MAT186 Calculus I - Fall 2020

Review Session **December 10, 2020**

$y'(t) =$ some stuff w/ t

Open book, open MATLAB, desmos

Contents

We'll start at 1:10

You have all 24 hours for both parts

Similar difficulty to the midterm but designed to take 180 minutes

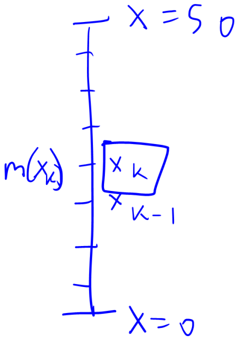
Topic focus is slightly more on the after-midterm stuff.

Quercus part and Crowdmark part

Do WeBWorK problems, PCE, suggested problems, past exam problems to practice

Riemann Sums

A box of 100 kilograms of sand is being pulled up a distance of 50 meters. The motion occurs at a constant rate over the course of one minute. The sand is leaking from the box at a rate of 1 kilogram per second. The box itself weighs 5 kilograms. Write and justify a definite integral expression for the amount of work done pulling the box up.



1.) Break it up. Divide $x\in[0,50]$ into n equally sized subintervals of length $\Delta x = \frac{50}{n}$

Endpoints $x_0, x_1 \dots x_n = 50$.

2.) ΔW_k = Work done pulling box from x_{k-1} to x_k.

Approx this w/ $\Delta W_k \approx mgh$. $\Delta x = x_k - x_{k-1} = \frac{50}{n}$

Let $m(x)$ be mass of box+sand when it's at height x.

$$m(x_k) = 5 + 100 - \boxed{(x_k\ m)\cdot \frac{1}{\frac{5}{6}}\frac{s}{m}\cdot\frac{1\ kg}{1\ s}}$$

box — starting amount of sand

50 m in 1 minute $= \frac{5}{6}$ m/s

time = $\frac{\text{dist}}{\text{velocity}}$ (x_k, $\frac{5}{6}$)

time · rate in kg/s = kg

$$m(x_k) = 105 - \frac{6}{5}x_k \quad (kg).$$

$$\Delta W_k \approx \left(105 - \frac{6}{5}x_k\right) g\,\Delta x$$

3.) $$W \approx \sum_{k=1}^{n}\left(105 - \frac{6}{5}x_k\right) g\,\Delta x$$

4.) $$W = \lim_{n\to\infty}\sum_{k=1}^{n}\left(105 - \frac{6}{5}x_k\right)\Delta x\, g = \int_{0}^{50}\left(105 - \frac{6}{5}x\right) g\,dx$$

x_n, x_0

We may ask you for this argument.

This is similar to WT5 and to the E4 readings.

Differential Equations

A population of rabbits, left without predators, grows at a rate equal to 10% of the current rabbit population per month. To help keep the population in check, we take rabbits out of the wild to allow students to pet them before finals. We do this at a constant rate of 25 rabbits per month. We started off with 300 rabbits in the population.

(a) Write a differential equation modeling the number of rabbits over time.

b) Are there any equilibrium solutions to this equation?

(c) Use the initial condition to write an initial value problem. What happens to the population of rabbits over time? Explain how you know.

$\boxed{P' = (\text{rate of rabbits in}) - (\text{rate of rabbits out})}$ IVP $P(0) = 300$

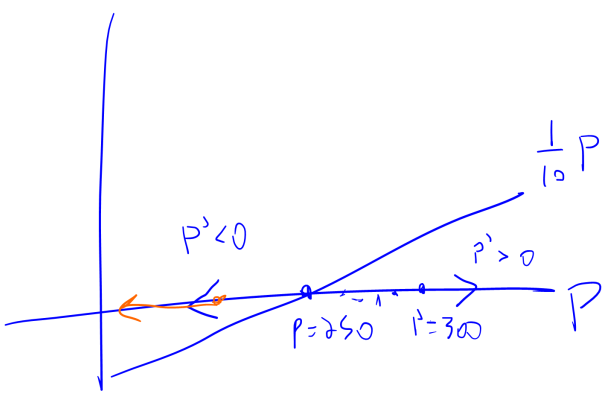
$P'(t) = \frac{1}{10} P(t) - 25$ rabbits/month

$P(t) = 0$ not a solution.
$\Downarrow$
$0 = \frac{1}{10} \cdot 0 - 25$ $\quad 0 = -25$

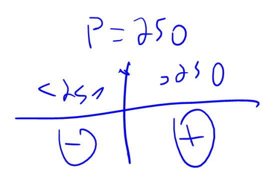
Eq Solns. Constant vals of $P(t)$ that are solutions.

