MAT 186 – CALCULUS I

WINTER TERM 2025

INSTRUCTOR

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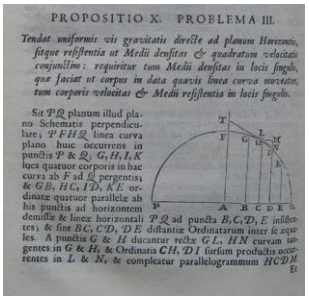
Please use either the above address or Piazza to reach me. Other methods tend to be much slower.

[Professor may not look precisely as pictured]

THE COURSE


PROPOSITIO X. PROBLEMA III.

Tendat uniformis vis gravitatis directe ad planum Horizontis, sitque resistentia ut Medii densitas & quadratum velocitatis conjunctim: requiritur tum Medii densitas in locis singulis, quæ faciat ut corpus in data quavis linea curva moveatur, tum corporis velocitas & Medii resistentia in locis singulis.

Sit PQ planum illud plano Schematis perpendiculare; $PFHQ$ linea curva plano huic occurrens in punctis P & Q; G, H, I, K loca quatuor corporis in hac curva ab F ad Q pergentis; & GB, HC, ID, KE ordinatæ quatuor parallelæ ab his punctis ad horizontem demissæ & lineæ horizontali PQ ad puncta B, C, D, E infinitæ; & sint BC, CD, DE distantiæ Ordinatarum inter se æquales. A punctis G & H ducantur rectæ GL, HN curvam tangentes in G & H, & Ordinatis CH, DI sursum productis occurrentes in L & N, & compleatur parallelogrammum $HCDM$

This course aims to teach you more than simply a set of mathematical tools for Engineering. Our goal is to help you understand some of the ideas behind how these tools work, so that you will be better able to **use** them; to show you how to **apply** calculus to your other courses; and to teach you how to **write** a proper solution. This last idea is extremely important – you cannot convince anyone that you are a good engineer if they cannot understand how you reach your conclusions.

We will run this course as an inverted class – lecture videos will be available on Quercus and we will use class time to work on problems that will help us to understand the material, its applications, and its writing.

LEARNING OBJECTIVES

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

There are certain goals that we hope to achieve in this class, of which the ability to take limits and derivatives is only the first – and definitely not the greatest. We want to be able to see you develop facility in writing proper solutions, communicating your ideas well, and applying all of these to simple scenarios in the real world. Below is a list of the goals we find the most vital for this course.

- Use the standard algorithmic topics in the course (differentiate, calculate, integrate, etc…)
- Understand the syntax of a solution
    - Needs meaningful feedback
    - Identify and meaningfully use correct syntax
- Plan an approach to a solution, and identify different solution paths
- Communicate a mathematical solution [live and on exam]
- Analyze solution with respect to physical or mathematical constraints
- Apply mathematical tools to a real-world situation without cues for which tools to use

A NOTE REGARDING YOUR HEALTH

Engineering at the University of Toronto is a demanding program. The amount of work and the rate at which assignments and tests come and go can be overwhelming. Many students find themselves experiencing physical, mental, or fiscal health issues that stop them from demonstrating their ability and their learning.

If you have any problems that significantly affect your performance, please seek out the assistance of the various services offered at this university. These are generally free and confidential and may be able to help you show your full ability in your courses, as well as enjoy your time here.

Do look for these resources as soon as you suspect that there is a problem. Do not worry about the problem being too minor – let the professionals decide if they can help. Also, it has been my experience that a lot of students avoid seeking help because they feel that they should not need "unfair advantages." **The purpose of these services is not to give an unfair advantage, but to help remove an unfair disadvantage.** It may be difficult to ask for help, but it can make all the difference during your time here.

Some general resources:
My SSP: https://www.studentlife.utoronto.ca/hwc/myssp
Student Life Website: http://www.studentlife.utoronto.ca
Health and Wellness Website: http://studentlife.utoronto.ca/hwc
Your department's Academic Advisor: http://undergrad.engineering.utoronto.ca/advising-support-services/academic-advising/
Accessibility Services: http://www.studentlife.utoronto.ca/as

If, at some point during the year, you find yourself feeling distressed and in need of more immediate support, visit the Feeling Distressed Webpage: **http://www.studentlife.utoronto.ca/feeling-distressed** for more resources.

