ECE110 ‘Condensed Notes’

Dear 2T3s (and upcoming froshes),

This document is basically my ‘condensed notes’, which is summary of whole course. It is based on professor Mojahedi’s lecture in 2018-2019 academic year. Hopefully it will guide your study for this course! **<u>Please do not rely on this note though</u>**, as curriculum can vary by year, and professors are doing examples which will aid your understandings on course materials. This note is written to show the concepts in relatively short time, not for giving examples. **<u>Make sure to go to your own lectures!!</u>**

You will see most of concepts in second year and onward! First part of ECE110 is repeated with calculus 3 concepts in ECE221, and second part is repeated and continued in ECE212 (then 231). So DO NOT forget those concepts!!

The note is taken in OneNote and snipped afterwards, so some points might be blurry! Also don’t freak out with 50+ pages, I had to break my notes for clear images. Good luck with your second semester!!

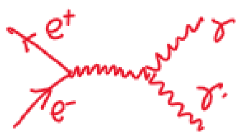
Unit 1 Condensed Notes

Friday, January 18, 2019 1:18 AM

Charge (q)

Types
- Positive $(q>0)$
- Neutral $(q=0)$
- Negative $(q<0)$

Properties
- Conserved.
  - $q_{initial} = q_{final}$



- Quantized
  - $\hookrightarrow q_0 = 1.602 \times 10^{-19}\ C$
  - $\hookrightarrow$ Every charge is <u>integer multiple</u> of q_0.

Coulomb Force

$$\vec{F} = \frac{kQq}{R^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}\hat{r}$$

- $k = 8.99 \times 10^9\ Nm^2/C^2$ Coulomb's constant
- $\varepsilon_0 = 8.854 \times 10^{-12}\ NC^2/m^2$ Permittivity

- Opposite Charge $\rightarrow$ Attract $(F<0)$ $\oplus\!\rightarrow\ \leftarrow\!\ominus$
- Like charge $\rightarrow$ Repel $(F>0)$ $\leftarrow\!\oplus\ \oplus\!\rightarrow$ $\leftarrow\!\ominus\ \ominus\!\rightarrow$

Unit 2 Condensed Notes

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$$\vec{F}_{q_1,q_2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{R^2}\hat{r}$$

$$\vec{E}_{q_2} = \frac{\vec{F}_{q_1,q_2}}{q_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{R^2}\hat{r}$$

- Assume $q_1 > 0$
- q_1 is too small → 'Test charge'
- Charge as 'particle'

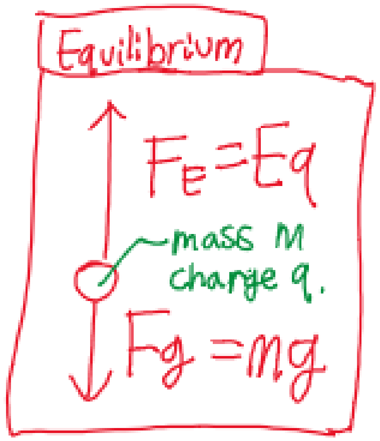
Particle motion

$$v = v_0 + at$$

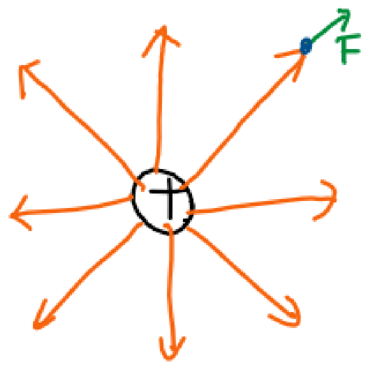
$$s = s_0 + v_0 t + \frac{1}{2}at^2$$

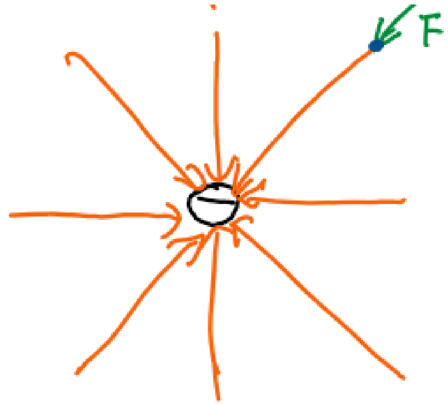
$$v^2 = v_0^2 + 2a(x - x_0)$$

$$s = \frac{(v_0 + v)t}{2}$$



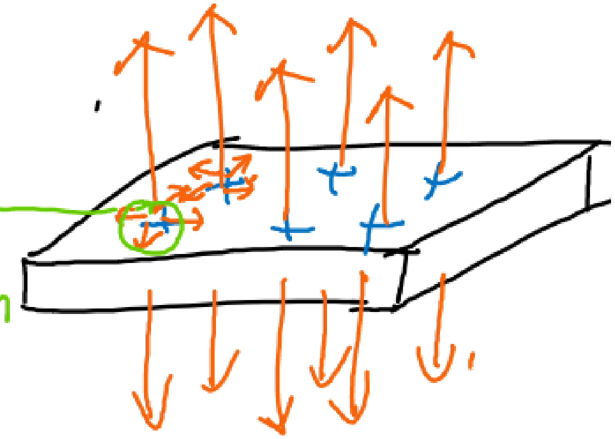
Field Line



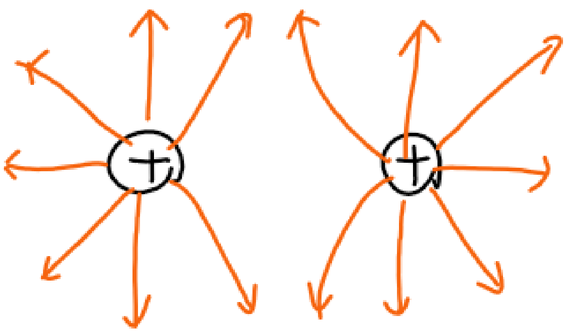


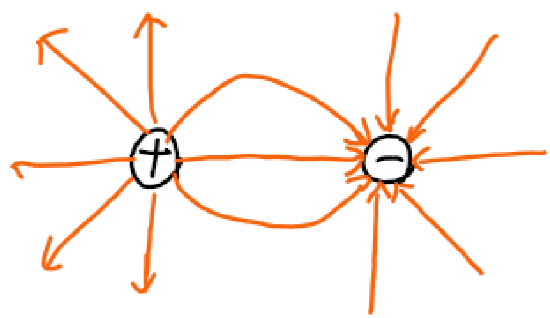
Start from positive

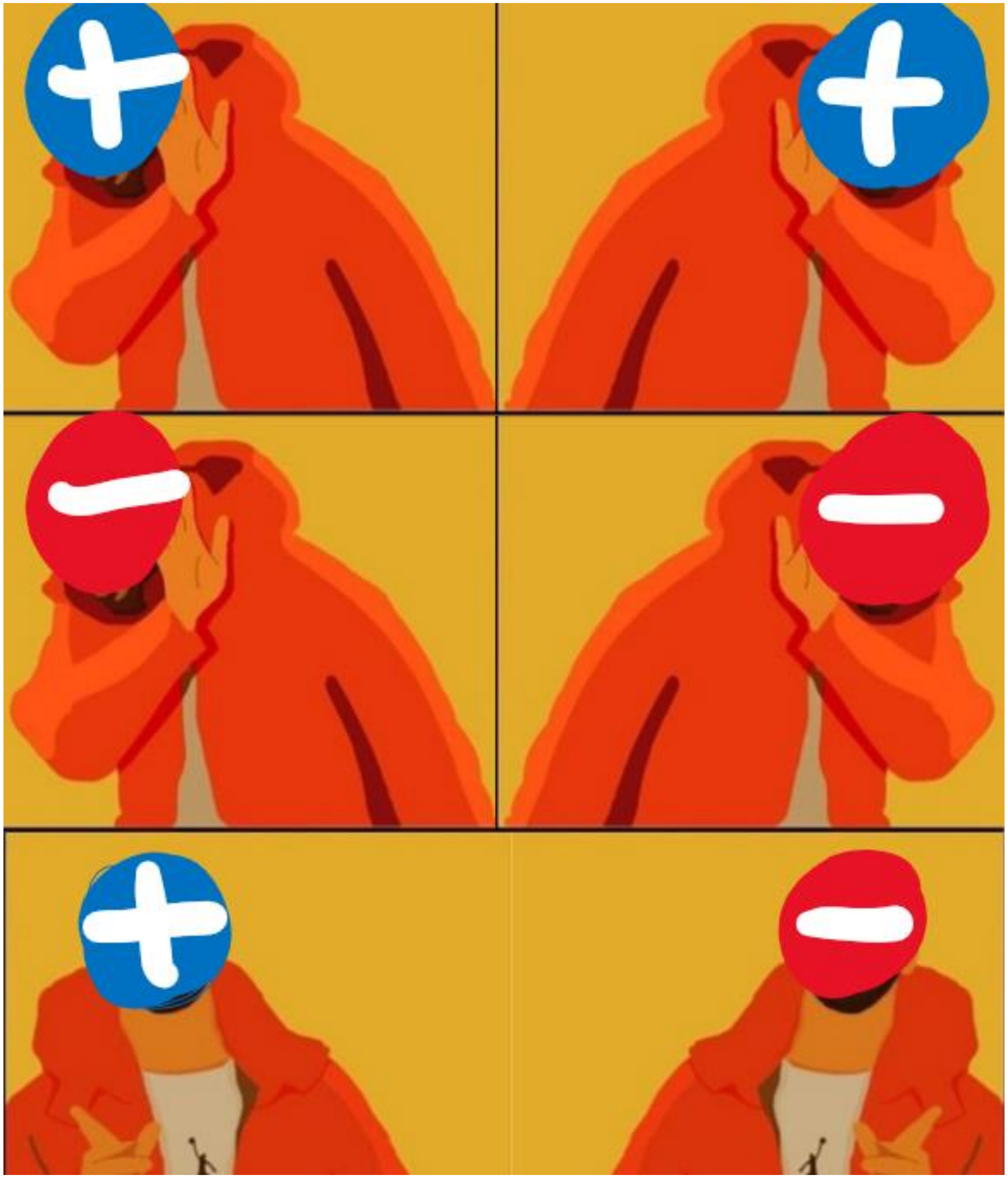
End at negative



Field on plate cancel out







Unit 3 Condensed Notes

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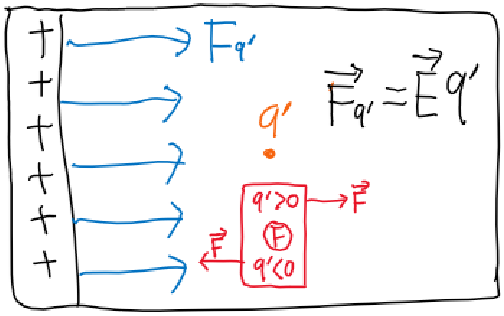


$\vec F_{q'} = \vec E q'$

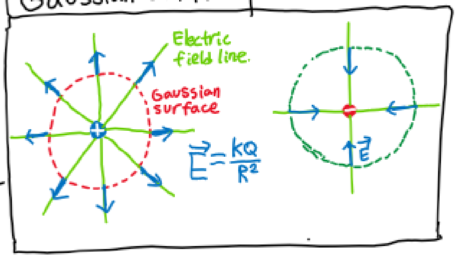
Gauss Law

$$\Phi = \oiint_S \vec E \, d\vec A = \frac{q_{enc}}{\varepsilon_0}$$

Flux — Surface integral (Closed surface) — Enclosed charge

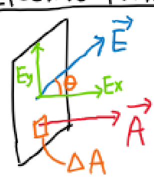
3.1 Electric Flux

Gaussian surface



$\vec E = \frac{kQ}{R^2}$

Electric flux



Flat surface

Uniform field

$$\Delta\Phi = (E\cos\theta)\Delta A = \vec E \cdot \Delta\vec A$$

(Total flux)

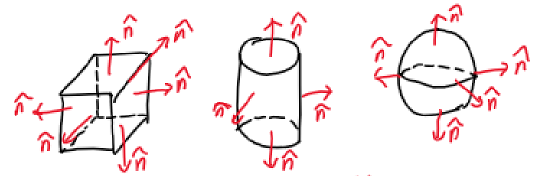
$$\Phi = \sum \vec E \cdot \Delta\vec A = \int \vec E \cdot d\vec A = EA\cos\theta$$

$$\Phi_{net} = \oiint_S \vec E \cdot d\vec A$$

(closed)

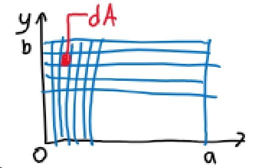
Surface Integral

$\oiint_S$ Closed Surface

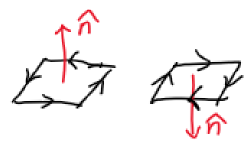


$\iint_S$ open surface

$\vec A = ab\,\hat n$



$d\vec A = dA\,\hat n$



⊙ out of the plane

⊗ Into the plane

$$\oiint_S \vec E \, d\vec A = \oiint_S E \, dA \cos(\angle \vec E, \hat n)$$

3.2 Gauss Law

$\varepsilon_0 \Phi = q_{enc}$ (enclosed)

$q_{enc} = \varepsilon_0 \oiint_S \vec{E}\, d\vec{A}$

Enclosed Charge (q_{enc})

- Total Charge.
- Charge entered into equation with algebraic sign.

$q_1 = 2nC$ $\quad$ $q_3 = -2nC$

$q_2 = 3nC$ $\quad$ $q_4 = -5nC$

$S_1 \rightarrow 2-2=0nC$, $\Phi = 0$

$S_2 \rightarrow 2+3=5nC$, $\Phi > 0$

$S_3 \rightarrow 2-2-5=-5nC$, $\Phi < 0$

Coulomb Force

$\vec{F}_{qQ} = \frac{KQq}{R^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}\hat{r}$

$\vec{E} = \frac{\vec{F}}{q} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

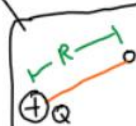
$\oiint_S \vec{E}\, d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$

Use RIGHT Gaussian surface!

$(\angle \vec{E}, \hat{n}) = 0 \rightarrow \cos(\angle \vec{E}, \hat{n}) = 1$

$\oiint_S \vec{E}\, d\vec{A} = \oiint_S E\, dA \cos(\angle \vec{E}, \hat{n}) = \frac{Q_{enc}}{\varepsilon_0}$

$E \oiint_S dA = \frac{Q_{enc}}{\varepsilon_0}$ $\quad$ $\boxed{E = \frac{Q}{4\pi r^2 \varepsilon_0}}$



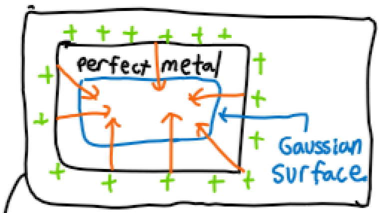
$\vec{F}_{q'} = \vec{E} q'$

↳ Point charge

- because of symmetry, electric field ($\vec{E}$) should be radial (same magnitude)

3.3 A charged Isolated Conductor

- **Isolated Conductor**
  - $\vec{E} = 0$ (inside isolated conductor)
    - If not, $\vec{E}$ exert forces on free e^-s ⇒ Current always exist (cannot work)
    - Internal $\vec{E}$ cancelled out



$\overrightarrow{E_{in\ metal}} = 0$ (inside perfect metal) → assumption, ideal state

↳ Electrostatic

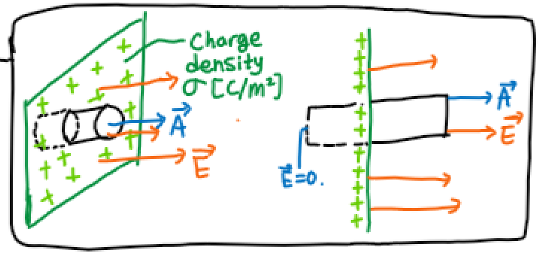
$\oint_S \vec{E}\, d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

$\vec{E} = 0 \rightarrow q_{enc} = 0$

no charge inside metal

→ All reside at Surface.

- **Isolated conductor with Cavity**
  - $\vec{E} = 0$ around cavity
    - All excess charge remains outer surface
    - No net charge on cavity walls
- **Conductor removed**
  - Enlarge cavity Until it consumes whole conductor (leaving charges)
    - $\vec{E}$
      - zero inside thin shell of charge
      - Unchanged in external points
        - $\vec{E}$ set up by charges (not conductor)
- **External Electric Field**



Let $\Phi = \oint_S \vec{E}\, d\vec{A} \approx EA$

$\Phi = \frac{q_{enc}}{\varepsilon_0}$, $q_{enc} = \sigma A$

$\Phi = EA = \frac{\sigma A}{\varepsilon_0}$

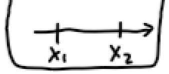
$E = \frac{\sigma}{\varepsilon_0}$

Charge Density

- **Linear density λ**

  $q_{tot} = \int \lambda\, dl$

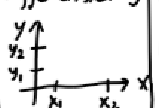
  $= \int \lambda(x)\, dx$



- **Surface density σ**

  $q_{tot} = \iint \sigma\, dA$

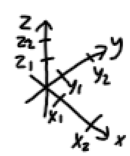
  $= \iint \sigma(x,y)\, dx\, dy$



- **Volume density ρ**

  $q_{tot} = \iiint \rho\, dV$

  $= \iiint \rho(x,y,z)\, dx\, dy\, dz$

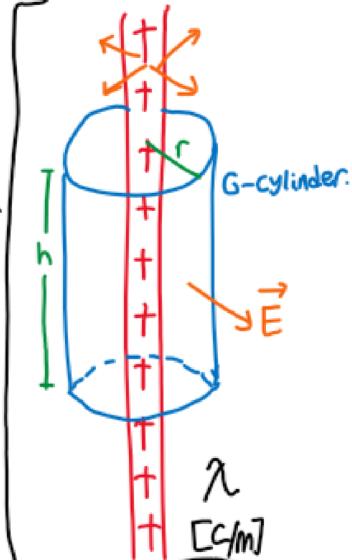


QType

Convert charge density into Qenc

Applying Gauss' Law

3.4 Cylinderical Symmetry

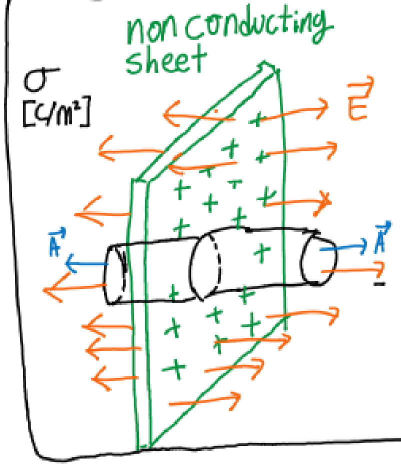


$$\varepsilon_0 \oiint_S \vec{E}\cdot d\vec{A} = q_{enc}$$
$$= \lambda h$$

$$\Phi_{tot} = \cancel{\Phi_{top}}^{\,0} + \cancel{\Phi_{bot}}^{\,0} + \Phi_{side}$$
$$= \varepsilon_0 E\, 2\pi r h = \lambda h$$

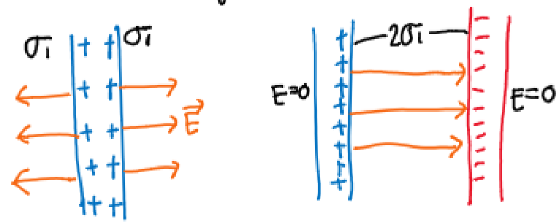
$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

