University of Toronto

Faculty of Applied Science and Engineering

Department of Mechanical and Industrial Engineering

FINAL EXAMINATION

April 20, 2007 9:30 a.m.

Exam Duration: 2.5 hours

First Year – Mechanical and Industrial Engineering

MIE100 – Dynamics

Calculator Type: 3

Exam Type: C

Examiners:

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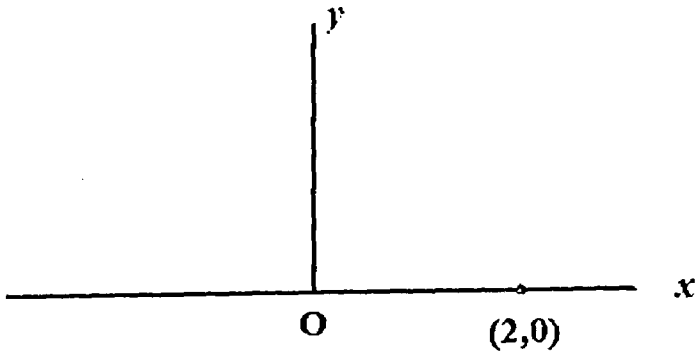
ANSWER ALL FIVE QUESTIONS

Question 1

At t= 0 a particle is located at (2,0) on a rectangular set of axes. Thereafter, its location is $(2, -\frac{t^3}{6})$ where t is in seconds and (x,y) are in meters.



a) What is the particle's velocity at t=2 seconds? Use the rectangular axes as given. (4 marks)

b) What is the particle's velocity at t=3 seconds? Use normal and tangential axes. (4 marks)

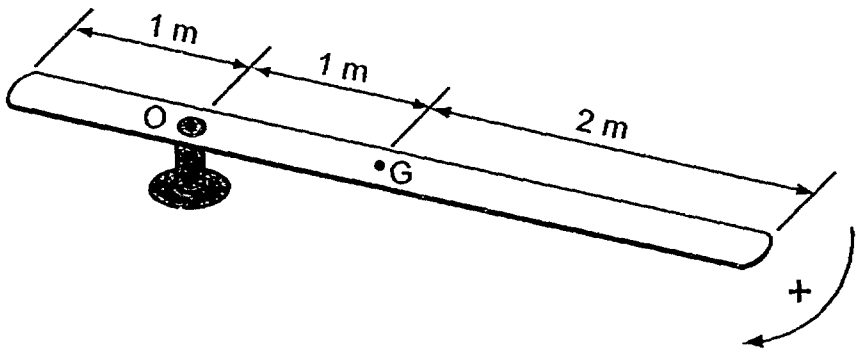
c) What is the particle's acceleration at t=4 seconds? Use polar co-ordinates centered at O. (6 marks)

d) If a second particle starts with zero velocity at (2,0) and its acceleration is $\vec{a} = +\frac{t^3}{6}\hat{j}$, then how far has it traveled in 5 seconds? (6 marks)

Question 2

A thin, uniform rod of mass 3 kg and length 4 m rotates in the *horizontal* plane about a pivot point, O. Kinetic friction between the rod and its pivot points results in a constant frictional moment about point O of unknown magnitude. The rod is initially at rest.

At t=0, a motor is turned on. The motor applies a constant clockwise moment of 100 N·m to the rod about point O. At t=3 seconds, the motor is turned off, and from that point the rod continues to rotate for another 45° before it comes to a stop.



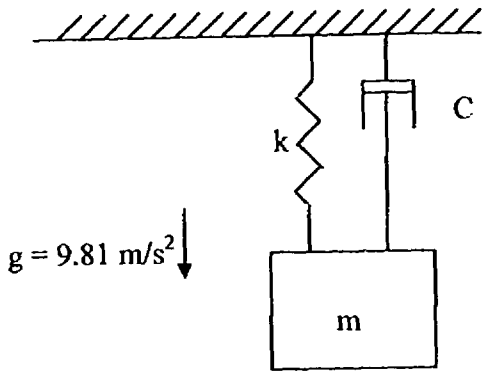
a) Determine the angular velocity when the motor is turned off at t=3 seconds. (8 marks)

b) Determine the magnitude of the moment due to friction. (8 marks)

c) Determine the total work done on the rod by non-conservative forces from t=0 until the rod stops rotating. (4 marks)

