University of Toronto
Faculty of Applied Science and Engineering
Department of Electrical and Computer Engineering

ECE110S – Electrical Fundamentals
Term Test 1 – February 13, 2018, 1:00 – 2:30 p.m.

$(e = 1.6 \times 10^{-19}$ C, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, g = 9.81 N/kg)

ANSWER ALL QUESTIONS ON THESE SHEETS, USING THE BACK SIDE IF NECESSARY.
1. Non-programmable calculators (Casio FX-991MS & Sharp EL-520X) are allowed.
2. You are allowed a one page (8.5" x 11") double-sided aid sheet.
3. For full marks, you must show methods, state UNITS and compute numerical answers when requested.
4. Write in PEN. Otherwise, no remarking request will be accepted.
5. There is one extra blank page at the end for rough work.

Last Name: _Full_

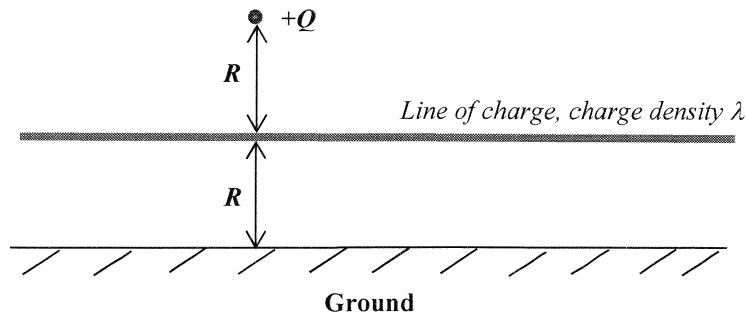**First Name:** _Solutions_

Student Number: _____

Tutorial Section:
(YOU LOSE ONE MARK FOR NOT MARKING YOUR TUTORIAL SECTION CORRECTLY)

☐	01	WB342	Wed. 4-6 p.m.
☐	02	HA403	Mon. 12-2 p.m.
☐	03	HA403	Thurs. 12-2 p.m.
☐	04	HA403	Fri. 4-6 p.m.
☐	05	GB405	Wed. 12-2 p.m.
☐	06	GB404	Tues. 3-5 p.m.
☐	07	BA1240	Fri. 4-6 p.m.
☐	08	SF3202	Fri. 4-6 p.m.
☐	09	BA1200	Mon. 4-6 p.m.
☐	10	LM162	Thurs. 1-3 p.m.
☐	11	SS2135	Thurs. 1-3 p.m.
☐	12	WB119	Fri. 12-2 p.m.

Question	Mark
1	
2	
3	
TOTAL	

Q1 [10 marks]

Figure below shows an infinitely long thin wire with uniform charge density λ [C/m] placed horizontally above the ground. A positive charge +Q (with mass m) is suspended above the charged wire (and ground) as shown.

$+Q$

R

Line of charge, charge density λ

R

Ground

a) What is the sign of the charges on the wire (positive or negative)? Justify your answer. (2 marks)

Positive $\vec{F_Q} = \vec{F_g}$

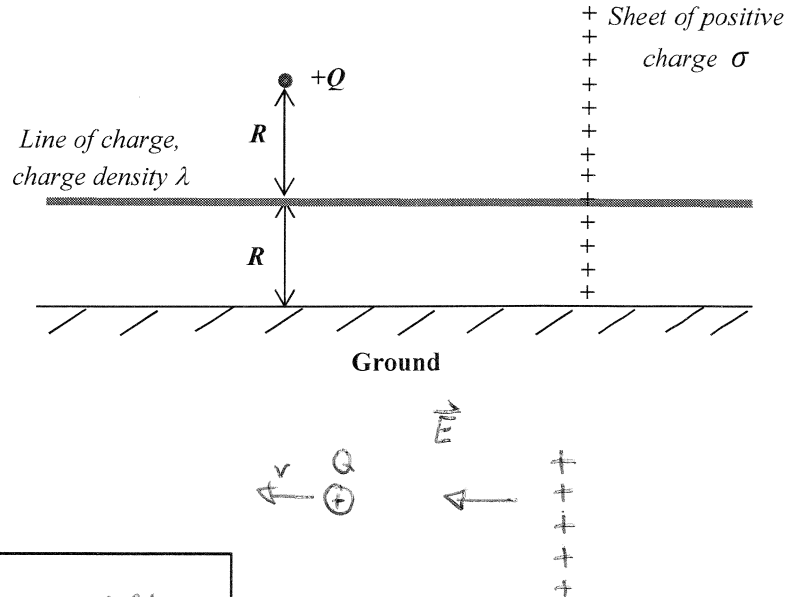**b)** Find an expression for the line charge density λ as a function of Q, m, and other parameters? (3 marks)

