2016.

Q1 [10 marks] Circle the correct answer or answers

Q1.a. Consider 5, equal positive charges placed on the vertices of a hexagon with sides of length R, as shown in the figure below. What is the value of the electric field at the centre point, P? (2 marks)

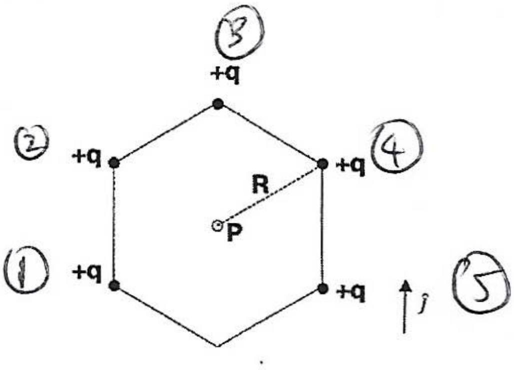
A) $\vec{E} = \frac{2kq}{R^2}\hat{j}$

B) $\vec{E} = \frac{-2kq}{R^2}\hat{j}$

C) $\vec{E} = \frac{kq}{R^2}\hat{j}$

D) $\vec{E} = \frac{-kq}{R^2}\hat{j}$

E) $\vec{E} = 0$



Where: $k = \frac{1}{4\pi\varepsilon_0}$

- 1 and 4 cancel (equal charge + distance but ~~different~~, opposite directions)
- 2 and 5 cancel as well
- 3 pushes down $\Rightarrow |\vec{E}| = \frac{kq}{r^2}$, downward, so $-\hat{j}$

Q1.b. The charge distribution is modified by the inclusion of the charge of -5q and the bottom apex as shown below. What is the value of electrical potential at point P? (2 marks)

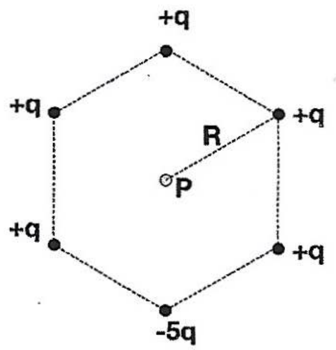
A) $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

B) $V = \frac{-5}{4\pi\varepsilon_0}\frac{q}{R}$

C) $V = 0$

D) $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

E) $V = \frac{10}{4\pi\varepsilon_0}\frac{q}{R^2}$



Potentials add up, $\boxed{\text{irrespective of direction}}$

Potential of one ~~position~~ positive +q: $V = \frac{kq}{R}$

However, negative charge does cause negative potential because voltage is dependent on charge polarity $\Rightarrow$ potential for $-5q$: $-\frac{5kq}{R}$

5 charges Add: $5\left(\frac{kq}{R}\right) - \frac{5kq}{R} = \phi$

Q1.c. Consider the cubic, closed Gaussian surfaces shown below. Which of the following situations has a zero net flux through the surface? (2 marks)

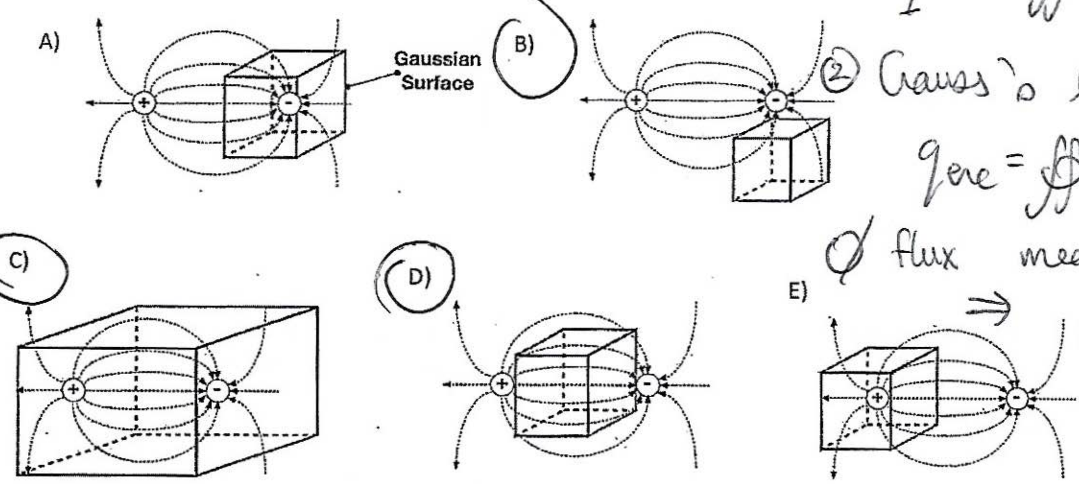
① Flux = $\Phi = \oiint \vec{E} \cdot d\vec{s}$

