ECE 110 TERM TEST 2

2024-03-12

First name (please write as legibly as possible within the boxes)

F	U	L	L													

Last name

S	O	L	U	T	I	O	N	S								

Student ID number

University of Toronto
Faculty of Applied Science and Engineering
Department of Electrical and Computer Engineering

ECE110S – Electrical Fundamentals

Term Test 2 – March 12, 2024

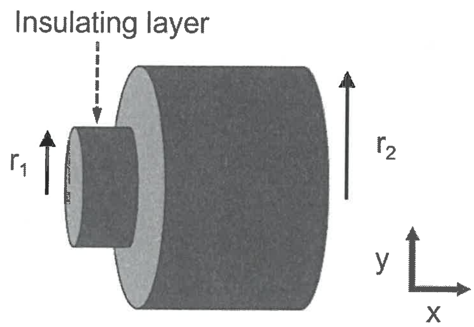
1:10 – 2:40 p.m. (1.5 hours)

($e = 1.6 \times 10^{-19}$ C, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $g = 9.8$ m/s^2)

Instructions:

1. Exam Type C3: a one-page (8.5" x 11") double-sided aid sheet is allowed. This aid sheet may be handwritten or computer-generated, including typewritten text, images, or other formats that fit within the paper. Using the template from FASE's website is not mandatory.
2. Non-programmable calculators (e.g., Casio FX-991MS & Sharp EL-520X) are permitted.
3. Please use a dark pen or pencil and ensure your handwriting is clear, as the paper will be scanned for grading.
4. For full marks, you must show methods, state UNITS, and compute numerical answers when requested.
5. An extra blank page is provided at the end. If you use this page for your answers, make sure to clearly indicate this within the relevant question area to ensure it is considered during the grading process.
6. TAs and instructors CANNOT answer inquiries regarding the exam questions. If you are unsure about the question, please state your assumptions in your answer.

Q1. Consider an infinitely long wire consisting of two concentric components. An inner solid cylinder with radius $r_1 = 1$ [cm], and a hollow cylindrical shell with an outer radius of $r_2 = 3$ [cm]. The current density within each component is defined by (1) $J_{Cylinder} = 250\ [\frac{\text{A}}{\text{m}^2}]$ (positive x-direction) and (2) $J_{Shell} = 500\ [\frac{\text{A}}{\text{m}^2}]$ (negative x-direction). Assume an infinitesimally thin insulating layer between the two components of the wire.



a) Determine the radius within the wire where the magnetic field is zero **[3 marks]**.

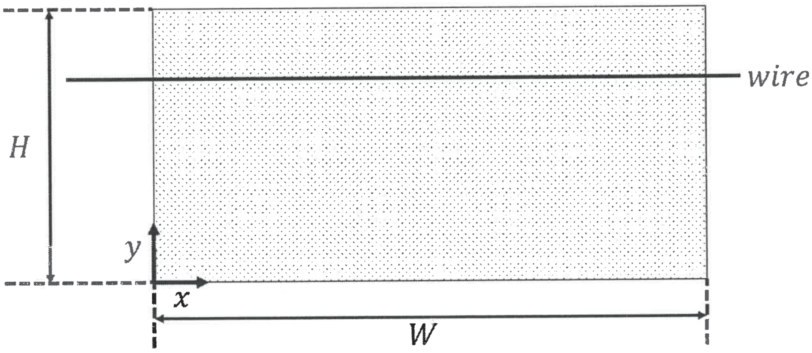
$I_{in} = \pi r_1^2 \cdot J_{cylinder} = 0.08 [A]$

$0.08 \doteq J_{shell} \cdot \pi (r^2 - 0.01^2)$

$r = 1.23$ [cm]

Q1 continued.

b) The infinitely long wire is then positioned above a uniform magnetic field ($B = 3.5 \times 10^{-7}$ [T]), which is defined within the rectangular area ($w = 3$ [m], $h = 1$ [m]). The direction of the magnetic field is pointing out of the plane (positive z-direction). Calculate the magnitude and direction of the magnetic force exerted on the wire. **[4 marks].**

