University of Toronto

Faculty of Applied Science and Engineering

MIE100—Dynamics

Final Examination

June 28, 2012, 9:30 am to 12:00 pm

Instructor: L. Sinclair

Aids Permitted: One non-programmable calculator

One 8 ½" by 11"aid sheet, any colour

This exam has 6 pages

Do all work in the exam booklet

Complete all five questions

Total Marks: 100

Q 1.

Bar AB of the mechanism shown in **Fig. 1** is rotating clockwise (positive) with a constant angular velocity of 3 radians/s.

a. Determine the velocity of point B at the instant shown. Express your answer in the rectangular co-ordinates provided. (5 marks)

b. Determine the angular velocity of bar BC at the instant shown. (15 marks)

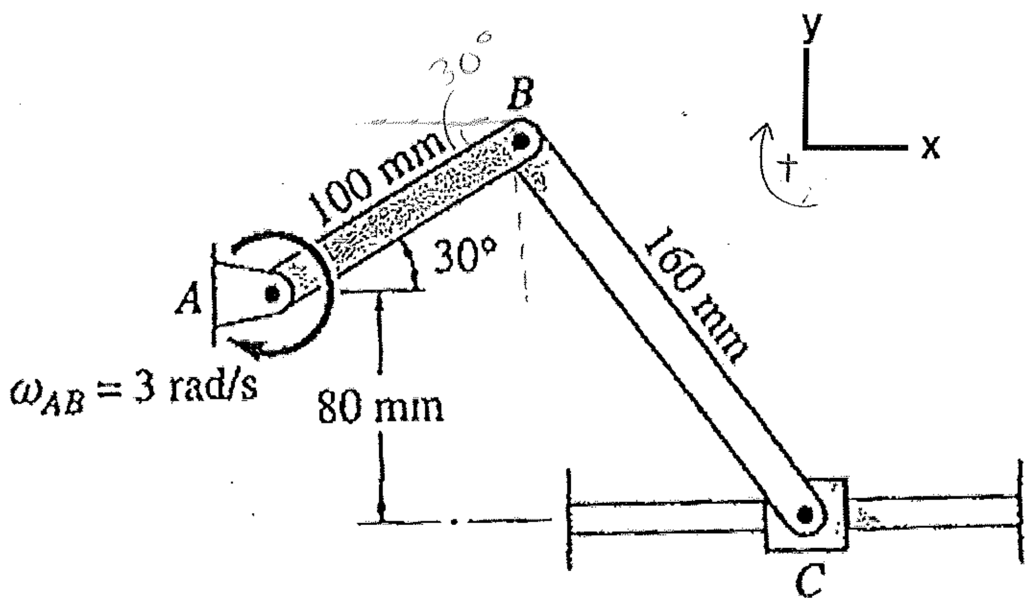


Fig. 1

Q2. A rope is wrapped around the uniform 50 kg pulley B and attached to the 20 kg block A. The coefficients of static and kinetic friction (μ_s and μ_k) between A and the incline are 0.25 and 0.20 respectively. The system is released from rest.

a. Draw a free body diagram of A <u>just before it starts to move</u>. You may use symbols for this part. Please make sure that your symbols are clear. (5 marks)

b. Determine the value (magnitude) of all the forces on A <u>just before it starts to move</u>. (10 marks)

c. Find the initial acceleration of A. Express your answer in standard rectangular co-ordinates. (5 marks)

