UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

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26 - Nov. - 2012	PS12 - Solution	

NAME		
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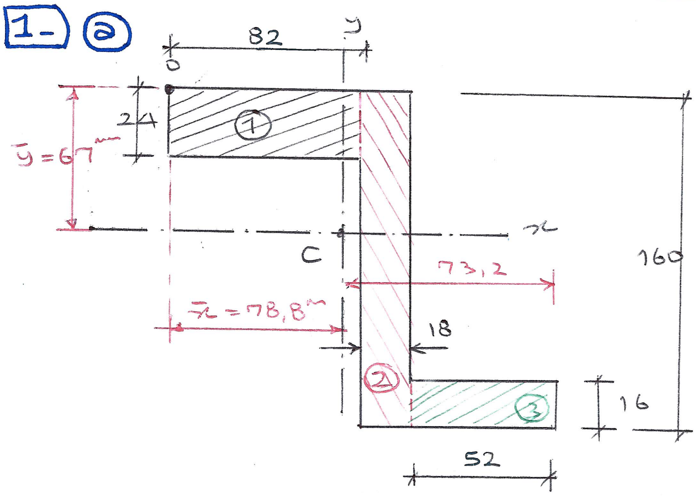
COURSE NO.	COURSE NAME	2
CIV100F	Mechanics	Stud. no ###1

Problem Set 12 (PS12)

$\boxed{\text{Solution}}$

$\boxed{1-}$ a)



• $\bar{x}$ and $\bar{y}$ are taken from PS9, Q1.

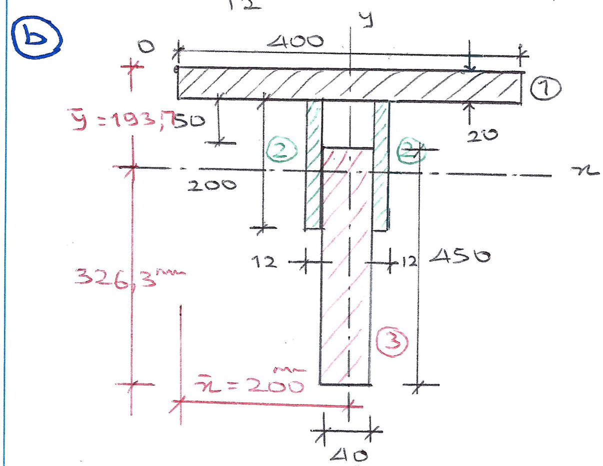
$C(\bar{x} = 78{,}8;\ \bar{y} = 67)^{mm}$

• $I_x = \dfrac{24^3 \cdot 82}{12} + (24 \cdot 82)\cdot(67-12)^2 + \dfrac{160^3 \cdot 18}{12} + (160 \cdot 18)(80-67)^2 + \dfrac{16^3 \cdot 52}{12} + (16 \cdot 52)(160-8-67)^2$

$= 18{,}707 \cdot 10^6\ mm^4$

• $I_y = \dfrac{82^3 \cdot 24}{12} + (82 \cdot 24)(78{,}8 - \dfrac{82}{2})^2 + \dfrac{18^3 \cdot 160}{12} + (18 \cdot 160)(100 - 78{,}8 - 9)^2 + \dfrac{52^3 \cdot 16}{12} + (52 \cdot 16)(73{,}2 - \dfrac{52}{2})^2 = 6{,}462 \cdot 10^6\ mm^4$

b)



• First, find the coordinates of C.

$\bar{y} = \dfrac{400 \cdot 20 \cdot 10 + 200 \cdot 12 \cdot 120 \times 2 + 450 \cdot 40 \cdot 295}{400 \cdot 20 + 200 \cdot 12 \times 2 + 450 \cdot 40}$

