UNIVERSITY OF TORONTO

Faculty of Applied Science and Engineering

CIV 100S – MECHANICS

Midterm Examination

Monday, 2nd March 2015

Time allowed: 1 ½ hours

NAME: _____M. SEICA_____

STUDENT NUMBER: _____Solutions_____DEPT. (CHE, ECE, etc.)_____

CIRCLE YOUR CALCULATOR MODEL:

CASIO FX991 **SHARP EL520**

Notes: 1. The 3 Questions are of the value shown below.

2. Make sure you have all 5 sheets of the examination paper. Page 5 is blank.

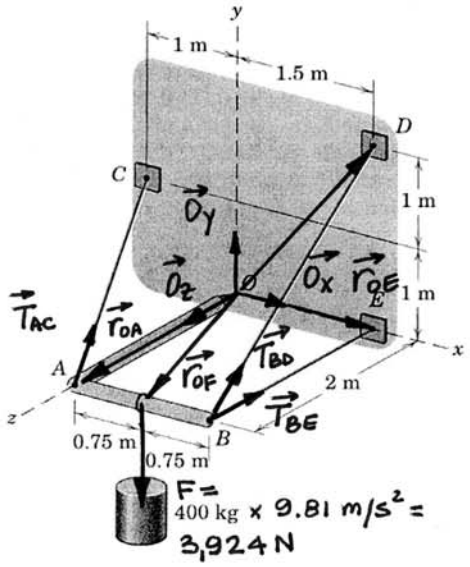
3. The only calculators permissible are listed above. Please circle your model.

4. No other paper will be accepted for marking or allowed on the desk.

5. Do not remove the staple.

DO NOT WRITE IN THIS SPACE.

1. The light right-angle boom which supports the 400-kg cylinder is supported by three cables and a ball-and-socket joint at *O* attached to the vertical *x-y* surface. Determine the reactions at *O* and the forces in the cables.



$\vec{T}_{AC} = T_{AC}(-0.4083\vec{i} + 0.4083\vec{j} - 0.8165\vec{k})$

$\vec{T}_{BD} = T_{BD}(0.7071\vec{j} - 0.7071\vec{k})$

$\vec{T}_{BE} = T_{BE}(-\vec{k})$

$\vec{F} = -3{,}924\vec{j}$ [N]

$\vec{r}_{OA} = 2\vec{k}$

$\vec{r}_{OD} = 1.5\vec{i} + 2\vec{j}$

$\vec{r}_{OE} = 1.5\vec{i}$

$\vec{r}_{OF} = 0.75\vec{i} + 2\vec{k}$

(i) $\Sigma\vec{M}_O = 0$

$$\vec{M}_O(T_{AC}) = \vec{r}_{OA} \times \vec{T}_{AC} = T_{AC}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 2 \\ -0.4083 & 0.4083 & -0.8165 \end{vmatrix} = -0.8166\,T_{AC}\vec{i} - 0.8166\,T_{AC}\vec{j}$$

$$\vec{M}_O(T_{BD}) = \vec{r}_{OD} \times \vec{T}_{BD} = T_{BD}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.5 & 2 & 0 \\ 0 & 0.7071 & -0.7071 \end{vmatrix} = -1.414\,T_{BD}\vec{i} + 1.061\,T_{BD}\vec{j} + 1.061\,T_{BD}\vec{k}$$

$$\vec{M}_O(T_{BE}) = \vec{r}_{OE} \times \vec{T}_{BE} = T_{BE}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.5 & 0 & 0 \\ 0 & 0 & -1 \end{vmatrix} = 1.5\,T_{BE}\vec{j}$$

$$\vec{M}_O(F) = \vec{r}_{OF} \times \vec{F} = (3{,}924)\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.75 & 0 & 2 \\ 0 & -1 & 0 \end{vmatrix} = 7{,}848\vec{i} - 2943\vec{k}\ [\text{Nm}]$$

$-0.8166\,T_{AC} - 1.414\,T_{BD} + 7{,}848 = 0$ $\therefore$ $T_{AC} = 4808 = 4{,}810\text{N}$

$-0.8166\,T_{AC} + 1.061\,T_{BD} + 1.5\,T_{BE} = 0$ $\therefore$ $T_{BE} = 655\text{N}$

