Solution

Last Name, First Name: ____

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University of Toronto
Department of Electrical & Computer Engineering
ECE110S – Electrical Fundamentals
Quiz 1A – January 30, 2008, 4:30-5:00 PM

($e = 1.6 \times 10^{-19}$ C, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, g=9.81N/kg)

Instructions: Non-programmable calculators allowed. No other aids. Answer in the space provided on these sheets. The back sides of these sheets can be used as well. For full marks you must show methods, state UNITS and compute numerical answers when requested. **Please write in PEN, not pencil.**

Note: $\hat{i} = \hat{x}$, $\hat{j} = \hat{y}$, $\hat{k} = \hat{z}$ are the unit vectors in the x, y, z, direction respectively.

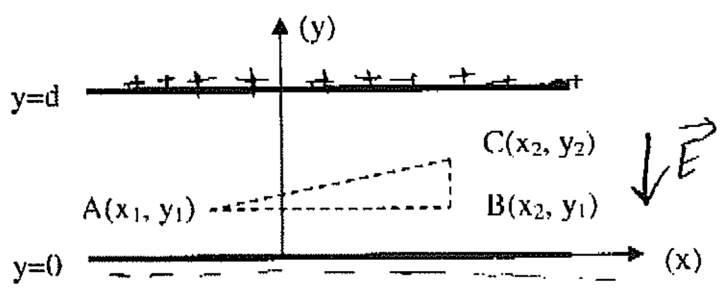
1. Electrostatics [10 marks]

A capacitor consists of two parallel plates of area A=5 cm^2 at a distance d=3 cm from each other. Between the plates, there is a uniform electric field $\vec{E} = -5\,\hat{j}\ V/m$.

(a) Calculate the potential difference ΔV between the plates. Clearly indicate which plate is positively and which is negatively charged. **[3 marks]**

(b) For the points A, B, C indicated, determine whether: $V_A - V_B$, $V_B - V_C$, $V_A - V_C$ are positive, negative or zero. **[3 marks]**

(c) Calculate the capacitance, the total charge on each of the two conductors and the energy stored in the capacitor. **[4 marks]**



(a) $\Delta V = |\vec{E}| \cdot d = (5) \cdot (3\times10^{-2}) = 0.15$ Volt

(b) $V_A - V_B = 0$

$V_B - V_C < 0$

$V_A - V_C < 0$

(c) $C = \varepsilon_0 \frac{A}{d} = \frac{8.85\times10^{-12} \times 5\times10^{-4}}{0.03} = 1.475\times10^{-13}$F

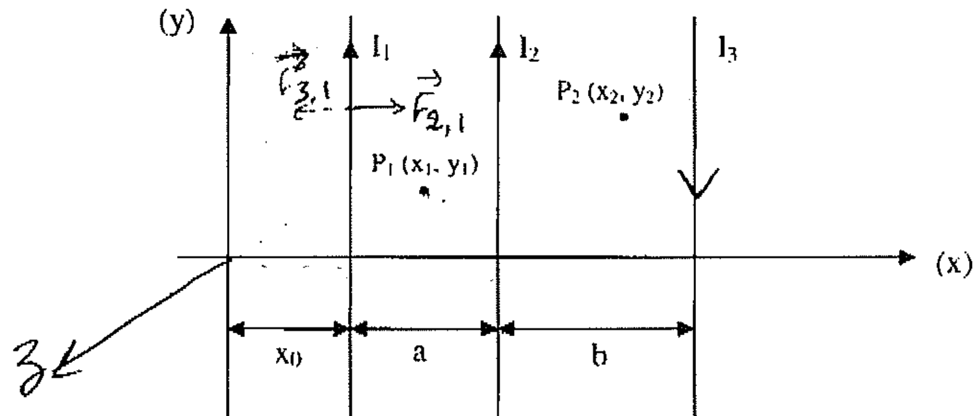
$Q = C \cdot \Delta V = 2.2125\times10^{-14}$ coulomb and $U = \frac{1}{2} C \Delta V^2 \simeq 1.66\times10^{-15}$ Joule.

2. Magnetostatics [10 marks]

Consider the system of three infinite, thin, wires parallel to the y-axis, carrying currents I_1=2 A, I_2=1 A, and I_3=2 A, respectively.

(a) Let a represent the distance between wires 1, 2 and let b represent the distance between wires 2, 3. Calculate the ratio b/a such that no net force is experienced by the first wire. Clearly indicate the corresponding direction of the current I_3. **[4 marks]**

(b) Assuming that I_3 is removed from the system, derive an expression for the net magnetic field (magnitude and direction) produced by wires 1, 2 at two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on the x-y plane (shown in the figure), as a function of their coordinates and x_0, a. **[6 marks]**



(a) $\vec{F}_{2,1} = + \frac{\mu_0 I_1 I_2 L}{2\pi a}\vec{i}$ so $\vec{F}_{3,1}$ should be directed to the left so I_3 is directed along the $(-\vec{j})$ and $\vec{F}_{3,1} = -\mu_0 \frac{I_1 I_3}{2\pi(a+b)}\vec{i}$

so $\vec{F}_{2,1} + \vec{F}_{3,1} = \vec{0} \Rightarrow \frac{I_2}{a} = \frac{I_3}{a+b} \Leftrightarrow \frac{a+b}{a} = \frac{I_3}{I_2} \Leftrightarrow 1 + \frac{b}{a} = 2 \Leftrightarrow \boxed{\frac{b}{a} = 1}$

(b) $\vec{B}_{P_1 (\text{due to } I_1)} = -\frac{\mu_0 I_1}{2\pi(x_1 - x_0)}\vec{K}$ and $\vec{B}_{P_1 (\text{due to } I_2)} = \frac{\mu_0 I_2}{2\pi(\cancel{x_0} - x_1 + a)}\vec{K}$ so

$\vec{B}_{P_1} = \left[\frac{\mu_0}{2\pi(x_1 - x_1 + a)} - \frac{2\mu_0}{2\pi(x_1 - x_0)}\right]\vec{K} = \left(\frac{2\times10^{-7}}{x_1 - x_1 + a} - \frac{4\times10^{-7}}{x_1 - x_0}\right)\vec{K}$ (Tesla)

$\vec{B}_{P_2 (\text{due to } I_1)} = \frac{-\mu_0 I_1}{2\pi(x_2 - x_0)}\vec{k}$ and $\vec{B}_{P_2 (\text{due to } I_2)} = \frac{-\mu_0 I_2}{2\pi(x_2 - x_0 - a)}\vec{k}$ so

$\vec{B}_{P_2} = \left(-\frac{4\times10^{-7}}{(x_2 - x_0)} - \frac{2\times10^{-7}}{(x_2 - x_0 - a)}\right)\vec{K}$ (Tesla)