

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, APRIL 2006
First Year - CIV, CHE, IND, LME, MEC, MMS

MAT 186H1S - CALCULUS I

Exam Type: A
Examiner: S. Cohen

SURNAME _____

GIVEN NAME _____

STUDENT NO. _____

SIGNATURE _____

Calculators Permitted: Casio 260, Sharp 520 or Texas Instrument 30

INSTRUCTIONS:

Attempt all questions.

Questions 1 through 6 are Multiple Choice;
circle the single correct choice for each question.
Each correct choice is worth 4 marks.

Question 7 consists of six short parts;
each part is worth 2 marks.

Question 8 is worth 12 marks. Show your work.

Questions 9 through 12 are long questions for
which you must show your work. Each long
question is worth 13 marks.

TOTAL MARKS: 100

Use the backs of the pages if you need more space.

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1. What is the equation of the normal line to the graph of $f(x) = \sqrt{x+2}$ at the point $(x, y) = (-1, 1)$?

(a) $y = 2x + 3$

(b) $y = -2x + 1$

(c) $y = -2x - 1$

(d) $y = 2x - 3$

2. The area of the region bounded by the two curves $f(x) = x^3$ and $g(x) = x^2 + 6x$ is given by

(a) $\int_{-2}^3 (f(x) - g(x)) \, dx$

(b) $\int_{-2}^3 (g(x) - f(x)) \, dx$

(c) $\int_{-2}^0 (f(x) - g(x)) \, dx + \int_0^3 (g(x) - f(x)) \, dx$

(d) $\int_{-2}^0 (g(x) - f(x)) \, dx + \int_0^3 (f(x) - g(x)) \, dx$

3. Let $f(x) = x^x$. The value of $f'(2)$ is

(a) 2

(b) $\ln 2 + 1$

(c) $4 \ln 2$

(d) $4 \ln 2 + 4$

4. Let $F(x) = \int_{-2}^{\cos x} \sqrt{5+t^2} dt$. Then $F'(\pi/2) =$

(a) $\sqrt{5}$

(b) $-\sqrt{5}$

(c) $\sqrt{5} + 2$

(d) $-\sqrt{5} - 2$

5. $\int_0^1 \frac{\arctan x}{1+x^2} dx =$

(a) $\int_0^1 u du$

(b) $\int_0^{\pi/4} u du$

(c) $\int_0^{45} u du$

(d) $\int_0^1 \frac{1}{1+u^2} du$

6. $\lim_{x \rightarrow 0} \frac{\sin^{-1} x - x}{x^3} =$

(a) 0

(b) $-\frac{1}{6}$

(c) $\frac{1}{6}$

(d) ∞

7. [2 marks each] Suppose f and g are continuous functions such that

$$\int_1^8 f(x) dx = -5, \int_1^8 g(x) dx = 7 \text{ and } \int_1^{15} f(x) dx = 3.$$

Find the values of the following. (Put your answers on the lines at the right of each question.)

(a) $\int_1^8 (3f(x) + 2g(x)) dx$

ANS: _____

(b) the average value of f on $[1, 8]$

ANS: _____

(c) $\int_8^{15} f(x) dx$

ANS: _____

(d) $\int_1^8 g(z) dz$

ANS: _____

(e) $\int_1^2 x^2 f(x^3) dx$

ANS: _____

(f) $\lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(\frac{n+7i}{n}\right) \frac{7}{n}$

ANS: _____

8. [12 marks] If 10 Newton-meters ($= 10$ Joules) of work are required to compress a (big) spring by 2 meters from its natural length, how much work must be done to compress it one more meter?

9. Suppose the velocity of a particle at time t is given by $v = 1 - t^2$, for $0 \leq t \leq 3$. Find the following:

(a) [3 marks] the average acceleration of the particle for $0 \leq t \leq 3$.

(b) [4 marks] the average velocity of the particle for $0 \leq t \leq 3$.

(c) [6 marks] the average speed of the particle for $0 \leq t \leq 3$.

10. [13 marks] Let $f(x) = \frac{3x^2 - 1}{x^3}$. Sketch the graph of $y = f(x)$, labelling all critical points, inflection points and asymptotes, if any.

- 11.(a) [6 marks] Find the volume of the solid generated by rotating around the y -axis the region in the plane bounded by the curves

$$y = e^{-x^2}, y = 0, x = 0, x = 1.$$

- 11.(b) [7 marks] Find the arc length of the curve $y = e^x + \frac{1}{4}e^{-x}$, from $x = 0$ to $x = 1$.

12. [13 marks] Consider an equilateral triangle with vertices at

$$(x, y) = (-5, 0), (5, 0) \text{ and } (0, 5\sqrt{3}).$$

What are the volume and surface area of the solid generated by rotating this triangle about the x -axis.