University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, APRIL, 2010

First Year - CHE, CIV, IND, LME, MEC, MSE

MAT187H1S - CALCULUS II

Exam Type: A

SURNAME: (as on your T-card) ____________________

GIVEN NAMES: ____________________

STUDENT NUMBER: ____________________

SIGNATURE: ____________________

Examiners:
D. Burbulla
A. Cabrera
P. Milgram
D. Raghavan

PAGE	MARK
MC	
Q7	
Q8	
Q9	
Q10	
Q11	
Q12	
Q13	
TOTAL	

Calculators Permitted: Casio 260, Sharp 520 or TI 30.

INSTRUCTIONS: Attempt all questions. Use the backs of the sheets if you need more space. Do not tear any pages from this exam. Make sure your exam contains 10 pages.

MARKS: Questions 1 through 6 are Multiple Choice; circle the single correct choice for each question. Each correct choice is worth 4 marks.

Questions 7, 8 and 9 are each worth 12 marks.

Questions 10 through 13 are each worth 10 marks.

TOTAL MARKS: 100

1. The solution to the initial value problem $\dfrac{dy}{dx} = \dfrac{2x}{3y^2}$, $y(0) = 5$ is

(a) $y = x^2 + 5$

(b) $y = x^{2/3} + 5$

(c) $y = \sqrt[3]{x^2 + 5}$

(d) $y = (x + 5)^{2/3}$

2. The general solution to the differential equation $\dfrac{dy}{dx} - \dfrac{y}{x} = 1$, for $x > 0$, is

(a) $y = x \ln x + Cx$

(b) $y = x \ln x + C$

(c) $y = x^2 + Cx$

(d) $y = \dfrac{\ln x}{x} + \dfrac{C}{x}$

3. The general solution to the differential equation $\dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} + 9y = 0$ is

(a) $y = C\, e^{-3x}$

(b) $y = A\, e^{-3x} + B\, x\, e^{-3x}$

(c) $y = A\, e^{-3x}\, \sin(3x) + B\, e^{-3x}\, \cos(3x)$

(d) $y = C\, x\, e^{-3x}$

4. What is the interval of convergence of the power series $\sum_{k=0}^{\infty} \left(\frac{3}{4}\right)^k (x+5)^k$?

(a) $\left(-\frac{3}{4}, \frac{3}{4}\right)$

(b) $\left(-\frac{15}{4}, \frac{15}{4}\right)$

(c) $\left(\frac{11}{3}, \frac{19}{3}\right)$

(d) $\left(-\frac{19}{3}, -\frac{11}{3}\right)$

5. The first four non-zero terms of the Maclaurin series for $\tan^{-1}\left(\frac{x^2}{2}\right)$ are:

(a) $\frac{x}{2} + \frac{x^3}{24} + \frac{x^5}{160} + \frac{x^7}{896}$

(b) $\frac{x}{2} - \frac{x^3}{24} + \frac{x^6}{160} - \frac{x^7}{896}$

(c) $\frac{x^2}{2} - \frac{x^6}{24} + \frac{x^{10}}{160} - \frac{x^{14}}{896}$

(d) $\frac{x^2}{2} + \frac{x^6}{24} + \frac{x^{10}}{160} + \frac{x^{14}}{896}$

6. The infinite series $\sum_{k=0}^{\infty} (-1)^k \frac{k^2}{\sqrt{k^6+1}}$

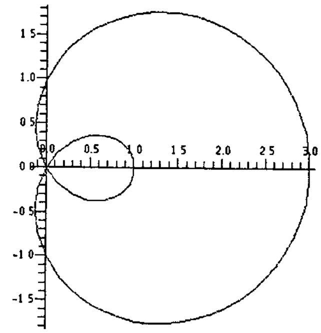
(a) converges absolutely.

(b) converges conditionally.

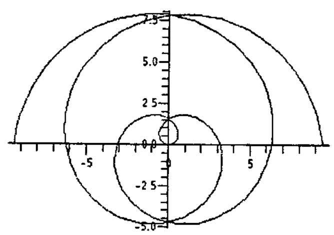
(c) diverges.

(d) is called the harmonic series.

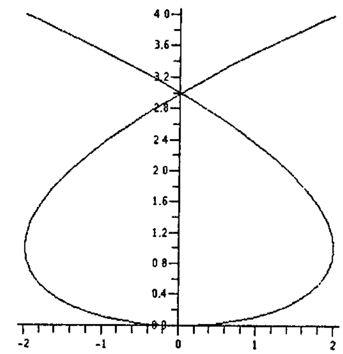
7. [2 marks for each graph.] Match the graphs A through F with their correct vector or polar equations, by putting the letter of the graph in the blank line beside the single appropriate vector or polar equation at the bottom of this page.



