FORMULA SHEET FOR MAT187

Trigonometric Identities.

- $\cos^2(x) = \dfrac{1+\cos(2x)}{2}$
- $\sin^2(x) = \dfrac{1-\cos(2x)}{2}$
- $\sin(x) = \cos\left(\dfrac{\pi}{2} - x\right)$

Applications of integration.

- Arc length for $y = f(x)$ $$\int_a^b \sqrt{1+\left(f'(x)\right)^2}\,dx$$
- Area of a surface of revolution $y = f(x)$ revolved around $x-$axis $$\int_a^b 2\pi f(x)\sqrt{1+\left(f'(x)\right)^2}\,dx$$
- Hyperbolic Functions $$\begin{cases} \sinh(x) = \dfrac{e^x - e^{-x}}{2} \\ \cosh(x) = \dfrac{e^x + e^{-x}}{2} \end{cases}$$

Power Series.

- Taylor Theorem $$\begin{cases} f(x) = p_n(x) + R_n(x) \\ p_n(x) = \displaystyle\sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k \\ R_n(x) = \dfrac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1} \end{cases}$$
- Binomial Series $$(1+x)^p = \sum_{k=0}^{\infty} \frac{p(p-1)\cdots(p-k+1)}{k!}x^k$$
- Sine Series $$\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}x^{2k+1}$$
- Cosine Series $$\cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}x^{2k}$$
- Logarithmic Series $$\ln(1-x) = -\sum_{k=1}^{\infty} \frac{1}{k}x^k$$

Vector-Valued Functions.

- Area of a polar function $r = f(\theta)$ $$\frac{1}{2}\int_\alpha^\beta \left(f(\theta)\right)^2 d\theta$$
- Length of a parametric curve $\vec{r}(t)$ $$\int_a^b \left|\vec{r}\,'(t)\right| dt$$
- Length of a polar curve $r = f(\theta)$ $$\int_\alpha^\beta \sqrt{\left(f(\theta)\right)^2 + \left(f'(\theta)\right)^2}\,d\theta$$
- Tangent and Normal vectors $$\vec{T}(t) = \frac{\vec{r}\,'(t)}{\left|\vec{r}\,'(t)\right|} \quad , \quad \vec{N}(t) = \frac{\vec{T}\,'(t)}{\left|\vec{T}\,'(t)\right|} \quad , \quad \vec{B}(t) = \vec{T}\times\vec{N}$$
- Curvature $$\kappa = \left|\frac{d\vec{T}}{ds}\right| = \frac{\left|\vec{T}\,'(t)\right|}{\left|\vec{r}\,'(t)\right|} = \frac{\left|\vec{v}\times\vec{a}\right|}{\left|\vec{v}\right|^3}$$