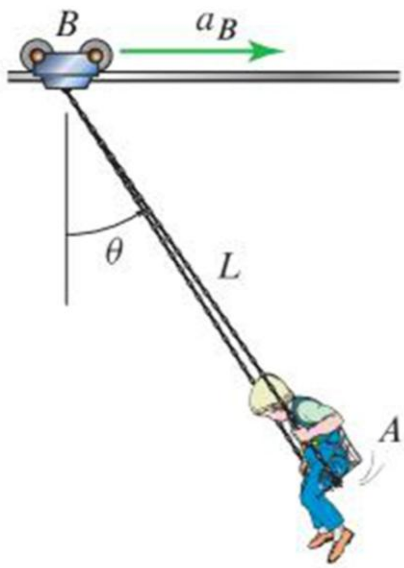
MIE100S – Winter 2017
Tutorial Problem 03a

A child A is swinging from a swing that is attached to a trolley that is free to move along a fixed rail. Letting L = 2.5 m, if at a given instant $a_B = 3.7\,m/s^2$, $\theta = 23°$, $\dot{\theta} = 0.45\frac{rad}{s}$ and $\ddot{\theta} = -0.2\,rad/s^2$, determine the magnitude of the acceleration of the child relative to the rail at that instant.



MIE100S – Winter 2017
Tutorial Problem 03a

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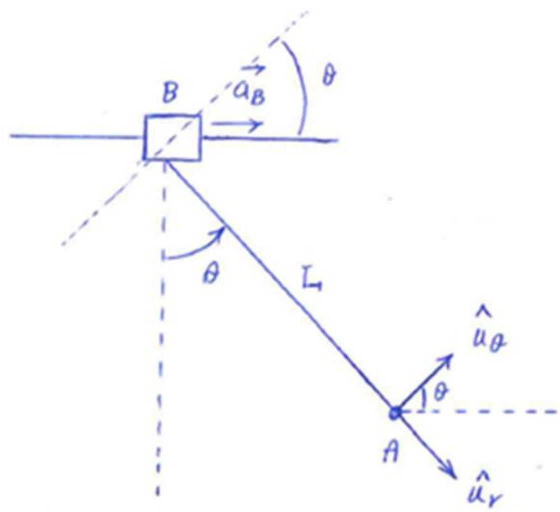


$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{u}_\theta \qquad r = L \Rightarrow \dot{r} = \ddot{r} = 0$$

$$\vec{a}_{A/B} = (-L\dot{\theta}^2)\hat{u}_r + (L\ddot{\theta})\hat{u}_\theta = (-2.5 \times 0.45^2)\hat{u}_r + (2.5 \times -0.2)\hat{u}_\theta \quad m/s^2$$

$$\vec{a}_{A/B} = -0.50625\ \hat{u}_r - 0.5\ \hat{u}_\theta \quad m/s^2$$

$$\vec{a}_B = (3.7 \times \cos 23°)\hat{u}_\theta + (3.7 \times \sin 23°)\hat{u}_r = 3.41\ \hat{u}_\theta + 1.44\ \hat{u}_r \quad m/s^2$$

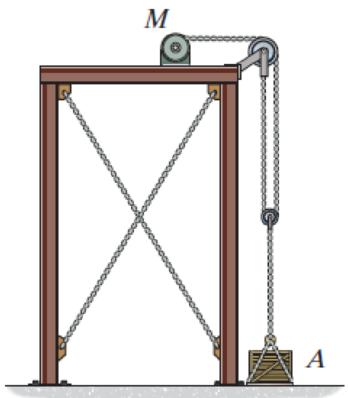
$$\vec{a}_A = \vec{a}_{A/B} + \vec{a}_B = (-0.50625 + 1.44)\hat{u}_r + (-0.5 + 3.41)\hat{u}_\theta \quad m/s^2$$

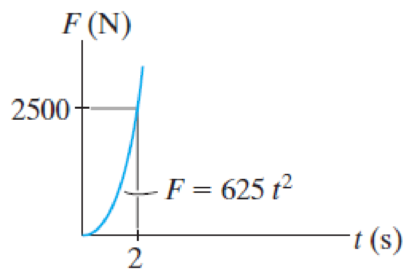
$$\vec{a}_A = (0.93375)\hat{u}_r + (2.91)\hat{u}_\theta \quad m/s^2$$

$$|\vec{a}_A| = \sqrt{(0.93375)^2 + (2.91)^2} = \boxed{3.06\ m/s^2}$$

MIE100S – Winter 2017
Tutorial Problem 03b

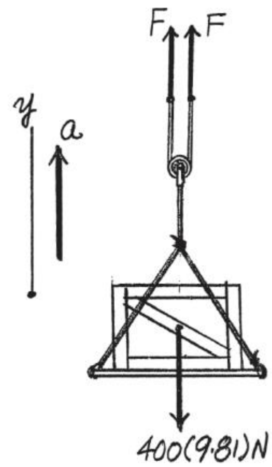
The force of the motor *M* on the cable is shown in the graph. Determine the velocity of the 400-kg crate *A* when t = 2 s.





MIE100S – Winter 2017
Tutorial Problem 03b-Solution

Free-Body Diagram: The free-body diagram of the crate is shown in Fig. *a*.



(a)

Equilibrium: For the crate to move, force 2**F** must overcome its weight. Thus, the time required to move the crate is given by

$+\uparrow \Sigma F_y = 0;$ $\qquad 2(625t^2) - 400(9.81) = 0$

$$t = 1.772 \text{ s}$$

Equations of Motion: $F = \left(625t^2\right)$ N. By referring to Fig. *a*,

$+\uparrow \Sigma F_y = ma_y;$ $\qquad 2\left(625t^2\right) - 400(9.81) = 400a$

$$a = (3.125t^2 - 9.81) \text{ m/s}^2$$

Kinematics: The velocity of the crate can be obtained by integrating the kinematic equation, $dv = adt$. For $1.772 \text{ s} \le t < 2 \text{ s}$, $v = 0$ at $t = 1.772$ s will be used as the lower integration limit. Thus,

$(+\uparrow)$

$$\int dv = \int adt$$

$$\int_0^v dv = \int_{1.772\text{ s}}^t \left(3.125t^2 - 9.81\right)dt$$

$$v = \left(1.0417t^3 - 9.81t\right)\Big|_{1.772\text{ s}}^t$$

$$= \left(1.0417t^3 - 9.81t + 11.587\right) \text{ m/s}$$

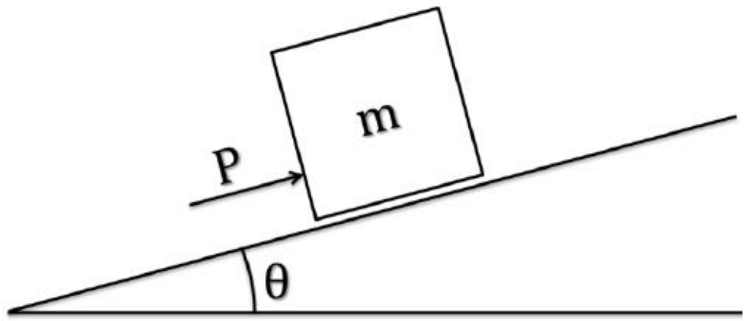
When $t = 2$ s,

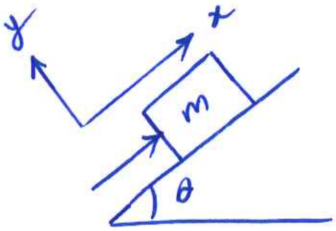
$$v = 1.0417(2^3) - 9.81(2) + 11.587 = 0.301 \text{ m/s} \qquad \textbf{Ans.}$$

MIE100S – Winter 2017
Tutorial Problem 04a

A 5-kg mass is placed on a rough inclined surface (θ=15º) to slide (μ_s = 0.45; μ_k = 0.4) subjected to a non-constant force $\vec{\boldsymbol{P}} = 5t\,\hat{i}$ Newtons, where t is measured in seconds.

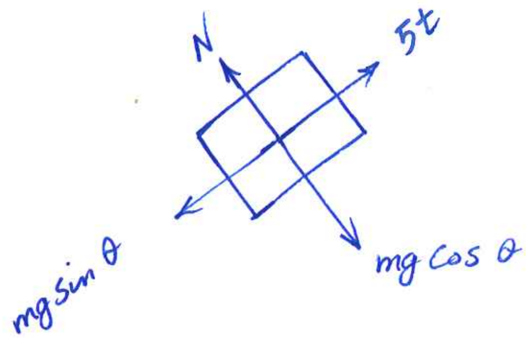
How much time will it take for the mass to reach a speed of 10 m/s?





