Velocity and acceleration
$$v = \frac{ds}{dt}$$
$$a = \frac{dv}{dt} = v\frac{dv}{ds}$$

Special cases
$(a = 0)$ $s = s_0 + v_0 t$

$$v^2 = v_0^2 + 2\int_{s_0}^{s} a(s)\, ds$$

(const a) $v = v_0 + a_0 t$
$$s = s_0 + v_0 t + \frac{1}{2}a_0 t^2$$
$$v^2 = v_0^2 + 2a_0(s - s_0)$$

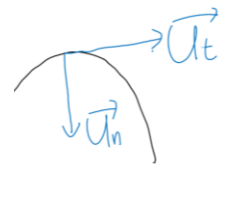
$$v = v_0 + \int_0^t a(t)\, dt$$
$$s = s_0 + v_0 t + \int_0^t \int_0^t a(t)\, dt\, dt$$

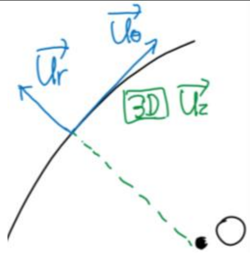
Rectangular coordinates
$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$
$$\vec{v} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}$$
$$\vec{a} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}$$
$$|\vec{v}| = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

SUVAT equation (a const)
$$v = v_0 + a_0 t$$
$$s = s_0 + v_0 t + \frac{1}{2}a_0 t^2$$
$$v^2 = v_0^2 + 2a_0(s - s_0)$$
$$s = \frac{(v + v_0)t}{2}$$

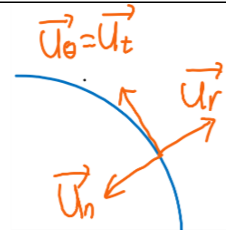
Projectile motion
$$v_{0x} = v_0 \cos\theta$$
$$v_{0y} = v_0 \sin\theta$$
$$a_x = 0$$
$$x = x_0 + v_0 t$$
$$a_y = a_{0y}$$
$$y = y_0 + v_{0y} t + \frac{1}{2}a_{0y} t^2$$

Normal-tangential system
$$\overrightarrow{v_{n-t}} = v\overrightarrow{u_t}$$
$$\overrightarrow{u_b} = \overrightarrow{u_t} \times \overrightarrow{u_n}$$
$$v = \frac{ds}{dt}$$
$$\vec{a} = \dot{v}\overrightarrow{u_t} + v\dot{\theta}\overrightarrow{u_n}$$
$$= \dot{v}\overrightarrow{u_t} + \frac{v^2}{\rho}\overrightarrow{u_n}$$
$$\rho = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{1.5}}{\left|\frac{d^2y}{dx^2}\right|}$$



Cylindrical $r - \theta$ system


$$\vec{r} = r\overrightarrow{u_r}$$
$$\vec{v} = \dot{r}\overrightarrow{u_r} + r\dot{\theta}\overrightarrow{u_\theta}$$
$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\overrightarrow{u_r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\overrightarrow{u_\theta}$$
$$\dot{\overrightarrow{u_r}} = \dot{\theta}\overrightarrow{u_\theta}$$
$$\dot{\overrightarrow{u_\theta}} = -\dot{\theta}\overrightarrow{u_r}$$

Circular motion


Dependent motion
- Rope has constant length
- Define good datum lines (fixed position)
- Find fixed length if possible
- Divide the rope into sections if needed

$$L_T = s_A + s_B$$
Then $v_A + v_B = 0$
$$a_A + a_B = 0$$

Relative motion
$$\overrightarrow{r_B} = \overrightarrow{r_A} + \overrightarrow{r_{B/A}}$$
$$\overrightarrow{v_B} = \overrightarrow{v_A} + \overrightarrow{v_{B/A}}$$
$$\overrightarrow{a_B} = \overrightarrow{a_A} + \overrightarrow{a_{B/A}}$$

Gravitational force
$$\vec{g} = -9.81\vec{j}\,\mathrm{ms^{-2}}$$
$$F = m\vec{g}$$

Frictional force (oppose motion)
Static $|F_{fsmax}| = \mu_s F_N$

Kinetic $|F_{fk}| = \mu_k F_N$

$F_{fk} > \mu_k F_N$ velocity decrease

$F_{fk} = \mu_k F_N$ velocity same

$F_{fk} < \mu_k F_N$ velocity increase

Spring force
$$F_s = -kx$$
k is spring constant

x is deviation from rest

Equilibrium (x-y-z)
$$\sum F_x = ma_x = m\ddot{x}$$
$$\sum F_y = ma_y = m\ddot{y}$$
$$\sum F_z = ma_z = m\ddot{z}$$

Equilibrium (n-t)
$$\sum F_n = ma_n = mv\dot{\theta} = \frac{mv^2}{\rho}$$
$$\sum F_t = ma_t = m\dot{v}$$

Equilibrium $(r - \theta)$
$$\sum F_r = ma_r = m\left(\ddot{r} - r\dot{\theta}^2\right)$$
$$\sum F_\theta = ma_\theta = m\,(2\dot{r}\dot{\theta} + r\ddot{\theta})$$

Work and motion
$$dU = \vec{F}\cdot d\vec{r} = F\cos\theta\, dr$$
$$U_{P>P'} = \int_P^{P'} dU = \int_P^{P'} \vec{F}\, d\vec{r} = \int_P^{P'} F\cos\theta\, ds$$