Off campus, immediate help is available 24/7 through **Good2Talk**, a post-secondary student helpline at **1-866-925-5454**.

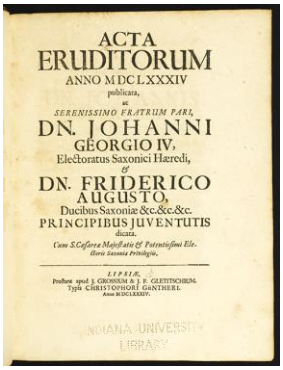
ACKNOWLEDGEMENT(S)

Mathematics is a subject that builds upon almost every element of its history. Numbers and operations created thousands of years ago, as the need grew for the ability to count, to add, and so on, are still used today in every possible context. Calculus as created in the 18th century is fundamentally what we use in this course.

Similarly, the land on which the University of Toronto is built has a rich history that we should not neglect. Humans have been on this land for over 15,000 years and their activities have helped to shape this unique city. The name Toronto is derived from a Wendat word that came to mean "meeting place," because of the frequent use of the area as grounds for nations to meet peacefully. The diversity of the modern city is an impressive reflection of its history.

The Toronto region is also a part of the Dish With One Spoon Wampum Belt Treaty. A wampum belt is made of two opposing-coloured shells and serves as a physical embodiment of the agreement between two peoples. To the left is a photo of a replica of the wampum belt that marks this particular treaty, in which multiple allied nations agreed to share the land and its resources equitably. The region involved in the treaty is vast, covering large sections of south Ontario, New York, Ohio, Michigan, and beyond. The metaphor of a dish with one spoon – the dish representing the area itself and the spoon representing our need to share it and to do so peacefully – is an ancient one, but its importance as we begin to see the effects of climate change is swiftly increasing.

TEXTBOOK

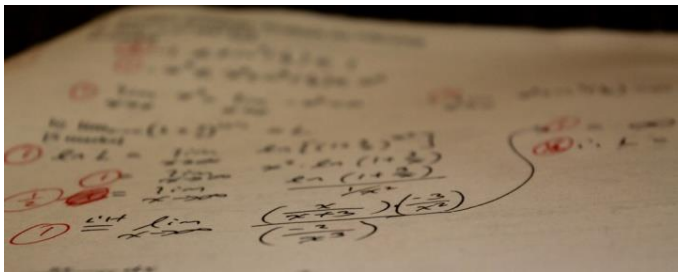

ACTA
ERUDITORUM
ANNO MDCLXXXIV
publicata,
SERENISSIMO FRATRUM PARI,
DN. JOHANNI
GEORGIO IV,
Electoratus Saxonici Hæredi,
&
DN. FRIDERICO
AUGUSTO,
Ducibus Saxoniæ &c.&c.&c.
PRINCIPIBUS JUVENTUTIS
dicata.
Cum S. Cæsarea Majestate & Potentissimi Electoris Saxonia Privilegio.
LIPSIÆ,
Prostant apud J. GROSSIUM & J. F. GLEITSCHIUM,
Typis CHRISTOPHORI GUNTHERI.
Anno MDCLXXXIV.
INDIANA UNIVERSITY
LIBRARY

The textbook we will use for the course is **Calculus, volumes 1 and 2** by **Edwin Herman and Gilbert Strang.** This is an open text (that is, a freely available electronic book), provided by Lyryx Learning. It can be found at:

https://openstax.org/details/books/calculus-volume-1
https://openstax.org/details/books/calculus-volume-2

If this book is not sufficient for your needs, the book **Calculus for Scientists and Engineers – Early Transcendentals**, by **Briggs, Cochran, and Gillett** is widely available on campus and is very good. We also recommend the textbooks by **Blank and Krantz** and by **Stewart**.

GRADES



Okay. This is about to get strange.

The grades in this course (other than the exam) will **not** be given in the usual manner. The next page contains a list of attributes that will be taught in the course – some are specific skills (e.g., knowing how to find an infinite limit), while others are more abstract (e.g., uses of geometry).

These attributes are divided into five categories:
Algebra basics; Mathematical writing; Calculus fundamentals; Calculus skills; and Higher math skills.

Each question and assignment during this semester will be given a score of 0 to 4 in some of these attributes (we will not let you know which ones we are looking for until after the due date, as identifying problems is an important skill). These scores are meant to show how well you demonstrated your ability in the question and are defined on the next page. Your final grade will depend on the marks you receive in the individual attributes.

