Midterm Test : MIE 100
March 8, 2007.

1. $m_b = .05\text{ kg}$

$m = 0.3\text{ kg}.$

$v_b = 100\text{ m/s}$

(a) linear momentum in horizontal direction is conserved during impact:

$$L_{xi} = L_{xf}$$

$$\Rightarrow m_b v_b = (m_b + m) v_f$$

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$$v_f = \frac{.05}{.35}(100) = 14.3\text{ m/s}.$$

3 2

-1 for no units

(b) $T_1 + \cancel{V_1}^{\,0} = \cancel{T_2}^{\,0} + V_2$ 3

$$T_1 = \frac{1}{2}(.35)(14.3)^2 = 35.7\text{ joules}.$$

$$V_2 = \cancel{2}\left\{\frac{1}{\cancel{2}}(80)x^2\right)$$

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$$\Rightarrow x = \sqrt{\frac{35.7}{80}} = 0.67\text{ m}$$

3 -1 for no units

perfect marks for (b) if everything is right except (a) answer.

2. (a) initial displacement:

$$x = \cancel{x_0}^{0} + \cancel{v_0 t}^{0} + \frac{1}{2}at^2$$

$$= \frac{1}{2}(3.2)(1.5)^2 = 3.6\,m$$

$\Rightarrow$ first stage of motion: move 3.6 m (2)

second stage of motion: move 1 m more.

v @ end of first stage of motion:

$$v^2 = 2as \Rightarrow v = \{2(3.2)(3.6)\}^{1/2} = 4.8\,m/s \quad (3)$$

$$\left[\text{or} \quad v_f = \cancel{v_i}^{0} + at = (3.2)(1.5) = 4.8\,m/s\right]$$

$\Rightarrow$ a_2 (for second stage of motion) (5)

$$0 = (4.8)^2 + 2a_2(1) \Rightarrow a_2 = -11.52\,m/s^2$$

(-1) for no units

properly a deceleration is +ve

please accept both +ve + -ve answers.

(b) maximum available friction force:

$$F_{f\,max} : \mu_s mg = .35\, m\, (9.81) = 3.43\, m$$

5

F_f needed to accelerate m @ $3.2\ m/s^2$.

5

$F_f = 3.2\, m$

but $3.2m < 3.43\, m$ ⇒ there is enough static friction to cause the mass to move with the belt in the first stage of motion.

(c) for the second stage of motion the package will slip and the friction force is identically $.25\, mg = 2.45\, m$.

5

$\Rightarrow a_{package} = -\ 2.45\ m/s^2$

but package starts at $v = 4.8\, m/s$

time for second stage of motion (go back to belt for this)

$$v_f = v_i + at$$

2

$$\Rightarrow 0 = 4.8 - 11.52t \Rightarrow t = 0.42s$$

$\Rightarrow$ package displacement for 2nd stage of motion

$$S = 4.8(0.42) - \frac{1}{2}(2.45)(.42)^2$$ ③

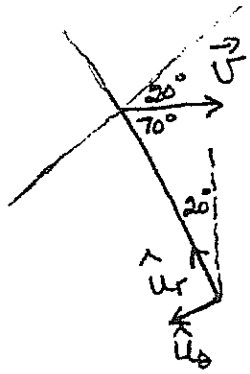
$$= \underline{\underline{1.80\ m}}$$

2nd correct answer.

$$0 = 4.8^2 - 2(2.45)S \qquad S = 4.70\ m.$$

3(a) $|\vec{v}| = 8$ m/s.

sketch



$v_r = -8\cos 70^\circ = -2.74$

$v_\theta = -8\sin 70^\circ = -7.52$

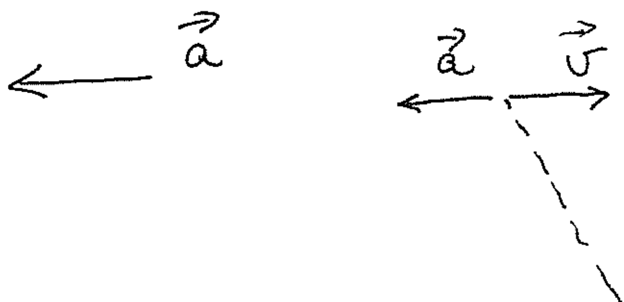
$\vec{v} = -2.74\,\hat{u}_r - 7.52\,\hat{u}_\theta$ m/s.

15

-2 for each incorrect sign

-1 for no units

(b)



anything horizontal to the left is acceptable

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(c) $d = r\cos\theta$

$0 = \dot{r}\cos\theta + r(-\sin\theta)\dot{\theta}$

but $r = \dfrac{d}{\cos\theta} \Rightarrow \quad 0 = \dot{r}\cos\theta - \dfrac{d\sin\theta}{\cos\theta}\,\dot{\theta}$

but $\dot{r} = |\vec{v}| \sin\theta$

$$\Rightarrow \quad 0 = |\vec{v}| \cancel{\sin}\theta \left(\cos\theta\right) - \frac{d \cancel{\sin}\theta}{\cos\theta} \dot{\theta}$$

$$\Rightarrow \quad |\vec{v}| = \frac{(d)(\dot{\theta})}{\cos^2\theta}$$

(15)

There are other ways to do this:

eg $\vec{v} = \sqrt{v_r^2 + v_\theta^2}$

$\vdots$