MIE100 Midterm Test
Thursday March 4, 2010
Start Time: 6:10pm Duration: 1 hr 30 min
Aids Allowed: Non-programmable calculator and one 8.5x11" aid sheet

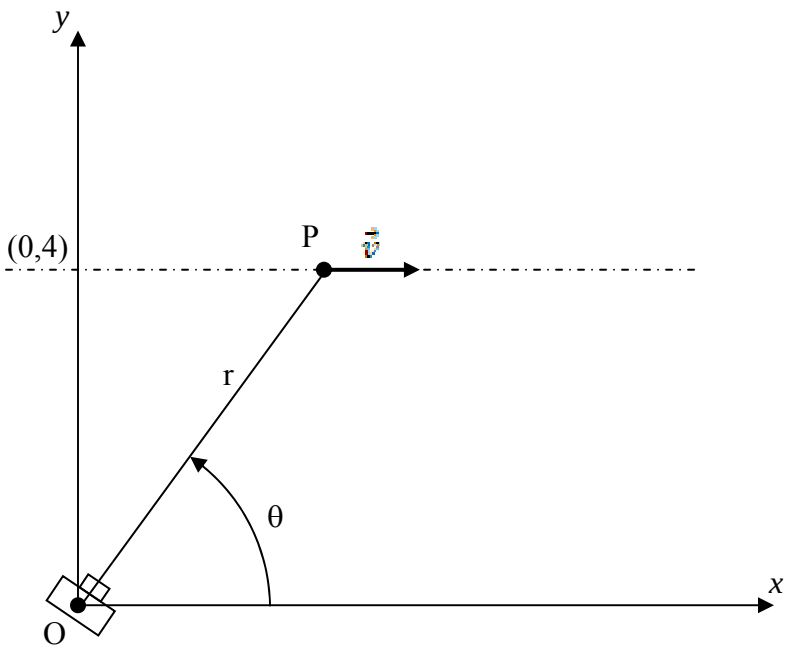
Instructions: Write your full name, student number, course information and tutorial section number on each exam booklet you use. You may keep this test paper. Answer all three questions.

Question 1

An object, P, travels along $y = 4$ with velocity $\vec{v} = (2x + 1)\hat{i}$ m/s. The scale on both the x and y axes is metres. A camera is positioned at the origin, O, and is aimed to track the motion of P.

(i) Using polar co-ordinates centered at O as shown, what is the rate of rotation of the camera, $\dot{\theta}$, when $\theta = 45^o$? [15 marks]

(ii) What is $\ddot{\theta}$ when $\theta = 45^o$? [20 marks]



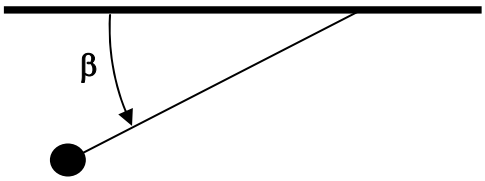
Question 2

A rock of mass 4 kg is tied to the end of a rope of length 5 meters. The other end of the rope is tied to a hook in the ceiling. Let β represent the angle measured in the counter-clockwise direction between the ceiling and the rope. The rock is released from rest when β = 0.

(i) What will be the speed of the rock when β reaches 60°? [10 marks]

(ii) What will be the magnitude of the **tangential** component of the acceleration when β reaches 70°? [10 marks]

(iii) What will be the tension in the rope when β reaches 60°? [10 marks]

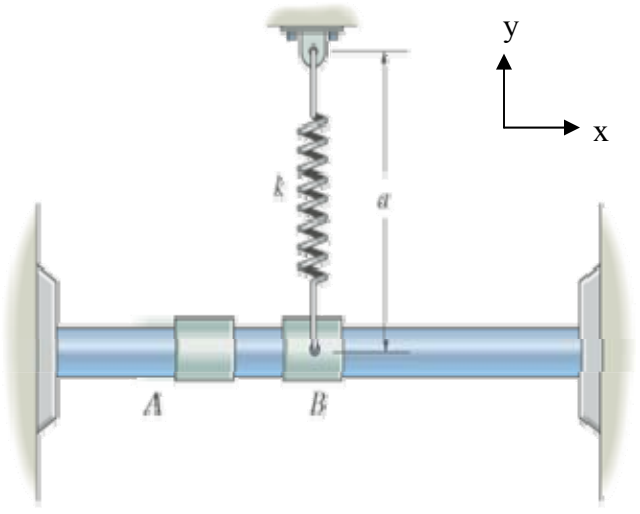


Question 3

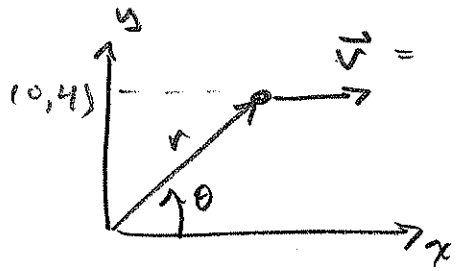
The collar B of mass 5.75 kg is initially at rest at the position shown. At this position, the spring (k=400 N/m) is unstretched and a = 0.9 m. The collar A of mass 0.5 kg strikes collar B at a velocity of 40 m/s to the right so that B slides a distance of 1.2 m on the smooth rod before momentarily stopping. Assume there is no friction.

