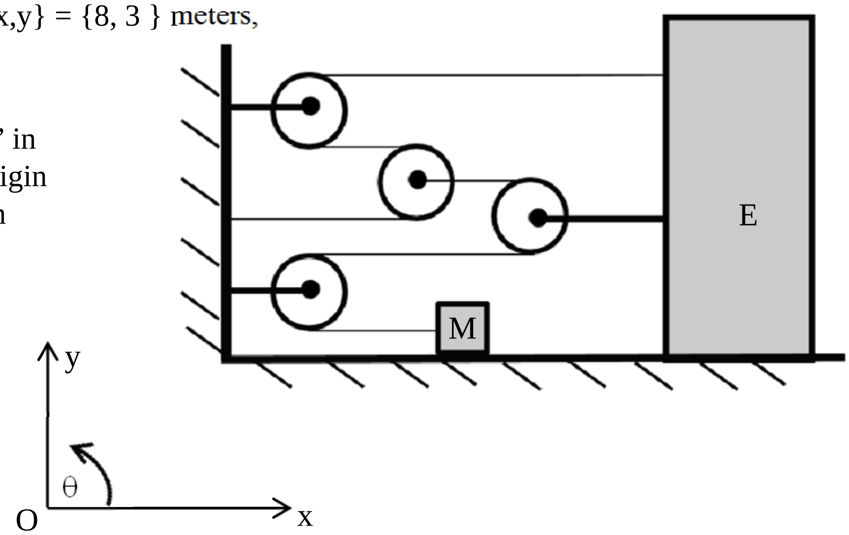
MIE100 Quiz – Jan 31, 2017

Allotted time: 80 minutes. Answer all questions

Answers to be placed in CrowdMark Booklets

1. The blocks "M" and "E" can slide horizontally on the floor. They are attached to a system of 4 pulleys and two ropes as shown. Assume both ropes remain taut at all times. Both blocks move only in the horizontal direction, without rotation. The acceleration of block "M" is not constant, and is given by the following formula in S.I. units: $\vec{a} = \frac{x}{50}\ \frac{m}{s^2}\ \hat{\imath}$

   At time t = 0, the position of block "M" is {x,y} = {8, 3 } meters, and its velocity is: $\vec{v} = -2\frac{m}{s}\ \hat{\imath}$



5 (a) Express the initial velocity of block "M" in polar (r - θ) coordinates, based on the origin "O" and direction of angle θ indicated in the diagram.

10 (b) Determine the velocity of block "E" at time t=0 in x-y coordinates.

10 (c) What will be the position of block "M" in x-y coordinates when its velocity reaches 2.3 m/s $\hat{\imath}$?

25 marks for question #1

2. The small particle P starts from rest at the point {x,y} = {-3, 3.5} meters at time t = 0. It then speeds up to the right at a constant rate of 19.6 m/s^2 as it follows the curved track. The rounded portion of the track has a radius of curvature of 3.5 m.

