University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, April, 2011
Duration: 2 and 3/4 hours
First Year - CHE, CIV, IND, LME, MEC, MMS

MAT186H1S - CALCULUS I
Exam Type: A

SURNAME: (as on your T-card) _____

GIVEN NAMES: _____

STUDENT NUMBER: _____

SIGNATURE: _____

Examiner:
T. Squires

Calculators Permitted: Casio 260, Sharp 520 or TI 30.

INSTRUCTIONS: Attempt all questions. Present your solutions in the space provided. Use the backs of the sheets if you need more space. Do not tear any pages from this exam. Make sure your exam contains 10 pages.

MARKS: Question 1 and 2 are worth 8 marks each, 4 marks for each part.

Question 3 is worth 12 marks, 7 marks for part (a) and 5 marks for part (b).

Questions 4, 5, 6 and 7 are each worth 12 marks.

Questions 8 is worth 12 marks, 7 marks for part (a) and 5 marks for part (b).

Questions 9 is worth 12 marks, 6 marks for part (a) and 6 marks for part (b).

TOTAL MARKS: 100

QUESTION	MARK
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
Q8	
Q9	
TOTAL	

1. [8 marks] Find the following:

(a) [4 Marks] $\frac{d}{dx}(\ln(x)\tan^{-1}(\frac{1}{x}))$

(b) [4 Marks] $\frac{dy}{dx}$ where $xy + \sin(y) = 1$.

2. [8 Marks] Find the following:

(a) [4 Marks] $y(t)$ if $\frac{dy}{dx} = \frac{1}{x}$ and $y(e) = 4$

(b) [4 Marks] $F''(x)$ if $F(x) = \int_0^x e^{t^2} dt$

3. [12 Marks] Let $f(x) = \ln(x)x$

(a) [7 Marks] Find $\lim_{x \to 0+} f(x)$

(b) [5 Marks] Find absolute maximum and minimum values of this function on the interval $(0, e]$.

4. [12 Marks] Consider $f(x) = e^{\sin(x)}$ on the open interval $(-3, 6)$. Use the first derivative to find the open intervals on which f is increasing and the open intervals on which f is decreasing. Find all the critical points of f in the interval $(-3, 6)$ and determine if they are relative maximum or relative minimum points.

5. [12 Marks] Find all the asymptotes to the graph of $f(x) = \dfrac{2x^2 - 1}{x^2 + 4}$, and sketch its graph.
You may assume that

$$f'(x) = \frac{18x}{(x^2+4)^2} \text{ and } f''(x) = \frac{72 - 54x^2}{(x^2+4)^3}.$$

Label the asymptotes, the critical points, and the inflection points, if any.

6. [12 Marks] What is the maximum volume of a cylindrical tube with closed ends if the surface area of the tube is exactly 24π cm^2? (The surface area of a cylindrical tube with closed ends is $S = 2\pi(rh + r^2)$ and the volume is $V = \pi r^2 h$.)

7. [12 Marks] A lighthouse is located on the eastern shoreline of a lake 4km directly across from a cottage located on the western shoreline of the same lake. The lighthouse performs 3 revolutions every minute. At what rate is the light moving across the western shoreline when it is 2km away from the cottage? Assume that the western and eastern shorelines are parallel, so that the line connecting the cottage to the lighthouse is perpendicular to both shorelines.

8. [12 Marks] Find the following

(a) [7 Marks] $\int_0^3 \frac{x}{\sqrt{x+1}} dx$. (Hint: Use substitution)

(b) [5 Marks] $\int_0^{\sqrt{\pi}} x \sin x^2 dx$

9. [12 Marks] Let $y = f(x) = \frac{1}{x}$. Let R be the region bound by the curves $y = f(x)$, $x = 1$, $x = 2$ and $y = 0$. Find the following

   (a) [6 marks] the volume of the solid generated by revolving the region R about the y-axis. Use whatever method you like.

   (b) [6 marks] the volume of the solid generated by revolving the region R about the line $y = -1$. Use the washer method.