$\frac{1}{10} P - 25 = 0 \Rightarrow P = 250$ only eq. solution.

P v. P'



reason for (c)



Where is $\frac{1}{10} P - 25$ negative or pos.?

$P'(0)$ is positive

$P'(t)$ will be positive for all $P > 250$

Rabbits $\to \infty$. Answer to (c)

Word Problems ⇔ Mathematical Expressions

A pickup truck is being used to transport sand. Let the time t be given in hours. Sand is leaking from a truck bed; there's 100 kg to start, and the rate of change of sand in the truck is $L(t)$ kilograms per hour. The fuel efficiency of this pickup truck is $F(m)$ miles per gallon, where m is the mass of sand in the pickup track. Assume both functions are differentiable and that $F(m)$ is invertible with inverse function $m(F)$.

1. When the fuel efficiency is 30 mi/gal, the mass of sand in the truck is half of what it is when the fuel efficiency is 20 mi/gal. Represent this statement with a single mathematical equation.

2. Interpret the following mathematical expression in one sentence and give its units:

$$F\left(100 + \int_0^5 L(t)\,dt\right)$$

3. Interpret the following mathematical expression in one sentence and give its units:

$$F'(10)$$

4. Will this expression be positive or negative? Explain why, and provide units for the expression.

$$\frac{d}{dT}F\left(100 + \int_0^T L(t)\,dt\right)$$

1.) $m(30) = \frac{1}{2}\left(m(20)\right)$

2.) Fuel consumption at time t in mi/gal

3.) Instantaneous rate of change of fuel efficiency with respect to the mass of sand in the truck, when there are 10 kg of sand in the truck. Units are (mi/gal)/kg

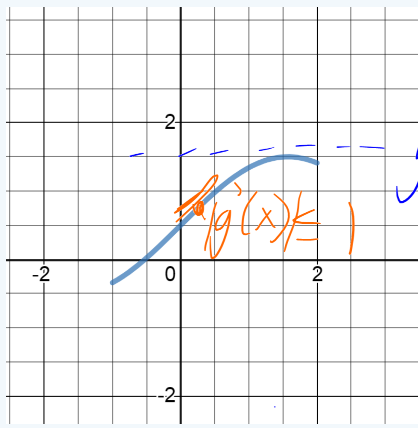
4.) Positive because it's the instantaneous rate of change of fuel efficiency over time and the truck is growing lighter. Units are (mi/gal)/hr

Inequalities and Approximation

Pictured is a graph of $g(x)$ on $[-1,2]$. Use the triangle inequality $|a+b| \le |a|+|b|$ to quickly put an upper bound on the function

$$f(x) = \left|\frac{d}{dx}[e^{x/2}g(x)]\right|$$

over this interval.



$|g'(x)| \le 1$

$|g(x)| \le 2$

so $e^{\frac{x}{2}} \le e^{\frac{2}{2}} < 3$

$$\left|\frac{d}{dx}\left[e^{x/2} g(x)\right]\right| = \left|\frac{1}{2}e^{\frac{x}{2}} g(x) + e^{\frac{x}{2}} g'(x)\right|$$

$$\le \frac{1}{2} e^{\frac{x}{2}} |g(x)| + e^{\frac{x}{2}} |g'(x)|$$

$$\le \frac{1}{2}\cdot 3 \cdot 2 + 3 \cdot 1 < 9$$

Limits

Find the following limit, fully justifying each step with mathematical principles we have covered.

$$\lim_{x \to \infty} \frac{3x + \cos(x)}{x + 1}$$

$$\lim_{x \to \infty} \frac{3x + \cos x}{x+1} = \lim_{x \to \infty} \frac{3 + \frac{\cos x}{x}}{1 + \frac{1}{x}}$$

$-1 \le \cos x \le 1$ all x

$-\frac{1}{x} \le \frac{\cos x}{x} \le \frac{1}{x}$ for all x large positive.

$$\therefore \underbrace{\frac{3 - \frac{1}{x}}{1 + \frac{1}{x}}}_{f(x)} \le \frac{3 + \frac{\cos x}{x}}{1 + \frac{1}{x}} \le \underbrace{\frac{3 + \frac{1}{x}}{1 + \frac{1}{x}}}_{g(x)}$$ for all large positive x

Since $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} g(x) = \frac{3+0}{1+0} = 3$

By squeeze $\lim_{x \to \infty} \frac{3x + \cos x}{x+1} = 3$.