First, let's see if it starts from rest:

$$\mu_s mg\cos\theta \overset{?}{>=<} mg\sin\theta$$



$\sum F_y = 0 \Rightarrow N = mg\cos\theta$

at $t = 0$: $\mu_s mg\cos\theta - mg\sin\theta = (0.45)(5)(9.81)(\cos 15) - (5)(9.81)(\sin 15) = 8.6 > 0$

$\downarrow$

It starts from rest

It has to overcome the static friction in $+x$ direction:

$$\sum F_x = 0 \Rightarrow 5t_1 - mg\sin\theta - \mu_s mg\cos\theta = 0$$

$$5t_1 - (5)(9.81)(\sin 15) - (0.45)(5)(9.81)(\cos 15) = 0$$

$\boxed{t_1 = 6.8\ s}$ → How long it takes before it starts to move

After it starts to move:

$$\sum F_x = ma_x \Rightarrow 5t - mg\sin\theta - \mu_k mg\cos\theta = m\frac{dv}{dt}$$

$$\int_{t_1}^{t}\left[\frac{5}{m}t - g(\sin\theta + \mu_k\cos\theta)\right]dt = \int_0^v dv$$

$$\left[\frac{5}{2m}t^2 - g(\sin\theta + \mu_k\cos\theta)\right]\Bigg|_{t_1}^{t} = v$$

$$\underbrace{\frac{5}{2m}}_{a} t^2 \underbrace{- g(\sin\theta + \mu_k \cos\theta)}_{b} t + \underbrace{\left(\frac{-5}{2m} t_1^2 + g(\sin\theta + \mu_k \cos\theta) t_1 - V \right)}_{c} = 0$$

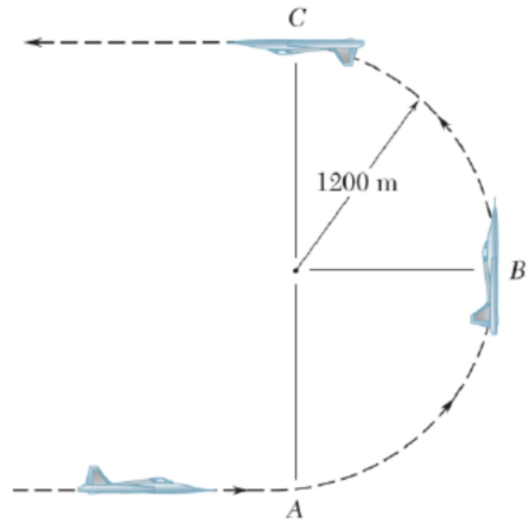
quadratic equation : $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(+) sign : acceptable because t cannot be less than t_1 $\Rightarrow$ $t > t_1$

$$\boxed{t = 10.8 \text{ s}}$$

MIE100S – Winter 2017
Tutorial Problem 04b

A 54-kg pilot flies a jet trainer in a half vertical loop of 1200 m radius so that the speed of the trainer increases at a constant rate. Knowing that the normal forces exerted on the pilot at points A and C are 1750 N and 703.5 N, respectively, determine the normal force exerted on her by the seat of the trainer when the trainer is at Point B.



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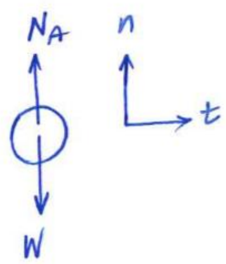
Tutorial Problem 04b-Solution

At A :

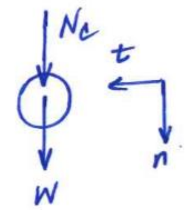


$$\sum F_n = ma_n \Rightarrow N_A - W = m\frac{v_A^2}{R}$$

$$1750 - (54)(9.81) = \frac{54\, v_A^2}{1200}$$

$$v_A = 164.7 \text{ m/s}$$

At C :



$$\sum F_n = ma_n \Rightarrow N_C + W = m\frac{v_C^2}{R}$$

$$703.5 + (54)(9.81) = 54\frac{v_C^2}{1200}$$

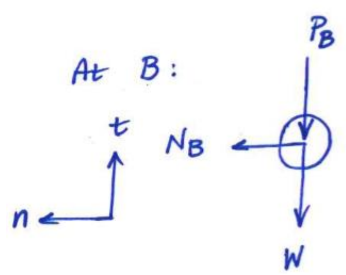
$$v_C = 165.5 \text{ m/s}$$

$$a_t : \text{const.} \Rightarrow v_C^2 = v_A^2 + 2a_t\,\Delta s_{AC}$$

$$\Delta s_{AC} = \pi R = 3769.9 \text{ m} \Rightarrow a_t = \frac{v_C^2 - v_A^2}{2\Delta s_{AC}} = 0.035 \text{ m/s}^2$$

$$v_B^2 = v_A^2 + 2a_t\,\Delta s_{AB} \Rightarrow v_B = 165.1 \text{ m/s}$$

At B :

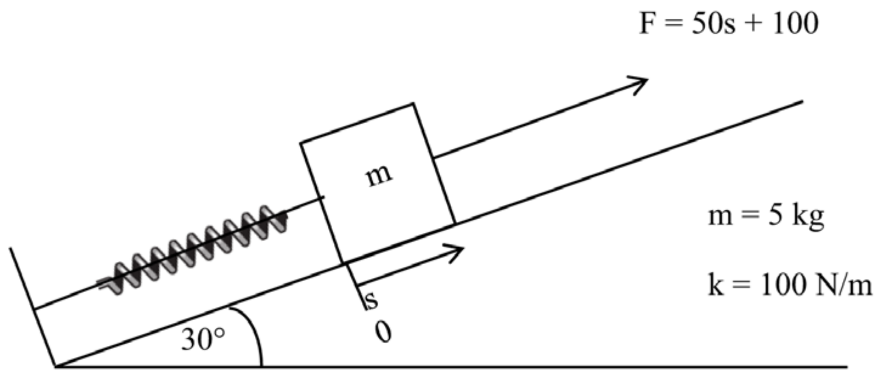


$$\sum F_n = ma_n \Rightarrow N_B = m\frac{v_B^2}{R}$$

$$N_B = (54)\frac{(165.1)^2}{1200} = 1226.6 \text{ N}$$

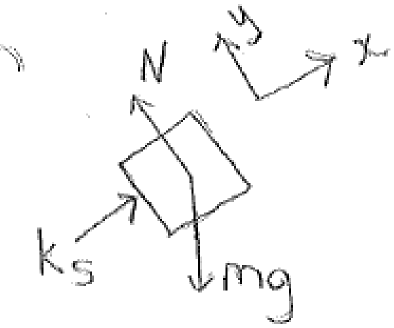
MIE100S – Winter 2017
Tutorial Problem 05a

The 5 kg block starts at rest on the incline in a system as shown above. A non-constant force, F, is applied to the block. The contact surface between the block and incline is smooth, and the attached spring-mass system is in equilibrium before the force is applied. Determine the block's speed when s = 0.5 m.



MIE100S – Winter 2017
Tutorial Problem 05a-Solution

$\sum F_x = 0 = ks - mg\sin 30 \quad \therefore s = 0.245\,m$



$T_1 = 0 \quad V_{e_1} = \frac{1}{2}ks^2 = 3.00\ J$

$V_{g_1} = 0$

$U_{1-2} = \int_1^2 F\,ds$

$= \int_0^{0.5} 50s + 100\,ds$

$= \frac{50s^2}{2} + 100s \Big]_0^{0.5}$

$= \frac{50}{8} + 50 = 56.25\ J$

$T_2 = \frac{1}{2} m v_2^2 = 2.5 v_2^2$

$V_{e_2} = \frac{1}{2}ks^2 = \frac{1}{2}k\,(0.5 - 0.245)^2 = 3.25\ J$

$V_{g_2} = mg\Delta h = mg\Delta s(\sin 30)$

$= 5(9.81)(0.5)\sin 30$

$= 12.26\ J$

$T_2 + V_{e_2} + V_{g_2} = T_1 + V_{e_1} + V_{g_1} + U_{1-2}$

$T_2 = U_{1-2} + V_{e_1} - V_{e_2} - V_{g_2}$

$= 56.25 + 3.00 - 3.25 - 12.26 = 43.74$

$\therefore v_2 = \sqrt{\frac{43.74}{2.5}} = 4.18\ m/s$

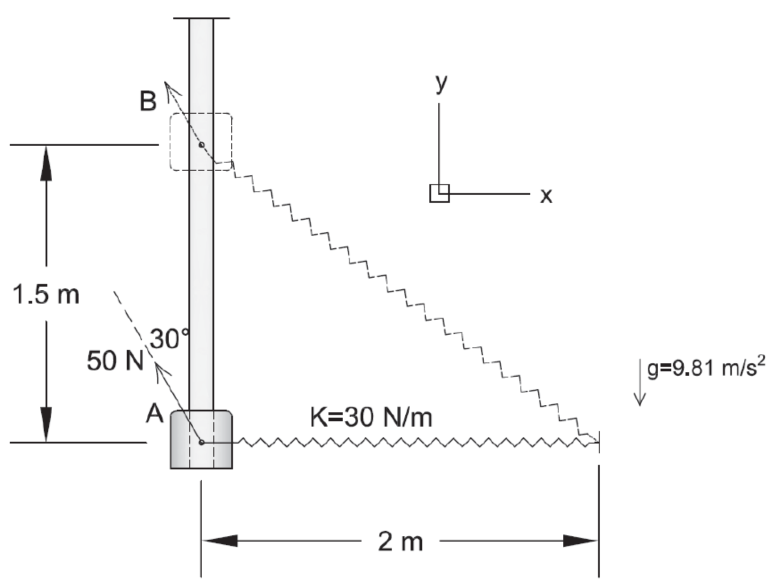
MIE100S – Winter 2017
Tutorial Problem 05b

The collar has a mass of 2 kg and is attached to the light spring, which has a stiffness of 30 N/m and an unstretched length of 1.5 m. The collar is released from rest at A and slides up the smooth rod under the action of the constant 50 N force.

a) What is the potential energy of the spring at time t=0?

b) Find the velocity of the collar as it passes position B.

c) Draw a free body diagram of the collar when it is in position B. Indicate the direction (as they act on the mass) of all forces in the plane of motion. It is not necessary to calculate their magnitudes.