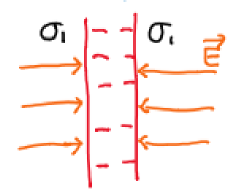
3.5 Planar Symmetry



$$\Phi = \Phi_{left} + \Phi_{right} + \cancel{\Phi_{side}}^{\,0}$$
$$= \oint_{SL} \vec{E}\, d\vec{A} + \oint_{SR} \vec{E}\, d\vec{A}$$
$$= 2EA$$
$$\Phi = \frac{q_{enc}}{\varepsilon_0}$$
$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

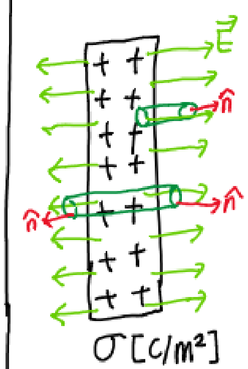
(Two conducting planes)





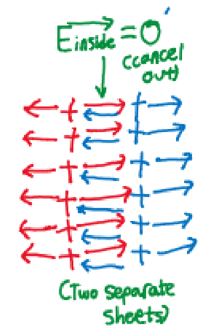
$$E = \frac{2\sigma_1}{\varepsilon_0}$$
$$= \frac{\sigma}{\varepsilon_0}$$

Conducting plate of surface charge density σ



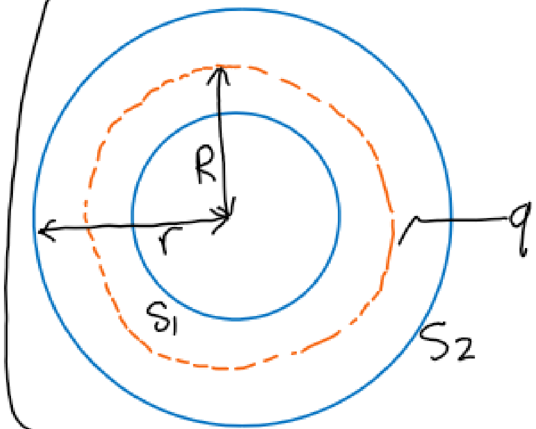
① $\boxed{E = \frac{\sigma}{\varepsilon_0}}$

② $\boxed{E = \frac{2\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}}$



③ $\boxed{E_{sheet} = \frac{\sigma}{2\varepsilon_0},\ E_{total} = \frac{\sigma}{\varepsilon_0}}$

3.6 Spherical Symmetry

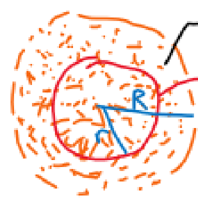
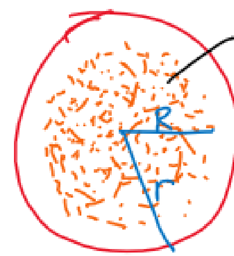


When $r \geq R_1$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

When $r < R$.

$$E = 0$$

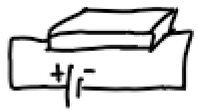
Internal Field	External Field
 $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2}}$ $\rho' = \frac{q'}{\frac{4}{3}\pi r^3}$	 $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$ $\rho = \frac{q}{\frac{4}{3}\pi r^3}$
Charge is uniformly spreaded $\frac{q'}{\frac{4}{3}\pi r^3} = \frac{q}{\frac{4}{3}\pi R^3}$ $q' = q\frac{r^3}{R^3}$	$\rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q\left(\frac{r}{R}\right)^3}{r^2}$ $\boxed{E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r}$ External

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Conductors

metals - good conductors

$\varepsilon \rightarrow$ $\leftarrow \ominus$



- charge move due to electrons

Shiny (reflect light)

Insulators

not good conductors

(e^-s tied)

Semiconductors

2 charge carriers

(act as +) hole

(-) Electron

4-1 Electric Potential

$$\Delta U = U_f - U_i = W_{appl} = -W_{field} \; [J]$$

- ΔU: Change in potential energy
- U_f: Potential Energy Final state
- U_i: Potential Energy Initial state
- W_{appl}: Work done externally (applied to system)
- W_{field}: Work done by field

$$\Delta U = U_f - U_i = \underbrace{W_{applied}}_{\text{applied work}} = -\underbrace{W_{field}}_{\text{Work done by field}} \underbrace{(= W)}_{\text{Book convention}}$$

(ΔE_p)

Initial	Initial
$\leftarrow\oplus \quad \oplus \xrightarrow{F}$	$\oplus\rightarrow \quad \leftarrow\ominus$
Final	**Final**
$\leftarrow\oplus\oplus\rightarrow$	$\oplus\ominus$
$W_{appl} > 0$ (Force need to be applied) $W_{field} < 0$ $\Delta U > 0$	$W_{appl} < 0$ $W_{field} > 0$ → work done by electric field $\Delta U < 0$

Conservation of Energy

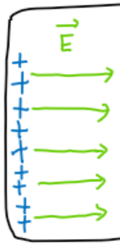
$$U_i + K_i + W_{appl} = U_f + K_f$$

(If assume $K_i = K_f$, $W_{appl} = U_f - U_i = -W_{field}$)

Potential	Gravitational	Electrical	Relationships
Work by external agent	initial (work applied, F_g) ⇒ final	initial (motion; F, F) ⇒ final (F, F)	$W_{appl} > 0 \Leftrightarrow W_{field} < 0$ $U_f > U_i \Rightarrow \Delta U > 0$
Work done by field	initial ⇒ final (F_g)	initial (F) ⇒ final (F, F)	$W_{appl} < 0 \Leftrightarrow W_{field} > 0$ $U_f < U_i \Rightarrow \Delta U < 0$

Electric Field

Whenever Charge goes against / along $\vec{E}$, $\Delta V > 0$ / $\Delta V < 0$



motion (repel) $\oplus \rightarrow$ Going to move along field line ($\vec{E}$)

motion (attraction) $\leftarrow \ominus$ Going to move against field line

$$\vec{E} = \frac{\vec{F}}{q} \ [N/m]$$ "Force experienced by charge q"

$$\Delta U = U_f - U_i = W_{appl} = -W_{field}$$

Work [J] = Force × distance [Nm]

Change in electrostatic potential: $\frac{\Delta U}{\boxed{q}} = \frac{U_f - U_i}{q} = \frac{W_{appl}}{q} = -\frac{W_{field}}{q}$

Enter q with its sign

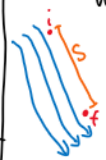
Electrostatic Potential = Voltage: $\Delta V = V_f - V_i \quad [J/C] \rightarrow [V]$

$\Delta U = U_f - U_i$
$-\Delta V = V_f - V_i$
"Change" of Ep and Voltage (NOT absolute value)

$$\boxed{\Delta V = \frac{\Delta U}{q} [V] \quad \Big| \quad \Delta U = q \Delta V}$$

E required to move e (1.602×10^{-19}C) in 1V ⇒ $\boxed{1eV = 1.602 \times 10^{-19} J}$

$$W_{field} = \vec{F}_{Final} \cdot \vec{S}$$ ($\vec{S}$: Displacement)



$$dW_{field} = \vec{F}_{Final}\, d\vec{S} = \vec{E} \cdot q\, d\vec{S}$$

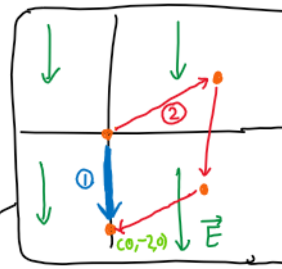
$$\int dW_{field} = \int_i^f \vec{E} \cdot q\, d\vec{S}$$

$$-W_{appl} = W_{field} = q \int_i^f \vec{E}\, d\vec{S}$$

$$\boxed{\Delta V = V_f - V_i = -\int_i^f \vec{E}\, d\vec{S}}$$

$d\vec{S}$: differential length element
↳ written in rectangular coordinate system (x, y, z)

$$d\vec{S} = d_x \hat{i} + d_y \hat{j} + d_z \hat{k}$$



① $\Delta V = V_{(0,-2,0)} - V_{(0,0,0)}$

$$= -\int_{(0,0,0)}^{(0,-2,0)} (E(-\hat{j}))(\cancel{dx\hat{i}} + dy\hat{j} + \cancel{dz\hat{k}})$$

$$= E_0 \int_{(0,0,0)}^{(0,-2,0)} dy = E_0 [y]_0^{-2} = -2E_0$$

$$\Delta V_① = \Delta V_②$$

$$\boxed{\Delta V = -E \Delta X}$$

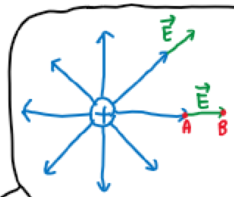
"Conservative field"

- Change in electrostatic potential and potential energy are path independent
- depends on initial and final values

4-2
-3

Equipotential Surfaces, electric field and potential due to charged particle

Potential Due to charged particle



"Potential difference between A and B?"

$r_2 \to \infty$

$V_{r_2} \to 0$

$$\boxed{V_{r\to\infty} = \frac{Q}{4\pi\varepsilon_0 r}}$$

Charge → Positive → Positive voltage

Charge → negative → Negative voltage

$$\Delta V = V_f - V_i = -\int_i^f \vec{E}\, d\vec{s}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a_r}$$

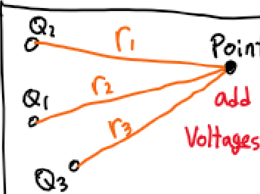
$d\vec{s} = dr\,\hat{a_r}$ (Spherical coordinate)

$$\Delta V = -\int_i^f \vec{E}\, d\vec{s} = -\int_{r_1}^{r_2} \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a_r}\, dr\,\hat{a_r}$$

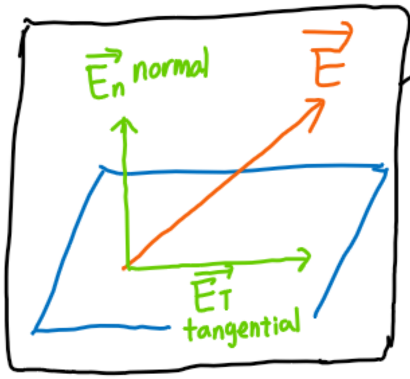
$$= -\frac{Q}{4\pi\varepsilon_0}\int_{r_1}^{r_2} \frac{1}{r^2}\, dr = -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r_1}^{r_2}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$\boxed{\Delta V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{kQ}{r_2} - \frac{kQ}{r_1}}$$



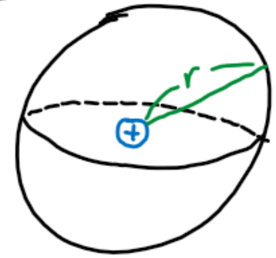
Equipotential surfaces



$$V_r = \frac{Q}{4\pi\varepsilon_0 r}$$

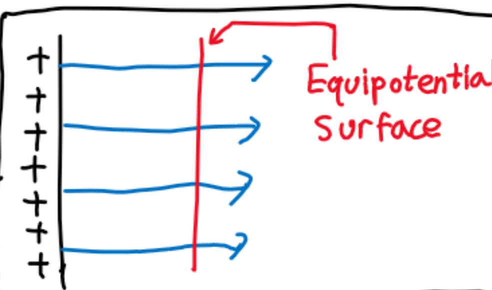
"Equipotential Surfaces"

Points in space that all points on equipotential surface has same electric potential or potential energy



ANY point on surface → same potential

Always perpendicular to electric field



4-7 Electric Potential Energy of a system of charged particles

$\Delta U = U_f - U_i$
$= q(V_f - V_i)$

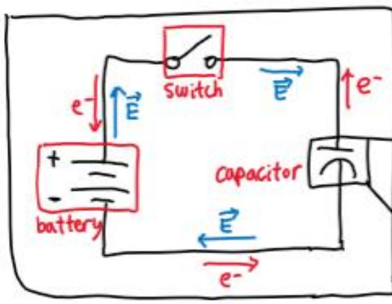
$V = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r}$ (V)

$U = q_1 \Delta V = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$

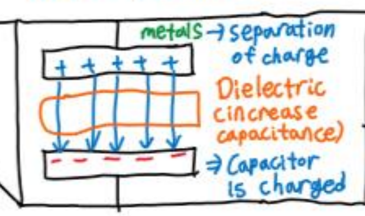
$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r} = \frac{k q_1 q_2}{r}$

assume $U_i = 0$
(assume q_1 located at $r=\infty$ initially)

5-1 5-2 Capacitance



Perfect wire
↳ Conduit to charge (electron) to move.

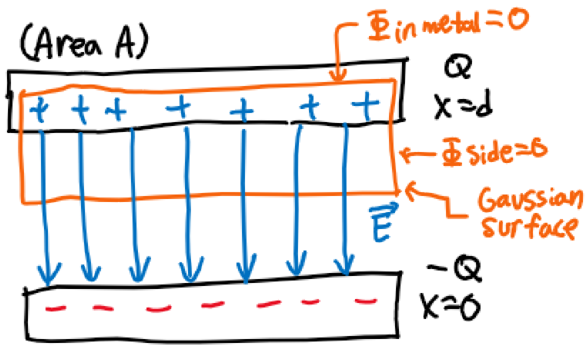


Capacitance = $\frac{Q}{V}$
→ NOT total charge.
↳ charge stored in one plate
↳ Potential Difference between two plates

$C = \frac{q}{V}$ [F] Farad.

* 1F is very large!

Parallel Plate Capacitor



- Electric Field

$$\Phi = \oint_S \vec{E}\, d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$(Q_{enc} = Q)$

$$\oint_B \vec{E}\, d\vec{A} = \frac{Q}{\varepsilon_0} \quad (\angle(\vec{E}\cdot d\vec{A}) = 0)$$

$$E \iint dA = EA = \frac{Q}{\varepsilon_0}$$

$$\boxed{E = \frac{Q}{A\varepsilon_0} = \frac{\sigma}{\varepsilon_0} \qquad \vec{E} = \frac{Q}{A\varepsilon_0}(-\hat{i})}$$

- Potential Difference

$$V = -\int \vec{E}\, d\vec{s}$$

$^*E\ [V/m]$

$$V_{x=d} - V_{x=0} = -\int_{x=0}^{x=d} \frac{Q}{A\varepsilon_0}(-\hat{i})\, dx\, \hat{i}$$

(remember $d\vec{s} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$)

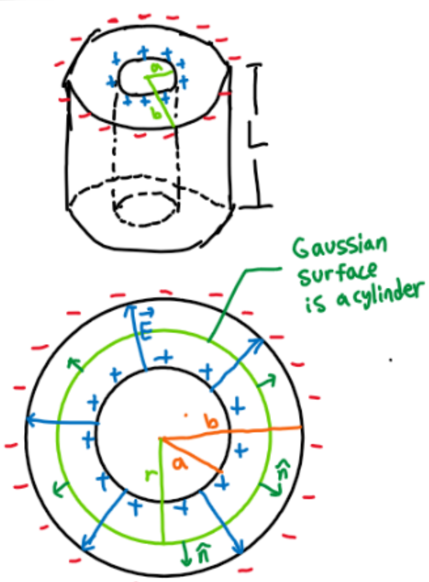
$$\boxed{V = \frac{Q}{A\varepsilon_0} d.}$$

$^*\varepsilon_0 = 8.854 \times 10^{-12}\,[F/m]$

$$\boxed{C_{\parallel} = \frac{Q}{V} = \frac{A\varepsilon_0}{d}\,[F]}$$

Let's say dielectric constant is 1. for now...

Cylinderical capacitor



- Electric Field

$$\oint \vec{E}\, d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \quad (Q_{enc} = Q)$$

$$\oint_{side} \vec{E}\, d\vec{A} = \frac{Q}{\varepsilon_0} \quad (\angle(\vec{E}, \hat{n}) = 0)$$

$$E \cdot 2\pi r L = \frac{Q}{\varepsilon_0}.$$

$$\boxed{E = \frac{Q}{2\pi\varepsilon_0 L r} \qquad \vec{E} = \frac{Q}{2\pi\varepsilon_0 L r}\hat{r}}$$

$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$ direction vector

$^*r$ is radius of Gaussian surface

- Potential Difference

$$V = V_{r=a} - V_{r=b} = -\int_{r=b}^{r=a} \vec{E}\, d\vec{s}$$

$(d\vec{s} = dr\,\hat{r} + d\phi\,\hat{\phi} + dz\,\hat{z})$ $d\vec{s} = dr\,\hat{r}$

$$V = -\int_b^a \frac{Q}{2\pi\varepsilon_0 L r}\hat{r}\, dr\, \hat{r}$$

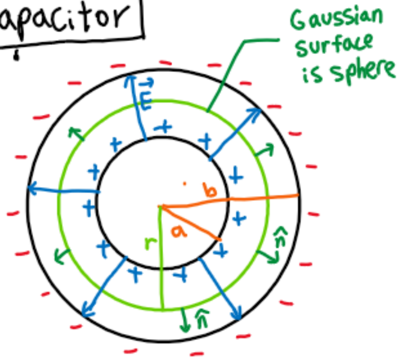
$$= -\frac{Q}{2\pi\varepsilon_0 L}\int_b^a \frac{1}{r}\, dr$$

$$= -\frac{Q}{2\pi\varepsilon_0 L}\ln\frac{a}{b}$$

$$\boxed{V = \frac{Q}{2\pi\varepsilon_0 L}\ln\frac{b}{a}}$$

$$\boxed{C_{cyl} = \frac{Q}{V} = \frac{2\pi\varepsilon_0 L}{\ln\frac{b}{a}}}$$

Spherical capacitor



Electric Field

$$\oiint_S \vec{E}\, d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

This is where $\vec{E} = \frac{kQ}{r^2}$ from.

