Final Exam 2021

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Learning Objectives

- What did we cover on the 2021 Final Assessment?

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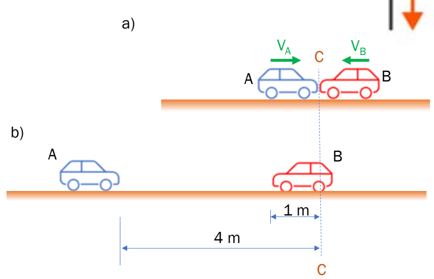
As shown in figure a, vehicle A and vehicle B collide at location C.
After the collision, both vehicles have their brakes locked and slide to new positions (Fig. b).

Given:

$m_A = m_B = 1.85\ kg$
$V_A = 9\ km/hr$ to the right before collision
$\mu_k = 0.3$ for both cars.

Find:
1) The speed of vehicle A immediately after collision;
2) The speed of vehicle B immediately after collision;
3) The speed of vehicle B just before the collision.



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1) After impact, car A

$T_1 = \frac{1}{2} m_A (v_A')^2$

$T_2 = 0$

$U_{1-2}=F_f \times 4$

$F_f = \mu_k m_A g$

$T_1 + U_{1-2} = T_2$

$\Rightarrow$ V'_A=4.852 m/s

2) After impact, car B

$T_1 = \frac{1}{2} m_B (v_B')^2$

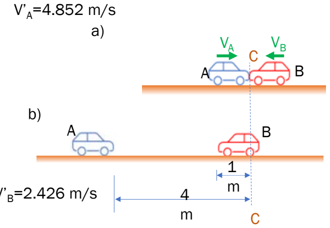
$T_2 = 0$

$U_{1-2}=F_f \times 1$

$F_f = \mu_k m_B g$

$T_1 + U_{1-2} = T_2$

$\Rightarrow$ V'_B=2.426 m/s



3) At C: $m_A v_A + mBvB = mAv_A' + mBvB'$ $\Rightarrow$ V_B=9.778 m/s

Particle momentum and work-energy

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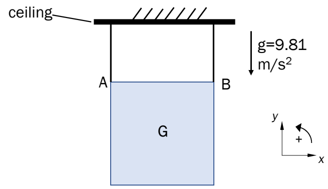
A square panel has mass 9 kg and a length of 2.5 meters on each side. Its radius of gyration about its center G is $k_G = 0.8\ meters$. The panel is initially hanging from the ceiling by two vertical wires attached to the panel at points A and B.

At time t=0, the wire attached at point A breaks.

What will be the angular acceleration α of the panel and the tension in the wire attached to point B immediately after the break occurs? Use the coordinate system shown in the diagram.



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Answer: Need 2 kinetic plus one kinematic equations

$\sum M_G = m\, k_G^2 \alpha$

$(T)(1.25) = (9)0.8^2\alpha$

$1.25T = 5.76\alpha$

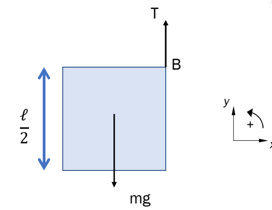
$T = 4.608\alpha$ (1)

$\sum F_y = ma_y$

$T\ - (9)(9.81) = 9a_y$

T - 88.29 = (9)(a_y) (2)

$a_y = \frac{\ell}{2}\alpha = -1.25\alpha$ (3)



Combine (1), (2), and (3) to yield:

$a_y\ = \ -(1.25)(\frac{T}{4.608})$

Rigid Body Kinetics

$\rightarrow T - 88.29 = -(9)(1.25)\left(\frac{T}{4.608}\right) = -2.4414T$

T= 25.65 Newtons and $\alpha = 5.57\,\frac{rad}{s^2}$

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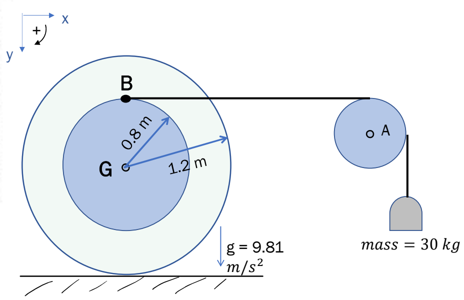

The 75kg-wheel has a radius of gyration with respect to G of 0.9 m. Pulley A is massless and runs on frictionless bearings.

The rope is wound around the inner hub of the wheel and the wheel is rolling initially at 10 rad/s counterclockwise. The wheel is rolling without slip on the ground.

Please use the coordinate system shown in the diagram.

a) Draw the free body diagrams.
b) How much time will it take for it to roll at 6 rad/s clockwise?
c) What are the initial (i.e. wheel is rolling at 10 rad/s counter clockwise) and final (i.e. wheel is rolling at 6 rad/s clockwise) speeds at the point B (as shown on the figure and at the top of the inner hub point at all times).

