2016 Final Q1 Soln

1. a) $T_1 + V_1 = T_2 + V_2$. 2

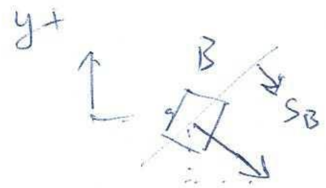
Initially stationary $\Rightarrow T_1 = 0$

Set initial positions of B, D datum $\Rightarrow V_1 = 0$

$\Rightarrow T_2 + V_2 = 0$

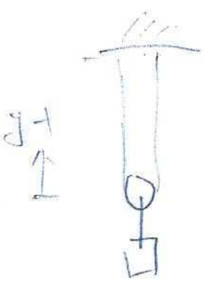
$(T_D + T_B)_2 + (V_D + V_B)_2 = 0$ 2

$\left(\frac{1}{2} V_{D_2}^2 m_D + \frac{1}{2} m_B V_{B_2}^2\right) + \left(m_D g\, y_{D_2} + m_B g\, y_{B_2}\right) = 0$



$y_{B_2} = -S_B \cdot \sin 30^\circ = -\frac{1}{2} S_B = -\frac{1}{2} \cdot 10 = -5$ 1

$y_{D_2} = \frac{1}{2} S_B = \frac{1}{2} \cdot 10 = 5$



$|\vec{V}_{B_2}| = 2|\vec{V}_{D_2}|$ $\quad V_{B_2} = 2V_{D_2}$ 1

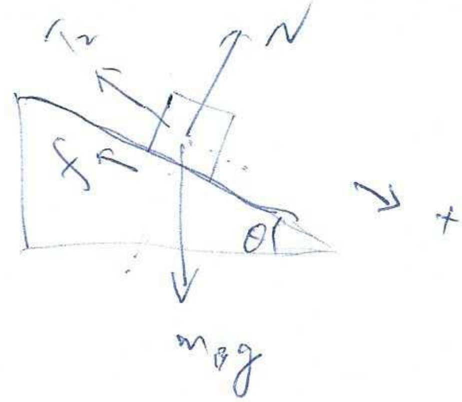
$\Rightarrow \frac{1}{2} \cdot 4 \cdot V_{D_2}^2 + \frac{1}{2} \cdot 20 \cdot 4V_{D_2}^2 + 4 \cdot g \cdot 5 + 20 \cdot g \cdot (-5) = 0$

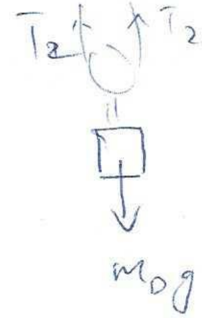
$\Rightarrow V_{B_2} = 8.645$ 1

$\vec{V}_{B_2} = V_{B_2} \cdot \cos 30^\circ \vec{i} - V_{B_2} \cdot \sin 30^\circ \vec{j}$

$= 7.487\vec{i} - 4.32\vec{j}$ (m/s) 1

1 b).





$\uparrow +$

$N = m_B g \cdot \cos 30°$ $\qquad$ $f_{s\,max} = \mu_s N = 0.4 \cdot 20 \cdot g \cdot \cos 30° = 4\sqrt{3}\, g$. 0.5

$f_K = \mu_K N = 0.2 \cdot 20 \cdot g \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}\, g$ 0.5

$$\left\{\begin{array}{l} (m_B g \cdot \sin 30°) - (T_2 + f) = m_B a_B \quad 1 \\ 2T_2 - m_D g = m_D a_D \quad 1 \\ a_B = 2 a_D \quad 1 \end{array}\right.$$

if stationary : $a_B = a_D = 0$.

$$\Rightarrow \left.\begin{array}{l} m_B g \cdot \frac{1}{2} - (T_2 + f) = 0 \\ 2T_2 = m_D (\cancel{a_D +} g) \end{array}\right\} \quad 1 \Rightarrow$$

$\Rightarrow \underline{f = 8g > 4\sqrt{3}\, g}$ 1 $\Rightarrow$ not stationary $\Rightarrow f = f_k$

$$\Rightarrow \left\{\begin{array}{l} m_B g \cdot \sin 30° - (T_2 + 2\sqrt{3}\, g) = m_B a_B = m_B 2 a_D \\ 2T_2 = m_D (a_D + g) \end{array}\right.$$

