MAT186 Calculus I
Term Test 1

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Instructions:

1. This test contains a total of 13 pages.
2. DO NOT DETACH ANY PAGES FROM THIS TEST.
3. There are no aids permitted for this test, including calculators.
4. Cellphones, smartwatches, or any other electronic devices are not permitted. They must be turned off and in your bag under your desk or chair. These devices may **not** be left in your pockets.
5. Write clearly and concisely in a linear fashion. Organize your work in a reasonably neat and coherent way.
6. Show your work and justify your steps on every question unless otherwise indicated. A correct answer without explanation will receive no credit unless otherwise noted; an incorrect answer supported by substantially correct calculations and explanations may receive partial credit.
7. For questions with a boxed area, ensure your answer is completely inside the box.
8. **The back side of pages will not be scanned nor graded**. Use the back side of pages for rough work only.
9. You must use the methods learned in this course to solve all of the problems.
10. DO NOT START the test until instructed to do so.

GOOD LUCK!

Multiple Choice: No justification is required. Only your final answer will be graded.

1. The inequality $-2 \leq x \leq 8$ can be expressed in the form $|x + a| \leq b$, where $a =$ ______ and $b =$ ______. [1 mark]

Indicate your final answer by **filling in exactly one circle** below (unfilled ○ filled ●).

- ○ $a = 5$, $b = 3$.
- ○ $a = 3$, $b = 5$.
- ○ $a = -3$, $b = 5$.
- ○ $a = -5$, $b = 3$.

2. Let $f(x) = x^2 - 1$. Select *all* the numbers below that are upper bounds for $|f(x)|$ on the interval $[-\frac{3}{4}, \frac{1}{3}]$. [2 marks]

You can fill in more than one option for this question (unfilled ○ filled ●).

- ○ $\frac{9}{10}$.
- ○ $\frac{7}{16}$.
- ○ $\frac{3}{2}$.
- ○ $\frac{11}{20}$.
- ○ 1.

Multiple Choice: No justification is required. Only your final answer will be graded.

3. One of your professors (we're not naming names) has noticed that student's raw scores on a linear algebra test is a decreasing function of time they spend on their phones. This same professor noticed that, given a raw score, they could determine how much time a student spent on their phone. Which of the functions below could represent this professor's observations? [1 mark]

Indicate your final answer by **filling in exactly one circle** below (unfilled ○ filled ●).

- ○ $f(t) = 100\sin(t + 50) + 100$.
- ○ $g(t) = 100 - \dfrac{(t-50)^2}{25}$.
- ○ $h(t) = 100\, e^{\frac{-t^2}{50}}$.
- ○ $k(t) = 100 - 2^{-t}$.

4. Recall that $\sin^{-1}(x)$ is the inverse of $\sin(x)$ whose domain is restricted to $[-\frac{\pi}{2}, \frac{\pi}{2}]$, and $\cos^{-1}(x)$ is the inverse of $\cos(x)$ whose domain is restricted to $[0, \pi]$. We define a new inverse function as follows:

$$\text{zin}^{-1}(x) \text{ is defined to be the inverse of } \sin(x) \text{ whose domain is restricted to } [\tfrac{\pi}{2}, \tfrac{3\pi}{2}].$$

Fill in the blank: A solution to the equation $\cos(\text{zin}^{-1}(x)) = \sin(\cos^{-1}(x))$ *does not exist* when ______. [2 marks]

You can fill in more than one option for this question (unfilled ○ filled ●).