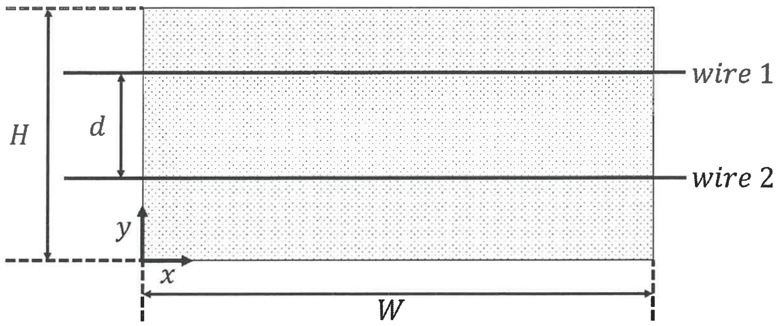


Total $I = I_{in} - I_{out} = 0.08 - 500\pi(0.03^2 - 0.01^2)$
$= 1.176$ [A] (neg. x-dir)

$F_B = I L \times B = 1.176 \cdot 3 \cdot 3.5 \times 10^{-7}$
$= 1.2348 \times 10^{-6}$ N [pos. y-dir]

Q1 continued.

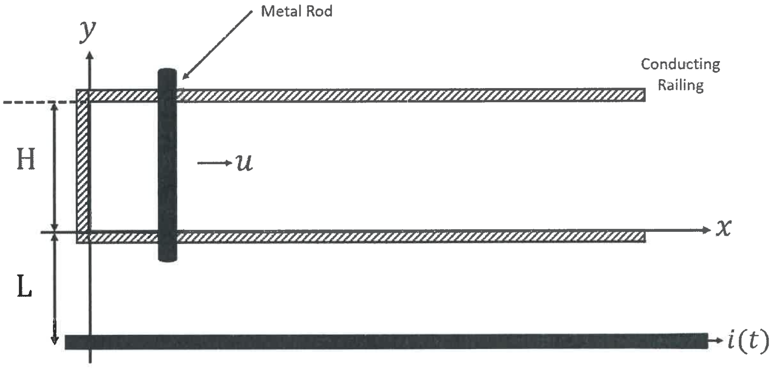
c) Consider a system where 2 parallel wires are located above the uniform magnetic field ($B = 3.5 \times 10^{-7}$ [T]), rectangular area ($w = 3$ [m], $h = 1$ [m]), where wire 2 is fixed in place and both wires are separated by a distance ($d = 40$ [cm]). If the current flowing through wire 1 is 0.2 [A] (positive x-direction), what current is needed in wire 2 to prevent the first wire from moving? **[3 marks]**



$$B = \frac{\mu_0 I_2}{2\pi d} = 3.5 \times 10^{-7}$$

$$I_2 = \frac{3.5 \times 10^{-7}}{\mu_0} 2\pi d = 0.7 [A] \text{ (neg. x-dir)}$$

Q2. A conductive U-shaped railing is placed in the x-y plane as shown below. A movable metal rod (touching the railing) is initially located parallel to the y-axis at $x = 0$ [m]. At $t = 0$ [s], the metal rod starts to move in the positive x-direction with a constant speed, $\boldsymbol{u}$ [m/s]. A infinite long straight wire carrying a current, $\boldsymbol{i(t)} = \mathbf{2}\boldsymbol{t}$ [A] lies parallel to the x-axis at $y = -L$ [m].



a) Determine the magnitude of the magnetic flux through the closed area formed between the conductive railing and the metal rod as a function of time. **[6 marks]**

$$\Phi_B = \int_0^H \vec{B} \cdot d\vec{A} = \int_0^H \frac{\mu_0 i(t)}{2\pi(y+L)} x\, dy = \frac{\mu_0 i(t) x}{2\pi} \int_0^H \frac{1}{y+L} dy$$

