MIE Final Exam April 2017.

1. (a) Label center of wheel "O"

$$\vec{a}_M = \vec{a}_o + (a_{M/o})_t \hat{e}_t + (a_{M/o})_n \hat{e}_n$$

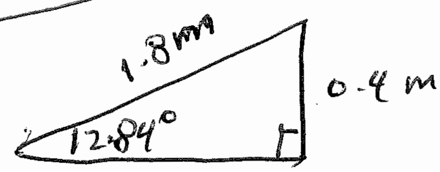
$$= \alpha R \hat{i} + \alpha R \hat{i} + \omega^2 R(-\hat{j})$$

$$= (12)(0.4)\hat{i} + (12)(0.4)\hat{i} + (3^2)(.4)(-\hat{j})$$

$$= 9.6\hat{i} - 3.6\hat{j}$$

$$= (9.6\hat{e}_t + 3.6\hat{e}_n) \text{ m/s}^2$$

(b)



$$\vec{V}_D = (3 s^{-1})(0.4\hat{i} - 0.4\hat{j})$$

$$= 1.2\hat{i} - 1.2\hat{j} \text{ m/s}.$$

$$\vec{V}_D = \vec{V}_B + \vec{V}_{D/B}$$

$$1.2\hat{i} - 1.2\hat{j} = V_B\hat{i} + (\omega_{DB})(1.8\cos 12.84°)(-\hat{j}) + (\omega_{DB})(1.8\sin 12.84°)(\hat{i})$$

solve for $\hat{j}$ components $\Rightarrow$ $\omega_{DB} = 0.684 \text{ s}^{-1}$

**NOTE: Major loss of marks if you say that point "D" moves vertically **

(c)

$$\omega\, d\omega = \alpha\, d\theta$$

$$\frac{\omega}{4\omega} d\omega = d\theta$$

$$1/4 \int_3^{\omega_{final}} d\omega = \int_0^{\pi} d\theta$$

$$1/4(\omega_{final} - 3) = \pi$$

$$\omega_{final} = 4\pi + 3 = 15.57 \text{ s}^{-1}$$

Question #2:

Perfectly elastic collision, coefficient of restitution $e = 1$

Treat as particles undergoing Direct Central Impact

Conservation of momentum:

$$(1) \quad m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$(2) \quad e = \frac{v_B' - v_A'}{v_A - v_B} = 1 \Rightarrow \boxed{v_B' = v_A' + 4}$$

$$(3.5\text{ kg})(4\text{ m/s}) + 0 = (3.5\text{ kg})(v_A') + (0.8\text{ kg})(v_A' + 4)$$

$$v_A' = \frac{(3.5)(4) - (0.8)(4)}{(3.5 + 0.8)}$$

$$v_A' = 2.517\text{ m/s}$$

$$v_B' = 6.517\text{ m/s}$$

$$\boxed{\begin{aligned} \vec{v}_A' &= 2.52\,\hat{i}\text{ m/s} \\ \vec{v}_B' &= 6.52\,\hat{i}\text{ m/s} \end{aligned}}$$

Check Kinetic Energies:

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2$$

$$28\text{ J} + 0 = \frac{1}{2}(3.5)(2.52)^2 + \frac{1}{2}(0.8)(6.52)^2$$

$$28 = 11.0395 + 16.9605$$

$$28\text{ J} = 28\text{ J} \quad ✓$$

Question # 3:

Conservation of Energy for Rigid Bodies applies here because the disk rolls without slip (and thus only static friction is present at the contact point).

