

University of Toronto  
FACULTY OF APPLIED SCIENCE AND ENGINEERING  
**FINAL EXAMINATION, APRIL, 2013**  
First Year - CHE, CIV, IND, LME, MEC, MMS

**MAT187H1S - CALCULUS II**

Exam Type: A  
Duration: 150 min.

SURNAME: (as on your T-card) \_\_\_\_\_  
GIVEN NAMES: \_\_\_\_\_  
STUDENT NUMBER: \_\_\_\_\_  
SIGNATURE: \_\_\_\_\_

**Examiners:**  
D. Bartosova  
D. Burbulla  
S. Cohen  
C. Dodd  
P. Milgram

**Calculators Permitted:** Casio 260, Sharp 520 or Texas Instrument 30. No other aids are permitted.

**INSTRUCTIONS:** Attempt all questions.

You must show your work and give full explanations to get full marks. Partial credit can be obtained for partially correct work, but NO credit may be given if your work is poorly presented, difficult to decipher, uses incorrect mathematical notation, or makes no sense.

Use the backs of the sheets if you need more space.

Do not tear any pages from this exam.

Make sure your exam contains 10 pages.

**Part Marks:** The value of each question is indicated in parentheses beside the question number.

**Total Marks:** 100

PAGE	MARK
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
Q8	
Q9	
TOTAL	

1. [12 marks] Find all the critical points of the function  $f(x, y) = 4x - 3x^3 - 2xy^2$ , and determine if they are maximum points, minimum points, or saddle points.

2. [10 marks] Newton's Law of Cooling states that

$$\frac{dT}{dt} = -k(T - A),$$

for some positive constant  $k$ , where  $T$  is the temperature of an object at time  $t$  in a room with constant ambient temperature  $A$ .

Suppose a freshly baked cake is taken out of the oven at 9:30 AM with temperature 200 C, and put on a table in a room with constant air temperature 20 C. If the temperature of the cake is 100 C at 9:50 AM, when will the temperature of the cake be 50 C ?

3. [12 marks] Find the general solution for each of the following differential equations:

(a) [7 marks]  $\frac{dy}{dx} + \frac{xy}{1+x^2} = 4x$

(b) [5 marks]  $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$

4. [10 marks] Suppose  $x = 16 \cos t + 16t \sin t$  and  $y = 16 \sin t - 16t \cos t$  are the parametric equations of a curve. Find the following:

(a) [5 marks] all points on the curve, for  $0 < t < 3\pi/2$ , at which the tangent line to the curve is horizontal or vertical.

(b) [5 marks] the length of the curve, for  $0 \leq t \leq 3\pi/2$ .

5.(a) [6 marks] Find the interval of convergence of the power series  $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k \tan^{-1} k}$ .

5.(b) [6 marks] Use series to find an approximation of  $\int_0^{1/2} \frac{dx}{\sqrt{1+x^4}}$  correct to within  $10^{-5}$ .

6. [10 marks] What is the initial velocity vector of a bullet if it is fired from ground level and hits a target 1 second later, given that the target is 150 m above ground level on a tower 300 m away as measured along the ground? Ignore air resistance; use  $g = 9.8 \text{ m/s}^2$ .

7. [12 marks] For  $0 \leq \theta \leq 2\pi$ , sketch the curves  $r_1, r_2$  with polar equations

$$r_1 = 2 + 2 \sin \theta; \quad r_2 = 2 + 2 \cos \theta,$$

and then find the area of the region outside the curve  $r_1$  but inside the curve  $r_2$ .

- 8.(a) [5 marks] Let  $f(x) = 1 - x - x^2 + x^3 - x^4 - x^5 + x^6 - x^7 - x^8 + x^9 - x^{10} - x^{11} + \dots$ .  
Show that this series converges absolutely for  $|x| < 1$  and then compute its sum.  
Hint: write  $f(x)$  as a sum of three infinite geometric series with common ratio  $x^3$ .

- 8.(b) [5 marks] Find the Maclaurin series of  $g(x) = \frac{1 + 2x}{1 + x + x^2}$ .

9. [12 marks] Consider the helix with vector equation  $\mathbf{r} = \cos(4t) \mathbf{i} + \sin(4t) \mathbf{j} + 3t \mathbf{k}$ .

(a) [6 marks] Calculate both  $\frac{d\mathbf{r}}{dt}$  and  $\left\| \frac{d\mathbf{r}}{dt} \right\|$ .

(b) [6 marks] Find an arc length parameterization of the curve, with reference point  $(1, 0, 0)$ , for which  $t = 0$ .