(i) Determine the velocity of collar A just after impact. [25 marks]

(ii) Find the magnitude of the average force exerted between A and B during the impact if the impact occurs over a duration of 0.002 s. [10 marks]

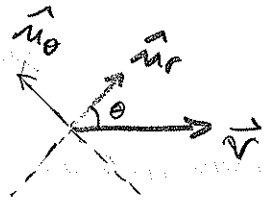


Q1.



$\vec{v} = (2x+1)\hat{i}$

(i)



$v_\theta = -v\sin\theta = r\dot{\theta}$

$\dot{\theta} = \dfrac{-v\sin\theta}{r}$

at position shown $\theta = 45°$:

$\tan\theta = \dfrac{y}{x}$

$x = \dfrac{y}{\tan\theta} = \dfrac{4}{\tan 45°} = 4\ m$) can be found by inspection

$r = \sqrt{x^2+y^2} = \sqrt{(4)^2+(4)^2} = 4\sqrt{2}\ m$

$\vec{v} = (2(4)+1)\hat{i} = 9\hat{i}\ m/s$

$\dot{\theta} = \dfrac{-9(\sin 45°)}{4\sqrt{2}}$

$\dot{\theta} = -1.125\ \dfrac{rad}{s}$

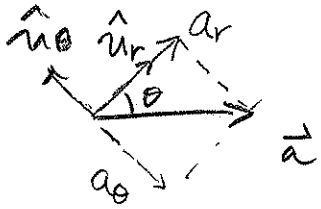
$\dot{\theta} = 1.125\ \dfrac{rad}{s}$ ↙ either

I(ii)

$$a = \frac{dv}{dt} = v\frac{dv}{dx} = (2x+1)\frac{d(2x+1)}{dx} = 2(2x+1)$$

$$\vec{a} = (2x+1)(2)\hat{i} \ \text{m/s}$$

($a_y = 0$ since motion is rectilinear)



$$a_\theta = (2\dot{r}\dot{\theta} + r\ddot{\theta})$$

first find $\dot{r}$ $\qquad v_r = v\cos\theta = \dot{r}$

at $\theta = 45°$:

$$\dot{r} = 9\cos 45° = 6.364 \ \text{m/s}$$

$$\vec{a} = (2(4)+1)(2)\uparrow = 18\uparrow \ \text{m/s}^2$$

$$a_\theta = -a\sin\theta = -18\cos 45°$$

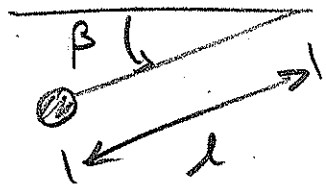
$$\therefore \quad -18\cos 45° = 2(6.364)(-1.125) + 4\sqrt{2}\,\ddot{\theta}$$

solve for $\ddot{\theta}$

$\ddot{\theta} = +0.281 \ \frac{\text{rad}}{\text{s}^2}$ $\quad \Longleftrightarrow$ either

or $\quad 0.281 \ \frac{\text{rad}}{\text{s}^2}\circlearrowleft$

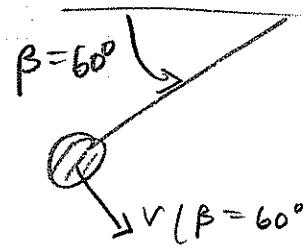
Q2



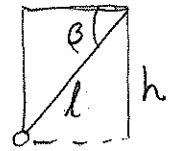
(i) Position 1

Position 2

datum



$$T_1 + V_1 + U_{1\to 2} = T_2 + V_2$$



$$0 = \frac{m V_{(\beta)}^2}{2} - mgl\sin\beta$$

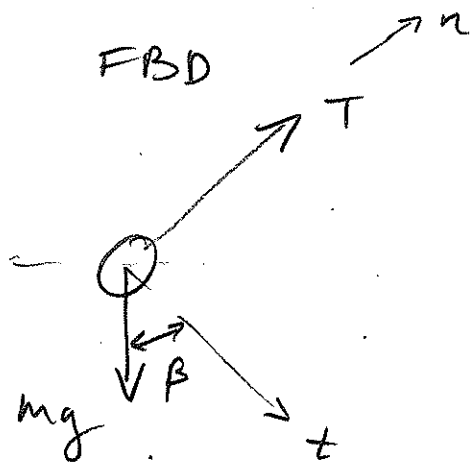
$$V_{(\beta)} = \left[2gl\sin\beta\right]^{\frac{1}{2}}$$