$$\vec{F_Q} = Q\vec{E_\lambda}$$
$$\hookrightarrow \vec{E_\lambda} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$\because \vec{F_Q} = \vec{F_g}$$

$$\therefore \frac{Q\lambda}{2\pi\varepsilon_0 r} = mg$$

$$\lambda = \frac{2\pi\varepsilon_0 r mg}{Q}$$

Q1 continued

c) Now, suppose at time $t = 0$, a uniform infinite sheet of positive charge with charge density σ [C/m²] is positioned as shown below. In which direction does the charge $+Q$ begin to move? Justify your answer. (2 marks)

+ *Sheet of positive*
+
+ *charge* σ
+
+
+

$+Q$

Line of charge, R
charge density λ

R

Ground

$\vec{E}$

v Q
$\leftarrow \oplus$ $\leftarrow$

moves from right to left

d) Find an expression for the magnitude of the velocity by which the charge $+Q$ moves in terms of Q, m, t and other parameters. (3 marks)

Hint: acceleration, $a = \dfrac{dv}{dt}$

$$\vec{F_\sigma} = \vec{E_\sigma}Q, \text{ where } \vec{E_\sigma} = \frac{\sigma}{2\varepsilon_o}$$

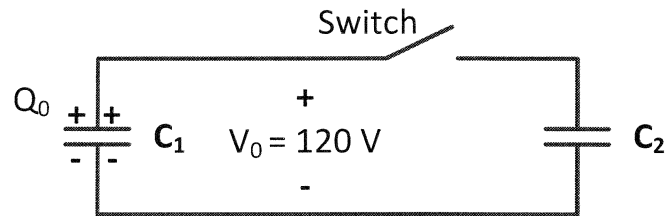$$\vec{F} = ma \longrightarrow \vec{F} = m\frac{dv}{dt}$$

$$\int m\,dv = \int_{t=o}^{t} \frac{\sigma}{2\varepsilon_o}Q\,dt$$

$$mv = \frac{\sigma}{2\varepsilon_o}Qt$$

$velocity = \dfrac{\sigma}{2m\varepsilon_o}Qt$

Q2 [10 marks]

a) The capacitor $C_1 = 8$ uF was initially charged to a potential difference $V_0 = 120$ V through a power supply and then disconnected from the power source. The capacitor $C_2 = 4$ uF was initially uncharged. What is the potential difference across each capacitor and what is the charge on each capacitor **after the switch is closed for a long time** (i.e. after charge no longer flows)? (4 marks)

Switch

Q_0

C_1 $V_0 = 120$ V C_2

$$Q_0 = C_1 V_0 = 960 \mu C$$

after switch closed

$$C_{total} = C_1 + C_2 = 12 \mu F$$

$$V_1 = V_2 = \frac{Q_0}{C_{total}} = 80 V$$

$$Q_1 = C_1 V_1 = 640 \mu C$$

$$Q_2 = C_2 V_2 = 320 \mu C$$

$V_1 = 80V$ $V_2 = 80V$ $Q_1 = 640 \mu C$ $Q_2 = 320 \mu C$

Q2 continued

b) A 5 nF parallel-plate capacitor is charged to 120 V. It is then disconnected from the battery and immersed in distilled water. Distilled water is a good insulator with a dielectric constant $\kappa = 80$.

i) What are the capacitance and voltage of the water-filled capacitor? (2 marks)

