



# ECE 110 TERM TEST 1

2024-02-08

First name (please write as legibly as possible within the boxes)

# SOLUTIONS

Last name

Student ID number



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University of Toronto  
Faculty of Applied Science and Engineering  
**Department of Electrical and Computer Engineering**

ECE110S – Electrical Fundamentals

Term Test 1 – February 08, 2024

7:10 – 8:40 p.m. (1.5 hours)

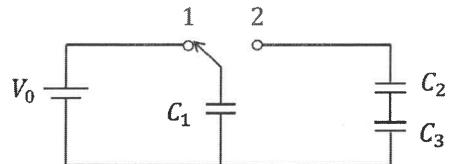
$(e = 1.6 \times 10^{-19} \text{ C}, \epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}, \mu_0 = 4\pi \times 10^{-7} \text{ H/m}, g = 9.8 \text{ m/s}^2)$

**Instructions:**

1. Exam Type C3: a one-page (8.5" x 11") double-sided aid sheet is allowed. This aid sheet may be handwritten or computer-generated, including typewritten text, images, or other formats that fit within the paper. Using the template from FASE's website is not mandatory.
2. Non-programmable calculators (Casio FX-991MS & Sharp EL-520X) are permitted.
3. Please use a dark pen or pencil and ensure your handwriting is clear, as the paper will be scanned for grading.
4. For full marks, you must show methods, state UNITS, and compute numerical answers when requested.
5. An extra blank page is provided at the end. If you use this page for your answers, make sure to clearly indicate this within the relevant question area to ensure it is considered during the grading process.
6. TAs and instructors CANNOT answer inquiries regarding the exam questions. If you are unsure about the question, please state your assumptions in your answer.



Q1. The capacitor  $C_1$  acquires a charge of  $Q_0$  when connected to a battery of voltage  $V_0$  as shown in the figure when the switch is in position 1. Then the switch is moved to position 2 connecting  $C_1$  to the initially uncharged capacitors  $C_2$  and  $C_3$ .



- a) Derive an expression for the voltage across  $C_3$  in terms of  $Q_0$ ,  $C_1$ ,  $C_2$  and  $C_3$ . [4 marks]

$$V_3 = \frac{Q_0 C_2}{C_1 (C_2 + C_3) + C_2 C_3}$$



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Q1 continued.

- b) If  $C_1$  is a cylindrical capacitor with a length of 3.6 cm, inner and outer conductor radii of 1 mm and 5 mm, respectively. Compute  $C_1$ . [1 mark]

$$C_1 = 1.242 \mu F$$

- c) If  $C_2$  is a parallel plate capacitor with plate area of 2 cm<sup>2</sup> and plate separation of 1 mm, and  $C_3$  is a parallel plate capacitor similar to  $C_2$  except it has a dielectric with dielectric constant of 3 between the plates. Compute  $C_2$  and  $C_3$ . [2 marks]

$$C_2 = 1.77 \mu F$$

$$C_3 = 5.31 \mu F$$

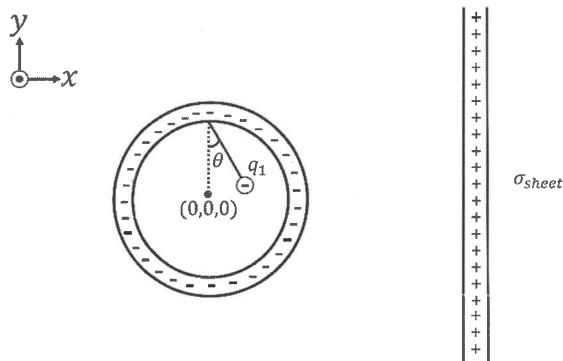
- d) If the battery voltage  $V_0$  is 5 volts. Compute the energy stored in  $C_2$  and the voltage across  $C_3$ . [3 marks]