Work by gravitational F
$$U_g = -W\Delta y$$
$$= -mg(y_2 - y_1)$$
*Always negative

Work by kinetic friction
*Against motion> negative
$$U_f = -F_f\Delta x$$

Work by spring
$$U_s = \int_{x_1}^{x_2} -kx\, dx$$
$$= -\frac{1}{2}k(x_2^2 - x_1^2)$$

Kinetic energy
$$T = \frac{1}{2}mv^2$$
$$T_1 + U_{1>2} = T_2$$
$$\frac{1}{2}m_i v_{i1}^2 + \int_{s_{i1}}^{s_{i2}} \overrightarrow{F_{it}}\, ds + \int_{s_{i1}}^{s_{i2}} \overrightarrow{f_{it}}\, ds = \frac{1}{2}m_i v_{i2}^2$$

Internal force is zero
If rigid body/particles connected by inextensible cable
$$\int_{s_{i1}}^{s_{i2}} \overrightarrow{f_{it}}\, ds = 0$$

Work done by force
$$U_g = -mg\Delta y$$
$$U_s = -\frac{1}{2}k(s_2^2 - s_1^2)$$
$$U_f = -F_{fk}\Delta s$$

Potential energy
$$V_g = mgh$$
$$V_s = \frac{1}{2}kx^2$$

Conservation of energy
$$T_1 + V_1 + U_{1>2} = T_2 + V_2$$
If $(\mathbf{U_{1>2}} = \mathbf{0})$
$$T_1 + V_1 = T_2 + V_2$$

Linear momentum
$$\vec{L} = m\vec{v}$$

Elastic collision
$$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2}$$

Inelastic collision
$$m_1 v_{i1} + m_2 v_{i2} = (m_1 + m_2)v_f$$

Conservation of momentum: Constant force
$$\int_{t_1}^{t_2} \vec{F}\, dt = \vec{F}\Delta t$$

Conservation of momentum: Avg force
$$\int_{t_1}^{t_2} \vec{F}\, dt = \overrightarrow{F_{avg}}\Delta t$$

Conservation of momentum: $\sum \boldsymbol{F} = \mathbf{0}$ || $\Delta t = 0$
$$m\overrightarrow{v_1} = m\overrightarrow{v_2}$$
$$\overrightarrow{L_1} = \overrightarrow{L_2}$$

Multiple particles
$$\sum m_i(\overrightarrow{v_{i1}}) + \sum \int_{t_1}^{t_2} \overrightarrow{F_i}\, dt = \sum m_i(\overrightarrow{v_{i2}})$$

Moment
$$\overrightarrow{\mu_0} = \overrightarrow{r_0} \times \vec{F}$$

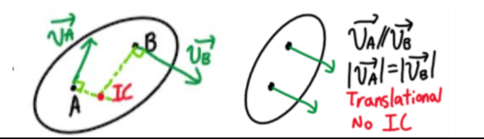
Angular momentum
$$\overrightarrow{H_0} = \overrightarrow{r_0} \times m\vec{v}$$
$$\left|\overrightarrow{H_0}\right| = r_0 mv\sin\theta = r_0 m v_\theta$$

Principle of angular momentum and impulse
$$\overrightarrow{H_{01}} + \int_{t_1}^{t_2} \sum \overrightarrow{\mu_0}\, dt = \overrightarrow{H_{02}}$$
$$\overrightarrow{\mu_0} = \frac{d\overrightarrow{H_0}}{dt}$$

Conservation of linear momentum
$$\sum H_{01i} = \sum H_{02i}$$

Rigid body motions
- Translation
- Fixed rotation
- General motion

Instantaneous centre of zero velocity
(Point where perpendicular vectors of velocities meet)



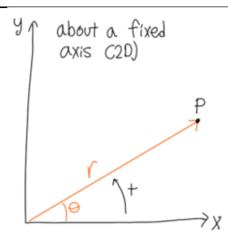
Fixed rotation
Angular displacement $\vec{\theta}$

Angular velocity $\vec{\omega}$

Angular acceleration $\vec{\alpha}$

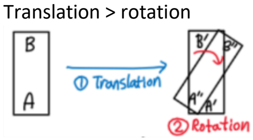
$$\overrightarrow{v_P} = \vec{\omega} \times \vec{r}$$
$$\overrightarrow{a_P} = \vec{\omega} \times \vec{v} + \vec{\alpha} \times \vec{r}$$

If α constant,
$$\omega = \omega_0 + \alpha_c t$$
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha_c\theta$$



General motion
Decompose the motion

Translation > rotation



$$\omega = \frac{v_B - v_A}{r_{B/A}}$$
$$\overrightarrow{v_B} = \overrightarrow{v_A} + \vec{\omega} \times \overrightarrow{r_{B/A}}$$

$$\overrightarrow{a_B} = \overrightarrow{a_A} + \vec{\omega} \times \overrightarrow{v_{B/A}} + \vec{\alpha} \times \overrightarrow{r_{B/A}}$$
$$= \overrightarrow{a_A} - \omega^2\overrightarrow{r_{B/A}} + \alpha\overrightarrow{r_{B/A}}$$
$$|a_{B/At}| = r\alpha$$
$$|a_{B/An}| = \omega^2 r$$

Translation
$$\overrightarrow{r_B} = \overrightarrow{r_A} + \overrightarrow{r_{B/A}}$$
$$\overrightarrow{v_B} = \overrightarrow{v_A}$$
$$\overrightarrow{a_B} = \overrightarrow{a_A}$$

Magnitude don't change

Direction don't change