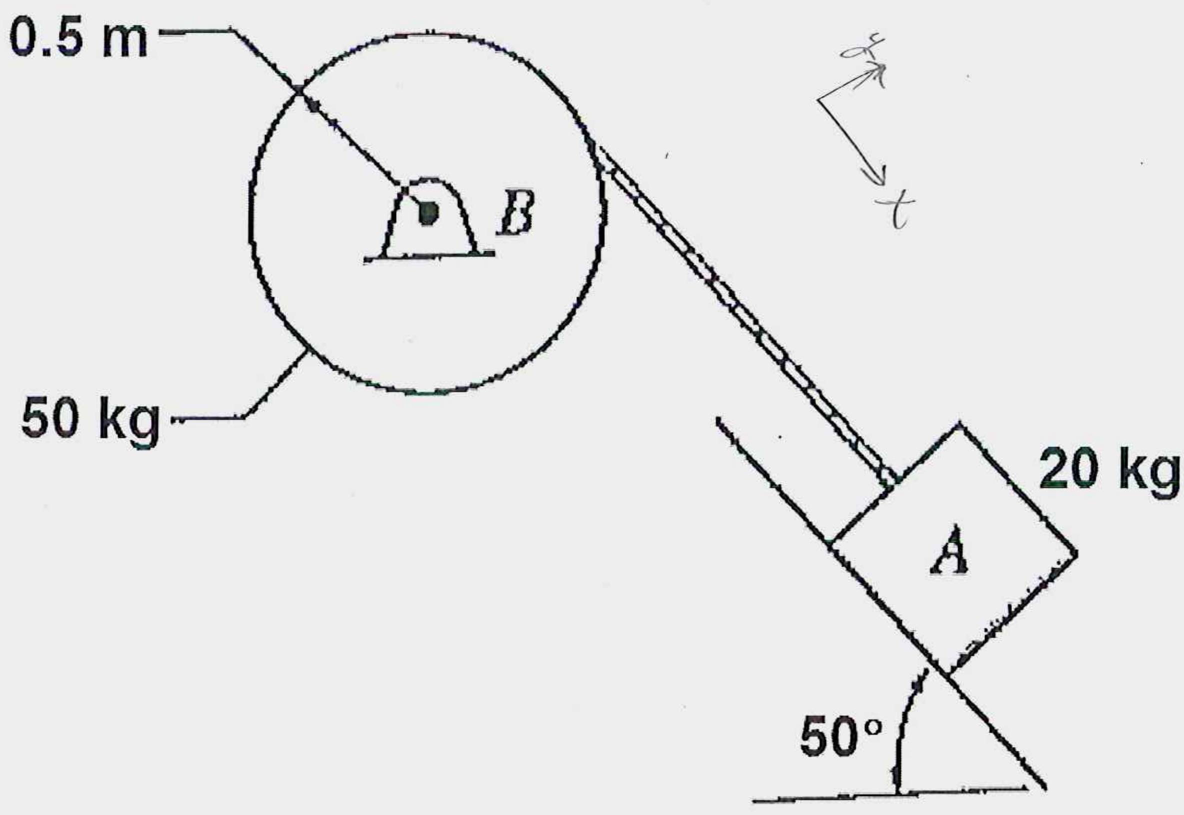


Fig. 2

Q3. The mass center of the 7 kg unbalanced wheel is located at G. The mass moment of inertia of the wheel about its geometric center A is $I_A = 0.3\ \text{kg}\cdot\text{m}^2$. In the position shown, the angular velocity of the wheel is + 3 radians/s and the horizontal spring is undeformed.

a. Find the mass moment of inertia of the wheel (in the position shown) about the point on the wheel which is in contact with the road. (5 marks)

b. Find the angular velocity of the wheel after it has rolled clockwise through 180° from this position. Assume that the wheel does not slip. (15 marks)

