UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, APRIL 2002
MAT 188 S – LINEAR ALGEBRA. FIRST YEAR: T-PROGRAM
EXAMINER: FELIX J. RECIO

INSTRUCTIONS:

1. ATTEMPT ALL QUESTIONS.
2. SHOW AND EXPLAIN YOUR WORK IN ALL QUESTIONS.
3. GIVE YOUR ANSWERS IN THE SPACE PROVIDED. USE BOTH SIDES OF PAPER, IF NECESSARY.
4. DO NOT TEAR OUT ANY PAGES.
5. USE OF NON-PROGRAMMABLE POCKET CALCULATORS, BUT NO OTHER AIDS ARE PERMITTED.
6. THIS EXAM CONSISTS OF NINE QUESTIONS. THE VALUE OF EACH QUESTION IS INDICATED (IN BRACKETS) BY THE QUESTION NUMBER.
7. THIS EXAM IS WORTH 50% OF YOUR FINAL GRADE.
8. TIME ALLOWED: 2 ½ HOURS.
9. PLEASE WRITE YOUR NAME, YOUR STUDENT NUMBER, AND YOUR SIGNATURE IN THE SPACE PROVIDED AT THE BOTTOM OF THIS PAGE.

PLEASE DO NOT WRITE HERE

QUESTION NUMBER	QUESTION VALUE	GRADE
1	10	
2	10	
3	10	
4	10	
5	10	
6	15	
7	10	
8	15	
9	10	
TOTAL:	100	

NAME: ______________________ , ______________________
(FAMILY NAME. PLEASE PRINT.) (GIVEN NAME.)

STUDENT No.: ______________ SIGNATURE: ______________________

1. a) (5 marks) Find parametric equations for the line that passes through the point $(1, -3, 2)$ and is parallel to the line of intersection of the planes $x - y + 3z = 1$ and $y - 2z = 2$.

   b) (5 marks) Find the coordinates of the point on the line $\mathbf{x}(t) = (1, 2, 3) + t(-1, 1, 2)$ closest to the point $(1, 2, -3)$.

2. (10 marks) Consider the linear system $\begin{cases} x + y = -1 \\ x - kz = k+1 \\ x + y - kz = k-1 \\ x - y + k^2 z = 2k+3 \end{cases}$, where k is a constant.

Find all the possible values of k, if any, for which this system has:

i) No solutions.

ii) Exactly one solution.

iii) Infinitely many solutions.

3. (10 marks) Consider the matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & 1 \\ 1 & 2 & 0 \end{bmatrix}$. Find all matrices M, if any, such that $A + MA = A^T$.

4. (10 marks) Solve the equation: $\det \begin{bmatrix} 0 & 1 & x & 0 \\ 1 & 0 & 0 & 2 \\ x & 0 & 2 & 1 \\ 0 & 2 & 1 & x \end{bmatrix} = 1$.

5. (15 marks) Let $\mathbf{S}$ be the subspace of $\mathbf{R}^5$ consisting of all vectors $\mathbf{v} = (x_1, x_2, x_3, x_4, x_5)$, such that $x_1 - x_2 = x_3 - x_4$, $x_2 - x_3 = x_4 - x_5$, and $x_3 - x_1 = x_5 - x_3$. Find the dimension of $\mathbf{S}$ and give a basis for this subspace.

6. (10 marks) Is the polynomial $1 - x - x^2 - x^3$ a linear combination of the polynomials $1 - x$, $x - x^2$, $x^2 - x^3$, $1 - x^2$, and $x - x^3$? Why or why not?

7. (10 marks) Let $\mathbf{C}[0,1]$ denote the inner product space consisting of all real valued functions which are continuous on the interval $[0,1]$, with the inner product defined as $(\mathbf{f},\mathbf{g}) = \int_0^1 \mathbf{f}(x)\mathbf{g}(x)\,dx$.
Find an orthogonal basis for the subspace of $\mathbf{C}[0,1]$ spanned by the functions $h_1(x) = x$, $h_2(x) = 2 + x$, and $h_3(x) = 12x^2$.

8. (15 marks) Given the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 3 & -1 \end{pmatrix}$. Find an invertible matrix P and a diagonal matrix D such that $P^{-1} A P = D$.

9. (10 marks) Solve the system of linear differential equations $\begin{cases} y_1' = 5y_1 + 4y_2 \\ y_2' = 6y_1 \end{cases}$.