University of Toronto

Faculty of Applied Science and Engineering

MIE100 – Dynamics

Final Examination

April 18, 2011, 2:00pm to 4:30pm

Instructors: *J. Postma, C. Simmons, A. Sinclair and L. Sinclair*

Aids Permitted: One non-programmable calculator
One 8 1/2" by 11" sheet of paper, any colour

Do all work in the exam booklet

Complete all_five_questions

Total Marks: 100

1(a). A projectile enters a resisting medium at $x = 0$ with an initial speed (v_o) of 270 m/s and travels 100 mm before coming to rest. Assume that the speed of the projectile is defined by the relation $v = v_o - kx$, where v is expressed in m/s and x is in meters.

Find the initial acceleration of the projectile.

1(b). The flight path of airplane B is a horizontal straight line that passes directly over a radar tracking station at A. Knowing that the airplane moves to the left with a constant velocity v_o, determine $d\theta/dt$ in terms of $|v_o|$, h and θ.

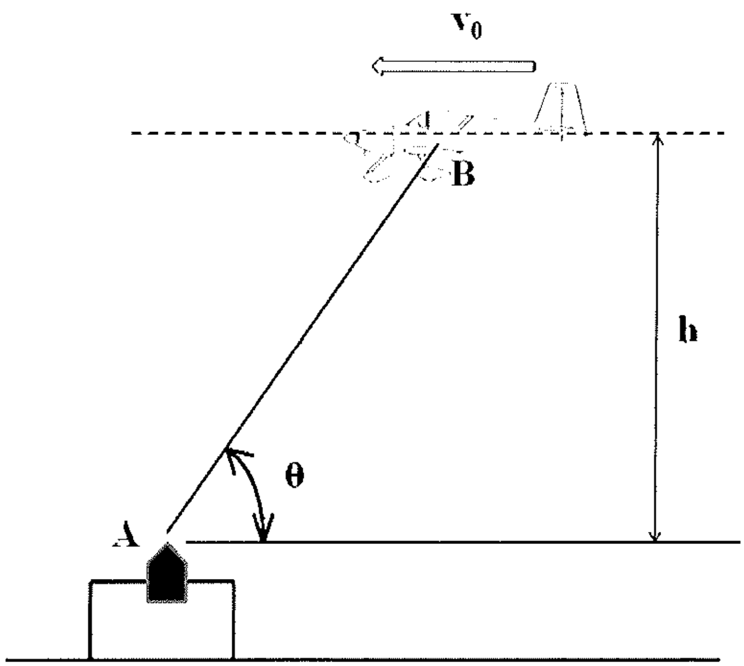


Figure 1b

2. A wheel on a 40° degree ramp is released from rest. The wheel has mass $m = 5$ kg, radius $R = 1$ m, and radius of gyration about its centre of mass, $k_G = 0.8$ m. The static and kinetic coefficients of friction between the wheel and ramp are $\mu_s = 0.3$ and $\mu_k = 0.28$, respectively. Use the rectangular coordinates aligned with the ramp as shown.

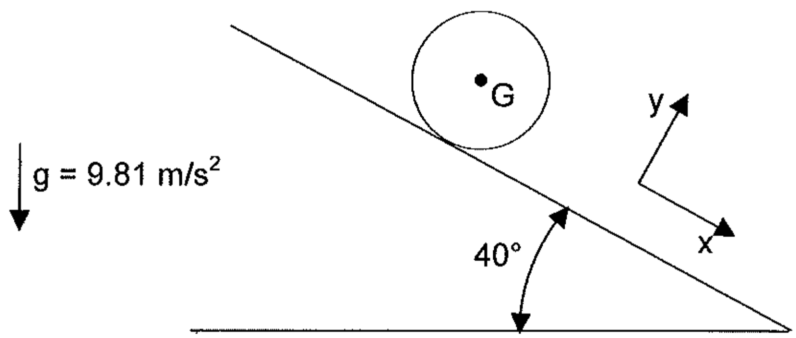


Figure 2

(a) Show that the wheel slips as it rotates.
(b) Determine α, the angular acceleration of the wheel.
(c) Determine $\overline{a_G}$, the acceleration of the centre of mass of the wheel.

