

*University of Toronto*

*Faculty of Applied Science and Engineering*

**MIE100 – Dynamics**

**Final Examination**

**April 23, 2012, 9:30 a.m. - noon**

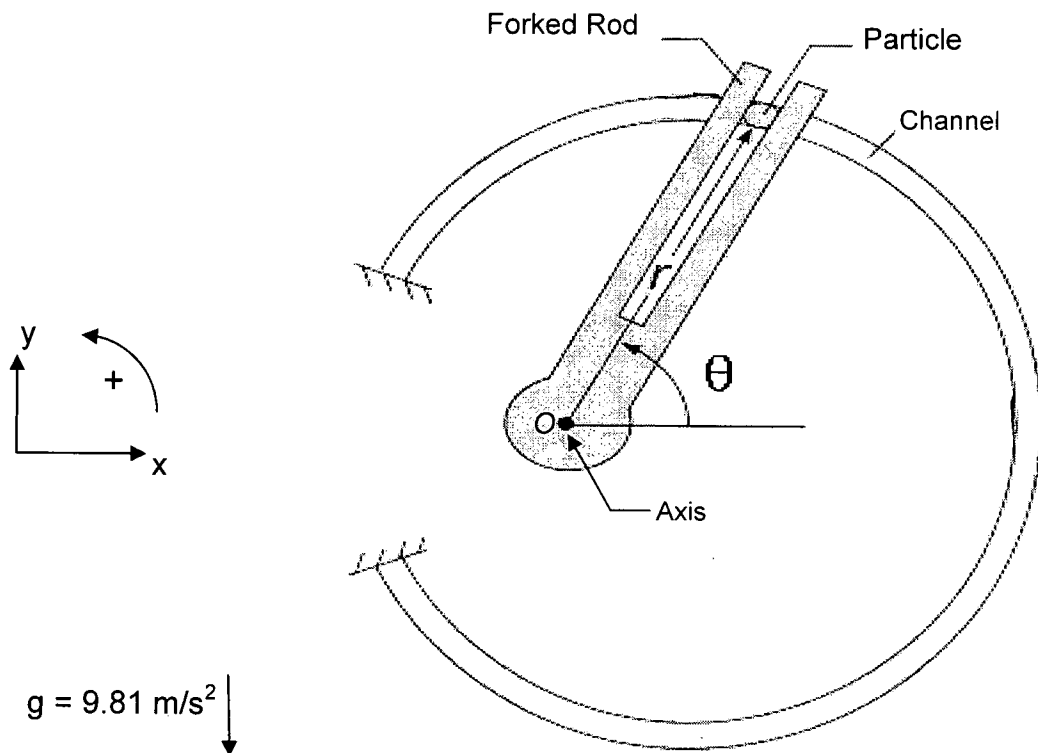
**Instructors:** *C. Simmons, A. Sinclair, L. Sinclair and P. Sullivan*

**Aids Permitted:** One non-programmable calculator  
One 8 1/2" by 11" sheet of paper, any colour

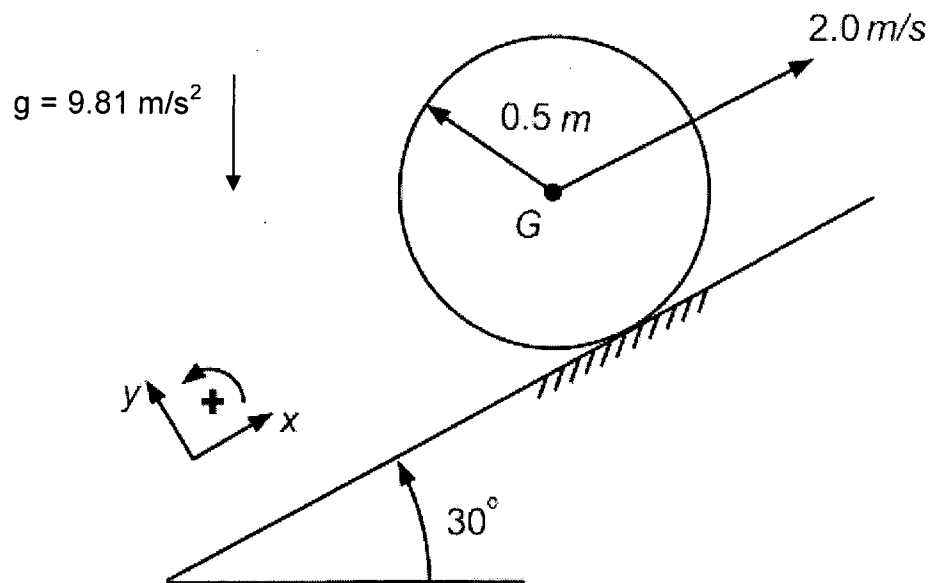
- Answers for each question must be given in the coordinate system specified in the question.
- The correct units must be specified in each final answer
- All rough work must be *neatly* shown to earn full credit for each question.
- This exam has six questions. Answer all six questions.
- Use a very dark pencil or pen.

