University of Toronto
Faculty of Applied Science and Engineering
Department of Electrical and Computer Engineering

ECE110S – Electrical Fundamentals
Term Test 2 – March 21, 2019, 6:30 – 8:00 p.m.

($e = 1.6 \times 10^{-19}$ C, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $g = 9.81$ N/kg)

ANSWER ALL QUESTIONS ON THESE SHEETS, USING THE BACK SIDE IF NECESSARY.

1. Non-programmable calculators (Casio FX-991MS & Sharp EL-520X) are allowed.
2. You are allowed a one page (8.5" x 11") double-sided aid sheet.
3. For full marks, you must show methods, state UNITS and compute numerical answers (when requested).
4. Write in PEN. Otherwise, no remarking request will be accepted.
5. There is one extra blank page at the end for rough work.

Last Name: Full

First Name: Solutions

Student Number: ______________________

Tutorial Section:
(YOU LOSE ONE MARK FOR NOT MARKING YOUR TUTORIAL SECTION CORRECTLY)

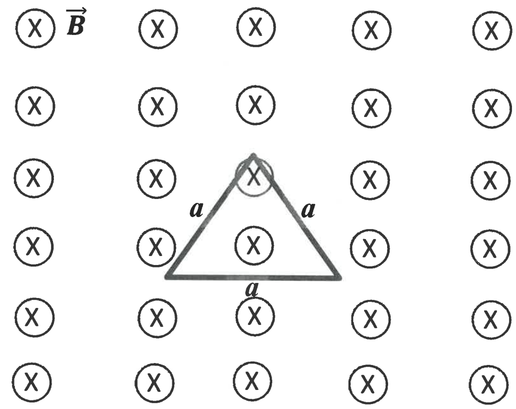
☐	01	HS106	Wed. 4-6 p.m.
☐	02	GB404	Wed. 12-2 p.m.
☐	03	SF1101	Wed. 4-6 p.m.
☐	04	SF3202	Fri. 10-12 p.m.
☐	05	HA410	Tues. 4-6 p.m.
☐	06	GB404	Fri. 12-2 p.m.
☐	07	GB248	Tues. 4-6 p.m.
☐	08	MP137	Thurs. 4-6 p.m.
☐	09	SF3201	Wed. 12-2 p.m.
☐	10	MY380	Wed. 4-6 p.m.
☐	11	SF1105	Wed. 4-6 p.m.

Question	Mark
1	
2	
3	
TOTAL	

Q1 [10 marks]

Figure shows a metallic wire loop in the shape of an equilateral triangle (all sides are equal having the length a centimeter) immersed in a magnetic field $\vec{B}$. The magnetic field ($\vec{B}$) is uniform in space, pointing into the page, but changes with time according to $t\,e^{-t}$ (T). Considering the time interval $0 < t < 1$ second, answer the following questions.

a) Derive an expression for the magnitude of the induced current in terms of a and t, assuming each side of the triangle has a resistance R (Ω). What is the direction of the induced current (clockwise or counter clockwise)? **(6 marks)**



$B = te^{-t}$

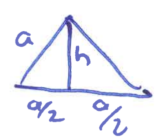
$\frac{dB}{dt} = (1-t)e^{-t}$ for $0<t<1$

$\frac{dB}{dt} > 0$ from $0<t<1$

$\Phi_B \uparrow$ $\therefore$ I_{ind} is CCW

$|\mathcal{E}_{ind}| = \left|-N\frac{d\Phi_B}{dt}\right|$ $\quad \Phi_B = \iint \vec{B}\cdot d\vec{A} = BA$

$A = \frac{ha}{2}$ where $h = \sqrt{a^2 - \frac{a^2}{4}}$



$= \sqrt{\frac{3a^2}{4}}$

$= \frac{\sqrt{3}}{2}a$

$A = \frac{\sqrt{3}}{4}a^2$ $\qquad \Phi_B = BA = te^{-t}\frac{\sqrt{3}}{4}a^2$

$|\mathcal{E}| = \left|-\frac{d\Phi_B}{dt}\right| = \left|\frac{\sqrt{3}}{4}a^2(1-t)e^{-t}\right|$

$= \frac{\sqrt{3}}{4}a^2(1-t)e^{-t}$ for $0<t<1$

$|I_{ind}| = \frac{|\mathcal{E}|}{3R} = \frac{1}{3R}\frac{\sqrt{3}}{4}a^2(1-t)e^{-t}$

