UNIVERSITY OF TORONTO

Faculty of Applied Science and Engineering

CIV100F and APS160F – MECHANICS

Midterm Examination – Sections 1, 2, 3, 4, 5, 6, 7, 8 and Online

Tuesday, 21st October 2014

Examiner: Staff in Civil Engineering

Time allowed: 1-½ hours

SURNAME: ________SEICA__________GIVEN NAME(S): _________M. _____________
(Please print clearly)

STUDENT NUMBER: _____Solutions______ DEPT. (ECE, Track One, etc.)____________

CIRCLE YOUR SECTION AND THE NAME OF YOUR INSTRUCTOR:

1. Grasselli, Giovanni
2. Grasselli, Giovanni
3. Briggs, Scott
4. Mercan, Oya
5. Guner, Serhan
6. Kamaleddine, Fouad
7. Sun, Min
8. Packer, Jeffrey

Online. Seica, Michael

CIRCLE YOUR CALCULATOR TYPE:

CASIO 991 **SHARP 520**

Notes:

1. **Ensure that you have all 5 sheets of the examination paper. Page 5 is blank.**
2. **Answer all three questions. The value of the questions is indicated below.**
3. **If you need more space for a question, please use the back of the preceding question. In all cases, please indicate clearly where your calculations are continued.**
4. **The only calculators permitted are listed above. Please circle your model.**
5. **No other paper will be accepted for marking or allowed on the desk.**
6. **Do not remove the staple.**

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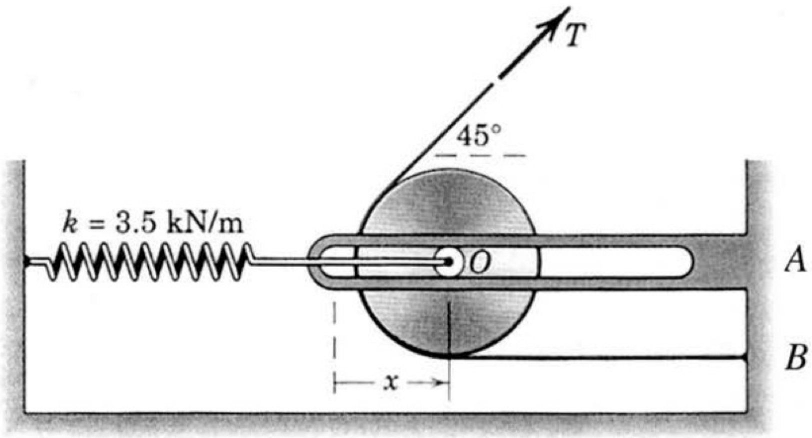
1. The spring of modulus $k = 3.5$ kN/m is loaded by a tension force of 35 N when the disc centre O is in the leftmost position (at $x = 0$). Determine the tension T required to position the disc centre at $x = 150$ mm. For that position, calculate the distance OA if the moment reaction at the fixed support at A is 60.9 Nm. The mass of the disc is 3 kg.



The force in a spring:

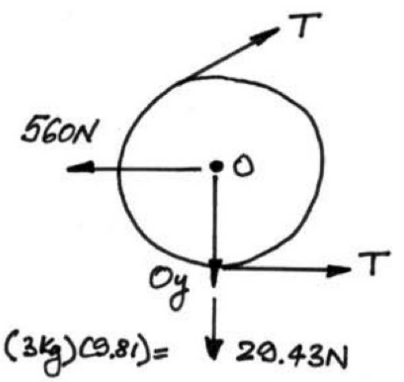
$F = k\Delta$

Extension of spring

The extension of the spring when the disc is at $x=0$: $\Delta_0 = \dfrac{35N}{3{,}500 N/m} = 10mm$.

The total extension of the spring at $x = 150mm$: $\Delta = 150 + 10 = 160mm$.

