

Algebra Solutions:

1. $\frac{11}{14} - \frac{12}{7} =$

[A] $\frac{-1}{14}$

[B] $\frac{5}{14}$

[C] $\frac{-1}{7}$

[D] $\frac{-13}{14}$

2. $\sqrt{1764}$ is

[A] an integer

[B] a number with finitely many decimal places

[C] an infinite non-repeating decimal

[D] an infinite repeating decimal

3. $6 \left(\frac{3 - 2(-1)^2}{\frac{14}{3} - (-5)} \right)^{-1} =$

[A] $\frac{18}{29}$

[B] 58

[C] 2

[D] 18

4. $\frac{9x^2y}{3x^7y^{-3} + 9x^2y} =$

[A] $\frac{3y^4}{x^5} + 1$

[B] $\frac{3y^4}{x^5 + 3y^4}$

[C] $\frac{3}{x^5y^{-4} + 1}$

[D] $\frac{3}{x^9y^{-4} + 3}$

Divide all of the terms on top and bottom by $3x^2$ and then multiply by y^3 .

5. not graded

6. If $y = 2x + 5$ and also $y = 2 - x$, then $(x, y) =$

A (1, 7)

B (3, -1)

C (-1, 3)

D (7, 1)

7. When $x = -8$, $(x)^{\frac{-2}{3}} - (x)^{\frac{1}{3}} + (x)^0 =$

A 7

B 3

C $\frac{13}{4}$

D Does not exist

$$\text{When } x = -8, \quad (-8)^{\frac{-2}{3}} = \frac{1}{((-8)^{\frac{1}{3}})^2} \quad \text{and} \quad (-8)^{\frac{1}{3}} = -2.$$

8. The region described by both $y \leq 3x + 5$ and $y \geq 2 - x$ contains the point:

A (1, 9)

B (0, 0)

C (3, -1)

D (4, 1)

Both solutions are correct.

9. $1 \leq |x| \leq 2$ describes the same x -values as

A $[1, 2]$

B $[-2, 2] \cup [-1, 1]$

C $[-2, -1] \cup [1, 2]$

D $(-\infty, -2] \cup [-1, 1] \cup [2, \infty)$

$|x| \leq 2$ implies $-2 \leq x \leq 2$ and $|x| \geq 1$ implies that $x \leq -1$ and $x \geq 1$.

10. not graded

Function Solutions:

11. $2c^2(3abc - 4ab) + 7abc - abc^2 - 5abc^3 =$

- [A] $6abc^3 - 2abc^2$ [B] $abc^3 - 5abc^2 + 7abc$ [C] $abc^3 - 6abc^2 + 7abc$ [D] $abc^3 - 9abc^2 + 7abc$

12. If $f(x) = \frac{|x-2|^{\frac{1}{2}}}{x-4} =$, then $f(-2) =$

- [A] $\frac{1}{3}$ [B] $\frac{-1}{3}$ [C] 0 [D] $\frac{1}{6}$

13. The function $f(x) = 4x^2 + 4x + 1$ has

[A] exactly 1 distinct root

[B] exactly 2 distinct roots

[C] exactly 3 distinct roots

[D] no roots

Using the formula for the roots of a quadratic equation (“quadratic formula”), we get:
 $x = \frac{-4 \pm \sqrt{16-16}}{2} = -2.$

14. The factored form of $x^3 + 2x^2 - x - 2$ includes the following factor:

- [A] $(x-2)$ [B] $(x+2)$ [C] $(2x-1)$ [D] $(x-1)^2$

$(-2)^3 + 2(-2)^2 - (-2) - 2 = -8 + 8 + 2 - 2 = 0$ Therefore, $(x+2)$ is a factor.

15. The domain of $f(x) = \sqrt{3-x}$ is equal to the range of:

[A] $g(x) = -2(x+1)^2 - 3$.

[B] $g(x) = -5(x+1)^2 + 3$.

[C] $g(x) = (x+1)^2 - 3$.

[D] $g(x) = 2(x+1)^2 + 3$.

which is equal to $(-\infty, 3]$.

16. Let $f(x) = \frac{-2x+1}{x+3}$. Then $f^{-1}(x) =$

[A] $\frac{x-2}{3x+1}$ [B] $\frac{x-3}{x+2}$ [C] $\frac{3x-1}{x+2}$

[D] $\frac{1-3x}{x+2}$

We set $x = \frac{-2y+1}{y+3}$ and solve for y .

17. If $\frac{a}{b} - 6 = 5b$, then

[A] a is a function of b , but b is not a function of a .

[B] b is a function of a , but a is not a function of b .

[C] a is a function of b , AND b is a function of a .

[D] Neither variable is a function of the other.