3. The oil pumping rig is driven by wheel OA which has radius $R = 0.750$ m and rotates about fixed axis O. Link AB is 2.50 m long. The distance from B to the fixed axis C is also 2.50 m. At the instant shown, wheel OA has angular velocity $\omega_{OA} = 1.00$ rad/s and angular acceleration $\alpha_{OA} = 0.500$ rad/s^2, both in the clockwise direction.

   (a) Determine the angular velocity of link AB, ω_{AB}, at the instant shown.

   (b) Determine the angular velocity of rod BCD, ω_{BCD}, at the instant shown.

   (c) Determine the acceleration of point A, $\overrightarrow{a_A}$, at the instant shown. Express your answer in the given x-y coordinates system.

   (d) Determine the angular acceleration of rod BCD, α_{BCD}, at the instant shown.

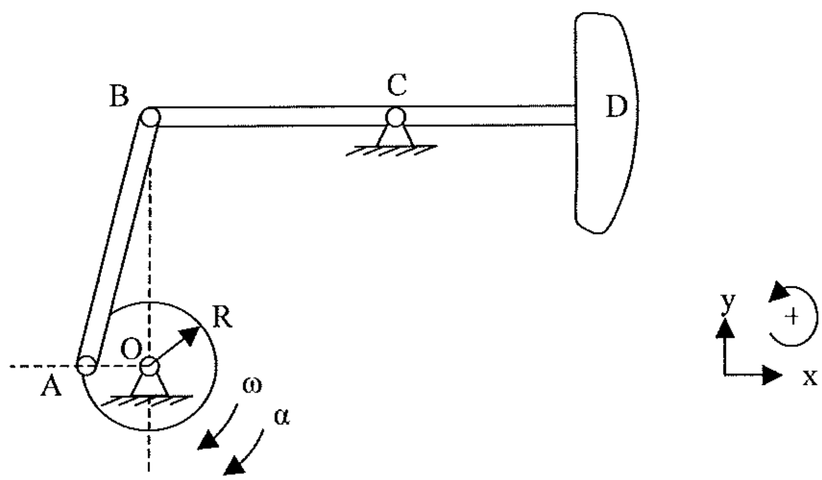


Figure 3

4. ABC is a long, thin, uniform rod of length 10 meters and mass 15 kg. It is pinned at point C. At the instant shown in the diagram where $\theta = 90°$, the velocity of point C is $-12\,\hat{j}\ {}^{m}/_{s}$.

(a) Find the acceleration of point C in x-y coordinates at the instant shown in the diagram.

(b) Find the angular momentum of the rod about point B at the instant shown in the diagram.

(c) What will be the speed of point C when $\theta = 180°$?

Note: The pinned bar rotates without friction

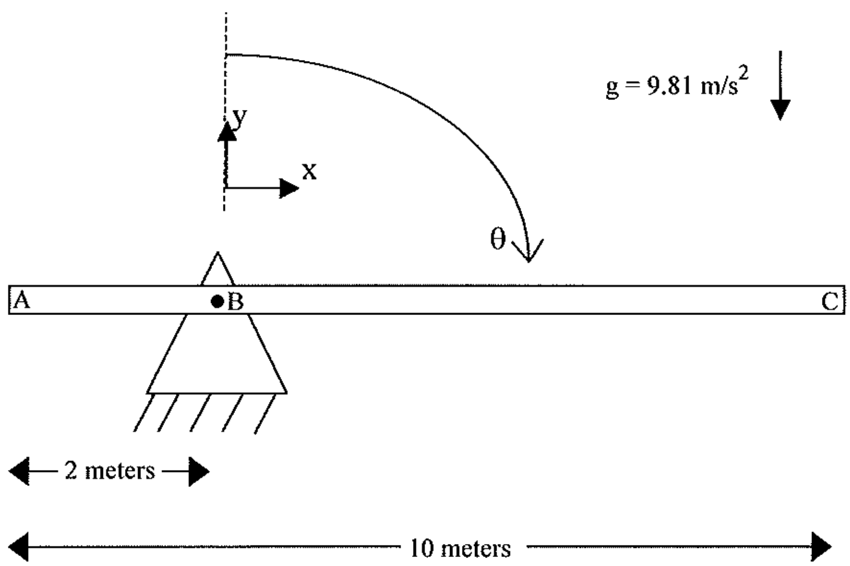


Figure 4

5. A mass is assembled with two springs and one damper as shown. A force (P) of 45 Newtons has been applied to the mass such that a static equilibrium has been reached. It is this static equilibrium position, which is shown in the diagram. At t = 0 seconds, P is removed. Neglect any rotational motion of the mass.

