UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

ECE 110H1 S -- ELECTRICAL FUNDAMENTALS
FINAL EXAMINATION, APRIL 12, 2016, 9:30 am

First Year -- Computer, Electrical, Industrial, Mechanical, Materials,
and Track One Engineering Programs.
Examiners – S. Aitchison, B. Bardakjian, A. Helmy, B. Wang and P. Yoo

( $e = 1.6 \times 10^{-19}$ C, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $g = 9.81$ N/kg )

NAME :
(PLEASE PRINT) _____ Family (Last) Name _____ Given (First) Name

STUDENT NUMBER : _____

EXAMINATION TYPE : A
CALCULATORS : Casio FX-991MS & Sharp EL-520X
DURATION : 2.5 hours
INSTRUCTIONS :

- Answer all five questions. Put the answers in the boxes provided.
- All work is to be done on these pages. Show steps, compute numerical results when requested and state units. Write down any assumption made.
- You may use the back of each page.
- Last blank page may be removed for rough work.

| Question | Mark |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| Total | |

**Q1** [**10 marks**] Circle the correct answer or answers

Q1.a. Consider 5, equal positive charges placed on the vertices of a hexagon with sides of length R, as shown in the figure below. What is the value of the electric field at the centre point, P? (2 marks)

A) $\vec{E} = \frac{2kq}{R^2}\hat{j}$

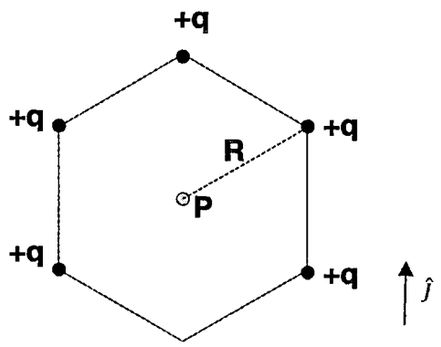
B) $\vec{E} = \frac{-2kq}{R^2}\hat{j}$

C) $\vec{E} = \frac{kq}{R^2}\hat{j}$

D) $\vec{E} = \frac{-kq}{R^2}\hat{j}$

E) $\vec{E} = 0$

Where: $k = \frac{1}{4\pi\varepsilon_o}$




Q1.b. The charge distribution is modified by the inclusion of the charge of -5q and the bottom apex as shown below. What is the value of electrical potential at point P? (2 marks)

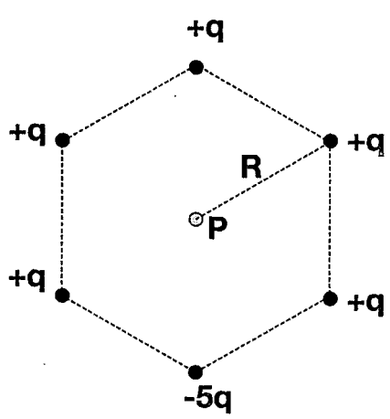
A) $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

B) $V = \frac{-5}{4\pi\varepsilon_0}\frac{q}{R}$

C) $V = 0$

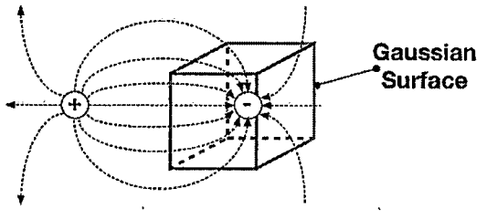
D) $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

E) $V = \frac{10}{4\pi\varepsilon_0}\frac{q}{R^2}$

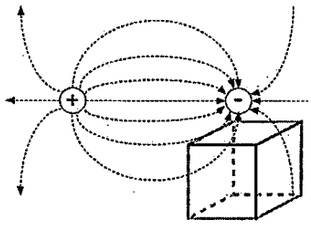


Q1.c. Consider the cubic, closed Gaussian surfaces shown below. Which of the following situations has a zero net flux through the surface? (2 marks)

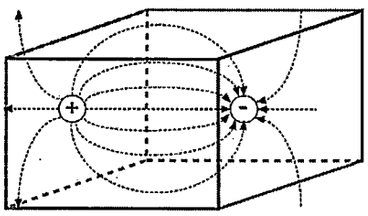
A)



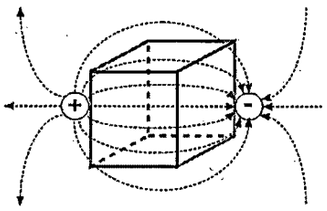

B)



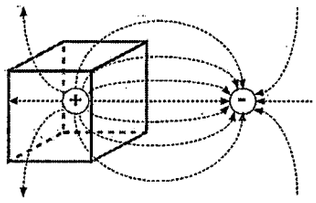
C)