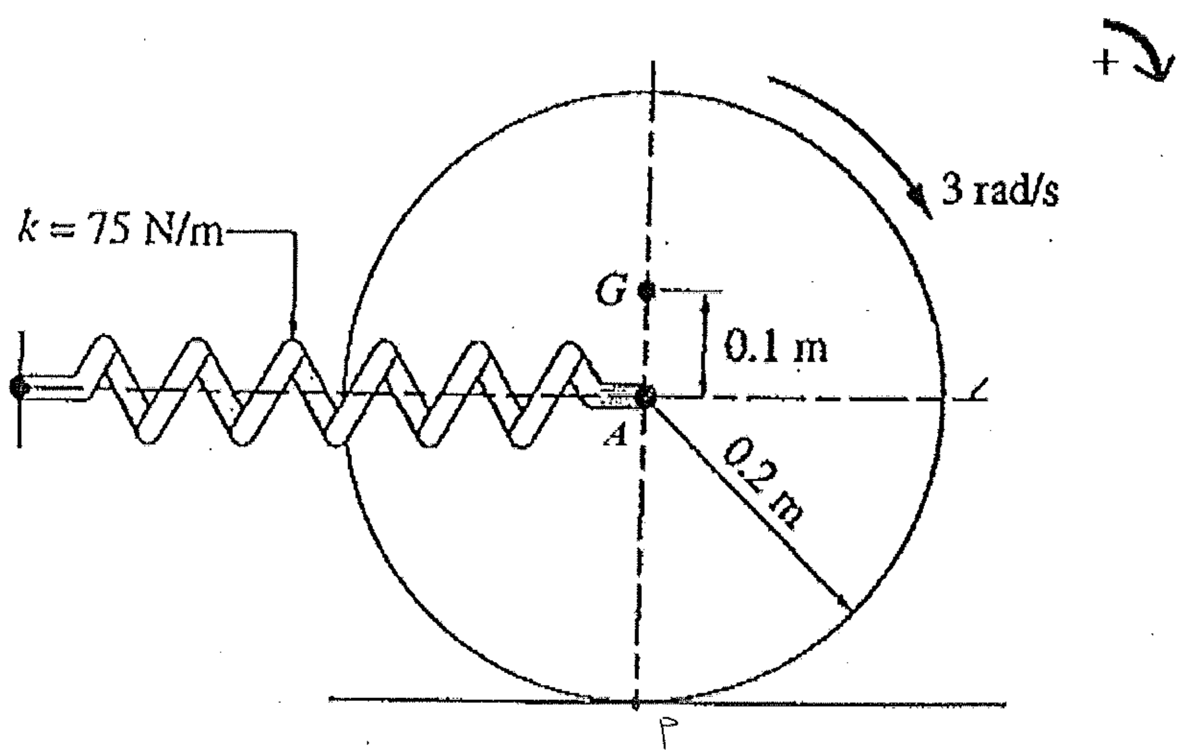


Fig. 3

Q4. A point starts at (0, -2) and travels at +45 degrees to the x axis in a standard xy plane. The point's speed (v in meters per second) is related to the time of travel as: $v = 3t^2$ where t is in seconds.

a. What is the location (x, y) of the point after one second? (5 marks)

b. What is the velocity of the point after 2 seconds? Express your answer in the rectangular co-ordinates given. (5 marks)

c. What is the acceleration of the point after 3 seconds? Express your answer in normal and tangential co-ordinates. (5 marks)

d. What is the end point (x, y) of the unit vector $\hat{u}_r$ (for a polar system centered at O) after the particle has traveled 5 meters? (5 marks)

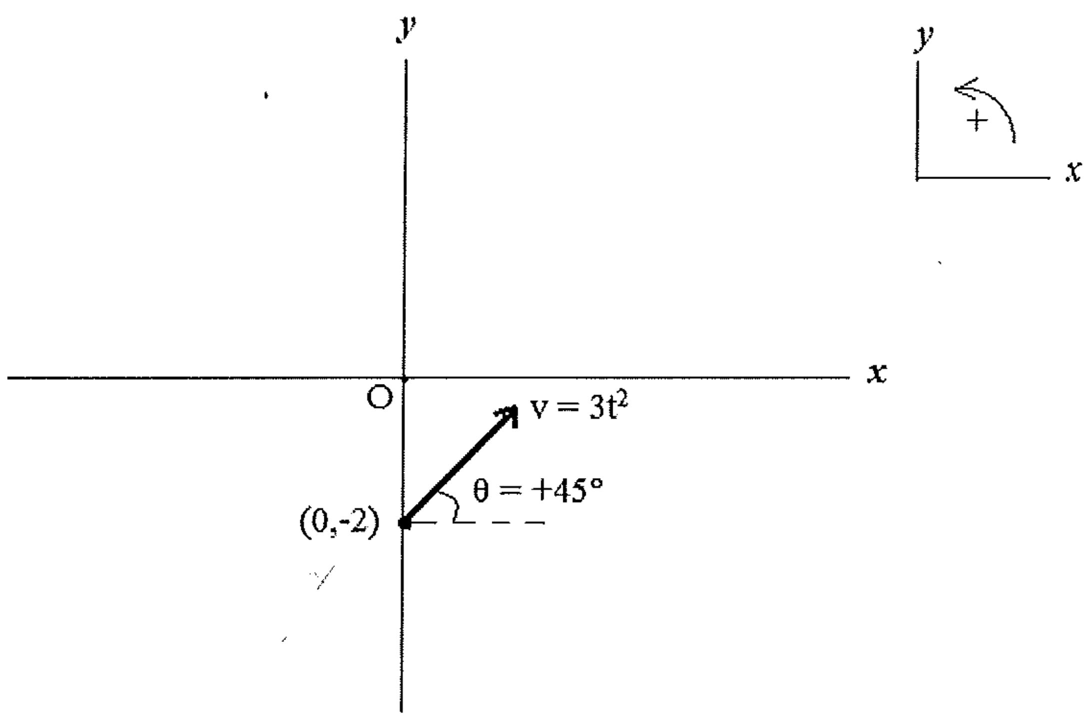


Fig. 4

Q5.