The exact calculations are complex may be subject to some changes if we feel that they need to be adjusted to better reflect the class' understanding of the material. The basic idea is:

(a) We take a weighted average of your scores on any given attribute to get your **attribute score**, also on a 0 to 4 scale (for example, if the average is at least 0.75, then the score is a 1).
(b) We use the attribute scores in a single category to compute the **category score** (this can vary for each category, but you may need 80% of the attributes to be at a score of 1 for the category score to be a 1).
(c) The five category scores are used to compute your **final grade** by taking a weighted average.

ASSIGNMENTS

Quizzes – There will be weekly quizzes in the tutorials. The quizzes will be **cumulative** – they will test not only the new material, but also the major points of previous work. Some of this will give the opportunity not just to improve previous performance on some attributes, but to fully replace it.

Group assignments – Three of these, due on February 7, March 7, and April 7. All work **must** be submitted by 11:59pm (EST) on its due date, or else be given a mark of zero. Late work must have prior approval.

Final examination – During the regular exam period. The exam will be marked in the old-fashioned way and the mark averaged with the term mark of the other work (50% each).

There will be opportunity to present solutions during lectures and this can earn **bonus marks**.

MEANING OF SCORES*

Score	Category	Definition
4	Excellent	Strong evidence of original thinking; good organization, capacity to analyze and synthesize; superior grasp of subject matter with sound critical evaluations; evidence of extensive knowledge base.
3	Good	Evidence of grasp of subject matter, some evidence of critical capacity and analytic ability; reasonable understanding of relevant issues.
2	Adequate	Understanding of the subject matter; ability to develop solutions to simple problems in the material.
1	Marginal	Some evidence of familiarity with subject matter and some evidence that critical and analytic skills have been developed.
0	Inadequate	Evidence of familiarity with only some subject matter; presence of some critical and analytic skills, but not sufficient for subsequent course.

* Material in this table adapted from the Arts and Science calendar.

COURSE ATTRIBUTES (MORE MAY BE ADDED ON QUERCUS, AS NEEDED)

Algebra Basics

1. Factoring polynomials
2. Algebra of exponents
3. Logarithms
4. Absolute values (taking cases)
5. Trigonometry
6. Domains of functions
7. Inverses (existence, finding, graphing)
8. Sigma notation (and basic sums)

Calculus Fundamentals

1. Basic limits
2. Limits to infinity
3. Trigonometric limits
4. Derivatives from limits
5. Tangent lines
6. Basic derivatives (product, quotient, chain rules)
7. Implicit differentiation
8. Diagrams for word problems
9. Getting equations from diagrams
10. Simple related rates
11. Finding extrema (local, absolute)
12. Intervals of increase/decrease
13. Concavity
14. Graphing polynomials
15. Simple optimization
16. Simple antiderivatives
17. Remembering to write +C
18. Riemann sums
19. Easy integrals using Riemann sums
20. Integral properties
21. FTC
22. Simple substitution
23. Solving separable DEs
24. Integral as net change
25. Area between curves
26. Simple applications of integrals

Calculus Skills

1. Challenging limits
2. Squeeze theorem
3. Asymptotes
4. Continuity (including ID of discontinuities)
5. IVT
6. Leibniz notation (especially for chain rule)
7. Advanced related rates
8. Logarithmic differentiation
9. Corners/cusps
10. Adv. graphing
11. Adv. optimization
12. Linear approximation
13. L'Hopital's rule (including when not to use)
14. L'Hopital for exponents
15. Advanced substitution
16. Modeling DEs
17. Volumes of rotation
18. Advanced applications of integrals

Mathematical Writing

1. Reasonably neat penmanship
2. Clear progression through solution
3. Proper use of English (especially word problems)
4. Interpreting solutions

Higher Math Skills

1. Use of theorems to work efficiently
2. Use of geometry to simplify problems
3. Correct use of units
4. Identify when a method fails (e.g., linear approximation)
5. Combining topics
6. Applying calculus to other courses

CALCULATORS

While you can easily disregard this advice, I recommend not using calculators during this course. Except for certain, obvious places, the arithmetic we do will be of the kind that it is better for you to be able to do yourself. It will give you a better understanding of the material and, once you have enough practice, will be faster.