**Total Marks: 100**

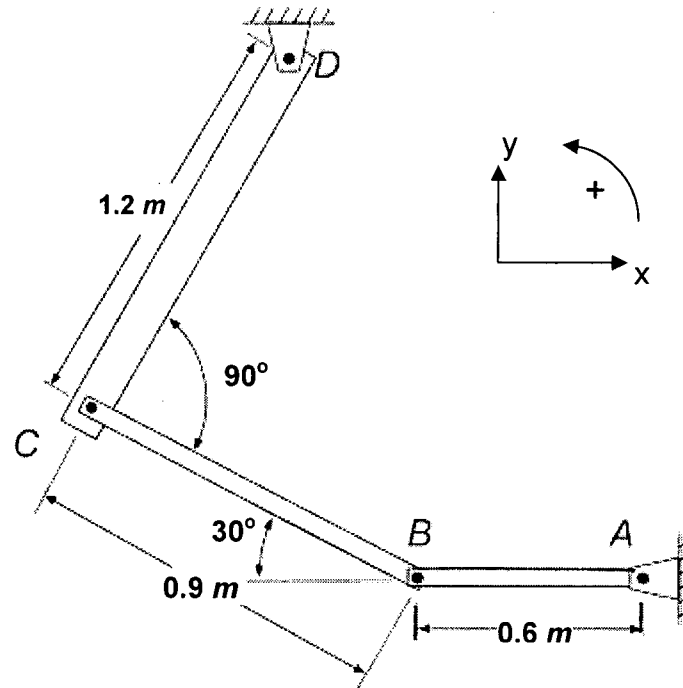
1. A 1 kg particle is pushed by a forked rod along a curved channel that guides its direction of motion. The channel's geometry is given by the equation  $r = 0.3(2 + \cos\theta)$ , where  $r$  is the distance in meters from the axis  $O$  of the forked rod to the channel. There is no friction. A small motor located at the axis  $O$  forces the forked rod to rotate at a constant angular speed of  $\dot{\theta} = -0.5 \text{ s}^{-1}$ .
  - a) Determine the acceleration  $\vec{a}$  of the particle in  $r$ - $\theta$  coordinates, when  $\theta = 90^\circ$ . (10 marks)
  - b) Determine the force that the channel exerts on the particle, when  $\theta = 0^\circ$ . (5 marks)



