UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, APRIL 2005

First Year – Mechanical, Industrial, Electrical, Computer

MIE100S – DYNAMICS

Exam Type: C

Examiners: M. Hoorfar, M. Popovic, C. Simmons, L. Sinclair

Answer all four questions.

This examination has **five** pages.

Permissible calculators: Casio260, Sharp520 or TI30

Each student is permitted a single 8 ½ x 11 inch aid sheet.

All answers must include very **CLEAR, NEAT** rough work.

Final answers must include the appropriate S.I. units (meter, second, kg, Newton, etc.).

Vectors in final answers must include the appropriate direction, using the coordinate system specified in the question.

Use $g = 9.81\ m/s^2$.

Question 1 (25 marks)

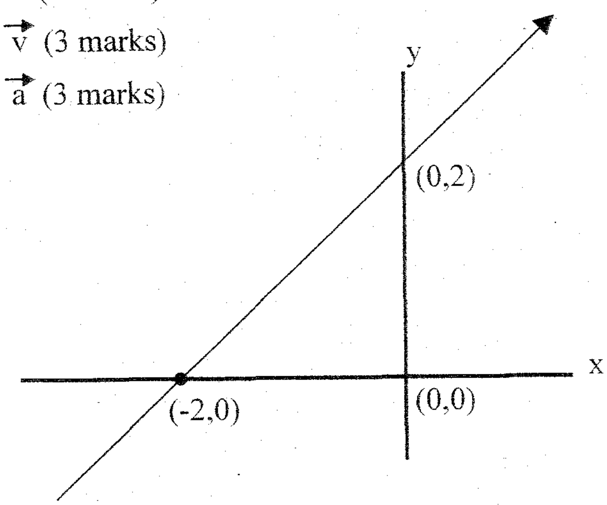
(a) A small particle is travelling along a straight line $Y = X + 2$ in the direction shown. X and Y are both measured in meters. At $t = 0$ the particle crosses the x axis with a constant speed of 2 m/s. Using a standard polar co-ordinate system centered at the origin shown, determine the following at $t = 0$.

(i) $\vec{r}$ (3 marks)

(ii) $\dot{r}$ (3 marks)

(iii) $\vec{v}$ (3 marks)

(iv) $\vec{a}$ (3 marks)



(b) An F16 combat aircraft has been damaged during a dogfight and is losing fuel. The pilot estimates that she must land within 25 minutes or risk running out of fuel completely. At $t = 0$ the plane is at an altitude of 5,000 meters, with a mass of 15,000 kg and has a velocity of 900 km/hr in the horizontal direction. In order to land, the pilot decelerates with a horizontal acceleration of -5.31×10^{-4} m/s^2. The vertical thrust (measured in Newtons) on the plane is 588.6v where v is the horizontal speed in meters per second. Neglect the mass loss of the fuel.

(i) Draw a large, neat and clear Free Body Diagram of the plane. (3 marks)

(ii) Will the pilot be able to land within the 25 minute time limit? (10 marks: Full marks will only be given here for a calculation of the actual landing time. Zero marks will be given for a yes or no guess.)

Question 2 (25 marks)

A thin circular disc with a hole in the middle is able to spin in the horizontal plane about a rod that passes through the hole and is fixed to the ground. However, the fit between the disc and rod is tight and this causes friction that exerts a constant moment, $M_O = 10$ N m, on the disc about its centre (point O). The disc has an outer radius, R_o, an inner radius, R_i, and a mass, m_d.

The disc is initially at rest. A very small bullet of mass m_b, travelling at speed of 100 m/s, strikes the disc on its edge and becomes embedded in the disc, as shown in the Figure. The collision lasts 0.01 seconds, during which time the constant frictional moment acts on the disc. Because the disc moves only in the horizontal plane, you may ignore the effects of gravity.

a) Show that the mass moment of inertia of the disc about its centre is $I_O = \frac{1}{2} \rho \pi t \, (R_o^4 - R_i^4)$, where ρ is the density of the disc and t is the thickness of the disc. Recall that $I_O = \frac{1}{2} mR^2$ for a solid thin circular disc of mass m and radius R. (5 marks)

b) Once the bullet is embedded in the disc, show that the centre of mass of the bullet+disc is a distance $\overline{OG} = \dfrac{m_b}{m_b + m_d} R_O$ away from the centre of the disc.

