

MAT187 - Calculus II

Term Test 1 – Group Part - February 5, 2019

Time allotted: 30 minutes

Aids permitted: None

Total marks: 10

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Instructions (READ CAREFULLY)

- DO NOT WRITE ON THE QR CODE AT THE TOP OF THE PAGES.
- Read all the instructions carefully.
- This test contains 8 pages and a detached **formula sheet**. Make sure you have all of them.
- You can use page 7 for rough work or to complete a question (**Mark clearly**).

DO NOT DETACH ANY PAGES.

- No calculators, cellphones, or any other electronic devices are allowed. If you have a cellphone with you, it must be turned off and in a bag underneath your chair.

GOOD LUCK!

MULTIPLE-CHOICE PART.**(4 marks)**ANSWER THESE QUESTIONS ON **PAGE 8**.

Only your answer on page 8 will be graded.

1. **(2 marks)** Consider the integral $\int_{-1}^{23} (-x^2 - 3x - 19) dx$. Choose the only correct ordering.

- (A) $M_{242} < \int_{-1}^{23} (-x^2 + 3x - 19) dx < M_{121}$.
- (B) $M_{242} < M_{121} < \int_{-1}^{23} (-x^2 + 3x - 19) dx$.
- (C) $M_{242} < \int_{-1}^{23} (-x^2 + 3x - 19) dx < T_{121}$.
- (D) $T_{121} < \int_{-1}^{23} (-x^2 + 3x - 19) dx < M_{242}$.
- (E) $M_{242} < T_{121} < \int_{-1}^{23} (-x^2 + 3x - 19) dx$.

2. **(2 marks)** To calculate the integral $\int_{-1}^6 \sec(x) dx$, we need to split the integral into several integrals. Select a correct way to split the integral.

- (A) $\int_{-1}^0 \sec(x) dx + \int_0^6 \sec(x) dx$
- (B) $\int_{-1}^{\frac{\pi}{2}} \sec(x) dx + \int_{\frac{\pi}{2}}^6 \sec(x) dx$
- (C) $\int_{-1}^{\frac{\pi}{2}} \sec(x) dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sec(x) dx + \int_{\frac{3\pi}{2}}^6 \sec(x) dx$
- (D) $\int_{-1}^{\frac{\pi}{2}} \sec(x) dx + \int_{\frac{\pi}{2}}^{\pi} \sec(x) dx + \int_{\pi}^{\frac{3\pi}{2}} \sec(x) dx + \int_{\frac{3\pi}{2}}^6 \sec(x) dx$

Continued...

LONG ANSWER PART

3. It's the 1960's and NASA is trying to find out how much work it will take **(6 marks)** to get a spaceship to escape the gravitational pull of Earth. This means that the spaceship should be able to keep going forever, getting further and further away from Earth.

The formula for work is $W = F d$, where F is the force acting on the object and d is the displacement.

The gravitational pull of Earth has magnitude $\frac{GMm}{r^2}$, where G is the universal gravitational constant, M is the mass of the Earth, m is the mass of the spaceship, and r is the distance from the centre of the Earth. You can denote the radius of the Earth as R .

- (a) (2 marks)** Find an integral formula for the work it takes to bring a spaceship from the surface of the Earth to a distance h from the surface.

Explain every step and define all your variables and constants.

Continued...

(b) **(1 mark)** Find an integral formula for the work it takes for a spaceship starting from the surface of the Earth to go on forever getting further and further away from Earth. Justify your answer.

Hint. You don't need to solve the integral.

(c) **(1 mark)** Is the amount of work from (b) finite or infinite? Justify your answer.

Continued...

(d) (2 marks) In reality, the mass of the spaceship also decreases, due to diminishing fuel.

Assume that the mass of the spaceship is given by

$$m(r) = \begin{cases} 4a - br & \text{if } br < 3a \\ a & \text{if } br = 3a \end{cases} \quad \begin{array}{l} \text{(burning fuel)} \\ \text{(ran out of fuel)} \end{array}$$

where a and b are positive constants.

Is the amount of work it takes for a spaceship starting from the surface of the Earth to go on forever, getting further and further away from Earth, **finite or infinite?** Justify your answer.

USE THIS PAGE TO CONTINUE OTHER QUESTIONS.

If you wish to have this page marked, make sure to refer to it in your original solution.

The end.