$1.061\,T_{BD} - 2943 = 0$ $\therefore$ $T_{BD} = 2774 = 2770\text{N}$

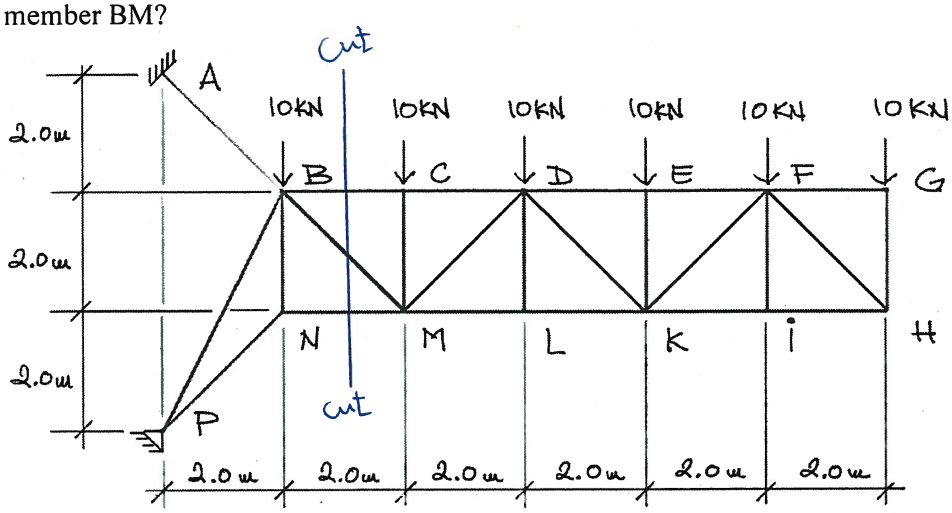
(ii) $\Sigma\vec{F} = 0$

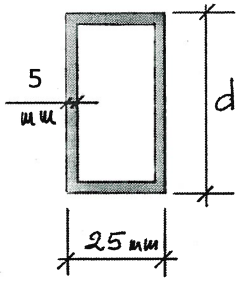
$(-0.4083)(4808) + O_x = 0$ $\therefore$ $O_x = 1963\text{N}$

$(0.4083)(4808) + (0.7071)(2774) - 3924 + O_y = 0$ $\therefore$ $O_y = 0\text{N}$

$(-0.8165)(4808) - (0.7071)(2774) - 655 + O_z = 0$ $\therefore$ $O_z = 6542 = 6540\text{N}$

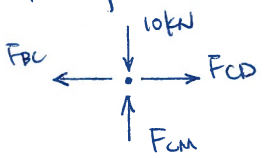
2. The truss shown is supported by a pin at *P* and a diagonal cable *AB*, at *B*. Determine the force in members *BM*, *NM*, *CM* and *CD*, and indicate whether they are in tension or compression. Members *CD* and *BM* are to be constructed from the same-size rectangular box cross section (shown), which is fabricated from 5 mm thick welded steel plate that can be cut in 5 mm increments. Knowing that the yield stress for steel is 350 MPa, the load factor for axially loaded bars is 2.25 and the modulus of elasticity for steel is 200,000 MPa, determine dimension *d* for the required section. Also, what is the elongation of member BM?





CROSS-SECTION OF TRUSS MEMBERS

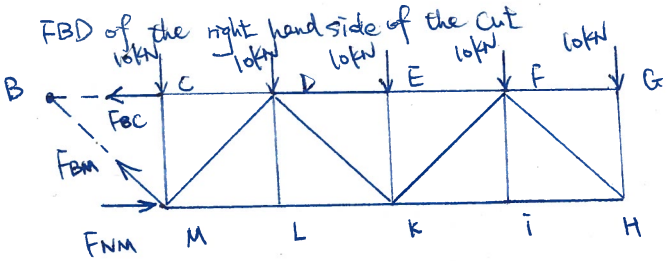
- FBD of Joint C



$F_{CM} = 10\,kN\ (C)$

$F_{BC} = F_{CD}$

- FBD of the right hand side of the Cut



$\Sigma M_B = 0$

$F_{NM}(2) - 10(2+4+6+8+10) = 0$

$F_{NM} = 150\,kN\ (C)$

$\Sigma F_y = 0 \quad \frac{F_{BM}}{\sqrt{2}} - (10)(5) = 0$

$F_{BM} = 70.71\,kN\ (T)$

$\Sigma F_x = 0 \quad -F_{BC} - \frac{F_{BM}}{\sqrt{2}} + F_{NM} = 0$

$-F_{BC} - 50 + 150 = 0$

$F_{BC} = 100\,kN\ (T)$

$\therefore F_{CD} = F_{BC} = 100\,kN\ (T)$

- Unfactored design force = $F_{CD} = 100\,kN$

$A_{req'd} = \frac{(100 \times 10^3\,N)(2.25)}{350\,N/mm^2} = 643\,mm^2$

$(d)(5)(2) + (15)(5)(2) \geq 643$

$d \geq 49.3\,mm$

$\therefore$ Use $d = 50\,mm$

$A_{actual} = (50)(5)(2) + (15)(5)(2) = 650\,mm^2$

- Elongation of member BM

$= \frac{(\text{unfactored force in BM})(\text{length of BM})}{(A_{actual})(E)}$

$= \frac{(70.71 \times 10^3\,N)(2\sqrt{2} \times 10^3\,mm)}{(650\,mm^2)(200{,}000\,N/mm^2)}$