② Gauss's law:

$q_{enc} = \oiint \vec{E} \cdot d\vec{s}$

∅ flux means ∅ charge

$\Rightarrow 0 = \oiint \vec{E} \cdot d\vec{s}$



Select boxes w/ no charge

Q1.d. Three capacitors are connected together in the circuit shown below. What is the total energy stored in the capacitors? (2 marks)

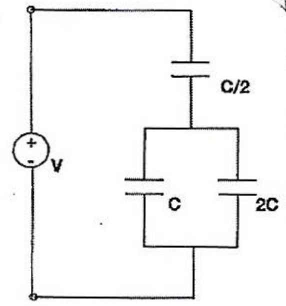
A) $\frac{1}{2}CV^2$

B) $\frac{3}{7}CV^2$

C) $\frac{3}{14}CV^2$

D) $\frac{3}{7}CV$

E) $\frac{6}{28}CV$



To find total energy, you can find the equivalent capacitance and then use $\frac{1}{2}CV^2$ to find energy*

① Combine parallel. $2C + C = 3C$

② Combine series now.

$C_{total} = \left[\frac{1}{C/2} + \frac{1}{3C}\right]^{-1}$

$= \frac{3}{7}C$

③ $\frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{3}{7}C\right)V^2 = \boxed{\frac{3}{14}CV^2}$

* Not required to know but you can prove this using energy conservation

Q1.e. An electron is placed mid-way between points A and B. The potential at point A is + 10 V, the potential at point B is -10 V and the potential at the midpoint is 0V. The electron will: (2 marks)

A) Not move since V = 0

B) Move towards point B with constant velocity

C) Accelerate towards point B

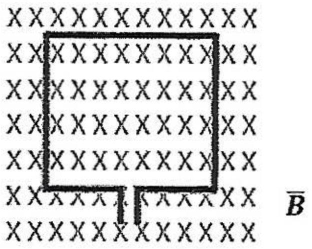
D) Move towards point A with constant velocity

E) Accelerate towards point A

① Potential exists in this system. This means an electric field exists, therefore any charge will have a force on it. Force implies acceleration.

② Positive charges go from high to low potential. This means negative charges must go from low to high. ⇒ 0V to 10V.

Q2 [10 marks] A single current loop, shown in the figure below, is exposed to a spatially uniform magnetic field $\mathbf{B} = \frac{2}{9\pi}\boldsymbol{sin(2\pi t)}$ tesla into the page as shown, where $\boldsymbol{t}$ is time in seconds.



Q2.a. Find the magnetic flux Φ_B passing through the wire loop as a function of time if the wire loop is a square in shape with dimensions 3 cm x 3 cm. (2 marks)

Area = 3cm x 3cm = $(0.03 cm)^2 = 9 \times 10^{-4} m^2$

A) $\Phi_B = 2 \times 10^{-6}\, sin(2\pi t)$ Wb

B) $\Phi_B = 5 \times 10^{-4}\, cos(2\pi t)$ Wb

C) $\Phi_B = \frac{5}{\pi} sin(2\pi t)$ Wb

D) $\Phi_B = \frac{2\times10^{-4}}{9\pi} sin(2\pi t)$ Wb

E) $\Phi_B = \frac{2\times10^{-4}}{\pi}\, sin(2\pi t)$ Wb

$\Phi_B = \oiint \vec{B} \cdot d\vec{s}$. (S is surface area, same as A)

$= \oiint \left(\frac{2}{9\pi} sin(2\pi t)\right) \cdot ds$ For a given t, flux will not ~~depend~~ depend on area,

$= \left(\frac{2}{9\pi} sin(2\pi t)\right) \iint ds$ Just multiply by area.

