# University of Toronto
# Faculty of Applied Science and Engineering
# Department of Electrical and Computer Engineering

## Final Examination- April 27, 2010

First Year – Computer, Electrical, Industrial, Mechanical, Materials, and Track-One (General) Engineering Program

## ECE110S – Electrical Fundamentals

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## Calculator Type: 3

## Exam Type: A

**ANSWER ALL QUESTIONS, USING THE BACK SIDE IF NECESSARY.**

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Notes:
1. Only non-programmable calculators are allowed.
2. For full marks, you must show methods, state UNITS, and compute numerical answers when requested.
3. To avoid ambiguity, write your final answers in the answer boxes provided.
4. There is one extra blank page at the end for rough work.

**(Useful Constants: $e = -1.6 \times 10^{-19}$ C, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $g = 9.81$ N/kg)**

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**Family Name:** ______________________________

**Given Names:** ______________________________

**Student Number:** ______________________________

| Question | Mark |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| **TOTAL** | |

**Question 1: [10 marks]**

Insert the *correct answer* in the *box* associated with each of the following:

(a) **(2 marks)** Gauss's law $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$ implies that:

(i) $\vec{E}$ must be the electric field due to the enclosed charge.
(ii) If $q = 0$, then $\vec{E} = 0$ everywhere on the Gaussian surface.
(iii) If the three particles enclosed by the Gaussian surface have charges of $+q$, $+q$, and $-2q$, then the integral is zero.
(iv) On the surface $\vec{E}$ is everywhere parallel to $d\vec{A}$
(v) If a charge is placed outside the Gaussian surface, then it cannot affect $\vec{E}$ at any point on the surface.

(b) **(2 marks)** Choose the correct statement:

(i) A proton tends to go from a region of low electrostatic potential to a region of high electrostatic potential.
(ii) The electrostatic potential of a negatively charged conductor must be positive.
(iii) If $\vec{E}$ is zero at a point P, then V must be zero at P.
(iv) If V = 0 at a point P, then $\vec{E}$ must be zero at P.
(v) None of the above is correct.

(c) **(2 marks)** Capacitors $C_1$ and $C_2$ are connected in parallel and a potential difference is applied to the combination. If the capacitor that is equivalent to the combination has the same potential difference, then the charge on the equivalent capacitor is the same as:

(i) The charge on $C_1$.
(ii) The sum of the charges on $C_1$ and $C_2$.
(iii) The difference of the charges on $C_1$ and $C_2$.
(iv) The product of the charges on $C_1$ and $C_2$.
(v) None of the above.

(d) **(2 marks)** Two parallel long wires carry the same current and repel each other with a force F per unit length. If both these currents are doubled and the wire separation tripled, the force per unit length becomes:

(i) 2F/9
(ii) 4F/9
(iii) 2F/3
(iv) 4F/3
(v) 6F

(e) **(2 marks)** A rectangular loop of wire is placed midway between two long straight parallel conductors as shown in Figure 1e. The reference direction for currents $i_1$ and $i_2$ are as indicated in Fig. 1e. If $i_1$ is increasing and $i_2$ is constant, then the induced current in the loop is:

(i) Zero.
(ii) Clockwise.
(iii) Counterclockwise.
(iv) Depends on $i_1 - i_2$.
(v) Depends on $i_1 + i_2$.

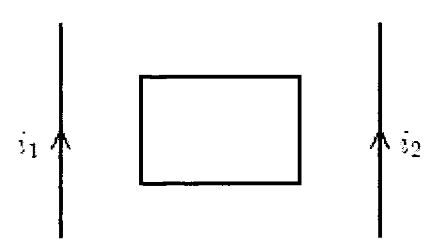


Figure 1e

**Question 2: [10 marks]**

A long co-axial cable consists of a solid inner conductor of radius $R_1 = 1$ mm that is concentric with a thin cylindrical metallic shell of radius $R_2 = 10$ mm as indicated in Figure 2. On a one-meter section of the cable, the charge on the inner conductor is $+2 \times 10^{-10}$ C, and the charge on the outer shell is $-5 \times 10^{-10}$ C.

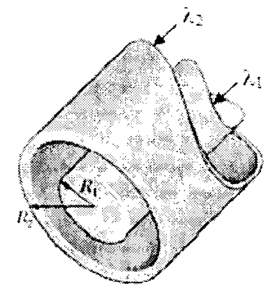

Figure 2

(a) **(1 mark)** State the surface charge density existing on the OUTER surface of the inner conductor.

| |
|---|
| Surface charge density: |
| on inner conductor = |

(b) **(2 marks)** State the surface charge densities existing on both the INNER and the OUTER surfaces of the outer cylindrical shell.

| |
|---|
| Surface charge densities on the outer shell: |
| on inner surface = |
| on outer surface = |

(c) **(3 marks)** Use Gauss's law to derive expressions for the electric field vector $\vec{E}(r)$ in all three regions: $r < R_1$; $R_1 < r < R_2$; and $r > R_2$.

| |
|---|
| For $r < R_1$, $\vec{E}(r) =$ |
| For $R_1 < r < R_2$, $\vec{E}(r) =$ |
| For $r > R_2$, $\vec{E}(r) =$ |

(d) **(4 marks)** Calculate the work done by the electric field to move a proton from the outer shell to the inner conductor.

