University of Toronto – Engineering

MIE100S – Applied Mechanics: Dynamics

Final Examination - April 2017

Exam length: 150 minutes

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General Instructions:

- Answer all questions in the CrowdMark booklet provided.
- All rough work must be *neatly* shown to earn credit for each question.
- You must use a pen or *dark* pencil.
- Answer all questions. Total marks on this exam = 100.
- For your *final answers*, express vector quantities **using the coordinate system specified in the question.**

This is a Type D examination. Permitted Aids:

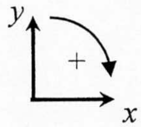
- Non-communicating/non-programmable calculator, such as Casio FX-991 or Sharp EL-520
- One 8 ½" x 11" aid sheet, any colour and format, brought to the test by the student. You may write on both sides of the sheet.

1. A wheel of radius R = 0.4 meters rolls to the right on the ground without slipping. At time t=0, the angular velocity of the wheel is $\omega = 3$ radians/second. The angular acceleration of the wheel is a function of the angular velocity, and is given by $\alpha = 4\omega$. (All variables are in SI units).

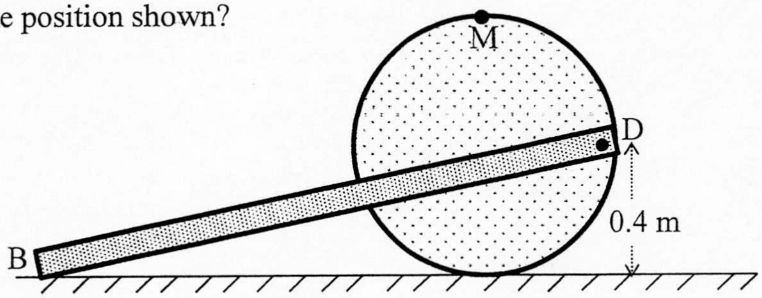
   A thin bar BD of length 1.8 meters is pinned at one end to the edge of the wheel; the other end of the bar drags along the ground. At time t=0, point D is at the position shown in the diagram, point M is at the top of the wheel.

6 (a) Determine the acceleration of point M at time t=0. Express your final answer in *normal-tangential* coordinates.

8 (b) Find the angular velocity ω of the *bar* at time t=0.

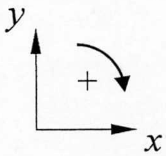


6 (c) What will be the angular velocity ω of the *wheel* when it has rotated 180 degrees from the position shown? (Hint: $\omega\, d\omega = \alpha\, d\theta$)

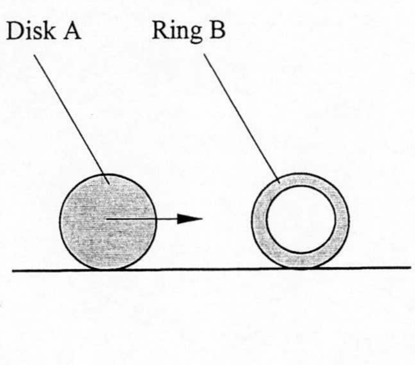


2. A uniform flat disk "A" of mass $m_A = 3.5$ kg hits a stationary ring "B" of mass $m_B = 0.8$ kg in a *perfectly elastic collision.* Both rigid bodies have zero angular velocity just before and after the collision. The coefficients of friction between the ground and rigid bodies are: $\mu_s = 0.3$ and

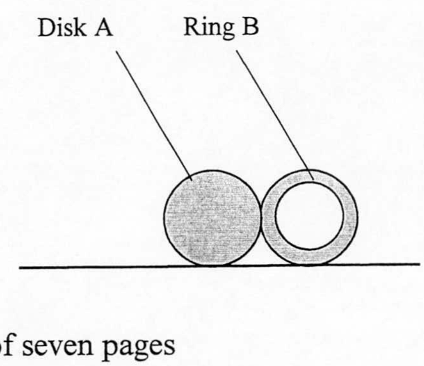
4 $\mu_k = 0.25$. If $(\vec{v_A})_G = 4.0\,\hat{\imath}$ m/s before the collision, determine the velocity of the center of mass of each body immediately after the collision.