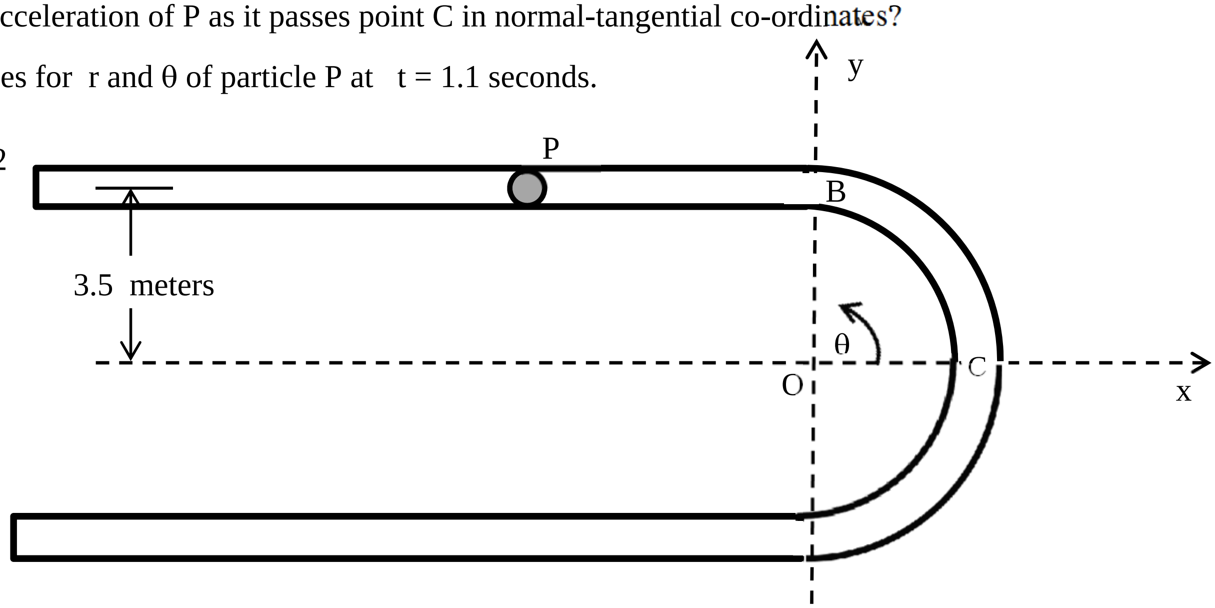
5 (a) What is the velocity of P in r-θ co-ordinates when it reaches point B? (The coordinate θ is measured relative to the x-axis in the direction shown in the diagram.)

5 (b) What are the values of $\ddot{r}$ and $\ddot{\theta}$ when particle P reaches point C, expressed in S.I. units?

5 (c) What is the acceleration of P as it passes point C in normal-tangential co-ordinates?

10 (d) Find the values for r and θ of particle P at t = 1.1 seconds.

25 marks for question #2



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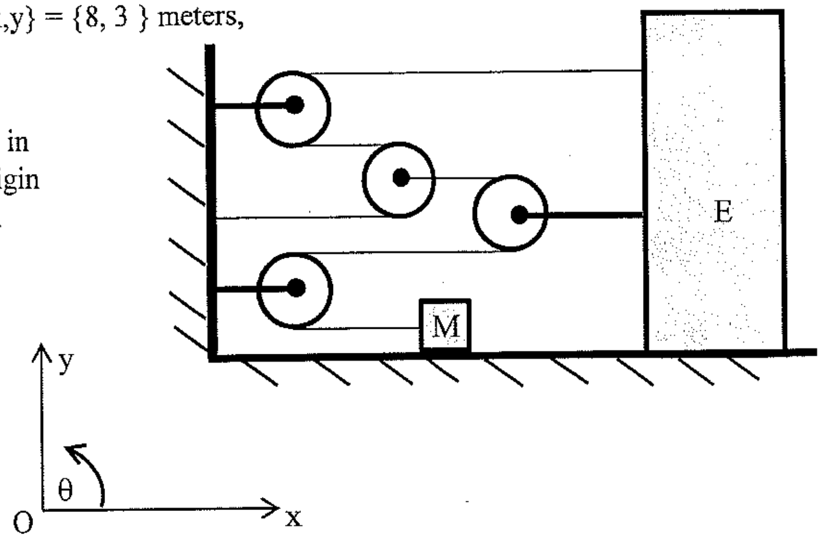