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Tutorial Problem 05b-Solution

$m = 2\,kg$

$k = 30\ N/m$

l_0 = Unstretched length of Spring $= 1.5\ m$

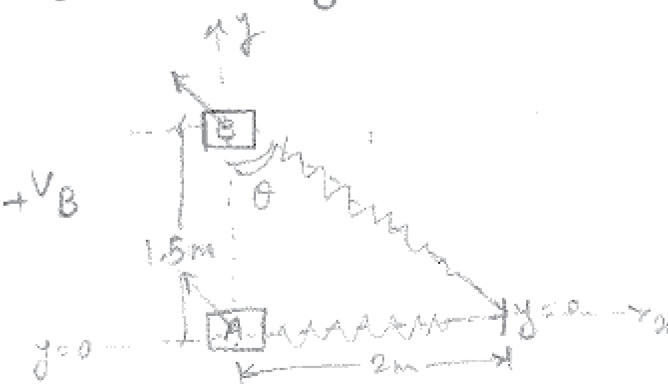
$V_A = 0\ m/s$ @ $t = 0$

a) A $t = 0$

$l_A = 2\ m$

$S_A = l_A - l_0 = 2 - 1.5 = 0.5\ m$

$V_e = \frac{1}{2} k S^2 = \frac{1}{2} \times 30 \times (0.5)^2 = 3.75\ J$



b) $T_A + V_A + \left(\sum U_{1-2}\right)_{Uncon} = T_B + V_B$

$T_A = \frac{1}{2} m V_A^2 = 0$

$V_A = mgh + \frac{1}{2} k S_A^2 = 0 + 3.75\,J = 3.75\,J$

$T_B = \frac{1}{2} m V_B^2 = V_B^2$

$S_B = l_B - l_0$

$l_B = \sqrt{2^2 + 1.5^2} = 2.5\ m$

$S_B = 2.5 - 1.5 = 1\ m$

$V_B = mgh_B + \frac{1}{2} k x_B^2 =$

$2 \times 9.81 \times 1.5 + \frac{1}{2} \times 50 \times 1^2 = 44.43 \text{ J}$

$\sum U_{1-2} = F \cos 30 (y_B - y_A) = 50 \cos 30 (1.5) = 64.95 \text{ J}$

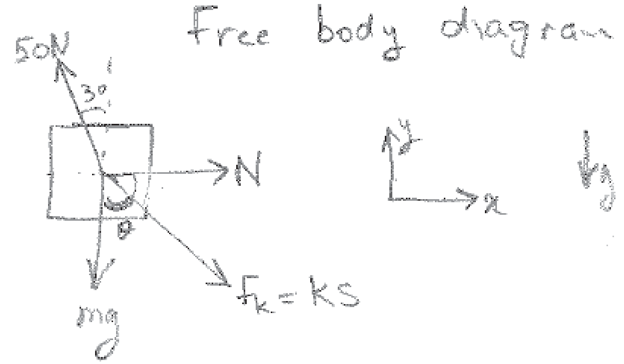
$T_A + V_A + \sum U_{1-2} = T_B + V_B$

$0 + 3.75 + 64.95 = V_B^2 + 44.43$

$V_B^2 = 24.27 \ (m^2/s^2)$

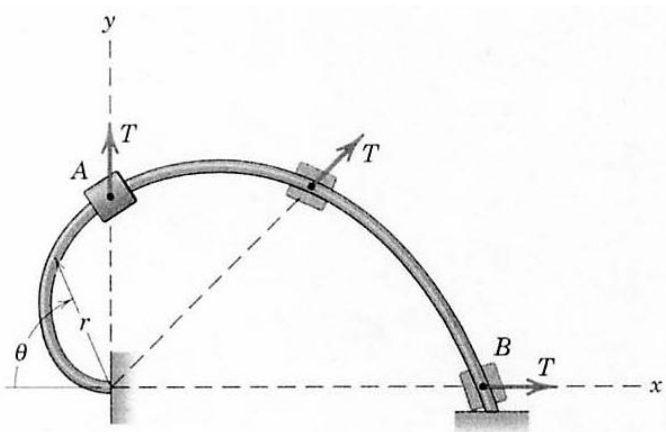
$V_B = 4.92 \ m/s$

c) Free body diagram in position B



MIE100S – Winter 2017
Tutorial Problem 06a

The 0.5 kg collar slides with negligible friction along the fixed spiral rod, which lies in the vertical plane. The rod has the shape of the spiral r=0.3θ, where r is in meters and θ is in radians. The collar is released from rest at A and slides to B under the action of a constant radial force T=10N. Calculate the magnitude of velocity of the slider as it reaches B.



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Tutorial Problem 06a-Solution

$m = 0.5$

$r = 0.3\theta$

$T = 10N$

$\cancelto{0}{T_1} + V_1 + U_T = T_2 + \cancelto{0}{V_2}$

$V_1 = V_{g_1} = mgh = mg\left(0.3\frac{\pi}{2}\right) = 0.5(9.81)(0.3)\left(\frac{\pi}{2}\right) = 2.31 \text{ joules}$

$T_2 = \frac{1}{2}(0.5)v^2$

$U_T = 10(\Delta r) \text{ but} \begin{cases} r_1 = 0.3\frac{\pi}{2} \qquad r_2 = 0.3\pi \\ \qquad \Rightarrow \Delta r = 0.3\frac{\pi}{2} \end{cases}$

$= 3\frac{\pi}{2} = 4.71 \text{ joules}$

$\Rightarrow 2.31 + 4.71 = 0.25v^2$

$\Rightarrow v = 5.3\text{m/s}$

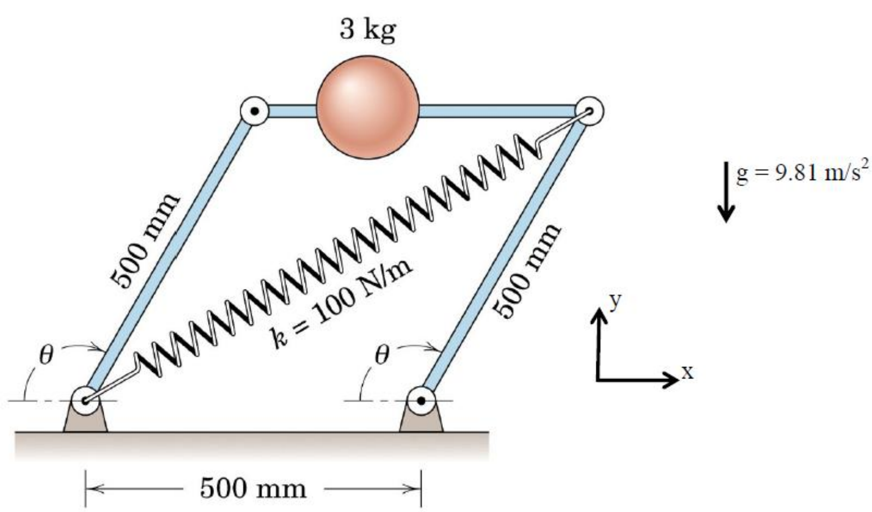
MIE100S – Winter 2017
Tutorial Problem 06b

A very small sphere of mass 3 kg is carried by the parallelogram linkage, where each side of the parallelogram has length 500 mm. The relaxed length of the spring is 400 mm. Ignore the mass of the link arms. At time t=0, $\theta = 90^{o}$ and the linkage is rotating in the clockwise direction such that the sphere has a speed of 4 m/s.

a) What is the potential energy of the spring at time t=0?

b) What will be the direction of travel of the sphere when θ gets extremely close to 180^{o}? (Express your answers in terms of the given x-y coordinate system.)

c) Find the speed of the sphere when θ gets extremely close to 180^{o}.



MIE100S – Winter 2017
Tutorial Problem 06b-Solution

Given :-

$m_1 = 3$ kg

$k = 100$ N/m

$l_1 = 500$ mm $= 0.5$ m

Relaxed length of spring $= 400$ mm $= 0.4$ m

$v_1 = 4$ m/s.

Soln

(a) At $t = 0$, $\theta = 90^\circ$

Actual length of spring at $t = 0$ $= \sqrt{2 l_1^2}$

$= \sqrt{2 \times (0.5)^2}$

$= 0.7071$ m

Hence the spring is stretched

$\therefore$ $s =$ stretched length − relaxed length

$= 0.7071 - 0.4$

$= 0.3071$ m

Hence, the elastic potential energy of the spring at $t = 0$ is,

$$V_e = \frac{1}{2} k s^2$$

$$= \frac{1}{2} \times 100 \times (0.3071)^2$$

$$= \underline{\underline{4.7155 \text{ Joules}}}$$

not so many decimals

(b) When θ gets extremely close to 180°, the direction of the sphere will be

−ve y, or negative j, or ↓ direction

all are fine.

(c) At $t = 0$, $\theta = 90^\circ$

$$T_1 = \frac{1}{2} m_1 v_1^2$$

$$= \frac{1}{2} \times 3 \times (4)^2$$

$$= 24 \text{ Joules}$$

$V_{e_1} = 4.7155$ Joules. ← do not penalize any mistake twice (grader)

$$V_{g_1} = +mgh = +3 \times 9.81 \times 0.5$$

$$= \underline{\underline{+14.71 \text{ Joules}}}$$

At θ close to 180°,

$$T_2 = \frac{1}{2} m_1 V_2^2$$
$$= 1.5 V_2^2$$

$$Ve_2 = \frac{1}{2} k s^2$$

stretched length of spring at $\theta \cong 180^\circ$ is 1 m.

