University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, APRIL 2003
Year 1 – Programs 1, 2, 3, 4, 6, 7, 8, 9

MAT 186H1S
CALCULUS I
Examiner: D. Burbulla

INSTRUCTIONS

Non-programmable calculator permitted; no other aids allowed.

Present your solutions to all of the following questions in the exam booklets supplied. The marks for each question are indicated in parantheses.

TOTAL MARKS: 100.

1. [15 marks] Find the following derivatives:

   (a) [5 marks] $\dfrac{dy}{dx}$ if $y = e^{\sin x} - \cos e^x$.

   (b) [5 marks] $\dfrac{dy}{dx}$ if $y = \text{Tan}^{-1} x + \text{Tan}^{-1}\left(\dfrac{1}{x}\right)$

   (c) [5 marks] $F'(2)$ if $F(x) = \displaystyle\int_0^{x^3} \sqrt{17 + t^2}\, dt$.

2. [15 marks] Find the following limits:

   (a) [5 marks] $\displaystyle\lim_{x \to 0} \frac{1 - 2x^2 - \cos(2x)}{x^2}$

   (b) [5 marks] $\displaystyle\lim_{x \to 1} \left(\frac{1}{\ln x} + \frac{1}{1 - x}\right)$

   (c) [5 marks] $\displaystyle\lim_{x \to 0} \left(2 - e^{-x}\right)^{\csc x}$

3. [15 marks] Find the following integrals:

   (a) [5 marks] $\displaystyle\int \frac{e^{3x}}{e^{3x}+10}\,dx.$

   (b) [5 marks] $\displaystyle\int_{-1}^{2} x^2(x^3+17)^{3/2}\,dx.$

   (c) [5 marks] $\displaystyle\int_{1}^{\infty} \left(\frac{1}{x^2}+\frac{4}{x^3}\right)\,dx.$

4. [15 marks] For this question, consider the curve with equation $x^{2/3}+y^{2/3}=1$.

   (a) [5 marks] Show by differentiating implicitly that $\dfrac{dy}{dx} = -\left(\dfrac{y}{x}\right)^{1/3}$.

   (b) [5 marks] Find the length of the curve from the point $(0,1)$ to the point $(1,0)$.

   (c) [5 marks] Find the volume of the solid obtained by revolving the curve for $-1 \le x \le 1$ around the x-axis.

5. [10 marks] A stone is dropped into a well and the sound of the stone striking the water is heard 3.1 sec later. If the speed of sound is 340 m/sec, how deep is the surface of the water in the well? (Use acceleration due to gravity $g = 9.8$ m/sec^2.)

6. [10 marks] Sketch the graph of $f(x) = \text{Sec}^{-1}\sqrt{x^2+1}$, indicating all critical points, all inflection points, and all asymptotes, if any.

7. [10 marks] Newton's Law of Cooling states that

$$\frac{dT}{dt} = k(T-A),$$

   where T is the temperature of an object at time t, A is the (constant) ambient temperature of the air surrounding the object, and k is a constant.

   A freshly baked pie is taken out of an oven with temperture 350° C and is placed on a table in a room with constant air temperature 20° C. If the pie cools by 100° C in 4 minutes, when will its temperature be 75° C?

8. [10 marks] A tank filled with water of density $\rho = 1000$ kg/m^3 has the shape of an inverted right circular cone, with radius 1 m at the top, and depth 3 m. Find the work done in pumping all of the water out of the tank and up to a horizontal pipe 2 m above the top of the tank. (Use acceleration due to gravity $g = 9.8$ m/sec^2.)