University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, DECEMBER, 2007
First Year - CHE, CIV, CPE, ELE, IND, LME, MEC, MMS

MAT 188H1F - LINEAR ALGEBRA
Exam Type: A

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PAGE	MARK
page 2	
page 3	
page 4	
page 5	
page 6	
page 7	
page 8	
page 9	
page 10	
TOTAL	

Calculators Permitted: Casio 260, Sharp 520 or TI 30.

Note: as in the textbook, 0 represents the zero vector, the zero matrix, or the zero number, depending on context.

INSTRUCTIONS: Attempt all questions. Use the backs of the sheets if you need more space. Do not tear any pages from this exam. Make sure your exam contains 10 pages.

MARKS: Questions 1 through 6 are Multiple Choice; circle the single correct choice for each question. Each correct choice is worth 4 marks.

Question 7 is worth 10 marks; 2 for each part.

Questions 8, 9 and 10 are each worth 10 marks.

Questions 11, 12 and 13 are each worth 12 marks.

TOTAL MARKS: 100

1. If U is a subspace of $\mathbb{R}^6$ and $\dim U = 2$, then $\dim U^{\perp}$ is

   (a) 2

   (b) 3

   (c) 4

   (d) 5

2. $\dim \left(\operatorname{span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 0 \\ -1 \end{bmatrix} \right\} \right)$ is

   (a) 2

   (b) 3

   (c) 4

   (d) 5

3. The minimum distance from the point $P(2, 0, 3)$ to the plane with equation $x = 6$ is

   (a) 2

   (b) 4

   (c) 6

   (d) 8

4. If the matrix $\begin{bmatrix} 1 & a & 0 \\ 0 & 2 & 2 \\ a & 12 & 3 \end{bmatrix}$ is invertible, then

(a) $a = 3$ or $a = -3$.

(b) $a = 3$.

(c) $a \neq 3$.

(d) $a \neq 3$ and $a \neq -3$.

5. Let

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

Which one of the following statements is true?

(a) A is diagonalizable and B is not diagonalizable.

(b) A is not diagonalizable and B is diagonalizable.

(c) Both A and B are diagonalizable.

(d) Both A and B are not diagonalizable

6. The equation of the plane passing through the point $(x, y, z) = (1, 0, -1)$ and containing the line $[x\ y\ z]^T = [2\ 3\ 4]^T + t[2\ 1\ 3]^T$ is

(a) $4x + 7y - 5z = 9$

(b) $2x + 3y + 4z = -2$

(c) $x - 2y + z = 0$

(d) $x - z = 6$

7. Decide if the following statements are True or False, and give a brief, concise justification for your choice. Circle your choice.

(a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^3$ True or False

(b) null $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \{0\}$ True or False

(c) span $\left\{ \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 7 \end{bmatrix} \right\} = \mathbb{R}^3$ True or False

(d) If A is a 3×3 matrix such that $\operatorname{adj} A = I$, then $A = I$. True or False

(e) $\dim(\operatorname{im} A) = \dim\left(\operatorname{im} A^T\right)$ True or False

8. Given that

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 & -1 \\ 2 & 0 & 3 & 0 & 1 \\ 4 & 0 & 7 & 2 & -1 \\ 0 & 0 & 1 & 3 & 1 \end{bmatrix} \text{ has reduced row-echelon form } R = \begin{bmatrix} 1 & 0 & 0 & 0 & 17 \\ 0 & 0 & 1 & 0 & -11 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

state the rank of A, and then find a basis for each of the following: the row space of A, the column space of A, and the null space of A.

9. Show that

$$U = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \mathbb{R}^4 \,\middle|\, \det \begin{bmatrix} x_1 & 0 & 3 & -1 \\ x_2 & 1 & 0 & 1 \\ x_3 & 0 & 0 & 1 \\ x_4 & -1 & 1 & 1 \end{bmatrix} = 0 \right\}$$

is a subspace and find a basis of U.

10. Let

$$A = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

be the matrix of the projection onto the line $y = 2x$. Find the eigenvalues and eigenvectors of A, and interpret your results geometrically.

11. Find an orthogonal matrix P and a diagonal matrix D such that $D = P^T AP$, if

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}.$$

12. Let $U = \operatorname{span}\left\{[\begin{array}{cccc} 0 & 1 & 0 & 1 \end{array}]^T, [\begin{array}{cccc} 1 & 0 & 0 & 1 \end{array}]^T, [\begin{array}{cccc} 1 & -1 & 0 & 0 \end{array}]^T\right\}$;
let $X = \left[\begin{array}{cccc} 1 & 1 & 0 & 1 \end{array}\right]^T$. Find:

(a) an orthogonal basis of U.

(b) $\operatorname{proj}_U(X)$.

13. Find the least squares approximating quadratic for the data points

$$(-1,0),(0,4),(1,1),(1,-2).$$