Name: ______________________________

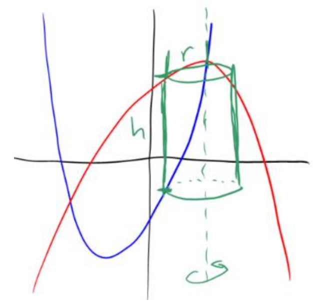
Student number: ____________________

MAT 186
Quiz 10

CF9	CS14

1. Find the volume of the shape created when the region between $y = 2x^2 + 5x - 2$ and $y = 4 + 2x - x^2$ is rotated about the line $x = 1$.

We begin by equating the functions and finding where they intersect.



$$2x^2 + 5x - 2 = 4 + 2x - x^2$$
$$3x^2 + 3x - 6 = 0$$
$$3(x+2)(x-1) = 0$$

Therefore, $x = -2,1$ are the points of intersection. The volume is:

$$\begin{aligned} V &= 2\pi \int_{-2}^{1} (-3x^2 - 3x + 6)(1 - x)dx \\ &= 6\pi \int_{-2}^{1} x^3 - 3x + 2dx \\ &= 6\pi \left(\frac{x^4}{4} - \frac{3x^2}{2} + 2x\right)_{-2}^{1} \\ &= \frac{81\pi}{2} \end{aligned}$$

2. Find the volume of the shape created when the same region from question 1 is rotated about the line $y = 3$.

This is corrected to the line $y = 5$, as the lower line intersects the graph.

In that case, we will be dealing with washers.

The outer radius will be $5 - (4 + 2x - x^2)$ and the inner radius $5 - (2x^2 + 5x - 2)$.

$$V = \pi \int_{-2}^{1} [5 - (4 + 2x - x^2)]^2 - [5 - (2x^2 + 5x - 2)]^2 \, dx$$

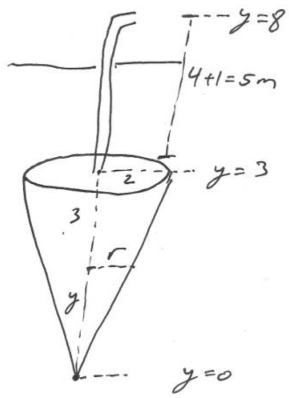
$$= \pi \int_{-2}^{1} (1 - 2x + x^2)^2 - (7 - 5x - 2x^2)^2 \, dx$$

Which is ugly enough to leave alone.

Continued on back.

3. A conical tank with height of 3 m and base radius of 2 m is buried under ground, with its base facing up and 4 m below ground level. How much work is required to empty the tank through a pipe 1 m above ground? Leave the density as ρ and the gravity as g.

CF8	CF9	CS18



From the diagram, we see the important measurements.

We can also see two similar triangles that allow us to find the cross-section area at any height y. The triangles give:

$$\frac{r}{y} = \frac{2}{3}$$

So, $r = \frac{2}{3}y$ and the area of the circle at that height is $A = \frac{4\pi}{9}y^2$.

The distance that the cross-section needs to be moved is $8 - y$, so the total work is

$$\int_0^3 \rho g \frac{4\pi}{9} y^2 (8 - y) dy$$
$$= \frac{4}{9}\pi\rho g \int_0^3 8y^2 - y^3 dy$$
$$= \frac{4}{9}\pi\rho g \left(\frac{8}{3}y^3 - \frac{1}{4}y^4\right)_0^3$$
$$= \frac{4}{9}\pi\rho g \left(72 - \frac{81}{4}\right)$$
$$= 4\pi\rho g \left(8 - \frac{9}{4}\right)$$
$$= \pi\rho g(32 - 9)$$
$$= 23\pi\rho g$$

The last few steps are there to demonstrate how this can be done without resorting to any three-digit number in your arithmetic.

[This one is optional. Solve it if you need better marks in the main attribute tested here. If you try this problem, it will not lower your mark.]

4. When two numbers are multiplied together, their product is 4. What is the smallest possible value for their sum?

Let the numbers be x and y. Then $xy = 4$, or $y = 4/x$.

The sum of the numbers is $x + y$, or $x + 4/x$.

$$S(x) = x + 4/x$$

$$S'(x) = 1 - 4/x^2$$

Set $S'(x) \geq 0$ to find both the critical values and areas of increase and decrease.

$$1 - \frac{4}{x^2} \geq 0$$

$$\frac{x^2 - 4}{x^2} \geq 0$$

$$\frac{(x-2)(x+2)}{x^2} \geq 0$$

No marks are deducted for assuming that the numbers are positive, but that is not actually asked for. Keep in mind that "smallest" means closest to zero.

On a number line, we get the function increasing for $x < -2$ and $x > 2$. It is decreasing on $-2 < x < 0$ and $0 < x < 2$.

At $x = -2$, we have a local maximum of -4. At $x = 2$, we have a local minimum of 4. As $x \to 0$ from either side, the sum diverges to $\pm\infty$. Therefore, these two sums $(4$ and $-4)$ are the smallest possible values.