LAST Name (as seen on ROSI): ______________________ Tutorial Number: ________

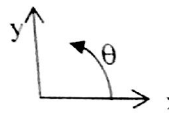
FIRST Name (as seen on ROSI): ______________________ Student Number: ________

MIE 100S - Quiz number 2a – January 26, 2015: 25 minutes

At time t = 0, a cannon ball is launched from the origin with an initial speed of 8 m/s, at an angle θ = 30° above the ground. Ignore the effects of air resistance and the rotation of the earth.

(a) At time t = 0, determine the acceleration in normal-tangential coordinates.

(b) At time t=0, determine $\ddot{r}$.



$$\vec{a} = \dot{v}\,\hat{u}_t + v\dot{\theta}\,\hat{u}_n = \dot{v}\,\hat{u}_t + v^2/\rho\,\hat{u}_n$$

$$\vec{v} = \dot{r}\,\hat{u}_r + r\dot{\theta}\,\hat{u}_\theta \qquad \vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{u}_\theta$$

g = 9.81

$a_t = \dot{v}$

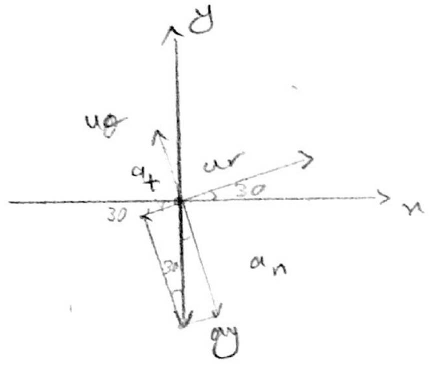
$V_x = V_0 \cos\theta$

$V_y = V_0 \sin\theta - gt$

at $t=0$ $V_x = 8\cos 30 = 6.92$, $V_y = 8\sin 30 - 0 \times 9.81 = 4$ m/s

$a_t = a_y \sin\theta = -9.81 \sin 30 = -4.905$ m/s²

$a_n = a_y \cos\theta = -9.81 \cos 30 = -8.4957$ m/s²



$a_r = \ddot{r} - r\dot{\theta}^2$

$v\dot{\theta} = a_n \longrightarrow 8\dot{\theta} = -8.4957$

$\dot{\theta} = -1.062$ rad/s

$u_r = |u_r|\cos 30\,\hat{i} + |u_r|\sin 30\,\hat{j} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

$u_\theta = |u_\theta|\sin 30\,\hat{i} + |u_\theta|\cos 30\,\hat{j} = -\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$

$$\left.\begin{aligned} a &= -9.81\hat{j} \\ a &= (\ddot{r} - r\dot{\theta}^2)\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right) + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\left(-\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) \end{aligned}\right\} \Longrightarrow$$

$a = \ddot{r}\left(\frac{\sqrt{3}}{2}\right)\hat{i} - r\dot{\theta}^2\frac{\sqrt{3}}{2}\hat{i} + \ddot{r}\times\frac{1}{2}\hat{j} - r\dot{\theta}^2\left(\frac{1}{2}\right)\hat{j} - r\ddot{\theta}\left(\frac{1}{2}\right)\hat{i} + \frac{\sqrt{3}}{2}r\ddot{\theta}\hat{j} - \dot{r}\dot{\theta}\hat{i} + \sqrt{3}\,\dot{r}$

in $\hat{i}$ direction: $\ddot{r}\left(\frac{\sqrt{3}}{2}\right) - \cancel{r\dot{\theta}^2 \times \frac{\sqrt{3}}{2}} - \cancel{\frac{r\ddot{\theta}}{2}} - \dot{r}\dot{\theta} = 0$

in $\hat{j}$ direction: $\frac{\ddot{r}}{2} - \cancel{r\dot{\theta}^2} + \cancel{\frac{\sqrt{3}}{2}r\ddot{\theta}} + \sqrt{3}\,\dot{r}\dot{\theta} = -9.81$

$$\Longrightarrow \begin{cases} \frac{\sqrt{3}}{2}\ddot{r} + 1.062\,\dot{r} = 0 \\ \frac{\ddot{r}}{2} - (\sqrt{3}) \times 1.062\,\dot{r} = -9.81 \end{cases} \longrightarrow \boxed{\begin{aligned} \ddot{r} &= -4.905 \\ \dot{r} &= 3.9998 \end{aligned}}$$