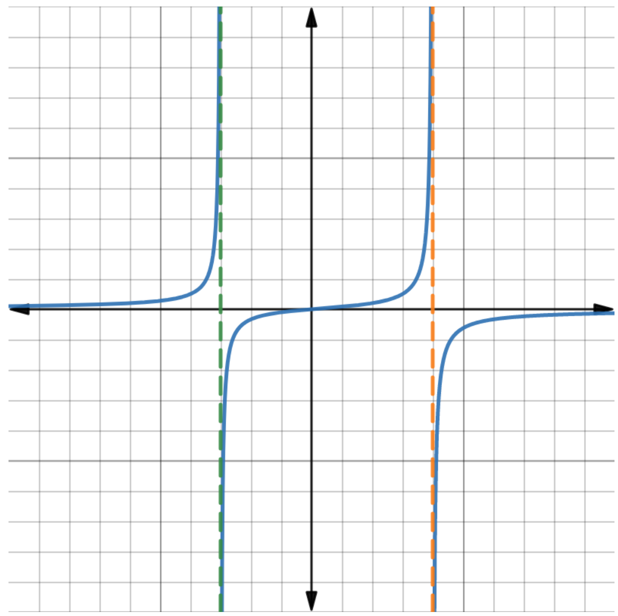
- ○ $x \in (-1, 0)$.
- ○ $x \in (0, 1)$.
- ○ $x = 1$.
- ○ $x = -1$.

Multiple Choice: No justification is required. Only your final answer will be graded.

5. Let a and b be positive constants, where $a \neq b$. The graph of a function $f(x)$ is pictured below, together with the vertical line $x = -a$, and the vertical line $x = b$. Which of the expressions below could represent the function $f(x)$? [1 mark]

Indicate your final answer by **filling in exactly one circle** below (unfilled ○ filled ●).

- ○ $f(x) = \dfrac{x}{(x-a)(x-b)}$.
- ○ $f(x) = \dfrac{x}{(x+a)(x-b)}$.
- ○ $f(x) = \dfrac{-x}{(x+a)(x-b)}$.
- ○ $f(x) = \dfrac{-x}{(x-a)(x-b)}$.



6. $\lim\limits_{x\to\infty} \left(\sqrt{x+\sqrt{x}} - \sqrt{x-\sqrt{x}}\right) =$ ______? [2 marks]

Indicate your final answer by **filling in exactly one circle** below (unfilled ○ filled ●).

- ○ -1
- ○ 0
- ○ 1
- ○ DNE ($\to \infty$)
- ○ DNE ($\to -\infty$)

Multiple Choice: No justification is required. Only your final answer will be graded.

7. Suppose that $f(x)$ and $g(x)$ are differentiable functions defined for all x, and satisfy the following properties:

- $f(-2) = g(-2)$
- $f'(x) < g'(x)$ for all x.

Which of the statements below are true? [2 marks]

You can fill in more than one option for this question (unfilled ○ filled ●).

○ $f(x) < g(x)$ for all $x > -2$.

○ $f(x) < g(x)$ for all $x < -2$.

○ The graphs of f and g do not intersect.

○ The graphs of f and g intersect at exactly one point.

○ The graphs of f and g intersect at more than one point.

8. Let $f(x)$ be differentiable for all x, and let $g(x)$ be the tangent line to f at a point $a \neq 0$. Which of the expression(s) below are equal to $f'(a)$? [2 marks]

You can fill in more than one option for this question (unfilled ○ filled ●).

○ $g'(a)$

○ $g(a+1) - g(a)$

○ $\lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$

○ $\frac{f(x) - f(a)}{x - a}$

○ $\lim_{h \to 0} \frac{f(a+h) - f(a-h)}{2h}$

True or False: Unsupported answers will not receive full credit. Organize your work in a reasonably neat and coherent way.

9. Let $f(x) = 2^x - \frac{10}{x}$. Then there exists a $c \in [-\frac{5}{2}, 3]$ such that $f(c) = 0$.

Indicate your final answer by **filling in exactly one circle** below (unfilled ○ filled ●).

○ True.

○ False.

[3 marks: 1 mark for correct final answer; 2 marks for explanation]

Short Answer: Unsupported answers will not receive full credit. Organize your work in a reasonably neat and coherent way. Put your final answer in the boxes provided.

10. Let $f(x)$ represent the fuel efficiency, in kilometres per litres, of a car travelling at a speed of x kilometres per hour.

