UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

First Year - MEC100S

DYNAMICS

Final Examination

Date: April 21, 2003 **Time: 9:30-12:00**

Instructions:

1. Answer all the questions.
2. 5 questions only and each question is worth 20%.
3. Only non-programmable calculators are allowed.

1. The blocks A and B shown in Fig.1 have a mass of 10 kg and 100 kg, respectively. Determine the distance B travels from the point where it is released from rest to the point where its speed becomes 2 m/s.

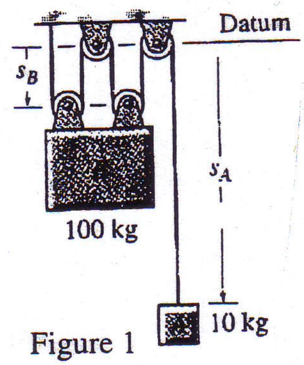


Figure 1

2. The road AB shown in Fig.2 is confined to move along the inclined planes at A and B. If point A has an acceleration of 3 m/s^2 and a velocity of 2 m/s, both directed down the plane at the instant the rod becomes horizontal, determine the angular acceleration of the rod at this instant.

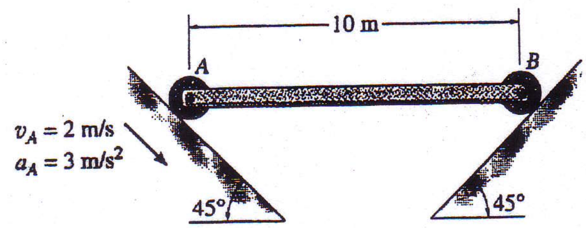


Figure 2

3. The upper body of a spinal cord injured patient in the wheel chair has a mass of 375 N, a center of gravity at G, and a radius of gyration about G of $K_G = 0.21$ m. By means of the seat belt the body segment is assumed to be pin- connected to the seat of the wheel chair at A. If a crash causes the wheelchair to decelerate at 15 m/s^2, determine the angular velocity of the body when it has rotation to $\theta = 30°$.

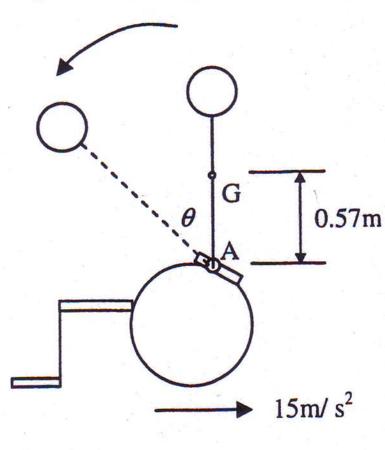


Figure 3

4. The 50 N cylinder rests on the 100 N dolly, Fig. 4. If the system is released from rest, determine the angular velocity of the cylinder in 2 s if:
   a) the cylinder does not slip on the dolly.
   b) the coefficients of static and kinetic friction between the cylinder and the dolly are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively.

   Neglect the mass of the wheels on the dolly

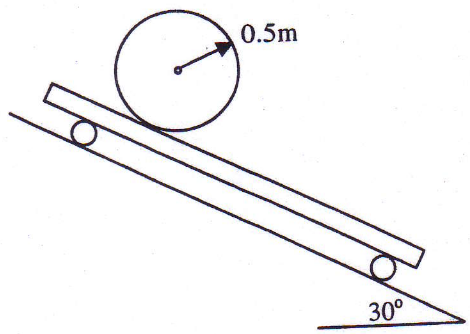


Figure 4

5. Calculate the natural frequency and damping ratio for the system in Fig.5 given the values $m = 10$ kg, $c =$ 100 kg/s, $k_1 = 4000$ N/m, $k_2 = 200$ N/m, and $k_3 = 1000$ N/m. Assume that no friction acts on the rollers. Is the system overdamped, critically damped, or underdamped?

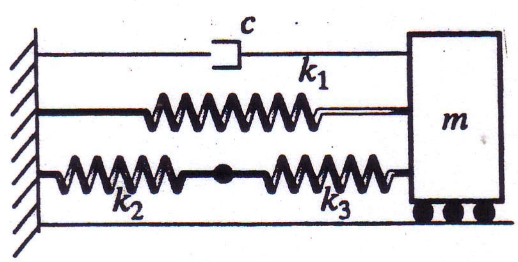


Figure 5

Rectilinear Motion

$a=\frac{dv}{dt}=\frac{d^2x}{dt^2}=v\frac{dv}{dx}$ $\quad x=x_0+v_0t+\frac{1}{2}at^2$ $\quad v=v_0+at$ $\quad v^2=v_0^2+2a(x-x_0)$

