UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATIONS, APRIL 2005
MAT 188 S – LINEAR ALGEBRA. FIRST YEAR: T-PROGRAM
EXAMINER: FELIX J. RECIO

INSTRUCTIONS:

1. ATTEMPT ALL QUESTIONS.
2. SHOW AND EXPLAIN YOUR WORK IN ALL QUESTIONS.
3. GIVE YOUR ANSWERS IN THE SPACE PROVIDED. USE BOTH SIDES OF PAPER, IF NECESSARY.
4. DO NOT TEAR OUT ANY PAGES.
5. USE OF NON-PROGRAMMABLE POCKET CALCULATORS, BUT NO OTHER AIDS ARE PERMITTED.
6. THIS EXAM CONSISTS OF SEVEN QUESTIONS. THE VALUE OF EACH QUESTION IS INDICATED (IN BRACKETS) BY THE QUESTION NUMBER.
7. THIS EXAM IS WORTH 50% OF YOUR FINAL GRADE.
8. TIME ALLOWED: 2 ½ HOURS.
9. PLEASE WRITE YOUR NAME, YOUR STUDENT NUMBER, AND YOUR SIGNATURE IN THE SPACE PROVIDED AT THE BOTTOM OF THIS PAGE.

PLEASE DO NOT WRITE HERE

QUESTION NUMBER	QUESTION VALUE	GRADE
1	12	
2	16	
3	12	
4	12	
5	14	
6	20	
7	14	
TOTAL:	100	

NAME: ______________________ , ______________________
(FAMILY NAME. PLEASE PRINT.) (GIVEN NAME.)

STUDENT No.: ______________________ **SIGNATURE:** ______________________

1. a) (5 marks) Let $A = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$. Compute $A\,(2B - C^T)$.

b) (7 marks) Let $M = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & 2 & -1 \end{bmatrix}$. Find all matrices X, if any, such that $MX = M^T + X$.

2. a) (8 marks) Consider the linear system $\begin{cases} x + y + 2z = 0 \\ x - y + kz = 4 \\ x + 2y + 4z = k \\ 2x + y + z = 3 \end{cases}$.

Find all the possible values of the constant k, if any, for which the given system has a unique solution and find the corresponding unique solution in each case.

b) (8 marks) Solve the linear system $\begin{cases} a + b + d + 2e = 1 \\ 2a + 2b - d + e = -1 \\ a + b + c + e = -1 \\ c + d + e = 0 \\ a + b - c - d = 0 \end{cases}$.

3. a) (6 marks) Consider the linear system $\begin{cases} 2x & + & 3y & + & z & = & 0 \\ 3x & - & 2y & - & z & = & 1 \\ 4x & + & 5y & + & 2z & = & -1 \end{cases}$.

Use Cramer's Rule to solve this system for z, without solving for the other two variables x and y.

b) (6 marks) Find all the values of b, if any, for which $det\begin{bmatrix} 0 & b & 1 & 1 \\ 1 & b & 0 & 1 \\ b & 1 & b & 1 \\ b & b & 2b & 1 \end{bmatrix} = 1$.

4. Consider the matrix $A = \begin{bmatrix} 5 & 3 & -3 \\ 0 & 2 & 0 \\ 6 & 6 & -4 \end{bmatrix}$.

a) (6 marks) Find all the eigenvalues of the matrix A.

b) (6 marks) Find an invertible matrix P and a diagonal matrix D such that $P^{-1}A\,P = D$.

5. a) (6 marks) Let $\vec{a} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$ and $\vec{c} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$. Find the angle between the vectors $\vec{a}$ and $\vec{b} \times \vec{c}$.

b) (8 marks) Let Π denote the plane that passes through the point $(-1, 0, 3)$ and is perpendicular to the line with scalar equations $\begin{cases} x = 4 - t \\ y = 5 + t \\ z = 7 - 2t \end{cases}$. Find the coordinates of the point on the plane Π closest to the point $(3, -4, 5)$.

6\. a) (6 marks) Let U be the set consisting of all vectors $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, where s is any real number.

Is U a subspace of $\mathbb{R}^3$? Why or why not?

b) (6 marks) Consider the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$.

Express the vector $\vec{w}$, if possible, as a linear combination of the vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$ and $\vec{v}_4$.

c) (8 marks) Find the dimension and a basis for the null space of the matrix $\begin{bmatrix} 1 & -1 & -1 & 0 & 1 \\ 2 & -2 & -1 & 1 & 1 \\ -1 & 1 & 2 & 1 & -2 \end{bmatrix}$.

7. a) (6 marks) Find an orthogonal basis for the subspace of $\mathbb{R}^4$ spanned by the vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix} \text{ and } \vec{v}_3 = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 5 \end{bmatrix}.$$

b) (8 marks) Find the best approximation to a solution of the inconsistent system $\begin{cases} x + y = 1 \\ x + 2y = 0 \\ -x + 2y = -1 \end{cases}$.