Faculty of Applied Science and Engineering
University of Toronto

MAT 188H1S – Linear Algebra
MONDAY, APRIL 14, 2014

FINAL EXAMINATION

LAST NAME: ____________________

FIRST NAME: ____________________

STUDENT NUMBER: ____________________

SIGNATURE: ____________________

Time allowed: 2 hours, 30 minutes
Total marks: 70
No calculators allowed.
Examiner: S. Cohen

Use the backs of pages when necessary,
indicating clearly where solutions continue.

FOR MARKER'S USE ONLY	
QUESTION	MARK
1	**/ 10**
2	**/ 10**
3	**/ 15**
4	**/ 15**
5	**/ 10**
6	**/ 10**
TOTAL	**/ 70**

1. [10 marks] Let $A = \begin{bmatrix} 0 & 2 & 4 & 1 & 1 & 0 \\ 0 & -1 & -2 & 2 & -3 & 1 \\ 0 & 1 & 2 & 1 & 0 & -1 \\ -0 & 2 & 4 & -1 & 3 & 2 \end{bmatrix}$.

Find the rank of A and determine bases for its row, column, and null spaces.

2. [10 marks] Solve the initial value problem:

$$f_1' = 2f_1 + 4f_2 \quad f_1(0) = 0$$
$$f_2' = 3f_1 + 3f_2 \quad f_2(0) = 1$$

3. Let $U = \operatorname{span}\left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 1 \\ -4 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix} \right\}$.

(a) [9 marks] Find an orthogonal basis for U.

(b) [4 marks] Let $X = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -2 \end{pmatrix}$.

Write X as the sum of two vectors, one from U and the other from $U^{\perp}$.

(c) [2 marks] Extend the basis from (a) into an orthogonal basis of $\mathbb{R}^4$.

4. [15 marks] Let $A = \begin{bmatrix} -1 & -2 & 2 \\ -2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$. Find an **orthogonal matrix** P and a diagonal matrix D such that $A = PDP^{-1}$.

[If you do not remember how to do this, you can simply diagonalize the matrix for part marks]

[More room on the next page]

[Extra room for the previous question]

5. [10 marks] Let $A = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \middle| \, 2a + b = c \right\}$. Show that A is a subspace of $\mathbb{R}^3$ and find a basis for it.

6. (a) [5 marks] Find a change of basis matrix to move from the basis $\left\{\begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}3\\-1\end{pmatrix}\right\}$ to $\left\{\begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}1\\-1\end{pmatrix}\right\}$.

(b) [5 marks] Let A be $n \times m$. If $A^3 = I$, show that $\text{rank}\,(A) = n$.