MAT187 - Calculus II - Winter 2019

Term Test 1 - February 5, 2019

MULTIPLE-CHOICE QUESTIONS

AND

FORMULA SHEET

Instructions:

- DO NOT OPEN until instructed to do so.
- **THIS PART WILL NOT BE COLLECTED.**
- This part contains 6 pages.

MULTIPLE-CHOICE PART. **(12 marks)**

ANSWER THESE QUESTIONS ON **PAGE 10** OF THE TEST.

1. (2 marks) When expanding the rational function $\dfrac{x^3+2x^2+3x+4}{(x+5)(x-3)^3\big((x+1)^2+9\big)^2}$ into partial fractions, which of the following terms will **NOT** be necessary in the decomposition?

(A) $\dfrac{A}{x+5}$

(B) $\dfrac{B}{(x-3)^2}$

(C) $\dfrac{C}{\big((x+1)^2+9\big)^2}$

(D) $\dfrac{Dx}{\big((x+1)^2+9\big)^2}$

(E) $\dfrac{Ex^2}{\big((x+1)^2+9\big)^2}$

2. (2 marks) Consider the integral $\displaystyle\int_{-1}^{23}\big(x^2+3x-19\big)\,dx$. Choose the only correct ordering of these numerical approximations.

(A) $M_{242} \quad < \quad \displaystyle\int_{-1}^{23}\big(x^2+3x-19\big)\,dx \quad < \quad M_{121}.$

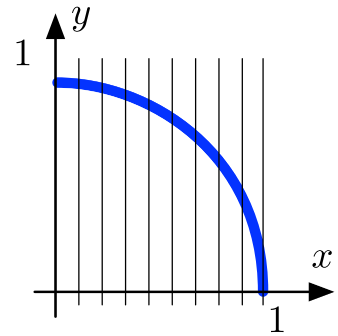
(B) $M_{242} \quad < \quad M_{121} \quad < \quad \displaystyle\int_{-1}^{23}\big(x^2+3x-19\big)\,dx.$

(C) $M_{242} \quad < \quad \displaystyle\int_{-1}^{23}\big(x^2+3x-19\big)\,dx \quad < \quad T_{121}.$

(D) $T_{121} \quad < \quad \displaystyle\int_{-1}^{23}\big(x^2+3x-19\big)\,dx \quad < \quad M_{242}.$

(E) $M_{242} \quad < \quad T_{121} \quad < \quad \displaystyle\int_{-1}^{23}\big(x^2+3x-19\big)\,dx.$

3. **(2 marks)** To find the mass of a quarter circular ring of radius 1 m with density $\rho(x, y) = (xy)^2$ kg/m, we need to slice the ring as in the figure.



We then need to approximate the ring with a simpler shape in each slice.

Select the simple shape that we should choose so that the resulting Riemann sum converges to the mass of the quarter ring.

(A) A horizontal line passing through the midpoint $\left(\frac{x_{i-1}+x_i}{2}, \sqrt{1 - \frac{(x_{i-1}+x_i)^2}{4}}\right)$.

(B) A straight line passing through both endpoints $\left(x_{i-1}, \sqrt{1 - x_{i-1}^2}\right)$ and $\left(x_i, \sqrt{1 - x_i^2}\right)$.

(C) A parabola passing through both endpoints $\left(x_{i-1}, \sqrt{1 - x_{i-1}^2}\right)$ and $\left(x_i, \sqrt{1 - x_i^2}\right)$.

(D) A cubic passing through both endpoints $\left(x_{i-1}, \sqrt{1 - x_{i-1}^2}\right)$ and $\left(x_i, \sqrt{1 - x_i^2}\right)$.

4. **(2 marks)** To calculate the integral $\displaystyle\int_{-2}^{5} \frac{1}{\sin(x)}\, dx$, we need to split the integral into several integrals. Select a correct way to split the integral.