a. Assume that the cage in the figure does not move. Assume too that the $x=0$ reference point is at the static equilibrium. Expressing your answer (for b too) using m and k, find the natural frequency (ω_n) of the vibratory motion of *m*. (10 marks)

b. Now move the cage by an amount $y(t) = Y \sin \omega t$ meters. If the steady state maximum displacement of *m* is 5 times Y, what is the forcing frequency ω? (10 marks)

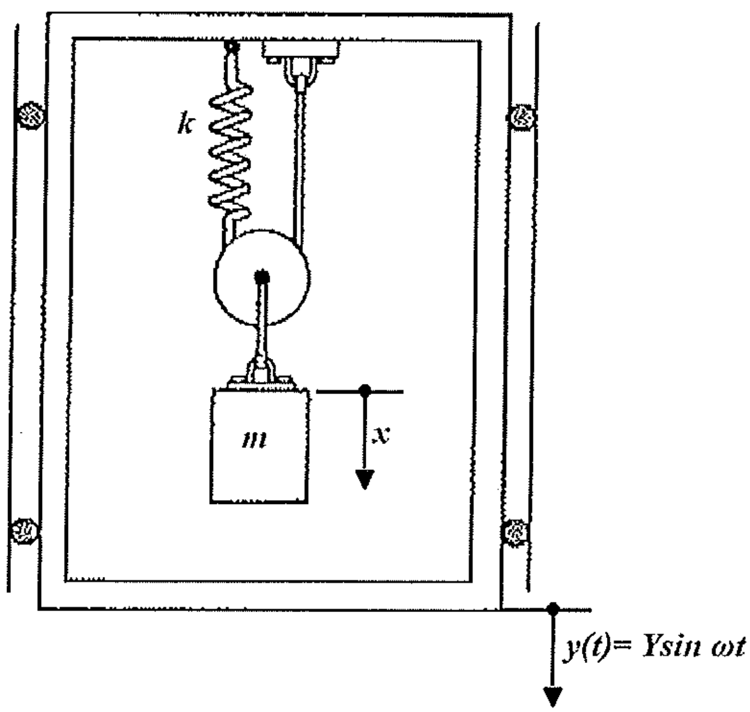


Fig.5

1) (a)

$$|\vec{v}_B| = r_{B/A}\,\omega = 0.1\,(3) = 0.3\ \text{m/s}.$$

$$\vec{v}_B = .3\cos 60°\,\hat{i} - .3\sin 60°\,\hat{j}$$

$$= 0.15\,\hat{i} - 0.26\,\hat{j}\ \text{m/s}.$$

(b) $\vec{a}_B = \vec{a}_C + \vec{a}_{B/C}$

$\vec{a}_B$: there is no tangential acc ($\alpha_{AB} = 0$)

$\Rightarrow$

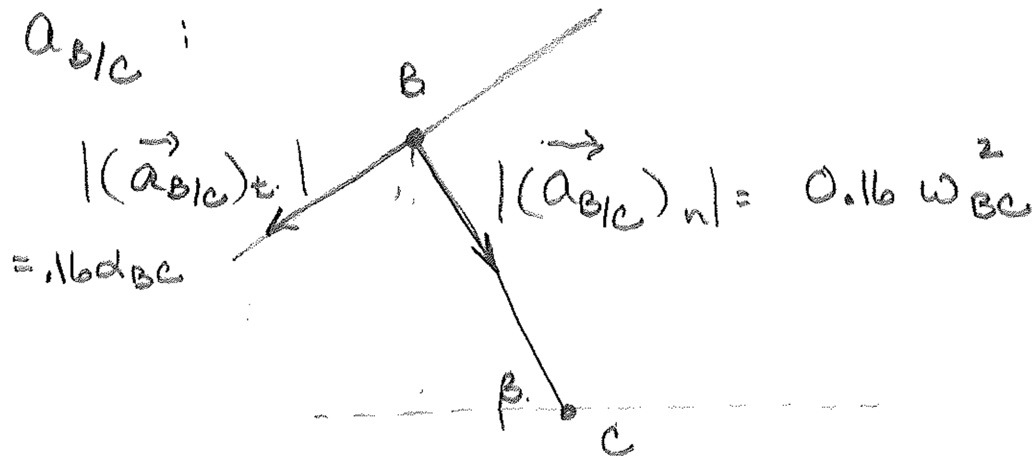
$r\omega^2 = 0.1\,(3)^2 = 0.9\ \text{m/s}^2$

