ECE 110 TERM TEST 1

2024-02-08

First name (please write as legibly as possible within the boxes)

FULL

Last name

SOLUTIONS

Student ID number

University of Toronto
Faculty of Applied Science and Engineering
Department of Electrical and Computer Engineering

ECE110S – Electrical Fundamentals

Term Test 1 – February 08, 2024

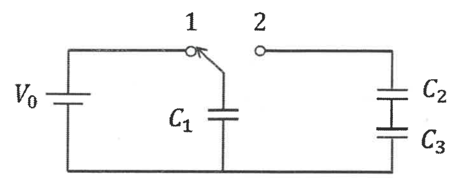
7:10 – 8:40 p.m. (1.5 hours)

($e = 1.6 \times 10^{-19}$ C, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $g = 9.8$ m/s^2)

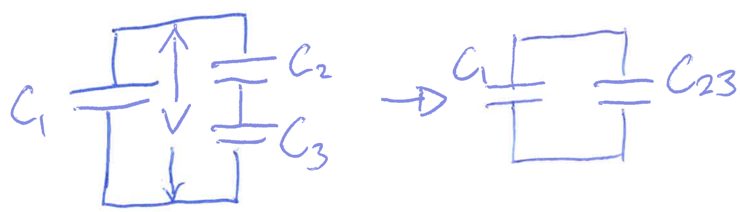
Instructions:

1. Exam Type C3: a one-page (8.5" x 11") double-sided aid sheet is allowed. This aid sheet may be handwritten or computer-generated, including typewritten text, images, or other formats that fit within the paper. Using the template from FASE's website is not mandatory.
2. Non-programmable calculators (Casio FX-991MS & Sharp EL-520X) are permitted.
3. Please use a dark pen or pencil and ensure your handwriting is clear, as the paper will be scanned for grading.
4. For full marks, you must show methods, state UNITS, and compute numerical answers when requested.
5. An extra blank page is provided at the end. If you use this page for your answers, make sure to clearly indicate this within the relevant question area to ensure it is considered during the grading process.
6. TAs and instructors CANNOT answer inquiries regarding the exam questions. If you are unsure about the question, please state your assumptions in your answer.

Q1. The capacitor C_1 acquires a charge of Q_0 when connected to a battery of voltage V_0 as shown in the figure when the switch is in position 1. Then the switch is moved to position 2 connecting C_1 to the initially uncharged capacitors C_2 and C_3.



a) Derive an expression for the voltage across C_3 in terms of Q_0, C_1, C_2 and C_3. **[4 marks]**



$$C_{23} = \frac{C_2 C_3}{C_2 + C_3}$$

$$Q_0 = (C_1 + C_{23})V$$

$$Q_{23} = Q_2 = Q_3 = C_{23}V$$

$$V_3 = \frac{Q_3}{C_3} = \frac{C_{23}}{C_3}V = \frac{C_2}{C_2+C_3}V$$

$$= \frac{C_2}{C_2+C_3}\left(\frac{Q_0}{C_1+C_{23}}\right)$$

$$= \frac{Q_0 C_2}{(C_2+C_3)\left(C_1 + \frac{C_2C_3}{C_2+C_3}\right)}$$

$$= \frac{Q_0 C_2}{C_1(C_2+C_3) + C_2C_3}$$

Q1 continued.

b) If C_1 is a cylindrical capacitor with a length of 3.6 cm, inner and outer conductor radii of 1 mm and 5 mm, respectively. Compute C_1. **[1 mark]**

$$C_1 = 2\pi\varepsilon_0 \frac{\ell}{\ln\left(\frac{b}{a}\right)} = 1.242\,pF$$

c) If C_2 is a parallel plate capacitor with plate area of 2 cm^2 and plate separation of 1 mm, and C_3 is a parallel plate capacitor similar to C_2 except it has a dielectric with dielectric constant of 3 between the plates. Compute C_2 and C_3. **[2 marks]**

$$C_2 = \varepsilon_0 \frac{A}{d} = 1.77\,pF$$

$$C_3 = 3C_2 = 5.31\,pF$$

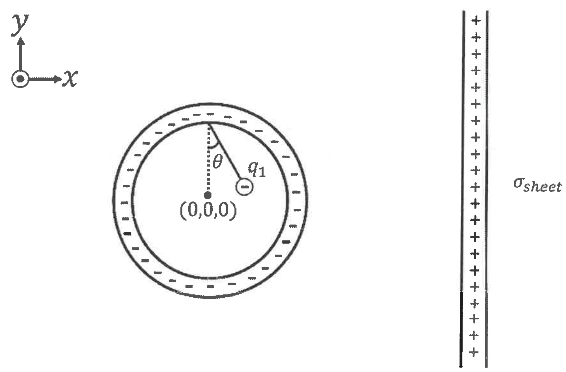
d) If the battery voltage V_0 is 5 volts. Compute the energy stored in C_2 and the voltage across C_3. **[3 marks]**

$$Q_0 = C_1 V_0 = 6.21\,pC \qquad C_{23} = \frac{C_2 C_3}{C_2 + C_3} = 1.33\,pF$$