Relaxed length $= 0.4$ m

$\therefore \quad s = 1 - 0.4$

$s = \underline{0.6 \text{ m}}$

$$Ve_2 = \frac{1}{2} \times 100 \times (0.6)^2$$
$$= 18 \text{ Joules.}$$

$$Vg_2 = 0.$$

$$T_1 + Ve_1 + Vg_1 = T_2 + Ve_2 + Vg_2$$

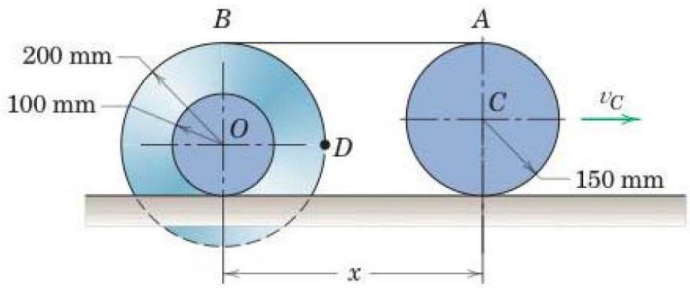
$$24 + 4.7155 + 14.71 = 1.5 V_2^2 + 18 + 0.$$

$$V_2 = \underline{4.11 \text{ m/sec}}$$

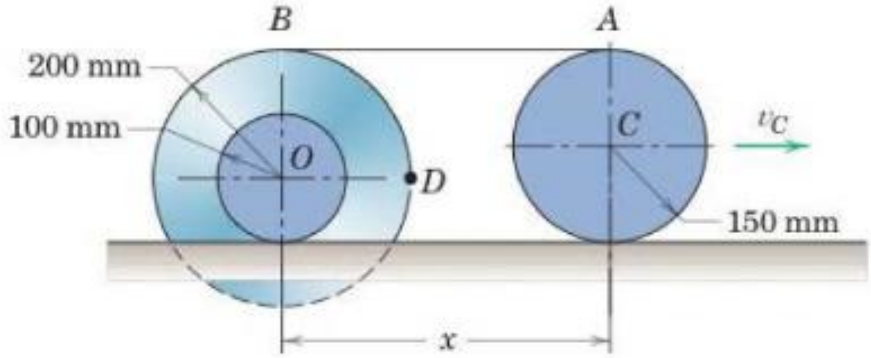
Speed of the sphere when θ gets close to 180° $= \underline{4.11 \text{ m/sec}}$

MIE100S – Winter 2017
Tutorial Problem 07a

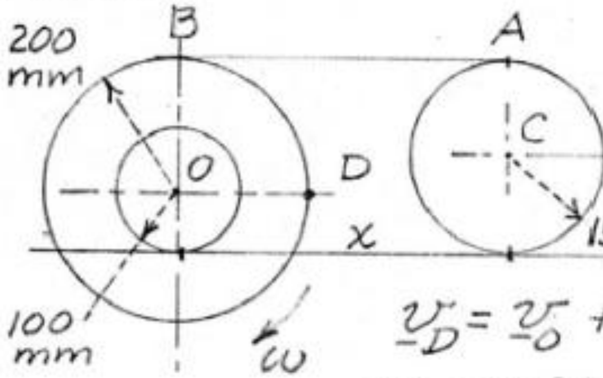
The centre C of the smaller wheel has a velocity $\overrightarrow{v_c}$ = 0.4 m/s in the direction shown. The cord which connects the two wheels is securely wrapped around the respective peripheries and does not slip. Calculate the velocity of point D when in the position shown. Also compute the change Δx which occurs per second if $\overrightarrow{v_c}$ is constant



MIE100S – Winter 2017
Tutorial Problem 07a-Solution



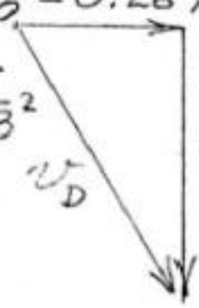
6/62 $v_B = v_A = 2v_C = 0.8\ m/s$, $\omega = \frac{v_B}{r} = \frac{0.8}{0.3} = \frac{8}{3}\ \frac{rad}{s}$



$v_O = \frac{0.1}{0.3} v_B = \frac{0.8}{3} = 0.267\ \frac{m}{s}$

$v_C = 0.4\ m/s$

$\underline{v}_D = \underline{v}_O + \underline{v}_{D/O}$, $v_{D/O} = \overline{OD}\,\omega = 0.2\left(\frac{8}{3}\right) = 0.533\ \frac{m}{s}$



$v_D = \sqrt{0.267^2 + 0.533^2}$

$\underline{= 0.596\ m/s}$

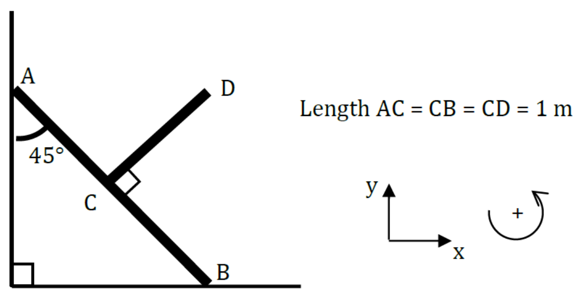
$\dot{x} = v_{C/O} = (v_C - v_O) = (0.4 - 0.267) = \underline{0.133\ m/s}$

MIE100S – Winter 2017
Tutorial Problem 07b

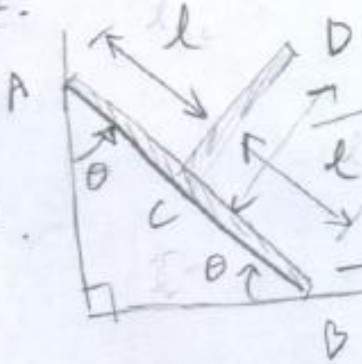
At the instant shown in the figure, point B of the T-shaped bar has a velocity of 1 $\hat{\imath}\,{}^{m}/_{s}$ and an acceleration of 2 $\hat{\imath}\,{}^{m}/_{s^2}$.

a) Determine the velocity of point D in x-y coordinates.

b) Determine the angular velocity of the T-shaped bar.



Q2.



$\theta = 45°$.

$\curvearrowright +$ $\uparrow y \; \rightarrow x$

$\vec{v}_B = v_B \hat{i}$ m/s $= 1 \hat{i}$ m/s.

$\vec{a}_B = 2$ m/s^2 $\hat{i}$.

$\overline{AC} = \overline{CB} = \overline{CD} = l = 1$ m

a) Find: $\vec{V}_D$

$$\vec{V}_D = \vec{V}_B + \vec{\omega} \times \vec{r}_{D/B}$$

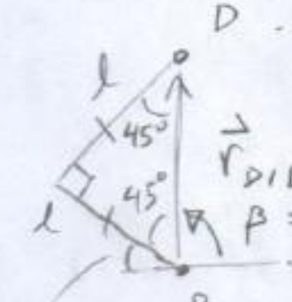
$$\vec{V}_D = v_B \hat{i} + \vec{\omega} \times r_{D/B} \hat{j}$$

$$\vec{\omega} = \omega \hat{k}$$

$$\vec{V}_D = V_B \hat{i} + \omega \hat{k} \times r_{D/B} \hat{j}$$

$$\vec{V}_D = V_B \hat{i} + \omega r_{D/B} \underbrace{(\hat{k} \times \hat{j})}_{-\hat{i}}$$

$$\vec{V}_D = (V_B + \omega r_{D/B}) \hat{i} \quad — \quad (1)$$



$\beta = 180° - 45° - 45°$

$\beta = 90°$

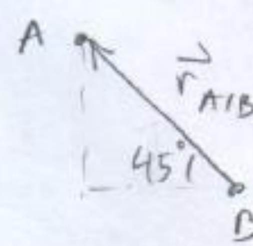
$$|r_{D/B}| = \sqrt{l^2 + l^2}$$

$$r_{D/B} = \sqrt{2}$$

$$\therefore \vec{r}_{D/B} = r_{D/B} \hat{j} = \sqrt{2} \hat{j} \text{ m}$$

Relative motion between A & B to find ω:

$$\vec{V}_A = \vec{V}_B + \vec{\omega} \times \vec{r}_{A/B}$$



$$\vec{r}_{A/B} = -r_{A/B} \cos 45° \hat{i} + r_{A/B} \sin 45° \hat{j}$$

Q2.

$$\vec{r}_{A/B} = -2\frac{\sqrt{2}}{2}\hat{i} + 2\frac{\sqrt{2}}{2}\hat{j} = -\sqrt{2}\hat{i} + \sqrt{2}\hat{j} \text{ m.}$$

$\rightarrow$ $\vec{v}_A = -v_A\hat{j}$

$\rightarrow$ $-v_A\hat{j} = v_B\hat{i} + \omega\hat{k} \times (-\sqrt{2}\hat{i} + \sqrt{2}\hat{j})$

$-v_A\hat{j} = v_B\hat{i} - \sqrt{2}\omega\hat{j} - \sqrt{2}\omega\hat{i}$

$\hat{i}$ - comp: $0 = v_B - \sqrt{2}\omega$

ans $\rightarrow$ $\boxed{\omega = \frac{1}{\sqrt{2}} = 0.71 \text{ rad/s} \circlearrowleft}$

part b).