Potential Difference

$$V = V_a - V_b = -\int_{r=b}^{r=a} \vec{E}\, d\vec{s}$$

$$= -\int_b^a \left(\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}\right) dr\,\hat{r}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_b^a$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)}$$

Capacitance

$$C = \frac{Q}{V} = \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}$$

$$\boxed{C_{sph} = \frac{4\pi\varepsilon_0 ab}{a-b} \quad (C>0)}$$

Capacitance of singlular sphere

$$C = \frac{4\pi\varepsilon_0 a}{1 - \frac{a}{b}}$$

$b \to \infty$,

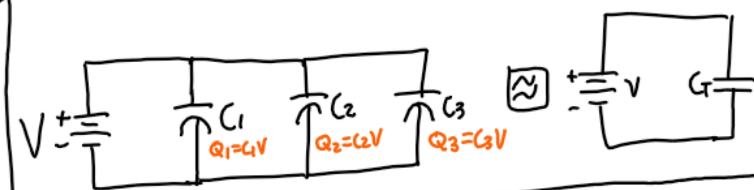
$$C_{\infty \to a} = 4\pi\varepsilon_0 a.$$

$$\boxed{C_{singsph} = 4\pi\varepsilon_0 r}$$

5-3 Capacitors in parallel/series

Parallel (||) Connection

Every capacitor has EQUAL voltage



$$Q_T = Q_1 + Q_2 + Q_3$$

$$\downarrow\ Q_T = C_T V$$

$$C_T = C_1 + C_2 + C_3$$

Series (↔) Connection

$$V_T = V_1 + V_2 + V_3$$

$$Q_T = Q_1 = Q_2 = Q_3$$

(conservation of energy)

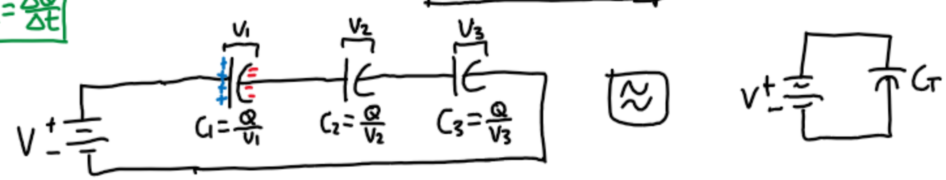
$$\boxed{I = \frac{\Delta Q}{\Delta t}}$$

$$V = \frac{Q}{C}.$$

$$V_T = \frac{Q}{C_T} = \sum \frac{Q}{C}$$

$$\frac{1}{C_T} = \sum \frac{1}{C}$$

$$\boxed{\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$



5-4 Energy stored in Electric Field

How much E can capacitor store?

$$\Delta V = \frac{\Delta U}{q} \qquad [C = \frac{q}{V} = \frac{A\varepsilon_0}{d}]$$

$$\Delta U = q\Delta V$$

$$U = \int dU = \int q\,dV = \int CV\,dV$$

$$\boxed{U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV}$$

Energy Density (u)

$$u = \frac{U}{\text{Volume}} \quad [J/m^3]$$

$$u_{\parallel} = \frac{U}{Ad}$$

$$u_{\parallel} = \frac{CV^2}{2Ad}$$

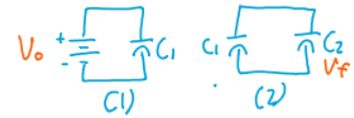
$$u_{\parallel} = \frac{K\varepsilon_0 A}{2d}\frac{E^2 d^2}{Ad}$$

$$\boxed{U = \frac{1}{2}CV^2 \quad C = \frac{\varepsilon_0 KA}{d} \quad V = Ed}$$

$$\boxed{u_{\parallel} = \frac{1}{2}K\varepsilon_0 E^2 = \frac{1}{2}K\varepsilon_0 \vec{E}\cdot\vec{E}}$$

EX

Capacitor C_1 is charged by battery (V_0). Then disconnected and connected to uncharged capacitor C_2. Final voltage of combination?



$Q_{(1)} = Q_{(2)}$, $V_0 \neq V_f$

$$C_1 V_0 = C_f V_f = (C_1 + C_2)V_f$$

$$V_f = \frac{C_1}{C_1 + C_2}V_0$$

5-5 Capacitor with a dielectric

"How can we store more E in capacitor?"

$C = \frac{\varepsilon_0 A}{d}$
- increase area
- decrease distance between 2 plates.

inefficient?

→ Put dielectric between two pieces of metals.

Dielectric constant

$K = \varepsilon_r$ "Relative permittivity" → As a number.

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m → vacuum)

$K_{Si} = 12$

$K_{Ge} = 16$

$K_{polyestorine} = 2.6$

$K_{vacuum} = 1$

($K > 1$ for anything else).

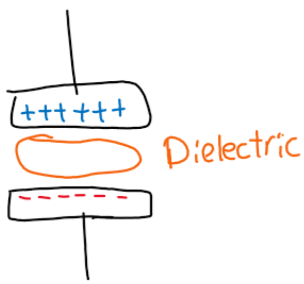
$$\boxed{\begin{aligned} C_{parallel} &= \frac{K\varepsilon_0 A}{d} \\ C_{cylinderical} &= \frac{K\varepsilon_0 2\pi L}{\ln(\frac{b}{a})} \\ C_{spherical} &= \frac{K\varepsilon_0 4\pi ab}{a-b} \end{aligned}}$$

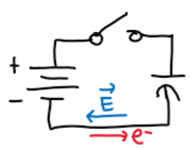
$$F = \frac{1}{4\pi K\varepsilon_0}\frac{q_1 q_2}{r^2}$$

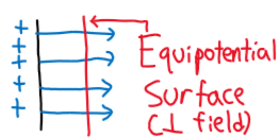
$$E = \frac{Q}{4\pi K\varepsilon_0 r^2} = \frac{\sigma}{K\varepsilon_0}$$

$$V = \frac{Q}{4\pi K\varepsilon_0 r}$$

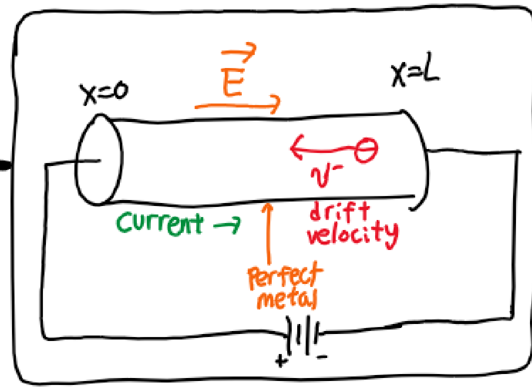
$$U = \frac{Qq}{4\pi K\varepsilon_0 r}$$







6-1 | 6-3 | 6-4 Electric current, Resistance and Ohm's Law



$$\boxed{C = \frac{Q}{V} = \frac{A\varepsilon_0}{d} \, [F]}$$

$$\boxed{R = \frac{V}{I} \, [\Omega]}$$ ohm

$$\boxed{I = \frac{dQ}{dt} \, [A]}$$ Ampere

$(\int dQ = \int I dt)$

I - current (movement of charges)

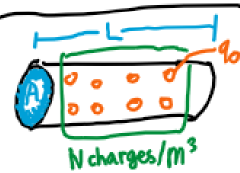
Doesn't have to be dependent on time

6-2 6-3 Resistivity and Current density

$$I = \iint \vec{J}\, d\vec{A} = JA$$

$$\boxed{J = \frac{I}{A}\ [A/m^2]}$$

Current density add up (never cancel out)



$$Q = q_0 N A L = q_0 N A V_d t$$

$$I = \frac{Q}{t} = q N A V_d$$

$$\boxed{\vec{J} = n\, q_0\, \vec{V}_d}$$

$\vec{J}$ – Current density $[A/m^2]$; n – Total charge per volume; q_0 – Elementary charge; V_d – drift speed

$$\boxed{\rho = \frac{|\vec{E}|}{|\vec{J}|}\ [\Omega m]}$$

ρ – Resistivity
$\vec{J}$ – Current density $[A/m^2]$
$\vec{E}$ – Electric Field $[V/m]$

$$\vec{J} = \frac{1}{\rho}\vec{E} = \sigma \vec{E}$$

$$\boxed{\sigma = \frac{1}{\rho}\ \left[\frac{1}{\Omega m}\right]}$$

conductivity

$$R = \frac{V}{I} = \frac{-\int_L^0 \vec{E}\, d\vec{s}}{JA} = \frac{-\int_L^0 E\hat{i} \cdot dx\hat{i}}{JA} = \frac{El}{JA}$$

$$R = \frac{El}{JA} = \rho \frac{l}{A}$$

6-5

Power in electric circuits

$dU = q\,dV$

$dV = R\,dI$

$\int^{U} dU = \int qR\,dI$

$\boxed{U = qRI}$

$Power = \frac{dU}{dt}$

$I = \frac{dq}{dt}$

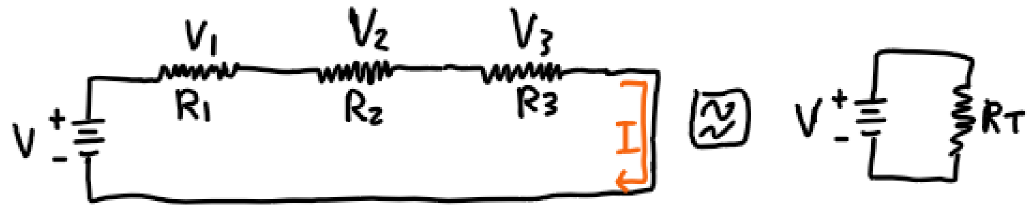
Power dissipated by resistor

$Power = \frac{dU}{dt} = RI\,\frac{dq}{dt}$

$\boxed{Power = I^2R = \frac{V^2}{R} = VI}$

6-A Parallel & series circuits

Series

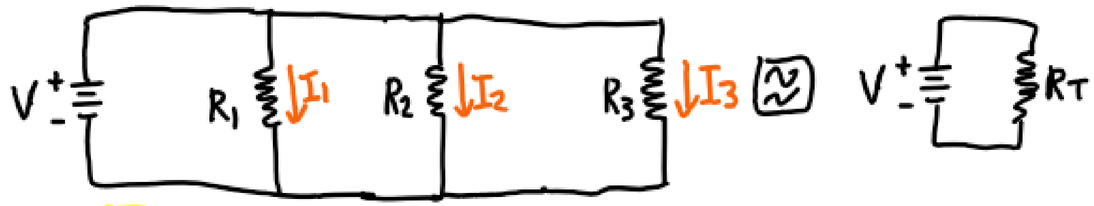


$$V = V_1 + V_2 + V_3$$
$$I = I_1 = I_2 = I_3$$
$$R = R_1 + R_2 + R_3$$

Kirchoff's Law

$$\Sigma V = 0$$

Parallel



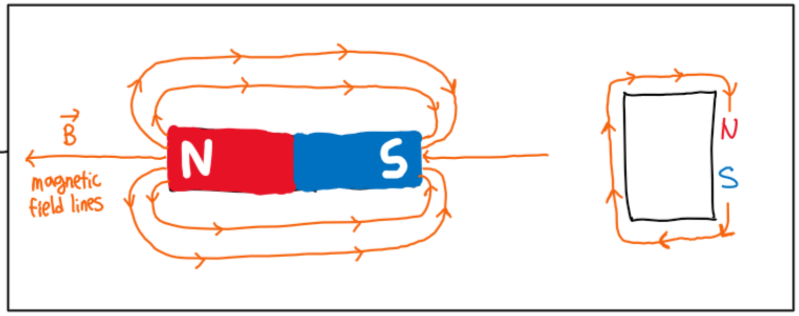
$$V = V_1 = V_2 = V_3$$
$$I = I_1 + I_2 + I_3$$
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

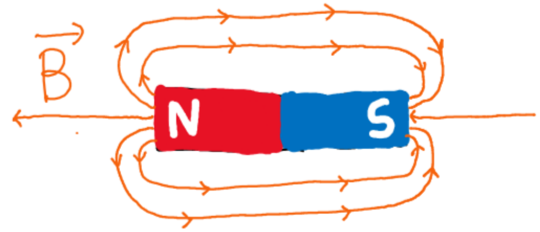
Kirchoff's Law

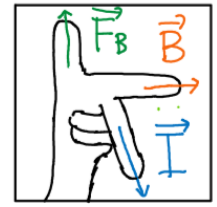
$$I_{in} = I_{out}$$
$$\Sigma I = 0$$

7-1 Magnetic Fields and Definition of $\vec{B}$

Magnetic field lines form a loop (no magnetic charge)







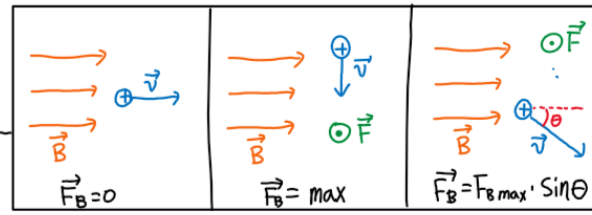
Current (movement of charge) produces magnetic field

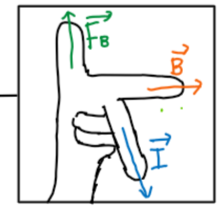
$\vec{F_E} = q\vec{E}$ Electric force

$$\boxed{\vec{F_B} = q\,\vec{v} \times \vec{B}}$$

$|\vec{F_B}| = q\,|\vec{v}|\,|\vec{B}|\,\sin\angle(\vec{v},\vec{B})$

q – Charge with algebraic sign
v – velocity of charge
B – magnetic field





$$\underset{\text{Lorentz Force}}{\vec{F}} = \vec{F_E} + \vec{F_B} = q\vec{E} + q(\vec{v} \times \vec{B})$$

Unit for magnetic field

$$\left[\frac{N \cdot S}{C \cdot m}\right] = \left[\frac{V}{m} \cdot \frac{S}{m}\right] = \left[\frac{Weber}{m}\right] = \left[\underset{\text{Tesla}}{T}\right]$$

↳ N/C = V/m ⤴

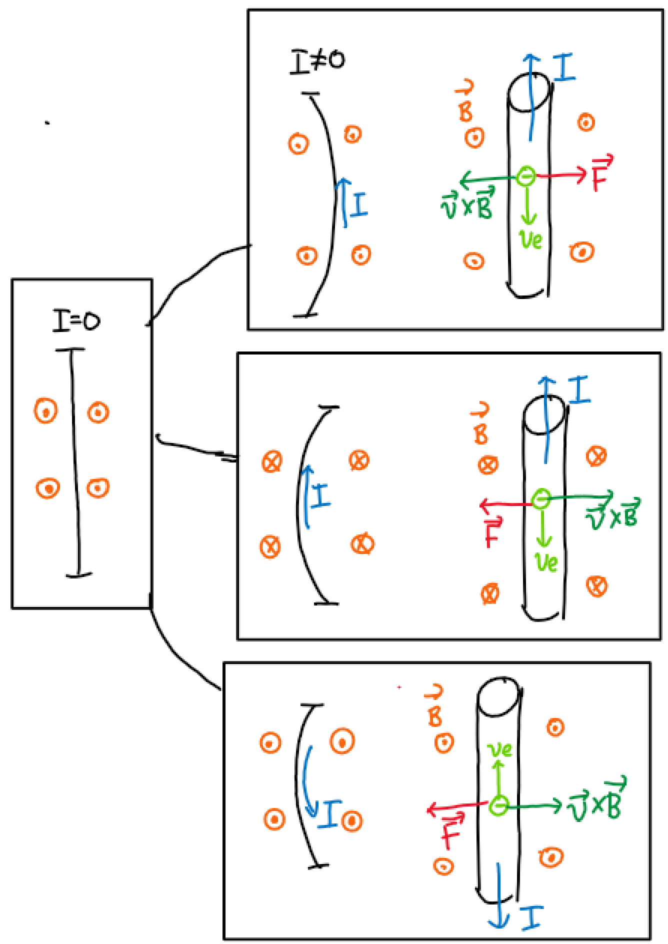
7-6 Magnetic force on a current-carrying wire

$$\vec{F}_B = q\vec{v}\times\vec{B}$$
$$F_B = Bqv\sin\theta$$

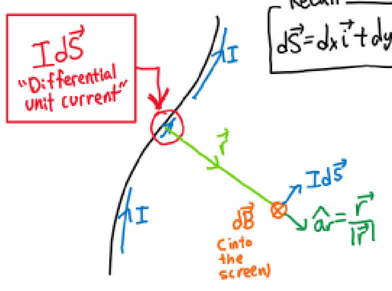
$$d\vec{F} = dq\,\vec{v}\times\vec{B} = I\,d\vec{\ell}\times\vec{B}$$
$$\vec{v} = \frac{d\vec{\ell}}{dt} \quad I = \frac{dq}{dt}$$

$$\vec{F}_B = I\,\vec{\ell}\times\vec{B}$$
$$F_B = BI\ell\sin\theta$$

"Wire is straight, $\vec{B}$ and I are constant"



8-1 Magnetic Field due to current



Recall
$d\vec{s} = dx\vec{i} + dy\vec{j} + dz\vec{k}$

Biot-Savart Law

$$d\vec{B} = \frac{\mu_0 I d\vec{s} \times \hat{a}_r}{4\pi r^2}$$

↳ magnetic field produced by small current $Id\vec{s}$

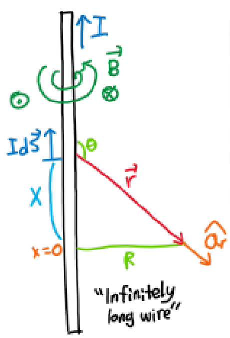
μ_0 - Permability of vacuum

$$\mu_0 = 4\pi \times 10^{-7} \, [H/m]$$

Henery

$$\left[\frac{H}{m} = \frac{Tm^2}{Am} = \frac{Tm}{A} = \frac{Vs}{A} \cdot \frac{1}{m}\right]$$

↔ ε_0 Permittibility



$$dB = \frac{\mu_0 I d\vec{S} \times \hat{a}_r}{4\pi r^2} = \frac{\mu_0 I dS \sin\theta}{4\pi r^2}$$

$$\int_{-\infty}^{\infty} dB = \int_{-\infty}^{\infty} \frac{\mu_0 I \sin\theta}{4\pi r^2} dS$$

$(dS = dX)$

$$B = \frac{\mu_0 I}{4\pi} \int_{x=-\infty}^{x=\infty} \frac{\sin\theta}{r^2} dX.$$

($r, \sin\theta$ depends on x)

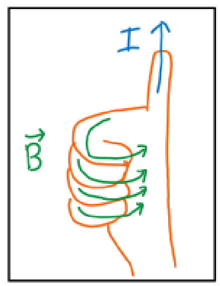
R - distance to measure

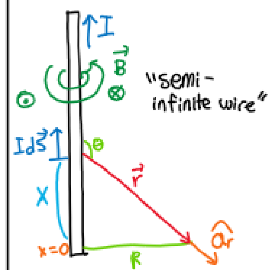
$r^2 = X^2 + R^2$

$\sin\theta = \sin(\pi - \theta) = \frac{R}{r} = \frac{R}{\sqrt{x^2+R^2}}$

$$B = \frac{\mu_0 I}{4\pi} \cdot 2 \int_0^{\infty} \frac{R \, dX}{(X^2+R^2)^{3/2}} = \frac{\mu_0 I}{2\pi} \left[\frac{X}{\sqrt{X^2+R^2}}\right]_0^{\infty}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi R}}$$





"semi-infinite wire"

$$B = \frac{\mu_0 I}{4\pi} \int_0^{\infty} \frac{\sin\theta}{r^2} dX$$

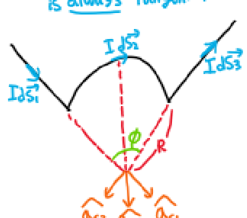
$r^2 = R^2 + X^2$

$\sin\theta = \frac{R}{\sqrt{R^2+X^2}}$

$$B = \frac{\mu_0 I}{4\pi} \int_0^{\infty} \frac{R \, dX}{(R^2+X^2)^{3/2}}$$

$$\boxed{B = \frac{\mu_0 I}{4\pi R}}$$

differential current is always tangential



Q $\vec{B}$ at centre of the curved section

$$d\vec{B}_T = d\vec{B}_1 + d\vec{B}_2 + d\vec{B}_3$$

$$d\vec{B} = \frac{\mu_0 I \, d\vec{s} \times \hat{a}_r}{4\pi r^2}$$

$d\vec{s}_1 \times \hat{a}_{r1} = 0 \rightarrow d\vec{B}_1 = 0$

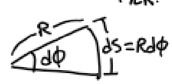
$d\vec{s}_3 \times \hat{a}_{r3} = 0 \rightarrow d\vec{B}_3 = 0$

$\angle(d\vec{s}_2, \hat{a}_{r2}) = 90°$

$$d\vec{B}_T = d\vec{B}_2 = \frac{\mu_0 I \, dS_2}{4\pi R^2}$$

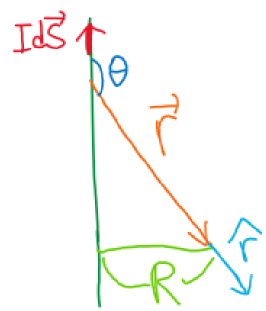
$$B_T = B_2 = \frac{\mu_0 I}{4\pi} \int_0^{\phi} \frac{dS_2}{R^2} = \frac{\mu_0 I}{4\pi} \int_0^{\phi} \frac{R \, d\phi'}{R^2} = \frac{\mu_0 I \phi}{4\pi R}.$$

