Practise Problem Set 1

MAT 187 - Summer 2025

These questions are meant for your own practice for quiz 1 and are not to be handed in. Some of these questions, or problems similar to these, may appear on the quizzes or exams. Therefore, solutions to these problems will not be posted but you may, of course, ask about these questions during office hours, or on Piazza.

Suggestions on how to complete these problems:

- Solution writing is a skill like any other, which must be practiced as you study. After you write down your rough solutions, take the time to write a clear readable solution that blends sentences and mathematical symbols. This will help you to retain, reinforce, and better understand the concepts.
- After you complete a practice problem, reflect on it. What course material did you use to solve the problem? What was challenging about it? What were the main ideas, techniques, and strategies that you used to solve the problems? What mistakes did you make at the first attempt and how can you prevent these mistakes on a Term Test? What advice would you give to another student who is struggling with this problem?
- Discussing course content with your classmates is encouraged and a mathematically healthy practice. Work together, share ideas, explain concepts to each other, compare your solutions, and ask each other questions. Teaching someone else will help you develop a deeper level of understanding. However, it's also important that reserve some time for self-study and self-assessment to help ensure you can solve problems on your own without relying on others.

1. Evaluate the following integrals:
   (a) $\int x^2 \sin(2x-3)dx$
   (b) $\int \ln(1-x^2)dx$
   (c) $\int \ln(x+\sqrt{x^2-1})dx$
2. Evaluate the following integrals:
   (a) $\int \frac{x^2}{(x^2+9)^{3/2}}dx$
   (b) $\int \sqrt{x^6-x^8}dx$
   (c) $\int \frac{1}{x^2\sqrt{x^2+1}}dx$

3. Evaluate the following integrals:
   (a) $\int \frac{16-7x}{(4x^2-1)(x^2-2x+2)}dx$
   (b) $\int \frac{x^6+3x^5-3x^3+6x^2+6x+1}{x^3+3x^2+2x}dx$
   (c) $\int \frac{4x}{(4x^2-4x+2)(x-1)^2}dx$

4. Given any constants a, b, c with $a > 0$, find a general formula for the integral
$$\int a^{bx} \sin(cx)dx$$
Consider the possible cases carefully.

5. Consider the integral
$$\int \frac{P(x)}{D(x)}dx$$
where $P(x)$ is a polynomial of degree n and
$$D(x) = (x-a)(x-b)$$
for two real numbers a, b such that $a \neq b$. Show that we can always evaluate this integral via the method of partial fractions.

6. A *reduction formula* is an equation that takes an integral involving a parameter n and returns a similar integral with a lower parameter (usually $n-1$ or $n-2$).
   (a) Find a reduction formula for $\int x^n e^x dx$ in terms of $\int x^{n-1} e^x dx$. Find a formula for $\int x^n e^x dx$ that does not include integrals.
   (b) Find a reduction formula for $\int x^n \sin(x)xdx$. Find a formula for $\int x^n \sin(x)dx$ that does not include integrals.
   (c) Find a reduction formula for $\int x^n \ln(x)xdx$. Find a formula for $\int x^n \ln(x)dx$ that does not include integrals.

7. A squirrel has buried an acorn in the lawn of King's College Circle in the hopes of eating it later. The probability that the acorn will sprout between the start of day n and the start of day m (where $0 \leq n \leq m$) is given by
$$\frac{4}{\pi}\int_n^m \frac{e^x}{e^{2x}+1}dx$$
What is the probability that the squirrel can return after seven days and dig up an unsprouted acorn?

8. A team of scientists is launching a rocket into low Earth orbit. When the rocket's fuel tank is empty, the rocket has mass m_r. The rocket burns fuel as it ascends, so after launch the mass of fuel in the rocket at height y above the **surface** of the Earth is given by the function
$$m_f(y) = \frac{10m_r}{1+y}$$
The rocket must fight against the gravitational force, denoted F, which is given by Newton's law of universal gravitation
$$F = \frac{GMm}{d^2}$$
where

- M is the mass of the Earth,
- m is the total mass of the rocket (including fuel),
- d is the distance from the rocket to the **centre** of the Earth, and
- G is the universal gravitational constant.

(a) Find a formula for d in terms of y and the radius of the Earth r_E (assume that the earth is perfectly spherical).

(b) Find a formula for m.

(c) Using the above answers, find a formula for the gravitational force as a function of y. You do not need to find the values of M and G.

(d) Use partial fractions to write the gravitational force as a sum of irreducible fractions.

(e) Write an integral formula for the amount of work required to lift the rocket to an altitude of A metres above the **surface** of the Earth. Note that *work*, as defined in physics, is the integral of force.

(f) Evaluate the integral formula you obtained in part (e).