$$\cancel{T_{A_1}}^{0} + V_{A_{g_1}} + V_{A_{s_1}} = T_{A_2} + \cancel{V_{A_{g_2}}}^{0} + \cancel{V_{A_{s_2}}}^{0}$$

$$mgh + \frac{1}{2}k(s-s_0)^2 = \frac{1}{2}I_G\omega^2 + \frac{1}{2}mv_G^2$$

$$v_G = \omega R.$$

$$mgh + \frac{1}{2}k(s-s_0)^2 = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega^2 + \frac{1}{2}m(\omega R)^2$$

$$\omega^2 = \frac{mgh + \frac{1}{2}k(s-s_0)^2}{\left(\frac{1}{4}+\frac{1}{2}\right)mR^2}$$

$$\omega^2 = \frac{(3.5)(9.81)(4) + \frac{1}{2}(900)(0.5-0.8)^2}{\frac{3}{4}(3.5)(1.3)^2}$$

$$\omega^2 = \frac{137.34 + 40.5}{4.43625}$$

$$\omega^2 = 40.088$$

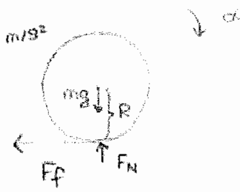
$$\boxed{\omega = 6.33 \text{ rad/s} \circlearrowright}$$

$$\boxed{\vec{v}_G = \omega R = 8.23\,\hat{i} \text{ m/s}}$$

Question 4: (If you run out of room, then continue on page 17 or 18)

$\Sigma M_G = I_G \alpha = F_f R$ $\quad$ $\Sigma F_x = -F_f = ma$ $\quad$ $a = -2.4525\ m/s^2$

↳ kinetic



$\Sigma F_y = N - mg = 0$ $\quad$ $N = mg$ $\quad$ $F_f = \mu_k N = 0.25(9.81)(0.8)$

$= 1.962\ N$

$\underbrace{0.8(0.9)^2}_{mk^2}(\alpha) = 1.8639$ $\quad$ $\alpha = 2.876\ rad/s^2$

when rolling w/o slip

① $v = r\omega$

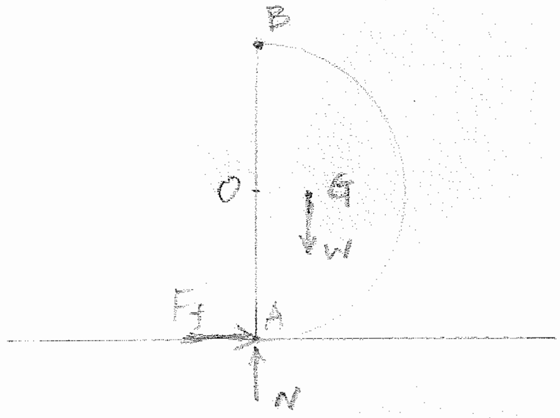
② $\omega = \omega_0 + \alpha t = 0 + 2.876t$ $\quad$ $r\omega = v = 2.876t \cdot r = 2.7322t$

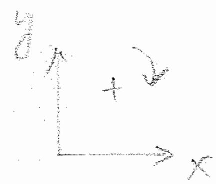
③ $v = v_0 + at = 2.4 - 2.4525t = 2.7322t$

$t = 0.4629\ s$

Q5 Solution:

a)





b)

$$\begin{cases} \Sigma F_x = m a_{Gx} & ① \\ \Sigma F_y = m a_{Gy} & ② \\ \Sigma M_A = I_A \alpha & ③ \end{cases}$$

x: $\Sigma F_x = F_f = \mu_s N$

y: $\Sigma F_y = N - W = N - mg$

Kinematics: $\omega = 0$,

$$\vec{a}_O = \vec{a}_A + \vec{a}_{O/A} = r\alpha\,\vec{i}$$

$$\vec{a}_G = \vec{a}_O + \vec{a}_{G/O} = r\alpha\,\vec{i} + (-\overline{OG}\,\alpha)\,\vec{j}$$

$\therefore$ ① $\Rightarrow$ $F_f = m r \alpha$ $\Rightarrow \mu_s = \dfrac{F_f}{N} = \dfrac{r\alpha}{g - \overline{OG}\,\alpha}$ ④

② $\Rightarrow$ $N = mg - m\,\overline{OG}\,\alpha$

5. (continued)

