

**MIE100S – Applied Mechanics: Dynamics**  
**Final Examination - Winter 2016**  
**Exam length: 150 minutes**

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**General Instructions:**

- Answer all questions in the CrowdMark booklet provided.
- Write your name, student number, and e-mail address on the CrowdMark Booklet
- **You must answer each question on the correct page of the CrowdMark booklet.**
- Do not tear any pages out of the CrowdMark booklet.
- All rough work must be *neatly* shown to earn credit for each question.
- You must use a pen or *dark* pencil.
- Answer all five questions. Total marks on this exam = 100.
- For your *final answers*, express vector quantities **using the coordinate system specified in the question.**

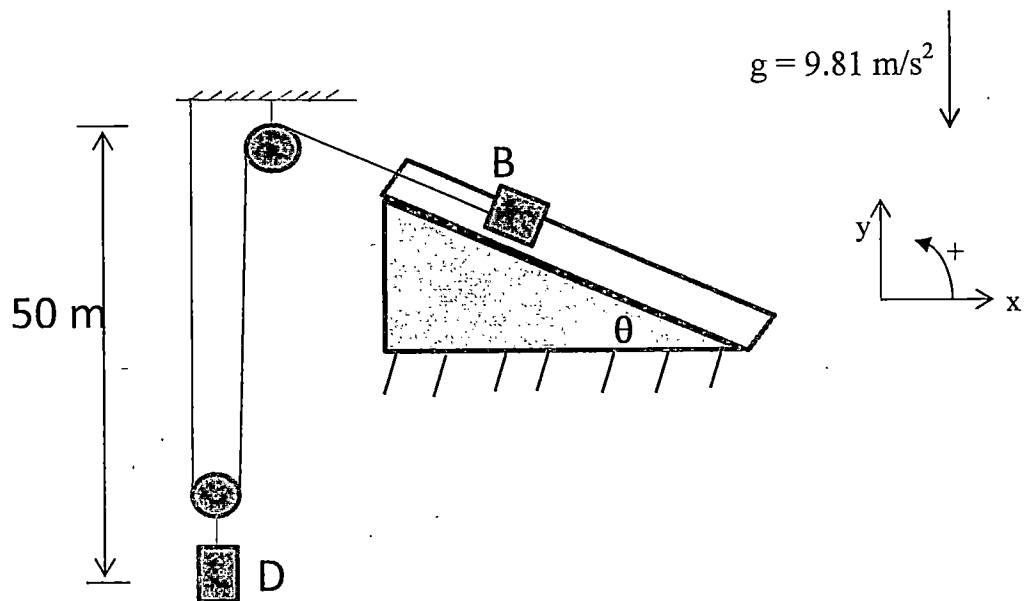
**This is a Type D examination. Permitted Aids:**

- Non-communicating/non-programmable calculator: Casio FX-991 or Sharp EL-520
- One 8 ½" x 11" aid sheet, any colour, brought to the test by the student. You may write on both sides of the sheet.

1. A small 20 kg block B is initially stationary on a long, sloped ramp with  $\theta = 30^\circ$ . A 4 kg block D is connected with B by a cable as shown in the diagram. Suppose that blocks B and D are released from rest at the positions shown in the diagram.

- 8 (a) **Assume there is no friction.** Find the velocity of B when it has slid 10 meters down the surface of the ramp.
- 8 (b) **Assume that  $\mu_k = 0.2$  and  $\mu_s = 0.4$ .** Will the block slide down the ramp? What will be its acceleration  $\vec{a}$ ?
- 4 (c) Suppose that blocks B and D are released from rest at the positions shown in the diagram at time  $t=0$ . **Assume there is no friction.** What will be the total linear impulse applied to D in the next 0.3 seconds?