$= \frac{a^2}{4\sqrt{3}R}(1-t)e^{-t}$

Q1 continued

b) Obtain an expression for the total power dissipated in the metallic loop. Show all your work and simplify your answer as much as possible. **(2 marks)**

$$P_{dis} = (I_{ind})^2 \cdot 3R = \frac{a^4}{16 \cdot 3R^2}(1-t)^2 e^{-2t} \cdot 3R$$

$$= \frac{a^4}{16R}(1-t)^2 e^{-2t}$$

c) Suppose that each side of the triangle is doubled in length, while the loop is still entirely immersed in the magnetic field. As compared to the part (a), does the induced current increase or decrease and by how much? Show all your work. **(2 marks)**

$R \rightarrow 2R$

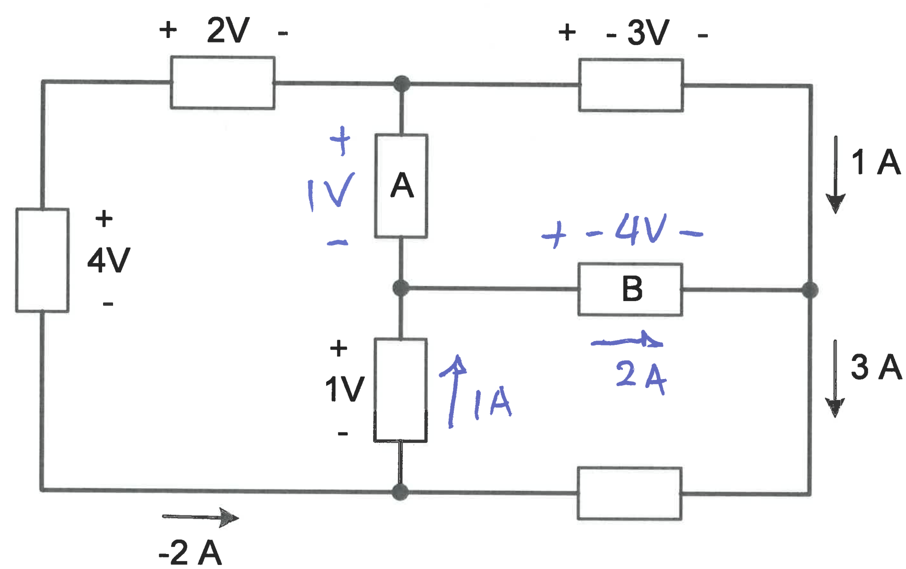
$a \rightarrow 2a$

$$\frac{a^2}{R} \rightarrow \frac{(2a)^2}{2R} = 2\frac{a^2}{R}$$

$$\therefore I_{ind}^{(new)} = 2 I_{ind}^{(old)}$$

Q2 [10 marks]

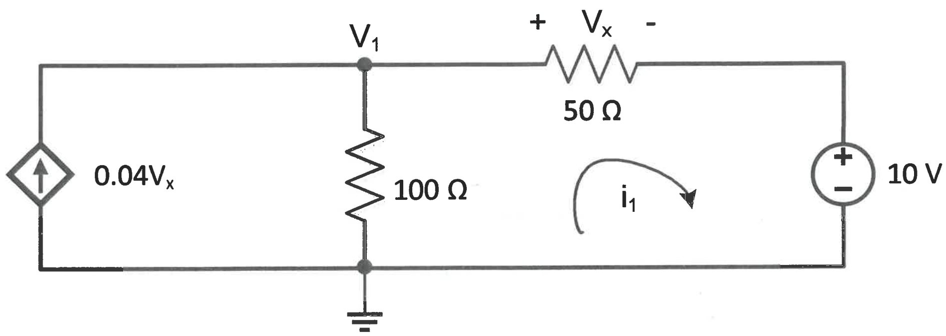
2.1 Consider the circuit shown below. Some element voltages and element currents are given. Calculate the powers for element A and element B? **(3 marks)**