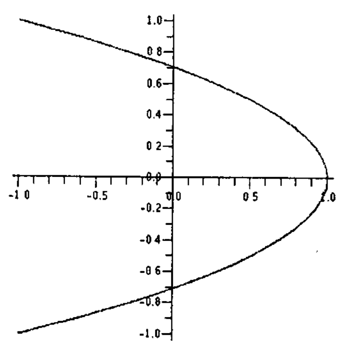
A



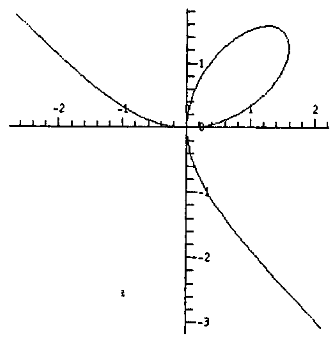
B



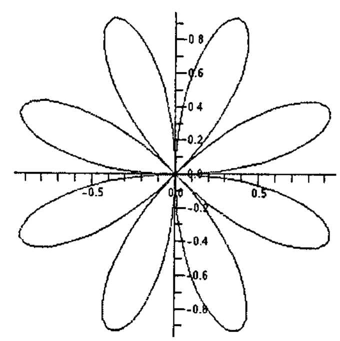
C



D



E



F

____ $r = 1 + 2\sin\theta$	____ $r = 1 - 2\cos\theta$	____ $r = 1 + 2\cos\theta$
____ $r = \cos 4\theta$	____ $r = \sin 4\theta$	____ $r = \cos 8\theta$
____ $r = \sin 8\theta$	____ $r = \theta$	____ $r = 1 - 2\sin\theta$
____ $\mathbf{r} = \cos 2t\,\mathbf{i} + \sin t\,\mathbf{j}$	____ $\mathbf{r} = \sin 2t\,\mathbf{i} + \cos t\,\mathbf{j}$	____ $\mathbf{r} = (t^3 - 3t)\,\mathbf{i} + t^2\,\mathbf{j}$
____ $\mathbf{r} = t^2\,\mathbf{i} + (t^3 - 3t)\,\mathbf{j}$	____ $\mathbf{r} = \dfrac{3t}{1+t^3}\,\mathbf{i} + \dfrac{3t^2}{1+t^3}\,\mathbf{j}$	____ $\mathbf{r} = \dfrac{t^3}{1+t^2}\,\mathbf{i} + \dfrac{2t^2}{1+t^2}\,\mathbf{j}$

8. **Given** that the acceleration of a particle at time t is $\mathbf{a} = 2\,\mathbf{i} - 9.8\,\mathbf{k}$, its velocity at $t = 0$ is $\mathbf{v}_0 = 30\,\mathbf{j} + 30\,\mathbf{k}$, and its position at $t = 0$ is $\mathbf{r}_0 = 10\,\mathbf{k}$, **find** the position and the speed of the particle when $z = 0$, for $t > 0$. (Use your calculator and approximate your answers to one decimal place.)

9. Find the area of the region that is inside both the circle with polar equation $r = 3\cos\theta$ and the cardioid with polar equation $r = 1 + \cos\theta$.

10. Approximate

$$\int_0^{0.5} \frac{dx}{(1+x^4)^{1/3}}$$

to within 10^{-5}, and explain why your approximation is correct to within 10^{-5}.

11. Find and classify all the critical points of the function $f(x, y) = x^2 + y^2 + \dfrac{2}{xy}$.

12. Consider the curve $\mathbf{r} = \sin e^t\,\mathbf{i} + \cos e^t\,\mathbf{j} + \sqrt{3}\,e^t\,\mathbf{k}$.

(a) [6 marks] Calculate both $\dfrac{d\mathbf{r}}{dt}$ and $\left\|\dfrac{d\mathbf{r}}{dt}\right\|$.

(b) [4 marks] Find an arc length parametrization of the above curve, with reference point $(0, -1, \sqrt{3}\,\pi)$, for which $t = \ln \pi$.

13. The point $(x, y) = (2, 4)$ is a critical point on the graph of each of the following curves. (You don't have to check this!) Determine in each case if the critical point is a relative maximum point, a relative minimum point, or neither.

(a) [2 marks] The graph of the function defined by the power series

$$y = 4 + \frac{1}{4}(x-2)^2 - \frac{1}{24}(x-2)^3 + \frac{1}{192}(x-2)^4 - \frac{1}{1920}(x-2)^5 + \cdots$$

(b) [4 marks] The curve with parametric equations $x = 1 + t^2$, $y = 2 + 3t - t^3$.

(c) [4 marks] The polar curve with polar equation $r = 3\sin\theta + 4\cos\theta$.