

First Name: \_\_\_\_\_

Last Name: \_\_\_\_\_

Student Number: \_\_\_\_\_

**MIE 200F – Test No. 9 – November 30, 1998****Test Duration – 20 minutes**

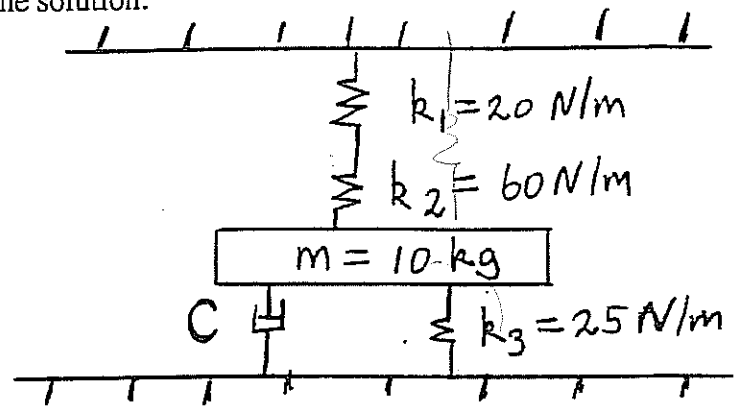
- (a) What is the natural frequency  $\omega_n$  of the system if the damper  $C = 0$ ?
- (b) What damper value  $C$  is required to reduce the vibration frequency by about 2%?

The equation:  $\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$  has the solution:

$$x(t) = C_1 \sin(\omega_d t + \phi) \exp(-\zeta\omega_n t),$$

where  $C_1$  and  $\phi$  are unknown constants,and the damped frequency  $\omega_d = \omega_n \sqrt{1-\zeta^2}$ 

$$\begin{aligned} K_{\text{total}} &= k_3 + \frac{k_1 k_2}{k_1 + k_2} \\ &= 25 + \frac{(20)(60)}{20+60} \\ &= 25 + 15 \\ &= 40 \text{ N/m} \end{aligned}$$



$$\begin{aligned} \text{(a)} \quad F &= -40x = m\ddot{x} \\ 10\ddot{x} + 40x &= 0 \\ \ddot{x} + 4x &= 0 \\ \omega_n^2 &= 4 \Rightarrow \omega_n = 2 \text{ s}^{-1} \end{aligned}$$

$$\text{(b)} \quad \frac{\omega_d}{\omega_n} = \sqrt{1-\zeta^2} = 0.98 \Rightarrow \zeta = 0.199$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0 \Leftrightarrow \ddot{x} + \frac{c}{m}\dot{x} + 4x = 0$$

$$\Rightarrow 2\zeta\omega_n = \frac{c}{m}$$

$$(2)(0.199)(2) = c/10$$

$$c = 7.96 \frac{\text{N}}{\text{m/s}}$$