Multiplying both sides by b we get: $a - 6b = 5b^2$ which is equivalent to $a = 5b^2 + 6b$. This is a parabola that passes the vertical line test when b is the variable, but not when a is.

18. If $f(x) = x^2 + 2x - 1$ and $g(x) = \frac{x}{\sqrt{x}}$, then $f(g(4)) =$

[A] 23

[B] 7

[C] $\sqrt{23}$

[D] $2\sqrt{2} + 3$

$f(g(4)) = f(2) = 7$

19. not graded

20. If $(1, 4)$ is on the graph of $y = f(x)$ then the graph of $y = f^{-1}(f(f^{-1}(x)))$ necessarily includes the point:

[A] $(1, 4)$ [B] $(-1, -4)$ [C] $(-4, -1)$ [D] $(4, 1)$

If $(1, 4)$ is on the graph of $y = f(x)$, then $(4, 1)$ is on the graph of $y = f^{-1}(x)$.

$$\text{So } f^{-1}(f(f^{-1}(4))) = f^{-1}(f(1)) = f^{-1}(4) = 1$$

Graphing Solutions:

21. The points $(-3, 1)$ and $(1, 9)$ are on the same line as

A $(0, 7)$ B $(0, 2)$ C $(-1, 4)$ D $(-1, 3)$

That line is given by the equation $y = 2x + 7$.

22. The set of points that are 4 units away from $(1, -2)$ form the graph of:

A $(x - 1)^2 + (y + 2)^2 = 16$

B $(x - 1)^2 + (y + 2)^2 = 4$

C $(x + 1)^2 + (y - 2)^2 = 2$

D $(x + 1)^2 + (y - 2)^2 = 4$

23. The vertex of the graph of $y = -2x^2 + 12x + 1$ is:

A $(-3, 19)$ B $(36, 8)$ C $(36, 19)$ D $(3, 19)$

$$y = -2x^2 + 12x + 1 = -2(x^2 - 6x) + 1 = -2(x^2 - 6x + 9 - 9) + 1$$

$$= -2(x^2 - 6x + 9) + 18 + 1 = -2(x - 3)^2 + 19 \text{ which has vertex } (3, 19).$$

24. A parabola with vertex $(1, -4)$ has an axis of symmetry given by the equation:

A $x = -4$ B $y = -4$ C $y = 1$ D $x = 1$

25. To transform the graph of $y = \sqrt{x}$ into the graph of $y = 3\sqrt{x - 2}$, we can

A stretch the graph by a factor of 3 in the x -direction and shift 2 units to the left.

B stretch the graph by a factor of 3 in the y -direction and shift 2 units to the left.

C stretch the graph by a factor of 3 in the x -direction and shift 2 units to the right.

D stretch the graph by a factor of 3 in the y -direction and shift 2 units to the right.

26. The graph of $y = 2x - 1$ intersects the graph of $x^2 + y^2 = 9$ at
- A exactly one point.
 B two distinct points.
 C no points but they pass within one unit of each other.
 D no points and they stay farther than one unit away from each other.

The line includes the point $(0, -1)$ which is strictly inside the circle. Therefore the graphs have to intersect when the line enters the circle and again when it exits.

27. The graph of $y = \frac{1}{x + \pi}$
- A intersects both the x -axis and y -axis.
 B intersects neither of the x -axis and y -axis.
 C intersects the x -axis, but not the y -axis.
 D intersects the y -axis, but not the x -axis.

$$y \neq 0.$$

28. If the point $(3, 4)$ is on the graph of $y = f(x)$, then the point $(-3, -4)$ is necessarily on the graph of

- A $y = f(-x)$ B $y = -f(-x)$ C $y = -f(x)$ D $y = f(x)$

29. not graded

30. The graph of $3(x - 5)^2 + (y + 3)^2 = 6$ is

- A a circle B an ellipse C a parabola D an hyperbola

$$\frac{(x - 5)^2}{2} + \frac{(y + 3)^2}{6} = 1$$

Trigonometry Solutions:

31. $\cos\left(\frac{\pi}{3}\right)$

[A] $\frac{1}{\sqrt{3}}$

[B] $\frac{\sqrt{3}}{2}$

[C] $\frac{1}{\sqrt{2}}$

[D] $\frac{1}{2}$

32. $\tan\left(\frac{n\pi}{3}\right) \geq 0$ when $n =$

[A] 1, 3, 5

[B] 2, 4, 6

[C] 1, -1, 0

[D] 1, 4, 7

33. $\sin\left(\frac{17\pi}{4}\right) =$

[A] $\sin\left(\frac{\pi}{4}\right)$

[B] $\sin\left(\frac{5\pi}{4}\right)$

[C] $\sin\left(\frac{7\pi}{4}\right)$

[D] $\sin\left(\frac{13\pi}{4}\right)$

34. $\sin(-\theta) - \cos(-\theta) =$

[A] $-\sin(\theta) - \cos(\theta)$ [B] $\sin(\theta) - \cos(\theta)$ [C] $\cos(\theta) - \sin(\theta)$ [D] $\sin(\theta) + \cos(\theta)$