$+\circlearrowright \quad \Sigma M_A = mg \cdot \overline{OG}$

$$I_A = I_G + m(\overline{AG})^2$$

$$= I_G + m(r^2 + \overline{OG}^2)$$

$$= 0.32\, mr^2 + mr^2 + m \cdot \left(\frac{4r}{3\pi}\right)^2$$

$$= 1.5\, mr^2$$

$\therefore$ (3) $\Rightarrow \quad mg\,\overline{OG} = (1.5\, mr^2)\,\alpha$

$$mg \cdot \frac{4r}{3\pi} = 1.5\, mr^2 \cdot \alpha$$

$$\Rightarrow \alpha = \frac{8}{9\pi}\frac{g}{r} \qquad (5)$$

$\left.\begin{matrix}(4)\\(5)\end{matrix}\right\} \Rightarrow \quad \mu_s = \dfrac{r \cdot \frac{8}{9\pi}\frac{g}{r}}{g - \frac{4r}{3\pi}\cdot\frac{8}{9\pi}\frac{g}{r}} = \underline{\underline{0.322}}$

c) $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} = 0 + (2r)\alpha\,\vec{i} = 2r \cdot \dfrac{8g}{9\pi r}\vec{i} = 0.566\, g\,\vec{i}$

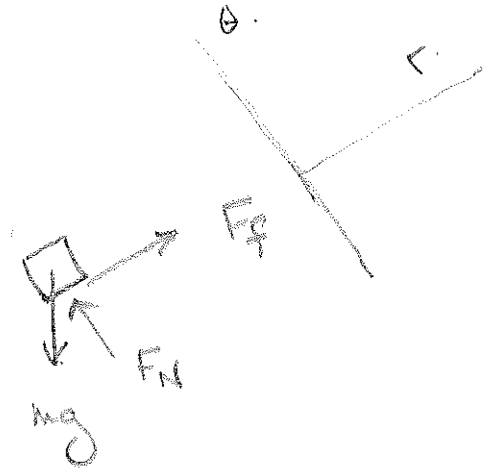
$$\underline{\underline{\vec{a}_B = 5.55\,\vec{i}\ m/s^2}}$$

Question 6 : Exam Solution 2017

$\omega = 3\ s^{-1}$

$m = 0.1$ kg

FBD @ 50°



$F_f = \mu_s F_N$

$\Sigma F_\theta = ma_\theta = m(\cancel{r\ddot{\theta}} + \cancel{2\dot{r}\dot{\theta}}) = 0$

$\Rightarrow F_N - mg\cos 50^\circ = 0 \Rightarrow F_N = .1(9.81)\cos 50^\circ = .63N$

$\Sigma F_r = ma_r = m(\cancel{\ddot{r}} - r\dot{\theta}^2) = m(-.45)(9) =$

$.1(-.45)(9) = -.405$

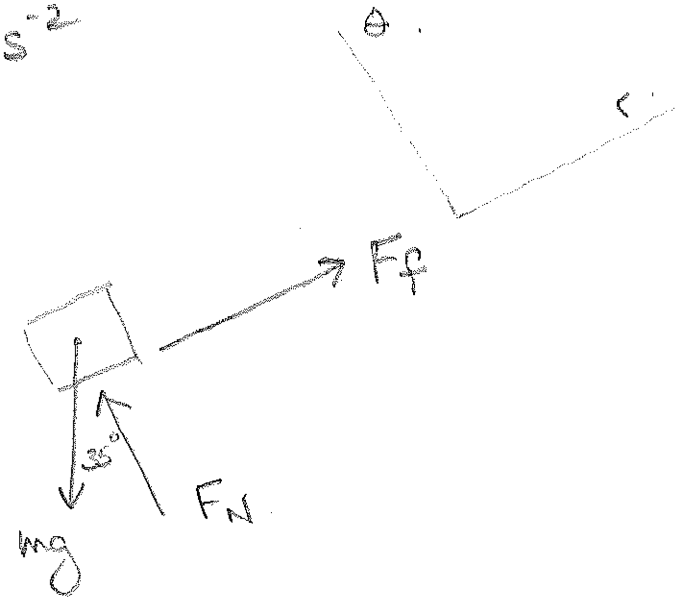
$\Rightarrow -.1(9.81)\sin 50^\circ + \mu_s(.63) = -.405$