D)



E)



Q1.d. Three capacitors are connected together in the circuit shown below. What is the total energy stored in the capacitors? (2 marks)

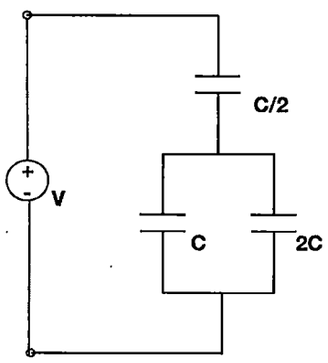
A) $\frac{1}{2}CV^2$

B) $\frac{3}{7}CV^2$

C) $\frac{3}{14}CV^2$

D) $\frac{3}{7}CV$

E) $\frac{6}{28}CV$




Q1.e. An electron is placed mid-way between points A and B. The potential at point A is + 10 V, the potential at point B is -10 V and the potential at the midpoint is 0V. The electron will: (2 marks)

A) Not move since V = 0

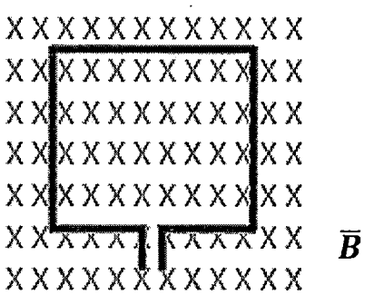
B) Move towards point B with constant velocity

C) Accelerate towards point B

D) Move towards point A with constant velocity

E) Accelerate towards point A

**Q2** [**10 marks**] A single current loop, shown in the figure below, is exposed to a spatially uniform magnetic field $\mathbf{B} = \frac{2}{9\pi}\boldsymbol{sin}(\mathbf{2\pi t})$ tesla into the page as shown, where $\boldsymbol{t}$ is time in seconds.




Q2.a. Find the magnetic flux $\boldsymbol{\Phi_B}$ passing through the wire loop as a function of time if the wire loop is a square in shape with dimensions 3 cm x 3 cm. (2 marks)

A) $\Phi_B = 2 \times 10^{-6}\, sin(2\pi t)$ Wb

B) $\Phi_B = 5 \times 10^{-4}\, cos(2\pi t)$ Wb

C) $\Phi_B = \frac{5}{\pi} sin(2\pi t)$ Wb

D) $\Phi_B = \frac{2\times10^{-4}}{9\pi} sin(2\pi t)$ Wb

E) $\Phi_B = \frac{2\times10^{-4}}{\pi}\, sin(2\pi t)$ Wb

Q2.b. Determine the magnitude of the *emf* generated in the wire loop. (2 marks)

A) $emf = 4 \times 10^{-4}\, cos(2\pi t)$ V

B) $emf = 0$ V

C) $emf = 5 \times 10^{4}\, cos(2\pi t)$ V

D) $emf = \frac{2\times10^{-4}}{9\pi} sin(2\pi t)$ V

E) $emf = \frac{1}{9\pi} tan(2\pi t)$ V

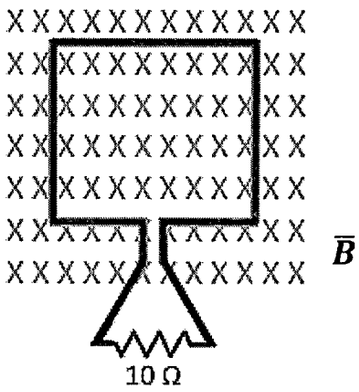
Q2.c. If the loop is connected to a 10 Ω resistor as shown in the figure below, what is the direction of the induced current in the current loop between **i)** $t = 0.0$ **s to** $t = 0.25$ **s** and **ii)** $t = 0.75$ **s to** $t = 1.0$ **s**. Indicate if the current is clockwise (CW) or counter clockwise (CCW). (2 marks)

A) i) CW and ii) CW

B) i) CCW and ii) CCW

C) i) CW and ii) CCW

D) i) CCW and ii) CW




Q2.d. Calculate the magnitude of the force on the top branch of the wire loop, if the loop is connected to a 10 Ω resistor and the wire loop is a square in shape with dimensions 3 cm x 3 cm. (4 marks)

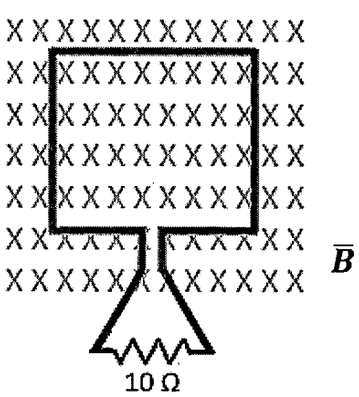
A) $F = 4 \times 10^{-7} sin(2\pi t)\ cos(2\pi t)$ N