$$\vec{a}_B = -.9\cos 30°\,\hat{i} - .9\sin 30°\,\hat{j}$$

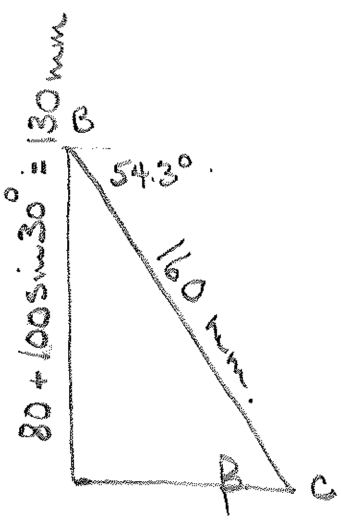
$$= -0.78\,\hat{i} - 0.45\,\hat{j}\ \text{m/s}^2.$$

$\vec{a}_C$: $\vec{a}_C = a_C\,\hat{i}$

$\vec{a}_{B/C}$:



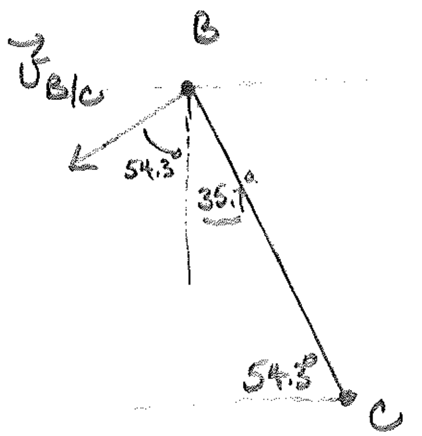
step 1: find β :



$\beta = \sin^{-1} \frac{130}{160} = 54.3°$

90

step 2: find ω_{BC}.



$$\vec{v}_B = \vec{v}_C + \vec{v}_{B/C}$$

$$0.15\,\hat{i} - 0.26\,\hat{j} = v_C\,\hat{i}$$

$$- .16\,\omega_{BC} \cos 35.7°\,\hat{i}$$

$$- .16\,\omega_{BC} \sin 35.7°\,\hat{j}$$

solving the $\hat{j}$ direction only for ω_{BC}

$V_{B/C} = \omega_{BC} r_{BC} = (2.78)(.18)$

$$-0.26 = -0.16\,\omega_{BC} \sin 35.7^\circ$$

$$\Rightarrow \omega_{BC} = \frac{0.26}{0.16 \sin 35.7^\circ} = 2.78\ s^{-1} \quad \text{ccw.}$$

(by observation)

keep going:

$$\Rightarrow |(\vec{a}_{B/C})_n| = .16(2.78)^2 = 1.24$$

$$\vec{a}_B = \vec{a}_C + \vec{a}_{B/C}$$

$$-0.78\,\hat{i} - 0.45\,\hat{j} = a_C\,\hat{i} + 1.24 \cos 54.3^\circ\,\hat{i}$$

$$-1.24 \sin 54.3^\circ\,\hat{j} - .16\,\alpha_{BC} \cos 35.7^\circ\,\hat{i} - .16\,\alpha_{BC} \sin 35.7^\circ\,\hat{j}$$

$\Rightarrow$ $\hat{i}$ direction: (not needed).

$$-.78 = a_C + .72 - .13\,\alpha_{BC}$$

$$a_C = -.78 - .72 + .13\,\alpha_{BC} = -1.5 + .13\,\alpha_{BC}$$

$\hat{j}$ direction:

$$-.45 = -1 - .093\,\alpha_{BC}$$

$$\Rightarrow \alpha_{BC} = \frac{-1 + .45}{.093} = -5.91\ s^{-2}$$

so the negative sign means I got the direction wrong for $(a_{B/C})_t$ – change signs in eqn:

$$-.78\,\hat{i} - .45\,\hat{j} = a_c\,\hat{i} + 1.24\cos 54.3\,\hat{i}$$
$$- 1.24\sin 54.3\,\hat{j} + .16\,\alpha_{BC}\cos 35.7\,\hat{i}$$
$$+ .16\,\alpha_{BC}\sin 35.7\,\hat{j}$$