$\Rightarrow \mu_s = .346/.63 = \underline{0.55}$

question 7

$\omega = 3\ s^{-1}$

$\alpha = 1.9\ s^{-2}$



$$\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$F_N - mg\cos 35^\circ = mr\ddot{\theta} + 0.$$

$$F_N = .1(.45)(1.9) + .1(9.81)\cos 35^\circ$$

$$= \underline{\underline{0.89\,N}}.$$

$$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2) = -mr\dot{\theta}^2$$

but use high school to find $\dot{\theta}^2$

$$\dot{\theta}^2 = 3^2 + 2(1.9)\left(\frac{35}{180} * 3.1415\right) = 11.3 \qquad \omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta.$$

$$F_f - .1(9.81)\sin 35^\circ = -.1(.45)(11.3)$$

$$\Rightarrow \underline{\underline{F_f = .054\,N}}. \qquad \underline{\underline{mg = 0.98\,N}}$$

8(a)

$$\Sigma M_0 = 3 \times 9.81 \times 2.5 - Kx \times 5 = 0$$

$$73.575 = 1250x$$

$$x = 0.05886\,m \text{ compressed}$$

Question 8b:

$I_o = \frac{1}{3} m L^2 = 25\, kg\, m^2$

$\Sigma M_O = I_o \ddot{\theta} = -kyL + F(t)\frac{L}{2}$

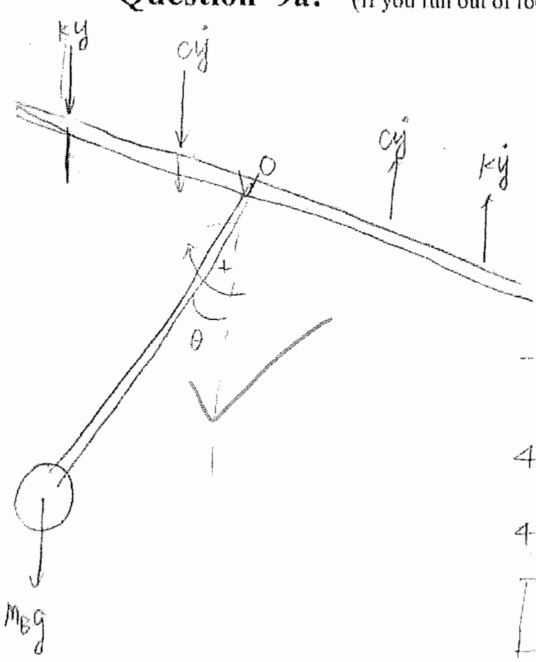
$I_o \ddot{\theta} + kL^2\theta = F(t)\frac{L}{2}$ $\quad w_f = 21\, s^{-1}$

$25\ddot{\theta} + 6250\theta = 25\sin(21t)$ $\quad w_n = \sqrt{\frac{6250}{25}} = 15.811\, s^{-1}$

$\theta_{max} = \frac{25}{6250} \frac{1}{1-(21)^2/(15.811)^2} = -0.00523\, rad$

$|\theta_{max}| = 0.005236$ radians

Question 9a: (If you run out of room, then continue on page 17 or 18)



(a) $I_0 = mr^2$

$= 10 \times l^2 = 10 \times 2^2 = 40\, kg.m^2$

$\Sigma M_0 = I_0 \alpha = 40\ddot{\theta}$

$-2ky \times 0.6L - 2c\dot{y} \times 0.3L - m_B g \times L \times \theta = 40\ddot{\theta}$

$-2k(\theta)(0.6L)(0.6L) - 2c(\dot{\theta})(0.3L)^2 - m_B g \times L \times \theta = 40\ddot{\theta}$

$40\ddot{\theta} + 2(0.3L)^2 c\dot{\theta} + 2k(0.6L)^2\theta + m_B g L\theta = 0$

$40\ddot{\theta} + \frac{18}{25}c\dot{\theta} + 446.4\theta + 196.2\theta = 0$

$\boxed{40\ddot{\theta} + 14.4\dot{\theta} + 642.6\theta = 0}$