MIE100S Dynamics – Spring 2012

Midterm Test – February 28, 2012

6:15pm – 7:45pm

COVER PAGE

General Instructions:

- Answer all questions in the exam booklets provided.
- Write your full ROSI name, student # and TUTORIAL # on your exam booklet(s).

Number of Pages:

- 4 (including cover page)

Number of Questions:

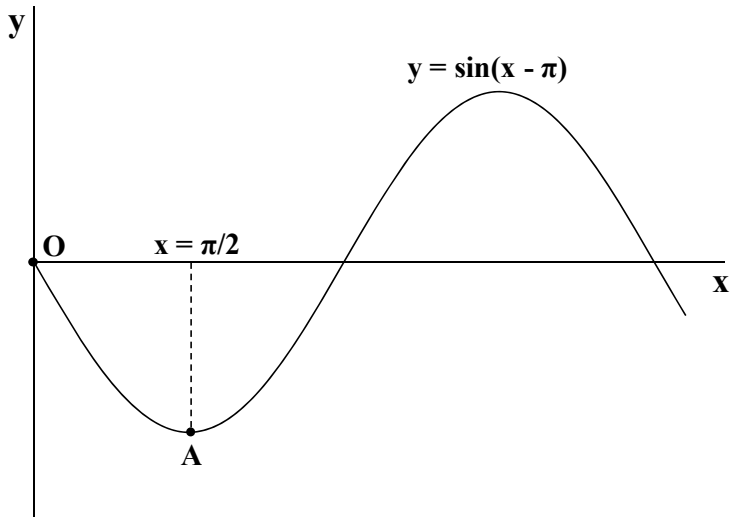
- 3 questions

Permitted Aids:

- Approved non-communicating/non-programmable calculator
- One aid sheet, any colour

1. A particle travels on a path described by y = sin(x-π), as shown. The speed of the particle along the path is constant at 0.4 m/s.

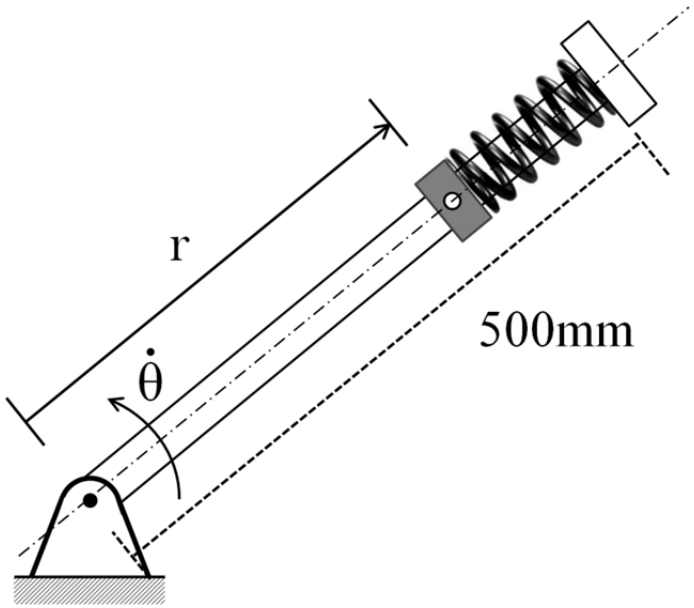
   (a) Determine the velocity ($\vec{v}$) of the particle **at A** expressed in the rectangular coordinates given.
   (b) Determine the velocity ($\vec{v}$) of the particle **at A** in polar (r-θ) coordinates, centered at point O indicated in the diagram.
   (c) Determine the acceleration ($\vec{a}$) of the particle **at A** in normal-tangential (n-t) coordinates.



An equation that you may find useful: $$\rho(x) = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

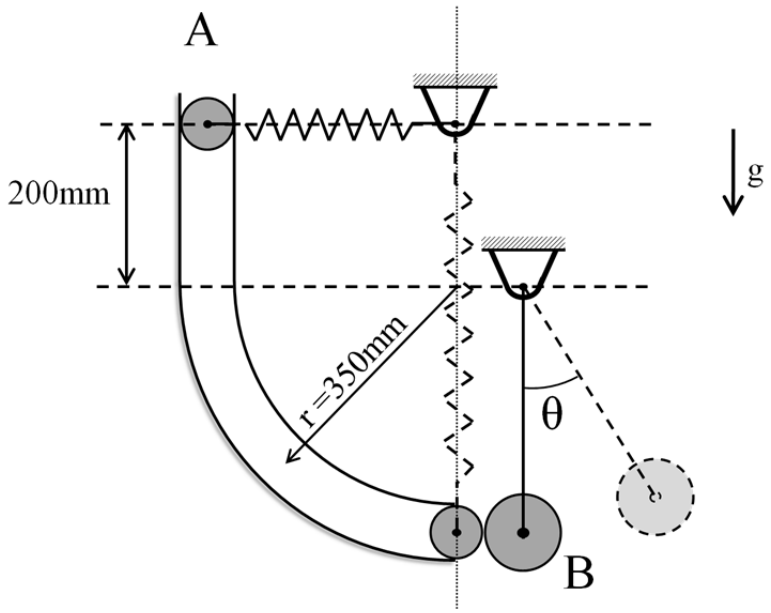
2. A collar of mass 0.1 kg is free to slide on a frictionless rod <u>in the horizontal plane</u> as shown. A spring is positioned to restrict the movement of the collar and has an un-stretched length (ℓ_o) of 125 mm and a spring constant of k = 20 N/m. The rod is rotating at a constant angular speed of 5 rad/s and r is constant.