(ϕ in radians)

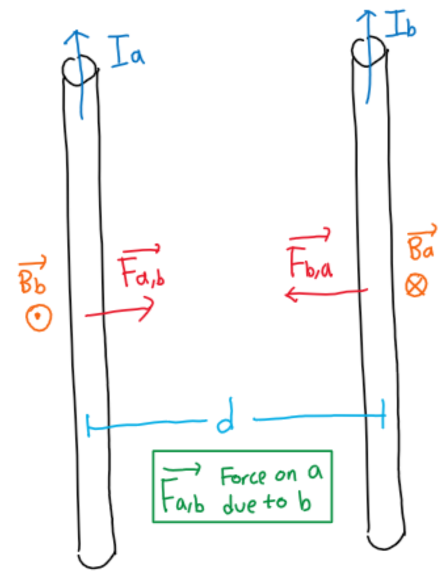


$\vec{B}$ at centre of loop

$$B = \frac{\mu_0 I \phi}{4\pi R}$$



8-2 Force between two parallel currents



$$B_a = \frac{\mu_0 I_a}{2\pi d}$$

$$\vec{F}_{b,a} = I_b \vec{l} \times \vec{B_a}$$

$$= I_b l B_a \underbrace{\sin(\vec{l}, \vec{B_a})}_{\sin 90^\circ = 1}$$

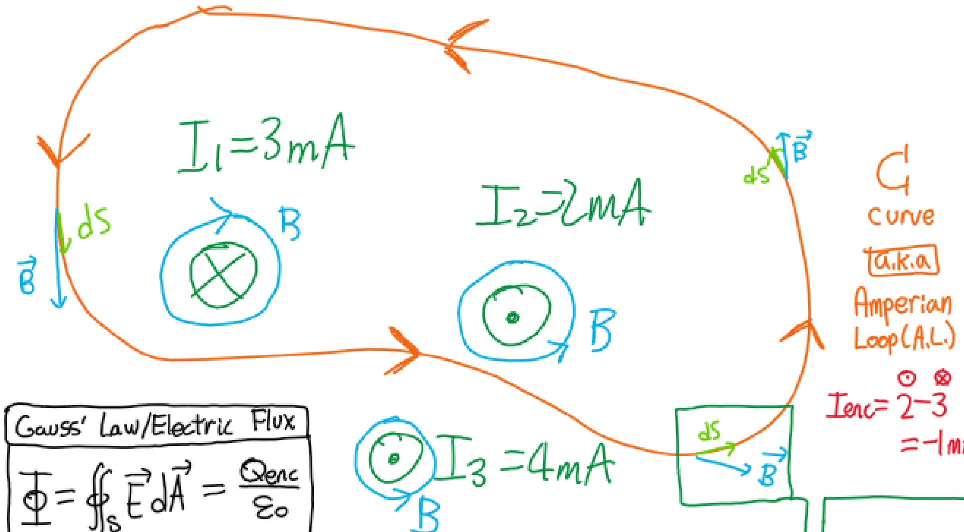
$$\vec{F}_{b,a} = I_b l B_a = I_b l \frac{\mu_0 I_a}{2\pi d}$$

$$\boxed{|\vec{F}_{b,a}| = |\vec{F}_{a,b}| = \frac{\mu_0 l I_a I_b}{2\pi d}}$$

Newton's 3rd law of motion

TIP $\boxed{\frac{\mu_0}{2\pi} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7}}$

8-3 Ampere's Law



$\odot \quad \otimes$

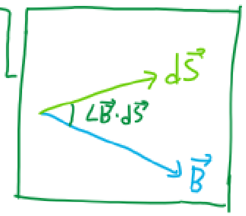
$I_{enc} = 2 - 3$
$= -1mA$

Gauss' Law/Electric Flux

$$\Phi = \oint_S \vec{E} d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\oint_C \vec{B} \cdot d\vec{S} = \mu_0 \underbrace{I_{enc}}_{\text{Enclosed current in loop (sign included)}}$$

Line integral

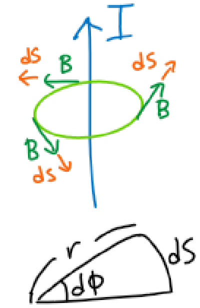


$$\oint_C \vec{B} \cdot d\vec{S} = \mu_0 I_{enc}$$

$$= \oint_C \overbrace{B \cos(\angle \vec{B}, d\vec{S})}^{\text{Projection of B along dS}} \, dS$$

$\vec{B} d\vec{S} = B dS \cos(\vec{B} \cdot d\vec{S})$

* $d\vec{S}$ always tangential to the curve



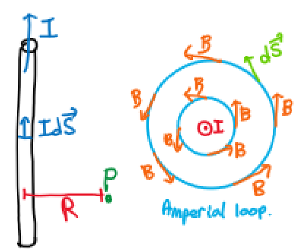
$$\oint_C \vec{B} d\vec{S} = \int B \cos(\vec{B}, d\vec{S}) dS = \mu_0 I_{enc}$$

$\cos(\angle \vec{B}, d\vec{S}) = 1$

$$\int B dS = \mu_0 I = \int_0^{2\pi} B R \, d\phi$$
$$= BR \cdot 2\pi$$

$$\boxed{B = \frac{\mu_0 I}{2\pi R}}$$

Choose ampherian loop/gausian surface CAREFULLY



$d\vec{S}$ is tangential

Magnitude of magnetic field remains same in loop

$$\oint_C \vec{B} \cdot d\vec{S} = \mu_0 I_{enc}$$
$$= \oint_C B \underbrace{\cos(\vec{B} \cdot d\vec{S})}_{\cos 0 = 1} dS$$

$\Rightarrow \mu_0 I = B \oint_C dS$

$(dS = R \, d\phi)$

$\mu_0 I = B \oint_C R \, d\phi$

$= BR \cdot 2\pi$

$$\boxed{B = \frac{\mu_0 I}{2\pi R}}$$

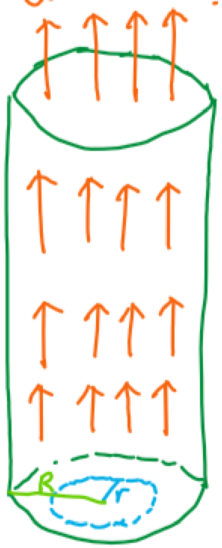
Q B if $0<r<R$?

(Magnetic field inside?)

Total current $= I$

$\vec{J} = J_0 \hat{k}$

(J_0 constant)



$$\oint_c \vec{B}\, d\vec{S} = \mu_0 I_{enc} \rightarrow \int B\, dS = \int_0^{2\pi} B r\, d\phi$$

$$= \int B \cos(\angle \vec{B}, d\vec{S})\, dS \qquad = 2\pi B r$$

$I_{tot} = \pi R^2 J_0$.

(total cylinder)

$$I_{enc} = \iint \vec{J}\, d\vec{A} = \pi r^2 J_0$$

$$\frac{I_{enc}}{I} = \frac{r^2}{R^2}$$

$$I_{enc} = \frac{r^2}{R^2} I$$

$$\oint \vec{B}\, d\vec{S} = 2\pi B r = \mu_0 I_{enc} = \mu_0 I_{tot} \frac{r^2}{R^2}$$

$$2\pi B r = \mu_0 I_{tot} \frac{r^2}{R^2}$$

$$\boxed{B = \frac{\mu_0 I_{tot}\, r}{2\pi R^2}}$$

B outside

$$B = \frac{\mu_0 I}{2\pi r}$$

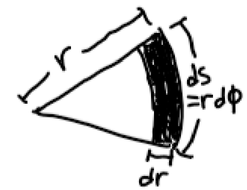
$$I_{enc} = \iint_S \vec{J}\, d\vec{A} = \iint_S J \cos(\vec{J}, d\vec{A})\, dA$$

$$= \iint_S J_0\, dA$$

$$= J_0 \int_{r=0}^{r} \int_0^{2\pi} r' dr' d\phi$$

$$= 2\pi \frac{r^2}{2} J_0 = \pi r^2 J_0$$

$$\boxed{I_{enc} = \pi r^2 J_0}$$



$\boxed{dA = r\, dr\, d\phi}$

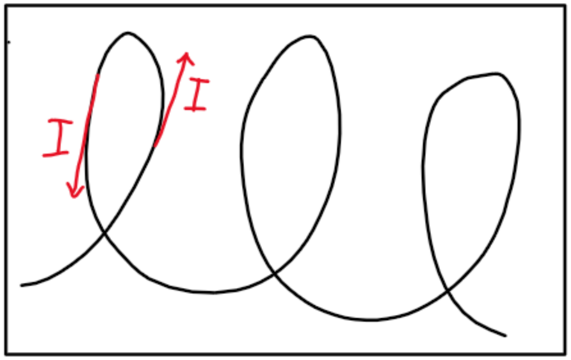
current depends on r, not ϕ

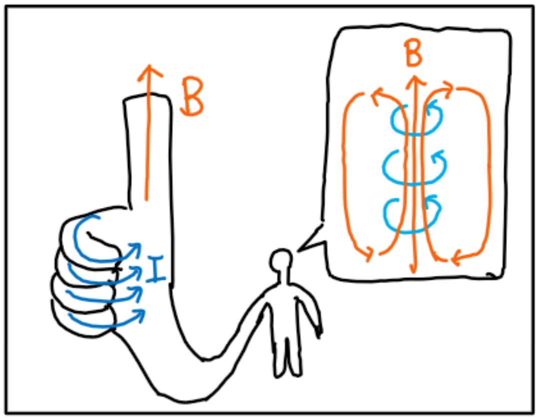
"J is $J(r)$ → not constant?"

$$I = \iint \vec{J}\, d\vec{A} = \iint J \cos(\angle \vec{J}\, d\vec{A})\, dA$$

$$= \iint J\, dA = \int_0^r \int_0^{2\pi} J(r) \cdot r\, dr\, d\phi$$

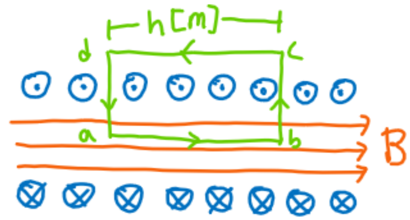
B-4 Solenoids





Ideal Solenoid

Magnetic field is ONLY inside loop.



→ No magnetic field outside
→ Cancelled

Let n "turns/meter"

ex 10 loops/cm
1000 loops/m.

$$\boxed{\begin{aligned} I_{enc} &= nhI \\ \mu_0 I_{enc} &= \mu_0 nhI \end{aligned}}$$

$$\oint \vec{B}\, d\vec{s} = \underbrace{\int_a^b \vec{B}\, d\vec{s}}_{\angle \vec{B}\cdot d\vec{s}=0} + \underbrace{\cancel{\int_b^c \vec{B}\, d\vec{s}}}_{\vec{B}\perp d\vec{s}} + \underbrace{\cancel{\int_c^d \vec{B}\, d\vec{s}}}_{\text{outside}} + \underbrace{\cancel{\int_d^a \vec{B}\, d\vec{s}}}_{\vec{B}\perp d\vec{s}}$$

$$= \int_a^b B\, ds = B(b-a) = Bh$$

Hence:

$\oint \vec{B}\, d\vec{S} = \mu_0 I_{enc} \rightarrow Bh = \mu_0 nhI$

$$\boxed{B = \mu_0 nI}$$

h doesn't matter

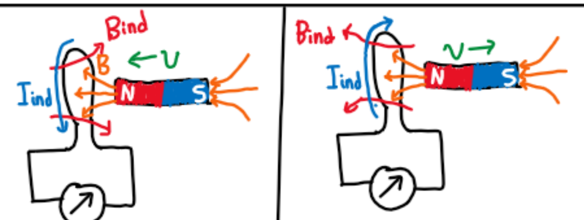
→ magnetic field in solenoid

9-1

Faraday's Law & Lenz's Law

$B = \frac{\mu_0 I}{2\pi r}$

Magnetic field induced by current $\longleftrightarrow$ Current induces magnetic field!



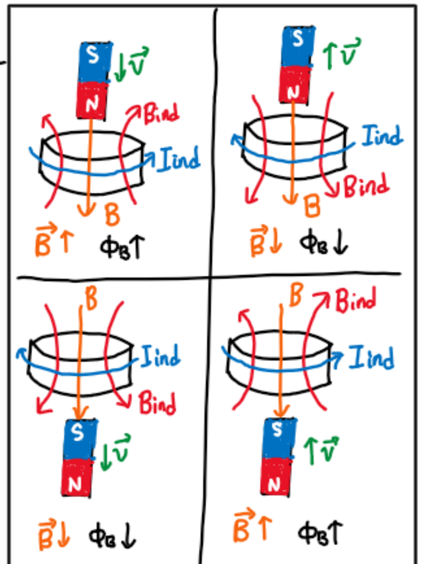
Φ_B More magnetic field comes in

Φ_B Less magnetic field comes in

Iind – induced current

EMF($\mathcal{E}$) – voltage

Bind – induced magnetic field



$$\mathcal{E} = -N\frac{\partial}{\partial t}\Phi_B = -N\frac{\partial}{\partial t}\iint \vec{B}\, d\vec{A}$$

<EMF in N loops>

$$\Phi_B = \iint \vec{B}\, d\vec{A} \quad [Wb]$$

Magnetic Flux; Open integral

$$\mathcal{E} = -\frac{\partial}{\partial t}\Phi_B = -\boxed{\frac{\partial}{\partial t}}\iint \vec{B}\, d\vec{A}$$

Partial derivative

Lenz' Law
Direction of induced current & EMF are against induced magnetic field

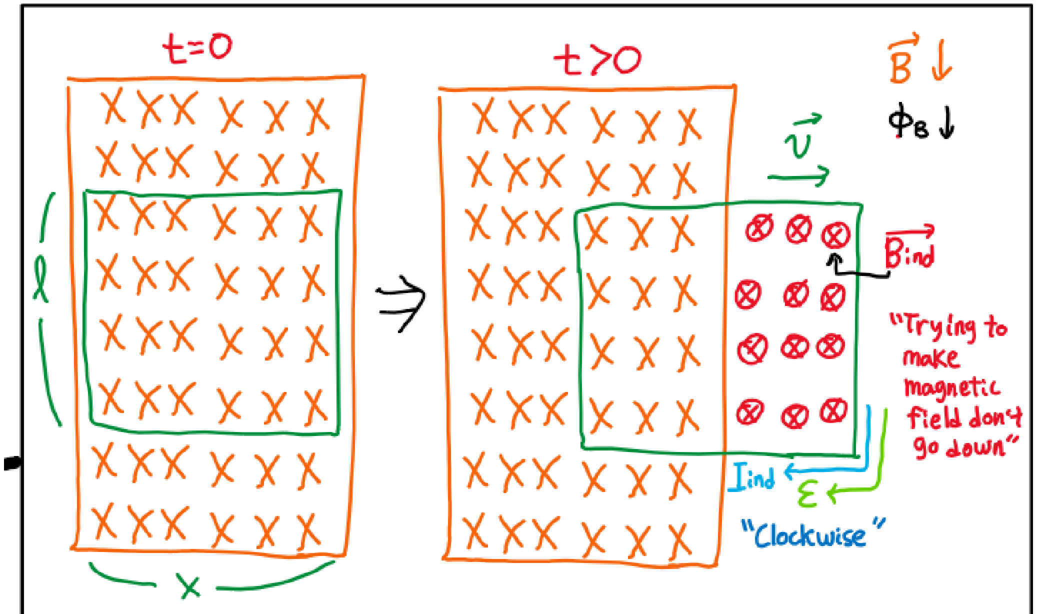
Φ_B is function of space & time

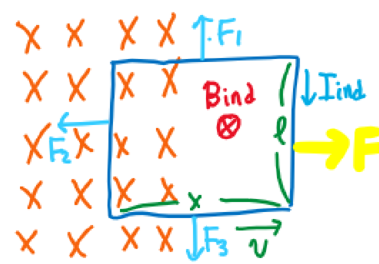
If Φ_B doesn't change over time, $\mathcal{E}$ (EMF) = 0

Faraday's Law
EMF is proportional to change in magnetic flux w/rt time

9-2

Induction & Energy Transfer





$P_{supp} = \vec{F}\cdot\vec{v}$

$\vec{F}_{wire} = I_{ind}(\vec{x}\times\vec{B}_{orig})$

$\vec{F}+\vec{F_2}=\vec{F}_{net}=m\vec{a}$

If $a=0$, $|\vec{F}|=|\vec{F_2}|$

$\vec{F_2}=I_{ind}\vec{L}\times\vec{B}$ ($L\perp B$)

$= I_{ind}LB = F$

$P_{supp} = Fv = I_{ind}LBv$

$= \frac{B^2L^2v^2}{R}$ (single loop)

$$|\mathcal{E}| = \left|-\frac{\partial}{\partial t}\Phi_B\right|$$

$I_{ind} = \frac{\mathcal{E}}{R}$ (Ohm's law)

$\Phi_B = \iint \vec{B}\,d\vec{A} = BLx$ ($\vec{B}\,/\!/\,d\vec{A}$)

$|\mathcal{E}| = -\frac{\partial}{\partial t}\Phi_B = BL\frac{dx}{dt} = BvL$

One loop

$\mathcal{E} = (-)Bv_x\ell$

N loops

$\mathcal{E} = (-)NBv_x\ell$

$I = \frac{NBv_x\ell}{R}$

[Power dissipated]

$P = \frac{V^2}{R} = VI = I^2R$

$= \frac{N^2B^2v^2\ell^2}{R}$ [W]

Power provided by external Agent

$P_{supply} = P_{dissipated}$

Conservation of energy

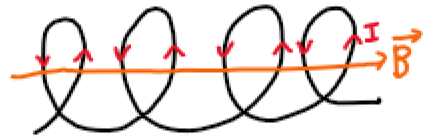
9-4

Inductors & Inductance

Capacitor $Q = CV$

Resistor $V = IR$

Solenoid (Loop) is an inductor



$B = \mu_0 n I$

[Inductance (L)]

$$L = \frac{N\Phi_B}{I} \quad [H]$$

N - total # of turns

$$\Phi_B = \iint \vec{B}\,d\vec{A} = BA = \mu_0 n I A$$

$$L = \frac{BAN}{I} = N\mu_0 n A$$

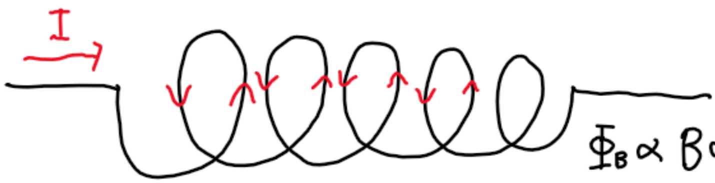
$$N = n\ell$$

$$L = \ell n^2 A \mu_0$$

$(\mu_0 = 4\pi \times 10^{-7}\ H/m)$

9-5

Self-Induction



$\Phi_B \propto B \propto I$

I increase / decrease $\Rightarrow \mathcal{E}_L$ left(←) / right(→) $\Rightarrow \Phi_B$ increase / decrease

$\mathcal{E}_L$ - Self induced emf

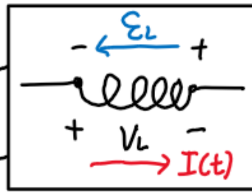
$$\boxed{\mathcal{E}_L = -N\frac{\partial}{\partial t}\Phi_B}$$

$$L = \frac{N\Phi_B}{I} \Rightarrow N\Phi_B = LI$$

$$\boxed{\begin{aligned}\mathcal{E}_L &= -N\frac{\partial}{\partial t}\Phi_B \\ &= -L\frac{d}{dt}I(t)\end{aligned}}$$

$$\boxed{V_L = -\mathcal{E}_L = L\frac{d}{dt}I(t)}$$

V_L - Voltage across inductor



9-7

Energy stored in magnetic field

$U = qV$ (Energy)

$$dU = dq\,V = q\,dV$$

$$I = \frac{dq}{dt} \Rightarrow dq = I\,dt$$

$$\text{Power} = \frac{dU}{dt} = IV$$

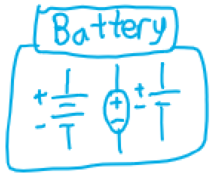
$$V = L\frac{d}{dt}I$$

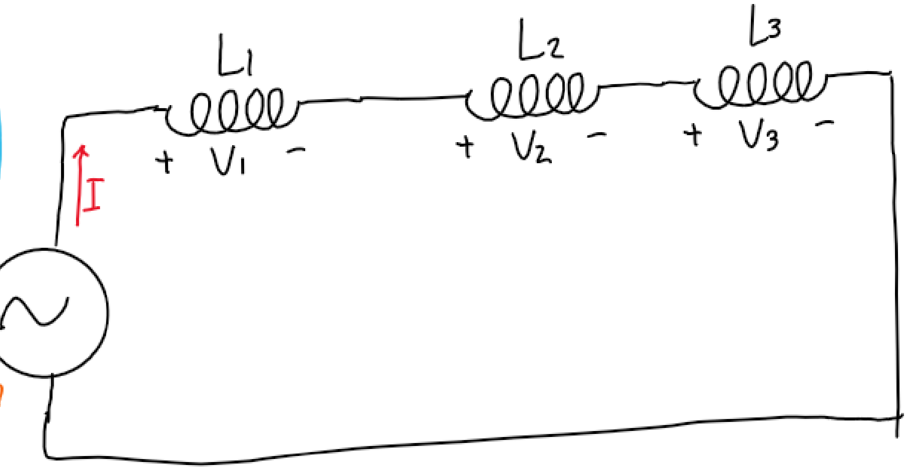
$$\frac{dU}{dt} = I\,L\left(\frac{dI}{dt}\right)$$

$$dU = I\,L\,dI$$

$$\boxed{U = \frac{1}{2}LI^2}$$

9-A Series & Parallel Inductors





Not a battery, AC circuit?

