Q.1 Given :- $m_A = 0.1$ kg

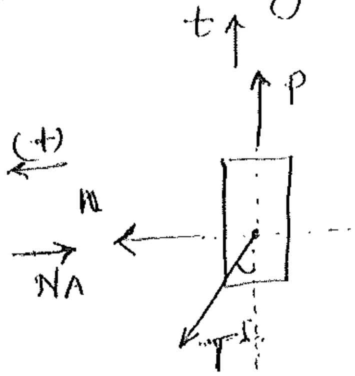
$m_B = 0.3$ kg

⊛ No weight motion (Horizontal plane)

Masses at rest when P is applied.

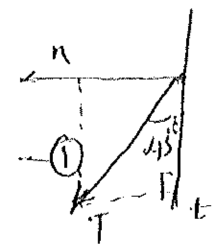
Soln :-

(a) Free body diagram of mass A.



$$\sum F_t = m_A a_t$$

$$P - T\cos 45^\circ = m_A a_t \quad \text{— (1)}$$

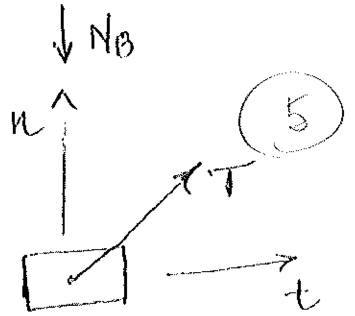


$$\sum F_n = m_A a_n$$

$$-N_A + T\sin 45^\circ = m_A a_n \quad \text{— (2)}$$

full marks: N_A in the opposite direction or different notation

(b) Free body diagram of mass B



5

$$\sum F_t = m a_t$$

$$T\cos 45^\circ = m_B a_t \quad \text{— (3)}$$

$$\sum F_n = m a_n$$

$$-N_B + T\sin 45^\circ = 0 \quad \text{— (4)}$$

5

full marks: opposite direction for N_B or different notation

part of (b).

(C) From eqⁿ (1), $P - T\cos 45^\circ = m_A a_t$

$4 - T\times(0.7071) = 0.1\, a_t$

$\Rightarrow \quad 0.1\, a_t + 0.7071 T = 4 \quad — (A)$

From eqⁿ (2), $-N_A + T\sin 45^\circ = m_a a_n$

Since, $a_n = 0$

$-N_A + 0.7071 T = 0.$

$0.7071\, T = N_A \quad — (B)$

From eqⁿ (3), $T\cos 45^\circ = m_B a_t$

$0.7071\, T = 0.3\, a_t \quad — (C)$

From eqⁿ (4), $-N_B + T\sin 45^\circ = 0$

$0.7071 T = N_B \quad — (D).$

Solving eqⁿs (A) & (C) we get

$0.1\, a_t + 0.7071\, T = 4$

$0.3\, a_t - 0.7071\, T = 0$

$0.4\, a_t = 4$

$a_t = 10 \text{ m/sec}^2$

Substituting a_t in eq^n (1) or (C)

we get , $T = 4.24\ N$

(d) Total acceleration, in normal and tangential coordinates is given as,

$$\vec{a} = a_t \hat{u}_t + a_n \hat{u}_n$$

$$\vec{a} = 10\,\hat{u}_t \quad m/s^2.$$

required for full marks.

Q.2

Given :- $m_1 = 3$ kg

$k = 100$ N/m

$l_1 = 500$ mm $= 0.5$ m

Relaxed length of spring $= 400$ mm $= 0.4$ m

$v_1 = 4$ m/s.

Soln

(a) At $t = 0$, $\theta = 90°$

Actual length of spring at $t = 0$ $= \sqrt{2 l_1^2}$

$= \sqrt{2 \times (0.5)^2}$

$= 0.7071$ m

Hence the spring is stretched

$\therefore$ δ = stretched length − relaxed length

$= 0.7071 - 0.4$

$= 0.3071$ m

Hence, the elastic potential energy of the spring at $t = 0$ is,

$$V_e = \frac{1}{2} k s^2$$

$$= \frac{1}{2} \times 100 \times (0.3071)^2$$

not so many decimals

$$= \underline{4.7155 \text{ Joules}}$$

(b) When θ gets extremely close to 180°, the direction of the sphere will be

$-ve\ y$, or negative j, or $\downarrow$ direction

all are fine.

(c) At $t = 0$, $\theta = 90^\circ$

$$T_1 = \frac{1}{2} m_1 v_1^2$$

$$= \frac{1}{2} \times 3 \times (4)^2$$

$$= 24 \text{ Joules}$$

$$V_{e_1} = 4.7155 \text{ Joules.}$$

$\leftarrow$ do not penalize any mistake twice (from (a))

$$V_{g_1} = +mgh = +3 \times 9.81 \times 0.5$$

$$= \underline{+14.71 \text{ Joules}}$$

At θ close to $180°$,

$$T_2 = \frac{1}{2} m_1 V_2^2$$

$$= 1.5\, V_2^2$$

$$Ve_2 = \frac{1}{2} k s^2$$

stretched length of spring at $\theta \cong 180°$ is 1 m.