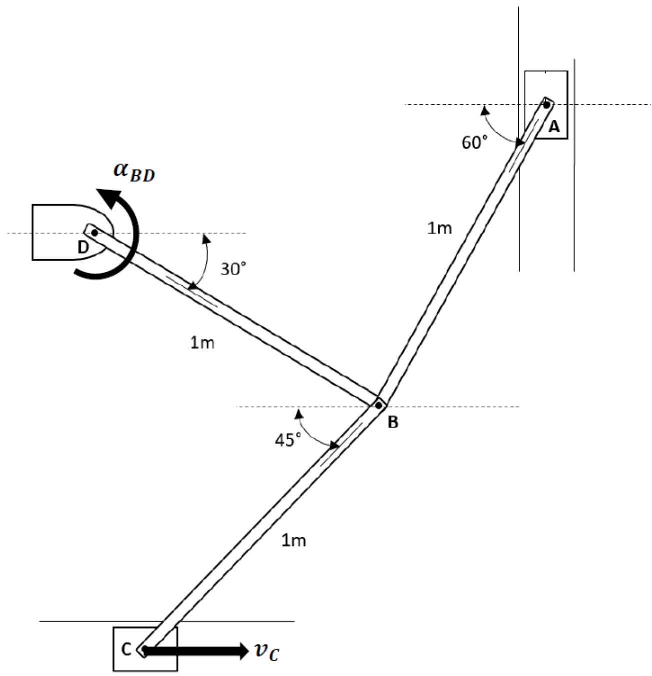
sub ω into 1)

$$\vec{v}_D = \left[v_B - r_{D/B}\,\omega\right]\hat{i} = \left[1 - \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)\right]\hat{i}.$$

$\boxed{\therefore\ V_D = 0 \text{ m/s.}}$ $\leftarrow$ ans. part b)

MIE100S – Winter 2017
Tutorial Problem 08a

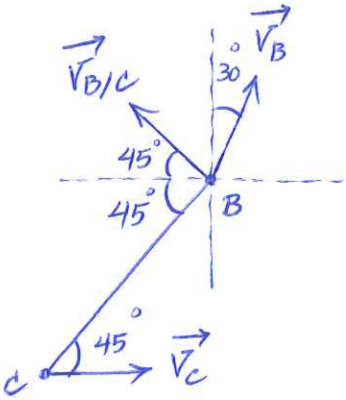
Given point C is moving to the right at $v_c = 0.5\,\frac{m}{s}$ and linkage BD has an angular acceleration of $\alpha_{BD} = 0.5\,\frac{rad}{s^2}$, in the direction shown, find the magnitude of acceleration of point B at the given instant.



MIE100 - Winter 2017

Tutorial Problem 08a - Solution

- link BC :



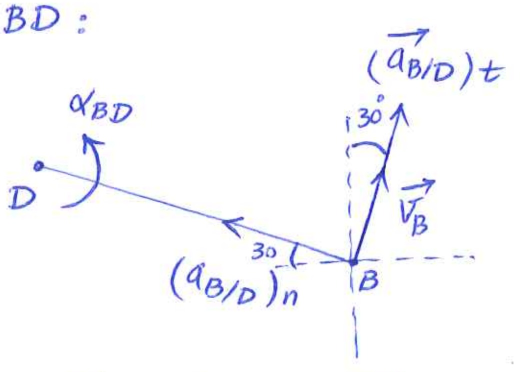
$$\vec{V}_B = \vec{V}_C + \vec{V}_{B/C} = \vec{V}_C + \vec{\omega}_{BC} \times \vec{r}_{BC}$$

$$V_B \sin 30 = V_C + (-\omega_{BC})(r_{BC}) \cos 45$$

$$V_B \cos 30 = (\omega_{BC})(r_{BC}) \sin 45$$

$$\left.\begin{aligned} \tfrac{1}{2} V_B &= 0.5 - \tfrac{\sqrt{2}}{2}\,\omega_{BC} \\ \tfrac{\sqrt{3}}{2} V_B &= \tfrac{\sqrt{2}}{2}\,\omega_{BC} \end{aligned}\right\} \qquad V_B = \underbrace{0.366}_{0.37}\ m/s \ , \quad \omega_{BC} = 0.45\ rad/s \circlearrowright$$

- link BD :



$$V_B = \omega_{BD}\, r_{BD}$$

$$\omega_{BD} = \frac{0.37}{1} = 0.37\ rad/s \ \circlearrowright$$

$$\vec{a}_B = \underbrace{\cancel{\vec{a}_D}}_{0} + \vec{a}_{B/D} = \vec{a}_{B/D}$$

$$(\vec{a}_{B/D})_t = (\alpha_{BD}\, r_{BD}) \sin 30\ \hat{i} + (\alpha_{BD}\, r_{BD}) \cos 30\ \hat{j} = 0.25\,\hat{i} + 0.43\,\hat{j}\ m/s^2$$

$$(\vec{a}_{B/D})_n = -(\omega_{BD}^2\, r_{BC}) \cos 30\ \hat{i} + (\omega_{BD}^2\, r_{BD}) \sin 30\ \hat{j} = -0.12\,\hat{i} + 0.07\,\hat{j}\ m/s^2$$

$$\vec{a}_B = (0.25 - 0.12)\,\hat{i} + (0.43 + 0.07)\,\hat{j} = \boxed{0.13\,\hat{i} + 0.5\,\hat{j}\ m/s^2}$$

$$\boxed{|\vec{a}_B| = 0.52\ m/s^2}$$

MIE100S – Winter 2017
Tutorial Problem 08b

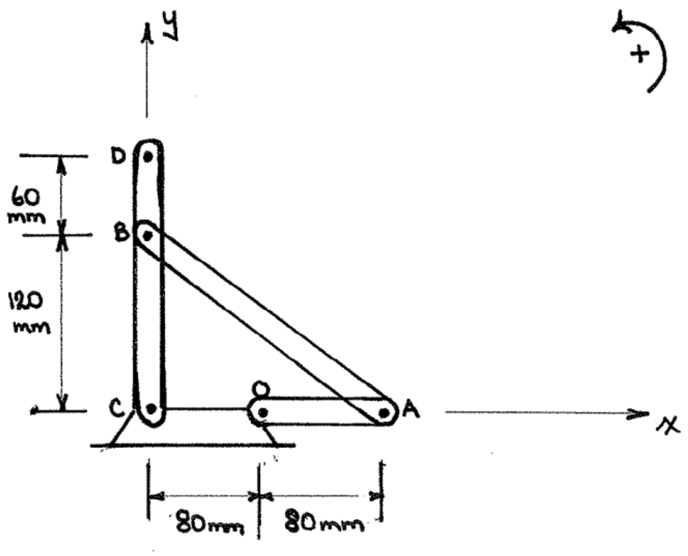
In the mechanism shown, the initial velocity of point D is $1\ \frac{m}{s}\ \hat{\imath}$, and the initial angular acceleration α_{CD} of bar CD is $-5\ \frac{rad}{s^2}$.

a. What is the initial velocity of point B, expressed in x-y coordinates?

b. What is the acceleration of point D? Express your answer in normal-tangential coordinates.

c. What is the initial velocity of point A, expressed in Cartesian coordinates?

d. What is the initial angular velocity ω_{AB} of bar AB?



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Tutorial Problem 08b-Solution

80mm | 80mm

$$\vec{v}_D = 1\,m/s\,\hat{i} = \cancelto{0}{\vec{v}_C} + \vec{\omega}_{CD}\times\vec{r}_{D/C} = \omega_{CD}\hat{k}\times(0.18m\,\hat{j}) = -\omega_{CD}\cdot 0.18m$$

$$\Rightarrow \omega_{CD} = \frac{-1m/s\,\hat{i}}{0.18m\,\hat{i}} = -5.5\bar{5}\ rad/s$$

a) $$\vec{v}_B = \cancelto{0}{\vec{v}_C} + \overset{\vec{\omega}_{CD}}{\cancel{\vec{\omega}_{BC}}}\times\vec{r}_{B/C} = -5.5\bar{5}\ rad/s\ \hat{k}\times(0.12m\,\hat{j})$$

$$\boxed{\vec{v}_B = 0.6\bar{6}\ m/s\ \hat{i} = 0.67\ m/s\ \hat{i}}$$

b) $$\vec{a}_D = \cancelto{0}{\vec{a}_C} + \vec{\alpha}_{CD}\times\vec{r}_{D/C} - \omega_{CD}^2\vec{r}_{D/C} = (-5\,rad/s^2\,\hat{k})\times(0.18m\,\hat{j}) - (-5.5\bar{5}\,\tfrac{rad}{s})^2(0.18m\,\hat{j})$$

$$\boxed{\vec{a}_D = -0.9\,m/s^2\,\hat{i} - 5.5\bar{5}\,m/s^2\,\hat{j} = 0.9\,m/s^2\,\hat{u}_t + 5.56\,m/s^2\,\hat{u}_n}$$

c)/d) $$\vec{v}_A = \cancelto{0}{\vec{v}_O} + \vec{\omega}_{OA}\times\vec{r}_{A/O} = \omega_{OA}\cdot 0.08\,m\,\hat{j}$$

$$\vec{v}_A = \vec{v}_B + \vec{\omega}_{AB}\times\vec{r}_{A/B} = 0.67\,m/s\,\hat{i} + \omega_{AB}\hat{k}\times(0.16m\,\hat{i} - 0.12m\,\hat{j})$$

$$= 0.67\,m/s\,\hat{i} + 0.12m\cdot\omega_{AB}\,\hat{i} + 0.16m\,\omega_{AB}\,\hat{j}$$

Equate $\hat{i}$ terms: $0 = 0.67\,m/s + 0.12m\cdot\omega_{AB}$

$$\boxed{\Rightarrow \omega_{AB} = -5.56\ rad/s}$$

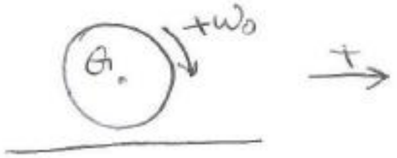
Sub in ∴ $$\boxed{\vec{v}_A = -0.89\ m/s\ \hat{j}}$$

MIE100S – Winter 2017
Tutorial Problem 09a

A uniform spinning disk drops a very short distance onto a flat horizontal surface. Initially, the disk will slip, and accelerate in horizontal direction. Eventually, it will roll without slipping, with constant velocity. Determine the final velocity of the center of the disk. Ignore the potential energy associated with this small vertical drop.

