

University of Toronto  
Department of Mathematics

**MAT186H1S**  
Calculus I

**FINAL EXAMINATION**  
Monday, April 16, 2012  
2:00 pm

Examiner: R. Burko

Duration: 2 1/2 hours

Permitted Calculators: Casio 260, Sharp 520, Texas Instrument 30

Total: 105 marks

[15 marks] 1. Evaluate the limits

$$(a) \lim_{x \rightarrow +\infty} \sqrt{x^2 + 1} - \sqrt{x^2 + 2x} \quad (b) \lim_{y \rightarrow 0^+} \frac{1 - \cos(\sqrt{y})}{y} \quad (c) \lim_{t \rightarrow +\infty} \left( \frac{t+1}{t-2} \right)^{2t}$$

[12 marks] 2. A farmer needs to put up a rectangular fenced area, as shown below, to contain his eight sheep. He wants to construct it so that each stall has equal dimension, and each sheep has as much space as possible. What is the maximum area he can enclose if he has exactly 200 m of fencing?



[8 marks] 3. Find  $f'(3)$  and  $g'(\frac{\pi}{2})$  if

$$f(x) = \int_0^{x^2} e^{-t^2} dt \quad \text{and} \quad g(x) = \int_x^{7x} \frac{\sin t}{t} dt.$$

[8 marks] 4. Find the area of the region bounded by the curves  $y = x^2 - 2x + 1$  and  $y = 5x - 9$ .

[20 marks] 5. Evaluate the integrals

$$(a) \int_0^3 |4-2x| dx \quad (b) \int_1^2 \frac{1+\ln x}{x} dx \quad (c) \int x \sin^2(x^2) \cos(x^2) dx \quad (d) \int \frac{dx}{e^x + e^{-x}}$$

(hint for (d): multiply and divide by  $e^x$ )

[8 marks] 6. Suppose  $f$  and  $g$  are integrable functions, with

$$\int_0^2 f(x) dx = 3, \quad \int_0^7 f(x) dx = -4, \quad \text{and} \quad \int_7^2 g(x) dx = 6.$$

Calculate  $\int_2^7 (3f(x) + 2g(x)) dx$ .

[14 marks] 7. Sketch the graph of  $f(x) = \frac{4x+4}{x^2+3}$ . Precisely label all  $x$ -intercepts,  $y$ -intercepts, critical points, vertical asymptotes, horizontal asymptotes, relative minima and relative maxima. Roughly label all inflection points by inspecting the graph.

[10 marks] 8. Find the equation of the line tangent to each curve at the given point.

$$(a) xy = 3y - 2x, \text{ at } (1, 1) \quad (b) y = x^{\tan x}, \text{ at } \left(\frac{\pi}{4}, \frac{\pi}{4}\right)$$

[10 marks] 9. A 6-foot man walks away from a 20-foot lamppost. At  $t = 0$  seconds, he is at rest at the base of the lamppost. His acceleration at  $t$  seconds is  $a(t) = 2t + \frac{12}{(t+1)^2}$  ft/s<sup>2</sup>.

- (a) What is his velocity as a function of time?
- (b) How fast is the length of his shadow increasing at  $t = 5$  seconds?