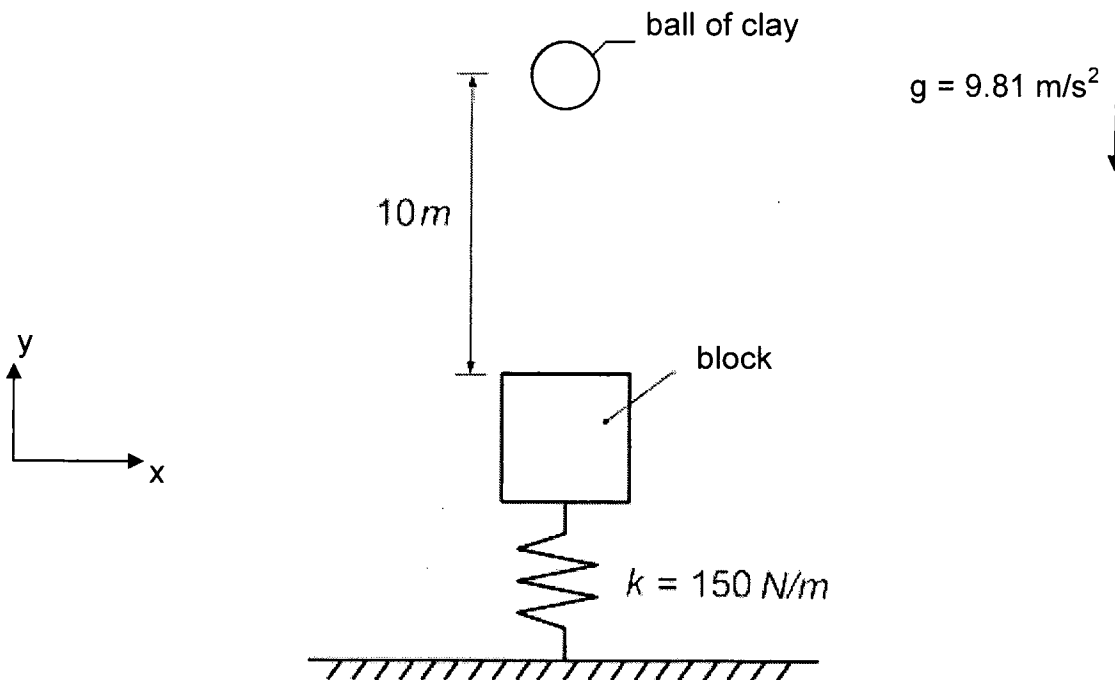
2. A uniform disk with a mass of  $5\text{ kg}$  and radius of  $0.5\text{ m}$  is moving up an inclined plane. At the instant shown, the center of the disk  $G$  has a speed of  $2.0\text{ m/s}$ .
- a) Determine the minimum coefficient of static friction required so that the disk does not start to slip. (5 marks)
  - b) Determine the angular acceleration of the disk at the instant shown, if  $\mu_s = 0.85$  and the disk rolls without slipping. (5 marks)
  - c) Determine the kinetic energy of the disk at the instant shown, if  $\mu_s = 0.85$  and the disk rolls without slipping. (5 marks)



3. Consider the mechanism shown in the figure below. At the instant shown, the link AB has an angular velocity of  $\omega_{AB} = -4 \text{ s}^{-1}$ , and an angular acceleration  $\alpha_{AB} = 0$ . At the instant shown below, find the following:
- a) The acceleration  $\vec{a}_B$  of point B in  $x$ - $y$  coordinates. (5 marks)
  - b) The location of the instantaneous center of zero velocity for link CB. Show your answer clearly in a diagram of link CB. (5 marks)
  - c) The angular velocity  $\omega_{CD}$  of link CD. (5 marks)