B) $F = \frac{1}{9\pi} \times 10^{-4} sin(2\pi t)\ cos(2\pi t)$ N

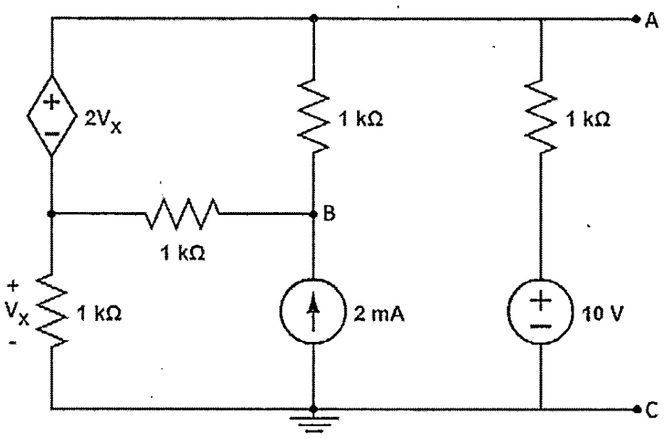
C) $F = \frac{12}{\pi} \times 10^{-2}\ cos(2\pi t)\ cos(2\pi t)$ N

D) $F = \frac{8}{3\pi} \times 10^{-7} sin(2\pi t)\ cos(2\pi t)$ N

E) None of the above



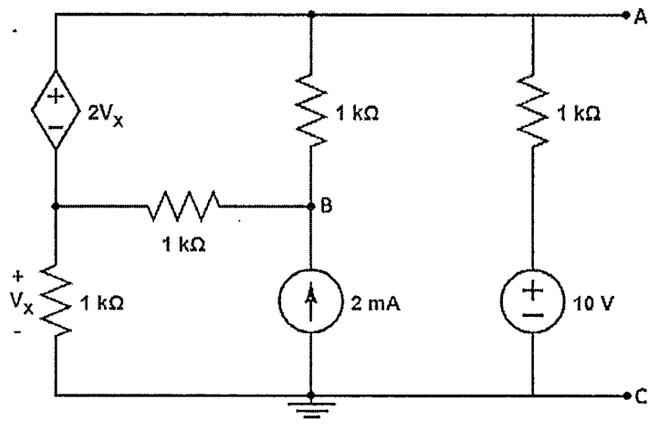
**Q3 [10 marks]** For the given circuit diagram below,




Q3.a. Using nodal analysis, find the voltage between nodes A and B. (5 marks)

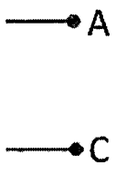
| $V_{AB} =$ |
|---|

**Q3** continued. (Circuit diagram has been duplicated for your convenience)




Q3.b. Find the Thevenin equivalent circuit between nodes A and C. (4 marks)

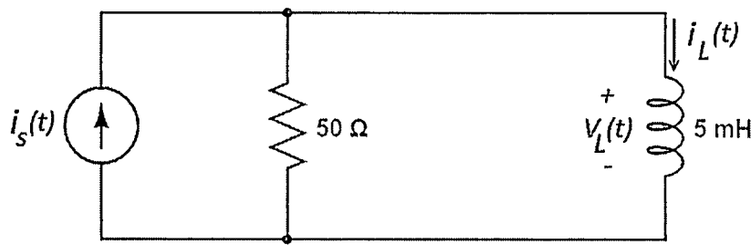
Thevenin Equivalent Circuit:

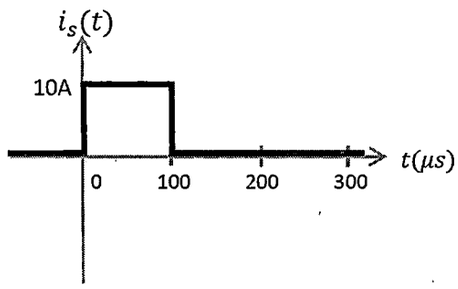



Q3.c. If a resistive load is connected between A and C, what is the maximum power that can be transferred from this circuit? (1 mark)

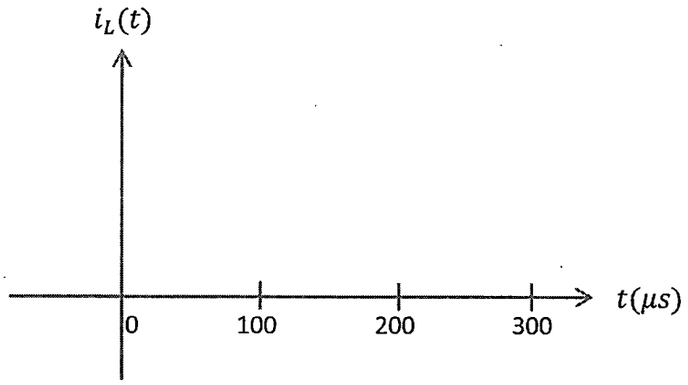
$P_{max} =$

**Q4** [**10 marks**] The parallel RL circuit is excited by the current source, $i_S(t)$ as shown. Assume that the initial energy stored in the inductor is zero.

