University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER, 2007

First Year - CHE, CIV, IND, LME, MEC, MMS

MAT 186H1F - CALCULUS I

Exam Type: A

SURNAME: ______________________

GIVEN NAMES: ______________________

STUDENT NUMBER: ______________________

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Calculators Permitted: Casio 260, Sharp 520 or TI 30.

INSTRUCTIONS: Attempt all questions. Use the backs of the sheets if you need more space. Do not tear any pages from this exam. Make sure your exam contains 11 pages.

MARKS: Questions 1 through 6 are Multiple Choice; circle the single correct choice for each question. Each correct choice is worth 4 marks.

Questions 7 through 10 each have two parts and are worth 10 marks; 5 marks for each part.

Questions 11 through 14 are each worth 9 marks.

TOTAL MARKS: 100

1. What is the equation of the tangent line to the graph of $f(x) = e^x$ at the point $(x, y) = (0, 1)$?

   (a) $y = x$

   (b) $y = x + 1$

   (c) $y = x - 1$

   (d) $y = x + e$

2. The arc length of the curve $f(x) = 2\cos x$ for $0 \leq x \leq \pi/2$ is given by

   (a) $\displaystyle\int_0^{\pi/2} \sqrt{1 - \sin^2 x}\, dx$

   (b) $\displaystyle\int_0^{\pi/2} \sqrt{1 + \sin^2 x}\, dx$

   (c) $\displaystyle\int_0^{\pi/2} \sqrt{1 - 4\sin^2 x}\, dx$

   (d) $\displaystyle\int_0^{\pi/2} \sqrt{1 + 4\sin^2 x}\, dx$

3. How many asymptotes – horizontal, vertical, or slant – are there to the graph of

$$y = x + 2 - \frac{2x - 2}{x^2 + 2x - 3}?$$

   (a) none

   (b) one

   (c) two

   (d) three

4. If $\sin\theta = \dfrac{1}{4}$ and $\cos\theta < 0$, then the exact value of $\cos(2\theta)$ is

(a) $\dfrac{\sqrt{15}}{8}$

(b) $\dfrac{7}{8}$

(c) $-\dfrac{\sqrt{15}}{8}$

(d) $-\dfrac{7}{8}$

5. $\displaystyle\int_1^{e^3} \frac{(4+\ln x)^2}{x}\,dx =$

(a) $\displaystyle\int_1^{e^3} u^2\,du$

(b) $\displaystyle\int_1^{3} u^2\,du$

(c) $\displaystyle\int_4^{7} u^2\,du$

(d) $\displaystyle\int_1^{7} u^2\,du$

6. $\displaystyle\lim_{x\to 0} \frac{e^x - x - 1}{x^2} =$

(a) 0

(b) $-\dfrac{1}{2}$

(c) $\dfrac{1}{2}$

(d) ∞

7. Find the following limits:

(a) $\lim_{x \to \infty} x \left(\frac{\pi}{2} - \tan^{-1} x \right)$.

(b) $\lim_{x \to 0^+} (1 + \sin^{-1}(3x))^{\csc(2x)}$

8. Find the following:

(a) $F'(\pi/2)$, if $F(x) = \int_0^{\cos x} e^{t^2}\, dt$.

(b) $\int_0^1 \frac{x+1}{x^2+1}\, dx$.

9. Suppose the velocity of a particle at time t is given by $v = 2t - t^2$, for $0 \leq t \leq 3$. Find the following:

(a) the average velocity of the particle for $0 \leq t \leq 3$.

(b) the average speed of the particle for $0 \leq t \leq 3$.

10. Let R be the region in the plane bounded by $x = 0, x = 2, y = 2x$ and $y = x^2$. Find the following:

(a) The volume of the solid obtained by revolving the region R about the x-axis.

(b) The volume of the solid obtained by revolving the region R about the line $x = -1$.

11. A storage tank, full of water with density ρ, is in the shape of an inverted cone, with radius at the top 2 m, and with perpendicular height 6 m. How much work is done in emptying the tank by pumping all the water up to a transfer pipe 1 m above the top of the tank?

12. One end of a rope is attached to the front of a boat, 1 m above sea level. The other end of the rope is pulled at a constant rate of 0.5 m/s from the top of a wharf 2 m above sea level, causing the boat to move toward the wharf. At what speed is the boat approaching the wharf when the horizontal distance between the front of the boat and the wharf is 5 m?

13. Let $f(x) = 3(x-2)^{1/3} - x$. Plot the graph of $y = f(x)$ on the interval $[0,4]$, indicating absolute minimum and maximum points, all critical points, and all inflection points, if any.

14. A school playing field is to have the shape of a rectangle with a semicircle attached at each of two opposite ends. The perimeter of the field is to be a 400 m running track. Find the dimensions of the field that

    (a) maximize the total area of the field.

    (b) maximize the area of the rectangular part.