$= \frac{2}{\cancel{9}\pi} sin(2\pi t) \cdot \cancel{9} \times 10^{-4} m^2 = \boxed{\frac{2}{\pi} sin(2\pi t) \times 10^{-4}.}$

...f don't need to know why this works. Just know that a surface integral $\oiint \vec{B}\cdot d\vec{s}$ is just (B·S) if B is a constant given a certain S.

Q2.b. Determine the magnitude of the *emf* generated in the wire loop. (2 marks)

A) $emf = 4 \times 10^{-4}\, cos(2\pi t)$ V

B) $emf = 0$ V

C) $emf = 5 \times 10^{4}\, cos(2\pi t)$ V

D) $emf = \frac{2\times10^{-4}}{9\pi} sin(2\pi t)$ V

E) $emf = \frac{1}{9\pi} tan(2\pi t)$ V

$emf = -\frac{d\phi_B}{dt}$

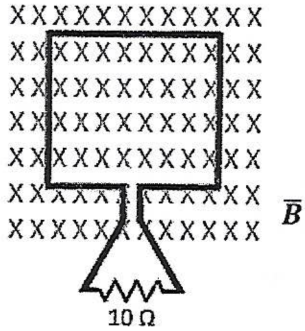
$= -\frac{d}{dt}\left[\frac{2\times10^{-4}}{\pi} sin(2\pi t)\right]$.

$= -\frac{2\times10^{-4}}{\pi} \cdot 2\pi\, cos(2\pi t)$

$= -4\times10^{-4}\, \cancel{sin}\, cos(2\pi t)$

$\Rightarrow |emf| = 4 \times 10^{-4} cos(2\pi t)$

Q2.c. If the loop is connected to a 10 Ω resistor as shown in the figure below, what is the direction of the induced current in the current loop between **i)** $t = 0.0$ **s to** $t = 0.25$ s and **ii)** $t = 0.75$ **s to** $t = 1.0$ **s**. Indicate if the current is clockwise (CW) or counter clockwise (CCW). (2 marks)



A) i) CW and ii) CW

B) i) CCW and ii) CCW

C) i) CW and ii) CCW

D) i) CCW and ii) CW

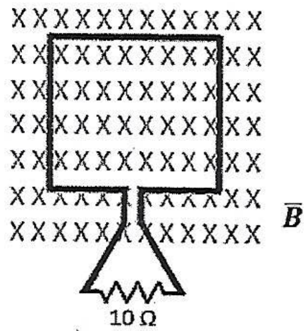
Any time both $t = 0 \rightarrow t = 0.25$ and $t = 0.75 \rightarrow t = 1$

will cause $\cos(2\pi t)$ to be positive (refer to 2b).

~~The ... emf will be negative~~

~~This~~ ~~m~~ By Lenz's law, since $\frac{d\Phi_B}{dt}$ is increasing, the loop will want to cause current that will create a magnetic field that will oppose the applied B. (ie opposite of into page, so out). By RHR on the loop, this means CCW.

Q2.d. Calculate the magnitude of the force on the top branch of the wire loop, if the loop is connected to a 10 Ω resistor and the wire loop is a square in shape with dimensions 3 cm x 3 cm. (4 marks)



\+ |emf| −

A) $F = 4 \times 10^{-7} sin(2\pi t)\, cos(2\pi t)$ N

B) $F = \frac{1}{9\pi} \times 10^{-4} sin(2\pi t)\, cos(2\pi t)$ N

C) $F = \frac{12}{\pi} \times 10^{-2}\, cos(2\pi t)\, cos(2\pi t)$ .N

D) $F = \frac{8}{3\pi} \times 10^{-7} sin(2\pi t)\, cos(2\pi t)$ N

E) None of the above

$|F| = BIL$.

$L = 0.03\,m$

$I = \frac{|emf|}{R} = \frac{4 \times 10^{-4} \cos(2\pi t)}{10\Omega} = 4 \times 10^{-5} \cos(2\pi t)$ A.