   (a) Draw a Free Body Diagram of the mass just after P has been removed.
   (b) Find the natural frequency of the system.
   (c) Find the damping ratio and the damped frequency of the system.
   (d) Find the maximum displacement of the mass relative to y = 0 as indicated on the diagram.

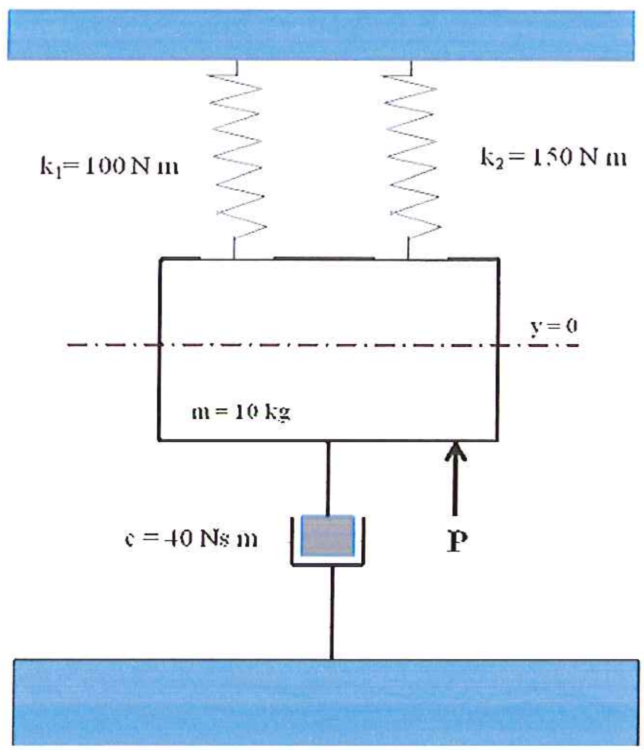


Figure 5

1(a)

$v_0 = 270$ m/s.

$v = v_0 - kx$

@ $x = 0.1$ m , $v = 0$

find a_0 (this is all scalar).

first find k:

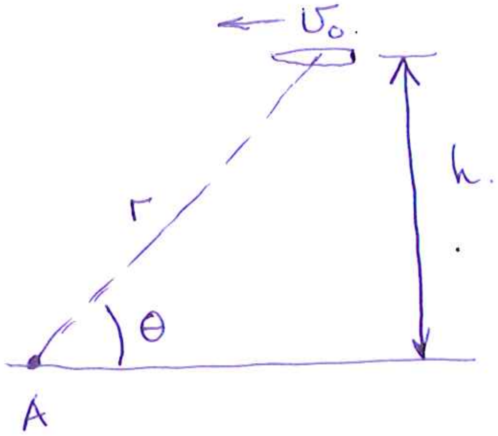
$0 = 270 - k(.1) \Rightarrow k = 2700$

$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = -k(v) = -k(v_0 - kx)$

but a_0 is the acceleration at $x = 0$

$\Rightarrow a_0 = -kv_0 = -2700(270) = 729$ km/s^2

1(b)



find $\dot{\theta}$ as a function of v_0, h, θ

$h = r\sin\theta \qquad \Rightarrow \qquad r = h/\sin\theta$

$0 = \dot{r}\sin\theta + r\cos\theta\,(\dot{\theta})$

but $\vec{v} = \dot{r}\,\hat{u}_r + r\dot{\theta}\,\hat{u}_\theta$

and $\dot{r} = v_r = -v_0\cos\theta$ (geometry)

$$\Rightarrow \dot{\theta} = \frac{-\dot{r}\sin\theta}{r\cos\theta} = \frac{-(-v_0\cancel{\cos\theta})\sin\theta}{(h/\sin\theta)\cancel{\cos\theta}}$$

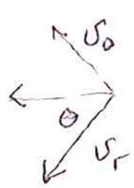
~~answer~~ $\theta = \tan^{-1}\frac{h}{x}$

$$= \frac{v_0}{h}\sin^2\theta$$

$\frac{v_0}{h}\cos^2 h = 7/10.$

another way.