Radius of the disk = R
Mass of the disk = M
Initial angular velocity = ω_0
Kinetic and static friction coefficients = μ_k and μ_s

MIE100S – Winter 2017
Tutorial Problem 09a-Solution



M, R

Find V_G @ rolling

$\Rightarrow \omega_f = V_G/R$

FBD

$F_f = \mu_k mg$

$$\Sigma F_x = m a_{G_x} \Rightarrow \mu_k mg = m a_G \Rightarrow a_G = \mu_k g$$

$$\Sigma M_G = I\alpha \Rightarrow -\mu_k mgR = \tfrac{1}{2} mR^2\alpha \Rightarrow \alpha = -\frac{2\mu_k g}{R}$$

But: $V_G = 0 + at \Rightarrow t = V_G/\mu_k g$

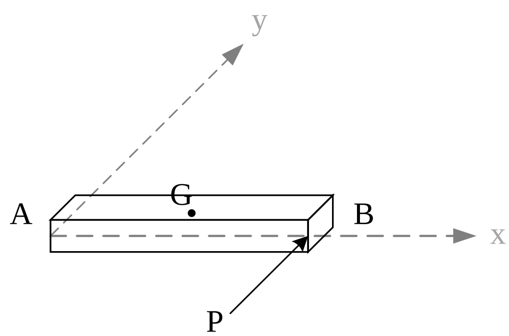
$$\omega_f = \omega_0 + \alpha t \Rightarrow t = \frac{\omega_f - \omega_0}{\alpha}$$

$$\Rightarrow \frac{V_G}{\mu_k g} = \frac{(V_G/R - \omega_0)}{-2\mu_k g/R} \Rightarrow 2V_G = -V_G + \omega_0 R$$

$$\Rightarrow V_G = \omega_0 R/3 \quad \longleftarrow$$

MIE100S – Winter 2017
Tutorial Problem 09b

A rod AB, with mass of 0.75 kg and length of 1.2 m, is at rest. A force of P = 2N is applied to one end of the rod. Find initial $\overrightarrow{a_A}$ in terms of supplied x-y.



x-y plane is horizontal

Neglect friction

$\omega_1 = 0$

$m_{AB} = 0.75$ kg

$l_{AB} = 1.2$ m

P = 2 N

MIE100S – Winter 2017
Tutorial Problem 09b-Solution

$$\sum F_x = m\, a_{G_x} = 0$$

$$\sum F_y = m\, a_{G_y} \Rightarrow 2 = 0.75\, a_{G_y} \Rightarrow a_{G_y} = 2.67 \ \frac{m}{s^2}$$

$$\Rightarrow \overrightarrow{a}_G = 2.67\, \hat{y} \ \frac{m}{s^2}$$

$$\sum M_G = I_G\, \alpha \qquad I_G = \frac{1}{12}(0.75)(1.2)^2 = 0.09 \ kg\,m^2$$

$$+\circlearrowleft \Rightarrow \alpha = \frac{2\,(0.6)}{0.09} = +13.3 \ s^{-1}$$

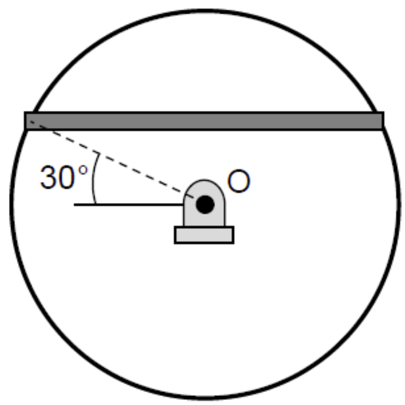
$$\text{but} \quad \overrightarrow{a}_A = \overrightarrow{a}_G + \overset{r\alpha}{\overrightarrow{a}_{A/G}}$$

$$= 2.67\,\hat{j} - 0.6\,(13.3)\,\hat{j} = -5.31\,\hat{j} \ \frac{m}{s^2}$$

get this from Physics.

MIE100S – Winter 2017
Tutorial Problem 10a

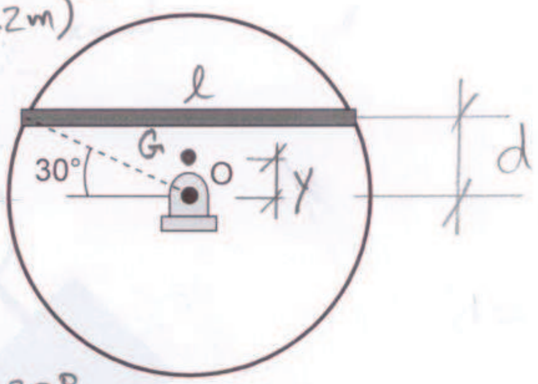
A uniform disk with a mass of 6 kg and radius of 0.2 m pivots without friction about a horizontal axis through point O. A long slender rod with a mass of 2 kg is fastened to the disk as shown. If the system is nudged from rest while in the position shown, determine its angular velocity $\boldsymbol{\omega}$ after it has rotated 180°.



MIE 100 - Winter 2017
Tutorial problem 10a - Solution

From a 2006 quiz

A uniform disk with mass 6 kg and radius 0.2 m pivots without friction about a horizontal axis through point O. A long slender rod with mass 2 kg is fastened to the disk as shown. If the system is nudged from rest while in the position shown, determine its angular velocity ω after it has rotated 180°.



$I_{disk_O} = \frac{1}{2}mr^2 = \frac{1}{2}(6\,Kg)(0.2m)^2$

$= 0.12\,Kg\cdot m^2$

$I_{bar_O} = I_{bar_{G_{bar}}} + md^2$

$= \frac{1}{12}ml^2 + md^2$

$l = 2r\cos 30° = 2(0.2)\cos 30°$

$= 0.346\ m$

$d = r\sin 30° = (0.2)\sin 30° = 0.1m$

$\Rightarrow I_{bar_O} = \frac{1}{12}(2Kg)(0.346m)^2 + (2Kg)(0.1m)^2$

$= 0.04\ Kg\cdot m^2$

$I_O = I_{bar_O} + I_{disk_O} = 0.16\ Kg\cdot m^2$

Centre of mass

$y = \dfrac{6(0) + 2(0.1)}{8} = 0.025m$

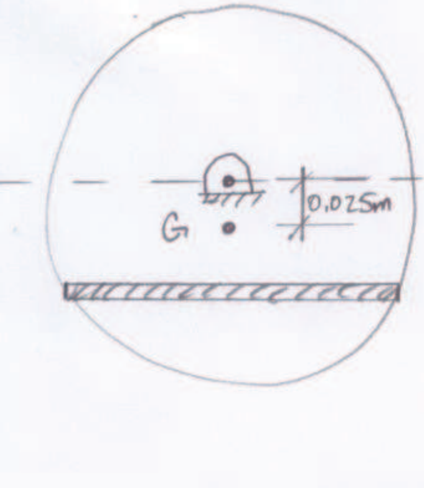
Conservation of energy

$T_1 + V_1 = T_2 + V_2$

180° rotation (state 2)

$T_1 = 0 \quad , V_1 = Wy \quad ; V_2 = W(-y)$

$T_2 = \frac{1}{2} I_O \omega_2^2$



— datum

$Wy = \frac{1}{2}I_O\omega_2^2 - Wy$

$\Rightarrow 2(9.81)(8Kg)(0.025m) = \frac{1}{2}(0.16Kg\cdot m^2)\omega_2^2$

$\boxed{\omega_2 = 7.0\ rad/s}$

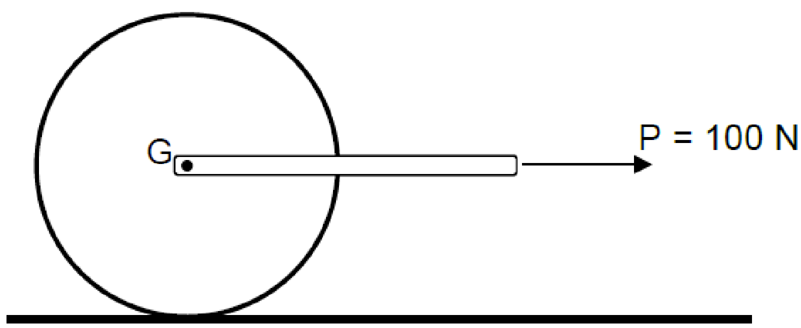
MIE100S – Winter 2017
Tutorial Problem 10b

A uniform disk with mass 24 kg and radius 0.3 m is initially at rest. It is then acted upon by a constant 100 N force as shown and rolls without slipping.