(a) What are the units of $f'(x)$? No justification is required for this part. [1 mark]

(b) Suppose $f'(101) = 0.74$. If you are driving at 101 kilometres per hour, should you speed up or slow down to be more fuel efficient? Briefly explain. Enter either "speed up" or "slow down" in the box provided as your final answer.
[3 marks: 1 mark for correct answer; 2 marks for explanation]

Answer:

Short Answer: Unsupported answers will not receive full credit. Organize your work in a reasonably neat and coherent way. Put your final answer in the boxes provided.

11. Let

$$f(x) = \begin{cases} 5^{2x-2} + 2, & x \leq 1 \\ \dfrac{3x^2 + x + 5}{Ax^2 + 2}, & 1 < x < 2 \\ \ln(Bx) + 1, & x \geq 2 \end{cases}$$

where A and B are positive constants.

(a) Find all values of A ad B such that $f(x)$ is continuous at $x = 1$ and $x = 2$. [5 marks]

A=

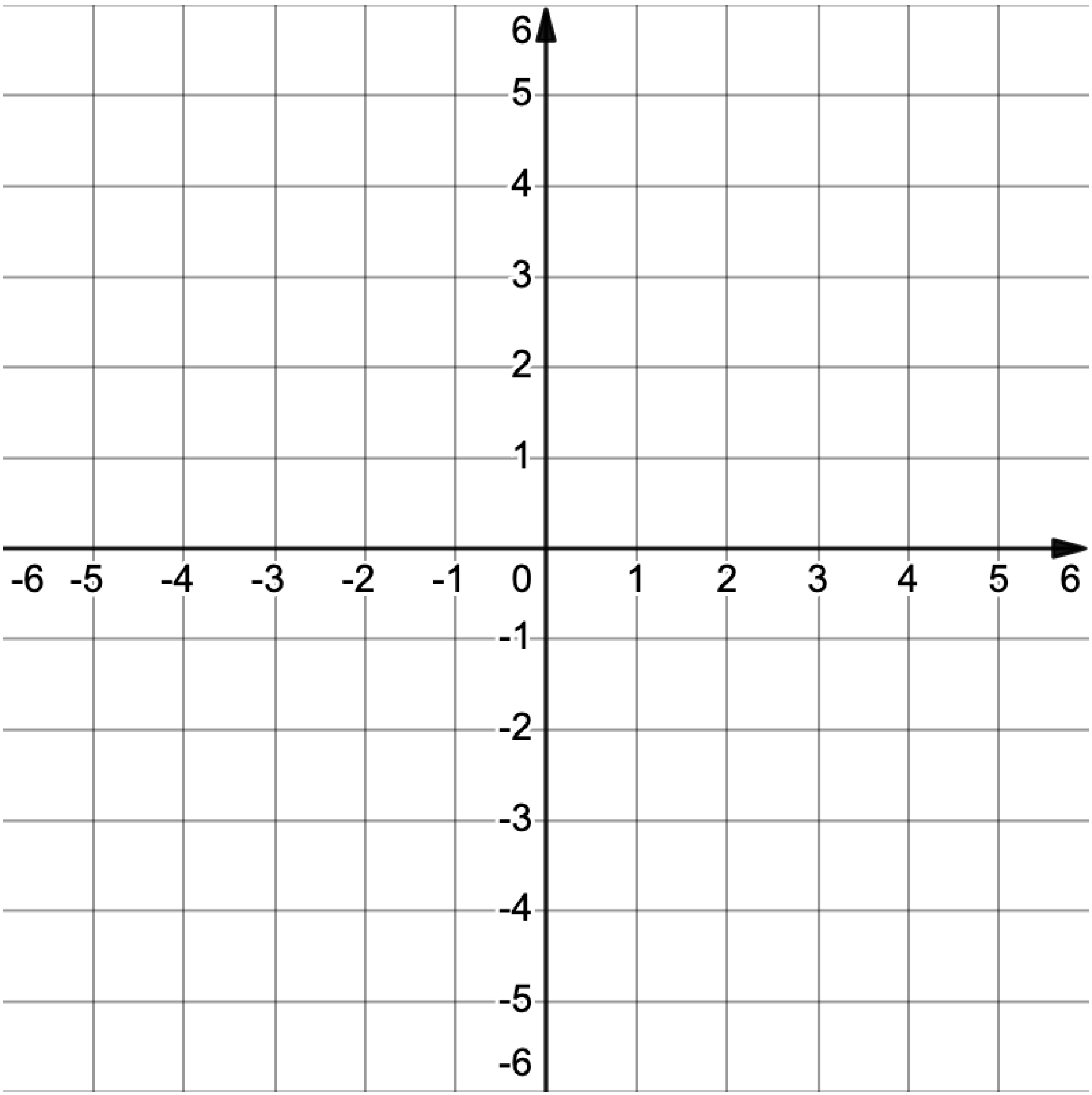
B=

(b) Use the limit definition of the derivative to write an explicit expression for $f'(-1)$. *Your answer should not involve the letter f. Do not attempt to evaluate or simplify the limit.* [2 marks]

$f'(-1)$=

12. On the axes below, sketch a well-labeled graph of a function f that satisfies the given properties. [5 marks]

- $f(-4) = 0$
- $f'(-5) < f(-4) - f(-5)$
- $\lim_{x \to -2^-} f(x) \to -\infty$
- $f'(x) = 0$ for $-2 < x < 0$
- $f'(x) = 2$ for $0 < x < 2$
- $\lim_{x \to 2} f(x)$ does not exist
- $\lim_{h \to 0} \dfrac{f(3+h) - f(3)}{h}$ does not exist
- f is continuous at $x = 3$
- f is increasing for $x > 3$
- $f''(x) < 0$ for $x > 3$



Short Answer: Unsupported answers will not receive full credit. Organize your work in a reasonably neat and coherent way. Put your final answer in the box provided.