Calculators will not be allowed in the quizzes and exam. Slide rules are acceptable. Really.

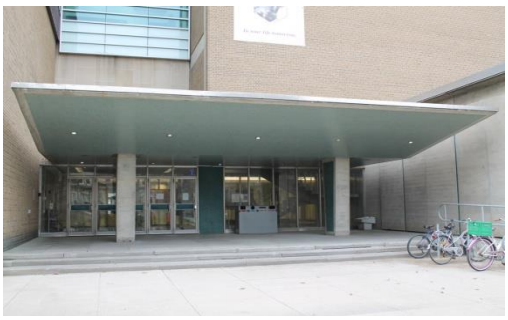
COURSE SCHEDULE - TENTATIVE

This schedule is subject to change. If we feel that the class needs more work in a certain field, we will adjust this as needed. For each section, we recommend doing as many exercises from the textbook as it takes to have a solid understanding of the different uses of the material in each section.

For each of the main parts of the course, we have included an example of a type of question that we might ask (except for the first one – I just think it's funny).

WEEK	SECTIONS	NOTES
Functions	 Notice the interesting maximum allowed height noted in this sign from the Boston Commons. Discuss briefly why using one-to-one functions for notation is important in engineering.	
Jan. 13 – 17	Chapter 1 **Do as many exercises as you need.** **Then, do a few more.**	
Jan. 20 – 24	Chapter 1 **Do as many exercises as you need.** **Then, do a few more.**	**Quiz 1**
Limits	   Three examples of 1-point perspective (one from Boston and two here at UofT) – the parallel lines in the picture all aim at a single point. Explain how 1-point perspective occurs in photos.	
Jan. 27 – 31	2.1-3, 4.6, 2.4, 3.1	**Quiz 2**

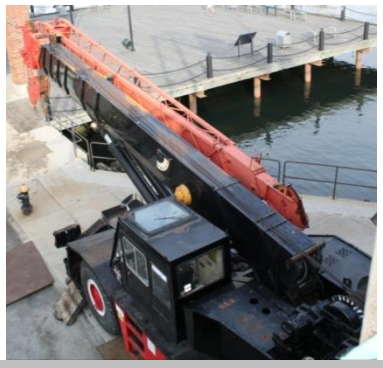
Derivatives



This is the canopy at the entrance to Lash Miller (the two figures in the left pictures are both UofT faculty members). How does the angle of the canopy affect the forces on the structure? How about when snow accumulates above?

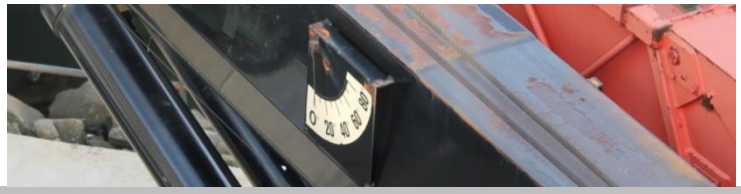
Feb. 3 – 7	4.6, 2.4, 3.1 – 3.3	**Quiz 3** **Project 1**
Feb. 10 – 14	3.4 – 3.9, 4.1 – 4.4	**Quiz 4**
Feb. 24 – 28	4.2, 4.3, 4.7	**Quiz 5**
Mar. 3 – 7	4.4 – 4.6, 4.8 – 4.10	**Quiz 6** **Project 2**

Integrals and Differential Equations



Note the inclinometer on the crane (pictured below).

How much work is required to lift the crane a further 20°?



Mar. 10 – 14	5.1 – 5.3	**Quiz 7**
Mar. 17 – 21	5.4 – 5.7, 6.1	**Quiz 8**
Mar. 24 – 28	6.2 – 6.6	**Quiz 9**
Mar. 31 – Apr. 7	**CV2:** 4.1 – 4.3	**Quiz 10** **Project 3**

PHOTO CREDITS

Bust of Newton – https://www.physics.utoronto.ca/~jharlow/teaching/phy151f13/
Page from the *Principia Mathematica* - http://newton.edwardworthlibrary.ie/books-by-newton/principia/
TI-30 calculator - https://en.wikipedia.org/wiki/Slide_rule
Wampum belt – http://archaeologymuseum.ca/wampum/
All others – Shai Cohen