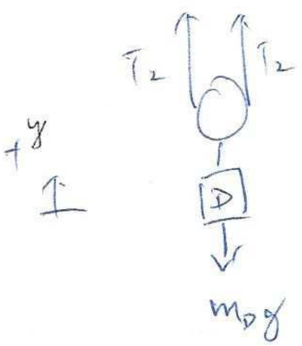
$\Rightarrow a_B = 2.119 \ m/s^2$ 1 $\Rightarrow \vec{a_B} = 2.119 \cdot \frac{\sqrt{3}}{2} \vec{i} - 2.119 \cdot \frac{1}{2} \vec{j}$ 1

$= 1.835 \vec{i} - 1.059 \vec{j} \ m/s^2$

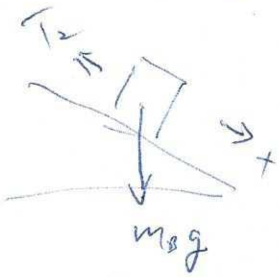
1 c)

$$\vec{L}_1 + \underbrace{\int_0^{0.3} \sum \vec{F} \, dt}_{1} = \vec{L}_2 .$$

$$\sum F = 2T_2 - m_D g \qquad 0.5$$



$$\left.\begin{cases} 2T_2 - m_D g = m_D a_D & (1) \\ m_B g \cdot \sin 30^\circ - T_2 = m_B \cdot a_B & (2) \\ a_B = 2 a_D \end{cases}\right] \; 1$$



$$\left.\begin{aligned} (1) &\Rightarrow a_D = \frac{2T_2 - m_D g}{m_D} \\ (2) &\Rightarrow a_D = \frac{\frac{1}{2} m_B g - T_2}{2 m_B} \end{aligned}\right\} \Rightarrow$$

$$\frac{2T_2 - m_D g}{m_D} = \frac{\frac{1}{2} m_B g - T_2}{2 m_B} \Rightarrow T_2 = \frac{5}{2.1} g = 23.36 \qquad 0.5$$

$$\Rightarrow \sum F = 2 \cdot \frac{5}{2.1} g - 4g = \frac{1.6}{2.1} g = \text{const.} \qquad 0.5$$

$$\Rightarrow \int \sum \vec{F} \, dt = \sum F \cdot \Delta t \, \vec{j} = \frac{1.6}{2.1} g \cdot 0.3 \, \vec{j} = \frac{1.6}{7} g = 2.242 \, \vec{j} \; (N.s)$$

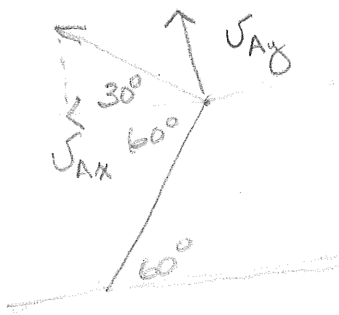
0.5

MIE100 - 2016 - Final exam answers #2

a) $r_A \omega_o = .08 * 10 = .8$

(5 marks)

$\Rightarrow \vec{v}_A = -.8\cos 30^\circ \hat{i} + .8 \sin 30^\circ \hat{j}$



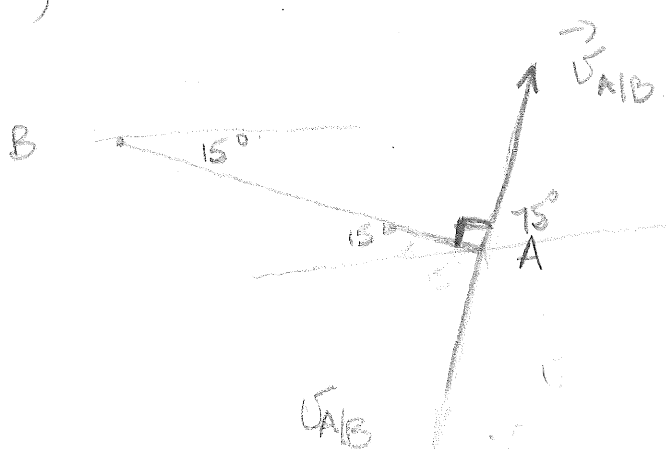
$\vec{v}_A = -.69\hat{i} + .4\hat{j}$ m/s

