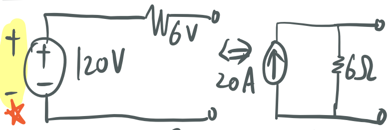
## Superposition & linearity

↳ You must include dependent sources!

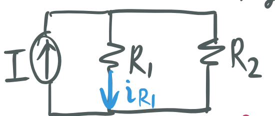
## Source Transformation:




## Voltage & Current Division


$V_{R_1} = V \cdot \frac{R_1}{R_1+R_2}$

$V_R = V \cdot \frac{R_x}{\Sigma R_j}$




$i_{R_1} = I \frac{R_2}{R_1+R_2}$

$i_R = I \cdot \frac{R_{eq}}{R_R}$

## Thevenin's Resistance

Find $R_{TH} = R_N$:

1. Indep Sources:

2. Only dep Sources:

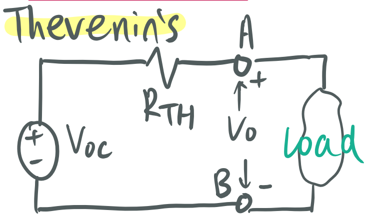
- Apply Indep Sources at terminal, Ohm's law

$R_{TH} = \frac{1V}{i_{find}}$ $R_{TH} = \frac{V_{find}}{1mA}$

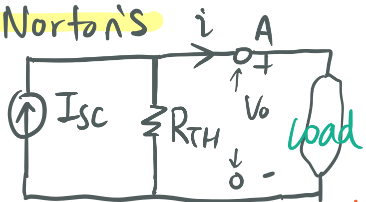
3. Both Indep & Dep:

$\boxed{R_{TH} = \frac{V_{oc}}{I_{sc}}}$

## Thevenin's




## Norton's




Look for Short Circuits!

## Maximum Power Transfer

$P_L = i^2 R_L = \left(\frac{V}{R_{TH}+R_L}\right)^2 R_L$


$R_{TH} = R_L$

## DC Analysis: L & C

**Capacitors:**

$i(t) = C\frac{dv}{dt}$ ] first

$i = 0$ in DC Analysis (open C)

$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)\,dx$

$P = VI = CV\frac{dv}{dt}$

$W_C = \frac{q^2}{2C} = \frac{1}{2}CV^2$

Series: $\frac{1}{C_{tot}} = \Sigma \frac{1}{C_j}$ (Q same, ΔV add)

Parallel: $C_{tot} = \Sigma C_j$ (ΔV same, Q add)

**Inductors:**

$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(x)\,dx$

$v(t) = L\frac{dI}{dt}$ if I no change $V(t) = 0$

$P = VI = \left(L\frac{dI}{dt}\right)I(t)$

$W_L = \frac{1}{2}LI^2$

Series: $L_{tot} = \Sigma L_j$

Parallel: $\frac{1}{L_{tot}} = \Sigma L_j$

## First Order Circuits

Step-by step Approach:

1. $x(t) = k_1 + k_2 e^{\frac{-t}{\tau}}$
2. Find $x(0^-)$ & $x(0^+)$
3. Find $x(\infty)$

$\tau = R_{TH} \cdot C$

$\tau = L / R_{TH}$

4. $\boxed{x(t) = x(\infty) + [x(0^+) - x(\infty)]e^{\frac{-t}{\tau}}}$

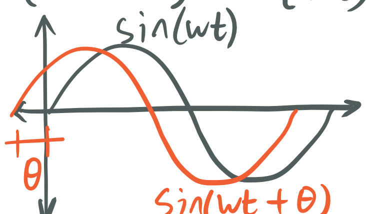
## Sinusoids

General form:

$x(t) = X_M \sin(wt)$, where $w = \frac{2\pi}{T} = 2\pi f$

$T$ [=sec] $f$ [= $\frac{\#cycles}{sec}$]

$x(w(t+T)) = x(wt)$




~~x~~ lags ~~x~~ by θ radians

$\cos(wt) = \sin(wt + 90°)$

$\sin(wt) = \cos(wt - 90°)$

$-\cos(wt) = \cos(wt \pm 180°)$

$-\sin(wt) = \sin(wt \pm 180°)$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

## Complex Numbers

$A = ze^{j\theta} = z(\cos\theta + j\sin\theta)$ (x = $\cos\theta$, y = $\sin\theta$)

$= z\angle\theta$

$z = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}(y/x)$

## AC Analysis:

Phasor representation:

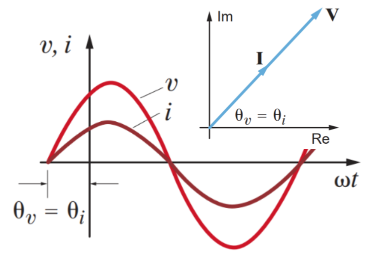
ex: $V(t) = V_m\cos(wt+\theta)$

$\rightarrow V_m\angle\theta$

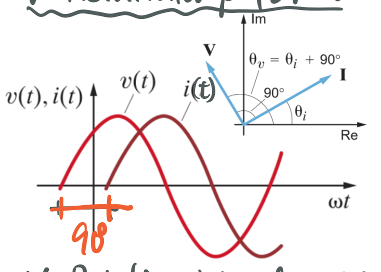
| Time Domain | Freq. Dom: |
|---|---|
| $A\cos(wt\pm\theta)$ | $A\angle\pm\theta$ |
| $A\sin(wt\pm\theta)$ | $A\angle\pm\theta - 90°$ |