   (a) Draw the free body diagram of the collar at the instant shown in the diagram. Indicate the exact directions that the forces exert on m. You may omit all forces that exert at right angles to the plane of motion.
   (b) Determine the radial distance, r (neglect the width of the collar).



3. A ball of mass $m_A = 2$ kg, connected to a spring with an un-stretched length (ℓ_o) of 200 mm and a spring constant of k = 100 N/m, falls from rest through a frictionless channel and collides with a larger ball of mass $m_B = 5$ kg, as shown. Ball A comes to rest immediately after impact.

   (a) Find the speed of ball A as it exits the channel just before impact with ball B.
   (b) Find the speed of ball B immediately after impact.
   (c) Find the maximum angle, θ, formed by ball B.



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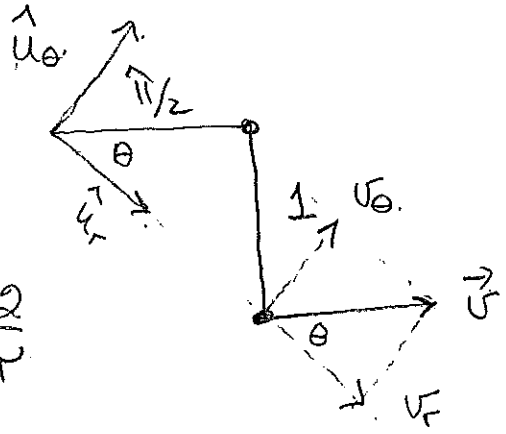
(a) $\vec{v} = 0.4\,\hat{i}$ m/s.

(b) geometry:



$$\theta = \tan^{-1}\frac{1}{\pi/2} = \tan^{-1}\frac{2}{\pi}$$

$$= 32.5^\circ.$$

$$\Rightarrow v_r = .4\cos 32.5^\circ = .4(.844) = .338$$

$$v_\theta = .4\sin 32.5^\circ = .4(.537) = .215$$

$$\Rightarrow \vec{v} = .338\,\hat{u}_r + .215\,\hat{u}_\theta \text{ m/s}.$$

(c) $a_t = 0$

$a_n = v^2/\rho$

$$\rho = \frac{\left[1 + \left(dy/dx\right)^2\right]^{3/2}}{d^2y/dx^2}$$ – given on test

$$y = \sin(x - \pi)$$

$$\frac{dy}{dx} = \cos(x - \pi)$$

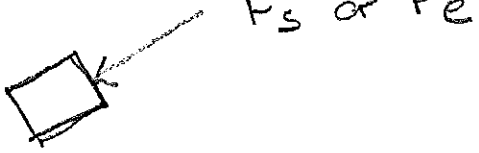
$$\frac{d^2y}{dx^2} = -\sin(x - \pi)$$

$$\Rightarrow \rho = \frac{\left[1 + \left[\cos(-\pi/2)\right]^2\right]^{3/2}}{-\sin(-\pi/2)} = \frac{1}{1} = 1.$$

$$\Rightarrow \vec{a} = \frac{(.4)^2}{1} \hat{u}_n = .16 \hat{u}_n \ m/s^2$$

2.

(a)



(b) $F_s = ks$

$r + l = 0.5\ m$

$l_0 - l = s \Rightarrow l = l_0 - s$

$r + l_0 - s = 0.5$

$s = r + l_0 - .5 = r + .125 - .5$

$= r - .375$

check signs with physics:

s is +ve ⇒ F_s is on FBD in correct direction.

$\Sigma F_n = ma_n$

$a_n = r\dot{\theta}^2 = r(25)$

$\Rightarrow 20(r - .375) = .1(25r)$

$20r - 7.5 = 2.5r$

$\Rightarrow r = 7.5/17.5 = 0.43\ m$

3. (a) $T_1 + V_1 + U_{1\to 2} = T_2 + V_2$

$T_1 = 0$

$V_1 = V_{e_1} + V_{g_1} = \frac{1}{2} k s_1^2 + m_A g h_1$

$= \frac{1}{2}(100)(.35 - .2)^2 + 2(9.81)(.2 + .35$

$= 50(.15)^2 + 2(9.81)(.55)$

$= 1.125 + 10.791 = 11.9$

$U_{1\to 2} = 0.$

$T_2 = \frac{1}{2} m_A v_2^2$ ← what we are asked to find.

$V_2 = V_{e_2} + \cancel{V_{g_2}}^{0} = V_{e_2}$

$= \frac{1}{2}(100)(.55 - .2)^2 = 50(.35)^2 = 6.125$

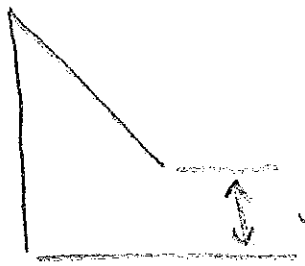
$\Rightarrow 11.9 = \frac{1}{2}(2) v_2^2 + 6.125$

$\Rightarrow v_2 = 2.40$ m/s.

(b) $m_A v_A + 0 = 0 + m_B v_B$.

$$\Rightarrow v_B = \frac{2}{5}(2.4) = 0.96 \text{ m/s}.$$

(c)



$h = l - l\cos\theta = l(1-\cos\theta)$

$$T_1 + \cancel{V_1}^{0} = \cancel{T_2}^{0} + V_2$$

$$\frac{1}{2}\cancel{5}(.96)^2 = \cancel{5}(9.81)(.35)(1-\cos\theta)$$

$$1 - \cos\theta = .1346$$

$$\cos\theta = .865 \Rightarrow \theta = 30^\circ$$