Tuesday October 11	START: 13:10	DURATION: 110 mins

University of Toronto

Faculty of Applied Science & Engineering

MIDTERM EXAMINATION I
MAT188H1F
Linear Algebra

EXAMINERS: D. Burbulla, S. Cohen, D. Fusca, F. Lopez, M. Palasciano, M. Pugh, B. Schachter, S. Uppal

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Instructions.

1. There are **55** possible marks to be earned in this exam. The examination booklet contains a total of 11 pages. It is your responsibility to ensure that *no pages are missing from your examination.* DO NOT DETACH ANY PAGES FROM YOUR EXAMINATION.
2. DO NOT WRITE ON THE QR CODE AT THE TOP RIGHT-HAND CORNER OF EVERY PAGE OF YOUR EXAMINATION.
3. For the full answer questions, WRITE YOUR SOLUTIONS ON THE FRONT OF THE QUESTION PAGES THEMSELVES. THE BACK OF EVERY PAGE WILL **NOT** BE SCANNED AND SEEN BY THE GRADERS.
4. Ensure that your solutions are LEGIBLE.
5. No aids of any kind are permitted. CALCULATORS AND OTHER ELECTRONIC DEVICES (INCLUDING PHONES) ARE NOT PERMITTED.
6. Have your student card ready for inspection.
7. There are no part marks for Multiple Choice (MC) questions.
8. You may use the two blank pages at the end for rough work. The last two pages of the examination WILL NOT BE MARKED unless you *clearly* indicate otherwise on the question pages.
9. For the full answer questions, show all of your work and justify your answers *but do not include extraneous information.*

Part I - Multiple Choice. Clearly indicate your answer to each question by circling your choice. Each question is worth 2 marks.

For each question, choose the BEST option from the given options.

1. Which of the following matrices are in row-echelon form?

(i) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(A) (ii) only
(B) (iii) only
(C) (i) only
(D) (ii) and (iii) only
(E) (i) and (ii) only

Answer: A
Solution:
i) This matrix is not in row-echelon form as there is a leading one in each row but each leading 1 is not to the right of the one above it.
ii) This matrix is in row echelon form.
iii) This matrix is not in row-echelon form as there is a row full of 0's that is not at the bottom of the matrix.

2. Let $\mathbf{x}, \mathbf{y}, \mathbf{z}$ be non-zero vectors in $\mathbb{R}^3$. Which of the following statements are TRUE?

(i) If $\mathbf{x} \times \mathbf{y} = \mathbf{x} \times \mathbf{z}$ then $\mathbf{y} = \mathbf{z}$.

(ii) $(\mathbf{x} \times \mathbf{y}) \times \mathbf{z} = \mathbf{x} \times (\mathbf{y} \times \mathbf{z})$.

(iii) $||\mathbf{x} \times \mathbf{y}|| = 0$ if and only if $\mathbf{x}$ and $\mathbf{y}$ are parallel.

(A) (ii) only
(B) (iii) only
(C) (i) and (iii) only
(D) (ii) and (iii) only
(E) none of (i), (ii), or (iii)

Answer: B
Solution:
i) This is not true. As a counterexample, we can take $\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. Since $||\mathbf{x}|| = 0$, $\mathbf{x} \times \mathbf{v}$ (where $\mathbf{v}$ is any vector in $\mathbb{R}^3$) will be equal to $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, in particular when $\mathbf{v} = \mathbf{y}$ or $\mathbf{z}$, but $\mathbf{y} \neq \mathbf{z}$.

ii) This is not true. As a counterexample, we can take $\mathbf{x} = \mathbf{y} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{z} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. Then $(\mathbf{x} \times \mathbf{y}) \times \mathbf{z} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ but $\mathbf{x} \times (\mathbf{y} \times \mathbf{z}) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$.

iii) This is true. We know that $||\mathbf{x} \times \mathbf{y}|| = ||\mathbf{x}||||\mathbf{y}|| \sin\theta$ where θ is the angle between $\mathbf{x}$ and $\mathbf{y}$. This being equal to 0 would imply that either $\mathbf{x}$ or $\mathbf{y}$ is the zero vector (in which case they are parallel as one is a multiple of another), or the sine of the angle between them is 0, meaning that angle is either 0 or π, meaning that one of the vectors is a (possible negative) multiple of another.