Curvilinear Motion

$\vec{v}=\frac{d\vec{r}}{dt}$ $\quad \vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ $\quad \vec{e}_n=\frac{d\vec{e}_t}{d\theta}$ $\quad \dot{\vec{e}}_r=\dot{\theta}\vec{e}_\theta$ $\quad \dot{\vec{e}}_\theta=-\dot{\theta}\vec{e}_r$

$\vec{a}=\frac{d\vec{v}}{dt}$ $\quad \vec{v}=\dot{x}\vec{i}+\dot{y}\vec{j}+\dot{z}\vec{k}$ $\quad \vec{v}=v\vec{e}_t$ $\quad \vec{v}=\dot{r}\vec{e}_r+r\dot{\theta}\vec{e}_\theta$

$\vec{a}=\ddot{x}\vec{i}+\ddot{y}\vec{j}+\ddot{z}\vec{k}$ $\quad \vec{a}=\frac{dv}{dt}\vec{e}_t+\frac{v^2}{\rho}\vec{e}_n$

$\vec{a}=\left(\ddot{r}-r\dot{\theta}^2\right)\vec{e}_r+\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)\vec{e}_\theta$

$\vec{r}_B=\vec{r}_A+\vec{r}_{B/A}$ $\quad \vec{v}_B=\vec{v}_A+\vec{v}_{B/A}$ $\quad \vec{a}_B=\vec{a}_A+\vec{a}_{B/A}$

Kinetics of Particles

$\sum\vec{F}=m\vec{a}$

$\sum F_x=ma_x$ $\quad \sum F_t=ma_t$ $\quad \sum F_r=ma_r$

$\sum F_y=ma_y$ $\quad \sum F_n=ma_n$ $\quad \sum F_\theta=ma_\theta$

$\sum F_z=ma_z$ $\quad \sum F_z=ma_z$

$V_e=\frac{1}{2}kx^2$ $\quad V_g=mgh$ $\quad V_g=-\frac{mgR^2}{r}$ $\quad T=\frac{1}{2}mv^2$

$T_1+U_{1-2}=T_2$ $\quad T_1+V_1=T_2+V_2$ $\quad \text{Power}=\vec{F}\cdot\vec{v}$

$\vec{L}=m\vec{v}$ $\quad \sum\vec{F}=\dot{\vec{L}}$ $\quad \int_1^2\sum\vec{F}\,dt=\vec{L}_2-\vec{L}_1$

$\vec{H}_O=\vec{r}\times m\vec{v}$ $\quad \sum\vec{M}_O=\dot{\vec{H}}_O$ $\quad \int_1^2\sum\vec{M}_O\,dt=\vec{H}_{O_2}-\vec{H}_{O_1}$

Systems of Particles

$\sum\vec{F}=m\bar{a}$ $\quad \vec{L}=\sum m\vec{v}=m\bar{v}$ $\quad \sum\vec{F}=\dot{\vec{L}}$ $\quad \int_1^2\sum\vec{F}\,dt=\vec{L}_2-\vec{L}_1$

$\vec{H}=\sum\vec{r}\times m\vec{v}$ $\quad \sum\vec{M}_O=\dot{\vec{H}}_O$ $\quad \sum\vec{M}_G=\dot{\vec{H}}_G$ $\quad \int_1^2\sum\vec{M}_O\,dt=\left(\vec{H}_O\right)_2-\left(\vec{H}_O\right)_1$

Kinematics of Rigid Bodies

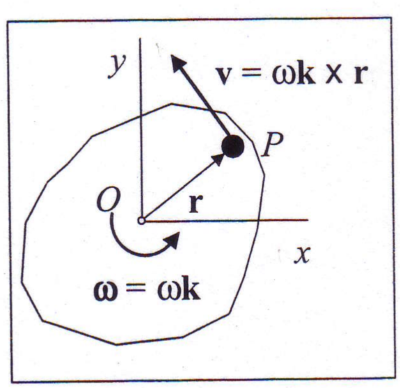
$\omega=\frac{d\theta}{dt}=\dot{\theta}$

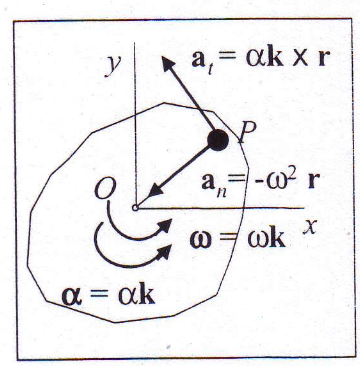
$\alpha=\frac{d\omega}{dt}=\dot{\omega}$ $\quad or \quad \alpha=\frac{d^2\theta}{dt^2}=\ddot{\theta}$

$\omega d\omega=\alpha d\theta$ or $\dot{\theta}\,d\dot{\theta}=\ddot{\theta}\,d\theta$

