UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 2014

DURATION: 2 AND 1/2 HRS

FIRST YEAR - CHE, CIV, CPE, ELE, ENG, IND, LME, MEC, MMS

MAT186H1F - Calculus I

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Exam Type: A. Aids permitted: Casio FX-991 or Sharp EL-520 calculator.

Full Name: _____
Last First

Student Number: _____

Signature: _____

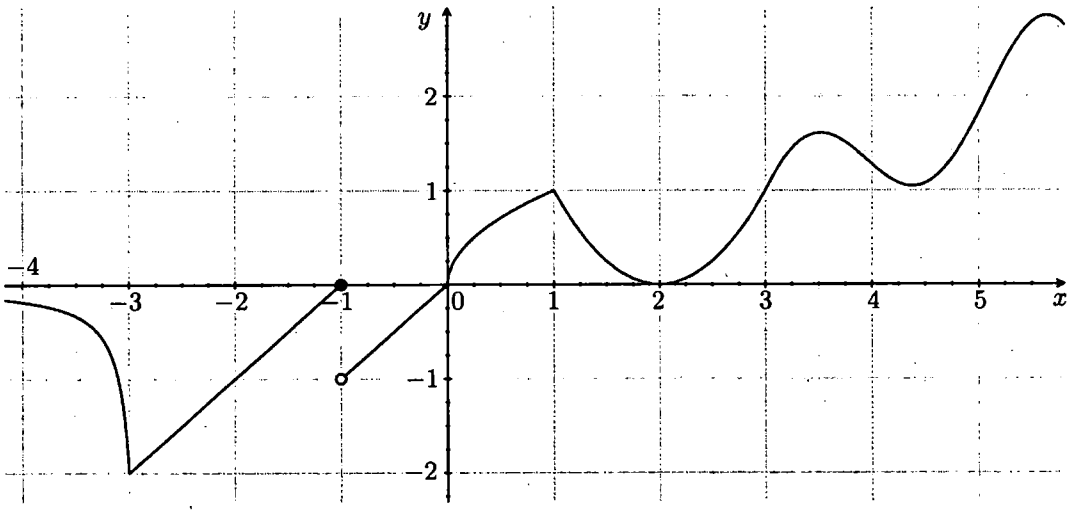
Instructions:

- ONLY THE **FRONT PAGES** WILL BE SCANNED. THE BACK PAGES WILL NOT BE SEEN BY THE EXAMINERS.
- DO NOT WRITE ON THE QR CODE AT THE TOP OF THE PAGES.
- This exam contains 10 pages (including this cover page). Make sure you have all of them. Do not tear any pages from this exam.
- You can use the back of the pages and page 10 for rough work.
- This exam consists of 8 questions. Each question is worth 10 marks. **Total Marks: 80**