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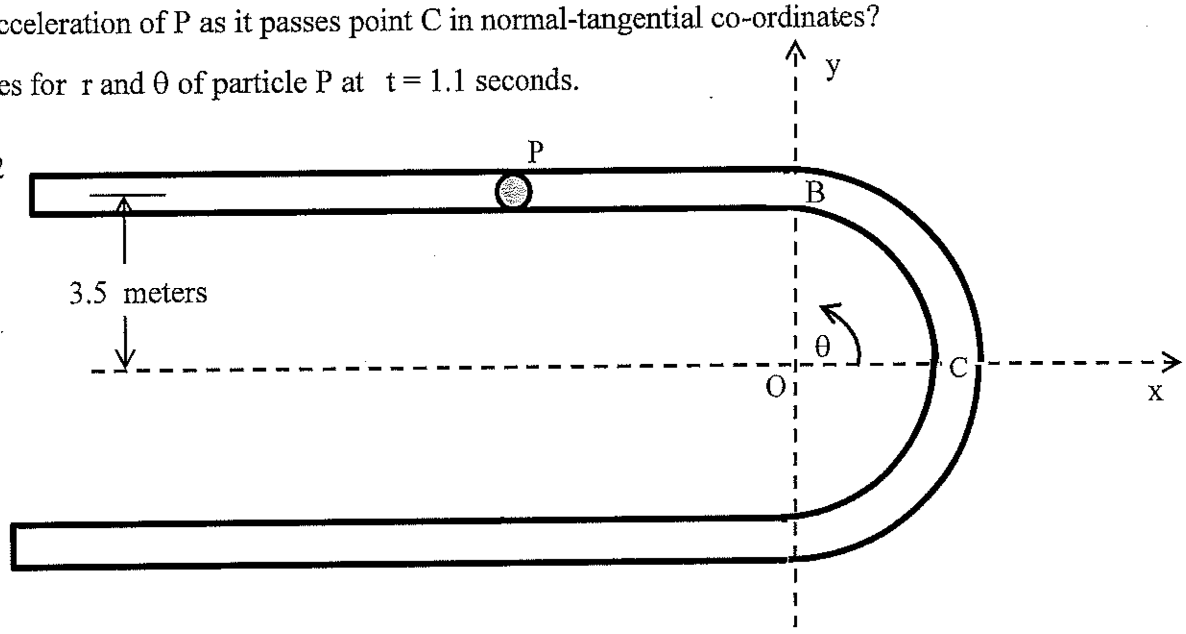
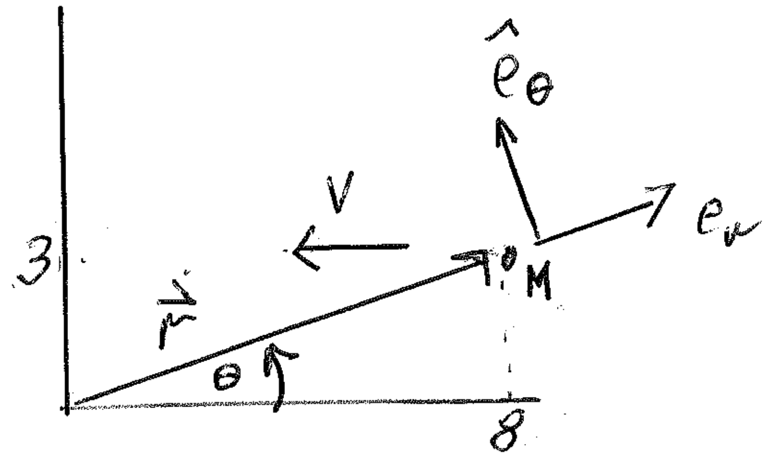


Answers - MIE 100 quiz
Jan 31/2017.

1 (a)