35. The radian measure of 15° is

[A] $\frac{\pi}{3}$ [B] $\frac{\pi}{4}$ [C] $\frac{\pi}{12}$ [D] $\frac{\pi}{24}$

$$15 \times \frac{\pi}{180} = \frac{\pi}{12}.$$

36. not graded

37. Let $f(x) = 3\cos(2x - 4) + 5$. The y -values of the graph of $y = f(x)$ are all between

A -3 and 3

B 2 and 8

C -7 and -1

D 3 and 5

The interval from -3 to 3 gets shifted upwards by 5 to get the interval from 2 to 8.

38. not graded

39. Let triangle ABC have a side of length $a = 6$ that is positioned across from angle $A = 30^\circ$. Let b be the side facing angle B . Then the value of the expression $\frac{\sin B}{b}$ is

A $\frac{1}{5}$

B 3

C $\frac{1}{12}$

D indeterminate. We do not have enough information.

This ratio is equal to $\frac{\sin A}{a}$ by the Sine Law.

40. A building has a 25m shadow. A line between the top of the building and the end of the shadow makes an angle of $\frac{\pi}{3}$ with the ground.

From this we can conclude that the height of the building is

A 25

B $25\sqrt{3}$

C 50

D $\frac{25}{3}$

$\tan \frac{\pi}{3} = \sqrt{3}$ This is also equal to $\frac{x}{25}$, where x is the height of the building.

Exponential and Logarithm Solutions:

41. $5 \ln(e^2) =$.

- [A] 0 [B] $5 \ln(2e)$ [C] 7 [D] 10

$5 \ln(e^2) = 5(\ln e)^2 = 10 \ln e = 10(1) = 10.$

42. If $A > 0$ and $B > 0$ then $\ln(A + B) =$

- [A] $(\ln A)(\ln B)$
[B] $\ln A + \ln B$
[C] $\frac{\ln A}{\ln B}$
[D] no other expression. This expression does not simplify.

43. If $2^w = 7$, then

- [A] $\log_2 w = 7$ [B] $\log_2 7 = w$ [C] $\log_7 w = 2$ [D] $\log_7 2 = w$

44. $\log_5\left(\frac{1}{125}\right) =$

- [A] $\frac{1}{3}$ [B] $\sqrt{3}$ [C] -3 [D] 0.3

45. The domain of the function $f(x) = \ln|x|$ is

- [A] $(-\infty, \infty)$ [B] $(0, \infty)$ [C] $(-\infty, 0)$ [D] $(-\infty, 0) \cup (0, \infty)$

46. The equation $\ln|x^2 + 4x - 21| - \ln|x + 7| = 1$ has

- A 1 solution B 2 solutions C 3 solutions D no solution

$$\ln\left|\frac{x^2 + 4x - 21}{x + 7}\right| = 1 \text{ which implies that } \ln\left|\frac{(x+7)(x-3)}{(x+7)}\right| = 1.$$

If $\ln(x-3) = 1$, then $e^{\ln(x-3)} = e^1$. So $x-3 = e$ and $x = e+3$ which turns $x^2 + 4x - 21$ and $x+7$ into positive expression. Therefore, the original equation is well defined and true.

47. The graphs of the function $f(x) = e^x$ and $g(x) = \ln x$

- A intersect only once.
 B intersect once on either side of the y -axis.
 C intersect once on either side of the x -axis.
 D do not intersect.

48. The solutions to $e^{-x}(x^2 + 3x - 40) \leq 0$ are contained inside the interval

- A $(-\infty, -8] \cup [5, \infty)$ B $[-8, 5]$ C $(-\infty, -5] \cup [8, \infty)$ D $[-5, 8]$

e^{-x} is always positive. $x^2 + 3x - 40 = (x+8)(x-5)$. This is less than or equal to zero between the roots $x = -8$ and $x = 5$.

49. If $2^{3x+1} = 8^{2x-1}$ then $x =$

- A -2 B $\frac{4}{3}$ C 5 D There is no solution to this equation.

$2^{3x+1} = (2^3)^{2x-1}$ which is equivalent to $2^{3x+1} = 2^{6x-3}$.

50. The following expression is equal to 2:

- A $-\log_2 4$ B $(e^{\ln 1})^2$ C $\log_2 \sqrt{2}$ D $e^{-\ln \frac{1}{2}}$

$$e^{-\ln \frac{1}{2}} = e^{\ln \frac{1}{2}^{-1}} = e^{\ln 2} = 2$$