$= 1.538\,mm$

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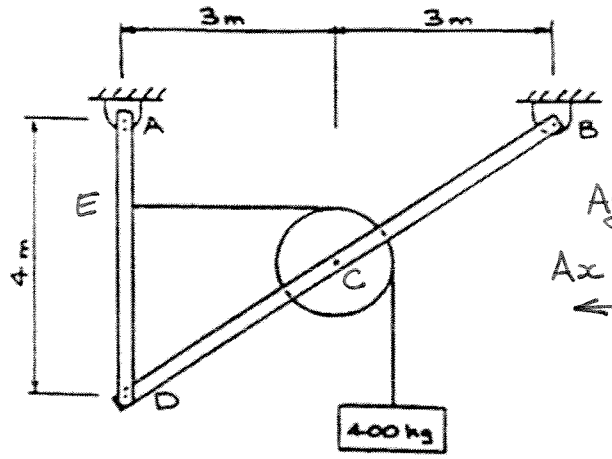
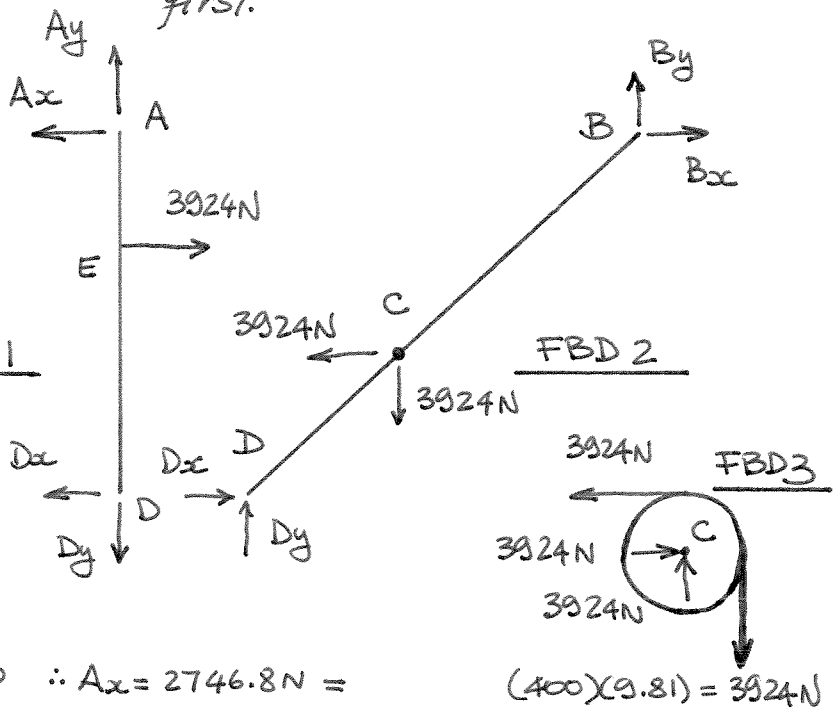
4. The pulley in the pin-connected frame shown has a diameter of 1.6 m. Determine the reaction components at A and B.



The frame is non-rigid if removed from its supports, so begin by considering the individual members first.



On FBD 1: $[\Sigma M_A = 0]$

$(3924)(2 - \frac{1.6}{2}) - D_x(4) = 0$

$\therefore D_x = 1177.2\ N \leftarrow$

$[\Sigma F_x = 0]$ $3924 - A_x - 1177.2 = 0$ $\therefore A_x = 2746.8N = 2750N \leftarrow$

On FBD 2: $[\Sigma M_B = 0]$

$(3924)(3) - (3924)(2) + (1177.2)(4) - D_y(6) = 0$ $\therefore D_y = 1438.8\ N \uparrow$

$[\Sigma F_x = 0]$ $1177.2 - 3924 + B_x = 0$ $\therefore B_x = 2746.8N = 2750N \rightarrow$

$[\Sigma F_y = 0]$ $1438.8 - 3924 + B_y = 0$ $\therefore B_y = 2485.2N = 2490N \uparrow$

On FBD 1:

$[\Sigma F_y = 0]$ $A_y - 1438.8 = 0$ $\therefore A_y = 1438.8N = 1439N \uparrow$