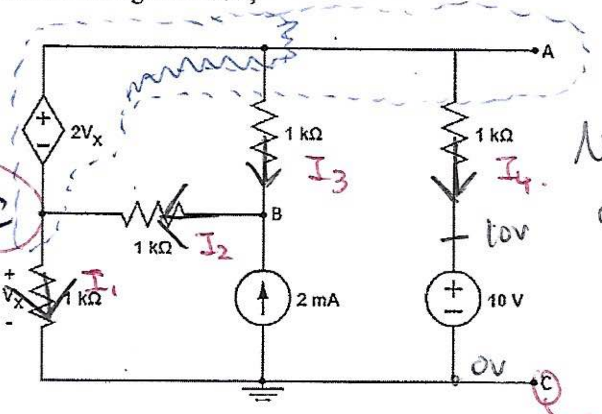
$B = \frac{2}{9\pi} \sin(2\pi t)$ Tesla.

$|F| = 3 \times 10^{-2}\,m \cdot 4 \times 10^{-5} \cos(2\pi t) \cdot \frac{2}{\frac{8\pi}{3}} \sin(2\pi t)$ ~~T~~ N

$= \frac{8}{3\pi} \times 10^{-7} \sin(2\pi t) \cos(2\pi t)$.

Q3 [10 marks] For the given circuit diagram below,

Dependent voltage source between C, A
⇒ create supernode



Note current directions. Set these to whatever you want.

C is somewhere different for this question, DISREGARD.

Q3.a. Using nodal analysis, find the voltage between nodes A and B. (5 marks)

$V_C = V_x$.

$V_{AB} =$ 2V.

Do analysis at B.

$-2mA - I_3 + I_2 = \phi \Rightarrow \cancel{V_B} - 2mA - \frac{V_A - V_B}{1k} + \frac{V_B - V_C}{1k} = \phi.$

$\Rightarrow -(V_A - V_B) + (V_B - V_C) = 2 \Rightarrow -V_A + 2V_B - V_C = 2.$ ①

Do analysis at supernode.

$I_1 + I_3 + I_4 - I_2 = \phi \Rightarrow \frac{V_C - 0}{1k} + \frac{V_A - V_B}{1k} + \frac{V_A - 10}{1k} - \frac{V_B - V_C}{1k} = \phi$

$\Rightarrow 2V_A - 2V_B + 2V_C = 10 \Rightarrow V_A - V_B + V_C = 5.$ ②

For dependent source, $V_A - V_C = 2V_x = 2V_C$.

$\Rightarrow V_A = 3V_C$ ③

Solve ①, ②, ③ $\Rightarrow V_A = 9V$

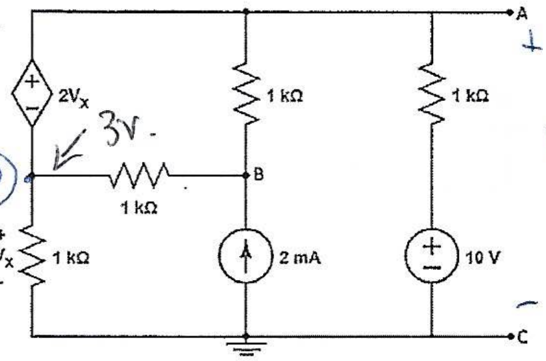
$V_B = 7V \Rightarrow V_{AB} = V_A - V_B = 2V.$

$V_C = 3V$

Used in 3b.

Q3 continued. (Circuit diagram has been duplicated for your convenience)

~~A~~ Thévenin equiv.. means ~~disconnect~~ ~~treat~~ treat Voc to be between A and C.



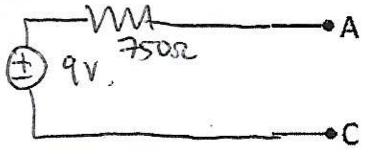
Note that C is ~~renamed~~ renamed to 0 in this part (refer to 3a.)

