

MAT 186S
Quiz 1 Solutions

1. Solve for x if $2x \geq \frac{3x+3}{2x+1}$.

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|-----|-----|-----|
| AB1 | AB6 | MW2 |
|-----|-----|-----|

We cannot just multiply by $2x + 1$, unless we take cases of when it is positive or negative. Far more efficient is to use addition and subtraction to collect the terms on one side:

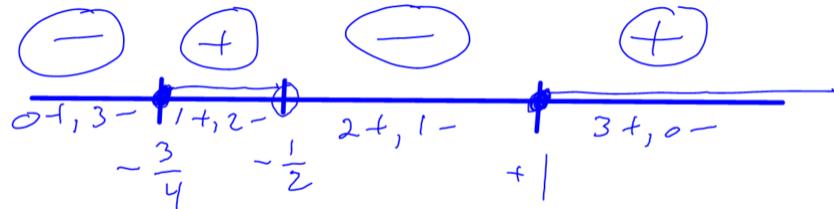
$$2x - \frac{3x+3}{2x+1} \geq 0$$

$$\frac{(2x)(2x+1) - (3x+3)}{2x+1} \geq 0$$

$$\frac{4x^2 - x - 3}{2x+1} \geq 0$$

$$\frac{(4x+3)(x-1)}{2x+1} \geq 0$$

This has zeroes at $x = -\frac{3}{4}, -\frac{1}{2}, +1$. Using a number line and **not** worrying about test points is far and away the best approach:



(Isn't this better than trying $x = -0.6$?)

Note that $x = -\frac{3}{4}$ and $x = +1$ give a value of zero and are in the solution set, while $x = -\frac{1}{2}$ leads to division by zero, so it is thrown out.

Therefore, the solution set is $-\frac{3}{4} \leq x < -\frac{1}{2}, x \geq 1$, or $\left[-\frac{3}{4}, -\frac{1}{2}\right) \cup [1, \infty)$.

2. Solve the inequality

$$4x^4 + 16x^3 + 15x^2 - 4x - 4 \leq 0$$

Show the work for factoring - that's where the marks are. If you could not find factors, show attempts that did not work.

Factoring the polynomial in any of a large number of ways gets us to:

$$(2x + 1)(2x - 1)(x + 2)^2 \leq 0$$

A number line quickly gives the domain as $x = -2$ or $-\frac{1}{2} \leq x \leq \frac{1}{2}$.

3. Solve for x if $|5x^2 + 7x - 6| \geq x^2 + 9x$.

We start by breaking the left-hand side into cases. Carefully.

Factoring, we get $5x^2 + 7x - 6 = (5x - 3)(x + 2)$. This lets us figure out the cases:

$$|5x^2 + 7x - 6| = \begin{cases} 5x^2 + 7x - 6 & x \leq -2, x \geq \frac{3}{5} \\ -(5x^2 + 7x - 6) & -2 < x < \frac{3}{5} \end{cases}$$

Case I: $5x^2 + 7x - 6 \geq 0$ (That is, $x \leq -2$ or $x \geq \frac{3}{5}$)

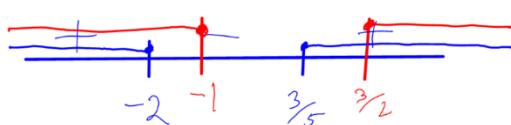
$$5x^2 + 7x - 6 \geq x^2 + 9x$$

$$4x^2 - 2x - 6 \geq 0$$

$$2(2x^2 - x - 3) \geq 0$$

$$2(2x - 3)(x + 1) \geq 0$$

Therefore, we have zeroes at $x = -1, \frac{3}{2}$. We need to use these and intersect them with our domain.



The diagram shows the domain for Case I in blue and the solutions to the resulting equation in red. What we keep is everything that is (a) in the solution set for the final equation **and** (b) within the domain for the case.

Therefore, we have $x \leq -2$ or $x \geq \frac{3}{2}$.

Case II: $5x^2 + 7x - 6 < 0$ (That is, $-2 < x < \frac{3}{5}$)

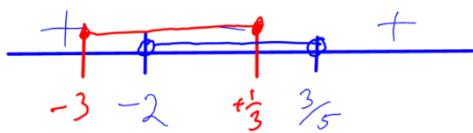
$$-(5x^2 + 7x - 6) \geq x^2 + 9x$$

(This is the best way of dealing with the negative – put it where the definition says and then get to work.)

$$0 \geq 6x^2 + 16x - 6$$

$$0 \geq 2(3x^2 + 8x - 3)$$

$$0 \geq 2(3x - 1)(x + 3)$$



The same approach as before yields $-2 < x \leq \frac{1}{3}$ as our solution set.

Combining the cases gives us the total solution: $x \leq \frac{1}{3}$ or $x \geq \frac{3}{2}$.

3'. If this is too difficult, then for part marks, solve $|4x - 14| \geq x^2 + 9x$.

We still need to take cases:

$$4x - 14 = \begin{cases} 4x - 14 & x \geq \frac{7}{2} \\ -(4x - 14) & x < \frac{7}{2} \end{cases}$$

Case I: $x \geq \frac{7}{2}$

$$4x - 14 \geq x^2 + 9x$$

$$x^2 + 5x + 14 \leq 0$$

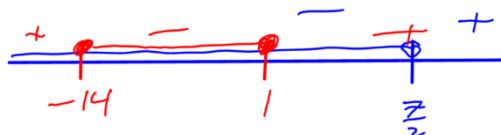
This one does not seem to factor, so we go to the quadratic equation, where we find that the discriminant is $b^2 - 4ac = 5^2 - 4(1)(14) = 25 - 56 = -29 < 0$. Therefore, the equation has no roots. Since it is positive at $x = 0$, it is positive for all x and there are no solutions.

Case II: $x < \frac{7}{2}$

$$-(4x - 14) \geq x^2 + 9x$$

$$0 \geq x^2 + 13x - 14$$

$$0 \geq (x + 14)(x - 1)$$



The number line indicates that the solution is $-14 \leq x \leq 1$, all of which is within the domain for this case.

Therefore, the solution is $-14 \leq x \leq 1$.