UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 2004
First Year - CIV, CHE, IND, LME, MEC, MMS

MAT 186H1F - CALCULUS I
Exam Type: A

Examiners

I. Alexandrova

D. Burbulla

E. Lawes

G. D. Lynch

SURNAME _____

GIVEN NAME _____

STUDENT NO. _____

SIGNATURE _____

INSTRUCTIONS:

Non-programmable calculators permitted.

Answer all questions.

Present your solutions in the space provided;
use the back of the **preceding** page if more
space is required.

TOTAL MARKS: 100

The value for each question is shown in
parentheses after the question number.

MARKER'S REPORT	
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Q7	
Q8	
Q9	
Q10	
TOTAL	

1. [10 marks; 5 marks for each part.] Find the following:

(a) $\dfrac{d}{dx}\left(e^{-3x}\sin(4x)\right)$

(b) $\dfrac{dy}{dx}$ at the point $(x, y) = (1, -1)$ if

$$\sin^{-1}(y^2 - x) = x^2 y + 1$$

2. [10 marks: 5 marks for each part.] Find the following:

(a) $F(x)$, if $F'''(x) = 1 + x$ and $F(0) = -1, F'(0) = 3$.

(b) $\int_1^{10} \frac{e^{-1/x}}{x^2} \, dx$

3. [8 marks; 4 marks for each part] The parts of this question are unrelated.

(a) Find the maximum and minimum values of $f(x) = x^{1/3} + 2x^{4/3}$ on the closed interval $[-1, 1]$.

(b) Four formulas are displayed below, in the column on the left. If the formula is incorrect, put the corrected formula in the corresponding space in the column on the right; if the formula is correct, leave the corresponding space on the right blank.

	Formula	Correction (if necessary)
(i)	$\int_b^a F'(x)\,dx = F(b) - F(a),$ if F' is a continuous function	
(ii)	$\dfrac{d}{dx}\left(\int_4^{x^2} f(z)\,dz\right) = 2xf(x),$ if f is a continuous function.	
(iii)	$\int\left(\dfrac{d}{dx}\sin\sqrt{x}\right)\,dx = \sin\sqrt{x}$	
(iv)	$\int \dfrac{1}{x}\,dx = \ln x + C$	

4. [12 marks; 2 marks for each part] Let $f(x) = \sqrt{x^2 - 4x} + x$. Decide if the following statements about f are True or False:

(a) f is continuous for all values of x.   True or False

(b) f has no critical points.   True or False

(c) f has no inflection points.   True or False

(d) There are no horizontal asymptotes
    to the graph of f.   True or False

(e) There are no vertical asymptotes
    to the graph of f.   True or False

(f) There are no vertical tangents
    to the graph of f.   True or False

5. [10 marks] Find the following limits:

(a) [4 marks] $\lim\limits_{x \to 0} \dfrac{x - \sin x}{x^3}$

(b) [6 HARD marks] $\lim\limits_{x \to \infty} \left(\dfrac{\pi}{2} - \tan^{-1} x \right)^{1/\ln x}$

6. [10 marks] The velocity of a particle at time, t, is given by $v = t^2 - 5t + 6$, for $0 \leq t \leq 3$. Find:

(a) [4 marks] the average acceleration of the particle.

(b) [6 marks] the average speed of the particle.

7. [10 marks] The region R in the plane is bounded by the three curves with equations

$$y = \sqrt{x}, y = -x \text{ and } y = 2.$$

Find the volume of the solid obtained by rotating the region R around the x-axis.

8. [10 marks] The killer whale's tank at the Sea Life Aquarium is in the shape of a solid of revolution obtained by rotating the ellipse with equation

$$\frac{x^2}{30^2} + \frac{y^2}{15^2} = 1$$

around the y-axis, for $-15 \leq y \leq 0$, where distances are measured in metres.

Periodically the water in the tank must be completely pumped out and replaced with clean water. How much work is done in pumping all the water in the tank up to the top of the tank and out? (You may assume the acceleration due to gravity is $g = 9.8$ m/sec^2 and that the density of water is $\rho = 1000$ kg/m^3. Also, assume all whales have been removed from the tank before the pumping begins, and that the tank is full of water.)

9. [10 marks] Find the area of the region bounded by the curves with equations

$$y = x^3 \text{ and } y = 6x - x^2.$$

10. [10 marks] Find the length of the curve $y = \dfrac{1}{2}x^2 - \dfrac{1}{4}\ln x$, for $1 \le x \le e$.