Ver. 1

$$\Phi_B = \frac{\mu_0 i(t) x}{2\pi} \int_0^H \frac{1}{y+L} d(y+L)$$

$$= \frac{\mu_0 i(t) x}{2\pi} \ln(y+L) \Big|_0^H$$

$$= \frac{\mu_0 \cdot 2t \cdot ut}{2\pi} \ln\left(\frac{H+L}{L}\right)$$

$$= \frac{\mu_0 u t^2}{\pi} \ln\left(\frac{H+L}{L}\right) [\mathrm{Wb}]$$

Ver. 2 let $w = y+L$

$$\Phi_B = \frac{\mu_0 i(t) x}{2\pi} \int_L^{H+L} \frac{1}{w} dw$$

$$= \frac{\mu_0 i(t) x}{2\pi} \ln(w) \Big|_L^{H+L}$$

$$= \frac{\mu_0 \cdot 2t \cdot ut}{2\pi} \ln\left(\frac{H+L}{L}\right)$$

$$= \frac{\mu_0 u t^2}{\pi} \ln\left(\frac{H+L}{L}\right) [\mathrm{Wb}]$$

Q2 continued.

b) Find the magnitude of the induced emf in the conductive path formed by the railing and the rod. **[2 marks]**

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt}\left(\frac{\mu_0 u t^2}{\pi}\ln\left(\frac{H+L}{L}\right)\right)$$

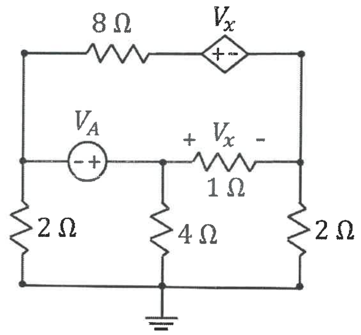
$$= \frac{2\mu_0 u t}{\pi}\ln\left(\frac{H+L}{L}\right) \; [V]$$

c) Circle the direction of the induced current in the conductive path. **[2 marks]**

Counter-clockwise

Clockwise

Q3. Consider the circuit below.

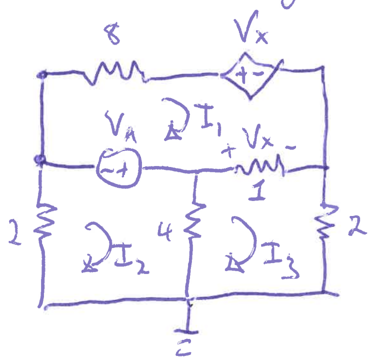


a) If $V_A = 0$ [V], determine V_x **[1 mark]**

$V_x = 0$ [V]

b) If $V_A = 8$ [V], determine V_x **[6 marks]**

Mesh Analysis



① $8I_1 + V_x - V_x + 8 = 0$

② $-8 + 4(I_2 - I_3) + 2I_2 = 0$

③ $V_x + 2I_3 + 4(I_3 - I_2) = 0$

$V_x = (I_3 - I_2)1$

$I_1 = -1$ [A]

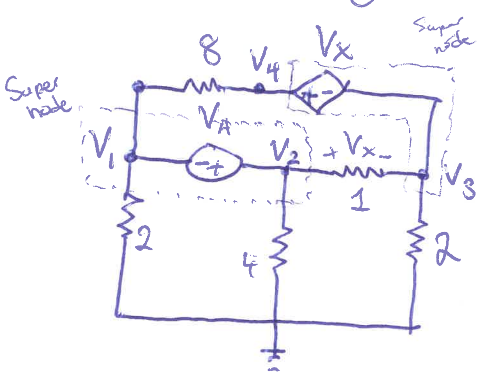
$I_2 = 2$ [A]

$I_3 = 1$ [A]

$V_x = 2$[V]

Q3 continued.

Nodal Analysis



$V_1 = -4[V]$

$V_2 = 4[V]$

$V_3 = 2[V]$

$V_4 = 4[V]$

$V_x = 2[V]$

① $\frac{V_1}{2} + \frac{V_2}{4} + \frac{V_1 - V_4}{8} + \frac{V_2 - V_3}{1} = 0$

② $\frac{V_4 - V_1}{8} + \frac{V_3 - V_2}{1} + \frac{V_3}{2} = 0$

$V_2 - V_1 = 8$

$V_4 - V_3 = V_x$

$V_2 - V_3 = V_x$

c) If $V_A = 8$ [V], determine the power of the voltage-controlled voltage source **[2 marks].** Is it being supplied or absorbed? Circle one answer **[1 mark]**

Supplied

Absorbed

$P = V_x \cdot I_1 = -2[W]$

This page is intentionally left blank. You may use this sheet for rough work.