$$\theta = \tan^{-1}\left(\tfrac{3}{8}\right) = 20.56^\circ$$

$$\vec{V} = -2\,m/s\ \hat{i}$$

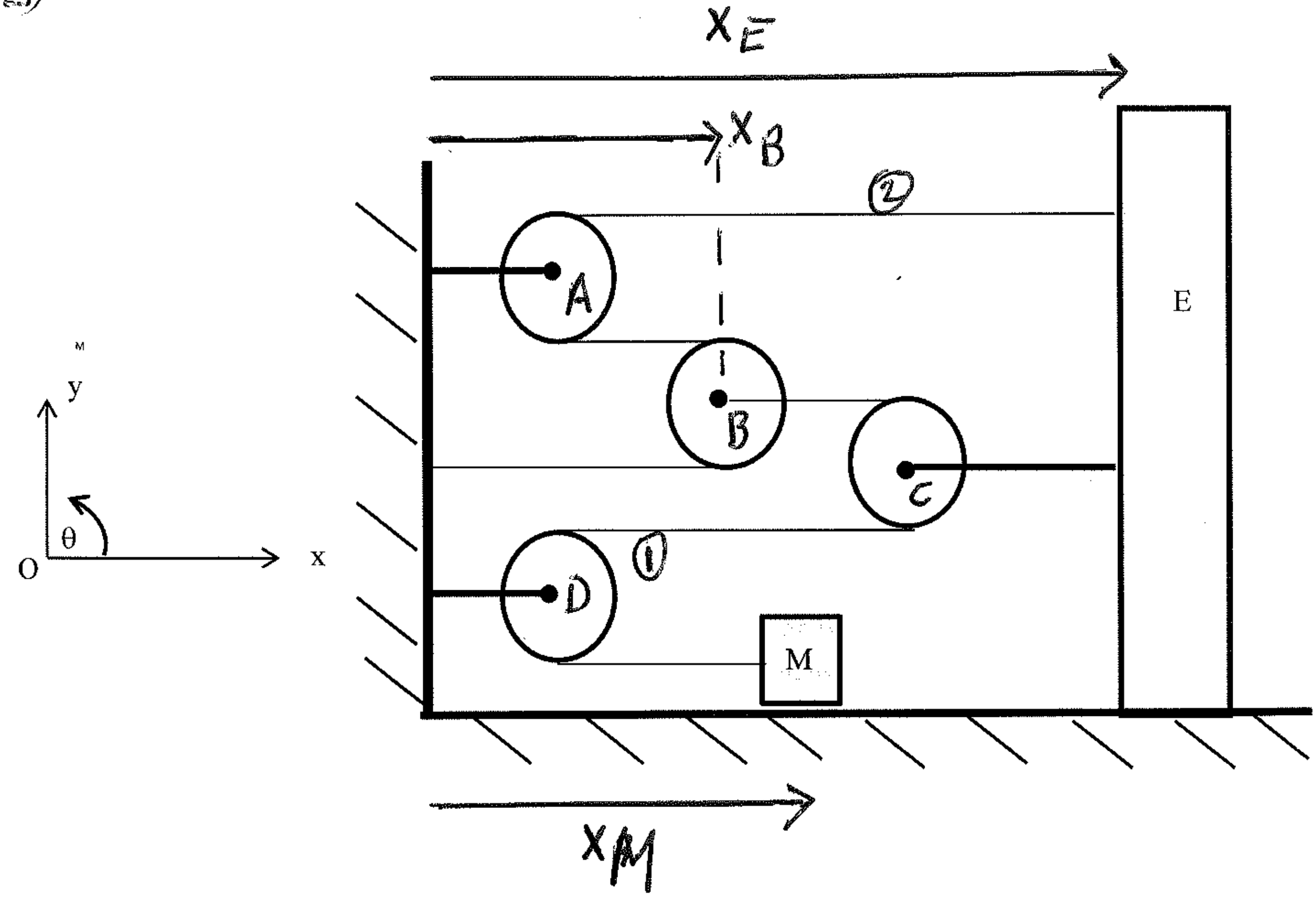
$$= (2\,m/s)\Big(\cos[180^\circ - 20.56^\circ]\ \hat{e}_r \quad + $$

$$- \cos[90^\circ - 20.56^\circ]\ e_\theta\Big]$$

$$= 2\,m/s\left(-0.936\ \hat{e}_r + 0.351\ \hat{e}_\theta\right)$$

$$= -1.872\ m/s\ \hat{e}_r + 0.702\ m/s\ \hat{e}_\theta$$

1(b)



