University of Toronto, Faculty of Applied Science and Engineering

MAT187H1S – Calculus II – Final Exam - April 16, 2019

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MULTIPLE-CHOICE QUESTIONS

AND

FORMULA SHEET

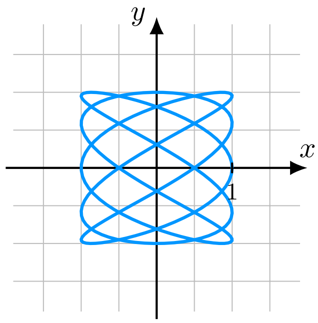
Instructions:

- DO NOT OPEN until instructed to do so.
- **THIS PART WILL NOT BE COLLECTED.**
- This part contains 6 pages.

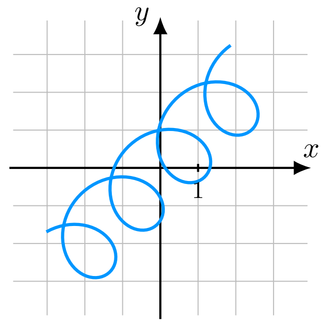
MULTIPLE-CHOICE PART. **(20 marks)**

ANSWER THESE QUESTIONS ON **PAGE 16** OF THE TEST.

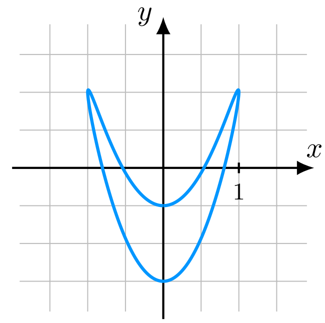
For questions **1.–4.**, consider the graphs:



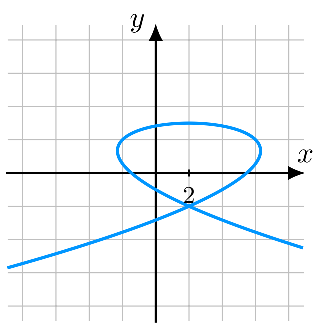
A



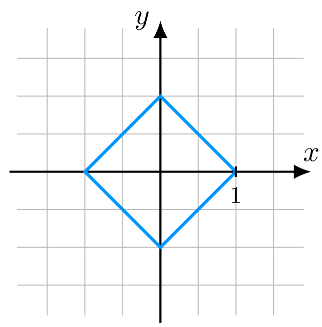
B



C



D



E

1. (2 marks) Which of the graphs **A–E** represents the following vector-valued function ?

$$\vec{r}_1(t) = \langle t + \sin(5t), t + \cos(5t) \rangle$$

2. (2 marks) Which of the graphs **A–E** represents the following vector-valued function ?

$$\vec{r}_2(t) = \langle \cos(5t), \sin(3t) \rangle$$

3. (2 marks) Which of the graphs **A–E** represents the following vector-valued function ?

$$\vec{r}_3(t) = \left\langle \sin(t), \frac{1}{2}\cos(t) - \cos(2t) \right\rangle$$

4. (2 marks) Which of the graphs **A–E** represents the following polar function ?

$$r = \frac{1}{|\cos(\theta)| + |\sin(\theta)|}$$

5. (2 marks) Consider the polar curve $r = 1 + \cos(3\theta)$. Select **ALL** of the following values of θ for which the curve passes through the origin.

(A) $\theta = 0$

(B) $\theta = \dfrac{\pi}{3}$

(C) $\theta = \dfrac{\pi}{2}$

(D) $\theta = \pi$

(E) $\theta = \dfrac{5\pi}{3}$

6. (2 marks) Consider the polar curve $r = 1 + \cos(3\theta)$.

What is the tangent line at $\theta = \dfrac{\pi}{3}$?

(A) $\theta = -\frac{\pi}{3}$

(B) $\theta = \dfrac{\pi}{3}$

(C) $\theta = \dfrac{\pi}{2}$

(D) $\theta = \pi$

(E) $\theta = \dfrac{5\pi}{3}$

7. (2 marks) The power series $\sum_{n=0}^{\infty} a_n(x-4)^n$ has radius of convergence $R = \pi$. What is the radius of convergence of

$$\sum_{n=0}^{\infty} a_n \left(\frac{x-4}{\pi}\right)^n \quad ?$$

(A) $R = 0$

(B) $R = \dfrac{1}{\pi}$

(C) $R = 1$

(D) $R = \pi^2$

(E) $R = \infty$

8. (2 marks) Calculate the limit

$$\lim_{x\to 0} \frac{e^x + \cos(x) + \sin(x) - \frac{2}{1-x} + 2x^2 + 2x^3 + 2\left(1 - \frac{1}{4!}\right)x^4 + 2\left(1 - \frac{1}{5!}\right)x^5}{x^6}.$$

(A) $-2 - \dfrac{2}{6!}$

(B) $-\dfrac{2}{6!}$

(C) -2

(D) 0

(E) The limit does not exist.

9. (2 marks) Consider two particles with positions

$$\vec{r}_1(t) = \langle 3\sin(t), 2\cos(t)\rangle$$
$$\vec{r}_2(t) = \langle -3+\cos(t), 1+\sin(t)\rangle$$

for $0 \leqslant t \leqslant 2\pi$.

How many times do their paths intersect?

(A) 0 times.
(B) 1 time.
(C) 2 times.
(D) 3 times.
(E) 6 times.