−1: no units

−1: each small error

b) $\vec{v}_A = \vec{v}_B + \vec{v}_{A|B}$

(10 marks)



$\vec{v}_{A|B} = .24\,\omega_{AB}\cos 75^\circ\,\hat{i} + .24\,\omega_{AB}\sin 75^\circ\,\hat{j}$

$\vec{v}_B = v_B\,\hat{i}$

$$-.69\hat{\imath} + .4\hat{\jmath} = v_B\hat{\imath} + .06\,\omega_{AB}\hat{\imath} + .23\,\omega_{AB}\hat{\jmath}$$

$\hat{\jmath}$ component only: $\quad +.4 = .23\,\omega_{AB}$

$$\Rightarrow \quad \omega_{AB} = +1.74\ s^{-1}$$

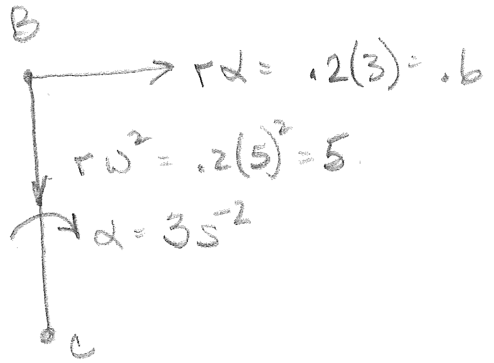
the math says positive, and clearly the motion is counterclockwise by observation

$$\Rightarrow \omega_{AB} = 1.74\,s^{-1}$$

-1 : no units

I did try not to penalize errors in (a).

c)
(5 marks)



$$\vec{a}_B = .6\hat{\imath} - 5\hat{\jmath}\ ms^{-2}$$

-1 : no units

-1 : each small error

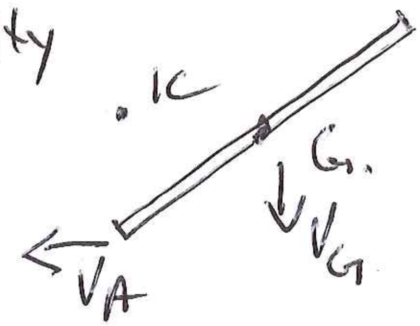
-4 or -5 : treating $\vec{a}$ as a scalar

-2 or -3 : treating $\vec{a}$ as both a vector and a scalar

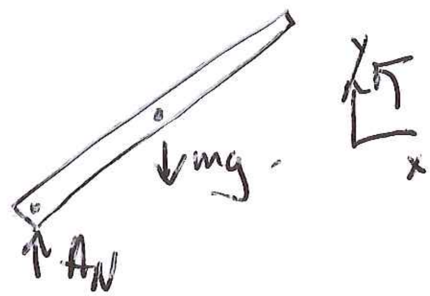
MIE 100 April 2016 Final exam.

Solution to problem 3:

(a) using center of zero velocity



(5) $v_G = \omega r$
$= (3)(\sqrt{3})$ m/s $\Downarrow$



(b)(i) $\Sigma F = m a_G.$

$A_N - mg = m a_G$

(all in y-direction.

(ii) $\Sigma M_G = I_G \alpha$

$-\sqrt{3} A_N = 1/12\, m L^2 \alpha$

$-\sqrt{3} A_N = 28\alpha$

(iii) relative motion;