$C_{water-filled} = $ 400 nF $\qquad\qquad V_{water-filled} = $ 1.5 V

$$C_{water-filled} = \kappa C_0 = 400 \text{ nF}$$

$$V = \frac{Q}{C}, \quad Q \text{ remains same} \rightarrow V \propto \frac{1}{C}$$

$$V_{water-filled} = \frac{120 V}{\kappa} = 1.5 V$$

ii) What are the energy stored in the capacitor **before** and **after** its immersion? (2 marks)
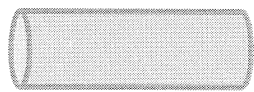
$U_{before} = $ 3.6×10^{-5} J $\qquad\qquad U_{after} = $ 4.5×10^{-7} J

$$U_{before} = \frac{1}{2} C_0 V_0^2 = \frac{1}{2} (5 nF)(120 V)^2 = 3.6 \times 10^{-5} J$$

$$U_{after} = \frac{1}{2} C_{water-filled} V_{water-filled}^2 = 4.5 \times 10^{-7} J$$

Q2 continued

c) Three current-carrying wires are shown below and each wire has a current of I_0. (2 marks)

Variation of radius:
$$r(x) = \frac{2a}{l}x + a$$

Case A
Length = l
Radius = a
Conductivity = σ_0

Case B
Length = l
Square side length = a
Conductivity = $2\sigma_0$

Case C
Length = l
Start/End Radius = a
Middle Radius = $2a$
Conductivity = σ_0

Circle either true (T) or false (F) for each of the following statement. (2 marks)

i) The largest current density is found in **Case C**.   T   (F)

ii) The resistance of **Case A** is larger than that of **Case B**.   T   (F)

c)

$$\text{Current density} = \frac{I_0}{\text{Cross Sec. } A}$$

$$A_{Case\,A} = \pi a^2 \qquad A_{Case\,B} = a^2 \qquad A_{Case\,C} = \pi a^2 \,(min)$$
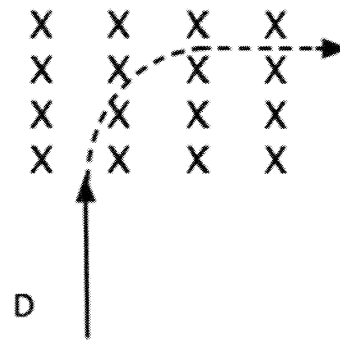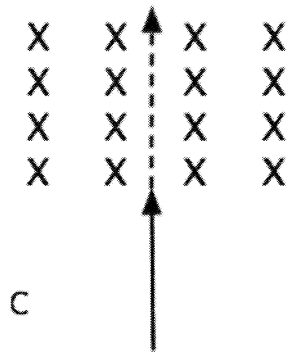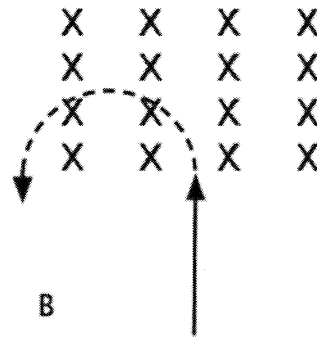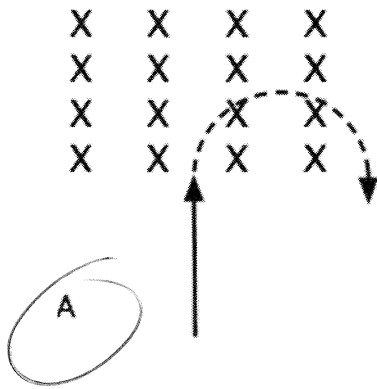
ii)

$$R = \frac{l}{\sigma A}$$

$$R_A = \frac{l}{\sigma_0 \pi a^2} \qquad R_B = \frac{l}{2\sigma_0 a^2}$$

Q3 [10 marks]
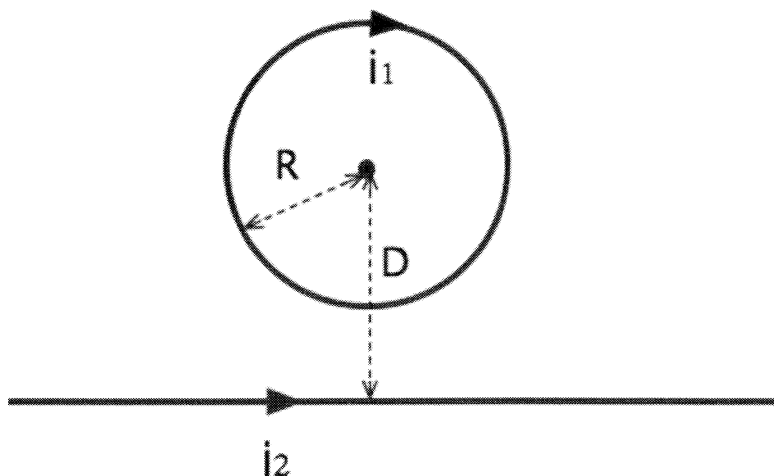
a) Consider a beam of electrons moving with a constant velocity v, through a magnetic field, B, directed into the page, as shown below.