Q3.b. Find the Thevenin equivalent circuit between nodes A and C. (4 marks)

To find Thevenin equivalent, we need

$\underline{V_{oc} = V_{th}}$, $\underline{R_{th}}$. $R_{th} = \frac{V_{oc}}{I_{sc}}$.

Thevenin Equivalent Circuit:

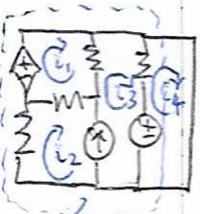


V_{oc} is just V_{AC} in this case. However we know from 3a that $V_D = 3V$, so

~~CAN~~ $V_x = 3 - 0 = 3V$, so $2V_x = 6$.

Therefore, to find V_A, we add $2V_x$ to V_D, so since $V_D = 3V$ and $2V_x = 6V$, $V_A = 9V$.

$V_C = \phi \Rightarrow V_{AC} = 9V = V_{oc}$. To find R_{th}, short circuit at V_{AC} and find I_{sc}. loop analysis



① $i_3 - i_2 = 2mA$. (current source).

$\boxed{V_x = -1k \cdot i_2.}$

$\Rightarrow$ loop 1: $-2(-1k \cdot i_2) + 1k(i_1 - i_3) + 1k(i_1 - i_2) = \phi$

② $\Rightarrow 2i_1 + i_2 - i_3 = \phi$.

$\Rightarrow$ Supermesh around first 3 loops.

$1k \cdot i_2 - 2(-1k \cdot i_2) + 1k(i_3 - i_4) + 10 = \phi$

$\Rightarrow 3i_2 + i_3 - i_4 = -10$. ③.

$\Rightarrow$ loop 4:

$-10 + (i_4 - i_3)1k = \phi \Rightarrow$ ④ $-i_3 + i_4 = 10$.

Solve 4 equations to get.

$i_1 = 1mA$
$i_2 = 0mA$
$i_3 = 2mA$
$i_4 = 12mA$

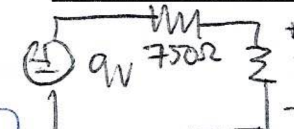
$i_4 = i_{sc} \Rightarrow R_{th} = \frac{V_{oc}}{i_{sc}} = \frac{9V}{12mA} = 750.$

Q3.c. If a resistive load is connected between A and C, what is the maximum power that can be transferred from this circuit? (1 mark)

Max power means:

$P_{max} = 27mW.$

$R_L = R_{th} \Rightarrow R_L = 750\Omega$.

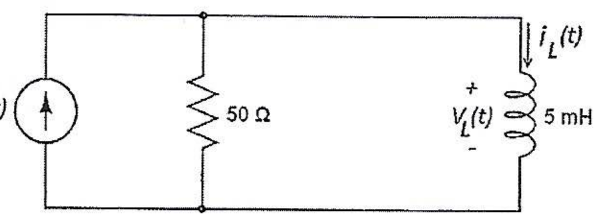


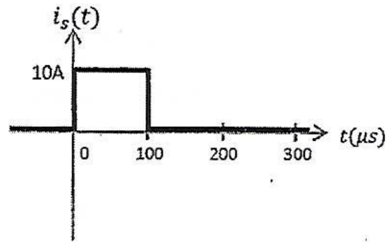
$\Rightarrow R_{tot} = 1500$

~~PMAX~~ $P = \frac{V^2}{R} = \cancel{\frac{(9V)^2}{1500\Omega}} = \frac{(4.5V)^2}{750\Omega} = \boxed{27mW}$

V across R_L is half so $V_{R_L} = \frac{9 \cdot 750}{750 + 750} = 4.$

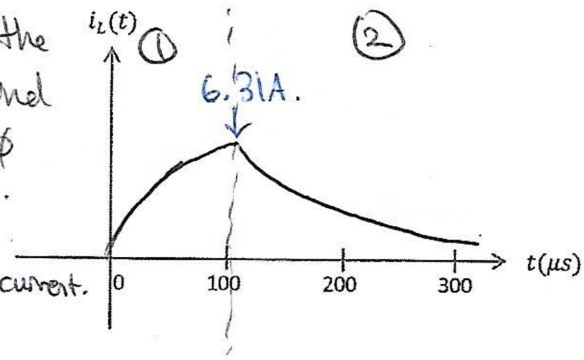
Q4 [10 marks] The parallel RL circuit is excited by the current source, $i_S(t)$ as shown. Assume that the initial energy stored in the inductor is zero.





