Practise Problem Set 2

MAT 187 - Summer 2025

These questions are meant for your own practice for quiz 2 and are not to be handed in. Some of these questions, or problems similar to these, may appear on the quizzes or exams. Therefore, solutions to these problems will not be posted but you may, of course, ask about these questions during office hours, or on Piazza.

Suggestions on how to complete these problems:

- Solution writing is a skill like any other, which must be practiced as you study. After you write down your rough solutions, take the time to write a clear readable solution that blends sentences and mathematical symbols. This will help you to retain, reinforce, and better understand the concepts.
- After you complete a practice problem, reflect on it. What course material did you use to solve the problem? What was challenging about it? What were the main ideas, techniques, and strategies that you used to solve the problems? What mistakes did you make at the first attempt and how can you prevent these mistakes on a Term Test? What advice would you give to another student who is struggling with this problem?
- Discussing course content with your classmates is encouraged and a mathematically healthy practice. Work together, share ideas, explain concepts to each other, compare your solutions, and ask each other questions. Teaching someone else will help you develop a deeper level of understanding. However, it's also important that reserve some time for self-study and self-assessment to help ensure you can solve problems on your own without relying on others.

1. For each of the following, evaluate the integral using the definition or show it diverges.
   (a) $\int_1^\infty \frac{\ln x}{x^2} dx$
   (b) $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$
   (c) $\int_0^\infty \frac{1}{1+x^2} dx$
   (d) $\int_0^\infty x e^{-x} dx$
   (e) $\int_0^1 \frac{\ln x}{\sqrt{x}} dx$
2. For each of the following, evaluate the integral using the definition or show it diverges.
   (a) $\int_{-1}^1 \frac{1}{\sqrt{|x|}}\, dx$
   (b) $\int_0^1 \frac{1}{x \ln x}\, dx$

(c) $\int_{-1}^{2} \frac{1}{(x-1)^2}\,dx$

3. For each of the following, use the comparison test to determine if the following converge or diverge.
   (a) $\int_{1}^{\infty} \frac{1}{x(\ln x)^2}\,dx$
   (b) $\int_{0}^{1} \frac{1}{\sqrt{x}+x}\,dx$
   (c) $\int_{0}^{1} \frac{1}{x^{0.9}(1-x)^{0.9}}\,dx$
   (d) $\int_{0}^{\infty} \frac{x}{1+x^3}\,dx$
   (e) $\int_{2}^{\infty} \frac{\ln x}{x^p}\,dx$ (where $p > 1$)

4. Consider the function
$$f(x) = \frac{4x}{(4x^2 - 4x + 2)(x-1)^2}$$
   (a) Evaluate the indefinite integral $\int f(x)dx$. (You may have already done this in Practise Problem 1)
   (b) Which of the following improper integrals converge?
$$\int_{-\infty}^{0} f(x)dx \qquad \int_{-10}^{1} f(x)dx \qquad \int_{1}^{10} f(x)dx \qquad \int_{10}^{\infty} f(x)dx$$

5. Consider the following improper integral.
$$I = \int_{0}^{\infty} \frac{\sqrt{x}-1}{\sqrt{x}(x^{3/2}+1)}dx$$
   (a) Write the improper integral I as a sum of limits of proper integrals.
   (b) Use the substitution $x = u^2$ to calculate the proper integrals you obtained in part (a).
   (c) Does the improper integral I converge or diverge? If it converges, find its value.

6. In a heat sink design, a thin metal *fin* conducts heat away from a base located at $x = 0$. The **heat flux** $q(x)$ along the length of the fin is modeled by:
$$q(x) = \frac{1}{\sqrt{x}}$$
   where x is the distance (in meters) from the base of the fin, and $q(x)$ is measured in **watts per meter.**

   Compute the *total heat conducted* (in watts) from the base to a point 1 meter away:

7. An infinitely long, straight wire lies along the x-axis and carries a uniform linear charge density of $\lambda = 1\,\mu\text{C/m}$. Let P be a point located a distance $a > 0$ meters above the origin on the y-axis.
   (a) Using Coulomb's law and symmetry, derive an expression for the vertical component of the electric field E_y at point P, due to a small element of charge $dq = \lambda\,dx$ at position x on the wire.
   (Hint: Use the formula for the electric field due to a point charge and express the vertical component in terms of x and a.)
   (b) Set up and evaluate the improper integral that gives the total vertical electric field E_y at point P. Does the integral converge?