$P_A = VI = (1V)(1A) = 1W$

$P_B = VI = (-4V)(2A) = -8W$

2.2 For the circuit given below,



a) Assume that you are solving this problem using nodal analysis technique, write a set of equations in terms of nodal voltages. **(2.5 marks)**

$$\frac{V_1}{100} + \frac{V_1 - 10}{50} - 0.04V_x = 0$$

$$V_x = V_1 - 10$$

b) Assume that you are solving this problem using mesh analysis technique, write a set of equations in terms of mesh currents. **(2.5 marks)**

$$100(i_1 - 0.04V_x) + 50i_1 + 10 = 0$$

$$V_x = 50i_1$$

c) Determine $\mathbf{V_x}$ by solving the set of equations in either **a)** or **b)** above. **(2 marks)**

from a) $V_1 = 20V$

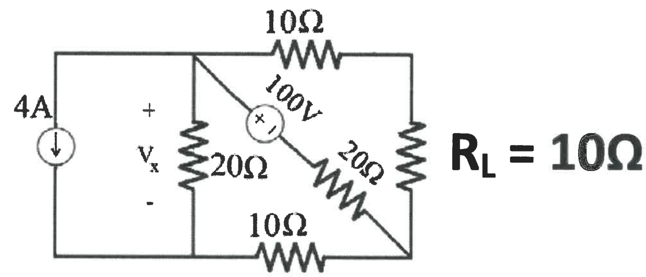
$$V_x = V_1 - 10 = 10V$$

from b) $i_1 = 0.2A$

$$V_x = 50i_1 = 10V$$

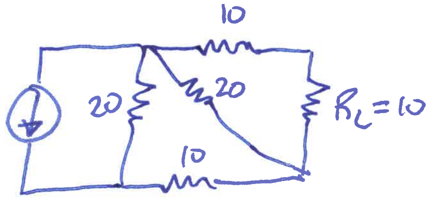
Q3 [10 marks]

Consider the circuit containing a current source and a voltage source as shown below.



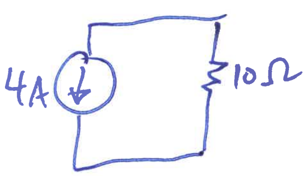
a) What is the contribution of the current source alone to:

i. The magnitude of V_x **(2 marks).**



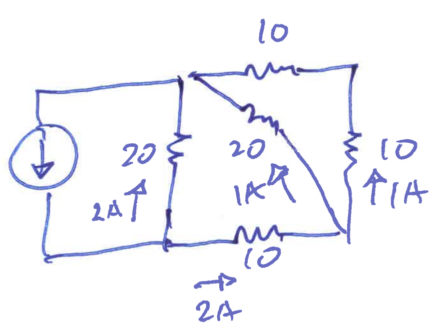
⇓

$V_{x_1} = -4 \cdot 10 = -40V$



ii. The power dissipated in the resistor R_L **(2 marks)**

$|I_{R_{L_1}}| = 1A$



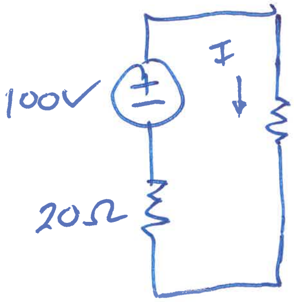
$P_{R_L} = (I_{R_{L_1}})^2 R$

$= 10W$

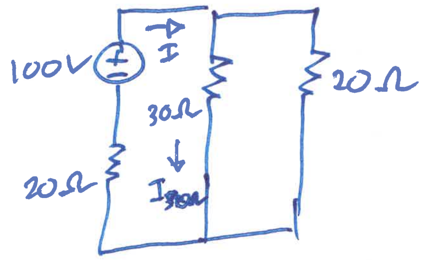
Current dividers

b) What is the contribution of the voltage source alone to:

i. The magnitude of V_x **(2 marks).**



$R_{eq} = 20 \| 30$
$= 12\,\Omega$



$$I = \frac{100}{32}\,A$$

$$I_{30\Omega} = \frac{20}{50} \cdot \frac{100}{32} = \frac{40}{32}\,A$$

$$V_{x_2} = \frac{40}{32} \times 20 = 25\,V$$

ii. The power dissipated in the resistor R_L **(2 marks)**

$$I_{R_{L_2}} = \frac{30}{50} \cdot \frac{100}{32} = \frac{15}{8}$$

$$P_{R_{L_2}} = (I_{R_{L_2}})^2 R_L$$

$$= \frac{225}{64} \cdot 10$$

$$= \frac{2250}{64} = \sim 35.1563\,\Omega$$

c) Compute:

i. The magnitude of V_x **(1 mark)**

$V_x = V_{x_1} + V_{x_2} = -40 + 25 = -15V$

ii. The total power dissipated in the resistor R_L **(1 mark)**

$I_{R_L} = I_{R_{L1}} + I_{R_{L2}} = 1.88 - 1 = 0.88A$

$P_{R_L} = (0.88)^2 \cdot 10 = 7.7W$