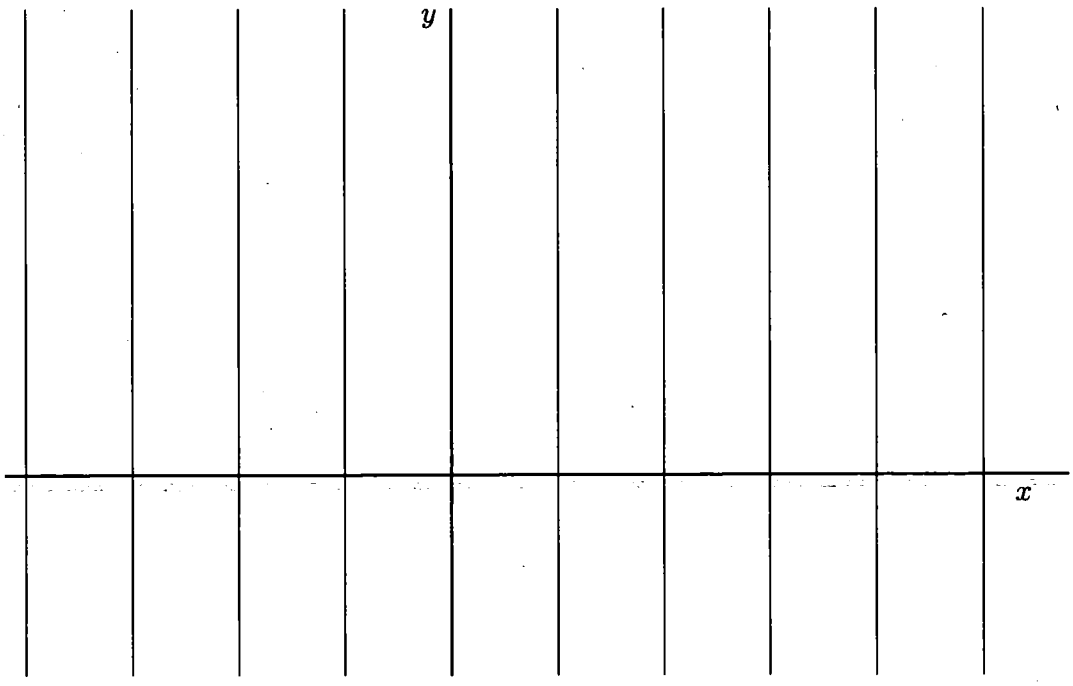
MAT186H1F – Final Exam

PART I : No explanation is necessary.

1. The graph of the function f is given below. Let $F(x) = \int_0^x f(t)\,dt$, for $-4 \le x \le 5$.



Plot the graph of F. In the horizontal direction use the same scale as the above graph; in the vertical direction it is the shape of your graph that is important, not the actual y-values.



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PART II : Present **complete** solutions to the following questions in the space provided.

2. Find the following limits:

(a) [3 marks] $\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(\frac{i\pi}{n}\right)\frac{\pi}{n}$

(b) [3 marks] $\lim_{x\to 0} \frac{\tan^{-1} x - x}{x^3}$

(c) [4 marks] $\lim_{x\to\infty} (x + e^x)^{2/x}$

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3. Find the following:

(a) [5 marks] $\int_{-3}^{4} \frac{x}{(4+x)^{1/3}}\,dx$

(b) [5 marks] $F'(2)$ if $F(x) = x\int_{8}^{x^3} \sqrt{36+t^2}\,dt$

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4. Let A be the area of the region between by $y = 1 + \ln x$ and $y = 0$ for $1 \leq x \leq e^2$.

   (a) [6 marks] Express the value of A in terms of one or more integrals with respect to x **and** in terms of one or more integrals with respect to y. (Draw a diagram!)

   (b) [4 marks] Find A.

5. Let $v = t^2 - 5t + 6$ be the velocity of a particle at time t, for $0 \leq t \leq 3$. Find:

   (a) [4 marks] the average velocity of the particle.

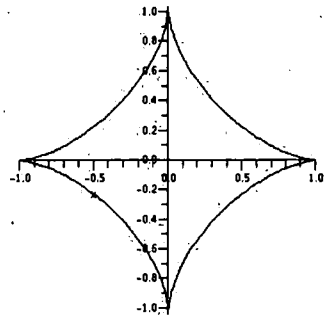
   (b) [6 marks] the average speed of the particle.

6. A storage tank's shape is formed by rotating the curve $y = \frac{x^4}{4}$, for $0 \leq x \leq 2$, around the y-axis. (Assume distances are measured in meters.)

   (a) [5 marks] What is the volume of the tank?

   (b) [5 marks] Suppose the tank is filled with water of density $\rho = 1000$ kg/m^3. How much work is needed to pump all the water out of the tank to an outflow pipe 1 meter above the top of the tank? Use $g = 9.8$ m/sec^2.

7. The graph of the astroid with equation $x^{2/3} + y^{2/3} = 1$ is shown to the right.



   (a) [5 marks] Find the total length of the astroid. NB: if your integrand is not defined at $x = 0$ don't worry; just ignore this minor technicality and proceed to integrate as usual.

   (b) [5 marks] Find the area of the surface generated by revolving the astroid about the x-axis.

8. At 1 PM a military jet is flying due east at 20 km/min. At that instant, at the same altitude and 300 km directly ahead of the military jet, a commercial air liner is flying due north at 10 km/min. When are the two planes closest to each other? What is the minimum distance between them?

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This page is for rough work; it will not be marked.

The end.