A baseball player releases a ball with an initial velocity of *35 m/s* at an angle 35₀ with the horizontal. If *t=0* is the time of release from the player's hand, determine the rate of change of the speed, at *t=1s*.

Hint: the rate of change of speed occurs along the line of the path (a_t). Use geometry to find the direction along the path.

A baseball player releases a ball with an initial velocity of *35 m/s* at an angle 35₀ with the horizontal. If *t=0* is the time of release from the player's hand, determine the rate of change of the speed, at *t=1s*.
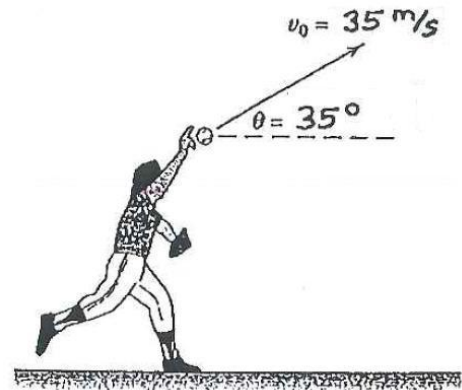
Hint: the rate of change of speed occurs along the line of the path (a$_t$). Use geometry to find the direction along the path.

$v_0 = 35 \, m/s$

$\theta = 35°$

Solution:

$$\dot{v} = a_t$$

horizontal: $x = x_o + v_{ox}t = (v_o \cos\theta)\,t$

vertical: $y = y_o + v_{oy}t + \frac{1}{2}a_y t^2 = (v_o \sin\theta)t - \frac{1}{2}gt^2$

$$V_x = \frac{dx}{dt} = v_o \cos\theta$$
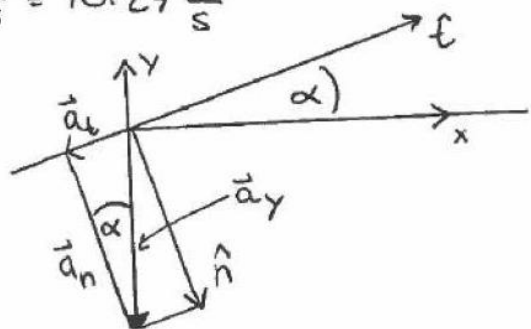
$$v_y = \frac{dy}{dt} = v_o \sin\theta - gt$$

at $t = 1$: $V_x = 35\,(\cos 35°)\,\frac{m}{s} = 28.67\,\frac{m}{s}$

$v_y = [35\,(\sin 35°) - 9.81]\,\frac{m}{s} = 10.27\,\frac{m}{s}$

$$\alpha = \arctan \frac{v_y}{v_x} = 19.7°$$

$a_x = 0$ (projectile motion)

$a_y = -9.81\,\frac{m}{s^2}$

$$\dot{V} = a_t = a_y \sin\alpha = -9.81 \sin(19.7°) = -3.31\,\frac{m}{s^2}$$
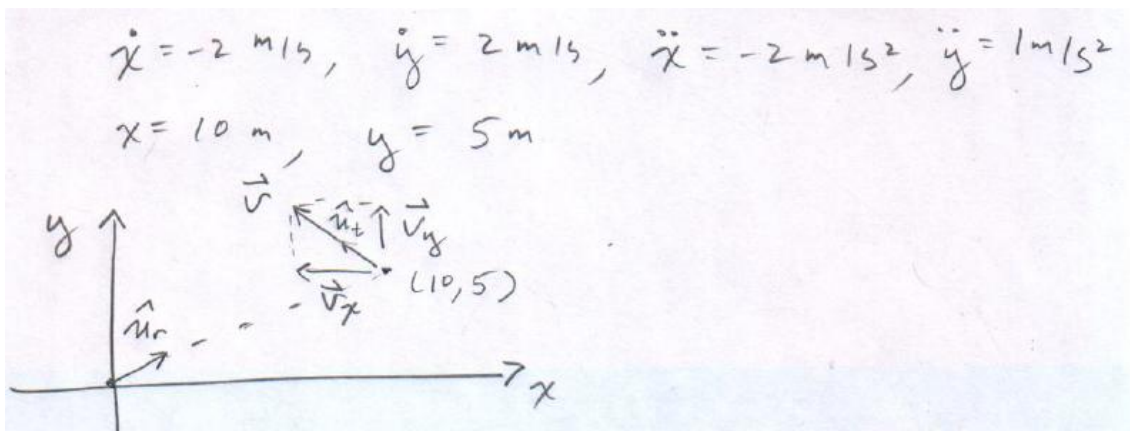
From MIE 100 Dynamics Midterm 2005