$v_\theta = r\dot{\theta}$ but $v_\theta = v_0\sin\theta$ & $r = h/\sin\theta$ $\Rightarrow \dot{\theta} = \frac{v_0\sin\theta}{h/\sin\theta}$



$= \frac{v_0\sin^2\theta}{h}$ as before

$\tan\theta = \frac{h}{x}$

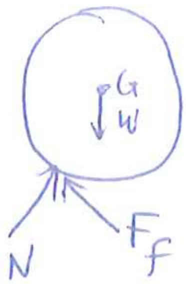
$x\tan\theta = h$ | harder.

$0 = \dot{x}\tan\theta + x.$

#2

$m = 5\,kg$
$R = 1\,m$
$K_G = 0.8\,m$
$\theta = 40°$

$\mu_s = 0.3$
$\mu_k = 0.28$



$\Sigma F_x = m a_x$

$-F_f + W\sin\theta = m a_x$ — (3)

$\Sigma F_y = m a_y$

$N - W\cos\theta = 0$

$N = mg\cos\theta$

Assume no slip,

$a_{Gx} = R\alpha$

$\therefore -F_f + mg\sin\theta = mR\alpha$ — (1)

Also, $\Sigma M_G = I_G\alpha$ +)

$F_f R = m K_G^2 \alpha$ — (2)

$F_f = \frac{m K_G^2}{R}\alpha$, sub into (1)

$\Rightarrow \alpha\left(mR + \frac{mK_G^2}{R}\right) = mg\sin\theta$

$$\alpha = \frac{Rg\sin\theta}{R^2 + K_G^2} = \frac{(1)(9.81)\sin 40}{1^2 + 0.8^2} = 3.85\,s^{-2}$$

$$F_f = \frac{mK_G^2 g\sin\theta}{R^2 + K_G^2} = \frac{(5)(0.8)^2(9.81)\sin 40}{1^2 + 0.8^2} = 12.3\,N$$

Check if $F_f < \mu_s N \Rightarrow \mu_s N = \mu_s mg\cos\theta = 11.3\,N \Rightarrow$ slips

If not, then slips and $F_f = \mu_k N = \mu_k mg\cos\theta$

$\Rightarrow$ From (2), $\alpha = \frac{F_f R}{mK_G^2} = \frac{\mu_k mg\cos\theta\, R}{mK_G^2} = \frac{\mu_k R g\cos\theta}{K_G^2} = 3.35\,s^{-2}$

From (3), $a_{Gx} = g\sin\theta - \frac{F_f}{m} = g\sin\theta - \mu_k g\cos\theta = g(\sin\theta - \mu_k\cos\theta)$
$= 4.2\,m/s^2$

#3

(a) $\omega_{OA} = -1\ s^{-1}$ $\curvearrowleft +$

$\alpha_{OA} = -0.5\ s^{-2}$

as defined in the problem.

Velocity analysis:

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}.$$

but $\vec{v}_B$ is vertical and $\vec{v}_A$ is vertical

$\Rightarrow \omega_{AB} = 0$ i.e. AB is in translation.

(b) but this also means ~~that~~ $\vec{v}_B = \vec{v}_A$.