$v=r\omega$

$a_n=r\omega^2$ $\quad a_t=r\alpha$

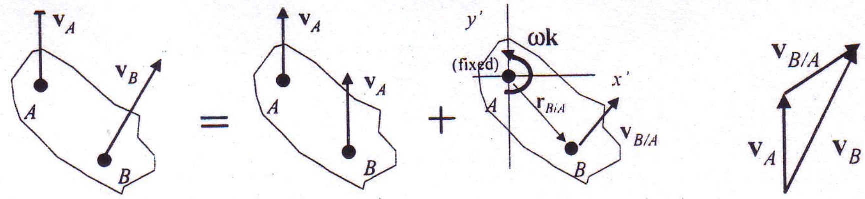




$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

$$v_{a/B} = r\omega$$

$$\vec{v}_{A/B} = \vec{\omega} \times \vec{r}$$

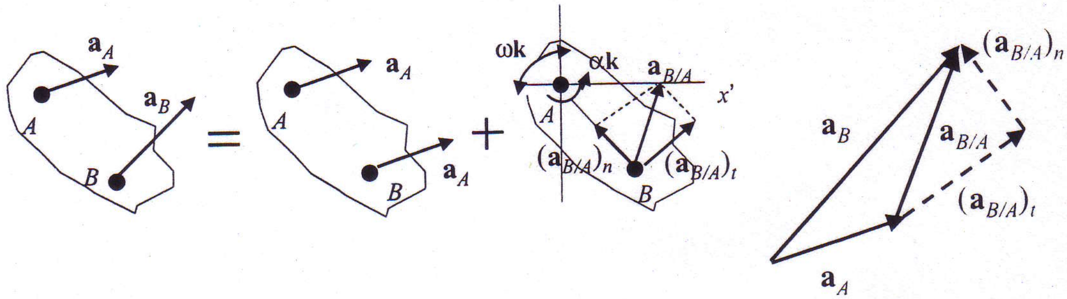


$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

$$\vec{a}_A = \vec{a}_B + (\vec{a}_{A/B})_n + (\vec{a}_{A/B})_t$$

$$(a_{A/B})_n = \frac{v_{A/B}^2}{r} = r\omega^2$$

$$(a_{A/B})_t = \dot{v}_{A/B} = r\alpha$$



Kinetics of Rigid Bodies

Equations of Motion

$\Sigma F_x = m\,\bar{a}_x \qquad \Sigma F_y = m\,\bar{a}_y \qquad \Sigma M_G = \bar{I}\,\alpha \qquad \Sigma M_o = I_o\alpha$

Energy

$T = \frac{1}{2}\,I_o\omega^2 \qquad T = \frac{1}{2}m\,v_G^2 + \frac{1}{2}I_G\omega^2 \qquad T_1 + \Sigma U_{1\text{-}2} = T_2$

Impulse and Momentum

$$\vec{L} = m\vec{v} \qquad \sum \vec{F} = \overrightarrow{\dot{L}} \qquad \int_1^2 \vec{F}\,dt = \vec{L}_2 - \vec{L}_1$$

$$H_O = I_O\omega \qquad \sum \vec{M}_O = \overrightarrow{\dot{H}_O} \qquad \int_1^2 \sum \vec{M}_O\,dt = I_O(\omega_2 - \omega_1)$$

$$H_G = \bar{I}\omega \qquad \sum \vec{M}_G = \dot{\vec{H}}_G \qquad \int_1^2 \sum \vec{M}_G\,dt = (\vec{H}_G)_2 - (\vec{H}_G)_1$$

Free Vibration

$$m\ddot{x} + c\dot{x} + kx = 0 \qquad \omega_n = \sqrt{\frac{k}{m}} \qquad c_c = 2m\sqrt{\frac{k}{m}} = 2m\omega_n \qquad \omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2}$$

$$m\lambda^2 + c\lambda + k = 0 \qquad \lambda_1 = -\frac{c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} \qquad \lambda_2 = -\frac{c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$c > c_c$ Overdamped $x = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$ $\qquad c = c_c$ Critically damped $x = (A + Bt)e^{-\omega_n t}$

$c < c_c$ Underdamped $\quad x = D[e^{-(c/2m)t}\sin(\omega_d t + \phi)]$

log decrement $\qquad \delta = \ln\left(\frac{x_1}{x_2}\right) = \dfrac{2\pi\left(\frac{c}{c_c}\right)}{\sqrt{1 - \left(\frac{c}{c_c}\right)^2}}$

Forced Vibration

$$m\ddot{x} + c\dot{x} + kx = P_m\sin(\omega_f t) \qquad x_p = X\sin(\omega t - \phi)$$

$$M = \frac{X}{P_m/k} = \frac{1}{\sqrt{[1 - (\omega_f/\omega_n)^2]^2 + [2(\frac{c}{c_c})(\omega_f/\omega_n)]^2}}$$

$$\phi = \tan^{-1}\left[\frac{2(\frac{c}{c_c})\omega_f/\omega_n}{1 - (\omega_f/\omega_n)^2}\right]$$