A point mass is located at $(x, y) = (10, 5)$ m with respect to a given set of rectangular axes. Following is some information about its motion:

$$\dot{x} = -2 \text{ m/s}, \qquad \dot{y} = 2 \text{ m/s}, \qquad \ddot{x} = -2 \text{ m/s}^2, \qquad \ddot{y} = 1 \text{ m/s}^2$$

Find:

  (i) The speed of the particle.

  (ii) The velocity of the particle, expressed in normal-tangential coordinates.

  (iii) The tangential acceleration a_t.

  (iv) The radius of curvature of the particle's path, ρ.

  (v) Sketch the direction of $\hat{u}_n$.

$$\dot{x} = -2 \ m/s, \quad \dot{y} = 2 \ m/s, \quad \ddot{x} = -2 \ m/s^2, \quad \ddot{y} = 1 \ m/s^2$$

$$x = 10 \ m, \quad y = 5m$$

i)

$$speed = |\vec{v}| = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(-2)^2 + (2)^2} = \sqrt{8}$$

$$\boxed{|\vec{v}| = 2\sqrt{2} \stackrel{\sim}{=} 2.83 \ \frac{m}{s}}$$

ii)

$\hat{u}_t$ is in the same direction
of $\vec{v}$'s and $\vec{v}$ has no component
normal to the path.

$$\therefore \quad \vec{v} = 0 \hat{u}_n + |\vec{v}| \hat{u}_t = 2.83 \hat{u}_t \ \frac{m}{s}$$

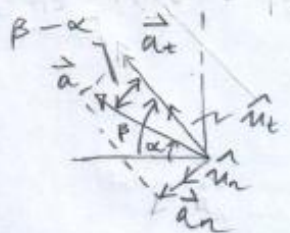**iii)** a_t is the component of $\vec{a}$
‖ to the path.

Acceleration vector:

$$\vec{a}_y = \ddot{y}\hat{j} = 1\hat{j} \ m/s^2$$

$$\vec{a}_x = \ddot{x}\hat{i} = -2\hat{i} \ m/s^2$$

$$\alpha = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$$

Velocity vector:

$$\vec{v}_y = \dot{y}\hat{j} = 2\hat{j} \ \frac{m}{s}$$

$$\vec{v}_x = \dot{x}\hat{i} = -2\hat{i} \ m/s$$

$$\beta = \tan^{-1}\left(\frac{2}{2}\right) = 45°$$

$$a_t = |\vec{a}| \cos(\beta - \alpha) = \sqrt{\ddot{x}^2 + \ddot{y}^2} \cos(\beta - \alpha)$$

$$a_t = \sqrt{(-2)^2 + (1)^2} \cos(45° - 26.6°)$$

$$\boxed{a_t = \sqrt{5} \cos(18.4°) = 2.12 \text{ m/s}^2}$$

iv)

$$a_n = \frac{|\vec{v}|^2}{\rho}$$

$$\rho = \frac{|\vec{v}|^2}{a_n}$$

$$a_n = |\vec{a}| \sin(\beta - \alpha) = \sqrt{5} \sin(18.4°)$$

$$a_n = 0.71 \text{ m/s}^2$$

$$\boxed{\rho = \frac{(2.83)^2}{0.71} = 11.28 \text{ m}}$$

v)

$$= 90° - 45°$$
$$= 45°.$$