Series

$$V = V_1 + V_2 + V_3$$

$$V = L\frac{dI}{dt} \rightarrow L_T\frac{dI}{dt} = (L_1 + L_2 + L_3)\frac{dI}{dt}$$

$$\boxed{L_T = L_1 + L_2 + L_3}$$

Parallel

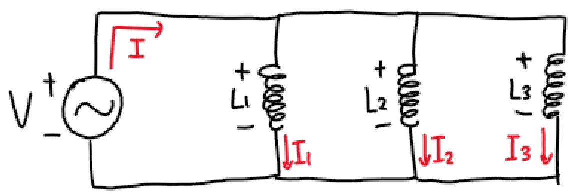
$$I = I_1 + I_2 + I_3$$

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} + \frac{dI_3}{dt}$$

$$V = L\frac{dI}{dt}$$

$$\frac{V}{L_T} = \frac{V}{L_1} + \frac{V}{L_2} + \frac{V}{L_3}$$

$$\boxed{\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$



10-R Basic Concepts

System of units

Basic SI units

Length	meter	m
Time	second	s
Current	Ampere	A
mass	Kilogram	kg
temper-ature	Kelvin	K
luminosity	Candela	cd

SI prefixes

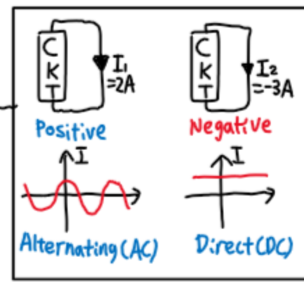
10^{-15}	10^{-12}	10^{-9}	10^{-6}	10^{-3}	1	10^{3}	10^{6}	10^{9}	10^{12}	10^{15}
femto	pico	nano	micro	milli		kilo	mega	giga	tera	Peta
f	p	n	μ	m		k	M	G	T	P

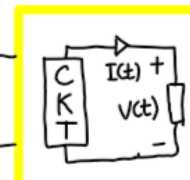
Basic Quantities

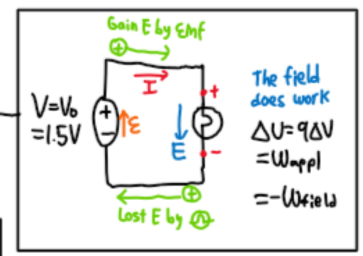
Polarity
└ the way which current is moving

$$I(t) = \frac{dq(t)}{dt}$$

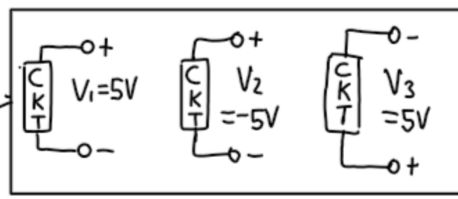
$$q(t) = \int_{-\infty}^{t} I(x)\,dx$$







$$V(t) = \frac{W(t)}{q}$$



$P_{dissipated}$ positive
$P_{supplied}$ negative
[Convention]

Positive current enters positive terminal of element
⇒ Power is positive

positive current enters negative terminal of element
⇒ Power is Negative

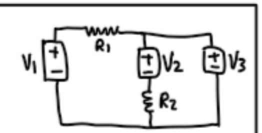
$$V = \frac{dW}{dq}$$

$$VI = \frac{dW}{dq}\frac{dq}{dt} = \frac{dW}{dt} = P$$

$$\Delta W = \int_{t_1}^{t_2} P\,dt = \int_{t_1}^{t_2} VI\,dt$$

$$\boxed{P = IV}$$

Battery can use power!
→ unit !!



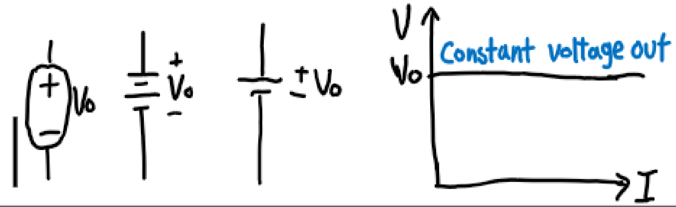
2 terminal devices

Resistor		$V = IR$
Capacitor		$I_C = C\frac{dV}{dt}$
Inductor		$V_L = L\frac{dI}{dt}$

10-3 Circuit elements

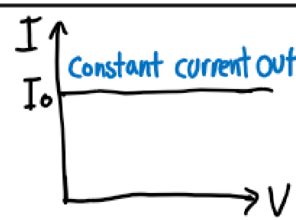
Independent sources

Independent voltage source



Independent current source

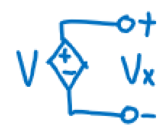
Transistors/ Diodes



Dependent sources

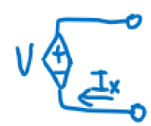
Dependent voltage source

Voltage-Control (dependent) Voltage source



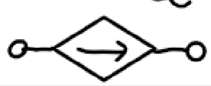
$$\boxed{V=\alpha\cdot V_x}$$

current-control (dependent) voltage source

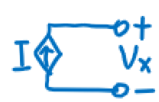


$$\boxed{V=r\cdot I_x}$$

Dependent current source

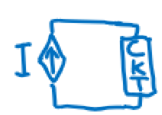


Voltage-Control (dependent) current source



$$\boxed{I=g\cdot V_x}$$

current-control (dependent) current source



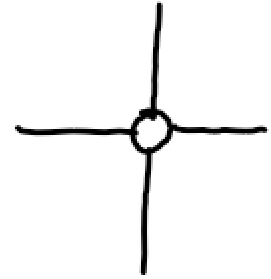
$$\boxed{I=\beta\cdot I_x}$$

10-F

Node
Branch
Lööp

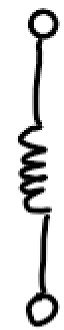
Node

Point on circuit that $\boxed{2+}$ elements come together
→ Kirchoff's current law



Branch

Section of circuit that contains only one element (and $\boxed{1+}$ terminals)
→ mathematical line (can be stretched)



Lööp

Closed path:
Start from one node
→ visit other node (ONLY once)
→ return back to origin node

→ Kirchoff's voltage law

11-1 Ohm's Law

$$V(t)=R\cdot I(t) \quad \boxed{R\geq 0}$$

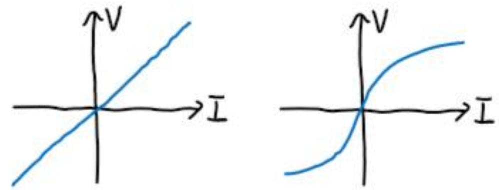
$$P(t) = V(t)\,I(t) = R\,I^2(t) = \frac{V^2(t)}{R}$$

$R=0 \quad V(t)=R\,I(t)=0$

$R=\infty \quad I(t)=\frac{V(t)}{R}=0$

$$I(t)=G\cdot V(t)$$

$$P(t)=\frac{I^2(t)}{G} = G\,V^2(t)$$



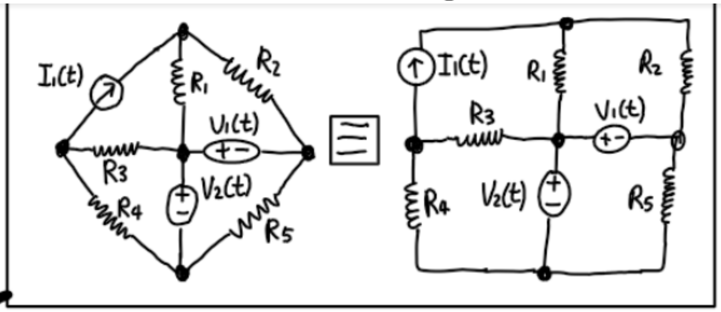
Ohmic

Non-ohmic

Conductance

$$G=\frac{1}{R}=\frac{I}{V}\ [S]$$

11-2 Kirchhoff's Laws



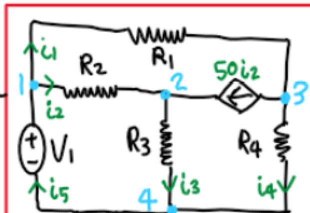
Kirchhoff's current law

Algebraic sum of current entering & leaving at each node must be zero

$$\sum_{j=1}^{N} I_j(t) = 0$$

$$\sum I_{enter} = \sum I_{leave}$$

Book	Current enter (+) Current leave (−)	NO DIFF
Prof	Current enter (−) Current leave (+)	



NODE1 $i_5 = i_1 + i_2$

NODE2 $(50i_2 + i_2) = i_3$

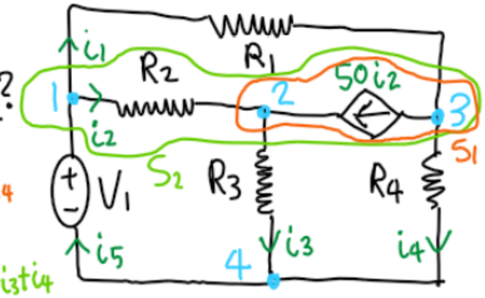
NODE3 $i_1 = 50i_2 + i_4$

NODE4 $i_3 + i_4 = i_5$

KCL for Closed surface?

Surface 1 $i_1 + i_2 = i_3 + i_4$

Surface 2 $\cancel{i_1} + i_5 = \cancel{i_1} + i_3 + i_4$



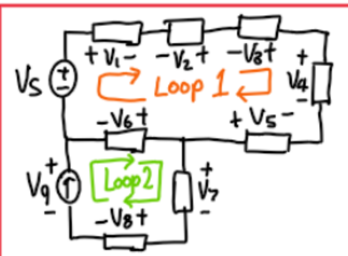
Kirchhoff's voltage law

Algebraic sum of voltages around a loop must be zero

$$\sum_{j=1}^{N} V_j(t) = 0$$

$$\sum V_{inc} = \sum V_{dec}$$

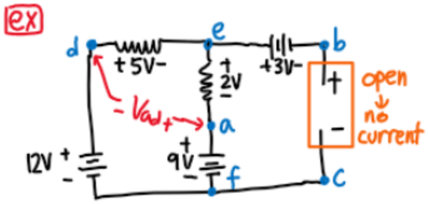
Book	V drop + V rise −	NO DIFF
Prof	V drop − V rise +	



Loop 1 ⇒ $V_1 + V_4 + V_6 = V_s + V_2 + V_3 + V_5$

Loop 2 ⇒ $V_9 + V_6 = V_7 + V_8$

ex



$V_{af}, V_{ad}, V_{bc}, V_{bf}$?

$V_{fd} = V_{de} + V_{ea} + V_{af}$

$12 = 5 + (-2) + V_{af}$

$V_{af} = 9V$

$V_{ad} + V_{fd} = V_{af}$

$V_{ad} + 12 = 9$

$V_{ad} = -3V$

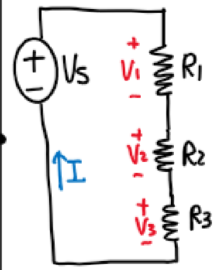
$V_{bc} + V_{eb} = V_{fa} + V_{ae}$

$V_{bc} + 3 = 9 - 2$

$V_{bc} = 4V = V_{bf}$

11-3
11-4
Single-Loop Circuits & Single-Node-Pair Circuits

SINGLE LOOP/ Voltage Divider



Voltages are distributed (unequally) across resistors

$$\boxed{I=\frac{V}{\Sigma R} \qquad V_i = V_S\frac{R_i}{\Sigma R}}$$

$\Sigma V_{inc} = \Sigma V_{dec}$

$V_S = V_1 + V_2 + V_3$

$\downarrow$

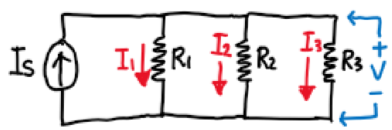
$R = R_1 + R_2 + R_3$

$I = I_1 = I_2 = I_3$

$IR = IR_1 + IR_2 + IR_3$

$$\boxed{I=\frac{V_S}{R_1+R_2+R_3} \qquad V_i=\frac{V_S R_i}{R_1+R_2+R_3}}$$

SINGLE NODE-PAIR/ Current Divider



$V = V_1 = V_2 = V_3$

$IR_1 = IR_2 = IR_3$

$\downarrow$

$\Sigma I_{enter} = \Sigma I_{leave}$

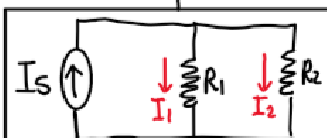
$I_S = I_1 + I_2 + I_3$

$\frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$$\boxed{V = I_S\left(\Sigma\frac{1}{R}\right)^{-1} \qquad I_i = I_S\frac{\frac{1}{R_i}}{\Sigma\frac{1}{R}}}$$

$$\boxed{V = I_S\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)^{-1} \qquad I_i = I_S\frac{\frac{1}{R_i}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}}$$

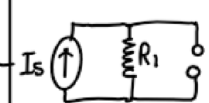


$I_1 = I_S\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}} = I_S\frac{R_2}{R_1+R_2}$

$\boxed{R_2 \to \infty}$

$I_1 = I_S\frac{R_2}{R_1+R_2} = I_S$

$I_2 = I_S\frac{R_1}{R_1+R_2} = 0$

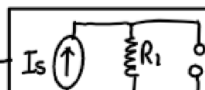


open circuit
→ no current
→ voltage exist

$\boxed{R_2 \to 0}$

$I_1 = I_S\frac{R_2}{R_1+R_2} = 0$

$I_2 = I_S\frac{R_1}{R_1+R_2} = I_S$

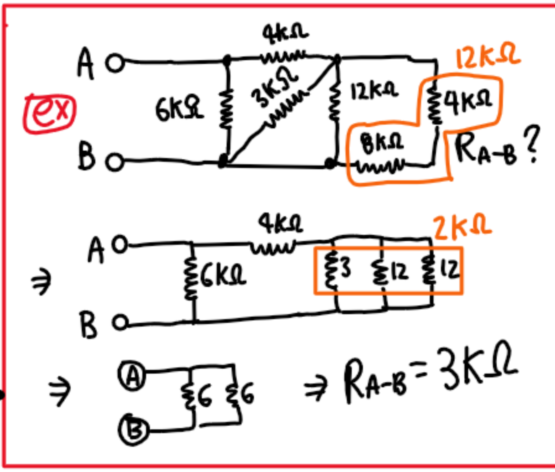


short circuit
→ no voltage across
→ current goes to R=0

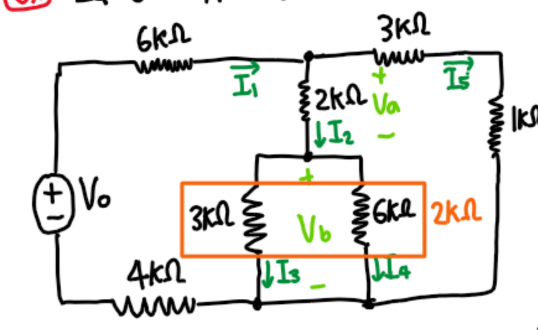
$V = I_1R_1 = I_2R_2 = 0$

11-5 Series and parallel resistor combinations

ex



ex $I_4 = 0.5mA$ V_o?