10. (2 marks) Consider the same particles with positions as in the previous question

$$\vec{r}_1(t) = \langle 3\sin(t), 2\cos(t)\rangle$$
$$\vec{r}_2(t) = \langle -3+\cos(t), 1+\sin(t)\rangle$$

for $0 \leqslant t \leqslant 2\pi$.

How many times do these particles collide?

(A) 0 times.
(B) 1 time.
(C) 2 times.
(D) 3 times.
(E) 4 times.

FORMULA SHEET FOR MAT187

Important Formulas.

- $\cos^2(x) = \dfrac{1+\cos(2x)}{2}$
- $\sin^2(x) = \dfrac{1-\cos(2x)}{2}$
- $\sin(x) = \cos\left(\dfrac{\pi}{2} - x\right)$
- $\lim\limits_{n\to\infty} \left(1+\dfrac{a}{n}\right)^n = e^a$

Trigonometric Integrals.

- $\displaystyle\int \sec(x) = \ln\big(\sec(x)+\tan(x)\big) + C$
- $\displaystyle\int \sec^3\, dx = \int \frac{\cos(x)}{\big(1-\sin^2(x)\big)^2}\, dx$
- $\displaystyle\int \frac{1}{1+x^2}\, dx = \arctan(x) + C$

Applications of integration.

- Arc length for $y = f(x)$ $\qquad \displaystyle\int_a^b \sqrt{1+\big(f'(x)\big)^2}\, dx$
- Area of a surface of revolution $y = f(x)$ revolved around $x-$axis $\qquad \displaystyle\int_a^b 2\pi f(x)\sqrt{1+\big(f'(x)\big)^2}\, dx$

Numerical Integration.

- Left-Hand Rule $\quad \displaystyle\int_a^b f(x)\, dx \approx L_n = \sum_{i=1}^{n} f(x_{i-1})\Delta x, \qquad |E_L| \leqslant \frac{b-a}{2}(\Delta x) \max_{x\in[a,b]} \big|f'(x)\big|$
- Right-Hand Rule $\quad \displaystyle\int_a^b f(x)\, dx \approx R_n = \sum_{i=1}^{n} f(x_i)\Delta x, \qquad |E_R| \leqslant \frac{b-a}{2}(\Delta x) \max_{x\in[a,b]} \big|f'(x)\big|$
- Midpoint Rule $\quad \displaystyle\int_a^b f(x)\, dx \approx M_n = \sum_{i=1}^{n} f\left(\frac{x_{i-1}+x_i}{2}\right)\Delta x, \qquad |E_M| \leqslant \frac{b-a}{24}(\Delta x)^2 \max_{x\in[a,b]} \big|f''(x)\big|$
- Trapezoid Rule $\quad \displaystyle\int_a^b f(x)\, dx \approx T_n = \sum_{i=1}^{n} \frac{f(x_{i-1})+f(x_i)}{2}\Delta x, \qquad |E_T| \leqslant \frac{b-a}{12}(\Delta x)^2 \max_{x\in[a,b]} \big|f''(x)\big|$

Differential Equations.

- Linear DE: $y' + p(t)y = g(t)$ $\qquad \mu(t) = e^{\int p(t)\, dt}$

$$y = \frac{1}{\mu(t)} \int \mu(t)g(t)\, dt + \frac{C}{\mu(t)}$$

- Separable DE: $g(y)\dfrac{dy}{dt} = h(t)$ $\qquad \displaystyle\int g(y)\, dy = \int h(t)\, dt$

Power Series.

- Taylor Theorem $$\begin{cases} f(x) = p_n(x) + R_n(x) \\ p_n(x) = \displaystyle\sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k \\ R_n(x) = \dfrac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1} \end{cases}$$
- Geometric Series (for $-1 < x < 1$) $$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$
- Exponential Series $$e^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k$$
- Binomial Series (for $-1 < x < 1$) $$(1+x)^p = \sum_{k=0}^{\infty} \frac{p(p-1)\cdots(p-k+1)}{k!} x^k$$
- Sine Series $$\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}$$
- Cosine Series $$\cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}$$
- Logarithmic Series (for $-1 \leqslant x < 1$) $$\ln(1-x) = -\sum_{k=1}^{\infty} \frac{1}{k} x^k$$

Vector-Valued Functions.

- Area inside a polar function $r = f(\theta)$ $$\frac{1}{2}\int_{\alpha}^{\beta} \big(f(\theta)\big)^2 \, d\theta$$
- Length of a parametric curve $\vec{r}(t)$ $$\int_a^b \big|\vec{r}\,'(t)\big| \, dt$$
- Length of a polar curve $r = f(\theta)$ $$\int_{\alpha}^{\beta} \sqrt{\big(f(\theta)\big)^2 + \big(f'(\theta)\big)^2} \, d\theta$$
- Unit tangent and normal vectors $$\vec{T}(t) = \frac{\vec{r}\,'(t)}{\big|\vec{r}\,'(t)\big|} \quad , \quad \vec{N}(t) = \frac{\vec{T}'(t)}{\big|\vec{T}'(t)\big|}$$
- Tangential and normal components of acceleration $$a_T = \tfrac{d\,|\vec{r}\,'(t)|}{dt} \quad , \quad a_N = \kappa |\vec{r}\,'(t)|^2$$
- Curvature $$\kappa = \left|\frac{d\vec{T}}{ds}\right| = \frac{\big|\vec{T}'(t)\big|}{\big|\vec{r}\,'(t)\big|} = \frac{\big|\vec{v} \times \vec{a}\big|}{\big|\vec{v}\big|^3}$$