-8, -2)$ is -5; there is no absolute minimum value of f on the interval $[-8, -2)$.

   (d) The absolute maximum value of f on the interval $[-8, -2)$ is -10; there is no absolute minimum value of f on the interval $[-8, -2)$.

4. The area of the region between the two curves with equations $y = 4x^2$ and $y = x^3$ for $0 \leq x \leq 6$ is given by

(a) $\displaystyle\int_0^6 (x^3 - 4x^2)\,dx$

(b) $\displaystyle\int_0^6 (4x^2 - x^3)\,dx$

(c) $\displaystyle\int_0^4 (x^3 - 4x^2)\,dx + \int_4^6 (4x^2 - x^3)\,dx$

(d) $\displaystyle\int_0^4 (4x^2 - x^3)\,dx + \int_4^6 (x^3 - 4x^2)\,dx$

5. The value of $\displaystyle\int_1^9 \frac{dx}{\sqrt{x}\,(x+1)^2}$ is given by

(a) $\displaystyle\int_1^3 \frac{2\,du}{(u^2+1)^2}$

(b) $\displaystyle\int_1^9 \frac{2\,du}{(u^2+1)^2}$

(c) $\displaystyle\int_1^3 \frac{du}{2(u^2+1)^2}$

(d) $\displaystyle\int_1^9 \frac{du}{2(u^2+1)^2}$

6. Suppose the position of a particle on the x-axis at time t is given by $x = -4t^2 + 8t - 5$. For which values of t is the particle slowing down?

(a) $t > 1$

(b) $t < 1$

(c) $t > 2$

(d) $t < 2$

7(a) [5 marks] Find $F'(2)$ if $F(x) = \int_4^{x^2} \sqrt{t^2+9}\, dt$.

7(b) [5 marks] Find $\int_1^3 \left(\frac{x^2+1}{x}\right) dx$.

8. Let $f(x) = xe^{-x^2}$ for which $f'(x) = e^{-x^2} - 2x^2e^{-x^2}$. Find the open intervals on which f is increasing and the open intervals on which f is decreasing. Find the critical points of f and determine if they are relative maximum or relative minimum points.

9. Let $f(x) = 6x^{1/3} + 3x^{4/3}$ for which

$$f''(x) = -\frac{4}{3}x^{-5/3} + \frac{4}{3}x^{-2/3}.$$

Find the open intervals on which f is concave up and the open intervals on which f is concave down. Find all the inflection points of f.

10. Suppose the velocity of a particle at time t is given by $v = 6t - 2t^2$, for $0 \leq t \leq 4$. Find the following:

(a) [4 marks] the average acceleration of the particle for $0 \leq t \leq 4$.

(b) [6 marks] the average speed of the particle for $0 \leq t \leq 4$.

11. Find the dimensions of the rectangle (centered at the origin) with maximum area that can be inscribed inside the ellipse with equation $\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$.

12(a) [6 marks] Find the volume of the solid of revolution obtained by revolving the region enclosed by $x = 0, x = \sqrt{3}, y = 0$ and $y = \dfrac{1}{1+x^2}$ about the y-axis.

12(b) [6 marks] Find the volume of the solid of revolution obtained by revolving the region enclosed by the curves $y = x, y = x^2, x = 0$ and $x = 1$ about the line with equation $y = -1$.

13(a) [6 marks] Find the area of the surface generated by revolving the curve with equation $y = 2\sqrt{1-x}$, for $-1 \leq x \leq 0$, about the x-axis.

13(b) [6 marks] Find all the asymptotes to the graph of $f(x) = \left(\frac{1}{x^2}\right)^{\pi/2 - \tan^{-1} x}$.