Before Impact



During Impact



$\downarrow$ g = 9.81 m/s^2

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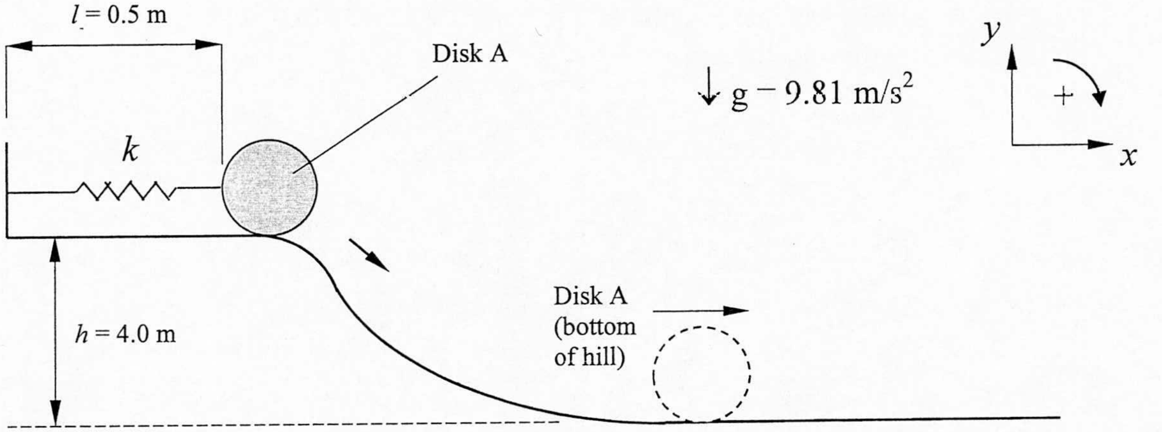
3. A uniform flat disk "A" of mass $m_A = 3.5$ kg and radius $R = 1.3$ m is initially at rest at the top of a hill. The spring with $k = 900$ N/m has a relaxed, undeformed length $l_0 = 0.8$ m, but has been compressed to a length $l = 0.5$ m, as shown in the diagram. Note that the spring is **not** attached to the disk.

The spring is released, (*i.e.* it returns to its relaxed length), and this causes the disk to roll down the hill and eventually reach a flat surface. The entire surface has $\mu_s = 0.3$ and $\mu_k = 0.25$. Assume the disk rolls down the hill *without slip*. Determine the disk's angular velocity ω and velocity of the centre of mass G, $(\vec{v_A})_G$, when it reaches the bottom of the hill.



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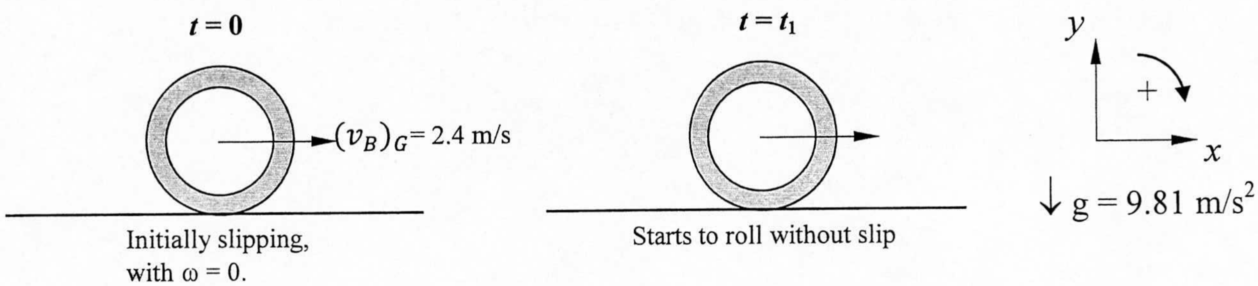
4. At time t=0, a ring "B" is translating on the ground with $(\vec{v_B})_G = 2.4\hat{\imath}$ m/s and no angular velocity (i.e., it is *slipping* on the surface). How much time does it take for the ring to stop slipping, and begin to roll without slipping? The coefficients of friction between the ground and rigid bodies are: $\mu_s = 0.3$ and $\mu_k = 0.25$. The ring "B" has mass $m_B = 0.8$ kg, radius $R = 0.95$ m, and radius of gyration with respect to the center of mass $k_G = 0.9$ meters.