13. We define the integer part $\mathrm{Int}(x)$ of a positive number $x > 0$ to be the largest integer number that is smaller than or equal to x. For example, $\mathrm{Int}(1.3) = 1$ and $\mathrm{Int}(\pi) = 3$. Determine

$$\lim_{x \to 0^+} \frac{x}{2} \mathrm{Int}\left(\frac{3}{x}\right).$$

[3 marks]

Hint: Give a certain theorem a squeeze.

$$\lim_{x \to 0^+} \frac{x}{2} \mathrm{Int}\left(\frac{3}{x}\right) = \boxed{}$$

IF NEEDED, USE THIS PAGE TO CONTINUE OTHER QUESTIONS.

If you wish to have this page marked, make sure to refer to it in your original solution.

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Formula Sheet for MAT186 Term Test 1

DO NOT DETACH THIS PAGE. IT MUST BE SUBMITTED WITH YOUR EXAM

Trigonometric Formulas:

- $\sin^2 x = \dfrac{1-\cos 2x}{2}$
- $\cos^2 x = \dfrac{1+\cos 2x}{2}$
- $\sin x = \cos\left(\frac{\pi}{2} - x\right)$
- $\tan x = \dfrac{\sin x}{\cos x}$
- $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
- $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

Logarithms and Exponentials:

- $\log_a a^x = x$
- $a^{\log_a x} = x$
- $a^{b+c} = a^b a^c$
- $\log_a bc = \log_a b + \log_a c$
- $(a^b)^c = a^{bc}$
- $\log_a b^c = c\log_a b$

Limits:

- $\lim\limits_{h\to 0}(1+ah)^{\frac{1}{h}} = e^a$
- $\lim\limits_{x\to\infty}\left(1+\dfrac{a}{x}\right)^x = e^a$
- $\lim\limits_{x\to 0}\dfrac{\sin x}{x} = 1$
- $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x} = 0$

The Squeeze Theorem:

- If $g(x) \leq f(x) \leq h(x)$ for all $x \neq a$ in some interval I containing a, and $\lim\limits_{x\to a} g(x) = l = \lim\limits_{x\to a} h(x)$, then $\lim\limits_{x\to a} f(x)$ exists and
$$\lim_{x\to a} f(x) = l$$

Intermediate Value Theorem (IVT):

- Let f be continuous on a closed interval $[a,b]$. If z is any real number between $f(a)$ and $f(b)$, then there is a number $c \in [a,b]$ such that $f(c) = z$.