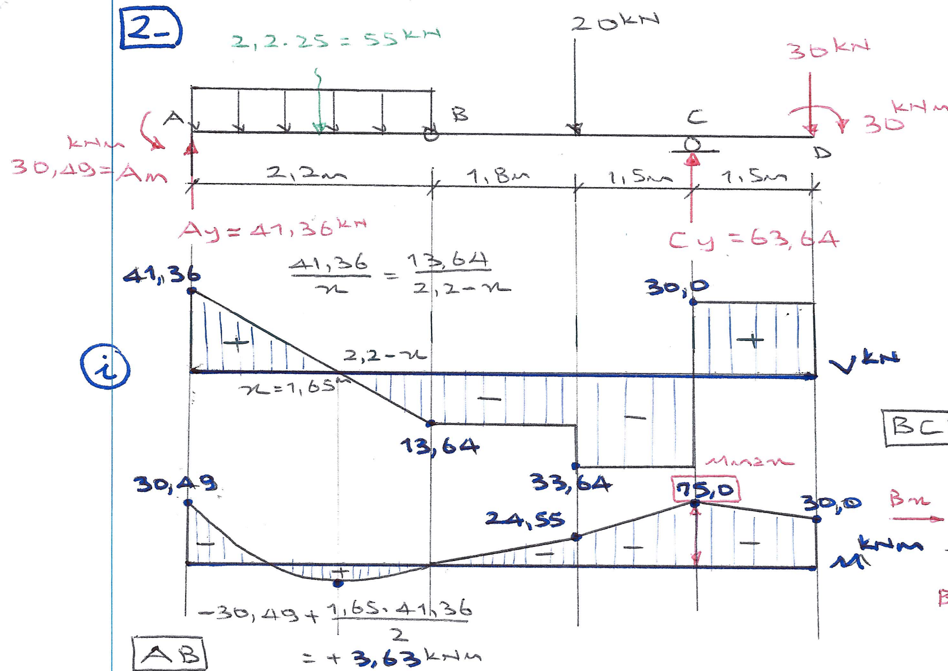
$= 193{,}7\ mm$

$C(\bar{x} = 200^{mm};\ \bar{y} = 193{,}7^{mm})$

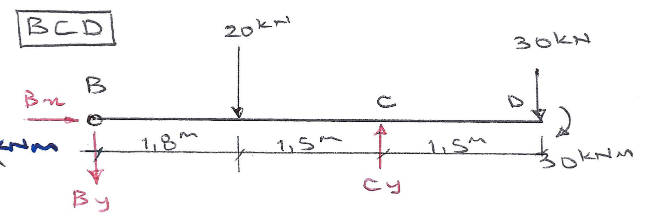
• $I_x = \dfrac{20^3 \cdot 400}{12} + (20 \cdot 400)(193{,}7 - 10)^2 + 2 \times \dfrac{200^3 \cdot 12}{12} + 2 \times (200 \cdot 12)(193{,}7 - 120)^2 + \dfrac{450^3 \cdot 40}{12} + (450 \cdot 40)\cdot(326{,}3 - \dfrac{450}{2})^2 = 0{,}801 \cdot 10^9\ mm^4$

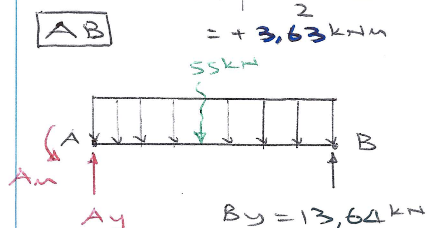
• $I_y = \dfrac{400^3 \cdot 20}{12} + (400 \cdot 20)\cdot(0)^2 + 2 \times \dfrac{12^3 \cdot 200}{12} + 2 \times (12 \cdot 200)(20+6)^2 + \dfrac{40^3 \cdot 450}{12} + (40 \cdot 450)\cdot(0)^2 = 0{,}112 \cdot 10^9\ mm^4$

2.



• First, need to find support rxns. There is no point on the global system to take moment (to eliminate 2 of 3 unknowns.) must isolate the parts from the hinge at B.





• $\Sigma M_B = 0 \Rightarrow 20^{kN} \cdot 1,8^{m} + 30^{kN} \cdot 4,8^{m} + 30^{kNm} = Cy \cdot 3,3^{m}$

$\Rightarrow Cy = 63,64$ kN

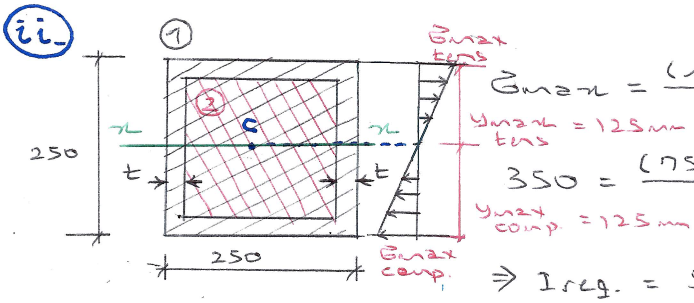
• $\Sigma F_y = 0 \Rightarrow By = 63,64 - 50 = 13,64^{kN}$

• $\Sigma M_A = 0 \Rightarrow 13,64 \cdot 2,2 + Am = 55 \cdot 1,1$

$\Rightarrow Am = 30,49$ kNm

• $\Sigma F_y = 0 \Rightarrow Ay = 55 - 13,64 = 41,36$ kN

ii.



$$\sigma_{max} = \frac{(M_{max} \cdot LF) \cdot y_{max}}{I}$$

$$350 = \frac{(75,0 \cdot 1,9) \cdot 125^{mm}}{I_{required}}$$

$\Rightarrow I_{req.} = 50,893 \cdot 10^6$ mm^4

$$I_x = I_{x_1} - I_{x_2} = \frac{250^3 \cdot 250}{12} - \frac{(250-2t)^3 (250-2t)}{12} = 50,893 \cdot 10^6 \text{ mm}^4$$

$(250-2t)^4 = 3295,5 \cdot 10^6 \Rightarrow t = 5,2$ mm $\Rightarrow$ use **t = 6,0 mm**

3-

$$\sigma_{max} = \frac{(M_{max} \cdot LF)}{S_{req}} \Rightarrow 300^{MPa} = \frac{50 \cdot 10^6 \, N \cdot mm \cdot 2{,}0}{S_{req}}$$

$$\Rightarrow S_{req} = 333{,}333 \cdot 10^3 \; mm^3$$

- From comp. notes p.37 table: need to select $S_x \geqslant S_{req_x}$

(i) **C310 × 31** ($S_x = 351 \cdot 10^3 \, mm^3$) is the lightest beam with $S_x \geqslant 333{,}333 \cdot 10^3 \, mm^3$. It weights 0,302 kN/m.

(ii) **C250 × 45** ($S_x = 337 \cdot 10^3 \, mm^3$) is the shallowest beam with $S_x \geqslant 333{,}333 \cdot 10^3 \, mm^3$. Its depth is about 250 mm. It weights 0,437 kN/m, 45% heavier than W310×31. Consequently this beam will cost 45% more!

~~ The End ~~ Last Problem Set.

Thank you for your hard work!

S. Guner.