

APS 104S, Mock Midterm Exam, January 2009
Solutions

1. When 2 gr of gas A is introduced into an initially evacuated flask which is maintained at constant temperature T , the pressure will be 1 bar. After adding 3 gr of gas B to the flask containing 2 gr of gas A at the same temperature T , the pressure will be 1.5 bar. Assume both gases behave as an ideal gas; calculate the ratio of the molar mass of gas B to that of gas A, i.e., (M_B/M_A) .

Solution:

For gas A alone: $P=P_A=n_ART/V=(2/M_A)(RT/V)=10^5 \text{ Pa} \rightarrow M_A=2RT/(10^5V)$

For mixture of A and B: $P_{\text{mix}}=P_A+P_B \rightarrow 1.5 \cdot 10^5=10^5+P_B \rightarrow P_B=0.5 \cdot 10^5 \text{ Pa}$

$P_B=(3/M_B)(RT/V)=0.5 \cdot 10^5 \rightarrow M_B=3RT/(0.5 \cdot 10^5V)$

$$\frac{M_B}{M_A} = \frac{\frac{3RT}{0.5 \cdot 10^5V}}{\frac{2RT}{10^5V}} = \frac{3}{2 \times 0.5} = 3$$

2. A piston/cylinder arrangement is filled with helium gas at 300 K and 1 bar. The initial volume is 1 m^3 . Heat is transferred to the system reversibly until its final volume is 4 m^3 . In this process, pressure varies with volume according to the following formula:

$$PV^2 = \text{const.} \quad \text{where } P \text{ is in Pa and } V \text{ is in m}^3$$

If helium behaves as an ideal gas with $C_v=3/2R$, calculate:

- the work produced by the system during this process
- the amount of heat added to the system during this process
- the change in the enthalpy of helium during this process
- Sketch this process in a P-V diagram and use that diagram to explain if an isothermal expansion to final volume of 4 m^3 produces more or less work than the work obtained in part (a)

Solution:

$$n = \frac{PV}{RT} = \frac{10^5 \times 1}{8.314 \times 300} = 40.1 \text{ mol}$$

$$PV^2 = \text{const.} \rightarrow \text{const.} = (10^5 \text{ Pa})(1 \text{ m}^3)^2 = 10^5 \text{ Pa.m}^6$$

$$W_{rev} = -\int_{V_1}^{V_2} P dV = -\int_{V_1}^{V_2} \left(\frac{10^5}{V^2}\right) dV = +10^5 \left(\frac{1}{V_2} - \frac{1}{V_1}\right) = 10^5 \left(\frac{1}{4} - 1\right) = -0.75 \times 10^5 J$$

$$P_2 V_2^2 = 10^5 \rightarrow P_2 = \frac{10^5}{4^2} = 6250 Pa$$

$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1} \rightarrow T_2 = \frac{P_2 V_2}{P_1 V_1} T_1 = 75 K$$

$$\Delta U = n C_v \Delta T = 40.1 * 3/2 R * (75 - 300) = -112520 J$$

$$\Delta U = Q + W \rightarrow Q = \Delta U - W = -37519.6 J$$

$$\Delta H = n C_p \Delta T = 40.1 * 5/2 R * (75 - 300) = -187533 J$$

W	ΔU	Q	ΔH
-0.75 10 ⁵ J	-112520 J	-37519.6 J	-187533 J

d. If you prepare a sketch in a P-V diagram, you will see that the isothermal expansion work is bigger than the work done in this process:

$$W_{rev} = -nRT \ln \frac{V_2}{V_1} = -40.1 * 8.314 * 300 * \ln(4/1) = -1.386 \times 10^5 J$$

$$P_2 = \frac{P_1 V_1}{V_2} = 25000 Pa \quad \leftarrow \text{If the process is isothermal}$$