### V Relationship for R

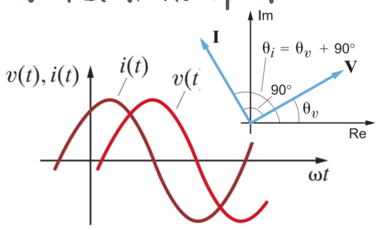
i and V always in phase.




### V Relationship for L:




### V Relationship for C:




Phasors (V)

R: $V = RI$

L: $V = jwLI$

C: $V = \frac{I}{jwC}$

## Impedence

$Z = \frac{V}{I}[\Omega]$ Impedence is not a phasor!

$= R + jX$

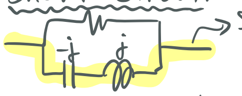
real: Resistive Component; imag: Reactive Component

$j = -1/j$

## Reactive component

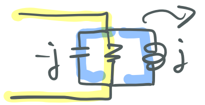
- Positive j ⇒ inductors
- Negative j ⇒ capacitors
- Negative R ⇒ check math

## Short Circuit



shorted $Z = 0$

## Open Circuit



open circuit $Z_p = \frac{1}{0} = \infty$

R: $Z = R$

L: $Z = jwL$ ($X_L$)

C: $Z = \frac{1}{jwC}$, $\frac{-1}{wC}$ ($X_C$)

Series: $Z_{tot} = Z_1 + Z_2 ..$

Parallel: $\frac{1}{Z_{tot}} = \Sigma\frac{1}{Z_j}$

Inductance (L) : Set values = $jwL$

Capacitance (C) : Set values = $\frac{1}{jwC}$

Power in AC circuit

$P_L = P_C = 0$

## Admittance:

$Y = \frac{1}{Z} = \frac{I}{V}[S]$

$Y = G + jB$ (G: Conductance, B: Susceptance.)

## Other formulas:

Circle: $S = r\theta$, θ=radians

Sphere $V = \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

Proton mass = neutron mass

$1.67 * 10^{-27}$ kg

Electron mass:

$9.11 * 10^{-31}$ kg

Boltzman's Constant:

$k_B = 1.38 * 10^{-23}$ J/K

Electron Charge magnitude:

$e = 1.60 * 10^{-19}$ C

Electron volt

$1\,eV = 1.60 * 10^{-19}$ J

Speed of Light

$c = 3 * 10^8 \frac{m}{s}$

Universal gravitational constant

$G = 6.67 * 10^{-11} \frac{N \cdot m^2}{kg^2}$

$\varepsilon_0 = 8.85 * 10^{-12}\ C^2/(N \cdot m^2)$

$k = 1/4\pi\varepsilon_0 = 9 * 10^9\ N \cdot m^2/C$

$\mu_0 = 4\pi * 10^{-7}$ T·m/A

Magnetic Constant

$k' = \frac{\mu_0}{4\pi} = 1 * 10^{-7} \frac{T \cdot m}{A}$

$1\,atm = 10^5 \frac{N}{m^2} = 10^5$ Pa

## Electric Potential

$V = \frac{-W_\infty}{q_0} = \frac{U}{q_0}$

$W_\infty$ = Work done by electric force.

## Electric Potential Energy

$U = qV$ (place $q$ at $V$)

Uniform $\vec{E}$: $\Delta V = -E\Delta X$

$\Delta U = q\Delta V = q(V_f - V_i)$

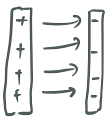
## Potential Due to a charged q

$V = \frac{kq}{r}$

$V_{tot} = k \sum \frac{q_i}{r_i}$

## Potential Energy & Density

$U = \frac{q^2}{2C} = \frac{CV^2}{2}$ $u = \frac{\varepsilon_0 E^2}{2}$



$q = CV$

## Current:

$i = \int \vec{J} \cdot d\vec{A}$ $i = \frac{dq}{dt}$

## Mechanical Energy:

$\Delta K = -q\Delta V$

$\Delta K = -q\Delta V + W_{app}$

$\vec{F} = q\vec{E}$ $\qquad W_{app} \downarrow q\Delta V$

## Summary of Gauss's Law

Surface of Charged conductor $E = \frac{\sigma}{\varepsilon_0}$

Line of Charge: $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

$\infty$ non-conducting sheet $E = \frac{\sigma}{2\varepsilon_0}$

outside a spherical shell of Charge $E = \frac{kq}{r^2}$

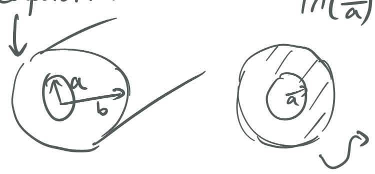
Inside a uniform spherical shell $E = 0$