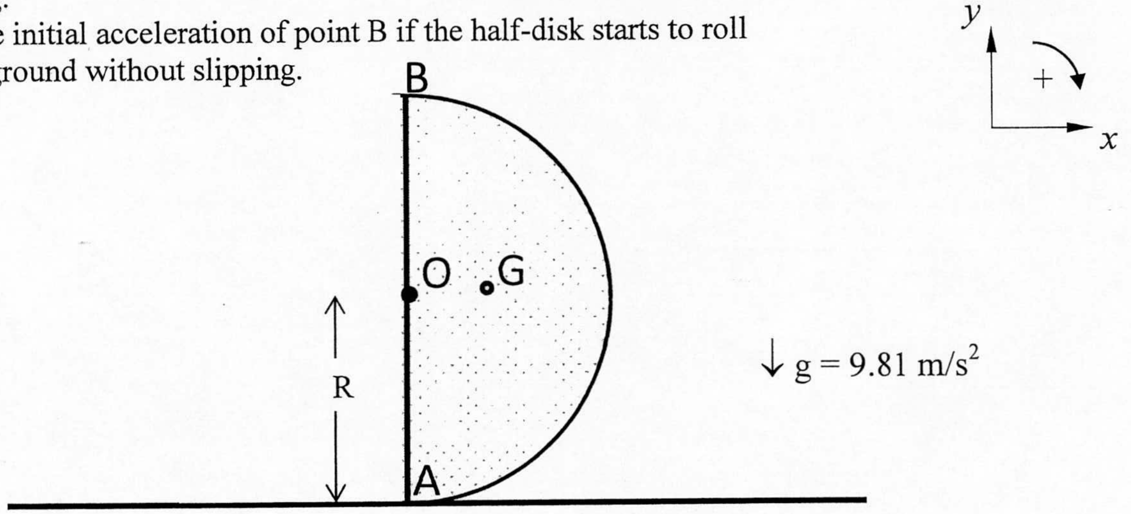
5. A uniform half-disk of mass m = 10 kg and radius R = 3 meters is on the ground and is released from rest at the position shown.

   OG = $4R/3\pi$, where "OG" is the distance from point "O" to point "G".
   I_G= 0.320 mR^2, where "G" is the center of mass of the half-disk

| 4 | (a) Draw a free body diagram of the disk. Clearly indicate the direction of all forces acting on the disk.

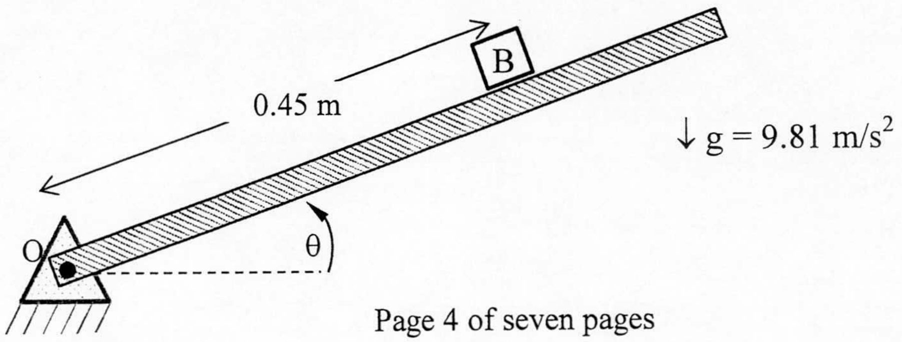
| 8 | (b) Find the minimum value of μ_s for which the half-disk starts to roll on the ground without slipping.

| 8 | (c) Find the initial acceleration of point B if the half-disk starts to roll on the ground without slipping.



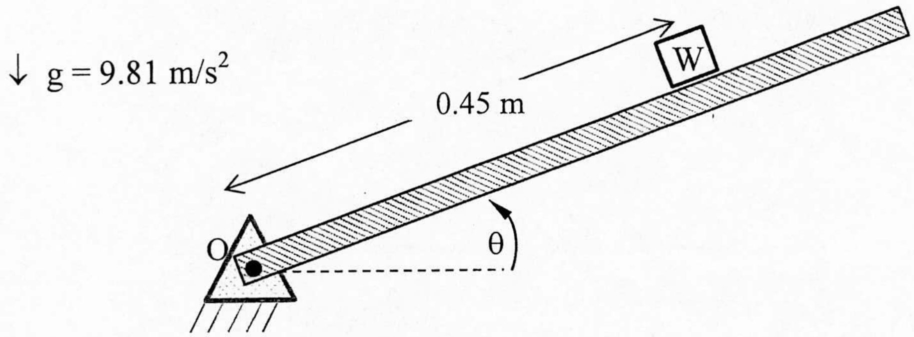
6. A bar is pinned at one end at point "O", such that it can rotate about a horizontal axis passing through "O" as shown in the diagram. A small motor at point "O" forces the bar to have a constant angular velocity of ω = 3 radians/second. As the bar passes the position θ = 0, a small block B of mass 0.1 kg is placed on the bar at a distance of 0.45 meters from "O".

| 8 | The mass starts to slip on the bar when θ reaches 50 degrees. Determine the value of the coefficient of static friction μ_s between B and the bar.



7. A bar is pinned at one end at point "O", such that it can rotate about a horizontal axis passing through "O" as shown in the diagram. A motor at point "O" controls the angular velocity of the bar. The motor forces the bar to have an angular velocity of $\omega = 3$ radians/second as the bar passes the position $\theta = 0$, and forces ω to increase at a constant rate of 1.9 radians/second^2. As the bar passes the position $\theta = 0$, a small block W of mass 0.1 kg is placed on the bar at a distance of 0.45 meters from "O". The coefficient of static friction between W and the bar is equal to $\mu_s = 0.60$.