3. For what value(s) of c is the set $\left\{ \begin{bmatrix} c \\ 1 \end{bmatrix}, \begin{bmatrix} c+2 \\ c \end{bmatrix} \right\}$ linearly dependent?

(A) $c = 0$
(B) $c = -1$
(C) $c = 2$
(D) (A) and (B) only
(E) (B) and (C) only

Answer: E
Solution:
For a set of 2 vectors to be linearly dependent, one must be a scalar multiple of the other.

A) This value of c does not work. When $c = 0$, the vectors are $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ which are clearly not scalar multiples. Thus, the vectors are linearly independent.

B) This value of c works. When $c = -1$, the vectors are $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and one vector is -1 times the other. Thus, the vectors are linearly dependent.

C) This value of c works. When $c = 2$, the vectors are $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ and the second vector is twice the first. Thus, the vectors are linearly dependent.

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Part I - Multiple Choice. Clearly indicate your answer to each question by circling your choice. Each question is worth 2 marks.

For each question, choose the BEST option from the given options.

4. Let $\mathbf{x}, \mathbf{y}, \mathbf{z}$ be vectors in $\mathbb{R}^3$. Which of the following statements are TRUE?

(i) If the set $S = \{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly independent, then the set $\{\mathbf{x}, \mathbf{y}\}$ is linearly independent.

(ii) If the set $S = \{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent, then the set $\{\mathbf{x}, \mathbf{y}\}$ is linearly dependent.

(iii) If the three sets $\{\mathbf{x}, \mathbf{y}\}$, $\{\mathbf{y}, \mathbf{z}\}$, and $\{\mathbf{x}, \mathbf{z}\}$ are linearly independent, then the set $S = \{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly independent.

(A) (i) only
(B) (ii) only
(C) (iii) only
(D) (i) and (ii) only
(E) (i) and (iii) only

Answer: A
Solution:
i) This is true.

Notice that if the set $\{\mathbf{x}, \mathbf{y}\}$ is linearly dependent then it is possible to form the zero vector using a non-trivial combination of $\mathbf{x}$ and $\mathbf{y}$. The zero vector can still be formed by the same linear combination of $\mathbf{x}$ and $\mathbf{y}$ with $\mathbf{z}$ taken with a coefficient of 0 which would mean that S is linearly dependent too.

ii) This is false. As a counterexample, we can take $\mathbf{x} = \mathbf{z} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. The set $\{\mathbf{x},\ \mathbf{y},\ \mathbf{z}\}$ is clearly linearly dependent as $\mathbf{x} - \mathbf{z} = \mathbf{0}$. But $\{\mathbf{x}, \mathbf{y}\}$ is linearly independent as $\mathbf{x}$ and $\mathbf{y}$ are not scalar multiples.

iii) This is false. As a counterexample, we can take $\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{z} = \mathbf{x} + \mathbf{y}$. Then the three sets $\{\mathbf{x}, \mathbf{y}\}$, $\{\mathbf{y}, \mathbf{z}\}$, and $\{\mathbf{x}, \mathbf{z}\}$ are linearly independent since no two $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are scalar multiples of each other, but $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent as $\mathbf{x} + \mathbf{y} - \mathbf{z} = \mathbf{0}$.

5. Suppose that the augmented matrix of a system of linear equations has been reduced to $\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$. Which of the following statements describes the set of solutions to the system?

(A) infinitely many solutions with three parameters
(B) infinitely many solutions with two parameters
(C) infinitely many solutions with one parameter
(D) unique solution
(E) no solutions

Answer: C
Solution:
We interpret this matrix as the augmented matrix of the system

$$\begin{aligned} x_1 + x_2 + x_4 &= 1 \\ x_3 + x_4 &= 1 \\ x_4 &= 0 \end{aligned}$$

We see that x_2 is the only free variable which as assigned as a parameter in the general solution. Hence, there are infinitely many solutions with one parameter.