$\Rightarrow \vec{v}_B = 1(.75)\,\hat{j} = 0.75\,\hat{j}$ m/s $\quad (|\vec{v}_A| = R_{OA}\,\omega_{OA})$

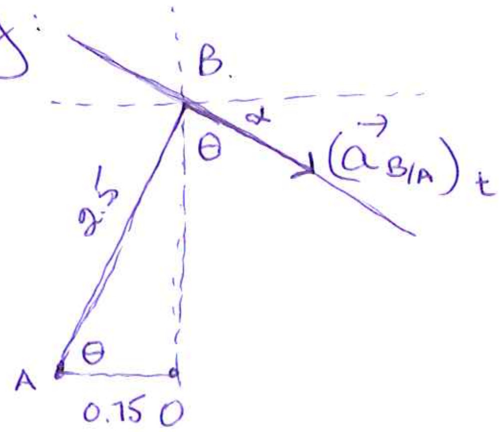
$\Rightarrow |\omega_{BC}| = 0.75/2.5 = 0.3\ s^{-1}$ by observation it is a clockwise rotation $\Rightarrow \omega_{BC} = -0.3\ s^{-1}$

(c) $\vec{a}_A = +.75(\omega)^2\,\hat{i} + .75(\alpha)\,\hat{j}$

$= .75\,\hat{i} + .375\,\hat{j}$ m/s^2

(d) $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$.

geometry:



$2.5\cos\theta = 0.75$

$\Rightarrow \theta = \cos^{-1}\dfrac{0.75}{2.5} = 72.5^\circ$.

$\Rightarrow \alpha = 17.5^\circ$.

$\vec{a}_B = 2.5(.3)^2\,\hat{\imath} + 2.5\,\alpha_{BC}\,\hat{\jmath}$

$\vec{a}_{B/A} = 2.5\,\alpha_{AB}\cos 17.5^\circ\,\hat{\imath} - 2.5\,\alpha_{AB}\sin 17.5^\circ\,\hat{\jmath}$

$\Rightarrow$ $\hat{\imath}$ component:

$2.5(.3)^2 = .75 + 2.5\,\alpha_{AB}\cos 17.5^\circ$

$\Rightarrow \alpha_{AB} = -.22\ s^{-2}$ $\Rightarrow$ I have an incorrect direction of α.

but now I think again and that must be right: look at the tight radius of point A + it is going to go to the left faster than B can.

$\Rightarrow$ still $\hat{i}$ component :

$$2.5(.3)^2 = .75 - 2.5\,\alpha_{AB}\cos 17.5^\circ .$$

$\Rightarrow \alpha_{AB} = + .22\ s^{-2}$ note that the # is the same

the math revealed the error.

but make the adjustment for $\hat{j}$ component .

$$2.5\,\alpha_{BC} = .375 + 2.5\,\alpha_{AB}\sin 17.5^\circ .$$

$$= .375 + .166$$

$\Rightarrow \alpha_{BC} = 0.216\ s^{-2}$

pinned bar question #4

(a) $\vec{v}_C = -12\hat{j}$ m/s $\Rightarrow$ $\omega = \frac{12}{8} = 1.5$ rad/s CW.

$\vec{a}_{C_n} = -\frac{v^2}{r}\hat{i} = -\frac{144}{8}\hat{i} = -18\hat{i}$ m/s^2.

to find the y component, find α.

$\Sigma M_B = I_B\,\alpha$.

$I_B = I_G + m|GB|^2 = \frac{1}{12}(15)(10)^2 + 15(3)^2$

$= 125 + 135 = 260$ kg m^2

the only force causing a moment is gravity.

$\circlearrowright^{+}$ $15(9.81)(3) = 260\,\alpha$

$\Rightarrow \alpha = 1.70\ s^{-2}$ $\qquad |BC|\alpha = 8(1.70) = 13.6$.

$\Rightarrow \vec{a}_C = -18\hat{i} - 13.6\hat{j}$ m/s^2 (direction is determined by observation).

(b) $H_B = I_B\,\omega$ but most of that was done above.

$= 260(1.5) = 390$ kg $\frac{m^2}{s}$

(c) use energy:

$$T_1 + U_{1\to 2} = T_2.$$

$$T_1 = \frac{1}{2} I_B \omega_1^2 = \frac{1}{2}(260)(1.5)^2 = 292.5$$

$$T_2 = \frac{1}{2} I_B \omega_2^2 = 130\,\omega_2^2$$

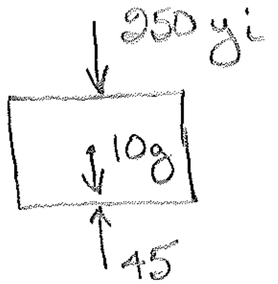
$$U_{1\to 2} = -mg\,\Delta h_G = -15(9.81)(-3) = 441.5$$

$$\Rightarrow \omega_2^2 = \frac{292.5 + 441.5}{130} = 5.65 \Rightarrow \omega_2 = 2.38\ s^{-1}$$

$$\Rightarrow |\vec{v}_c| = 2.38(8) = 19\ m/s.$$

$$\vec{v}_c = -19\hat{\imath}\ m/s$$

5(a) static equilibrium before P removal.



