University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, DECEMBER, 2010
Duration: 2 and 1/2 hours
First Year - CHE, CIV, IND, LME, MEC, MMS

MAT186H1F - CALCULUS I
Exam Type: A

SURNAME: (as on your T-card) _____

GIVEN NAMES: _____

STUDENT NUMBER: _____

SIGNATURE: _____

Examiners:
D. Burbulla
Y. Burda
S. Cohen
B. Wang

QUESTION	MARK
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
Q8	
TOTAL	

Calculators Permitted: Casio 260, Sharp 520 or TI 30.

INSTRUCTIONS: Attempt all questions. Present your solutions in the space provided. Use the backs of the sheets if you need more space. Do not tear any pages from this exam. Make sure your exam contains 10 pages.

MARKS: Question 1 is worth 24 marks, 6 marks for each part.

Question 2 is worth 10 marks, 4 marks for part (a) and 6 marks for part (b).

Questions 3, 5 and 7 are each worth 10 marks.

Questions 4, 6 and 8 are each worth 12 marks.

TOTAL MARKS: 100

1. Find the following:

(a) $\int \left(\frac{1}{1+x^2} + 3\sin x + \frac{1}{x} \right) dx$

(b) the average value of $f(x) = e^{-2x}$ on the interval $[0, 4]$.

(c) the distance travelled by a particle for $0 \leq t \leq 4$ if its velocity at time t is given by $v = \sqrt{t} - 1$.

(d) $\lim_{x \to 1} (4x - 3)^{\tan(\pi x/2)}$

2. Find the following:

(a) $F'(2)$ if $F(x) = \int_2^{x^2} e^{\sqrt{t}}\, dt.$

(b) $\int_0^1 \frac{x}{\sqrt{4-3x^2}}\, dx.$

3. Let $f(x) = 4x^{1/3} - x^{4/3}$, for which you may assume

$$f''(x) = -\frac{4}{9}\frac{(2+x)}{x^{5/3}}.$$

Find the open intervals on which f is concave up and the open intervals on which f is concave down. Find all the inflection points of f, if any.

4. Consider $f(x) = \sin^2 x - \cos x$ on the open interval $(-\pi, 3\pi)$. Find the open intervals on which f is increasing and the open intervals on which f is decreasing. Find all the critical points of f in the interval $(-\pi, 3\pi)$ and determine if they are relative maximum or relative minimum points.

5. Find all the asymptotes to the graph of $f(x) = \dfrac{x^2}{x-1}$, and sketch its graph. You may assume that

$$f'(x) = \frac{x(x-2)}{(x-1)^2} \text{ and } f''(x) = \frac{2}{(x-1)^3}.$$

Label the asymptotes, the critical points, and the inflection points, if any.

6. An open box is to be made from a 1 m by 3 m rectangular piece of sheet metal by cutting out squares of equal size from the four corners and bending up the sides. Find the maximum volume that the box can have.

7. A conical water tank with vertex down has a radius of 3 m at the top and is 8 m high. If water flows into the tank at a rate of 1 m^3/min how fast is the depth of the water increasing when the water is 5 m deep? (The volume of a cone is given by $V = \pi r^2 h/3$.)

8. Let $f(x) = x^2 - 1$ for $1 \leq x \leq 2$. Find the following:

   (a) [6 marks] the volume of the solid generated by revolving the region between the curves $y = 3$ and $y = f(x)$ for $1 \leq x \leq 2$ about the y-axis.

   (b) [6 marks] the area of the surface generated by revolving the curve $y = f(x)$ about the y-axis for $1 \leq x \leq 2$.