The force in the spring at $x = 160mm$: $F = (3{,}500 N/m)(0.16 m) = 560 N$

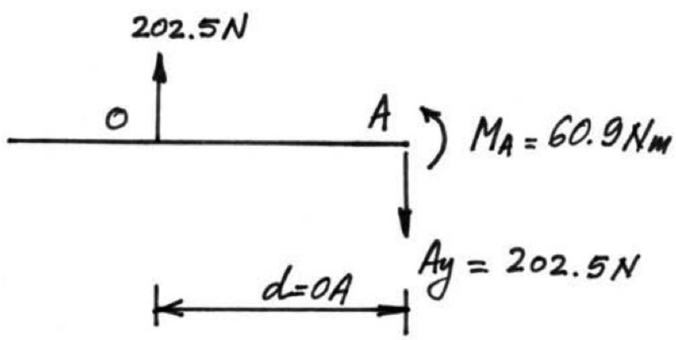


On the FBD of the disc: $\Sigma F_x = 0$

$T(1+\cos 45°) = 560N$ $\therefore$ $T = 328 N$

$\Sigma F_y = 0$ $(328N)\sin 45° - O_y - 29.43N = 0$

$\therefore$ $O_y = 202.5N \downarrow$



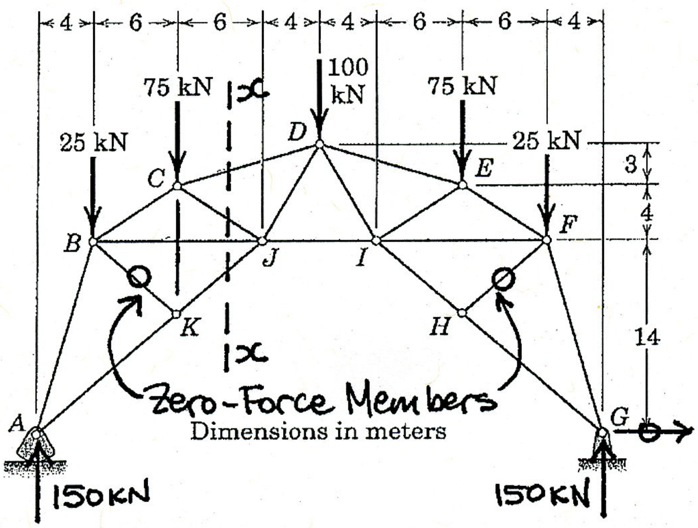
On the FBD of the cantilever OA:

$\Sigma M_A = 0$

$60.9 Nm - (202.5N)(d) = 0$

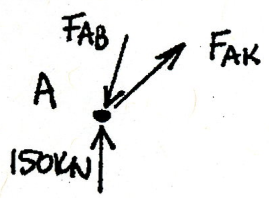
$\therefore$ $d = 300 mm$

2. Determine the forces in members *CD*, *JK*, *EI* and *FI* of this arched roof truss.



The truss has symmetric geometry and the applied forces are also symmetric; therefore, the forces in members are also symmetric (i.e. $F_{CD} = F_{DE}$, $F_{JK} = F_{HI}$, $F_{CJ} = F_{EI}$, $F_{BJ} = F_{FI}$), as are the support reaction forces.

- From the equilibrium of joint A:



$[\Sigma F_x = 0]$ $F_{AK}\left(16/\sqrt{16^2+14^2}\right) - F_{AB}\left(4/\sqrt{4^2+14^2}\right) = 0$

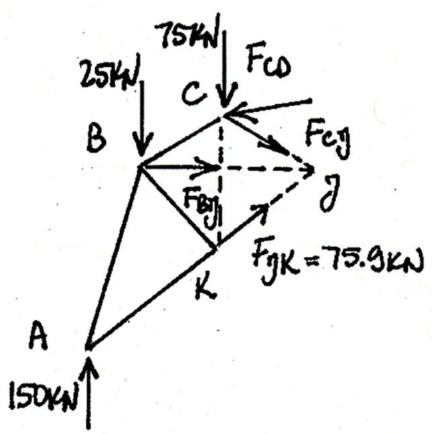
$0.7526\, F_{AK} = 0.2747\, F_{AB}$ $\quad F_{AB} = 2.74\, F_{AK}$

$[\Sigma F_y = 0]$ $150 + F_{AK}\left(14/\sqrt{16^2+14^2}\right) - F_{AB}\left(14/\sqrt{4^2+14^2}\right) = 0$

$150 + 0.6585\, F_{AK} - 0.9615\, F_{AB} = 0$

$1.9757\, F_{AK} = 150$ $\quad \therefore F_{AK} = \underline{\underline{F_{JK} = 75.9 \text{ kN (T)}}}$

- Cut the truss along x–x and use the LHS FBD:



$[\Sigma M_J = 0]$ $-(150)(16) + F_{CD}\left(10/\sqrt{10^2+3^2}\right)(4) +$

$+ F_{CD}\left(3/\sqrt{10^2+3^2}\right)(6) + (25)(12) + (75)(6) = 0$

$\therefore \underline{\underline{F_{CD} = 297 \text{ kN (C)}}}$

$[\Sigma M_C = 0]$ $-(150)(10) + (25)(6) + F_{BJ}(4) +$

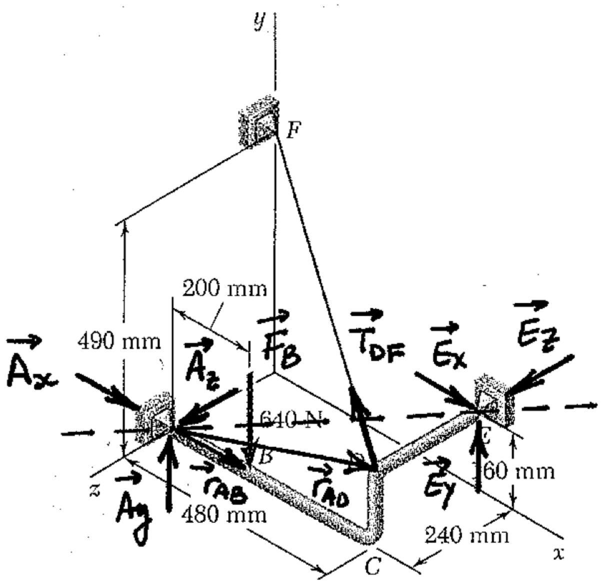
$+ (75.9)\left(16/\sqrt{16^2+14^2}\right)\left(18 - (10)(14/16)\right) = 0$

$\therefore F_{BJ} = \underline{\underline{F_{FI} = 205 \text{ kN (T)}}}$

$[\Sigma F_y = 0]$ $150 - 25 - 75 - (297)\left(3/\sqrt{10^2+3^2}\right) + (75.9)\left(14/\sqrt{16^2-14^2}\right) -$

$- F_{CJ}\left(4/\sqrt{4^2+6^2}\right) = 0$ $\quad \therefore F_{CJ} = \underline{\underline{F_{EI} = 26.4 \text{ kN (T)}}}$

3. The pipe *ACDE* is supported by ball-and-socket joints at *A* and *E* and by the wire *DF*. Determine the tension in the wire when a force with a magnitude of 640N is applied at *B*, as shown. The weight of the pipe is small and can be ignored.



$$\vec{AE} = 480\vec{i} + 160\vec{j} - 240\vec{k}$$

$$AE = \sqrt{(480)^2 + (160)^2 + (-240)^2} = 560\text{mm}$$

$$\vec{\lambda}_{AE} = \frac{\vec{AE}}{AE} = \frac{480\vec{i} + 160\vec{j} - 240\vec{k}}{560}$$

$$\therefore \vec{\lambda}_{AE} = \frac{6\vec{i} + 2\vec{j} - 3\vec{k}}{7}$$

$$\vec{r}_{AB} = 200\vec{i}$$

$$\vec{r}_{AD} = 480\vec{i} + 160\vec{j}$$

$$\vec{DF} = -480\vec{i} + 330\vec{j} - 240\vec{k}$$

$$DF = \sqrt{(-480)^2 + (330)^2 + (-240)^2} = 630\text{mm}$$

$$\therefore \vec{T}_{DF} = T_{DF}\frac{\vec{DF}}{DF} = T_{DF}\frac{-480\vec{i} + 330\vec{j} - 240\vec{k}}{630} \qquad \therefore \vec{T}_{DF} = T_{DF}\frac{-16\vec{i} + 11\vec{j} - 8\vec{k}}{21}$$

$$\vec{F}_B = -(640)\vec{j}\ \text{N}.$$

Summing moments about line AE will eliminate the unknown reaction forces at A and E and will solve directly for $\vec{T}_{DF}$:

$$\therefore \Sigma\vec{M}_{AE} = 0 \qquad \vec{\lambda}_{AE} \cdot (\vec{r}_{AD} \times \vec{T}_{DF}) + \vec{\lambda}_{AE} \cdot (\vec{r}_{AB} \times \vec{F}_B) = 0$$

$$\begin{vmatrix} 6 & 2 & -3 \\ 480 & 160 & 0 \\ -16 & 11 & -8 \end{vmatrix} \frac{T_{DF}}{(21)(7)} + \begin{vmatrix} 6 & 2 & -3 \\ 200 & 0 & 0 \\ 0 & -640 & 0 \end{vmatrix} \frac{1}{7} = 0$$

By expanding the determinants:

$$-1120\,T_{DF} + 384\times10^3 = 0 \qquad \therefore T_{DF} = 343\,\text{N}$$