$$V_{(60°)} = \left[2(9.81)(5)\sin 60°\right]^{\frac{1}{2}}$$

$$V_{(60°)} = 9.217 \ \frac{m}{s}$$

2

(ii)



$$\overset{+}{\searrow} \sum F_t = \cancel{m} g \cos\beta = \cancel{m}\, a_t(\beta)$$

$$a_t(\beta) = g\cos\beta$$

$$a_t(70°) = 9.81 \cos 70° = 3.355 \ \frac{m}{s^2}$$

(iii)

$$\overset{+}{\nearrow} \sum F_n = -mg\sin\beta + T(\beta) = m\, a_n(\beta) = m\frac{v^2(\beta)}{l}$$

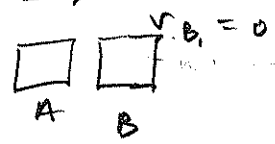
$$T(\beta) = m\left(g\sin\beta + \frac{v^2(\beta)}{l}\right)$$

$$T(60°) = 4\left[9.81\sin 60° + \frac{(9.217)^2}{5}\right]$$

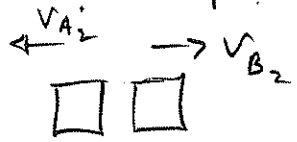
$$T(60°) = 101.95 \ N$$

Q3

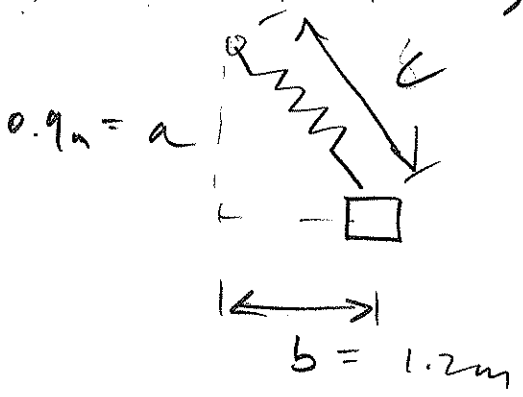
Position 1 (just before impact)



Position 2 (just after impact)



Position 3 (spring at max stretch)



$$C = \sqrt{a^2 + b^2}$$

Work / Energy for collar B between 2 & 3

$$T_2 + V_2 = T_3 + V_3$$

($V_2 = 0$, $T_3 = 0$)

$$\frac{m_B v_{B_2}^2}{2} = \frac{1}{2} k \delta^2 = \frac{1}{2} k \left[(a^2 + b^2)^{\frac{1}{2}} - a \right]^2$$

$$v_{B_2} = \left[\frac{k}{m_B} \left[(a^2 + b^2)^{\frac{1}{2}} - a \right]^2 \right]^{\frac{1}{2}}$$

$$v_{B_2} = \sqrt{\frac{k}{m_B}} \left[(a^2 + b^2)^{\frac{1}{2}} - a \right]$$

$$v_{B_2} = \sqrt{\frac{400}{5.75}} \left[\sqrt{(0.9)^2 + (1.2)^2} - 0.9 \right]$$

$$v_{B_2} = 5.00\ m/s.$$

Conservation of momentum law

1 → 2

$$m_A v_{A_1} + \cancel{m_B v_{B_1}}^{\,0} = m_A v_{A_2} + m_B v_{B_2}$$

$$v_{A_2} = \frac{m_A v_{A_1} - m_B v_{B_2}}{m_A}$$

$$v_{A_2} = \frac{0.5(40) - (5.75)(5.00)}{0.5}$$

$$v_{A_2} = -17.5 \frac{m}{s}$$

$$\boxed{v_{A_2} = 17.5 \frac{m}{s} \leftarrow}$$

either

$$\boxed{v_{A_2} = -17.5\,\hat{\imath}\ m/s}$$

(ii) Impulse + momentum law

1 + 2 for collar A

FBD: $m_A g$, F, N_A

$m_A v_{A_1}$ + $\bar{F}\Delta t$ = $m_A v_{A_2}$

$$m_A v_{A_1} + \bar{F}\Delta t = + m_A v_{A_2}$$

$$\bar{F} = \frac{-m_A v_{A_1} + m_A v_{A_2}}{\Delta t}$$

$$\bar{F} = \frac{-0.5(40) + (0.5)(17.5)}{0.002} = -14.375\ kN$$

magnitude: $F = 14.4\ kN$