Faculty of Applied Science and Engineering
University of Toronto

MAT 186 H1S - CALCULUS I
WEDNESDAY, APRIL 20, 2016

FINAL EXAMINATION

FAMILY NAME: _____

GIVEN NAME: _____

STUDENT NUMBER: _____

SIGNATURE: _____

Time allowed: 2 hours, 30 minutes
Total marks: 80
No calculators allowed.
Examiner: S. Cohen
Use the backs of pages when necessary,
indicating clearly where solutions continue.

FOR MARKER'S USE ONLY	
QUESTION	MARK
1	/ 15
2	/ 15
3	/ 20
4	/ 8
5	/ 7
6	/ 15
TOTAL	/ 80

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1. Some warm-up questions. Justify your answers fully.

a) Evaluate $\lim_{x\to 2} \frac{x^3-5x+2}{2x^2-3x-2}$.

[3 marks]

b) Evaluate $\frac{d}{dx}(3x^2 + x - 3)$ using the limit definition of the derivative.

[3 marks]

c) Evaluate $\lim_{x\to 3} \frac{|x-3|}{x-3}$.

[3 marks]

d) Find the equation of the tangent line of $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$ at the point $(0, 1/2)$.

[3 marks]

e) Evaluate $\int_0^1 \sin(\pi t) + (2t-3)^2 dt$.

[3 marks]

2. Tougher questions. Evaluate the following:

a) $\lim_{x\to 0} x^4 \cos^3\left(\frac{1}{x}\right)$

[5 marks]

b) $\lim_{x\to 0^+} x^{\sqrt{x}}$

[5 marks]

c) $\int_0^{\pi/2} \sin x \; e^{\cos x} dx$

[5 marks]

3. Graph the function $f(x) = \frac{3x-2}{|x-1|}$. Organize your solution well and include all of the important values. You have this page and the next for this problem.

[20 marks]

[Continue Question 3 on this page.]

4. Using calculus (NOT linear algebra), find the point on the line segment from $(1,1)$ to $(5,3)$ that is closest to $(2,-3)$.

[8 marks]

5. A ball is thrown straight up in the air with a velocity of 25 m/s. To an observer standing 30 m away and whose eye level is the same as the point of release for the ball, how quickly is the angle of elevation to the ball changing 4 seconds into its flight? [Use $g = 10\ \text{m/s}^2$.]

[7 marks]

6. (a) Use integrals to find the area of the triangle with vertices $(-1,1), (0,3)$, and $(2,1)$.

[4 marks]

(b) Find the volume of the solid created if the triangle above is rotated about $x = 3$.

[5 marks]

(c) For the solid from part (b), how much work is required to fill it with liquid pumped up from the $x-$ axis?

[6 marks]