1(b) Assume no rotation

$$\frac{d}{dt}\left\{\text{Length of rope ①} = x_M + x_E + (x_E - x_B)\right\}$$

$$0 = V_M + 2V_E - V_B \qquad *$$

$$\frac{d}{dt}\left\{\text{Length of rope ②} = x_B + x_B + x_E\right\}$$

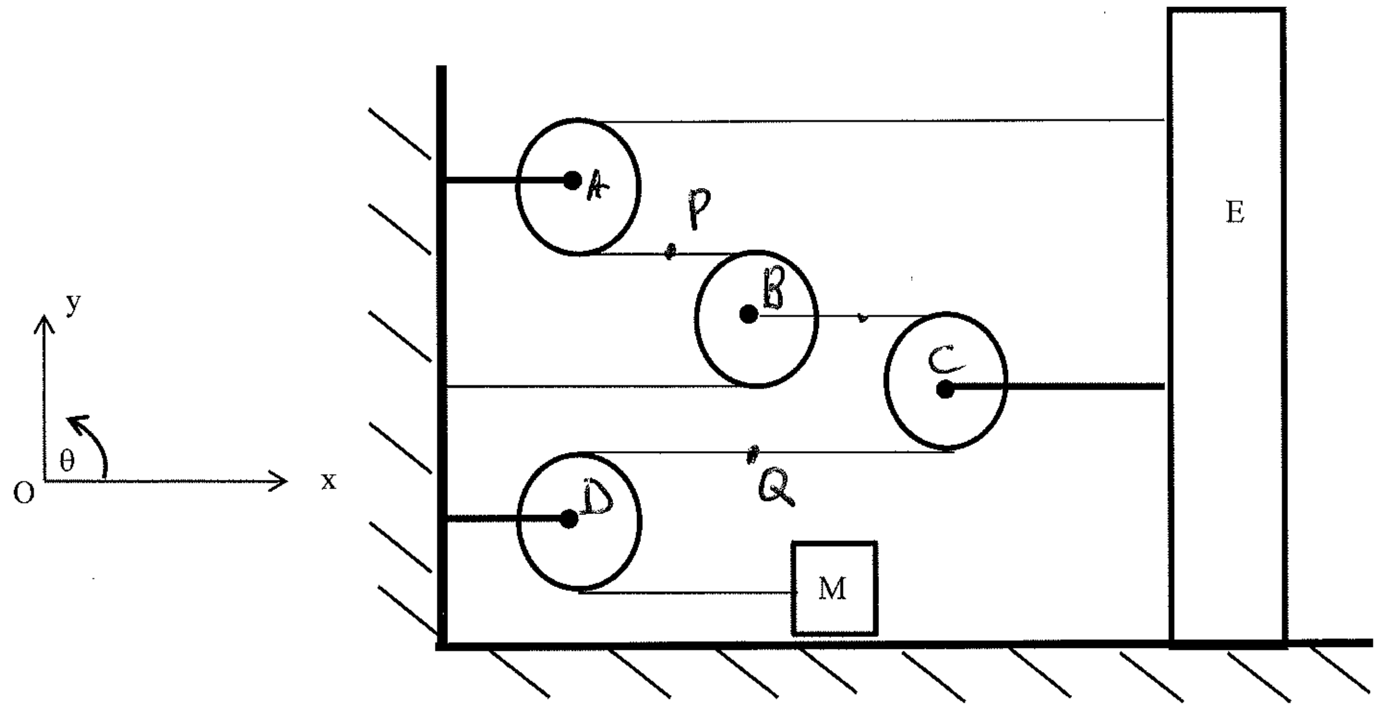
$$0 = 2V_B + V_E \Rightarrow V_B = -\frac{V_E}{2}$$

substitute this last equation into *

$$\Rightarrow \quad 0 = V_M + 2V_E + V_E/2$$

$$\vec{V}_E = -\tfrac{2}{5}\vec{V}_M = \left(-\tfrac{2}{5}\right)(-2\hat{i}) = +0.8 \text{ m/s } \hat{i}$$

Alternative solution to 1(b)



(b) Alternative method:

The speed at the center of each wheel is the average of the speed at the top & bottom of each wheel.