i) Which of the following diagrams correctly describes the motion of the electron beam. Circle the correct answer. (1 mark)

X   X   X   X   X   X   X   X
X   X   X   X   X   X   X   X
X   X   X   X   X   X   X   X
X   X   X   X   X   X   X   X

A   B

X   X   X   X   X   X   X   X
X   X   X   X   X   X   X   X
X   X   X   X   X   X   X   X
X   X   X   X   X   X   X   X

C   D

Q3 continued

b) A circular loop of radius $R = 1$ cm, carries a current, $i_1 = 10$ mA, in a clockwise direction as shown in the figure below. At a distance of 2 cm from the centre of the loop there is a straight wire carrying current $i_2 = 25$ mA.

i) What is the magnitude and direction of the magnetic field at the centre of the loop? (3 marks)

$$B = 3.78 \times 10^{-7} T \left(\text{into page}\right)$$

$$B_{loop} = \frac{\mu_0 i_1}{2R} = 2\pi \times 10^{-7} T \ (\text{into page})$$

$$B_{total} = B_{loop} - B_{wire}$$

$$B_{wire} = \frac{\mu_0 i_2}{2\pi D} = 2.5 \times 10^{-7} T \left(\text{out of page}\right)$$

ii) What value and direction of i_2 would produce zero-magnetic field at the center of the circle? (2 marks)

$$i_2 = 62.8 \, mA \quad \text{from left to right}$$

$$|B_{loop}| = |B_{wire}|$$

$$\frac{\mu_0 i_1}{2R} = \frac{\mu_0 i_2}{2\pi D}$$

$$i_2 = \frac{i_1 \pi D}{R} \quad \text{for } B = 0$$

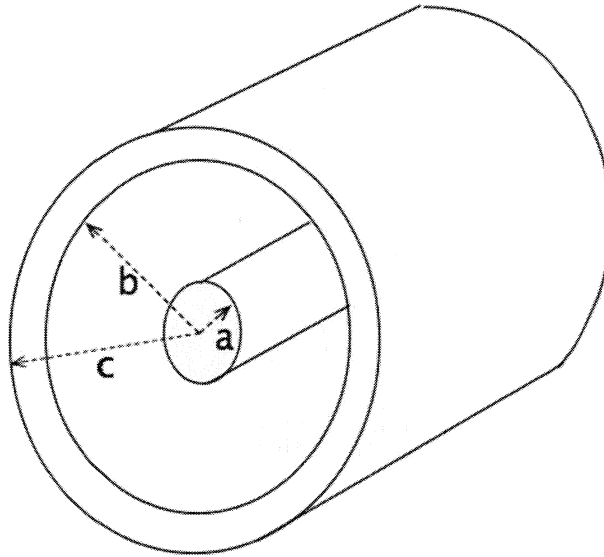$$i_2 = 20\pi \, mA \text{ or } 62.8 \, mA$$

left to right

Q3 continued

c) Consider the two concentric conductors as shown below. The inner conductor has a radius of $a = 1$ cm, while the outer conductor has an inner radius of $b = 3$ cm, and an outer radius of $c = 4$ cm. The inner conductor carriers a current of $I = 40$ mA and the outer conductor carries the same current in the opposite direction.

i) What is the magnitude of the magnetic field at a radius of $r = 2$ cm (between the inner and outer conductors)? (2 marks)

$$B = 4 \times 10^{-7} \, T$$

$$\oint \vec{B} \cdot ds = \mu_0 i$$

$$\oint \vec{B} \cdot ds \rightarrow B \oint ds = B(2\pi r)$$

$$B = \frac{\mu_0 i}{2\pi r} = 4 \times 10^{-7} \, T$$

ii) What is the value of B at a radius of $r = 5$ cm? (2 marks)

$$B = 0$$

Ampere's Law outside both conductors

$$\oint \vec{B} \cdot ds = \mu_0 i_{enc} = 0$$

$$\therefore B = 0$$