$\hat{j}$ direction:

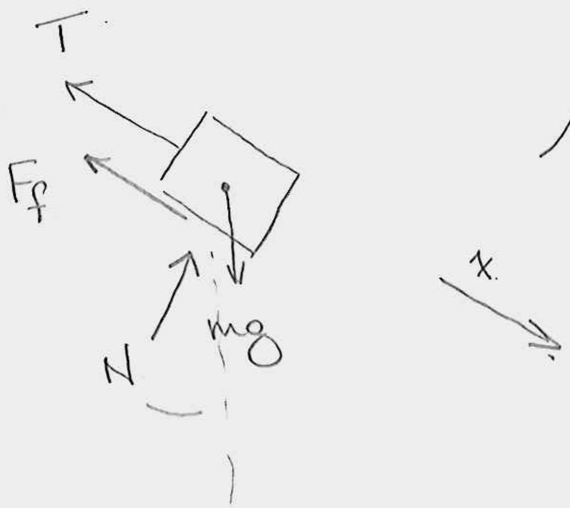
$$-.45 = -1.24\sin 54.3 + .16\,\alpha_{BC}\sin 35.7$$

$$\alpha_{BC} = \frac{-.45 + 1}{.093} = +5.97\ s^{-2}$$

$$\Rightarrow \quad \alpha_{BC} = 5.91\ s^{-2} \quad CW$$

(2)

(a)



$\mu_s = 0.25$

(b) $N = mg \cancel{\sin}\ \cos 50° = \cancel{150}$ N. $196.2(.643) = 126.2$

$mg = 196.2$ N.

$F_f = 0.25(\cancel{150}) = \cancel{37.5}$ N. 31.5

to find T: we must add in the pulley.

$\Sigma M_O = I_O \alpha$ where O is the center of pulley B.

$I_O = \frac{1}{2} mR^2 = \frac{1}{2} 50(.5)^2 = 6.25$ kgm^2.

$T(.5) = 6.25\,\alpha$ but $\alpha = .5 a_A$.

$$T(0.5) = 6.25\,(0.5\,a_A)$$

$$T = 6.25\,a_A.$$

but $\left(\sum F_A\right)_A = m_A a_A.$

$$196.2\,(\cos 40^\circ) - 6.25\,a_A - 31.5 = 20\,a_A$$

$$a_A = \frac{150 - 31.5}{26.25} = 4.51 \text{ m/s.}$$

$$a_A = 2.90\,\hat{i} - 3.45\,\hat{j} \text{ m/s}^2.$$

$$T = 6.25\,(4.51) = 28.2 \text{ Newtons.}$$

#3.

(a) $I_A = 0.3 \text{ kgm}^2$

$I_A = I_G + md^2$

$I_G = 0.3 - 7(.1)^2 = 0.3 - 0.07 = 0.23$.

$I_{IC} = \underbrace{.23 + 7(.3)^2}_{\text{Parallel Axis Theorem.}} = 0.86 \text{ kgm}^2$.

(b) $T_1 + \cancel{V_{e_1}}^{0} + V_{g_1} = T_2 + V_{e_2} + V_{g_2}$

$T_1 = \frac{1}{2}(.86)(3)^2 = 3.87$ joules.

$V_{g_1} = mgh_G = 7(9.81)(.3) = 20.6$ joules.

$T_2 = \frac{1}{2} I_{IC} \omega_2^2$

we are asked for this.

$I_{IC} = .23 + 7(.1)^2 = .3$.

new location of G.

(7A)

$V_{e_2} = \frac{1}{2} k s^2$

but s is the distance A has moved

= ½ of circumference.

$= \pi(.2) = \cancel{6.28\,m}.\quad 0.628\,m$

$\Rightarrow V_{e_2} = \frac{1}{2}(75)\cancel{6.2}(0.628)^2 = 14.8 \text{ joules}.$

$V_{g_2} = 7(9.81)(.1) = 6.87 \text{ joules}.$

$\Rightarrow 3.87 + 20.6 = \frac{1}{2}(.3)\omega_2^2 + 14.8 + 6.87.$

$\Rightarrow \omega_2^2 = \frac{2.8}{.15} \quad \Rightarrow \quad \omega = 4.3\ s^{-1}$