From wheel A, $V_A = \dfrac{V_E + V_P}{2} \Rightarrow V_P = -V_E$

From wheel B, $V_B = \dfrac{V_P + 0}{2} \Rightarrow V_B = \dfrac{V_P}{2} = \dfrac{-V_E}{2}$

From wheel C, $V_E = \dfrac{V_B + V_Q}{2} \Rightarrow V_Q = 5/2\, V_E$

From wheel D, $V_M = -V_Q = -5/2\, V_E$

$$V_E = -\tfrac{2}{5} V_M = \left(-\tfrac{2}{5}\right)\left(-2\hat{\imath}\right)$$

$$\vec{V}_E = 0.80 \text{ m/s } \hat{\imath}$$

1(c)

$$\int a\,dx = \int v\,dv$$

$$\int_{8}^{x_f} \frac{x}{50}\,dx = \int_{-2}^{2.3} v\,dv$$

$$\left.\frac{x^2}{100}\right|_{8}^{x_f} = \left.\frac{v^2}{2}\right|_{-2}^{2.3}$$

$$\frac{x_f^2 - 64}{100} = \frac{2.3^2 - (-2)^2}{2}$$

$$x_f^2 - 64 = 64.5$$

$$x_f = 11.3$$

Final position is $\{x, y\} = \{11.3, 3\}$ meters

2 (a)

$$V^2 = V_0^2 + 2a\Delta S$$

$$V^2 = 0 + (2)(19.6)(3)$$

$$V^2 = 117.6$$

$$V = 10.84$$

In the specified coordinate system,

$$\vec{V} = -10.84 \text{ m/s } \hat{e}_\theta$$

2 (b)

$\ddot{r} = 0$ for circular motion around the origin

$$a_\theta = r\ddot{\theta} + 2\cancel{\dot{r}}\dot{\theta}$$

$$-19.6 = 3.5\ddot{\theta} + 0$$

$$\ddot{\theta} = -5.6 \ s^{-2}$$

(or -5.6 radians$/s^2$)

2(c)

At point C, total distance traveled is

$$\Delta s = 3 + \frac{(2\pi)(3.5)}{4}$$

$$= 3 + 5.5 = 8.5 \text{ meters.}$$

$$v^2 = v_0^2 + 2 a_0 \Delta s$$

$$= 0 + (2)(19.6)(8.5) = 333.2$$

$$v = 18.25 \text{ m/s.}$$

$$a_n = \frac{v^2}{\rho} = (18.25)^2 / 3.5 = 95.2$$

$$a_t = 19.6$$

$$\Rightarrow \vec{a} = 95.2\, \hat{e}_n + 19.6\, \hat{e}_t \text{ m/s}^2$$

2 (d) Distance traveled after 1.1 s:

$$\Delta S = V_0 t + \tfrac{1}{2} a_0 t^2$$

$$= 0 + (\tfrac{1}{2})(19.6)(1.1)^2$$

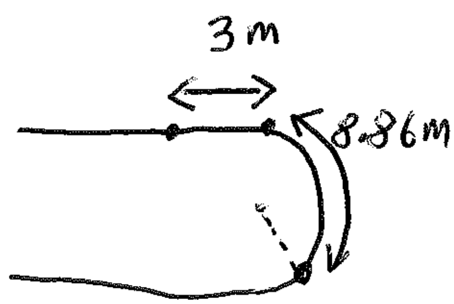
$$\Delta S = 11.86 \text{ m}$$

Distance traveled along arc:

$$11.86 - 3 = 8.86 \text{ m}$$

Fraction of a circle traveled:

$$\frac{8.86}{2\pi R} = \frac{8.86}{(2)(\pi)(3.5)} = 0.40$$



$$\text{Angle } \theta = \frac{\pi}{2} - \frac{8.86}{R}$$

$$= \frac{\pi}{2} - 2.53$$

$$\theta = -0.96 \text{ radians} \quad (\text{or } -55^\circ)$$

Location of P at t = 1.1s is $\begin{cases} r = 3.5\text{m} \\ \theta = -0.96 \text{ radians } (-55^\circ) \end{cases}$

or