$V_o = 36V$

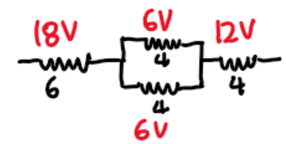
$V_b = 0.5mA \times 6k\Omega$
$= 3V$

$I_2 = I_3 + I_4$
$= 1.5mA$

$V_a = 3V$

$V_a + V_b = 6V$

$I_1 = I_2 + I_5 = 3mA$

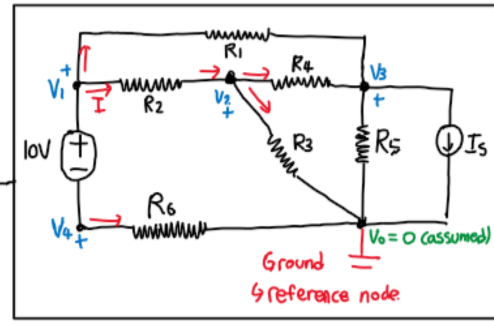


12-1 Nodal Analysis

Nodal Analysis

- Calculate/Solve for "voltages at nodes"
  - systems of linearly coupled, algebraic equations
  - set one node as 'reference' and calculate voltages in other nodes

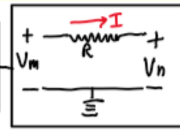
- $\boxed{\#\text{ equations needed} = \underset{(\#\text{nodes})}{N} - \underset{\text{reference}}{1} - \underset{(\#\text{ of voltage sources})}{N_v}}$ } Independent or dependent "constraint"



$V_1 = V_4 + 10V$

- Node 1
  $\frac{V_1-V_2}{R_2} + \frac{V_1-V_3}{R_1} + \frac{V_4-V_0}{R_6} = 0$
- Node 2
  $\frac{V_1-V_2}{R_2} = \frac{V_2-V_3}{R_4} + \frac{V_2-V_0}{R_3}$
- Node 3
  $\frac{V_3-V_0}{R_5} + I_s = \frac{V_2-V_3}{R_4} + \frac{V_1-V_3}{R_1}$

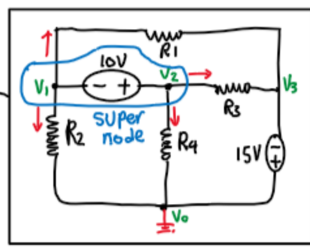
- Use Kirchoff's current law to make systems of equations



$I = \frac{V_m - V_n}{R}$

Supernode

- two nodes connected by voltage source
- none of them are reference node

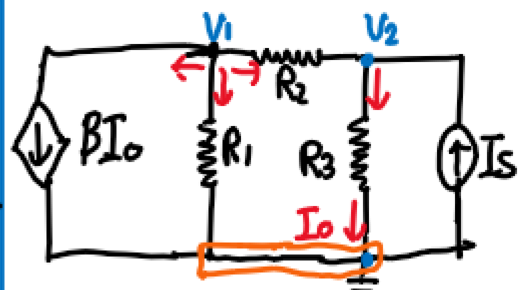


4 nodes, 2 voltage source
→ 1 equation
$V_3 = -15V. \quad V_2 = V_1 + 10V.$

$\frac{V_1-V_3}{R_1} + \frac{V_1-V_0}{R_2} + \frac{V_2-V_0}{R_4} + \frac{V_2-V_3}{R_3} = 0$

$\frac{V_1+15}{R_1} + \frac{V_1}{R_2} + \frac{V_1+10}{R_4} + \frac{V_1+25}{R_3} = 0$

Dependent source



3 nodes, no voltage source
→ 2 equations.

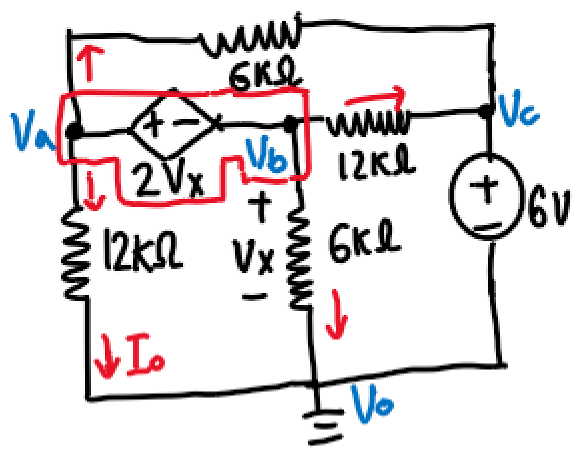
∘ Node 1

$$\beta I_o + \frac{V_1-V_0}{R_1} + \frac{V_1-V_2}{R_2} = 0$$

∘ Node 2

$$\frac{V_1-V_2}{R_2} + I_s = \frac{V_2-V_0}{R_3} = I_o$$

Dependent source + supernode



4 nodes, 2 voltage sources
→ 1 equation.

$V_c = 6V$

$$\frac{V_b-V_o}{6k\Omega} + \frac{V_a-V_o}{12k\Omega} + \frac{V_a-V_c}{6k\Omega} + \frac{V_b-V_c}{12k\Omega} = 0$$

$V_b = V_x \quad V_a = 3V_x.$

$V_a = 4.5V$

$I_o = 0.375mA$

12-2 Loop (mesh) Analysis

Mesh Analysis

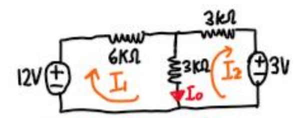
Use Kirchoff's voltage law

$$\begin{pmatrix}\text{\# Equations} \\ \text{Needed}\end{pmatrix} = \begin{pmatrix}\text{\# of Independent} \\ \text{loops}\end{pmatrix} - \underbrace{N_I}_{\text{(\# of current source)}}$$

N_I: Independent or dependent "Constant"

$$= \underbrace{B}_{\text{Branch}} - \underbrace{N}_{\text{Node}} + 1 - N_I$$

ex



2 equations needed.

Loop 1 $\Sigma V_{dec} = \Sigma V_{inc}$

$6k\Omega \cdot I_1 + 6k\Omega(I_1 - I_2) = 12$

$12k\Omega \cdot I_1 - 6k\Omega \cdot I_2 = 12$

Loop 2 $\Sigma V_{dec} = \Sigma V_{inc}$

$6k\Omega(I_2 - I_1) + 3k\Omega \cdot I_2 + 3 = 0$

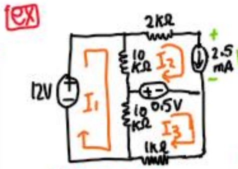
$-6k\Omega \cdot I_1 + 9k\Omega \cdot I_2 = -3$

$I_1 = \frac{5}{4}mA$

$I_2 = \frac{1}{2}mA$

$I_o = \frac{3}{4}mA$

ex



$\#Eq = 3 - 1 = 2$

$I_2 = 2.5mA$

• Loop 1 $\Sigma V_{inc} = \Sigma V_{dec}$

$10k\Omega(I_1 - I_2) + 10k\Omega(I_1 - I_3) = 12$

$20k\Omega \cdot I_1 - 10k\Omega \cdot I_3 = 12 + 2.5$

• Loop 3

$0.5 + 1k\Omega \cdot I_3 + 10k\Omega(I_3 - I_1) = 0$

$-10k\Omega \cdot I_1 + 11k\Omega \cdot I_3 = -0.5$

$I_1 = 3.35mA \quad I_3 = 3mA.$

Loop 2

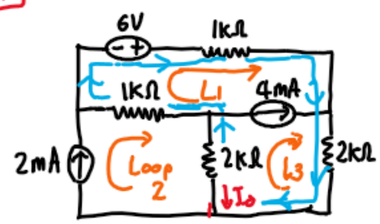
$V_o + 10k\Omega(I_2 - I_1) + 2k\Omega(I_2) = 0.5$

$V = 4V$

Super mesh

When current source is shared between two loops

ex



3 loop - 2 source = 1 eq

• supermesh ($L_1 + L_3$)

$I_1 + 4mA = I_3$

$1k\Omega \cdot I_1 + 2k\Omega \cdot I_3$

$+ 2k\Omega(I_3 - I_2)$

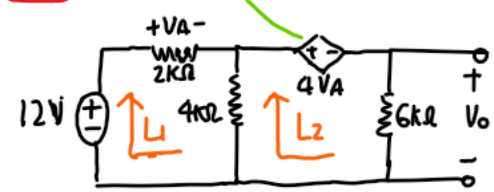
$+ 1k\Omega(I_1 - I_2) = 0$

$I_1 = -\frac{2}{3}mA \quad I_3 = \frac{10}{3}mA$

$I_o = I_2 - I_3 = -\frac{4}{3}mA$

ex

Relate controlling variable to current



• Loop 1

$\underbrace{2k\Omega \cdot I_1}_{V_A} + 4k\Omega(I_1 - I_2) = 12$

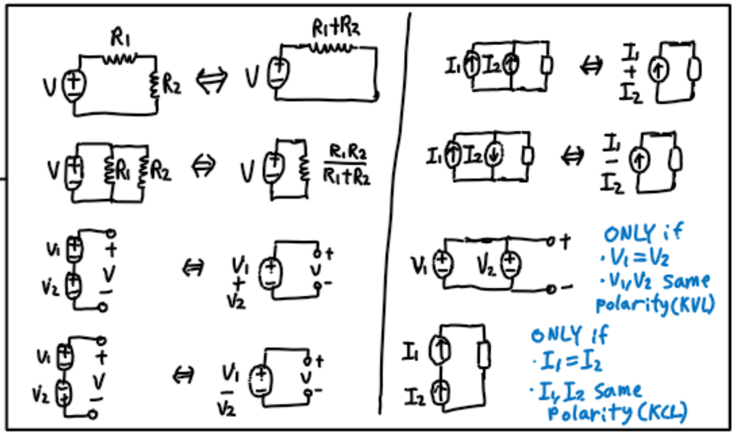
• Loop 2

$4V_A + 6k\Omega(I_2) + 4k\Omega(I_2 - I_1) = 0$

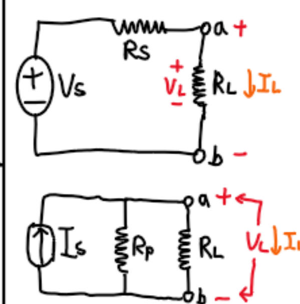
$I_2 = -\frac{12}{19}mA \quad V_o = -\frac{72}{19}V$

13-1 More techniques

Equivalence



Source transformation



Real battery has internal resistance

$$\boxed{V_L = V_S \frac{R_L}{R_L + R_S}}$$

$$\left[\begin{array}{l} V_S = I_S \times R_S \\ R_S = R_P \end{array}\right.$$

(To make circuits equivalent)

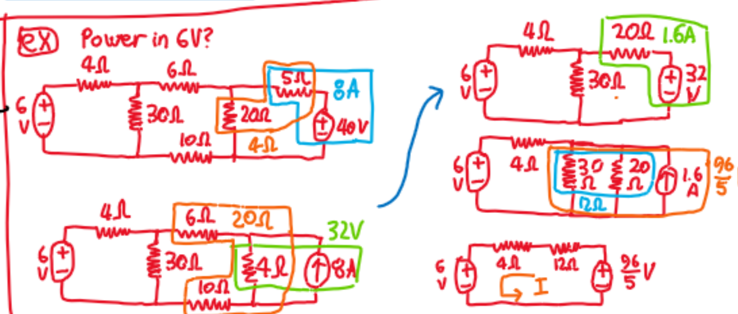
$$I_L' = I_S \frac{R_P}{R_P + R_L}$$

$$V_{ab}' = I_S \frac{R_P R_L}{R_P + R_L}$$

$$= \frac{V_S}{R_P} \frac{R_P R_L}{R_P + R_L} = V_{ab}$$

Equivalent!

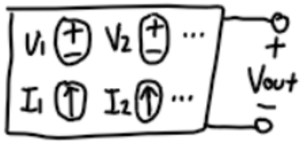
ex Power in 6V?



$$I = \frac{\frac{96}{5} - 6}{16} = 0.825A$$

$P = VI = 4.95W$ dissipated (centering + side)

Linearity



$V_{out} = \alpha_1 V_1 + \alpha_2 V_2 + \dots + B_1 I_1 + B_2 I_2 + \dots$

ex) Linear network
- 1 independent current source
- 1 independent voltage source

V_s		I_s		V_{out}
$10V\alpha$	+	$0AB$	=	3.5V
$10V\alpha$	+	$1AB$	=	0.5V
$4V\alpha$	+	$1AB$	=	?

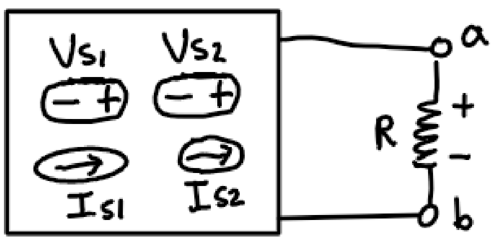
$\alpha = 0.35$ $B = -3$ $? = -1.6V$

ex) Use linearity to find I_o when $I_s = 6mA$

Assume $I_o = 1mA$
V across 3kΩ ⇒ 3V
" 6kΩ ⇒ 3V

I across 2kΩ
↳ 1.5mA
I_s + I across 12kΩ
↳ 1.5mA
V across I_s
$= 1.5 \times 2 + 6 \times 0.5 = 6V$
V across 12kΩ = 6V
↳ 0.5mA
$I_s = 2mA$ ($I_s : I_o = 2:1$)
$I_o = 3mA$

13-2 Superposition



Deactivate all sources except one source!
(Repeat for each voltage/current source)

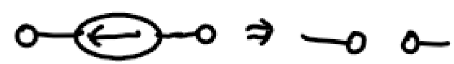
Superposition holds for I & V but not power

$I_1 \rightarrow$ current through R due to V_1 only
$I_2 \rightarrow$ current through R due to V_2 only
...

$I_{tot} = I_1 + I_2 + \dots$
$P_{tot} = I_{tot}^2 R$
$= (I_1 + I_2 + \dots)^2 R$
→ Quadratic relationship

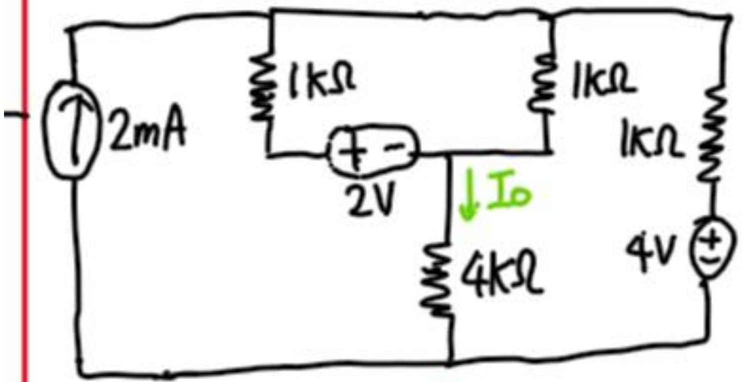
Deactivate source?

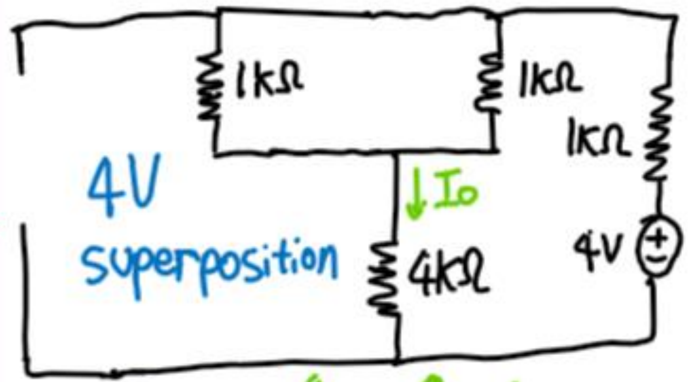
Voltage source ⇒ Short
voltage ✗
current ✓



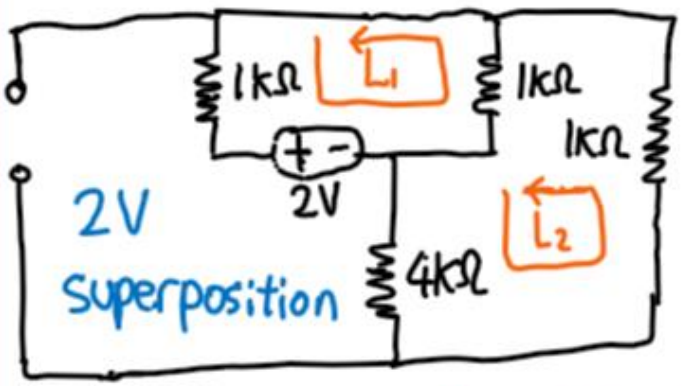
current source ⇒ open
voltage ✓
current ✗

ex $I_{total} = I_{o\text{-}2V} + I_{o\text{-}4V} + I_{o\text{-}2mA} = \frac{10}{11}\,mA$





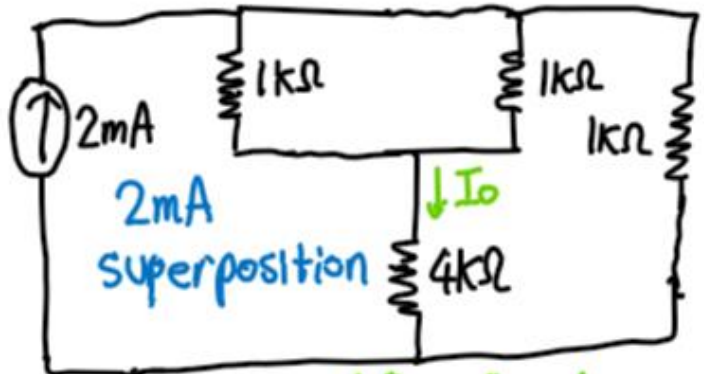
$I_{o\text{-}4V} = \frac{4}{5.5} = \frac{8}{11}\,mA$



L_1: $I_1 \cdot 1k\Omega + 2 + (I_1 - I_2)\,1k\Omega = 0$

L_2: $(4+1)k\Omega \cdot I_2 + (I_2 - I_1)\,1k\Omega = 0$

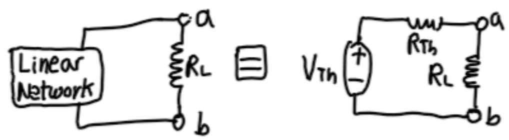
$\left[\begin{array}{cc|c} 2 & -1 & -2 \\ -1 & 6 & 0 \end{array}\right]$ $I_2 = -\frac{2}{11}A = I_{o\text{-}2V}$



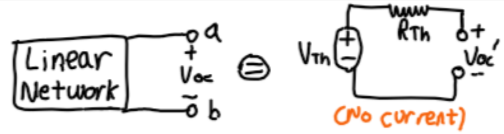
$I_{o\text{-}2mA} = \frac{1 \cdot 2}{5.5} = \frac{4}{11}\,mA$

13-3 Thevenin / Norton Theorem

Thevenin Theorem



$$R_{Th} = \frac{V_{openc}}{I_{shortc}} \qquad V_{Th} = V_{openc}$$



$V_{oc}' = V_{Th} = V_{oc}$

$I_{sc} = \frac{V_{Th}}{R_{Th}} = \frac{V_{oc}}{R_{Th}}$

① Change R_L into open circuit and get V_{oc}

② Change R_L into short circuit and get I_{sc}

③ $R_{Th} = \frac{V_{oc}}{I_{sc}}$

Norton Theorem

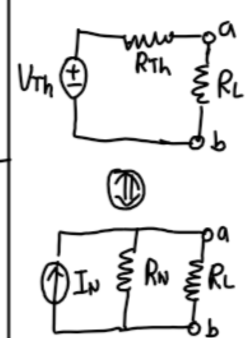
If there are independent sources

⓪ Get V_{oc}.