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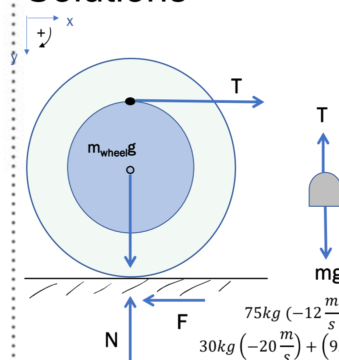
Solutions

Rigid Body Momentum



(b) Initial speed of G is $V_{G,\,initial}$ = -1.2 x (10) = -12 m/s and final speed of G is $V_{G,\,final}$ = 1.2 x (6) = 7.2 m/s

$$V_B = V_G + V_{B/G}$$

$$V_{B,initial} = -12 - 0.8 * \omega_i = -20\frac{m}{s}$$

$$V_{B,final} = 7.2 - 0.8 * \omega_f = 12\frac{m}{s}$$

(c) time, use principle of impulse & momentum

$$mV_{G,x1} + \Sigma \int_{t1}^{t2} Fdt = mV_{G,x2}\ (1)$$

$$mV_{G,y1} + \Sigma \int_{t1}^{t2} Fdt = mV_{G,y2}\ (2)$$

$$I_G\omega_1 + \Sigma \int_{t1}^{t2} M_G dt = I_G\omega_2 (3)$$

$$75kg\ (-12\frac{m}{s}) + (T - F)t = 75kg * \left(7.2\frac{m}{s}\right)$$

$$30kg\left(-20\frac{m}{s}\right) + \left(9.8\frac{m}{s^2} * 30kg\ - T\right)t\ =\ 30kg\left(12\frac{m}{s}\ \right)$$

$$75\ kg(0.9m^2)(-10\frac{rad}{s}) + (0.8T + 1.2F)t\ =\ 75kg(0.9m^2)\left(6\frac{rad}{s}\right)$$

$t\ =\ 7.86s$

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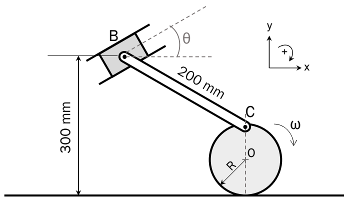

The wheel is rolling without slipping with a constant angular velocity of 5 rad/s. Link BC is connected to slider B. If point C is right at the top edge of the wheel, find the velocity and acceleration of the slider B for the position shown where θ=30° and R=100 mm.

Use the coordinate system shown

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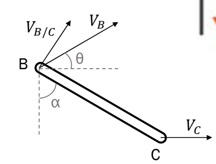
$$0.3 = \overline{BC}\cos\alpha + 2R \quad \Rightarrow \quad \alpha = cos^{-1}\left(\tfrac{0.3-2(0.1)}{0.2}\right) \quad \Rightarrow \quad \alpha = 60°$$

$$V_C = 2R\omega = 2(0.1)(5) = 1\ {}^{m}/_{s}$$

$$\overrightarrow{V_B} = \overrightarrow{V_C} + \vec{V}_{B/C}$$

$$V_B \cos 30°\,\hat{\imath} + V_B \sin 30°\,\hat{\jmath} = 1\,\hat{\imath} + 0.2\,\omega_{BC}\cos 60°\,\hat{\imath} + 0.2\,\omega_{BC}\sin 60°\,\hat{\jmath}$$

$$\left.\begin{aligned} V_B\cos 30° &= 1 + 0.2\,\omega_{BC}\cos 60° \\ V_B \sin 30° &= 0.2\,\omega_{BC}\sin 60° \end{aligned}\right\} \Rightarrow \begin{aligned} &\boxed{V_B = 1.73\ {}^{m}/_{s}} \\ &\omega_{BC} = 5\ {}^{rad}/_{s}\ \ CW \end{aligned}$$



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$$\overrightarrow{a_B} = \overrightarrow{a_C} + (\vec{a}_{B/C})_t + (\vec{a}_{B/C})_n$$

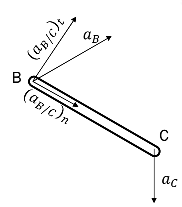
$$\overrightarrow{a_C} = \vec{a}_o + \alpha \times \vec{r}_{C/O} - \omega^2\vec{r}_{C/O} = -R\omega^2\,\hat{\jmath} = -(0.1)5^2\hat{\jmath} = -2.5\,\hat{\jmath}\ \frac{m}{s^2}$$

$$(a_{B/C})_t = \overline{BC}\alpha_{BC}$$

$$(a_{B/C})_n = \overline{BC}\omega_{BC}^2$$

$$a_B\cos 30°\,\hat{\imath} + a_B\sin 30°\,\hat{\jmath} = -2.5\,\hat{\jmath} + 0.2\,\alpha_{BC}\cos 60°\,\hat{\imath} + 0.2\,\alpha_{BC}\sin 60°\,\hat{\jmath}$$
$$+(0.2)(5)^2\sin 60°\,\hat{\imath} - (0.2)(5)^2\cos 60°\,\hat{\jmath}$$

$$\left.\begin{aligned} a_B\cos 30° &= 0.2\,\alpha_{BC}\cos 60° + (0.2)(5)^2\sin 60° \\ a_B\sin 30° &= -2.5 + 0.2\,\alpha_{BC}\sin 60° - (0.2)(5)^2\cos 60° \end{aligned}\right\} \Rightarrow \begin{aligned} &\boxed{a_B = 12.5\ {}^{m}/_{s^2}} \\ &\alpha_{BC} = 64.951\ {}^{rad}/_{s^2}\ CW \end{aligned}$$