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Part II - Short Answer Questions. Write your solutions in the space provided below each question.

1. Let $\mathbb{P}$ be the plane $2x_1 + x_3 = 3$.

(a) Find parametric equations of the line $\mathbb{L}$ that contains the point $(-1, 6, 0)$ and is orthogonal to $\mathbb{P}$. [2 marks]

Answer:

$$\begin{aligned} x_1 &= -1 + 2t \\ x_2 &= 6 \\ x_3 &= t \end{aligned}$$

for any $t \in \mathbb{R}$.

Solution:

Since the equation for $\mathbb{P}$ is $2x_1 + 0x_2 + 1x_3 = 3$, then the vector $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ is normal to the plane. Thus, any line with $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ as a direction vector will be orthogonal to the plane. Since we are given the point that the line must contain, we can write the vector equation of the line as

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \\ 6 \end{bmatrix} + t \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

for any $t \in \mathbb{R}$. Equating components gives the parametric equations.

Note: This question specifically asked for parametric equations so if your answer was the vector equation you were deducted 0.5 points.

(b) Find the (shortest) distance from the point $(-1, 6, 0)$ to $\mathbb{P}$ and determine the point on $\mathbb{P}$ closest to $(-1, 6, 0)$. [6 marks]

Answer: Distance: $\sqrt{5}$ Point: (1, 6, 1)
Solution:

Let Q be the point $(-1, 6, 0)$. Take any point P on the plane. For simplicity we choose P to be $(0, 0, 3)$. Then, the shortest distance is

$$\begin{aligned} ||\text{proj}_{\mathbf{n}} \vec{PQ}|| &= || \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix} || \\ &= \sqrt{5} \end{aligned}$$

To find the point on the plane nearest to the point (-1, 6, 0), we simply subtract (-2, 0, -1) from (-1, 6, 0) to get (1, 6, 1). You may find drawing a picture will help you visualize the solution.

2. (a) Suppose that the angle θ between the vectors $\begin{bmatrix} 1 \\ 7 \\ c \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$ is given by $\cos(\theta) = \frac{1}{3}$. Find c. [4 marks]

Answer: $c = -\frac{47}{12}$.
Solution:

We know that $\begin{bmatrix} 1 \\ 7 \\ c \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} = || \begin{bmatrix} 1 \\ 7 \\ c \end{bmatrix} || \, || \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} || \cos\theta$. Thus, we get that $12 + c = \sqrt{50 + c^2}$ so $144 + 24c + c^2 = 50 + c^2$. This is equivalent to $144 + 24c = 50$ which gives $c = -\frac{47}{12}$.

2. (b) Does the plane containing the points $P(4, 0, 5)$, $Q(2, 2, 1)$, and $R(1, -1, 2)$ pass through the origin? Support your answer. [4 marks]

Answer: Yes.
Solution:

We start by finding the normal vector $\mathbf{n}$ to the plane. We know that $\mathbf{n} = (\vec{PQ}) \times (\vec{PR}) = \begin{bmatrix} -2 \\ 2 \\ -4 \end{bmatrix} \times \begin{bmatrix} -3 \\ -1 \\ -3 \end{bmatrix} = \begin{bmatrix} -10 \\ 6 \\ 8 \end{bmatrix}$. This means that the equation of the plane containing the three points is given by

$$-10x_1 + 6x_2 + 8x_3 = d$$

for some d. Since the point $P(4, 0, 5)$ is on the plane, we have $d = -10(4) + 6(0) + 8(5) = 0$. Hence the equation of the plane is

$$-10x_1 + 6x_2 + 8x_3 = 0$$

which indeed passes through the origin since $-10(0) + 6(0) + 8(0) = 0$.

3. (a) Let $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ be a set of vectors in $\mathbb{R}^n$. Define $\text{span}\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$. [2 marks]

Word definition:

$\text{span}\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is the set of all possible linear combinations of the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_k$.

Math definition:

$\text{span}\{\mathbf{v}_1, \ldots, \mathbf{v}_k\} = \{c_1\mathbf{v}_1 + \cdots + c_k\mathbf{v}_k \mid c_1, \ldots, c_k \in \mathbb{R}\}$.