Q4.a. Determine and sketch the inductor current, $i_L(t)$. Indicate the current values at $t = 0$ and at $t = 100\ \mu s$. (3 marks)

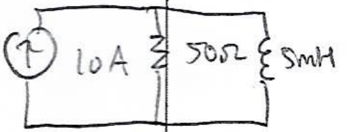
⇒ Separate into 2 sections: before the currents go to ~~10~~ 10 and after it drops to ϕ



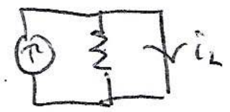
Part 1: $i(t) = k_1 + k_2 e^{-\frac{R}{L}t}$.

Initially uncharged, so no current.

$i_L(0^+) = i_L(0^-) = \phi$.



At $t = \infty$, becomes short circuit ⇒



So, $i_L = 10A$ because the resistor will be ignored by current.

$k_1 = 10$.

At $t = \phi$, $i_L = \phi$ so $i_L = k_1 + k_2 e^{-\frac{R}{L}(0)}$ ⇒ $k_2 = i(0) - k_1$

$= 0 - 10$

$k_2 = -10$.

⇒ $i_L = 10 - 10e^{-\frac{R}{L}t}$ $(R = 50\Omega,\ L = 5mH)$

$\boxed{i_L = 10 - 10e^{-10000t}}$ (draw graph to $100\mu s$).

~~from before 100~~ ⇒ Part 2. Treat $t = 100\mu s$ as starting point, so zero time. Once source disconnects, current dissipates exponentially. Draw on graph.

$i_L(\infty) = \phi$, ~~k~~ $i_L(100\mu s) = 6.31A$

⇒ $i_L = 6.31e^{-10000(t - 100\mu s)}$.

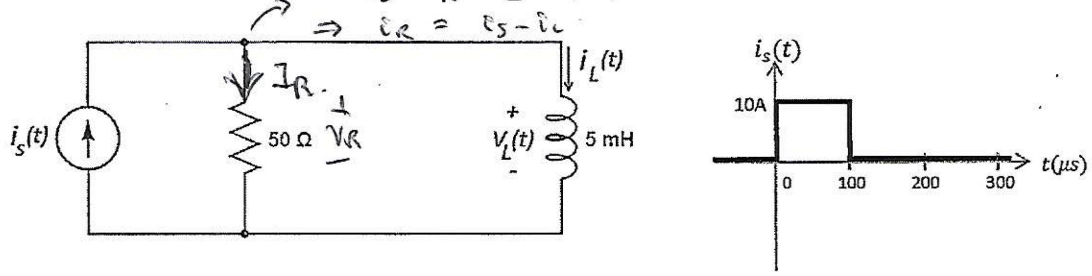
right translation.

don't forget the translation

Q4 continued. (Circuit diagram has been duplicated for your convenience)

$-i_S + i_R + i_L = 0$

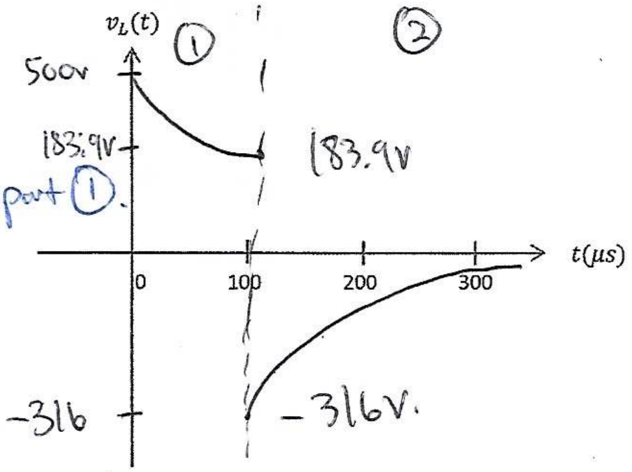
$\Rightarrow i_R = i_S - i_L$



Q4.b. Calculate the inductor voltages at $t = 0+$ and at $t = 100\ \mu s$, and sketch the inductor voltage, $v_L(t)$. (7 marks)

Separate into 2 parts as well.

