University of Toronto
Department of Mathematics

MAT186H1S
Calculus I

FINAL EXAMINATION
Monday, April 22, 2013
2:00 pm

Examiner: R. Burko

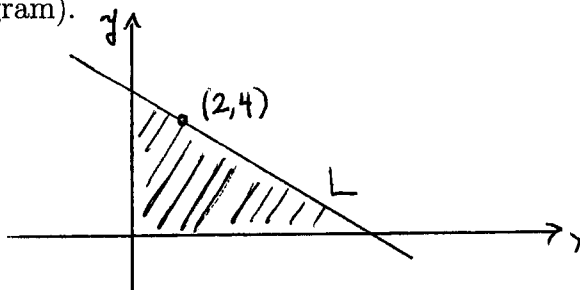Duration: 2 1/2 hours

Permitted Calculators: **Casio 260, Sharp 520, Texas Instrument 30**

Total: 100 marks

[12 marks] **1.** Evaluate the limits

$$\textbf{a)} \lim_{t \to +\infty} \frac{3e^{2t} + 4e^t}{5e^t - e^{-t} + 6e^{2t}} \qquad \textbf{b)} \lim_{y \to 0^+} \frac{1}{y} - \frac{1}{\sin \pi y} \qquad \textbf{c)} \lim_{x \to 1^+} (2 \ln x)^{3/\ln(x-1)}$$

[12 marks] **2.** Find the equation of the line L that passes through the point $(2, 4)$, has negative slope, and minimizes the area of the triangle enclosed by L and the coordinate axes (see diagram).

[8 marks] **3.**

$$\textbf{a) Find } f'(2) \text{ if } f(x) = x \int_{10}^{5x} \frac{1}{\sqrt{t^3 - t}} dt.$$

$$\textbf{b) Calculate } \int_{\sqrt{\pi/2}}^{\sqrt{\pi}} g'(t^2) 2t\, dt \text{ if } g(x) = \frac{\sin x}{x}.$$

[10 marks] **4.** Scarface (SF) and Diamond Jim (DJ) are standing at a point along the perimeter of a circular field of radius 1 km. SF begins running at 3 km/h through the diameter of the field. At the same moment, DJ begins running at 2π km/h counterclockwise along the edge of the field. How fast is the distance between the two runners changing after 10 minutes? [Hint: Use the triangle formula $C^2 = A^2 + B^2 - 2AB\cos\theta$]

[20 marks] **5.** Evaluate the following:

a) The total area between $y = x^2 - 2$ and the interval $[0, 3]$ on the x-axis.

b) The average value of $f(x) = e^{-2x+1}$ on the interval $[0, \ln 2]$.

c) $\displaystyle\int \frac{\ln x}{x(\ln x - 1)}dx$ d) $\displaystyle\int_0^1 \frac{x^2}{3 + x^6}dx$

[8 marks] **6.** Suppose f is an even function (i.e. $f(-x) = f(x)$) and integrable on the interval $[-3, 3]$. Suppose also that

$$\int_{-3}^{-2} f(x)dx = -3, \quad \int_0^2 f(x)^2dx = 4, \quad \text{and} \quad \int_0^3 f(x)^2dx = 5.$$

Calculate

a) $\displaystyle\int_2^3 f(x)(3 - f(x))dx$ b) $\displaystyle\int_{-3}^3 (f(x) + x)^2dx$

[14 marks] **7.** Sketch the graph of $f(x) = e^{x-2x^2}$. Label all $x-$intercepts, $y-$intercepts, critical points, vertical asymptotes, horizontal asymptotes, relative minima, relative maxima and inflection points.

[8 marks] **8.** For the curve $5y - 3xy + 2x = 0$, find the following:

a) The equation of the tangent line at the point (2,4).

b) $\dfrac{d^2y}{dx^2}$ at the point (1,1).

[8 marks] **9.** Suppose $F(x)$ is an antiderivative of $\dfrac{\sin\sqrt{x}}{\sqrt{x}}$ on $(0, \infty)$, $F(x) = \sqrt{x}g(x)$, and $G(x)$ is an antiderivative of $g(x)$ on $(0, \infty)$. If $F(\pi^2) = 1$ and $G(\pi^2) = 0$, find the function $G(x)$.