3. (b) Show that $\text{span}\left\{\begin{bmatrix}2\\4\\3\end{bmatrix}, \begin{bmatrix}2\\7\\6\end{bmatrix}\right\} \subseteq \text{span}\left\{\begin{bmatrix}2\\1\\0\end{bmatrix}, \begin{bmatrix}-2\\2\\3\end{bmatrix}, \begin{bmatrix}0\\3\\3\end{bmatrix}\right\}$. Support your answer. [6 marks]

Solution:

To show that $\text{span}\left\{\begin{bmatrix}2\\4\\3\end{bmatrix}, \begin{bmatrix}2\\7\\6\end{bmatrix}\right\} \subseteq \text{span}\left\{\begin{bmatrix}2\\1\\0\end{bmatrix}, \begin{bmatrix}-2\\2\\3\end{bmatrix}, \begin{bmatrix}0\\3\\3\end{bmatrix}\right\}$ we must show that if $\mathbf{x}$ is a vector in $\text{span}\left\{\begin{bmatrix}2\\4\\3\end{bmatrix}, \begin{bmatrix}2\\7\\6\end{bmatrix}\right\}$, then $\mathbf{x}$ is also in $\text{span}\left\{\begin{bmatrix}2\\1\\0\end{bmatrix}, \begin{bmatrix}-2\\2\\3\end{bmatrix}, \begin{bmatrix}0\\3\\3\end{bmatrix}\right\}$. In other words, if $\mathbf{x}$ can be written as a linear combination of $\begin{bmatrix}2\\4\\3\end{bmatrix}$, and $\begin{bmatrix}2\\7\\6\end{bmatrix}$, then $\mathbf{x}$ can also be written as a linear combination of $\begin{bmatrix}2\\1\\0\end{bmatrix}$, $\begin{bmatrix}-2\\2\\3\end{bmatrix}$, and $\begin{bmatrix}0\\3\\3\end{bmatrix}$. This will be true if each of the vectors $\begin{bmatrix}2\\4\\3\end{bmatrix}$ and $\begin{bmatrix}2\\7\\6\end{bmatrix}$ can themselves be written as a linear combination of $\begin{bmatrix}2\\1\\0\end{bmatrix}$, $\begin{bmatrix}-2\\2\\3\end{bmatrix}$, and $\begin{bmatrix}0\\3\\3\end{bmatrix}$.

Notice, for example, that $\begin{bmatrix}2\\4\\3\end{bmatrix} = 2\begin{bmatrix}2\\1\\0\end{bmatrix} + \begin{bmatrix}-2\\2\\3\end{bmatrix}$ and $\begin{bmatrix}2\\7\\6\end{bmatrix} = 3\begin{bmatrix}2\\1\\0\end{bmatrix} + 2\begin{bmatrix}-2\\2\\3\end{bmatrix}$ (other combinations are possible) so the result follows.

In fact, though the question does not ask you to show this, $\text{span}\left\{\begin{bmatrix}2\\4\\3\end{bmatrix}, \begin{bmatrix}2\\7\\6\end{bmatrix}\right\} = \text{span}\left\{\begin{bmatrix}2\\1\\0\end{bmatrix}, \begin{bmatrix}-2\\2\\3\end{bmatrix}, \begin{bmatrix}0\\3\\3\end{bmatrix}\right\}$. Can you explain why?

4. (a) Define what it means for a subset W of $\mathbb{R}^n$ to be a subspace of $\mathbb{R}^n$. [3 marks]

Solution:
W is a subspace of $\mathbb{R}^n$ if

1. W is non-empty.

2. W is closed under addition. i.e. For all $\mathbf{x}, \mathbf{y} \in W$, $\mathbf{x} + \underset{\sim}{y} \in W$.

3. W is closed under scalar multiplication. i.e. For all $\mathbf{x} \in W$, and $c \in \mathbb{R}$, $c\mathbf{x} \in W$.

