University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, APRIL 2003
First Year - Programs 1,2,3,4,6,7,8,9

MAT 187H1S
Calculus II

Examiners

D. Burbulla

M. De Oliveira

P. Garfield

G. Karali

C. Lun

SURNAME ______________________________

GIVEN NAME ______________________________

STUDENT NO. ______________________________

SIGNATURE ______________________________

INSTRUCTIONS:

Non-programmable calculators permitted.

No other aids permitted.

Answer all questions.

Present your solutions in the space provided; use the back of the **same** page if more space is required.

TOTAL MARKS: 100

The value for each question is shown in parentheses after the question number.

MARKER'S REPORT	
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
Q8	
TOTAL	

1. (20 marks: each part is worth 5 marks) Find the following:

(a) the unit tangent vector to the curve with parametric equations

$$x = t^2, y = \sqrt{t}, z = \ln t$$

at $t = 1$.

(b) $\displaystyle\int_0^\infty \frac{1}{(1+x^2)^{3/2}}\,dx.$

(c) the interval of convergence, including endpoints (if any), of the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln n}{n} x^n$.

(d) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} 2^{2n}}{(2n+1)!} x^{2n+1}$. (HINT: what is the Maclaurin series for $\sin x$?)

2. (10 marks) Solve for A as a function of t if

$$\frac{dA}{dt} + \frac{A}{100 + 2t} = \frac{1}{5}$$

and $A = 20 \quad$ when $t = 0$.

3. (10 marks; 5 marks for each part.) Find the general solution, y as a function of x, for each of the following differential equations:

(a) $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = 0.$

(b) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 5y = 0.$

4. (10 marks; each part is worth 5 marks) Find the following:

(a) the exact sum (*not* a decimal approximation) of $\sum_{n=0}^{\infty} \frac{1}{(n+1)3^n}$.

(b) the value of $\int_0^{0.5} (1+x^2)^{2/3}\, dx$ correct to within 10^{-4}.

5. (10 marks) Find the critical points of $f(x,y) = 2x^2 + 8xy + y^4$ and at each critical point determine whether f has a relative maximum point, a relative minimum point, or a saddle point.

6. (20 marks) For this question, consider the cardioid with polar equation $r = 1 + \sin\theta$.

(a) (4 marks) Plot the graph of the cardioid.

(b) (6 marks) Find the Cartesion **or** polar coordinates of all the critical points on the graph of the cardioid.

7. (10 marks) Do the following infinite series converge or diverge? Justify your answer.

(a) (3 marks) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n\sqrt{n}}$

(b) (3 marks) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^{n^2}$

(c) (4 marks) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}$

(c) (5 marks) What is the area of the region enclosed by the cardioid?

(d) (5 marks) Find the length of the cardioid for $0 \leq \theta \leq 2\pi$.

8. (10 marks) A boy stands on a cliff 50 m high that overlooks a river 85 m wide. If he can throw a stone at 20 m/sec, can he throw it across the river? (Assume the acceleration due to gravity is $9.8 \mathrm{m/sec}^2$.)