Rigid Body Kinematics

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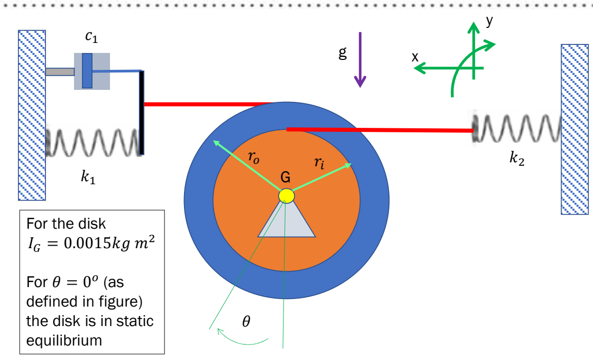

$r_o = 0.12m$
$r_i = 0.06m$
$k_1 = 130\,\frac{N}{m}$
$k_2 = 390\,\frac{N}{m}$
$c_1 = 55\,\frac{Ns}{m}$

What is the θ of this system at $t = 0.02\ seconds$?

The system is released from rest with an initial displacement of θ=0.03 radians

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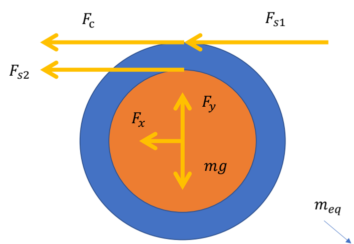
Solution - FBD



$$\sum M_G \quad -(F_c + F_{s1})r_o - F_{s2}r_i = I_G\alpha_D$$

$$F_{s1} = k_1 r_o \theta$$

$$F_{s2} = k_2 r_i \theta$$

$$F_c = c r_o \dot{\theta}$$

$$\alpha_D = \ddot{\theta}$$

$$\overbrace{I_G}^{m_{eq}}\ddot{\theta} + \overbrace{c r_o^2}^{c_{eq}}\dot{\theta} + \underbrace{(k_1 r_o^2 + k_2 r_i^2)}_{k_{eq}}\theta = 0$$

$$c_c = 2\sqrt{k_{eq} m_{eq}} = 2\sqrt{(k_1 r_o^2 + k_2 r_i^2) I_G}$$

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Solution

$$\zeta = \frac{c_{eq}}{c_c} = 5.65$$

$$\omega_n = \sqrt{\frac{k_1 r_o^2 + k_2 r_i^2}{I_G}} = 46.63\,\frac{rad}{s}$$

$$\theta = e^{-\zeta\omega_n t}\left(Ae^{\sqrt{\zeta^2-1}\,\omega_n t} + Be^{-\sqrt{\zeta^2-1}\,\omega_n t}\right)$$

Probably easier to just write the numbers

$$\dot{\theta} = -\zeta\omega_n\, e^{-\zeta\omega_n t}\left(Ae^{\sqrt{\zeta^2-1}\,\omega_n t} + Be^{-\sqrt{\zeta^2-1}\,\omega_n t}\right) + e^{-\zeta\omega_n t}\left(A\sqrt{\zeta^2-1}\,\omega_n e^{\sqrt{\zeta^2-1}\,\omega_n t} - B\sqrt{\zeta^2-1}\,\omega_n e^{-\sqrt{\zeta^2-1}\,\omega_n t}\right)$$

$$\theta(0) = 0.05rad = A + B$$

$$\dot{\theta}(0) = -(A+B)\zeta + A\sqrt{\zeta^2-1} - B\sqrt{\zeta^2-1}$$

Using the initial conditions

$$A = \frac{\theta(0)}{2}\left(1 + \frac{\zeta}{\sqrt{\zeta^2-1}}\right) = 0.0302$$

$$B = \frac{\theta(0)}{2}\left(1 - \frac{\zeta}{\sqrt{\zeta^2-1}}\right) = -0.0002$$

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Solution

$$\theta(t) = e^{-264t}\left(0.0302e^{259.83t} - 0.0002e^{-259.83t}\right)$$

$$\theta(0.02sec) = 1.59deg$$

Or

$$\theta(0.02sec) = 0.0278rad$$

$$0.0302e^{-4.17t} - 0.0002e^{-523.83t} = \theta(t)$$

Free Damped Vibrations

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Conclusions

- Be careful with how you arrange things
- Look at your work and DRAW DRAW DRAW

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