University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, APRIL, 2009
First Year - CHE, CIV, IND, LME, MEC, MMS

MAT187H1S - CALCULUS II
Exam Type: A

SURNAME: (as on your T-card) _____

GIVEN NAMES: _____

STUDENT NUMBER: _____

SIGNATURE: _____

Examiners:
D. Burbulla
D. Krepski
P. Milgram
Y. Zong

PAGE	MARK
MC	
Q7	
Q8	
Q9	
Q10	
Q11	
Q12	
Q13	
TOTAL	

Calculators Permitted: Casio 260, Sharp 520 or TI 30.

INSTRUCTIONS: Attempt all questions. Use the backs of the sheets if you need more space. Do not tear any pages from this exam. Make sure your exam contains 10 pages.

MARKS: Questions 1 through 6 are Multiple Choice; circle the single correct choice for each question. Each correct choice is worth 4 marks.

Questions 7, 8 and 9 are each worth 12 marks.

Questions 10 through 13 are each worth 10 marks.

TOTAL MARKS: 100

1. The position $x(t)$ of a particle at time t changes according to the differential equation

$$4x''(t) + 7x'(t) + 3x(t) = 0.$$

The motion of the particle is

   (a) simple harmonic motion.

   (b) underdamped.

   (c) critically damped.

   (d) overdamped.

2. What is the length of the polar curve with polar equation $r = e^\theta$, for $0 \leq \theta \leq 1$?

   (a) $e - 1$

   (b) $\sqrt{2}$

   (c) $\sqrt{2}(e - 1)$

   (d) $\sqrt{2}e$

3. What is the interval of convergence of the power series $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{4^n \ln n}(x - 1)^n$?

   (a) $(-3, 5)$

   (b) $[-3, 5]$

   (c) $[-3, 5)$

   (d) $(-3, 5]$

4. How many critical points are there on the curve with parametric equations

$$x = t^3 - 12t, \; y = \ln(t^2 + 1),$$

for $t \in \mathbb{R}$?

(a) 1

(b) 2

(c) 3

(d) 4

5. What is the area bounded by one loop of the curve with polar equation $r^2 = 4\cos\theta$?

(a) 1

(b) 2

(c) 4

(d) 8

6. What is the arc length of the curve with parametric equations

$$x = t\sin t; y = t\cos t; z = \frac{1}{3}(2t)^{3/2},$$

for $0 \leq t \leq 2$?

(a) 1

(b) 2

(c) 3

(d) 4

7. [12 marks; 4 for each part.] Decide if the following infinite series converge or diverge. Summarize your work at the right by marking your choice, and by indicating which convergence/divergence test you are using.

(a) $\sum_{n=1}^{\infty} \dfrac{4n^{5/2} + 3n^2 - 3n}{n^3 + n^{3/2} + \sqrt{n}}$

    ○ Converges   ○ Diverges

    by _____

(b) $\sum_{n=0}^{\infty} \dfrac{\dots(n^2 + 3n - 6)}{n^2 + 1}$

    ○ Converges   ○ Diverges

    by _____

(c) $\sum_{n=1}^{\infty} \dfrac{n^{2n}}{(2n)!}$

    ○ Converges   ○ Diverges

    by _____

8. [12 marks] An artillery gun with muzzle velocity (initial speed) of 1000 ft/sec is located atop a seaside cliff 500 ft high. At what inclination angle(s) should it fire a projectile in order to hit a ship at sea 20,000 ft from the base of the cliff? (Neglect air resistance; use $g = 32$ ft/sec^2.)

9. [12 marks] Solve for y as a function of x if $y = 1$ when $x = 0$ and

$$\frac{dy}{dx} + \frac{y}{1+x} = \arctan x.$$

10. [10 marks] Approximate $\int_0^{0.5} (1 + x^6)^{3/2}\, dx$ to within 10^{-6}, and explain why your approximation is correct to within 10^{-6}.

11. [10 marks] Find and classify the critical points of the function $f(x, y) = 2x^4 + y^2 - 4xy$.

12. [10 marks] The acceleration of a Maserati sports car is proportional to the difference between 250 km/hr and its velocity v. If this sports car can accelerate from rest to 100 km/hr in 10 sec, how long will it take for the car to accelerate to 200 km/hr?

13. [10 marks] Find the fifth degree Taylor polynomial of $f(x) = e^{\sin x}$ at $a = 0$.