Determine:

(a) The velocity of its centre of mass, G, after it has moved 1.8 m

(b) The friction force required to prevent slipping

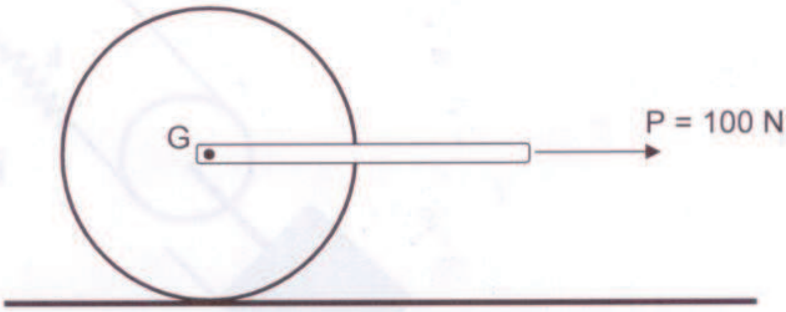


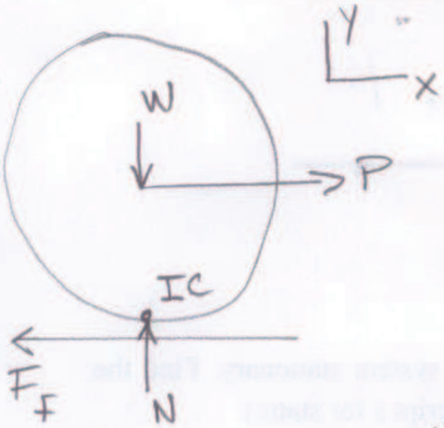
MIE 100 - Winter 2017
Tutorial problem 10b - Solution

A textbook problem

A uniform disk with mass 24 kg and radius 0.3 m is initially at rest. It is then acted upon by a constant 100 N force as shown and rolls without slipping. Determine:

(a) the velocity of its centre of mass, G, after it has moved 1.8 m; and
(b) the friction force required to prevent slipping.





$$T_1 + U_{1\to 2} = T_2 \quad , \; T_1 = 0$$

$$T_2 = \frac{1}{2} I_{IC}\,\omega_2^2$$

$$I_{IC} = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2 = \frac{3}{2}(24\,kg)(0.3\,m)^2$$

$$= 3.24\ kg\cdot m^2$$

$$U_{1\to 2} = P\Delta x = (100N)(1.8m) = 180\ N\cdot m$$

$$\Rightarrow \emptyset + 180N\cdot m = \frac{1}{2}(3.24\ kg\cdot m^2)\omega_2^2 \Rightarrow \omega_2 = 10.541\ rad/s$$

Since no slipping, $v_G = \omega r = (10.541)(0.3)$

$$\boxed{v_G = 3.162\ m/s \rightarrow} \quad (a)$$

$$\overset{+}{\rightarrow}\sum F_x = m(a_G)_x \longrightarrow P - F_f = m(a_G)_x$$

$$v_G^2 = v_0^2 + 2(a_G)_x(\Delta x)$$

$$3.16^2 = \emptyset^2 + 2(a_G)_x(1.8)$$

$$(a_G)_x = 2.78\ m/s^2$$

$$100\ N - F_f = (24\,kg)(2.78\ m/s^2)$$

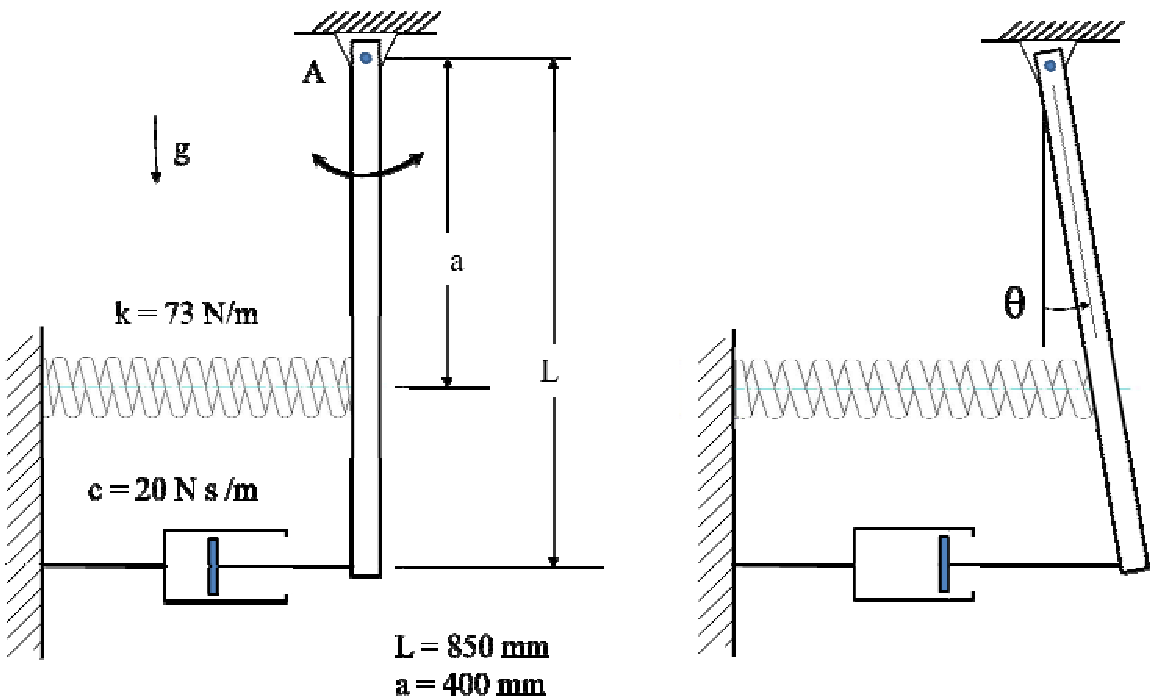
$$\Rightarrow \boxed{F_f = 33.3\ N} \quad (b)$$

MIE100S – Winter 2017
Tutorial Problem 11a

The homogeneous slender rod of mass m = 16 kg and length of L = 850 mm is supported by a pin at A and is able to rotate about A. The bar is connected to a spring and damper as shown. The bar is initially in equilibrium with the spring relaxed at the position shown in the diagram on the left.

a) What is the moment of inertia of the rod about A?

b) What is the differential equation for the motion of the rod in terms of θ? Assume the motion only involves small angles of θ.

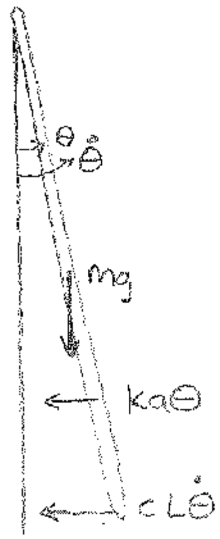


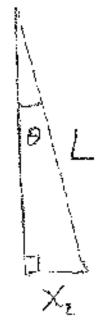
MIE 100 - Winter 2017
Tutorial problem 11a - Solution

Q5

a) $I_A = \frac{1}{3} m L^2 = \frac{1}{3}(16)(0.85)^2 = 3.853 \ \text{kg m}^2$

b)





$x_1 = a\sin\theta$

$x_1 \approx a\theta$

$x_2 = L\sin\theta$

$\dot{x}_2 = L\cos\theta\,\dot{\theta}$

$\dot{x}_2 \approx L\dot{\theta}$

$\Sigma M_A = I_A \ddot{\theta}$

$-mg\frac{L}{2}\theta - ka\theta(a) - cL\dot{\theta}(L) = I_A\ddot{\theta}$

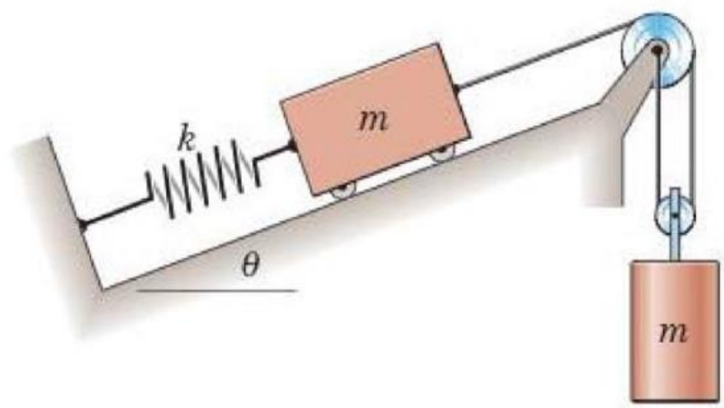
$I_A\ddot{\theta} + cL^2\dot{\theta} + \left[\frac{mgL}{2} + ka^2\right]\theta = 0$

$3.853\ddot{\theta} + 20(0.85)^2\dot{\theta} + \left[\frac{16(9.81)(0.85)}{2} + 73(0.4)^2\right]\theta = 0$

$3.853\ddot{\theta} + 14.45\dot{\theta} + 78.388\,\theta = 0$

MIE100S – Winter 2017
Tutorial Problem 11b

Calculate the natural frequency ωn of the system shown in the figure. The mass and the friction of the pulleys are negligible.