① Deactivate sources.
- voltage source → short
- current source → open

② Replace R_L into virtual voltage source. Using series/parallel, find equivalent resistor (R_{Th})

③ $R_{Th} = R_N$, $I_N = I_{sc}$



$R_{Th} = R_N = \frac{V_{oc}}{I_{sc}}$

$I_N = \frac{V_{oc}}{V_{oc}/I_{sc}} = I_{sc} = \frac{V_{Th}}{R_{Th}}$

If Question asks

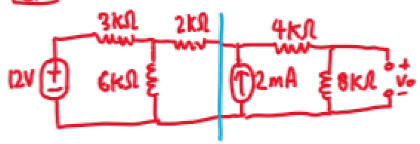
I_o → Use loop, mesh, ... analysis methods

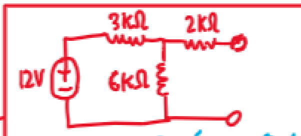
V_{Th} → Replace R_L into V_{oc} ($= V_{Th}$)

R_{Th} → [V_s → short, I_s → open, R_L → voltage (imaginary)] → Get net resistance

Multiple Thevenin usage

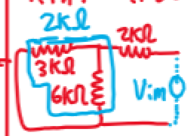
ex

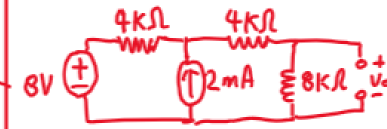




$V_{oc} = 12 \cdot \frac{6}{3+6} = 8V$

$R_{Th}? = 4k\Omega$



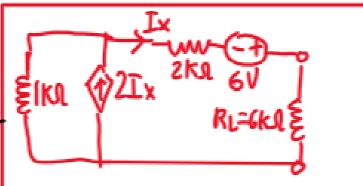


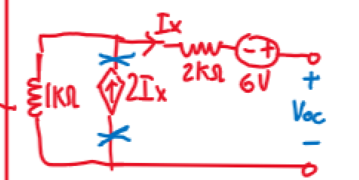
$R_{Th} = 8k\Omega$

$V_{8k\Omega} = 16V$

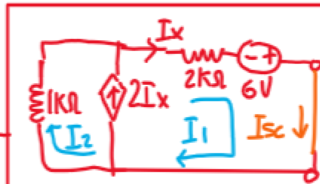
$V_o = 8V$

Controlling variables in dependent sources cannot be separated





$I_x = 0, 2I_x = 0 \quad V_{oc} = 6V$

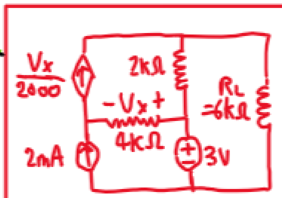


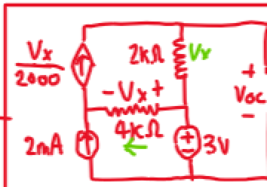
supermesh

$2I_1 + 1I_2 = 6$

$I_{sc} = 6mA$

$R_{Th} = 1k\Omega$





$V_x = 4k\Omega \times I_4$

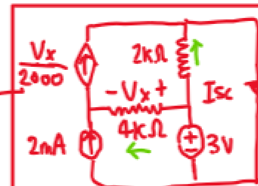
$I_4 + 2mA = \frac{V_x}{2000}$

$V_x + 8V = 2V_x$

$V_x = 8V$

$V_{oc} = \frac{V_x}{2k} + 2k + 3$

$= 11V$



$V_x = 8V$

$I_2 = 1.5mA$

$\frac{V_x}{2k} + I_2 = I_{sc}$

$I_4 + 2mA = \frac{V_x}{2k}$

$I_{sc} = 5.5mA$

$R_{Th} = 2k\Omega$

ex V_{Th}, R_{Th} in terms of R_L

$V_s = 50V$, $R_1 = 10\Omega$, $I_s = 1.5A$, $R_2 = 10\Omega$, $R_3 = 20\Omega$

○ $R_L \to V_{oc}$

supermesh.

$I_1R_1 + I_2(R_2+R_3) = V_s$

$I_2 - I_1 = I_s$

$10R_1 + 30R_2 = 50$

$I_2 - I_1 = 1.5$

$I_2 = \frac{13}{8}A$

current divider

$V_{Th} = V_{oc} = V_{ab}$

$= \frac{13}{8} \times 20 = 32.5V.$

○ $R_L \to I_{sc}$

$I_2 - I_1 = I_s$

$R_1I_1 + R_2I_2 = V_s$

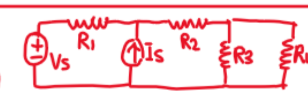
$I_2 - I_1 = 1.5$

$10I_1 + 10I_2 = 50$

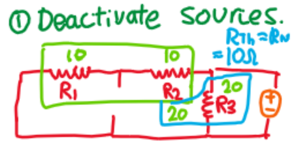
$I_{sc} = I_2 = 3.25A$

$R_{Th} = \frac{V_{oc}}{I_{sc}} = 10\Omega.$

Using Norton

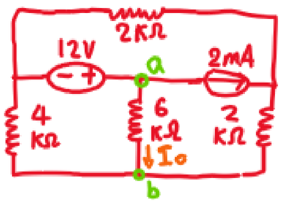


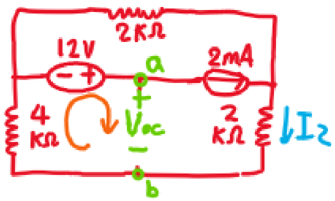
① Deactivate sources.



② Virtual voltage source.

ex Thevenin/Norton
→ Find I_o

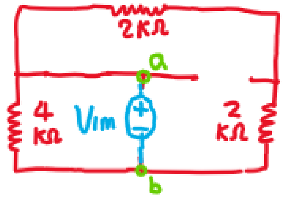




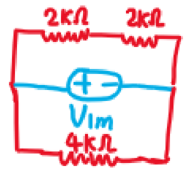
$$I_2 = 2mA \cdot \frac{2k\Omega}{(2+6)k\Omega} = 0.5mA$$

$$V_{oc} + \tfrac{1}{2}mA \cdot 4k\Omega = 12V$$

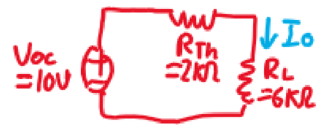
$$V_{oc} = V_{Th} = 10V.$$



⇓



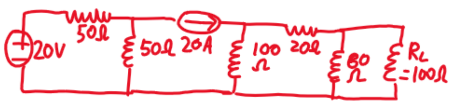
$R_{Th} = 2k\Omega$ ⇒



$$I_o = \frac{10V}{(2+6)k\Omega} = 1.25mA.$$

$$P_{dissipated} = I_o^2 R.$$

EXAM Thevenin
→ power in R_L.



$R_{Th} = \frac{1}{\frac{1}{120}+\frac{1}{80}} = 48$

IMAGINARY voltage source.

$V_{Th} = V_{oc}$
$= 80 \cdot 10$
$= 800V.$

$$I_L = \frac{V_{oc}}{R_{Th}+R_L} = 5.405A$$

$$V_{RL} = I_L R_L = 540.54V$$

$$P_{RL} = V_{RL} I_L = 2921.84W$$

13-4 Maximum Power Transfer

$I_L = \frac{V_{Th}}{R_{Th}+R_L}$

$P_L = I_L^2 R_L$

$= \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$

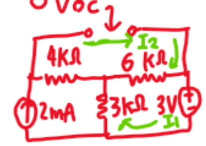
Max. P_L in terms of R_L

$\frac{dP_L}{dR_L} = V_{Th}(R_L+R_{Th})(R_L-R_{Th})$

$P_{L\,max} = P_{out\,max} = \frac{V_{Th}^2 R_{Th}}{4 \cdot R_{Th}^2} = \frac{V_{Th}^2}{4R_{Th}}$

$\boxed{ex}$ R_L for maximum power transfer

- V_{oc}



Loop analysis
→ $V_{oc} = 10V$

- R_{Th}

$R_{Th} = 6k\Omega$

-

When $R_L = R_{Th} = 6k\Omega$,

$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{100V^2}{24k\Omega^2}$

Efficiency $= \frac{P_{out}}{P_{in}}$ $\frac{\text{(dissipated by load)}}{\text{(supplied)}}$

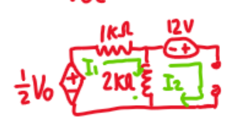
$P_{in} = \frac{V_{Th}^2}{R_L+R_{Th}}$

$P_{out} = \frac{V_{Th}^2 R_L}{(R_L+R_{Th})^2}$

Do NOT compare efficiency with maximum power transfer

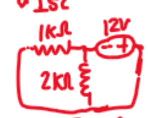
$\boxed{ex}$ R_L for max power transfer

- V_{oc}



KVL, voltage divider
→ $V_{oc} = 18V (= V_{Th})$

- I_{sc}

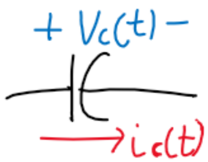


$R_T = \frac{2}{3}k\Omega$

$I_{sc} = 18mA$

$R_{Th} = \frac{V_{oc}}{I_{sc}} = 1k\Omega$

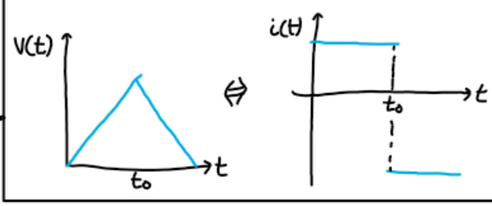
14-1 Capacitors



$q(t) = C \cdot V_c(t)$

$i_c(t) = \frac{dq(t)}{dt}$

$= C \cdot \frac{dV_c(t)}{dt}$



$dV = \frac{1}{C}\, i(t)\, dt$

$$\boxed{V(t) = \int_{t'=t_0}^{t'=t} \frac{1}{C}\, i(t')\, dt'}$$

$\int dV = \frac{1}{C}\int_{-\infty}^{t'=t} i(t')\, dt'$

$V = \frac{1}{C}\int_{-\infty}^{t'=t_0} i(t')\, dt'$

$+ \frac{1}{C}\int_{t'=t_0}^{t'=t} i(t')\, dt'$

$$\boxed{V = V(t_0') + \frac{1}{C}\int_{t'=t_0}^{t'=t} i(t')\, dt'}$$

$P = \frac{dW}{dt} = V(t)\, i(t)$

$dW = C V(t)\, dV$

$(W = U = \frac{1}{2} C V^2)$

$\int dW = \int_{-\infty}^{t'=t} C V\, dV$

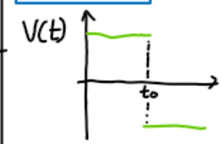
$W(t) = C\int_{-\infty}^{t'=t_0} V\, dV$

$+ C\int_{t'=t_0}^{t'=t} V\, dV$

$$\boxed{W_c(t) = \frac{1}{2} C [V(t)]^2}$$

What if V is discontinuous?

$i \to \infty$, not possible

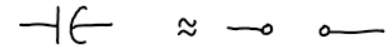


V_c must be continuous

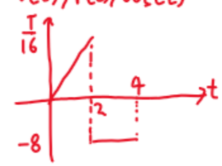
$V_c(t_0^-) = V_c(t_0^+)$

What if There is DC singnal $i_c = 0$

To DC signal, capacitor looks like open circuit



ex Current through Capacitor $C = 4F$. $V(t), P(t), W_c(t)$

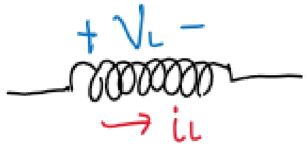


$t[s]$	$i(t)$	$V(t)$	$P(t)$	$W(t)$
$(-\infty, 0)$	0	0	0	0
$[0, 2)$	$8t$	$0 + \frac{1}{C}\int i(t)dt = t^2$	$8t^3$	$2t^4$
$[2, 4)$	-8	$-2t+8^{(1)}$	$16t-128$	$2(8-2t)^2$
$[4, \infty)$	0	0	0	0

$(1) = V(2) + \frac{1}{C}\int_2^t i(t)dt$

$= 4 + \frac{1}{4}\left[-8t\right]_2^t = -2t + 8$

14-2 Inductors



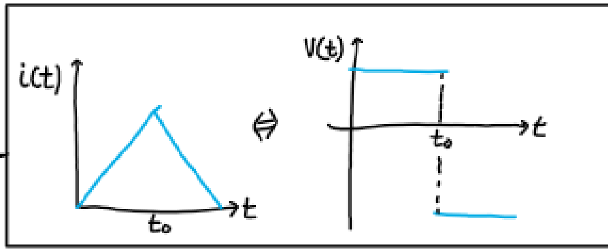
$$L = \frac{N\Phi_B}{i}$$

$$Li = N\Phi_B$$

$$L\frac{di}{dt} = N\frac{d\Phi_B}{dt}$$

$$= V_L = -\mathcal{E}$$

$$\boxed{V_L = L\frac{di_L}{dt}}$$



$$di = \frac{1}{L}V(t)\,dt$$

$$\boxed{i(t) = i(t_0) + \frac{1}{L}\int_{t'=t_0}^{t'=t} V(t')\,dt'}$$

$$P(t) = V(t)I(t)$$

$$= i(t)L\frac{di}{dt}$$

$$dW = L\,i(t)\,di$$

$$W = \int_{-\infty}^{t'=t} L i\,di$$

$$\boxed{W = \frac{1}{2}L[i(t)]^2}$$

What if I is discontinuous?

$V(t) \to \infty \Rightarrow$ impossible.

I_L should be <u>continuous</u>

$$i_L(t_0^-) = i_L(t_0^+)$$

What if There is DC signal $V_L = 0$

Inductor regarded as <u>short circuit</u>

14-3 Capacitor – Inductor combinations

RE CAP

	SERIES	PARALLEL
Inductor	$L_T = \sum_i L_i$	$\frac{1}{L_T} = \sum_i \frac{1}{L_i}$
Capacitor	$\frac{1}{C_T} = \sum_i \frac{1}{C_i}$	$C_T = \sum_i C_i$
Resistor	$R_T = \sum_i R_i$	$\frac{1}{R_T} = \sum_i \frac{1}{R_i}$

15-1 Introduction to transient circuits

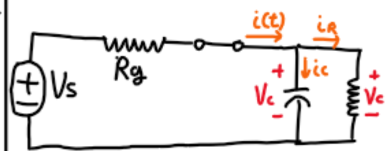
$$C\frac{dV_c(t)}{dt} + \frac{V_c(t)}{R} = 0$$

$$\frac{dV_c(t)}{dt} + \frac{V_c(t)}{RC} = 0$$

$$V_c(t) = V_0 e^{-\frac{t}{RC}}$$

15-2 First-order transient circuits

Resistor – Capacitor



If (battery Removed / switch open)
↳ $V(t)=0$
↳ Capacitor hold charge.

At $t=0$, switch closed. $V(t)$?

$i_c(t) = C\frac{dV_c(t)}{dR}$

$i_R(t) = \frac{V_c(t)}{R}$

$i(t) = i_c(t) + i_R(t)$

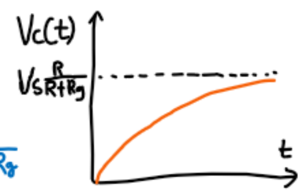
$V_c(0^-) = V_c(0^+) = 0$

$$\boxed{V_c(t) = V_s\frac{R}{R+R_g}(1-e^{t/\tau})}$$

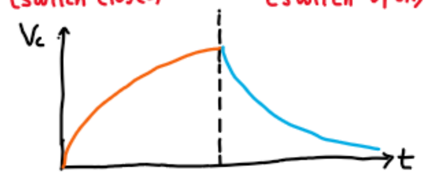
$$C\frac{R_gR}{R_g+R}\frac{dV_c(t)}{dt} + V_c(t) = V_s\frac{R}{R+R_g}$$

$$\tau = C\cdot\frac{R_gR}{R_g+R} = R_{Th}\cdot C$$

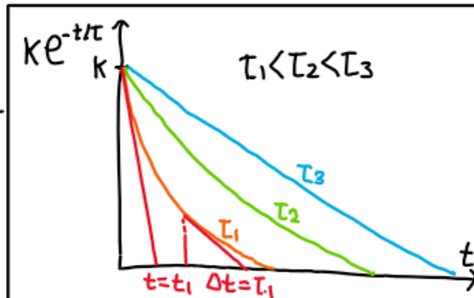
$$V_c(\infty) = V_s\frac{R}{R+R_g}$$

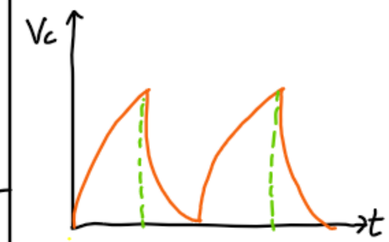


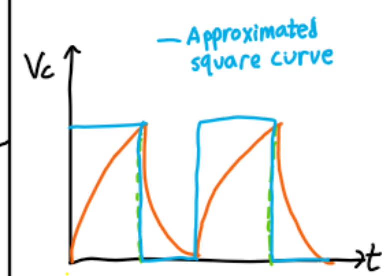
Capacitor charged (switch closed) VS Capacitor discharged (switch open)



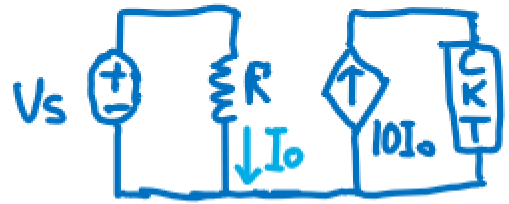
$\boxed{V_c = V_{max}(1-e^{t/\tau})}$ $\boxed{V_c = V_{max}e^{-t/\tau}}$





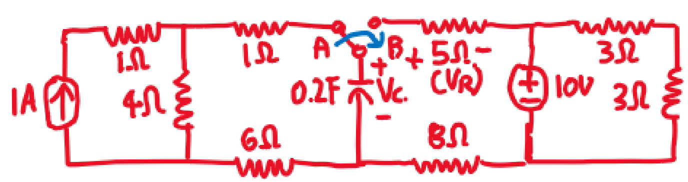


Do not confuse (dependent source)

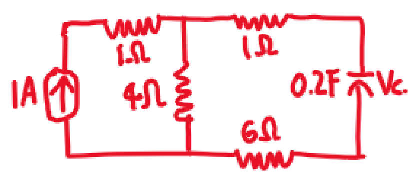


$I_o = \frac{V_s}{R}$ $\quad 10I_o = 10\frac{V_s}{R}$

ex At $t=0$, switch $A \to B$
Find $V_c(t)$ & $V_R(t)$ for $0 \le t < \infty$

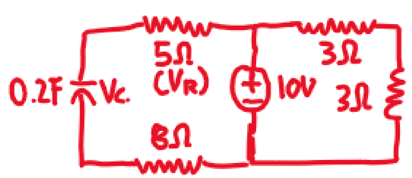


∘ $-\infty < t < 0^-$



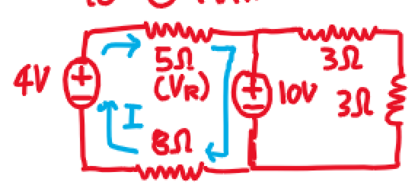
$V_R(0^-) = 0V$
$V_c(0^-)$
$= 1A \cdot 4\Omega = 4V$
(No current passing)

∘ $0^+ \le t < \infty$



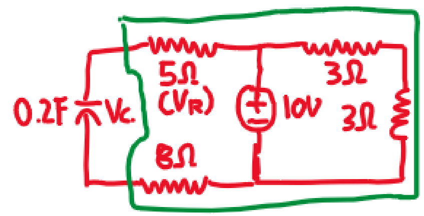
$V_c(\infty) = 10V$
$V_R(\infty) = 0V$
(no current)

• $t = 0^+$. Replace capacitor to ⊕ 4V...

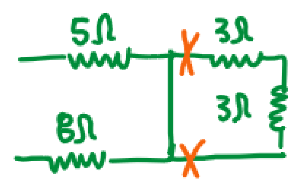


$13I + 10 = 4$

$V_R(0^+) = I \cdot 5 = -\frac{6}{13} \cdot 5$
$= -2.3V.$



Linear Network...
R_{Th}?