Hint: $\int_b^a \frac{dr}{r} = \ln\left(\frac{a}{b}\right)$ for $a$>0 and $b$>0

| |
|---|
| Work = |

**Question 3: [10 marks]**

A large field coil is driven by a sinusoidal voltage of 100 kHz with an amplitude of 5V, and can be treated as an ideal solenoid. A small test coil is inserted into the center of the field coil and has negligible effect on the field coil. An induced *emf* is measured on the test coil.

The **field coil** parameters are: Coil radius $r_f$ = 4.0 cm; Coil length $l_f$ = 1.0 m; Total number of turns $N_f$ = 500; Radius of the copper wire, $r$ = 1.0 mm; Resistivity of copper at room temperature, $\rho = 1.72\times10^{-8}\ \Omega\text{m}$.

The **test coil** parameters are: Coil radius $r_t$ = 1.3 cm; Coil length $l_t$ = 16 cm; Total turns $N_t$ =160.

Hint: Some of the useful equations are: $L \equiv N\phi/i$; $B = \mu_o n\, i$; $R = \rho\, l/A$

Answer the following questions:

(a) **(2 marks)** Find $L_f$, the inductance of the field coil.

$L_f$ =

(b) **(2 marks)** Find $R_f$, the resistance of the field coil.

$R_f$ =

(c) **(2 marks)** If the field coil is modeled by an inductor in series with a resistor, with the inductance and resistance values found in part (a) and (b), find the phasor current **I** in Figure 3c.

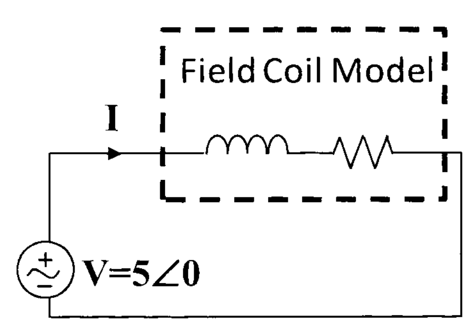


Figure 3c

**I** =

(d) **(2 marks)** Find the time-domain expression for the magnetic field *B(t)* inside the field coil.

| |
|---|
| $B(t) =$ |

(e) **(2 marks)** Find the time-domain expression for the induced *emf* $\mathscr{E}(t)$ on the test coil.

| |
|---|
| Induced *emf* on test coil $\mathscr{E}(t) =$ |

## Question 4: [10 marks]

**Part A**

**(4 marks)** Determine $V_o$ for the circuit given in Figure 4a.

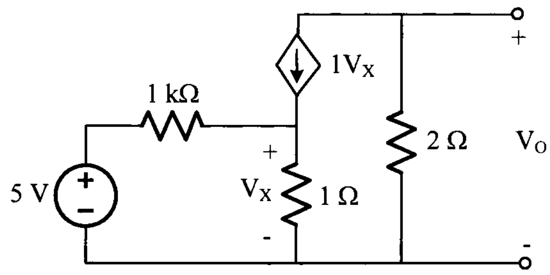


Figure 4a

| $V_o =$ |
|---|
| |

**Part B**
From the circuit given in Figure 4b, determine the following:

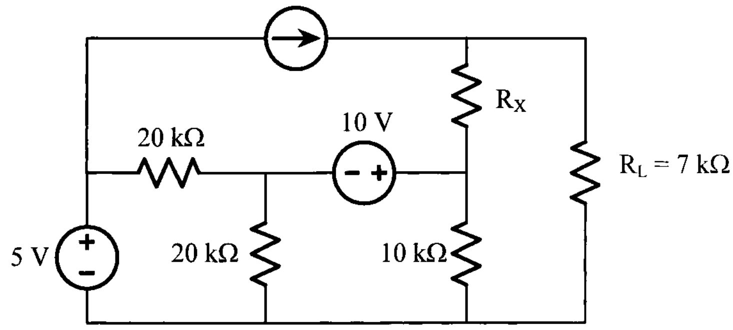


Figure 4b

| |
|---|
| (a) $R_X$ = |
| (b) $P_{max}$ = |

(a) **(2 marks)** The value of $R_X$ for maximum power transfer to the load, $R_L$.

(b) **(4 marks)** The maximum power $P_{max}$ absorbed by the load, $R_L$.