Question 3

The mass m=10 kg is suspended from the ceiling by a spring of stiffness k=90 N/m, in parallel with a dashpot of strength C. The mass moves vertically, without rotation.



a) For what range of values of C is it possible for the mass to experience damped, unforced oscillations? (4 marks)

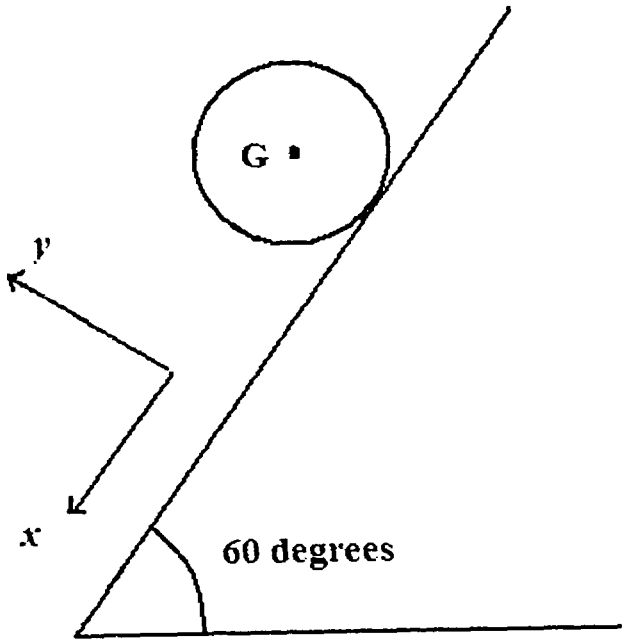
b) Find the approximate value of C required such that the amplitude of unforced oscillations decreases by 1% in each cycle. (Note that for this very lightly damped system $\omega_n \approx \omega_d$.) (4 marks)

c) Suppose the mass undergoes unforced oscillations of amplitude 0.2 meters, with dashpot C=0. What are the maximum and minimum values of kinetic energy of mass in each cycle? (4 marks)

d) Suppose the mass undergoes oscillations of amplitude 0.2 meters, with dashpot C=0. What are the maximum and minimum values of potential energy of the spring in each cycle? (4 marks)

e) An earthquake makes the ceiling oscillate vertically with amplitude of 8mm and a frequency of 4 cycles/second. Assume C=0. What is the amplitude of vibration of the mass? (4 marks)

Question 4

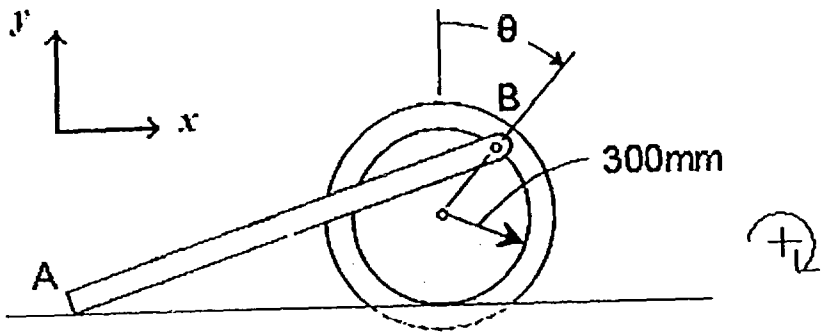


A 10-kg wheel with radius 200mm and radius of gyration of 180mm about its centre G, is released from rest on a surface which is inclined 60 degrees from the horizontal line. The wheel slips as it rotates. The kinetic coefficient of friction (μ_k) between the wheel and the inclined surface is 0.3. Use rectangular co-ordinates aligned with the incline as shown.

a) Determine the acceleration $\bar{a}_G$ of the centre G of the wheel. (10 marks)

b) Determine the wheel's angular acceleration (α). (10 marks)

Question 5



The wheel rolls to the right on its inner hub without slipping. Its center has a constant velocity of 1.5 m/s. The length of rod AB is 1.2 m.

a) Determine the angular velocity of rod AB (ω_{AB}) when $\theta = 0^\circ$ (5 marks)

b) Determine the angular velocity of rod AB (ω_{AB}) when $\theta = 90^\circ$ (5 marks)

c) At the instant when $\theta = 90^\circ$ determine the acceleration of point B. Use the rectangular co-ordinates given. (5 marks)

d) At the instant when $\theta = 90^\circ$ determine the acceleration of point A. Use the rectangular co-ordinates given. (5 marks)