12 Draw a free body diagram of block W when θ is equal to 35 degrees. Indicate the direction of each force on the free body diagram, and determine the magnitude of each force. (You may assume that block B is not slipping).

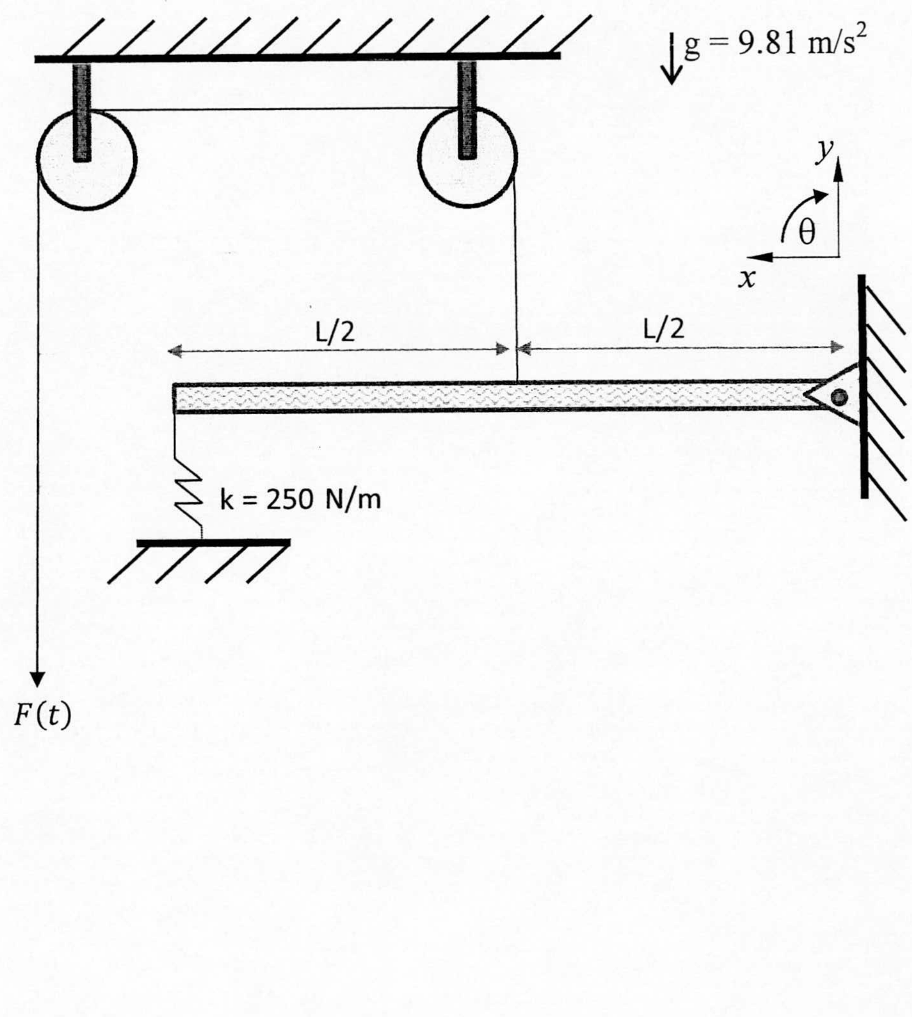


8. The system shown consists of a spring, pulley and long slender rod of length L = 5 meters and mass m = 3 kg. There is a spring with stiffness k = 250 N/m attached to the rod as shown in the diagram. The rope is being pulled by a force $F(t)$.

Ignore the mass of the pulleys. Assume that the system is in static equilibrium when the bar is horizontal ($\theta = 0$) and F(t) = 0. Assume that the rope does not stretch and it does not go slack. Assume that oscillation amplitudes are very small.

4 (a) Determine the amount of compression or extension of the spring, under static conditions when $\theta = 0$ and F(t) =0.

6 (b) If $F(t) = 10 \sin(21t)$, determine the maximum angular deflection θ_{max} of the rod under steady state conditions. All quantities are expressed in SI units



9. A pendulum consists of a small sphere B of mass 10 kg, attached to a bar in the shape of a "T" which is pinned at O and has negligible mass. The T-shaped bar is composed of a horizontal bar of length 4 meters, welded to one vertical bar of length 2 meters. The horizontal bar is connected to two ceiling supports by two identical springs of stiffness k= 155 N/m, and two dashpots (dampers) of strength $c = 20 \frac{N}{m/s}$. The springs are relaxed, and the angular orientation θ of the bar is zero when the vertical bar hangs straight down and sphere B is aligned directly under the pin at O as shown in the diagram.

5 (a) What is the equation of motion of this system for its angular rotation θ?

5 (b) What dashpot value, c, would make the system critically damped?

100 marks total for exam