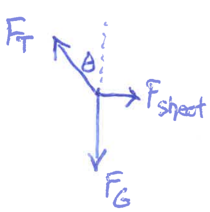
$$V = \frac{Q_0}{C_1 + C_{23}} = 2.41\,V \qquad Q_2 = Q_3 = Q_{23} = C_{23}V = 3.21\,pC$$

$$U_2 = \frac{Q_2^2}{2C_2} = 2.9\,pJ \qquad V_3 = \frac{Q_3}{C_3} = 0.6\,V$$

Q2. Consider a stationary non-conducting spherical shell, with a radius of 1 m that has a negative charge uniformly distributed on its surface. As shown in the figure, a negatively charged particle with a charge $q_1 = -5 \times 10^{-8}\ C$ and a mass of $5 \times 10^{-6}\ kg$ is suspended by a thin insulating thread that is 1 m in length. The angle θ between the thread and the vertical axis is 30°. The origin of the coordinate system is set at the center of the spherical shell. An infinitely large uniformly charged non-conducting sheet is positioned 3 m away from the origin ($x = 3\ m$) in parallel with the y-z plane (shown in cross-section in the figure below). Assume gravitational acceleration is 9.8 m/s^2. The z-axis is perpendicular to the page with the positive direction coming out of the page.



a) What is the surface charge density of the sheet σ_{sheet}? **[6 marks]**



$F_{sheet} = F_T \sin\theta = \frac{1}{2} F_T$

$F_G = F_T \cos\theta = \frac{\sqrt{3}}{2} F_T$

$F_G = mg = 4.9 \times 10^{-5}\ N$

$F_T = 5.65 \times 10^{-5}\ N$

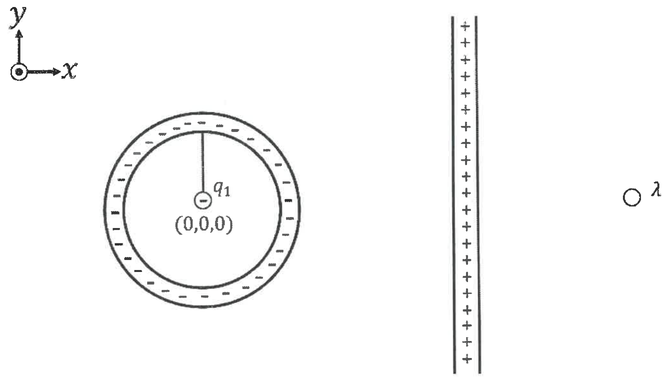
$F_{sheet} = 2.825 \times 10^{-5}\ N$

$F_{sheet} = qE_{sheet}$

$E_{sheet} = \frac{q\sigma}{2\varepsilon_0}$

$\sigma = 1 \times 10^{-8}\ C/m^2$

Q2 continued.



b) Now consider adding a uniformly charged, non-conducting, infinitely-long line placed along the z-axis, centered at the coordinates (10,0,0). If q_1 is located at the origin of the coordinate system (0,0,0). What is the line charge density λ? **[4 marks]**

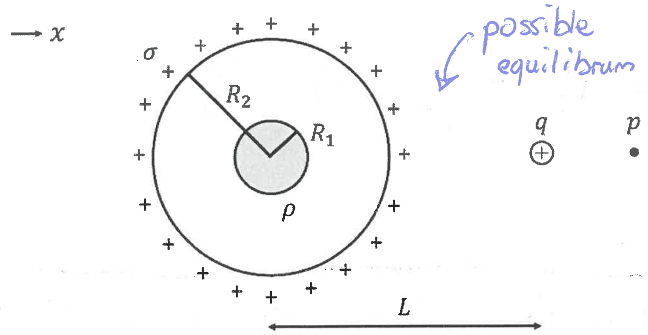
$$F_{line} = -F_{sheet} = -2.825\times10^{-5}\ N$$

$$F_{line} = q\vec{E}_{line}$$

$$E_{line} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$\lambda = -3.14\times10^{-7}\ C/m$$

Q3. Figure show a non-conducting uniformly charged shell of radius $R_2 = 2\ cm$ with charge density $\sigma = \frac{1}{32\,\pi}\frac{nC}{cm^2}$. A uniformly charged solid sphere has charge density $\rho = \frac{3}{8\,\pi}\frac{nC}{cm^3}$,with radius $R_1 = 1\ cm$, is located inside the shell and is co-centered with it. A positive point charge $q = 1\ nC$ is located at $L = 10\ cm$. All of the above charges are fixed in position.



possible equilibrum pos.

a) We wish to place a positive charge q_0 outside of the shell such that q_0 is at equilibrium. Calculate the location at which q_0 must be placed. Show all your work and justify your answer. **[7 marks]**

$$\vec{F}_{q_0,\sigma} + \vec{F}_{q_0,\rho} = \vec{F}_{q_0,q}$$

$$Q_\sigma = 4\pi(R_2)^2\sigma = \tfrac{1}{2}\ nC$$

$$Q_\rho = \frac{4}{3}\pi(R_1)^3\rho = \frac{1}{2}\ nC$$

Let x be the dist. from center of the shell/sphere to location of q_0 then

$$\frac{q_0 \cdot Q_\sigma}{4\pi x^2} + \frac{q_0 \cdot Q_\rho}{4\pi x^2} = \frac{q_0 \cdot q}{4\pi (L-x)^2}$$

$$\frac{1}{2x^2} + \frac{1}{2x^2} = \frac{1}{(L-x)^2} \Rightarrow x = \pm(L-x)$$

$$x = 5\,cm$$

$O = L$ not acceptable sol.

Q3 continued.

b) With q_0 in place, and assuming $q_0 = 1\,nC$, what is total electrostatic potential (voltage) at the location P, where P is $2\ cm$ to the right of the charge q. **[3 marks]**

$$V_T = V_\sigma + V_p + V_q + V_{q_0}$$

$$= \frac{Q_\sigma}{4\pi\varepsilon_0(L+2)} + \frac{Q_p}{4\pi\varepsilon_0(L+2)} + \frac{q}{4\pi\varepsilon_0(2)} + \frac{q_0}{4\pi\varepsilon_0(\frac{L}{2}+2)}$$

$$= 652.976\ V$$

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