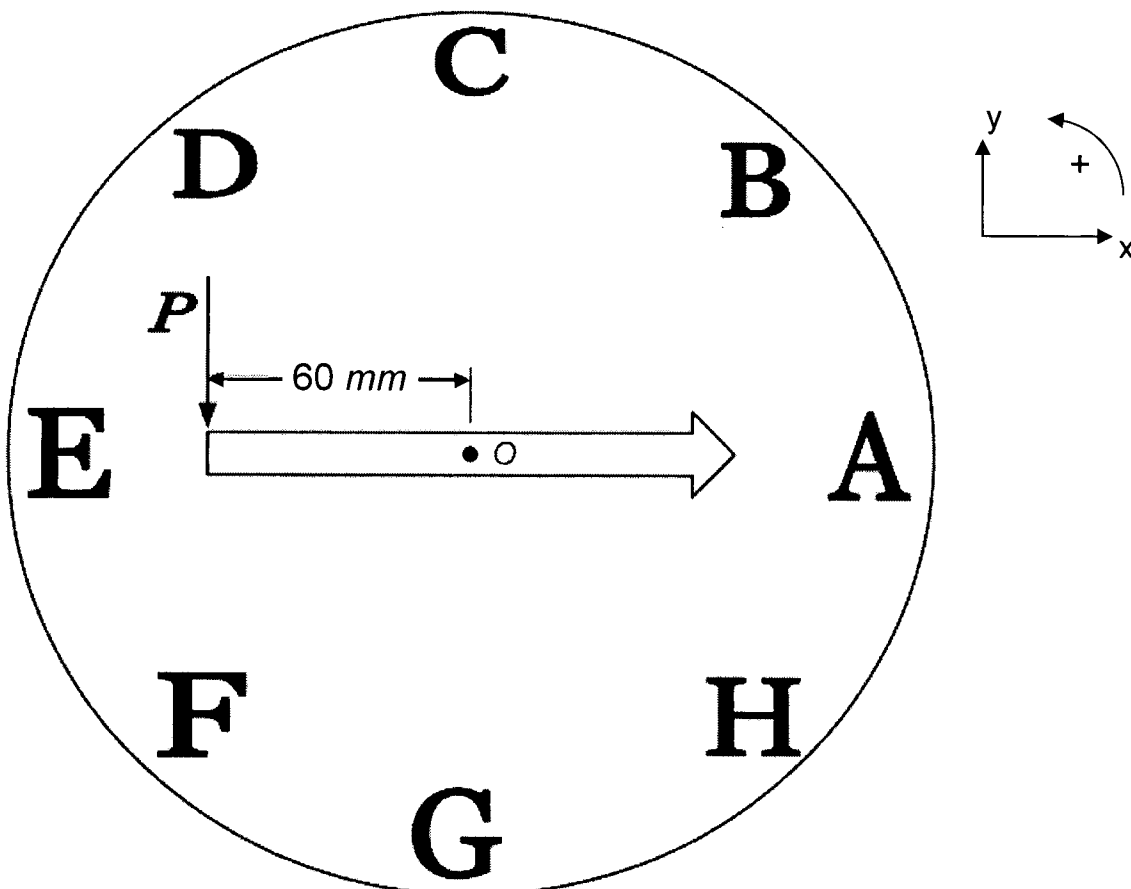
4. A  $1.0 \text{ kg}$  block is at rest on top of a spring with a stiffness  $k = 150 \text{ N/m}$ . A  $0.5 \text{ kg}$  ball of clay is released from rest at a height of  $10 \text{ m}$  above the top surface of the block. The clay hits the  $1.0 \text{ kg}$  block and sticks to it.
- a) What is the total amount of kinetic energy of the block with clay stuck onto it, immediately after the collision? (*5 marks*)
- b) Determine the force exerted by the spring on the block, at the instant when the block reaches its lowest point. (*10 marks*)



5. You are playing a game where you spin an arrow on a circular game board by applying a force with magnitude  $P = 10\text{ N}$  to the arrow with your hand for an extremely short time  $\Delta t$ , as shown below. The game board is fixed and lying flat on a table such that gravity has no effect.

The arrow is initially at rest when the force is applied at the instant shown. The arrow spins about its centre of mass at point  $O$ , and its rotation is resisted by a constant frictional moment of magnitude  $M_o = 0.08\text{ N}\cdot\text{m}$ . The arrow has mass of  $0.25\text{ kg}$  and its radius of gyration about point  $O$  is  $k_o = 0.03\text{ m}$ .

- (a) Determine the time duration  $\Delta t$  required for application of the force so that the arrow will make exactly two full rotations before coming to rest pointing at the letter "A". (10 marks)
- (b) What is the angular impulse about  $O$  given to the arrow by your hand? (5 marks)
- (c) How much work is done by the frictional moment  $M_o$ ? (5 marks)



6. A non-uniform bar has a mass  $m$  and a moment of inertia  $I_G$  about its centre of mass at  $G$ . The bar is pinned at point  $O$  at a distance  $L$  from point  $G$ . As the bar rotates about  $O$ , the pin joint exerts a resistive moment on the bar with a magnitude  $c\dot{\theta}$  that is proportional to its angular speed.

The bar is released from rest at  $\theta = 5$  degrees from its equilibrium position at time  $t = 0$ , resulting in the motion plotted below.

- Draw a free body diagram of the oscillating bar and derive the second-order differential equation of motion for  $\theta(t)$  in terms of the parameters  $m$ ,  $L$ ,  $I_G$ , and  $c$ . Assume that the amplitude of oscillations is small. (5 marks)
- Estimate a value for the *damped* natural frequency  $\omega_d$ . (5 marks)
- Estimate the damping ratio,  $c/c_c$ . (5 marks)
- By how many percent would the period of oscillation  $\tau$  increase or decrease if the resistive component  $c$  were removed from the system? (5 marks)