$$U_2 = 2.9 \mu J$$

$$V_3 = 0.6 V$$



Q2. Consider a stationary non-conducting spherical shell, with a radius of 1 m that has a negative charge uniformly distributed on its surface. As shown in the figure, a negatively charged particle with a charge  $q_1 = -5 \times 10^{-8} C$  and a mass of  $5 \times 10^{-6} kg$  is suspended by a thin insulating thread that is 1 m in length. The angle  $\theta$  between the thread and the vertical axis is  $30^\circ$ . The origin of the coordinate system is set at the center of the spherical shell. An infinitely large uniformly charged non-conducting sheet is positioned 3 m away from the origin ( $x = 3 m$ ) in parallel with the y-z plane (shown in cross-section in the figure below). Assume gravitational acceleration is  $9.8 m/s^2$ . The z-axis is perpendicular to the page with the positive direction coming out of the page.



- a) What is the surface charge density of the sheet  $\sigma_{sheet}$ ? [6 marks]

$$\sigma = 1 \times 10^{-8} C/m^2$$

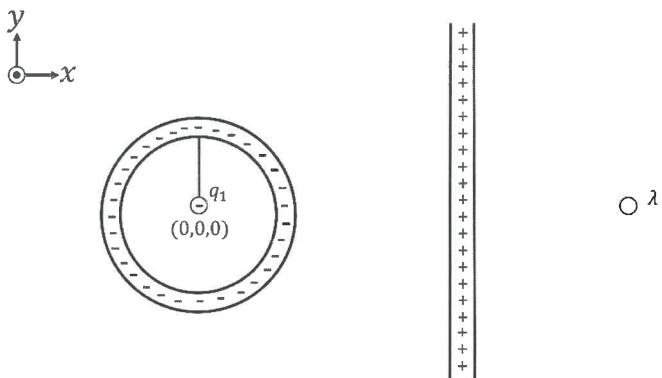


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Q2 continued.

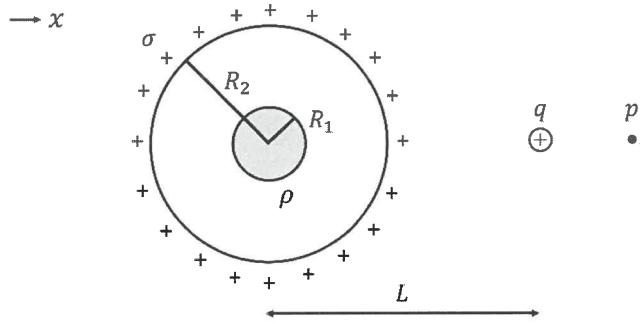


- b) Now consider adding a uniformly charged, non-conducting, infinitely-long line placed along the z-axis, centered at the coordinates (10,0,0). If  $q_1$  is located at the origin of the coordinate system (0,0,0). What is the line charge density  $\lambda$ ? [4 marks]

$$\lambda = -3.14 \times 10^{-7} \text{ C/m}$$



Q3. Figure show a non-conducting uniformly charged shell of radius  $R_2 = 2 \text{ cm}$  with charge density  $\sigma = \frac{1}{32\pi} \frac{nC}{\text{cm}^2}$ . A uniformly charged solid sphere has charge density  $\rho = \frac{3}{8\pi} \frac{nC}{\text{cm}^3}$ , with radius  $R_1 = 1 \text{ cm}$ , is located inside the shell and is co-centered with it. A positive point charge  $q = 1 \text{ nC}$  is located at  $L = 10 \text{ cm}$ . All of the above charges are fixed in position.



- a) We wish to place a positive charge  $q_0$  outside of the shell such that  $q_0$  is at equilibrium. Calculate the location at which  $q_0$  must be placed. Show all your work and justify your answer. [7 marks]

$$x = 5 \text{ cm}$$



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Q3 continued.

- b) With  $q_0$  in place, and assuming  $q_0 = 1 \text{ nC}$ , what is total electrostatic potential (voltage) at the location  $P$ , where  $P$  is 2 cm to the right of the charge  $q$ . [3 marks]

$$V_p = 652.976 \text{ V}$$

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