FACULTY OF APPLIED SCIENCE AND ENGINEERING
University of Toronto
FINAL EXAM, MONDAY, APRIL 19, 2010

MAT 186S
Calculus I

Examiner: S. Cohen
Duration: 2 hours, 30 minutes
STANDARD CALCULATORS ALLOWED

Total Marks: 90

Family Name:

Given Name(s):

Please sign here:

Student ID number:

For Markers Only

Question	Marks
1	/ 10
2, 3	/ 10
4	/ 10
5	/ 9
6	/ 9
7	/ 10
8, 9	/ 14
10	/10
11	/ 8
TOTAL	**/ 90**

1. Evaluate the limits:

a) $\lim\limits_{x \to \infty} x^{(1/x)}$

[5 marks]

b) $\lim\limits_{x \to -\infty} \sqrt{4x^2 + 2} + 2x - 1$

[5]

2. Find the derivative of $f(x) = \sqrt{x^2 + 2}$ using the limit definition of derivative.

[6]

3. Show whether or not $f'(2)$ exists for the function:

$$f(x) = \begin{cases} 2\sin^2(\pi x) & x < 2 \\ x^3 - 12x & x \geq 2 \end{cases}$$

[4]

4. Find $\frac{dy}{dx}$:

a) $y = \dfrac{x^3 - 1}{2x + 1}$

[3]

b) $y = \cos\left(5x^2\sqrt{2x - 1}\right)$

[3]

c) $y = (4x - 1)^{x^2+1}$

[4]

5. Find the point on the graph of $y = x^2 - 4x + 1$ that is closest to $(8, 0)$.

[9]

6. Find the dimensions of the right circular cylinder of the largest volume that fits in a sphere of radius R.

[9]

6. Find $\int_{-2}^{2} 6x^2 + x - 1 \, dx$ using Riemann sums and then check using integrals.

[10]

8. Find the area between $f(x) = 2x^4 - x^2 + 3$ and $g(x) = 2x^4 - x^3 + 2x + 3$.

[7]

9. Let a function be defined by: $F(x) = x \cdot \displaystyle\int_{x}^{x^2-2} \sqrt{t^2 + 5}\, dt$ Evaluate $F'(2)$.

[7]

10. Let a solid be created by rotating the areas between $y = x^3 - x$ and the x-axis around the line $x = 1$. Find the volume of this solid.

[10]

11. Find the surface area generated when the curve $y = \sqrt{x + 3}$, from $x = -1$ to $x = 3$, is rotated around the x-axis.

[8]