4. (b) Consider the subset $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mid \frac{x_1}{2} = \frac{x_2}{3} \text{ and } \frac{x_2}{3} = \frac{x_3}{4} \right\}$ of $\mathbb{R}^3$. Determine whether W is a subspace of $\mathbb{R}^3$. Support your answer. [5 marks]

Answer: W is a subspace of $\mathbb{R}^3$.
Solution:

1. Notice that $\mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is in W since $\frac{0}{2} = \frac{0}{3}$ and $\frac{0}{3} = \frac{0}{4}$ so that indeed W in non-empty.

2. We show that if $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ are in W, then $\mathbf{x} + \mathbf{y} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \end{bmatrix} \in W$. Now, $\mathbf{x} + \mathbf{y} \in W$ iff $\frac{x_1 + y_1}{2} = \frac{x_2 + y_2}{3}$ and $\frac{x_2 + y_2}{3} = \frac{x_3 + y_3}{4}$. Since

$$\frac{x_1 + y_1}{2} = \frac{x_1}{2} + \frac{y_1}{2} = \frac{x_2}{3} + \frac{y_2}{3} = \frac{x_2 + y_2}{3},$$

and

$$\frac{x_2 + y_2}{3} = \frac{x_2}{3} + \frac{y_2}{3} = \frac{x_3}{4} + \frac{y_3}{4} = \frac{x_3 + y_3}{4}$$

we have that indeed $\mathbf{x} + \mathbf{y} \in W$. Notice that the second to last equality in each line above is true because $\mathbf{x}$ and $\mathbf{y}$ are in W.

3. We show that if $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in W$ and $c \in \mathbb{R}$, then $c\mathbf{x} = \begin{bmatrix} cx_1 \\ cx_2 \\ cx_3 \end{bmatrix} \in W$. Now, $c\mathbf{x} \in W$ iff $\frac{cx_1}{2} = \frac{cx_2}{3}$ and $\frac{cx_2}{3} = \frac{cx_3}{4}$. Since

$$\frac{cx_1}{2} = c\frac{x_1}{2} = c\frac{x_2}{3} = \frac{cx_2}{3},$$

and

$$\frac{cx_2}{3} = c\frac{x_2}{3} = c\frac{x_3}{4} = \frac{cx_3}{4}$$

we have that indeed $c\mathbf{x} \in W$. Notice that the second to last equality in each line above is true because $\mathbf{x}$ is in W.

5. Consider a linear system of equations whose augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & a & b \end{array}\right].$$

For what values of a and b will the system have:

(i) *infinitely many* solutions; (ii) a *unique* solution; (iii) *no* solution? [8 marks]

Solution:
We start by reducing the matrix to (check this!)

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & a-5 & b-4 \end{array}\right].$$

i) The system will have infinitely many solutions if the above matrix contains an entire row of zeros. i.e. if $a = 5$ and $b = 4$.

ii) The system will have a unique solution if the number of leading variables (pivots) = the total number of variables. This happens if $a \neq 5$ (b can be any real number).

iii) The system will have no solution if the above matrix contains a row of the form $\left[\begin{array}{ccc|c} 0 & 0 & 0 & c \end{array}\right]$, where $c \neq 0$. This happens if $a = 5$ and $b \neq 4$.

6. Is it possible to find a 2×2 matrix whose rows are linearly independent but whose columns are linearly dependent? Prove your answer. [5 marks]

Answer: No.
Solution:
We suppose such a matrix exists and arrive at a contradiction. Call this matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Case 1: None of the entries in the matrix are 0. Since the columns are linearly dependent, we get that $b = ka$ and $d = kc$, for some scalar k. Our matrix is then

$$\left[\begin{array}{cc} a & ka \\ c & kc \end{array}\right].$$

We notice that the rows of this matrix is linearly dependent as the second row is equal to $\frac{c}{a}$ times the first row. Thus, it is not possible to find such a matrix in this case. Notice, however, that the above argument does not hold if $a = 0$!

Case 2: If $a = 0$, then $A = \begin{bmatrix} 0 & b \\ c & d \end{bmatrix}$. Since the columns are linearly dependent, it must be that $b = 0$ too. Hence, $A = \begin{bmatrix} 0 & 0 \\ c & d \end{bmatrix}$. But then the rows are linearly dependent too since the first row is 0 times the second.

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