



UNIVERSITY OF TORONTO  
FACULTY OF APPLIED SCIENCE AND ENGINEERING  
FINAL EXAMINATION, DECEMBER 2015  
DURATION: 2 AND 1/2 HRS

FIRST YEAR - CHE, CIV, CPE, ELE, ENG, IND, LME, MEC, MMS  
**MAT186H1F - Calculus I**

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Exam Type: A.

Aids permitted: Casio FX-991 or Sharp EL-520 calculator.

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**Instructions:**

- DO NOT WRITE ON THE QR CODE AT THE TOP OF THE PAGES.
- This exam contains 12 pages, including this cover page, printed two-sided. Make sure you have all of them. Do not tear any pages from this exam.
- This exam consists of eight questions, some with two parts. Attempt all of them. Each question is worth 10 marks. Marks for parts of a question are indicated in the question. **Total Marks: 80**
- PRESENT YOUR SOLUTIONS IN THE SPACE PROVIDED. You can use pages 10, 11 and 12 for rough work. If you want anything on pages 10, 11 or 12 to be marked you must indicate in the relevant previous question that the solution continues on page 10, 11 or 12.



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1. Find the following:

(a) [5 marks]  $\int_0^4 x \sqrt{16 - x^2} dx$

(b) [5 marks]  $F'(2)$ , if  $F(x) = \int_{-x}^x \sqrt{32 + t^2} dt$



2. Let  $v = t^2 - 4t + 3$  be the velocity of a particle at time  $t$ , for  $0 \leq t \leq 3$ . Find:

(a) [4 marks] the average velocity of the particle.

(b) [6 marks] the average speed of the particle.



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3. The region bounded by the curves  $y = 3x$  and  $y = x^2$  is revolved about the  $x$ -axis. Find the volume of the solid that is generated.



4. Let  $A$  be the area of the region between by  $y = \pi/2$  and  $y = \sin^{-1} x$  for  $-1 \leq x \leq 1$ .

- (a) [6 marks] Express the value of  $A$  in terms of one or more integrals with respect to  $x$  and in terms of one or more integrals with respect to  $y$ . (Draw a diagram!)

- (b) [4 marks] Find  $A$ .



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5. Let  $r > 0$ . Let  $V$  be the volume of the solid of revolution generated by revolving the region in the  $xy$ -plane bounded by  $y = \frac{1}{1+x^2}$ ,  $y = 0$ ,  $x = r$  and  $x = r + 1$ , about the  $y$ -axis.

(a) [4 marks] Write down the definite integral that gives the value of  $V$ .

(b) [6 marks] Which value of  $r$  will maximize the value of  $V$ ?



6. A water tank is shaped like an inverted cone with height 6 m and radius 1.5 m at the top. If the tank is full, how much work is required to pump the water to the level of the top of the tank, and out of the tank? (Assume the density of water is  $\rho = 1000 \text{ kg/m}^3$  and that  $g = 9.8 \text{ m/sec}^2$ .)



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7. Consider the curve  $y = \frac{2}{3} x^{3/2}$ , for  $0 \leq x \leq 8$ .

(a) [5 marks] Find the length of the curve.

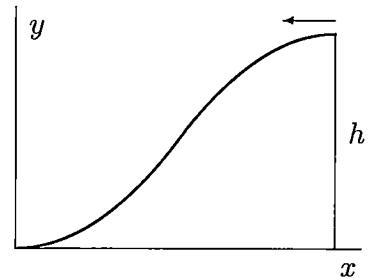
(b) [5 marks] Find the area of the surface generated by revolving the curve about the  $y$ -axis.



8. Suppose that when an airplane approaches an airport for landing its path must satisfy the following five conditions:

- (i) Altitude must be  $y = h$  meters when descent begins.
- (ii) Smooth touchdown occurs at  $x = 0$ .
- (iii) Constant horizontal speed  $u$  m/sec must be maintained throughout.
- (iv) At no time must the vertical acceleration in absolute value exceed a fixed positive constant  $k$ .
- (v) The path must be a cubic polynomial with zero slope at the beginning and end of the descent.

How far from the landing strip must the descent begin? Your answer will be in terms of  $u, h$  and  $k$ .





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