



UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, APRIL 2015
DURATION: 2 AND 1/2 HRS
FIRST YEAR - CHE, CIV, CPE, ELE, ENG, IND, LME, MEC, MMS
MAT188H1S - Linear Algebra
EXAMINER: D. BURBULLA

Exam Type: A.

Aids permitted: Casio FX-991 or Sharp EL-520 calculator.

Full Name: _____

Last

First

Student Number: _____**Signature:** _____**Instructions:**

- ONLY THE FRONT PAGES WILL BE SCANNED. THE BACK PAGES WILL NOT BE SEEN BY THE EXAMINERS.
- DO NOT WRITE ON THE QR CODE AT THE TOP OF THE PAGES.
- This exam contains 10 pages (including this cover page). Make sure you have all of them. Do not tear any pages from this exam.
- You can use the back of the pages and page 10 for rough work.
- This exam consists of 8 questions. Each question is worth 10 marks.

Total Marks: 80

**PART I :** No explanation is necessary.

1. Big Theorem, Final Exam Version: Let $\mathcal{A} = \{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n\}$ be a set of n vectors in \mathbb{R}^n , let

$$A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \dots & \mathbf{a}_n \end{bmatrix}$$

be the matrix with the vectors in \mathcal{A} as its columns, and let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ be the linear transformation defined by $T(\mathbf{x}) = A\mathbf{x}$. Decide if the following statements are equivalent to the statement, “ A is invertible.” Circle Yes if the statement is equivalent to “ A is invertible,” and No if it isn’t.

Note: +1 for each correct choice; -1 for each incorrect choice; and 0 for each part left blank.

- | | |
|--|-------------|
| (a) $\det(A) \neq 0$. | Yes No |
| (b) A is diagonalizable. | Yes No |
| (c) The reduced echelon form of A is I , the $n \times n$ identity matrix. | Yes No |
| (d) T is one-to-one. | Yes No |
| (e) $\mathbf{0}$ is not in $\text{row}(A)$. | Yes No |
| (f) $\text{range}(T) = \mathbb{R}^n$. | Yes No |
| (g) $\text{row}(A) = \text{span}(\mathcal{A})$. | Yes No |
| (h) $\dim(\text{col}(A)) + \dim(\text{null}(A)) = n$. | Yes No |
| (i) $A = A^T$. | Yes No |
| (j) $\lambda = 0$ is not an eigenvalue of A . | Yes No |

**PART II :** Present **COMPLETE** solutions to the following questions in the space provided.

2. Find the following:

(a) [2 marks] $\dim(S^\perp)$, if S is a subspace of \mathbb{R}^7 and $\dim(S) = 3$.(b) [2 marks] $\det(-3A^T B^2)$, if A and B are 3×3 matrices with $\det(A) = 1$ and $\det(B) = 2$.(c) [2 marks] $\text{proj}_{\mathbf{a}} \mathbf{u}$, if $\mathbf{u} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$ and $\mathbf{a} = \begin{bmatrix} 6 \\ -3 \\ 2 \end{bmatrix}$.(d) [2 marks] $\begin{bmatrix} 4 & 3 \\ 6 & 5 \end{bmatrix}^{-1}$.(e) [2 marks] $\det \begin{bmatrix} 4 & 3 & 10 & 6 \\ 13 & 5 & e & 10 \\ -9 & 4 & \pi & 8 \\ 14 & -1 & \sqrt{2} & -2 \end{bmatrix}$.



3. For any real numbers a, b, c , let $A = \begin{bmatrix} 1 & a & b \\ -a & 1 & c \\ -b & -c & 1 \end{bmatrix}$ and let $B = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$.

(a) [6 marks] Determine for which values of a, b, c , if any, A and B are invertible.

(b) [4 marks] If either A or B is invertible, find its inverse.



4. Suppose A is an $n \times n$ invertible matrix such that $A^2 = 5A$.

(a) [4 marks] What are the possible eigenvalues of A ?

(b) [2 marks] What must A be if it is invertible?

(c) [4 marks] Suppose that A is diagonalizable. Find the rank and nullity of A in terms of the multiplicities of its eigenvalues.



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5. Find the solution to the system of linear differential equations

$$\begin{aligned}y'_1 &= -\frac{3}{2}y_1 + \frac{1}{2}y_2 \\y'_2 &= y_1 - y_2\end{aligned}$$

where y_1, y_2 are functions of t , and $y_1(0) = 5$, $y_2(0) = 4$.



6. Find an orthogonal matrix P and a diagonal matrix D such that $D = P^T AP$, if

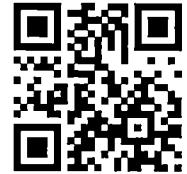
$$A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}.$$



7. Let $S = \text{span} \left\{ [1 \ 1 \ 0 \ 1]^T, [0 \ 1 \ 1 \ 1]^T, [1 \ 0 \ 1 \ 1]^T \right\}$.

(a) [5 marks] Find an orthogonal basis of S .

(b) [5 marks] Let $\mathbf{x} = \begin{bmatrix} 1 & 5 & 3 & 1 \end{bmatrix}^T$. Find $\text{proj}_S(\mathbf{x})$.



8. The parts of this question are unrelated. Each part is worth 5 marks.

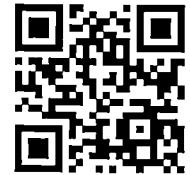
(a) Suppose $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4$ are four vectors in \mathbb{R}^4 and $\det[\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3 \ \mathbf{a}_4] = 5$. Find the value of $\det[\mathbf{a}_1 \ 3\mathbf{a}_2 + 6\mathbf{a}_4 \ \mathbf{a}_3 \ 2\mathbf{a}_4 - 8\mathbf{a}_2]$.

(b) Let $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ be three linearly independent vectors in \mathbb{R}^4 . Define $T : \mathbb{R}^4 \rightarrow \mathbb{R}$ by $T(\mathbf{x}) = \det[\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3 \ \mathbf{x}]$. Show that T is a linear transformation and find $\text{range}(T)$.

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This page is for rough work; it will only be marked if you indicate you want it marked.