$$\sum F_y = 0$$

$$-250\, y_i - 10(9.81) + 45 = 0.$$

$$\Rightarrow y_i = \frac{53.1}{-250} = -0.212\,\text{m}.$$

$\Rightarrow$ the springs are stretched 212 mm.

but the neutral level of the vibration is where

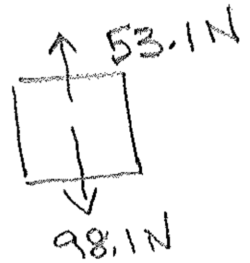
$mg = ky \qquad 10(9.81) = 250\, y_e \qquad y_e = 0.392\,\text{m}.$

$\Rightarrow$ our initial displacement is $.392 - .212 = .18\,\text{m}$

upward.

(a) answer:

↑ this is for (d).



keep going:

(b). $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{250}{10}} = 5\ s^{-1}$

(c) $C_c = 2\omega_n m = 100 \Rightarrow c/c_c = 0.4$

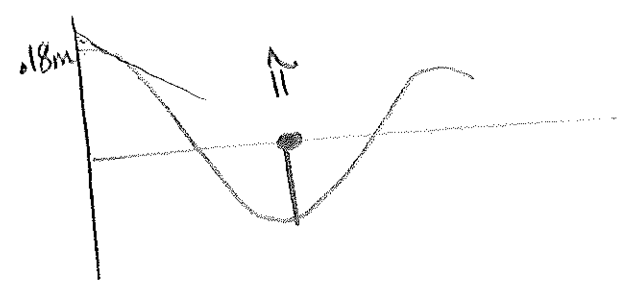
$$\omega_d = \omega_n\sqrt{1-(c/c_c)^2} = 4.58.$$

So our formula for damped (free) vibration is:
(underdamped)

is: $x = D\left(e^{-c/2m\, t}\right) \sin\left(\omega_d t + \phi\right)$

but from the physics: $D = 0.18$.
$\phi = -\pi/2$

$$y = 0.18 \exp\left[-\frac{40}{20} t\right] \cos\left(4.58 t\right)$$



∴ max displacement occurs @

$\omega_d t = \pi$

$t = \frac{3.14}{4.58} = 0.686\ s$.

$x = 0.18\ e^{-1.37} \cos \pi$

$= -.046 \Rightarrow$ total deflection $= 0.225\ m$.
downward.

james method:

$$x = D e^{-\frac{c}{2m}t} \sin(\omega_d t + \phi)$$

$x(0) = +0.18$

$\dot{x}(0) = 0$

but we still want an answer @ $\omega_d t = \pi$

$\Downarrow$

$c = 40$

$2m = 20$

$\omega_d = 4.58$

$t = 0.686s$

$$x = 0.18 = D e^{0} \sin(0 + \phi)$$

$$0.18 = D \sin\phi$$

$$\dot{x}(t) = D\left[-\frac{c}{2m} e^{-c/2m\, t} * \sin(\omega_d t + \phi) + e^{-c/2m\, t} * \omega_d \cos(\omega_d t + \phi)\right]$$

$$0 = D\left[-\frac{c}{2m} * \sin\phi + \omega_d \cos\phi\right]$$

$$\Rightarrow 2 \sin\phi = 4.58 \cos\phi$$

$$\Rightarrow \tan\phi = \frac{4.58}{2} \quad \Rightarrow \phi = 66.4^{\circ}$$

$$\Rightarrow D = \frac{0.18}{.92} = 0.196$$

answer:

$$x = 0.196\, e^{-1.37} * \sin(180 + 66.4^\circ)$$

$$= 0.196(.25)(-.916) = .045$$

Same answer.