UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, APRIL 2003
MAT 188 S – LINEAR ALGEBRA. FIRST YEAR: T-PROGRAM
EXAMINER: FELIX J. RECIO

INSTRUCTIONS:

1. ATTEMPT ALL QUESTIONS.
2. SHOW AND EXPLAIN YOUR WORK IN ALL QUESTIONS.
3. GIVE YOUR ANSWERS IN THE SPACE PROVIDED. USE BOTH SIDES OF PAPER, IF NECESSARY.
4. DO NOT TEAR OUT ANY PAGES.
5. USE OF NON-PROGRAMMABLE POCKET CALCULATORS, BUT NO OTHER AIDS ARE PERMITTED.
6. THIS EXAM CONSISTS OF EIGHT QUESTIONS. THE VALUE OF EACH QUESTION IS INDICATED (IN BRACKETS) BY THE QUESTION NUMBER.
7. THIS EXAM IS WORTH 50% OF YOUR FINAL GRADE.
8. TIME ALLOWED: 2 ½ HOURS.
9. PLEASE WRITE YOUR NAME, YOUR STUDENT NUMBER, AND YOUR SIGNATURE IN THE SPACE PROVIDED AT THE BOTTOM OF THIS PAGE.

PLEASE DO NOT WRITE HERE

QUESTION NUMBER	QUESTION VALUE	GRADE
1	10	
2	15	
3	10	
4	10	
5	10	
6	15	
7	15	
8	15	
TOTAL:	100	

NAME: ______________________, ______________________

(FAMILY NAME. PLEASE PRINT.) (GIVEN NAME.)

STUDENT No.: ______________________ SIGNATURE: ______________________

1. Let L_1 be the that passes through the points $(6, 5, -3)$ and $(0, -4, 3)$.

   Let L_2 be the that passes through the points $(3, 1, 8)$ and $(-2, 1, -2)$.

   a) (5 marks) Find an equation of the plane that contains the line L_1 and is parallel to the line L_2.

   b) (5 marks) Find the coordinates of the point on the line L_1 which is closest to the line L_2.

2. a) (10 marks) Solve the linear system $\begin{cases} \quad\;\; b + c + d = 2 \\ a + b \qquad\;\; - d = 0 \\ 2a \qquad - c - 4d = -2 \\ a + 2b + 2c \qquad\;\; = 4 \\ -a + b \qquad\;\; + 3d = 0 \end{cases}$

b) (5 marks) Find all values of k, if any, for which the system $\begin{cases} x_1 \qquad\quad + 2x_3 - 3x_4 = 1 \\ x_1 - 2x_2 \qquad\quad + kx_4 = 0 \\ x_1 + 2x_2 + 4x_3 - x_4 = 4 \end{cases}$ is inconsistent.

3. (10 marks) Consider the matrix $A = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Find all matrices M, if any, such that $MA = M + A^T$.

4) (10 marks) Let $A = \begin{bmatrix} 5 & 1 & 1 & 0 \\ 1 & 5 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 5 \end{bmatrix}$ and let M be another 4×4 matrix such that $\det(A M^3) = 1$.

Compute $\det M$.

5. Given the matrix $A = \begin{bmatrix} 1 & 2 & 0 & -1 & 0 \\ 1 & 3 & -1 & 0 & 1 \\ 0 & 1 & -1 & 1 & -1 \\ 1 & 0 & 2 & -3 & 0 \end{bmatrix}$

a) (5 marks) Find a basis for its column space.

b) (5 marks) Find a basis for the solution space of $A\,\mathbf{x} = \mathbf{0}$.

6. (15 marks) Let $\mathbf{C}[0,1]$ denote the inner product space consisting of all real valued functions which are continuous on the interval $[0,1]$, with the inner product defined as $(\mathbf{f},\mathbf{g})=\int_0^1 \mathbf{f}(x)\mathbf{g}(x)\,dx$.
Find an orthonormal basis for the subspace of $\mathbf{C}[0,1]$ spanned by the functions $h_1(x)=1$, $h_2(x)=2x+1$, and $h_3(x)=3x^2$.

7. (15 marks) Given the matrix $A = \begin{bmatrix} 1 & 3 & -3 \\ -1 & -3 & 1 \\ 1 & 1 & -3 \end{bmatrix}$. Find an invertible matrix P and a diagonal matrix D such that $P^{-1} A P = D$.