LAST Name: ______________

First Name: ______________

Student Number: ______________

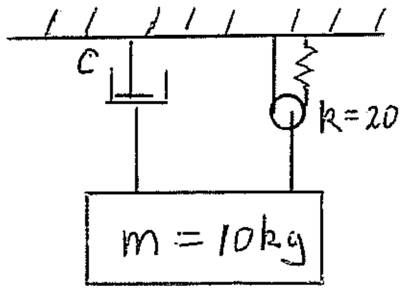
MIE 200F - Quiz number 8b - November 29/01
quiz duration = 25 minutes

A block is supported by the system of dampers and springs as shown. Assume that the mass can move only in the vertical direction, with no rotation. Values of all dampers and springs are given in SI units.

(a) For what values of the damper "C" will the system be underdamped?

(b) What value of damper "C" is required to reduce the magnitude of the oscillations by a factor of 10 within 30 seconds?

$g = 9.81\ m/s^2 \downarrow$



The equation $\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = 0$, has the solution

$x(t) = C_1 \cos(\omega_d t) \exp(-\zeta\omega_n t) + C_2 \sin(\omega_d t) \exp(-\zeta\omega_n t)$

$= C_3 \sin(\omega_d t + \varphi) \exp(-\zeta\omega_n t)$

where $\omega_d = \omega_n (1-\zeta^2)^{1/2}$

$+\downarrow$ (a) $F = -c\dot{x} - (4)(20)x = m\ddot{x}$

$10\ddot{x} + c\dot{x} + 80x = 0$

$\ddot{x} + \frac{c}{10}\dot{x} + 8x = 0 \iff \ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 = 0$

$\Rightarrow \omega_n = \sqrt{8}$; $C_c = 2m\omega_n = 56.6\ N/m/s.$

system is underdamped for $c < 56.6\ N/m/s$

(b) $\exp(-(\zeta)(\omega_n)(30\text{seconds})) = 1/10$

take ln of both sides:

$-(\zeta)(\omega_n)(30) = -2.303$, $\omega_n = \sqrt{8}$

$\zeta = 0.027$

$c/10 = 2\zeta\sqrt{8} \Rightarrow c = 1.53\ N/m/s$

OR

$\exp\left(-\frac{c}{2m} t\right)\Big|^{t=30\text{seconds}} = 0.1$

$\left(-\frac{c}{2m}\right)(30) = -2.303 \Rightarrow c = 1.53\ N/m/s$

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