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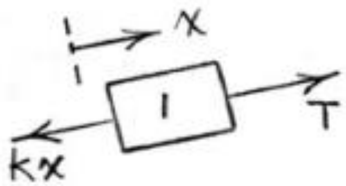
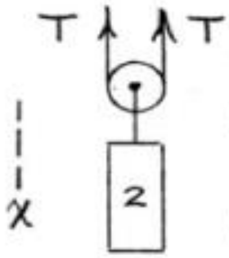
Tutorial Problem 11b-Solution



Free body diagrams show dynamic forces only. x is the displacement from equilibrium.

From constraint, $a_2 = \frac{1}{2} a_1$



$\sum F_x = m\ddot{x}$:

① $-kx + T = m\ddot{x}$

② $-2T = m\left(\frac{1}{2}\ddot{x}\right)$

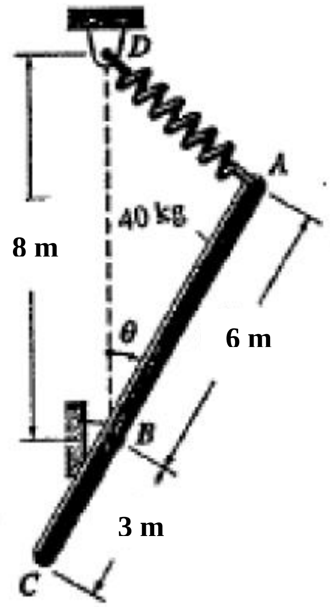
Eliminating T : $\ddot{x} + \frac{4k}{5m} x = 0$

$$\omega_n = \sqrt{\frac{4k}{5m}}$$

MIE100S – Winter 2017
Tutorial Problem 11a

The uniform 40 kg slender bar AC rotates in a vertical plane about the pin at B. The ideal spring AD has a spring constant k=20 N/m and an undeformed length of L_0=3m. The bar is released at rest in the position $\theta = 0$. When the bar reaches the horizontal position, find:

a) Angular velocity of the bar.
b) Force at pin B.



MIE100S - Winter 2017
Tutorial problem 11a - Solution

4. This is an energy problem - we are given distance (not Δt) information.

we also assume that the motion actually occurs. $L_0 = 3m$.

$$T_1 + V_{1g} + V_{e_1} + U_{1\to2} = T_2 + V_{2g} + V_{2e}$$

$T_1 = 0$

$V_{e_1} = \frac{1}{2}kx^2 = \frac{1}{2}(20)(1)^2 = 10$ joules.

$V_{2g} = 0$

$T_2 = \frac{1}{2}I_B\omega^2 = \frac{1}{2}\left\{\frac{1}{12}(40)(9)^2 + 40(1.5)^2\right\}\omega^2$

$= 180\,\omega^2$

$U_{1\to2} = 0$

$V_{g_1} = mgh_G = 40(9.81)(1.5) = 588.6$ joules.

V_{e_2} : sketch.

$V_{e_2} = \frac{1}{2}(20)(7^2) =$

490 joules.

(a) $\Rightarrow \quad 588.6 + 10 = 180\omega^2 + 490$

$\Rightarrow \quad \omega = 0.78\ s^{-1}$

(b) force at pin B:

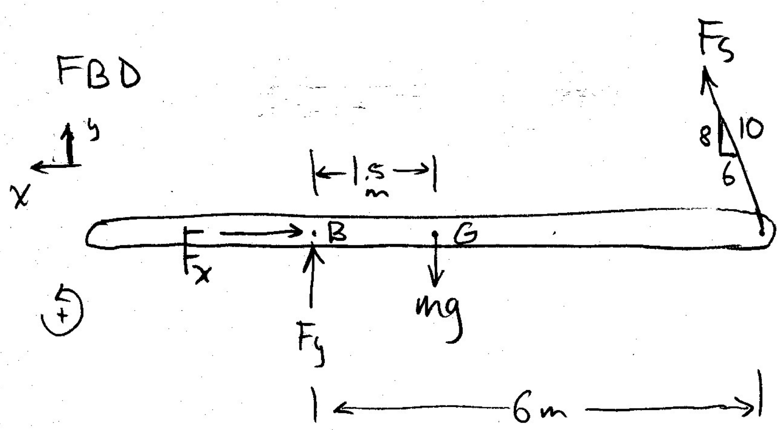
Fixed axis rotation:

(i) $\Sigma M_B = I_B \alpha$

(ii) $\Sigma F_x = m a_{Gx}$

(iii) $\Sigma F_y = m a_{Gy}$

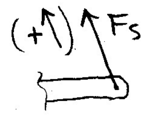
FBD



Note that $a_{Gx} = a_{Gn} = \omega^2 r$

$a_{Gy} = a_{Gt} = \alpha r$

where r is 1.5m.



(i) $\frac{8}{10} F_s(6) - mg(1.5) = I\alpha$

$\alpha = \dfrac{\frac{8}{10}(140)(6) - 40(9.81)(1.5)}{360}$

$\alpha \doteq 0.2317$ rad/s. (↺+)

$F_s = -ks$

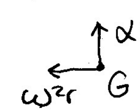
$F_s = -20(-7)$

$F_s = 140N$

S is negative in this localized coordinate system, making F_s +ve as shown.

$a_{Gx} = \omega^2 r = 0.7767^2(1.5) = 0.905 \quad (\vec{a}_G)_x = 0.905$ ↑ m/s²

$a_{Gy} = \alpha r = 0.2317(1.5) = 0.3475 \quad (\vec{a}_G)_y = 0.3475$ ↑ m/s²



(ii) $-F_x + F_s\left(\frac{6}{10}\right) = m a_{Gx}$

$F_x = 140\left(\frac{6}{10}\right) - 40(0.905)$

$F_x = 47.8\ N$

(iii) $F_y - mg + \frac{8}{10} F_s = m a_{Gy}$

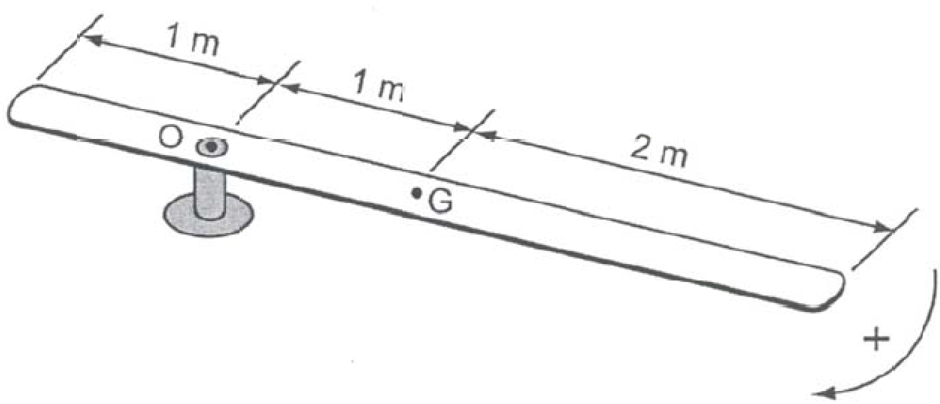
$F_y = 40(0.3475) + 40(9.81) - \frac{8}{10}(140)$

$F_y = 294.3\ N$

MIE100S – Winter 2017
Tutorial Problem 11b

A thin, uniform rod of mass $3kg$ and length $4m$ rotates in the *horizontal* plane about a pivot point, O. Kinetic friction between the rod and its pivot points, results in a constant frictional moment about point O of unknown magnitude. The rod is initially at rest.

At t=0, a motor is turned on. The motor applies a constant clockwise moment of 100 $N.m$ to the rod about point O. At t=3s, the motor is turned off and from that point the rod continues to rotate for another 45° before it comes to a stop.



a) Determine the angular velocity when the motor is turned off at $t=3s$.

b) Determine the magnitude of the moment due to friction.

c) Determine the total work done on the rod by the non-conservative forces from $t=0$ until the rod stops rotating.

2. $m = 3kg$ $\quad M_M = 100 N\cdot m$

$l = 4m$

(a) + (b) use energy and angular momentum.

from $0 \to 3s$: $0 + (M_M - M_f)3 = I_O\omega_3$.

from $3s \to$ stop : $\frac{1}{2} I_O \omega_3^2 - M_f\left(\frac{\theta}{4}\right) = 0$

$$I_O = I_G + md^2 = \frac{1}{12}(3)(4)^2 + 3(1)^2$$

$$= 7 \text{ kgm}^2.$$

$3(100) - 3M_f = 7\omega_3 \Rightarrow M_f = 100 - \frac{7}{3}\omega_3.$

$3.5\omega_3^2 - .785 M_f = 0$.

$\Rightarrow 3.5\omega_3^2 - .785(100 - 2.33\omega_3) = 0$.

$\Rightarrow 3.5\omega_3^2 + 1.83\omega_3 - 78.5 = 0$.

$$\omega_3 = \frac{-1.83 \pm \sqrt{(1.83)^2 + 4(3.5)(78.5)}}{7}$$

$$= \frac{-1.83 \pm 33.2}{7} = 4.48 \ s^{-1}$$

using the + answer only.

$\Rightarrow M_f = 100 - \frac{7}{3}(4.48) = 89.5 \ N\cdot m$ (in the -ve direction)

(c) zero.