**Question 5: [10 marks]**
Assume the energy stored in each capacitor is zero and both switches are in positions as indicated on Figure 5. At $t = 0$, the switch on the left is closed. Once the capacitor $C_1$ is fully charged (at $t = t_1$), the switch on the right moves from 1 to 2.

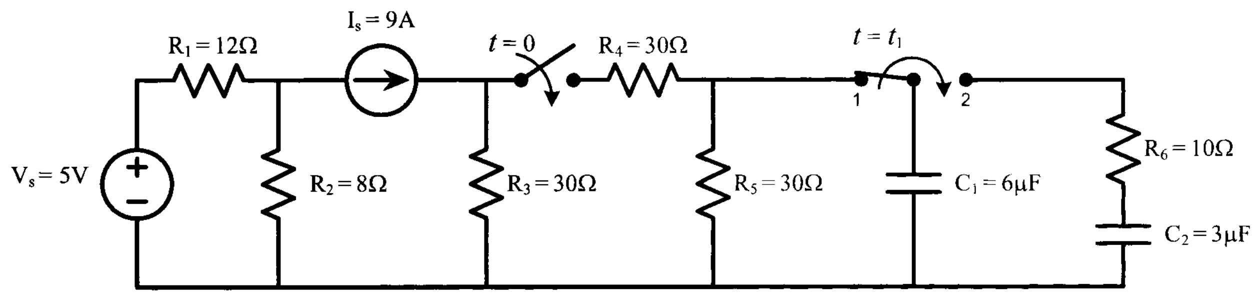


Figure 5

(a) **(3 marks)** Calculate the time constant $\tau_1$ of charging $C_1$ for $0 < t < t_1$.

$\tau_1 =$

(b) **(2 marks)** Calculate the voltage across the fully charged $C_1$.

$V_{C1}(t_1) =$

(c) **(2 marks)** Find the maximum current at capacitor $C_2$

$I_{max\_C2} =$

(d) **(1 mark)** Determine the charge of $C_2$ at $t = t_1$ and at $t = \infty$.

| |
|---|
| $Q_{C2}(t_1) =$ |
| $Q_{C2}(\infty) =$ |

(e) **(2 marks)** Determine the total energy stored in both capacitors at $t = t_1$ and at $t = \infty$.

| |
|---|
| $U_{total}(t_1) =$ |
| $U_{total}(\infty) =$ |

(f) **(1 mark)** In (e), why is the total stored energy different for $t = t_1$ and $t = \infty$?

| |
|---|
| The difference in stored energy is due to: |

**Question 6: [10 marks]**

Figure 6 shows an AC circuit with excitation $i(t) = 4\sqrt{2}\cos\left(1 \times 10^8\, t - \frac{\pi}{4}\right)$ A.

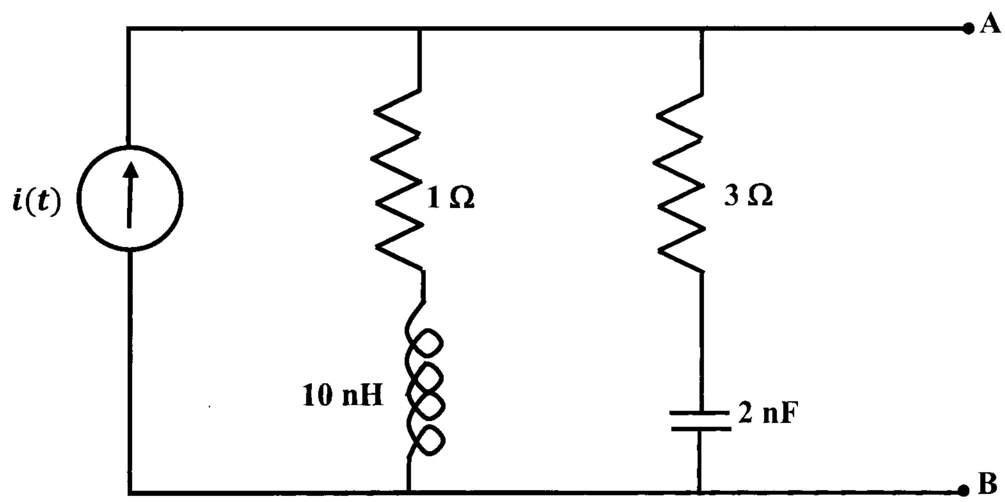


Figure 6

(a) **(3 marks)** Redraw the circuit in frequency domain, indicating all relevant phasor and impedance values.

(b) **(3 marks)** Find the average power dissipated by the 3Ω resistor.

$P_{3\Omega} =$

(c) **(1 mark)** Find the average power dissipated by the inductor.

$P_{10nH} =$

(d) **(3 marks)** What should be the value of a load impedance to be connected across terminal A-B for maximum average power transfer?

$\mathbf{Z}_{AB} =$

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