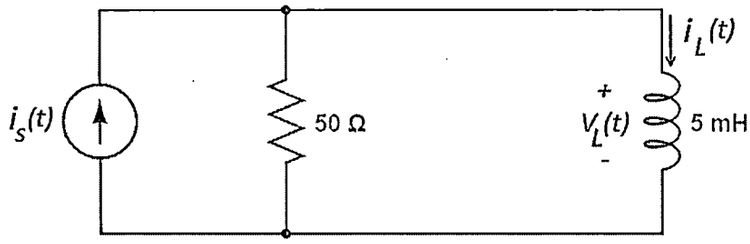


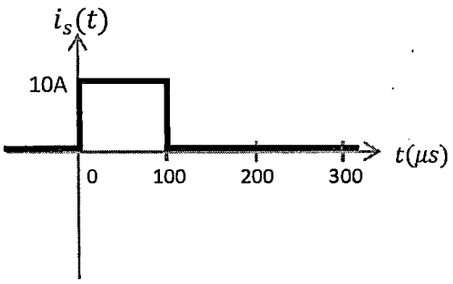




Q4.a. Determine and sketch the inductor current, $i_L(t)$. Indicate the current values at $t = 0$ and at $t = 100\ \mu$s. (3 marks)

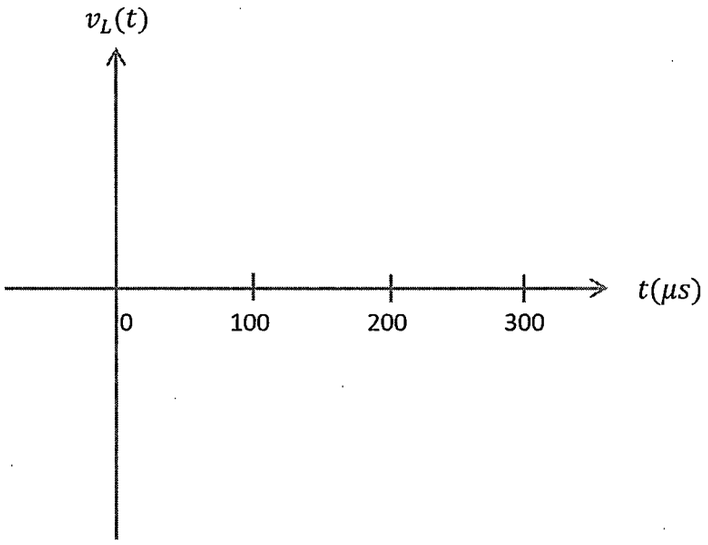


**Q4** continued. (Circuit diagram has been duplicated for your convenience)

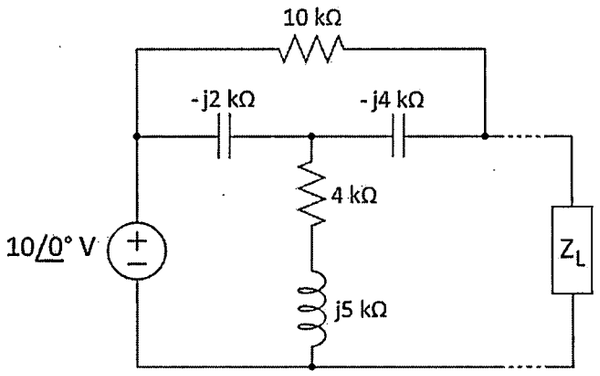



Q4.b. Calculate the inductor voltages at $t = 0+$ and at $t = 100\ \mu$s, and sketch the inductor voltage, $v_L(t)$. (7 marks)



**Q5** [**10 marks**] Given the following circuit.




Q5.a. Determine the open circuit voltage phasor ($\mathbf{V_{OC}}$) for the Thevenin equivalent circuit as seen from the load ($Z_L$). (7 marks)

| $\mathbf{V_{oc}} =$ |
|---|

Q5.b. Determine the Thevenin equivalent impedance as seen from the load ($Z_L$). (3 marks)

| $Z_{Th} =$ |
|---|

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