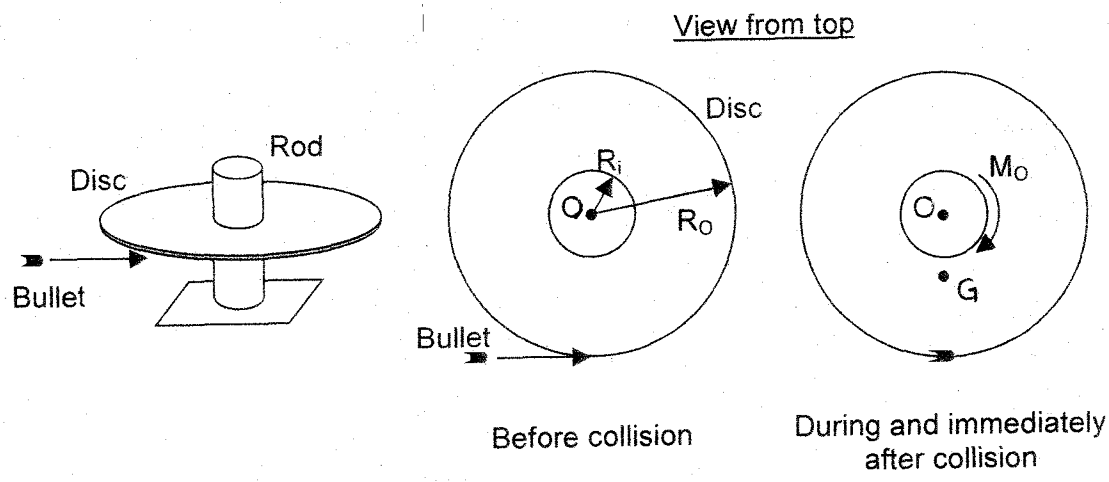
(5 marks)

For parts (c) through (e), use the following parameters in your calculations: $m_b = 0.1$ kg; $m_d = 14.4$ kg; $R_o = 1$ m; $R_i = 0.624$ m; and $I_O = 10$ kg m^2.

c) Calculate the angular velocity of the bullet+disc immediately after the collision. (5 marks)

d) Calculate the velocity of the centre of mass of the bullet+disc immediately after the collision. (5 marks)

e) Determine and justify whether or not the disc will stop spinning before the bullet returns to its original position. (5 marks)



Question 3 (30 marks)

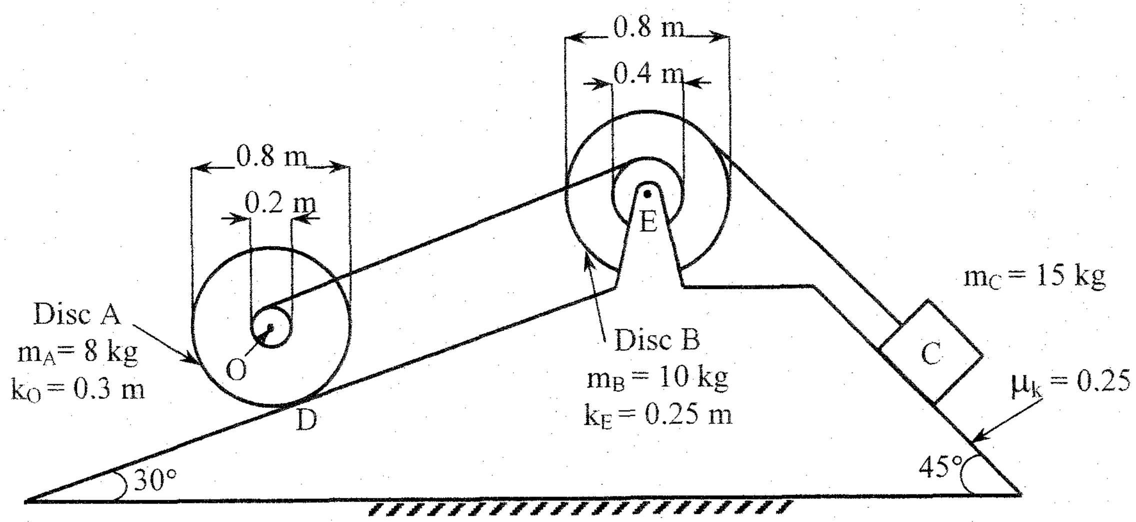
The mass C moves down and hence the disc A **rolls up** the incline **without slipping**.

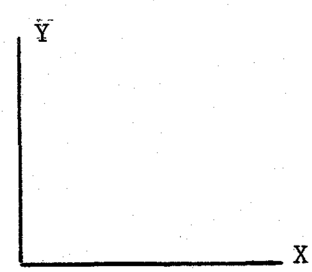
(a) Determine the acceleration of the mass center of disk A (i.e. $\vec{a}_O$) and the tensions in the ropes. (18 marks)

(b) Determine the angular acceleration of the disc B (i.e. α_B), and the acceleration of the mass C (i.e. $\vec{a}_C$). (4 marks)

(c) Determine the force of friction between the incline and disc A. (4 marks)

(d) Draw large and clear **Free Body Diagrams (FBD)** of the **disc A** and **mass C**. Show the actual direction of the forces. (4 marks)





Question 4 (20 marks)

The 1.2 kg bob of a simple pendulum of length L = 600mm is suspended by a string from a 1.4 kg collar C. The collar is forced to move according to the relation $x_c = d \sin(\omega_o t)$, with an amplitude d = 10mm and frequency $f_o = 0.5$ Hz. Determine the following.

(a) The natural frequency of the pendulum (5 marks)
(b) The amplitude of the motion of the bob (5 marks)
(c) The force that must be applied to collar C to maintain the motion (10 marks)