$R_{Th} = 13\Omega.$
$\tau = R_{Th} \cdot C = 2.6s$

Answer. $0^+ \le t < \infty$

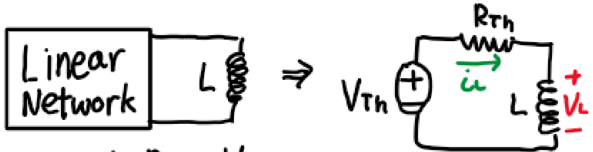
$V_c(t) = 10 - 6e^{-t/2.6}$

$V_R(t) = -2.3e^{-t/2.6}$

$$V(t) = V(\infty) + (V(t_0) - V(\infty))\, e^{-t/\tau}$$

$$\tau = R_{Th} \cdot C = \frac{L}{R_{eq}}$$

Resistor – Inductor



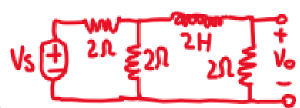
$$V_L + i_L R_{Th} = V_{Th}$$

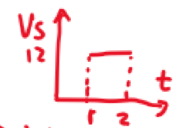
$$L \frac{di_L}{dt} + i_L R_{Th} = V_{Th}$$

$$\frac{L}{R_{Th}} \frac{di_L}{dt} + i_L = \frac{V_{Th}}{R_{Th}}$$

$$\boxed{\tau = \frac{L}{R_{Th}}}$$

ex) Calculate $V_o(t)$ for time interval $-\infty < t < 1$, $1 < t < 2$, $2 < t < \infty$





① $-\infty < t < 1$

$V_s(t) = 0$ $V_o(1^-) = 0$

$V_o(1^-) \neq V_o(1^+)$

Not necessary but conservation of charge

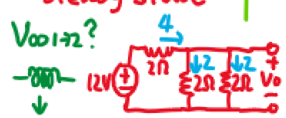
$I_o(1^-) = I_o(1^+)$

$V_o(1^-) = V_o(1^+) = 0$

② $1 < t < 2$

• Steady state

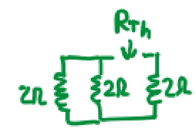
$V_{\infty\, 1 \to 2}$?



$V_o(\infty) = 4V$

• Time constant

R_{Th}? $\tau = \frac{L}{R_{Th}}$



$R_{Th} = 3\Omega$ $\tau = \frac{2}{3}s$

$V_o(t) = 4(1 - e^{-3(t-1)/2})$

$V_o(2^-) = 3.11V.$

③ $2 < t < \infty$

$V_o(\infty) = 0$, $\tau = \frac{2}{3}$

$V_o(2^-) = V_o(2^+) = 3.11V$

$V_o(t) = 3.11 e^{-3(t-2)/2}$

16-A Complex Numbers

$$z = x + jy = |z|e^{j\theta} = \sqrt{x^2+y^2}\,e^{j\tan^{-1}(y/x)} = |z|\angle\theta$$

Rectangular: $x + yi$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$-\frac{\pi}{2} \le \tan^{-1}\theta \le \frac{\pi}{2}$$

Polar: $r(\cos\theta + j\sin\theta)$

Euler (exponential): $re^{j\theta}$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$j = \sqrt{-1} = 1\angle\frac{\pi}{2} = e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2}$$

$$-j = 1\angle-\frac{\pi}{2} = e^{-j\frac{\pi}{2}} = \frac{1}{j}$$

$$1 = e^{j0} = 1\angle 0$$

$$-1 = j \cdot j = e^{j\pi} = 1\angle\pi$$

$$Ae^{j\theta} = A(\cos\theta + j\sin\theta)$$

$$\sin(\omega t + \theta) = \cos\left(\omega t + \theta - \frac{\pi}{2}\right)$$

$$\cos(\omega t + \theta) = \sin\left(\omega t + \theta + \frac{\pi}{2}\right)$$

$$-\sin(\omega t + \theta) = \sin(\omega t + \theta - \pi)$$

$$-\cos(\omega t + \theta) = \cos(\omega t + \theta + \pi)$$

$$z = x + jy$$

$$\mathrm{Re}[z] = x$$

$$\mathrm{Im}[z] = y$$

$\bar{z} = x - jy$

Conjugate

$$z_1 = x_1 + y_1 j$$

$$z_2 = x_2 + y_2 j$$

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

$$z_1 * z_2 = (x_1x_2 - y_1y_2) + j(x_2y_1 + x_1y_2)$$

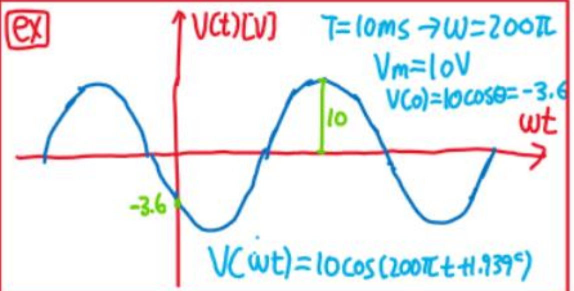
$$\frac{z_1}{z_2} = \frac{z_1 \cdot \bar{z}_2}{z_2 \cdot \bar{z}_2} = \frac{(x_1x_2 + y_1y_2) + j(x_2y_1 - x_1y_2)}{x_2^2 + y_2^2}$$

$$z = |z|e^{j\theta} = |z|\angle\theta$$

$$z_1 z_2 = |z_1 \cdot z_2|\angle\theta_1 + \theta_2$$

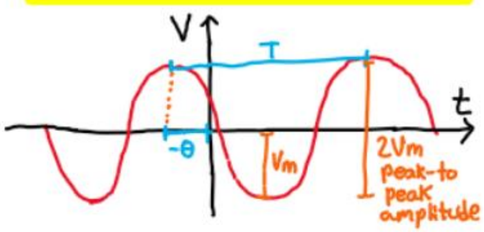
$$\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|}\angle\theta_1 - \theta_2$$

16-1 Sinusodials

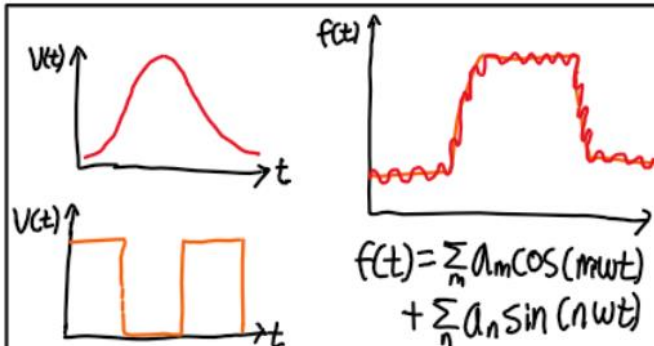


⟨Sinusodial Functions⟩

$$V(t)=\underbrace{V_m}_{\text{max voltage}}\cos(\underbrace{\omega}_{\text{Angular Freq.}}t+\underbrace{\theta}_{\text{Phase angle}})$$



$\omega=\frac{2\pi}{T}=2\pi f \quad T=\frac{1}{f}$



$f(t)=\sum_n a_m\cos(m\omega t)+\sum_n a_n\sin(n\omega t)$

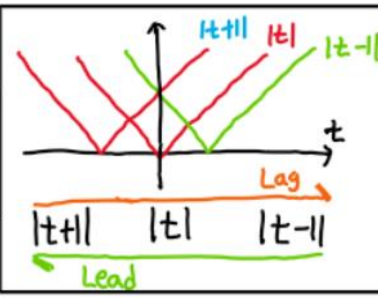
Fourier Transform

To get approximating function by summation of sine/cosines

(Fourier Series)

More functions added → more precision

Lead/Lag



lead/lag of Sinusodial

- Same frequency (f)
- All functions → all sine $\boxed{\text{or}}$ cosine
- All amplitudes → all positive $\boxed{\text{or}}$ negative
- $0 \leq$ separation $< 180°$

ex
$i_1(t)=2\sin(400t+45°)$
$i_2(t)=\cos(400t+10°)$
$=\sin(400t+100°)$
$i_3(t)=-\frac{1}{4}\sin(400t+60°)$
$=\frac{1}{4}\sin(400t-120°)$

← Lag   lead →

i_3 165° i_1 55° i_2

16	2 3 4 5	Complex forcing functions, phasors, impedence and admittance

$$v(t) = V_m \cos(\omega t + \theta_v)$$
$$= \mathrm{Re}[V_m e^{j(\omega t + \theta_v)}]$$
$$= \mathrm{Re}[V_m e^{j\theta_v} e^{j\omega t}]$$
$$= \mathrm{Re}[\underline{V} e^{j\omega t}]$$

$$\underline{V} = V_m e^{j\theta_v} = V_m \angle \theta_v$$

phasor

$$i(t) = I_m \sin(\omega t + \theta_i)$$
$$v(t) = V_m \sin(\omega t + \theta_v)$$
$$= V_m \cos(\omega t + \theta_v - \tfrac{\pi}{2})$$
$$= \mathrm{Re}[V_m e^{j(\omega t + \theta_v - \pi/2)}]$$
$$= \mathrm{Re}[V_m e^{j(\theta_v - \pi/2)} e^{j\omega t}]$$
$$= \mathrm{Re}[\underline{V} e^{j\omega t}]$$
$$\underline{V} = V_m e^{j(\theta_v - \pi/2)} = V_m \angle \theta_v - \tfrac{\pi}{2}$$

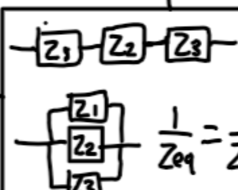
$$Z = \frac{\underline{V}}{\underline{I}} \; [\Omega]$$

impedance

$$= Z_{Re} + j Z_{Im}$$
$$= R + jX$$

$$\underline{Y} = \frac{1}{Z} = G + jB$$

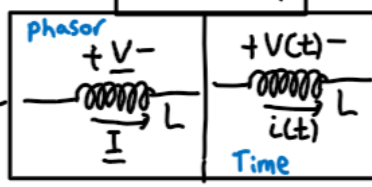
Admittance, Conductance, Susceptance



$$Z_{eq} = Z_1 + Z_2 + Z_3$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

Inductor

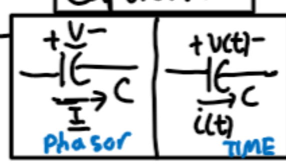


$$v(t) = V_m \cos(\omega t + \theta_v) = \mathrm{Re}[\underline{V} e^{j\omega t}]$$
$$i(t) = I_m \cos(\omega t + \theta_i) = \mathrm{Re}[\underline{I} e^{j\omega t}]$$
$$v(t) = L \frac{di(t)}{dt}$$
$$\mathrm{Re}[\underline{V} e^{j\omega t}] = \mathrm{Re}[L \tfrac{d}{dt} \underline{I} e^{j\omega t}]$$
$$= \mathrm{Re}[L j\omega \underline{I} e^{j\omega t}]$$
$$\underline{V} = j\omega L \underline{I}$$

$$Z_L = j\omega L \; [\Omega]$$

$$\underline{I} = \frac{\underline{V}}{j\omega L} = \frac{V_m}{\omega L} \angle \theta_v - \tfrac{\pi}{2}$$
$$= I_m \angle \theta_i \quad (\theta_i = \theta_v - \tfrac{\pi}{2})$$
$$I_m = \frac{V_m}{\omega L}$$
$$i(t) = \frac{V_m}{\omega L} \cos(\omega t + \theta_v - \tfrac{\pi}{2})$$

Capacitor



$$v(t) = V_m \cos(\omega t + \theta_v) = \mathrm{Re}[\underline{V} e^{j\omega t}]$$
$$i(t) = I_m \cos(\omega t + \theta_i) = \mathrm{Re}[\underline{I} e^{j\omega t}]$$
$$i(t) = C \frac{dv(t)}{dt}$$
$$\mathrm{Re}(\underline{I} e^{j\omega t}) = C \underline{V} \tfrac{d}{dt} \mathrm{Re}(e^{j\omega t})$$
$$= \mathrm{Re}(C \underline{V} j\omega e^{j\omega t})$$

$$Z_C = \frac{1}{j\omega C} = -\frac{j}{\omega C} \; [\Omega]$$

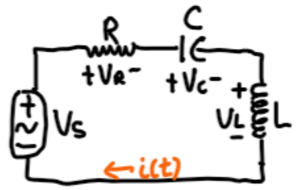
$$I_m \angle \theta_i = \omega C \angle \tfrac{\pi}{2} V_m \angle \theta_v$$
$$\theta_i = \theta_v + \tfrac{\pi}{2}$$
$$I_m = \omega C V_m$$
$$i(t) = V_m \omega C \cos(\omega t + \theta_v + \tfrac{\pi}{2})$$

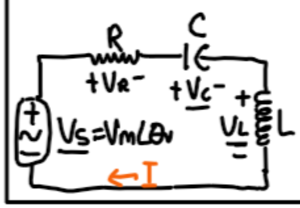
Resistor

$$Z_R = R \; [\Omega]$$
$$V_R(t) = I_R(t) \cdot R$$

Case

$V_s = V_m \cos(\omega t + \theta_v)$





$\underline{V_s} = \underline{V_R} + \underline{V_C} + \underline{V_L}$

$\underline{V_R} = R\underline{I_R}$ $\quad \underline{V_C} = -\frac{j}{\omega C}\underline{I_C}$

$\underline{V_L} = j\omega L\,\underline{I_L}$

$V_m\angle\theta_v = \underline{I}(R + j(\omega L - \frac{1}{\omega C}))$

$\underline{I} = \dfrac{V_m\angle\theta_v}{R + j(\omega L - \frac{1}{\omega C})}$ $\quad \boxed{\phi = \tan^{-1}\frac{\omega L - \frac{1}{\omega C}}{R}}$

$= \dfrac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}\angle\theta_v - \phi$

$i(t) = \dfrac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}\cos(\omega t + \theta_v - \phi)$

ex

$V_s(t) = 120\sin(377t + 60)$

$\downarrow$

$120\cos(377t - 30)$

$\downarrow$

$120\angle -30°$

$Z_C = -\frac{j}{377 \cdot 50 \cdot 10^{-6}}$

$= -j53[\Omega]$

$Z_R = 20\Omega$

$Z_L = j\omega L$
$= j \cdot 37 \cdot 40 \cdot 10^{-3}$
$= j15[\Omega]$

$Z_T = \frac{(20 + j15)(-j53)}{20 + (15 - 53)j}$

$= \frac{795 - j1060}{20 - j38}$

$= 30.5 + j4.89$

$= 30.9\angle 9.11°[\Omega]$

ex 2 elements in series

$\omega = 1000/s$ $\quad Z_{eq} = 5 + j4$

$\omega = 1300/s$ $\quad Z_{eq}?$

$Z_L = j\omega L$ $\quad Z_C = -\frac{j}{\omega C}$

$Z_R = R$ positive imaginary $\rightarrow$ inductor.

$R = 5$ $\quad Z_L \rightarrow j4 = j1000L$

$L = 0.004$

$\boxed{Z = 5 + j5.2}$ $\quad j1300L = j5.2$

ex For the circuit shown, what should be value of L if current has same phase angle as Vs?

$V_s = 12\cos(1000t + 75)$

$Z_L = j1000L$

$Z_C = -10j$

$L = 10mH$

$\underline{I} = \frac{V_s}{4 + j1000L - 10j}$

$= \frac{12\angle 75}{\sqrt{4^2 + (1000L - 10)^2}\angle\phi}$

$\phi = \angle\tan^{-1}\frac{1000L - 10}{4}$

Past exam

$\rightarrow 10\cos(200t - 90)$

$V_s(t) = 10\sin(200t)[V]$

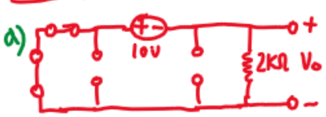
$i_s(t) = 5\cos(200t)[mA]$

a) Contribution of DC voltage to Vo

b) Impedances

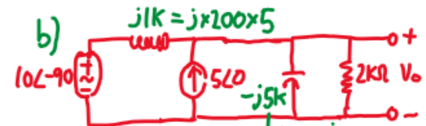
c) contribution of AC sources to Vo(t), use superposition

a)



$V_o^{DC} = -10V$

b)



$\hookrightarrow -\frac{j}{200 \times 10^{-6}}$

c)

$\boxed{V_s}$

$Z_T = j + \frac{2(-j5)}{2 - j5}$

$V_{o1} = 10\angle -90\left(\frac{\frac{-10j}{2 - j5}}{Z_T}\right)$

$= 10\angle -90(0.899 - j0.562)$

$= 10.6\angle -122$

$\boxed{I_s}$

$\frac{1}{Z_T} = \frac{1}{j1k} + \frac{1}{-j5k} + \frac{1}{2k} = \frac{8 + 5j}{10jk}$

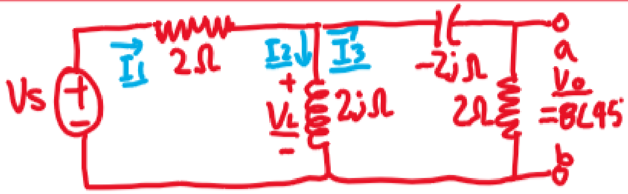
$V_{o2} = 5\angle 0 \cdot Z_T = 5.3\angle 58$

$V_{oT} = 5.3\angle -122$

$\boxed{V_o(t) = Re(5.3^{-j122}e^{j200t}) = 5.3\cos(200t - 122°)}$

16-7 Basic Analysis w/ Kirchhoff's Laws

ex



a) Voltages/currents shown

$I_3 = \frac{8\angle 45}{2} = 4\angle 45$

$V_L = V_{2-2j} = (2\sqrt{2}+2\sqrt{2}j)(2-2j)$
$= 8\sqrt{2}\angle 0$

$I_2 = \frac{8\sqrt{2}\angle 0}{2\angle 90} = 4\sqrt{2}\angle -90$

$I_1 = (2\sqrt{2}+2\sqrt{2}j) - 4\sqrt{2}j$
$= 4\angle -45$

$V_s = I_1 \cdot 2 + V_L$
(KVL) $= 2\cdot 4\angle -45 + 8\sqrt{2}\angle 0$
$= 12\sqrt{2} - 4\sqrt{2}j$
$= 8\sqrt{5}\angle -18.435$

b) Z_{eq} for a-b

$((2\|2j)+-2j)\|2$

$Z_{eq} = \frac{1}{\frac{1}{\frac{1}{\frac{1}{2}+\frac{1}{2j}}-2j}+\frac{1}{2}}$

$= 0.8 - 0.4j$

c) V_{Th}? ($=V_{oc}$)
$= 8\angle 45$

EXAM TYPE

$V_s(t) = V_0\cos(\omega t + \phi_0)$

$\omega_0 = \frac{1}{\sqrt{CL}}$
Resonance frequency

a) Circuit into phasor domain

$V_s(t) = V_0\angle\phi_0$ $\quad Z_C = -\frac{j}{\omega C}$
$Z_R = R$ $\quad Z_L = j\omega L$

b) $Z = \frac{V_s}{I_s}$. ω_0 for $Z \in \mathbb{R}$?

$\frac{1}{Z_{eq}} = \frac{1}{R} + j\omega C - \frac{j}{L\omega}$
$= \frac{L\omega + j(\omega^2 CRL - R)}{RL\omega}$

$Z_{eq} = \frac{RL\omega}{L\omega + j(\omega^2 CRL - R)}$
$= \frac{RL\omega[L\omega - j(\omega^2 CRL - R)]}{(L\omega)^2 + (\omega^2 CRL - R)^2}$

$R(\omega^2 CL - 1) = 0$ $\quad R=0$ or $\omega^2 CL = 1$

$Z_C|_{\omega=\omega_0} = -\frac{j}{\omega_0 C}$
$= -\sqrt{\frac{L}{C}}\,j$

$Z_L|_{\omega=\omega_0} = j\omega_0 L$
$= j\sqrt{\frac{L}{C}}$

Phase difference
180°

At $\omega = \omega_0$, $Z_L = -Z_C$
$I_L Z_L = I_C(-Z_C)$
$I_L = -I_C$