(1): We know $V_L = V_R$, so



$V_R = 50\Omega \cdot I_R = V_L(t)$.

$= 50\Omega\,(i_S - i_L)$. Recall i_L for part (1).

$= 50(10 - (10 - 10e^{-10000t}))$

$= \cancel{500}\ 500e^{-10000t}$.

Draw on graph.

$V_L(t = 100\mu s) = 500e^{-10000(100\mu s)} = 183.9V$.

(2): Disconnect source.

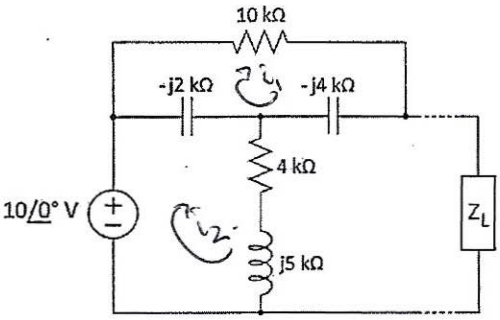
$\boxed{V_L = V_R = -50\Omega \cdot i_L}$ by loop analysis on this loop.

Recall i_L from part 2.

$\Rightarrow V_L = 50(6.31e^{-10000(t-100\mu s)})$.

$= 316e^{-10000(t-100\mu s)}$.

Q5 [10 marks] Given the following circuit.



Q5.a. Determine the open circuit voltage phasor (V_{OC}) for the Thevenin equivalent circuit as seen from the load (Z_L). (7 marks)

V_{oc} is voltage across $4k + j5k$ and $-j4k$.

V_{oc} = $\boxed{10.3 + j3.2\,V.}$

Use loop analysis to find currents.

LOOP 1: $-10 - j2k(i_2 - i_1) + 4k\,i_2 + j5k\,i_2 = \phi.$

$$\Rightarrow [2j]i_1 + [4 + j3]i_2 = 10$$

LOOP 2: $10k\,i_1 + (-j4k)i_1 - j2k(i_1 - i_2) = \phi.$

$$\Rightarrow [10 - j6]i_1 + [j2]i_2 = \phi$$

Solve currents.

$\Rightarrow \quad i_1 = \dfrac{-10j}{31+3j}, \quad i_2 = \dfrac{30+50j}{-3+31j},$ these are in mA.

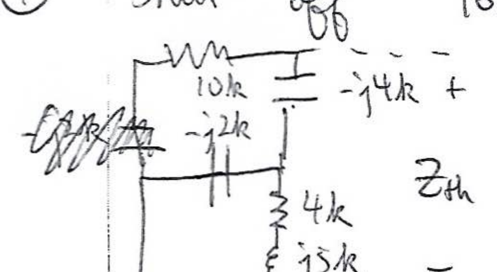
$V_{oc} = i_2[4k + j5k] + i_1[-j4k] = 10.3 + j3.2\,V.$

Q5.b. Determine the Thevenin equivalent impedance as seen from the load (Z_L). (3 marks)

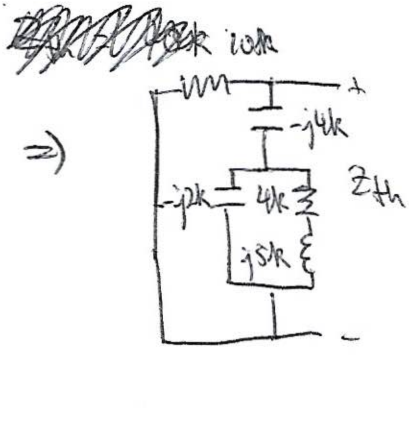
~~No dep~~ only independent source.

Z_{Th} = $\boxed{3.144 - j4.175\,k\Omega.}$

① Shut off 10∠0°.



⇒ ⇒



$\Rightarrow Z_{th} = 10k \,//\, [j4k + (4k + j5k)/$

$= \underline{3.144 - j4.175\,k\Omega}$