$\Rightarrow \cancel{4.5}^{\,3.87} + 20.6 = \cancel{\tfrac{1}{2}}^{\,.43}\,\omega^2 + 6.87 + 14.8$

$\Rightarrow \cancel{25.4}^{\,24.5} - 21.67 = \cancel{\tfrac{1}{2}}^{\,.43}\,\omega^2$

$\Rightarrow \omega = \cancel{2.62}^{\,2.57}\ s^{-1}$

(4) (a) $v = \frac{ds}{dt}$ $\qquad \int_0^1 v\,dt = \int_0^s ds.$

$$\int_0^1 3t^2\,dt = s = \frac{3t^3}{3}\Big|_0^1 = 1 \text{ meter}$$

$\Delta x = +0.71$ m.

$\Delta y = +0.71$ m.

$\Rightarrow$ location: $(.71, -1.29)$.

(b) $\vec{v} = 12(.707)\hat{i} + 12(.707)\hat{j} = 8.5\,\hat{i} + 8.5\,\hat{j}$ m/s.

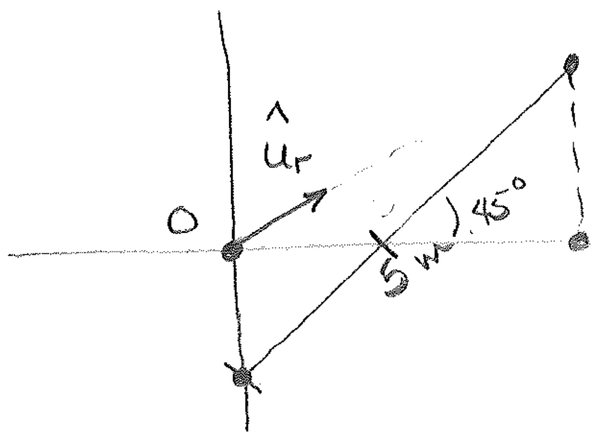
$[3(2)^2 = 12]$

(c) $\vec{a} = a_t \hat{u}_t$. but $a_t = \frac{dv}{dt} = 6t$.

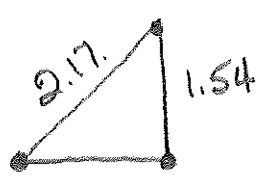
@ 3s: $a_t = 18 m/s^2$.

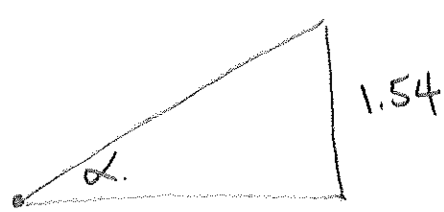
$\vec{a} = 18 \hat{u}_t \ m/s^2$.

(d)



as the point crosses the x-axis: it has travelled $\sqrt{8}$ m = 2.83 m ⇒ past the x-axis it has travelled 2.17 m.



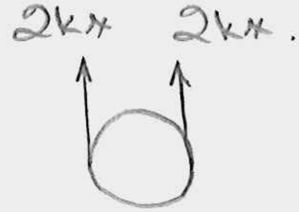


$2 + 1.54 = 3.54$.

$$\alpha = \tan^{-1} \frac{1.54}{3.54} = 23.5°$$

$$\Rightarrow \hat{u}_r = (1 \cos 23.5°, \ 1 \sin 23.5°) = (.92, .40)$$

5. (a) if we start at static eqm. we can neglect gravity: displace x: the spring will displace $2x$



Tension = $2kx$.

$$m\ddot{x} + 4kx = 0 \Rightarrow \omega_n = \sqrt{\frac{4k}{m}} = 2\sqrt{\frac{k}{m}}$$

(b). $$5 = \frac{1}{1-\left(\frac{\omega_0}{\omega_n}\right)^2} \Rightarrow 1-\left(\frac{\omega_0}{\omega_n}\right)^2 = 0.2$$

$= .89\,\omega_n$

$$\frac{\omega_0}{\omega_n} = \sqrt{.8} = .89 \Rightarrow \omega_0 = 0.89\,(2)\sqrt{\frac{k}{m}}$$

✓ $= 1.79\sqrt{\frac{k}{m}}$

$\cancel{C = \frac{F_0/k_{eq}}{1-(\omega_0/\omega_n)^2}}$

~~the pulley arrangements makes $k_{eq} \to 4k$.~~

~~4k. + $\omega_n = \sqrt{\frac{4k}{m}}$.~~