Inside uniform sphere of Charge $E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r$

## Capacitors:

// capacitor $C = \frac{\varepsilon_0 A}{d}$

cylindrical capacitor $C = 2\pi\varepsilon_0 \frac{L}{\ln(\frac{b}{a})}$



Spherical Capacitor with Concentric spherical plate a & b $C = 4\pi\varepsilon_0 \frac{ab}{b-a}$

Isolated sphere $C = 4\pi\varepsilon_0 R$

$\Delta V = -\int_i^f \vec{E} \cdot d\vec{s}$

Insert Dialectric to C

Q remains $C' = KC$

## Drift Speed

$\vec{J}\left[\frac{A}{m^2}\right] = (ne)\vec{V}_d$

ne = carrier charge density

$n = \frac{\text{Charge carrier}}{\text{Unit volume}}$

## Resistivity

$\rho[\Omega m] = \frac{1}{\sigma} = \frac{E}{J}$

$\vec{E} = \rho\vec{J}$

Resistance of wire:

$R = \frac{\rho L}{A}$

Change of $\rho$ with temperature

$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$

$\rho_0$ = resistivity at $T_0$

$\alpha$ = temperature coefficient of resistivity for material

## Magnetic field:

$\vec{F}_B = q\vec{v} \times \vec{B}$

$\vec{B} [= T]$

## Ampere's Law:

$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$

## Biot Savart Law:

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{s} \times \hat{r}}{r^2}$

## Long straight wire $\vec{B}$

outside $B = \frac{\mu_0 i}{2\pi R}$

Inside $B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$

R = radius of wire.

## ideal solenoid:

$B = \mu_0 i n$

## Parallel current

$F_{ba} = \frac{\mu_0 L i_a i_b}{2\pi d}$

Straight wire:

$\vec{F}_B = i\vec{L} \times \vec{B}$

$d\vec{F}_B = i\, d\vec{L} \times \vec{B}$

## Power of wire

Derivation

$|\varepsilon| = \frac{d\Phi_B}{dt} = \frac{d}{dt} BLX$

$= BL\frac{dx}{dt} = BLV$

$i = \frac{BLV}{R}$

$P = Fv = \frac{B^2L^2v^2}{R}$

$F_B = ILB$ $\varepsilon = |BLV|$

## Thermal Energy Rate:

$P = \left(\frac{BLV}{R}\right)^2 = \frac{B^2L^2V^2}{R}$

## Circular Arc $\vec{B}$

$B = \frac{\mu_0 i \phi}{4\pi R}$

$\phi$ = angle in radians

## Power:

$P = iV = i^2R = \frac{V^2}{R}$ (resistive dissapation)

## Magnetic flux

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\Phi_B = BA$ ($B \perp$ to $A$) ($B$ uniform)

## Faraday's Law of Induction

$\varepsilon = \frac{-d\Phi_B}{dt}$

Coil of N turns:

$\varepsilon = -N \cdot \frac{d\Phi_B}{dt}$

## Lenz's Law

Induced emf will either oppose / same direction depending on magnetic flux.

## Self induction

$\boxed{\varepsilon_L = -L \cdot \frac{di}{dt}}$

## Inductance

$L = \mu_0 n^2 A l$

## Inductor w/ N loops

$L = \frac{N\Phi_B}{i}$

## Magnetic Energy

L with i

$U_B = \frac{1}{2} L i^2$

$\frac{\mu_0 N^2 A}{l} = \mu_0 n^2 l A$

$l$ = Solenoid's length

N = num of turns

## Supply & absorb

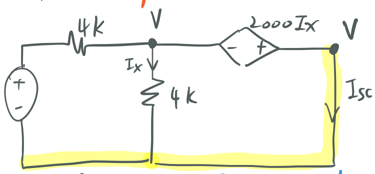
supply → pump from low to high

absorb → vice versa

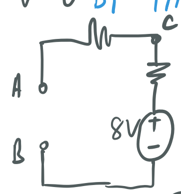
## Circuit Analysis Special Cases:

- Choose ground
- Analyze each KVL or KCL individually.
- Write down unknowns / what values it can solve for.
- Go for 1 unknown first

* Direction of current is important!!



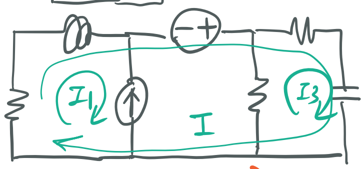

V=0 BY INSPECTION

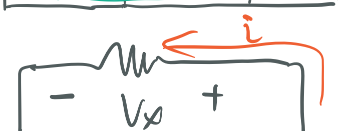



$V_{ab} = 8V$!

no closed loop

$V_{ac} = 0V$!




high V to low V, not the other way around