x-direction: $a_G = a_A - 2\omega^2 \cos 30° - 2\alpha \sin 30°$

$0 = a_A - 18\sqrt{3} - \alpha$

y-direction: $a_G = a_A - 2\omega^2 \sin 30 + 2\alpha \cos 30°$

$a_G = 0 - 9 + 1.73\alpha$

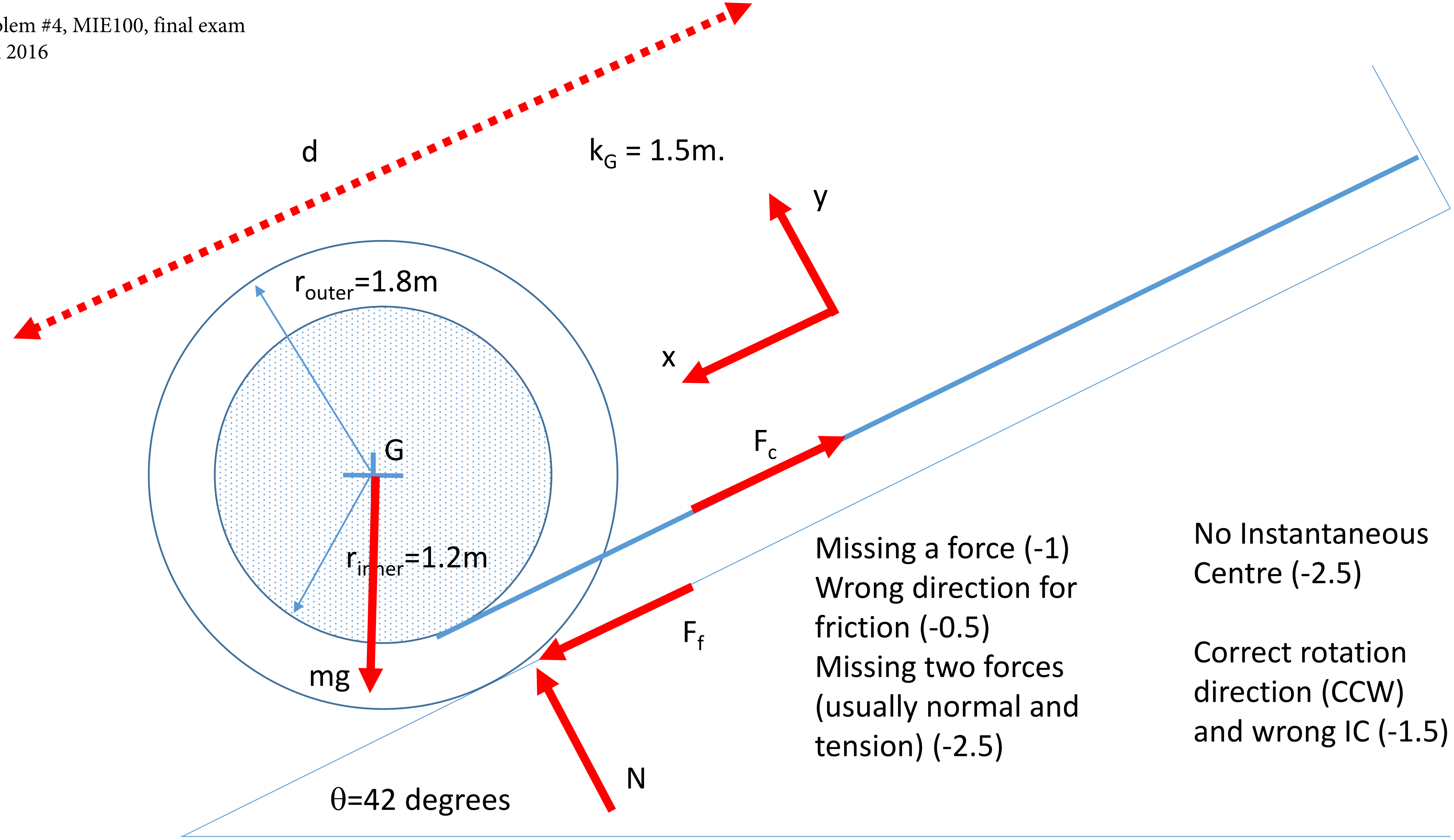
(iv) 4 Eqns, 4 unknowns

(10) Get this far, & you get at least 8 or 9 out of 10

$a_G = -8.43$ m/s^2; $\alpha = -0.324\ s^{-2}$; $A_N = 5.23$ Newtons

(5) (c) I_c is at point A, {-2, 0}

Problem #4, MIE100, final exam
april 2016



Part A (Remove from 5)

Missing a force (-1)
Wrong direction for friction (-0.5)
Missing two forces (usually normal and tension) (-2.5)

Part B (Remove from 5)

No Instantaneous Centre (-2.5)

Correct rotation direction (CCW) and wrong IC (-1.5)

Part C (Final Mark given)