(A) $\displaystyle\int_{-2}^{0} \frac{1}{\sin(x)}\, dx + \int_{0}^{5} \frac{1}{\sin(x)}\, dx$

(B) $\displaystyle\int_{-2}^{\pi} \frac{1}{\sin(x)}\, dx + \int_{\pi}^{5} \frac{1}{\sin(x)}\, dx$

(C) $\displaystyle\int_{-2}^{0} \frac{1}{\sin(x)}\, dx + \int_{0}^{\pi} \frac{1}{\sin(x)}\, dx + \int_{\pi}^{5} \frac{1}{\sin(x)}\, dx$

(D) $\displaystyle\int_{-2}^{0} \frac{1}{\sin(x)}\, dx + \int_{0}^{3} \frac{1}{\sin(x)}\, dx + \int_{3}^{\pi} \frac{1}{\sin(x)}\, dx + \int_{\pi}^{5} \frac{1}{\sin(x)}\, dx$

5. (2 marks) Consider the initial-value problem

$$\begin{cases} y'(x) = y(x) + x^2 e^x + 1 - e^{-1} \\ y(-1) = -1 \end{cases}$$

Select the only correct statement.

(A) The solution is increasing at $x = -1$.

(B) The solution is decreasing at $x = -1$.

(C) The solution has a local maximum at $x = -1$.

(D) The solution has a local minimum at $x = -1$.

(E) The solution has an inflection point at $x = -1$.

6. (2 marks) Note that the initial-value problem

$$\begin{cases} y'(x) = \big(y(x)\big)^2 \\ y(0) = 0 \end{cases}$$

has the solution $y(x) = 0$. This should help you to determine the following:

If a Separable initial-value problem has a solution, then the method we learned to solve Separable ODEs...

(A) never finds a solution.

(B) sometimes finds a solution.

(C) always finds a solution.

(D) always finds all solutions.

FORMULA SHEET FOR MAT187

Trigonometric Identities.

- $\cos^2(x) = \dfrac{1+\cos(2x)}{2}$
- $\sin^2(x) = \dfrac{1-\cos(2x)}{2}$
- $\sin(x) = \cos\left(\dfrac{\pi}{2} - x\right)$

Trigonometric Integrals.

- $\displaystyle\int \sec(x) = \ln\big(\sec(x) + \tan(x)\big) + C$
- $\displaystyle\int \sec^3\, dx = \int \frac{\cos(x)}{\big(1-\sin^2(x)\big)^2}\, dx$
- $\displaystyle\int \frac{1}{1+x^2}\, dx = \arctan(x) + C$

Applications of integration.

- Arc length for $y = f(x)$ $\qquad \displaystyle\int_a^b \sqrt{1 + \big(f'(x)\big)^2}\, dx$
- Area of a surface of revolution $y = f(x)$ revolved around $x-$axis $\qquad \displaystyle\int_a^b 2\pi f(x)\sqrt{1 + \big(f'(x)\big)^2}\, dx$

Numerical Integration.

- Left-Hand Rule $\displaystyle\int_a^b f(x)\, dx \approx L_n = \sum_{i=1}^n f(x_{i-1})\Delta x,\qquad |E_L| \leqslant \frac{b-a}{2}(\Delta x) \max_{x\in[a,b]} \big|f'(x)\big|$
- Right-Hand Rule $\displaystyle\int_a^b f(x)\, dx \approx R_n = \sum_{i=1}^n f(x_i)\Delta x,\qquad |E_R| \leqslant \frac{b-a}{2}(\Delta x) \max_{x\in[a,b]} \big|f'(x)\big|$
- Midpoint Rule $\displaystyle\int_a^b f(x)\, dx \approx M_n = \sum_{i=1}^n f\left(\frac{x_{i-1}+x_i}{2}\right)\Delta x,\qquad |E_M| \leqslant \frac{b-a}{24}(\Delta x)^2 \max_{x\in[a,b]} \big|f''(x)\big|$
- Trapezoid Rule $\displaystyle\int_a^b f(x)\, dx \approx T_n = \sum_{i=1}^n \frac{f(x_{i-1})+f(x_i)}{2}\Delta x,\qquad |E_T| \leqslant \frac{b-a}{12}(\Delta x)^2 \max_{x\in[a,b]} \big|f''(x)\big|$

Differential Equations.

- Separable DE: $g(y)\dfrac{dy}{dt} = h(t)$ $\qquad \displaystyle\int g(y)\, dy = \int h(t)\, dt$