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UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, April 2007

MAT186H1 S – CALCULUS I

Calculator Type: Casio 260, Sharp 520, or Texas Instrument 30
Exam Type: C

Examiner – S. Cohen

Name: _____

Student Number: _____

Signature: _____

For Marker's Use Only	
Question	**Mark**
1	/ 20
2	/ 18
3	/ 10
4	/ 15
5	/ 14
6	/ 10
7	/ 10
8	/ 12
9	/ 26
Total	**/ 135**

1. [20 marks total] Find the following limits, or show that they do not exist.

(a) [5 marks] $\lim_{x\to 0} \dfrac{x-\tan x}{x^3}$

(b) [4 marks] $\lim_{x\to -\infty} \left(\dfrac{3x^2-2x}{3x+1} - \dfrac{x^2-1}{x-4} \right)$

(c) [5 marks] $\lim_{x \to 0} \frac{\arctan\left(x^2\right)}{\sin^2 x}$

(d) [6 marks] $\lim_{x \to \infty} \left(\frac{3+x}{x} \right)^{2x+1}$

2. [18 marks total] Evaluate y' in each of the following and simplify where possible.

(a) [4 marks] $xy^2 \sin y = 0$

(b) [4 marks] $\tan y + xe^{4y} = 5xy$

(c) [4 marks] $y = (x^2 - 1)^{\sqrt{x}}$

(d) [6 marks] $y = \arcsin x - \arctan\left(\frac{x}{\sqrt{1-x^2}}\right)$

3. [10 marks total] Given functions f and g such that $\int_1^4 f(x)\,dx = 5$, $\int_4^6 f(x)\,dx = 2$, $\int_1^4 g(x)\,dx = -2$, and $\int_1^6 g(x)\,dx = 1$, evaluate the following:

(a) [2 marks] $\int_1^4 2g(x) - 3f(x)\,dx$

(b) [2 marks] $\int_4^6 g(x)\,dx$

(c) [3 marks] The average value of f on [1,6].

(d) [3 marks] $\int_1^2 xf(x)dx$

$xf(x^2)$

4. [15 marks total] Evaluate the following integrals.

(a) [4 marks] $\int \frac{e^{1/x}}{x^2} dx$

(b) [4 marks] $\int_0^1 \frac{x^3}{1+x^4}\,dx$

(c) [7 marks] $\int \frac{\sqrt{x}}{1+x^3}\,dx$

5. [14 marks total] Let $f(x) = x^5 - 10x^3 - 10x^2$

(a) [4 marks] Find T_4 of f on $[-2,2]$

(a) [6 marks] Find $f'(x)$ and its absolute extrema on $[-2,2]$

(a) [4 marks] Evaluate the greatest possible amount of error in the estimate from part (a).

6. [10 marks] Let a triangle be built by drawing a straight line from the positive x-axis to the positive y-axis and passing through the point $(2,5)$. Find the equation of the line that minimizes the area of the triangle.

7. [10 marks] Water is being pumped into a swimming pool which is 10m wide, 20m long, 1m deep at the shallow end, and 3m deep at the deep end. If the water level is rising at 1cm/min when the depth is 1m at the deep end, at what rate is water being pumped into the pool?

8. [12 marks total] The region R in the plane is bounded by the functions

$$y = \sqrt{x}, \quad y = -x, \quad y = 2.$$

The three-dimensional figure S is then created by rotating R around the x-axis.

(a) [6 marks] Draw the region R (on the plane) and find its area.

(b) [6 marks] Find the perimeter of R (some lengths can be found without integrals, but not all).

9. [26 marks total] Let the solid S be created by rotating the region R (from above) around the x-axis.

(a) [8 marks] Find the volume of S using shells.

(b) [8 marks] Find the volume of S using discs.

(c) [10 marks] Find the surface area of S (some parts are easy enough to do without integrals – but, again, not all).