University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, APRIL, 2009

First Year - CHE, CIV, IND, LME, MEC, MMS

MAT186H1S - CALCULUS I

Exam Type: A

SURNAME: (as on your T-card) _____

GIVEN NAMES: _____

STUDENT NUMBER: _____

SIGNATURE: _____

Examiner:
D. Burbulla

Calculators Permitted: Casio 260, Sharp 520 or TI 30.

INSTRUCTIONS: Attempt all questions. Use the backs of the sheets if you need more space. Do not tear any pages from this exam. Make sure your exam contains 10 pages.

MARKS: Questions 1 through 6 are Multiple Choice; circle the single correct choice for each question. Each correct choice is worth 4 marks.

Question 7 covers two pages and is worth 24 marks.

Question 8 is worth 12 marks, 6 marks for each part.

Questions 9, 10, 11 and 12 are each worth 10 marks.

TOTAL MARKS: 100

PAGE	MARK
MC	
Q7	
Q8	
Q9	
Q10	
Q11	
Q12	
TOTAL	

1. What is the equation of the tangent line to the graph of $f(x) = e^{2x}$ at the point $(x, y) = (0, 1)$?

   (a) $y = 2x$

   (b) $y = 2x + 1$

   (c) $y = 2x - 1$

   (d) $y = x + 1$

2. The volume of the solid of revolution obtained by revolving the function $f(x) = x^2$, for $0 \leq x \leq 1$, around the x-axis, is equal to

   (a) $\frac{\pi}{2}$

   (b) $\frac{\pi}{3}$

   (c) $\frac{\pi}{4}$

   (d) $\frac{\pi}{5}$

3. How many vertical asymptotes are there to the graph of $f(x) = \dfrac{\sin(x^2 - 1)}{x^4 - 5x^2 + 4}$?

   (a) 1

   (b) 2

   (c) 3

   (d) 4

4. If $\sin\theta = \frac{1}{4}$ and $\cos\theta < 0$, then the exact value of $\sin\left(\frac{\pi}{4} + \theta\right)$ is

(a) $\frac{1-\sqrt{15}}{4\sqrt{2}}$

(b) $\frac{1+\sqrt{15}}{4\sqrt{2}}$

(c) $\frac{-1+\sqrt{15}}{4\sqrt{2}}$

(d) $\frac{-1-\sqrt{15}}{4\sqrt{2}}$

5. $\int_1^2 \frac{\sqrt{4+x^6}}{x}\,dx =$

(a) $\frac{1}{3}\int_1^8 u\sqrt{4+u^2}\,du$

(b) $\frac{1}{3}\int_1^8 \frac{\sqrt{4+u^2}}{u}\,du$

(c) $\frac{1}{3}\int_1^2 u\sqrt{4+u^2}\,du$

(d) $\frac{1}{3}\int_1^2 \frac{\sqrt{4+u^2}}{u}\,du$

6. $\lim_{x\to 0} \frac{\tan x - x}{x^3} =$

(a) $\frac{1}{3}$

(b) $\frac{1}{6}$

(c) $-\frac{1}{3}$

(d) $-\frac{1}{6}$

7. [24 marks] Let $f(x) = \dfrac{x^2 + x - 1}{x - 1}$, for which you may assume

$$f'(x) = \frac{x(x-2)}{(x-1)^2} \text{ and } f''(x) = \frac{2}{(x-1)^3}.$$

(a) [4 marks] Find the open intervals on which f is increasing; decreasing.

(b) [4 marks] Find the critical points of f and determine if each critical point is a relative maximum or a relative minimum point.

(c) [4 marks] Find the open intervals on which f is concave up; concave down.

(d) [6 marks] Find all asymptotes to the graph of f, if any.

(e) [6 marks] Sketch the graph of f labeling all critical points, inflection points and asymptotes, if any.

8. [12 marks; 6 for each part] Consider the curve $y = x^2 - \frac{1}{8}\ln x$, for $1 \leq x \leq 2$. Find the following:

(a) The length of the curve.

(b) The surface area of the solid of revolution obtained by revolving the curve about the y-axis.

9. [10 marks] Suppose the velocity of a particle at time t is given by $v = 4t - t^2$, for $0 \leq t \leq 6$. Find the following:

   (a) [5 marks] the average velocity of the particle for $0 \leq t \leq 6$.

   (b)[5 marks] the average speed of the particle for $0 \leq t \leq 6$.

10. [10 marks] A spherical storage tank with radius 1 m is full of water with density ρ. How much work is done in pumping the water from the top half of the tank up to a transfer pipe 2 m above the top of the tank?

11. [10 marks] Find the following limits:

(a) [5 marks] $\lim_{x\to\infty} x\left(e^{3/x}-1\right)$.

(b) [5 marks] $\lim_{x\to 0}\left(1+\sin^{-1}(x^2)\right)^{3/x^2}$.

12. [10 marks] Find the following:

(a) [5 marks] $F'(e)$, if $F(x) = \int_0^{\ln x} \sqrt{3+t^5}\, dt$.

(b) [5 marks] $\int_1^e \frac{dx}{x[1+(\ln x)^2]}$.