UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 2018

DURATION: 2 AND 1/2 HRS

FIRST YEAR - CHE, CIV, CPE, ELE, ENG, IND, LME, MEC, MMS

MAT186H1F - Calculus I

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Exam Type: A.

Aids permitted: Casio FX-991 or Sharp EL-520 calculator.

Full Name: _____

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Instructions:

- DO NOT WRITE ON THE QR CODE AT THE TOP OF THE PAGES.
- This exam contains 12 pages, including this cover page, printed two-sided. Make sure you have all of them. Do not tear any pages from this exam.
- This exam consists of eight questions, some with many parts. Attempt all of them. Each question is worth 10 marks. Marks for parts of a question are indicated in the question. **Total Marks: 80**
- Do not approximate your answers unless specifically instructed to do so.
- Present your solutions in the space provided; give full and complete explanations! You can use pages 10, 11 and 12 for rough work. If you want anything on pages 10, 11 or 12 to be marked you must indicate in the relevant previous question that the solution continues on page 10, 11 or 12.

1. Find the following:

(a) [3 marks] $\lim_{x \to 1} \dfrac{\ln x}{x^3 - x}$

(b) [3 marks] the equation of the tangent line to the graph of $f(x) = \tan^{-1} x$ at the point $(1, \pi/4)$.

(c) [4 marks] all the inflection points on the graph of $y = x^{7/3} - 14\,x^{1/3}$

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2. Find the following:

(a) [3 marks] $\displaystyle\int_0^{\pi/2} 3\cos x\,\sqrt{1+3\sin x}\,dx$

(b) [2 marks] $\displaystyle\int_{-\pi/2}^{\pi/2} e^{-x^2}\sin x\,dx$

(c) [5 marks] $\displaystyle\int_0^{\pi} x^2\cos x\,dx$

Continued...

3. Let $v = t^2 - 5t + 6$ be the velocity of a particle at time t, for $0 \leq t \leq 3$. Find:

(a) [4 marks] the average velocity of the particle.

(b) [6 marks] the average speed of the particle.

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4. Let V be the volume of the solid obtained by revolving the region bounded by $y = \sin x$ and $y = 1$, for $0 \leq x \leq \pi/2$, around the y-axis.

   (a) [3 marks] Use the method of shells to express the value of V as one integral with respect to x.

   (b) [3 marks] Use the method of discs to express the value of V as one integral with respect to y.

   (c) [4 marks] Find the value of V.

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5. A tank is full of water. The tank is a square-based pyramid oriented with the tip of the pyramid pointing upward. Each side of the base of the pyramid is 4 m long, and the pyramid is 8 m tall. How much work is required to empty the tank by pumping all the water up to a height 1 m above the top of the tank? (Assume the density of water is ρ and that the acceleration due to gravity is g; leave your answer in terms of ρ and g.)

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6. Consider the curve with equation $y = \dfrac{x^3}{12} + \dfrac{1}{x}$ for $1 \leq x \leq 4$.

(a) [5 marks] Find the length of the curve.

(b) [5 marks] Find the area of the surface generated by revolving the curve about the y-axis.

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7. Let A be the area of the region bounded by the graphs of $f(x) = x + 2|x|$ and $g(x) = m\,x + 1$. Find the value of m that minimizes the value of A.

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8. Gauss's error function, erf, is defined for all x by $\operatorname{erf}(x) = \dfrac{2}{\sqrt{\pi}} \displaystyle\int_0^x e^{-t^2}\, dt$. The first four values of $\operatorname{erf}(n)$, for $n = 1, 2, 3, 4$ are: $\operatorname{erf}(1) \approx 0.842701$, $\operatorname{erf}(2) \approx 0.995322$, $\operatorname{erf}(3) \approx 0.999978$, $\operatorname{erf}(4) \approx 0.999999$.

(a) [2 marks] Find the value of $\displaystyle\int_1^2 e^{-t^2}\, dt$ correct to four decimal places.

(b) [2 marks] Find $\operatorname{erf}'(x)$ and determine for which values of x the error function is increasing.

(c) [2 marks] Show that $\operatorname{erf}(-x) = -\operatorname{erf}(x)$.

(d) [4 marks] Find the value of $\displaystyle\int_0^1 \operatorname{erf}(x)\, dx$ correct to four decimal places.

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