Name: ______________________________

Student number: ____________________

MAT 186
Quiz 8

1. Evaluate the following limit:

AB3	CF6	CS13

$$\lim_{x\to\infty} \frac{x^2 - \ln(2/x)}{3x^2 + 2x}$$

The limit is of the form ∞/∞, so L'Hopital is probably the way to go. Ideally, you should simplify first:

$$\begin{aligned}
\lim_{x\to\infty} \frac{x^2 - \ln(2/x)}{3x^2 + 2x} &= \lim_{x\to\infty} \frac{x^2 - \ln 2 + \ln x}{3x^2 + 2x} \\
&\overset{L'H}{=} \lim_{x\to\infty} \frac{2x - 0 + 1/x}{6x + 2} \\
&= \lim_{x\to\infty} \frac{2 + 1/x^2}{6 + 2/x} \\
&= 1/3
\end{aligned}$$

2. Another fun limit:

AB3	AB5	CF6	CS13	CS14

$$\lim_{\theta\to(\pi/2)^-} (\tan\theta)^{\cos\theta}$$

This limit is of the form ∞^0, so L'Hopital's rule applies.

$$\begin{aligned}
L &= \lim_{\theta\to(\pi/2)^-} (\tan\theta)^{\cos\theta} \\
\ln L &= \lim_{\theta\to(\pi/2)^-} \cos\theta \ln(\tan\theta) \\
&= \lim_{\theta\to(\pi/2)^-} \frac{\ln(\tan\theta)}{\left(\frac{1}{\cos\theta}\right)} \\
&\overset{L'H}{=} \lim_{\theta\to(\pi/2)^-} \frac{\left(\frac{1}{\tan\theta}\cdot\sec^2\theta\right)}{\left(\frac{-1}{\cos^2\theta}\cdot(-\sin\theta)\right)} \\
&= \lim_{\theta\to(\pi/2)^-} \frac{\cos\theta}{\sin^2\theta} = 0 \\
\therefore L &= e^0 = 1
\end{aligned}$$

Continued on back.

AB8	CF18	CF19

3. For the function $f(x) = 6x - 2$, over the interval $[-2, 2]$, find the area under the graph using Riemann sums.

For the Riemann sum, we take $\Delta x = \frac{2-(-2)}{n} = \frac{4}{n}$.

Therefore, $x_k = a + k\Delta x = -2 + \frac{4}{n}k$.

It is easiest to use a right-hand sum:

$$\begin{aligned}
\int_{-2}^{2} 6x - 2\,dx &= \lim_{n\to\infty} \sum_{k=1}^{n} (6x_k^* - 2)(\Delta x) \\
&= \lim_{n\to\infty} \sum_{k=1}^{n} \left(6\left(-2 + \frac{4}{n}k\right) - 2\right)\left(\frac{4}{n}\right) \\
&= \lim_{n\to\infty} \sum_{k=1}^{n} \left(-14 + \frac{24}{n}k\right)\left(\frac{4}{n}\right) \\
&= \lim_{n\to\infty} \sum_{k=1}^{n} \left(-\frac{56}{n} + \frac{96}{n^2}k\right) \\
&= \lim_{n\to\infty} \left(\sum_{k=1}^{n} \left(-\frac{56}{n}\right) + \sum_{k=1}^{n} \left(\frac{96}{n^2}k\right)\right) \\
&= \lim_{n\to\infty} \left(-56 + \frac{96}{n^2} \cdot \frac{n(n+1)}{2}\right) \\
&= -56 + 48 \\
&= -8
\end{aligned}$$

CF16

4. Verify your answer from question 3, using antiderivatives.

$$\begin{aligned}
\int_{-2}^{2} 6x - 2\,dx &= (3x^2 - 2x)_{-2}^{2} \\
&= (12 - 4) - (12 + 4) \\
&= -8
\end{aligned}$$