Wrong distance (3m) - 9.5 with everything else correct

Missing gravity 8

Right idea but not complete 7

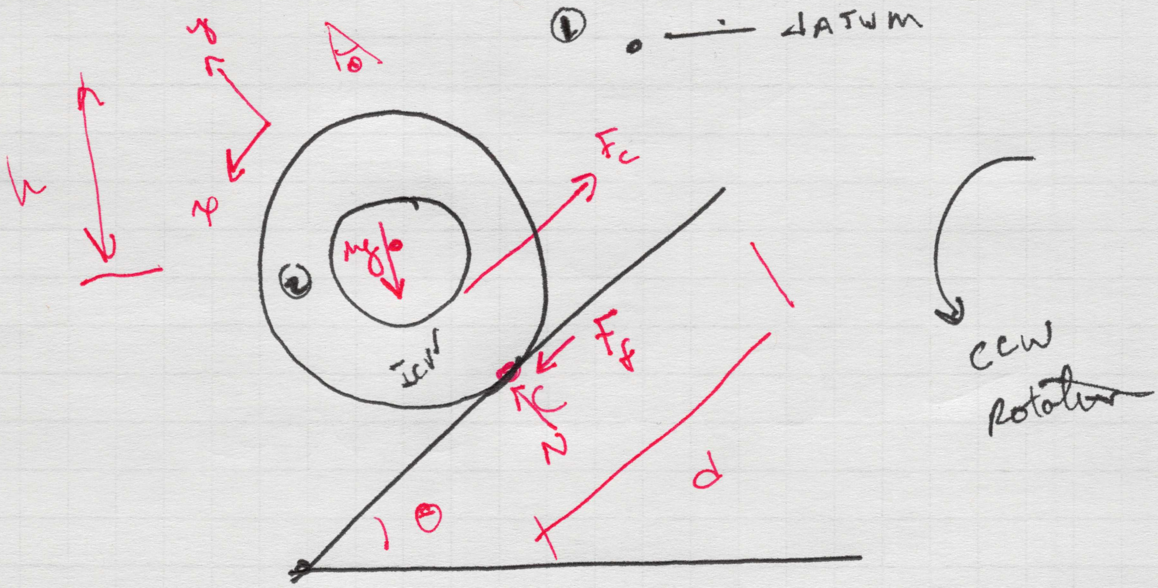
Mercy Marks 3

Mercy marks with correct moment of inertia 4

No friction work - 7.5 marks (everything else good)

Some idea of what to do- 5

Energy solution but confused - 6.5



$$T_1 + V_1 + (U_{1-2})_{NC} = T_2 + V_2$$

$$T_1 = \frac{1}{2} I_O \omega_{s_1}^2 \qquad T_2 = \frac{1}{2} I_O \omega_{s_2}^2$$

$$I_O = m[k_G^2 + \rho^2] = 738\ kgm^2$$

① $V_1 = 0$

$V_2 = -mgd\sin\theta$

$U_{1-2} = -\mu_k mg\, l\cos\theta$

Kinetics

$\underline{N = mg\cos\theta}$ $\qquad F_f = \mu_k mg\cos\theta$

$$\boxed{U_{1-2} = T_2 + V_2}$$

$$l = \frac{dR}{\rho} = 4.5m$$

$$v_{G_2} = \sqrt{\frac{(U_{1-2} + mgd\sin\theta)\rho^2}{\frac{1}{2}m(k_G^2 + \rho^2)}} = \underline{\underline{3.079m}}$$

Question # 5 :

(a) $\omega_n = \sqrt{\frac{k_{eq}}{m}}$ (c not needed for natural frequency)

(series) $\frac{1}{k_{23}} = \frac{1}{k_2} + \frac{1}{k_3} = \frac{1}{2000} + \frac{1}{1500} = \frac{7}{6000}$

(parallel) $k_{eq} = k_1 + k_{23} = 1000 + \frac{6000}{7} = 1857 \text{ N/m}$

$\omega_n = \sqrt{\frac{1857 \text{ N/m}}{40 \text{ kg}}} = \boxed{6.81 \text{ rad/s}}$

(b) Initial conditions $y(t=0) = 0.12 \text{ m}$

$\dot{y}(t=0) = 0.5 \text{ m/s}$

$c = 280 \text{ N·s/m}$

$\omega_d = \sqrt{\frac{k_{eq}}{m} - \left(\frac{c}{2m}\right)^2} = \sqrt{\frac{1857 \text{ N/m}}{40 \text{ kg}} - \left(\frac{280}{80}\right)^2}$