Relaxed length = 0.4 m

$$\therefore\ s = 1 - 0.4$$

$$s = \underline{0.6\ m}$$

$$Ve_2 = \frac{1}{2} \times 100 \times (0.6)^2$$

$$= 18 \text{ Joules.}$$

$$Vg_2 = 0.$$

$$T_1 + Ve_1 + Vg_1 = T_2 + Ve_2 + Vg_2$$

$$24 + 4.71550 + 14.71 = 1.5\, V_2^2 + 18 + 0.$$

$$V_2 = \underline{4.11\ m/sec}$$

Speed of the sphere when θ gets close to $180°$ = $\underline{4.11\ m/sec}$

Q.3

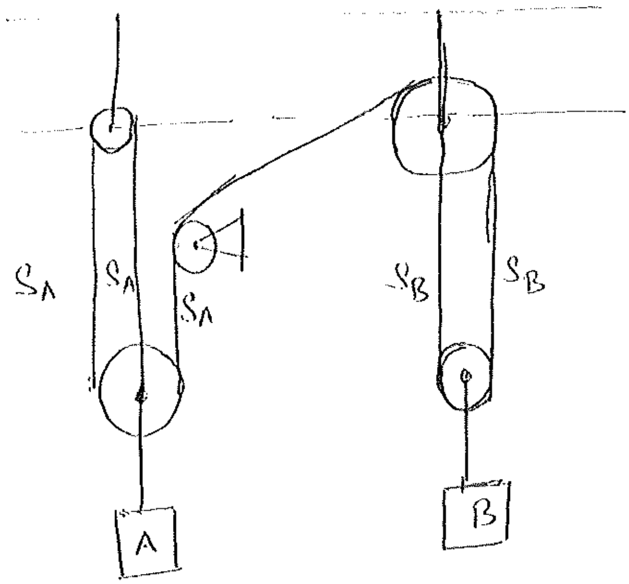
Given :- At $t = 0$

$v_A = 0.4$ m/s $\downarrow$

$m_A = 5$ kg

$m_B = 4$ kg

Sol^n :-



$3S_A + 2S_B = l_T - l_c$

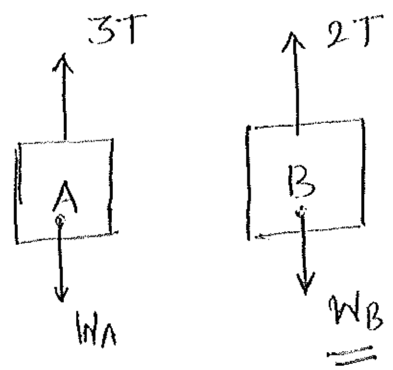
$3v_A + 2v_B = 0$ — ⑧

$v_A = -\frac{2}{3} v_B$ — ①

Also, $a_A = -\frac{2}{3} a_B$ — ②

any notation (y, h, s, x, etc is fine).

ⓐ



← I usually use mg not W but either is fine.

ⓑ From eq^n ①

$$v_B = -\frac{3}{2} \times v_A$$

$$= -\frac{3}{2} \times 0.4$$

$$v_B = \underline{-0.6 \text{ m/sec}} \quad @ \ t = 0$$

↙ this assumes +ve downward: any correct indication of physical direction is OK.

(c) $\Sigma F = ma \Rightarrow$ Accleration remains constant even at $t = 2$ sec

+ve downward.

From F.B.D of m_A.

$-3T + W_A = m_A a_A$.

$-3T + (9.81 \times 5) = 5 a_A$.

$-3T + 5a_A = 49.05$ — (3)

From F.B.D of m_B.

$T = \frac{5a_A - 49.05}{3}$

$-2T + W_B = m_B a_B$.

$T = \frac{4a_B - 39.24}{2}$

$-2T + (9.81 \times 4) = 4 a_B$

$-2T + 4a_B = 39.24$ — (4)

Solving eq^n (3) & (4) for T we get.

$-3T + 5a_A = 49.05$

$-2T + 4a_B = 39.24$

$-10a_A + 12a_B = 19.62$ — (5)

Substituting eq^n (2) in eq^n (5) we get

$-10 \times \left(-\frac{2}{3} a_B\right) + 12a_B = 19.62$

$a_B = +1.0511 \ m/s^2$

$$a_A = -\frac{2}{3}(+1.051)$$

$$= -0.7006 \text{ m/s}^2$$

~~From eqn~~ . $a_A = -0.7006 \text{ m/s}^2$

$a_B = +1.0511 \text{ m/s}^2$

(d)

From eqn (3)

one can also use $F_y = m a_y$.

$$3T - 5 \times 0.7006 = 49.05$$

$$\underline{T = 17.52 \text{ N}}$$

From principle of impulse and momentum for a ~~system~~ mass B we have,

$$m_B v_{B_1} + \int_{t_1}^{t_2} \Sigma F \, dt = m_B v_{B_2}$$

$$+ m_B v_{B_1} + m_B g \Delta t - 2T \Delta t = m_B v_{B_2}$$

$$+ 4 \times (-0.6) + 4 \times 9.81 \times 2 - 2 \times 17.52 \times 2 = 4 \times v_{B_2}$$

$$\therefore \boxed{v_{B_2 @ t = 2 sec} = \cancel{2.417 \text{ m/sec}} \; \cancel{+} \; 1.5 \text{ m/sec} \quad \downarrow}$$

downward

~~wrong~~ ~~physically.~~ ~~if a_B is~~ ~~-ve but it~~ ~~is~~

extra solution for 3(d).

$$\frac{5a_A - 49.05}{3} = \frac{4a_B - 39.24}{2}$$

$$10a_A - 98.1 = 12a_B - 117.75$$

$$\Rightarrow 12a_B - 10a_A = +19.65$$

$$12a_B - 10\left(-\frac{2}{3}a_B\right) = 19.67$$

$$\Rightarrow a_B = \frac{19.67}{12 + 6.67} = +1.05$$

$\downarrow +$

$v_{Bi} = -0.6$ m/s

$v_{Bf} = ?$

$a_B = +1.05$

$\Delta t = 2s$

$v = v_0 + at$

$-.6 + 1.05(2) = +1.5$ $\downarrow$ downwards