$\omega_d = \underline{\underline{5.846 \text{ rad/s}}}$ (damped , underdamped specifically)

$c_{crit} = 2m\omega_n = 2(40)(6.81) = 544.8 \text{ N·s/m}$ ✓

$y(t) = De^{-(c/2m)t} \sin(\omega_d t + \phi)$

$y(t) = De^{-(280/80)t} \sin(5.846t + \phi)$

$$y(0) = D \sin\phi = 0.12$$

$$y'(t) = -3.5De^{-3.5t} \sin(5.846t + \phi)$$

$$+ De^{-3.5t} \cos(5.846t + \phi)(5.846)$$

$$y'(0) = -3.5D\sin\phi + 5.846D\cos\phi = 0.5$$

Solve D and ϕ:

$$-3.5(0.12) + 5.846(0.12)\frac{\cos\phi}{\sin\phi} = 0.5$$

$$\tan\phi = \frac{5.846(0.12)}{0.5 + 0.12(3.5)} = 0.7625$$

$$\phi = 37.326^\circ = 0.65147 \text{ rad}$$

$$D = \frac{0.12}{\sin(0.65147 \text{ rad})} = 0.1979 \text{ m}$$

$$y(t = 0.2s) = 0.1979 e^{-3.5(0.2)} \sin(\underbrace{5.846(0.2) + 0.65147}_{1.82067})$$

$$\boxed{y(t = 0.2s) = 0.0952 \text{ m}}$$

(c) $$x_p = \frac{F_o/k}{1 - (\omega_o/\omega_n)^2} \sin\omega_o t = C\sin\omega_o t$$ where C is amplitude.

$$C = \frac{200/1857}{1 - (5/6.81)^2} = \boxed{0.2337 \text{ m}}$$

5(b) Alternative form of solution

$$y(t) = C_1 e^{-(c/2m)t} \sin(\omega_d t) + C_2 e^{-(c/2m)t} \cos(\omega_d t)$$

$$y(0) = 0.12 \text{ m} = \cancel{C_1(0)} + C_2(1) \Rightarrow \boxed{C_2 = 0.12 \text{ m}}$$

$$\dot{y}(t) = C_1\left(-\frac{c}{2m}\right)e^{-(c/2m)t} \sin(\omega_d t) + C_1 e^{-(c/2m)t} \cos(\omega_d t)\,\omega_d$$

$$+ C_2\left(-\frac{c}{2m}\right)e^{-(c/2m)t} \cos(\omega_d t) - C_2 e^{-(c/2m)t} \sin(\omega_d t)\,\omega_d$$

$$\dot{y}(0) = \cancel{C_1(-3.5)(1)(0)} + C_1(1)(1)(5.846)$$

$$+ C_2(-3.5)(1)(1) - \cancel{C_2(1)(0)(5.846)} = 0.5 \text{ m/s}$$

$$C_1 = \frac{0.5 + 3.5(0.12)}{5.846} = \boxed{0.15737 \text{ m}}$$

$$y(t) = 0.1574\, e^{-3.5t} \sin(5.846t) + 0.12\, e^{-3.5t} \cos(5.846t)$$

$$y(0.2) = 0.1574\, e^{-0.7} \sin(1.1692) + 0.12\, e^{-0.7} \cos(1.1692 \text{ rad})$$

$$y(0.2) = 0.07193 + 0.02329$$

$$\boxed{y(0.2) = 0.09522 \text{ m}}$$ same answer.

If form is $y(t) = D e^{-(c/2m)t} \cos(\omega_d t + \phi)$

$$0.12 = D \cos\phi$$

$$\dot{y}(0) = 0.5 = (-3.5) D \cos\phi - 5.846 \sin\phi\, D$$

$$0.5 = (-3.5)(0.12) - 5.846(0.12)\tan\phi$$

$$\tan\phi = -1.311 \Rightarrow \phi = -52.67^\circ = \underline{-0.9193 \text{ rad}}$$

$$D = \underline{0.1979}$$

$$\longrightarrow y(t) = 0.1979\, e^{-3.5t} \cos(5.846t - 0.9193)$$

$$y(0.2) = 0.0952 \text{ m} \checkmark$$