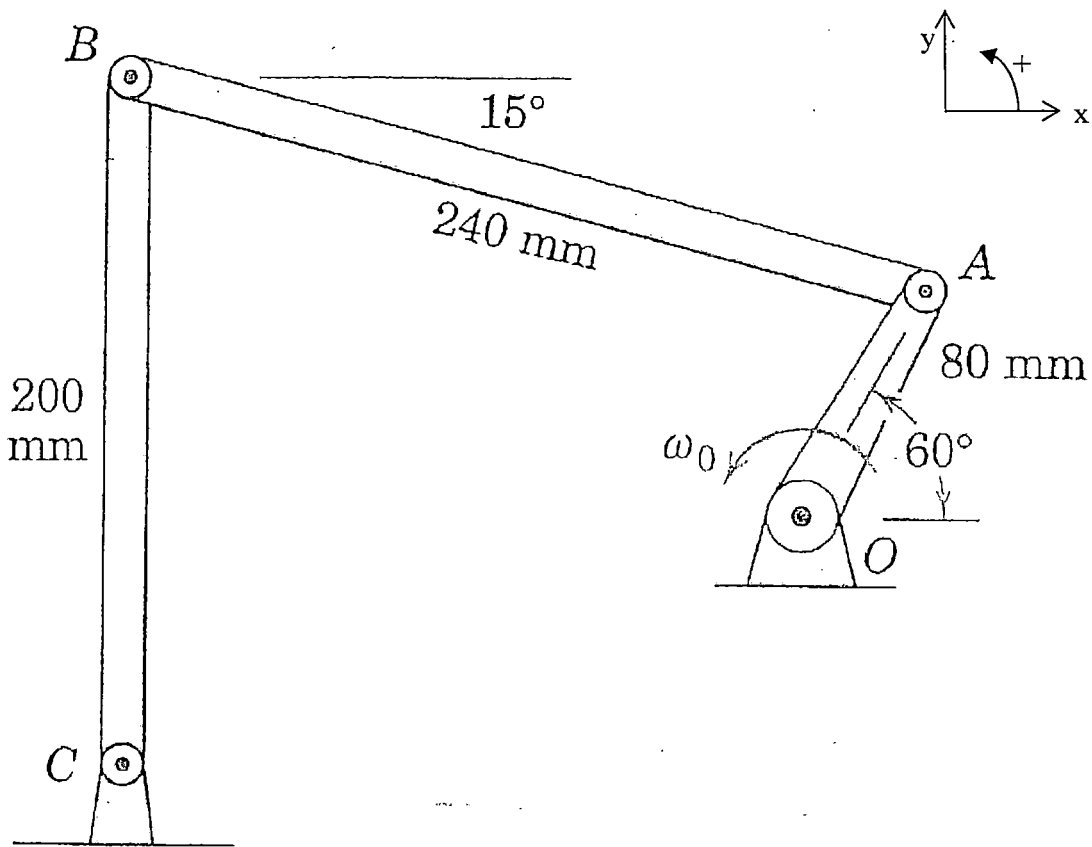
20 marks for question #1



2. Answer each part of this question for the instant shown in the diagram.

- 5 (a) If  $\omega_0 = 10$  radians/s, determine the velocity of point A.
- 10 (b) If  $\omega_0 = 10$  radians/s, determine the angular velocity  $\omega$  of bar AB.
- 5 (c) If bar CB has  $\omega = 5$  radians/s and  $\alpha = -3$  radians/s<sup>2</sup>, determine the acceleration of point B.

**20 marks for question #2**



3. The long, thin uniform rod AB was released from rest at a high altitude and allowed to drop onto a frictionless floor. Its center of mass is at G. The length of the rod is 4 meters, and its mass is 21 kg. At the instant shown in the diagram, the "A" end of the rod has now contacted the floor, and is sliding to the left along the floor. At the instant shown in the diagram, the angular speed of the rod is  $\omega = -3$  radians/s.

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(a) At the instant shown in the diagram, find the velocity of the center of the rod.

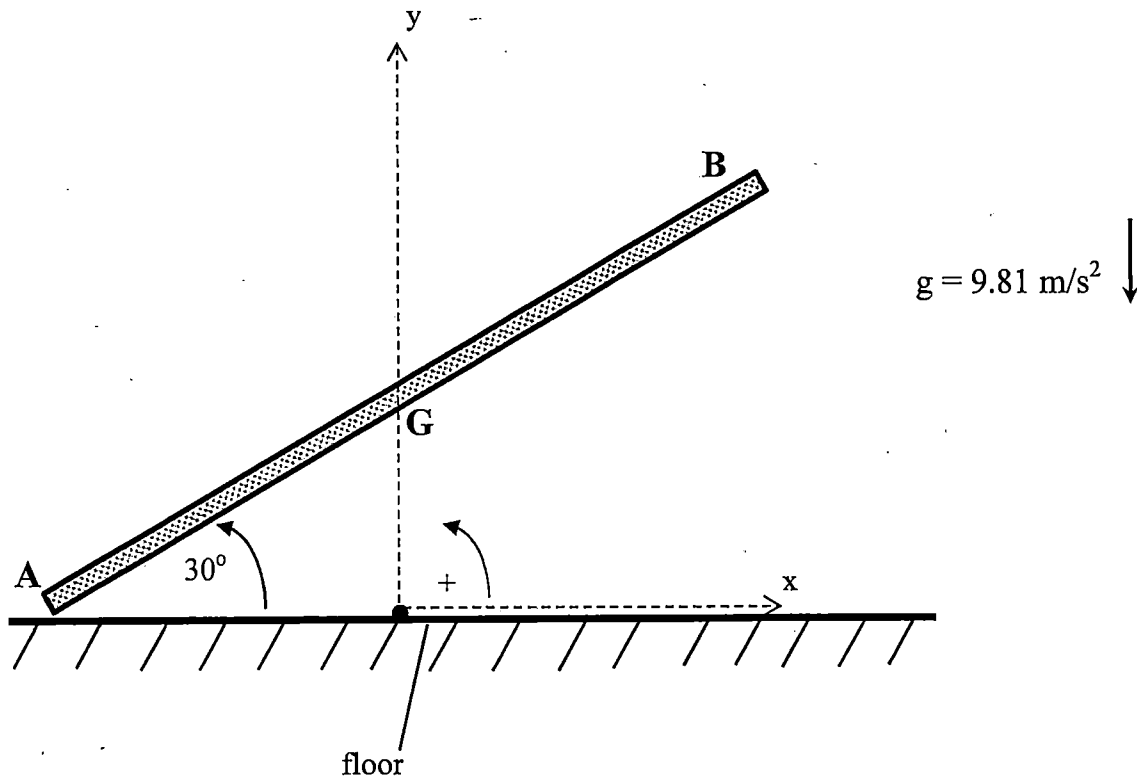
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(b) At the instant shown in the diagram, find the angular acceleration  $\alpha$  of the rod.

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(c) At the instant before the "B" end of the rod hits the ground, what will be the location of the instantaneous center of zero velocity of the rod? Express your answer in terms of the  $\{x,y\}$  coordinate system shown in the diagram.

20 marks for question #3



4. A 200 kg spool with inner radius  $r_{\text{inner}} = 1.2\text{m}$  and outer radius  $r_{\text{outer}} = 1.8\text{m}$  is released from rest on an incline of  $\theta = 42^\circ$ . A rope is wound around the inner hub of the spool as shown. The centre of the spool  $G$  corresponds to its centre of mass. Its radius of gyration is  $k_G = 1.5\text{m}$ . The coefficient of kinetic friction between the spool and the incline is  $\mu_k = 0.23$ .

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- a) Draw a Free Body Diagram of the spool. Clearly indicate the direction of the friction force acting on the spool.

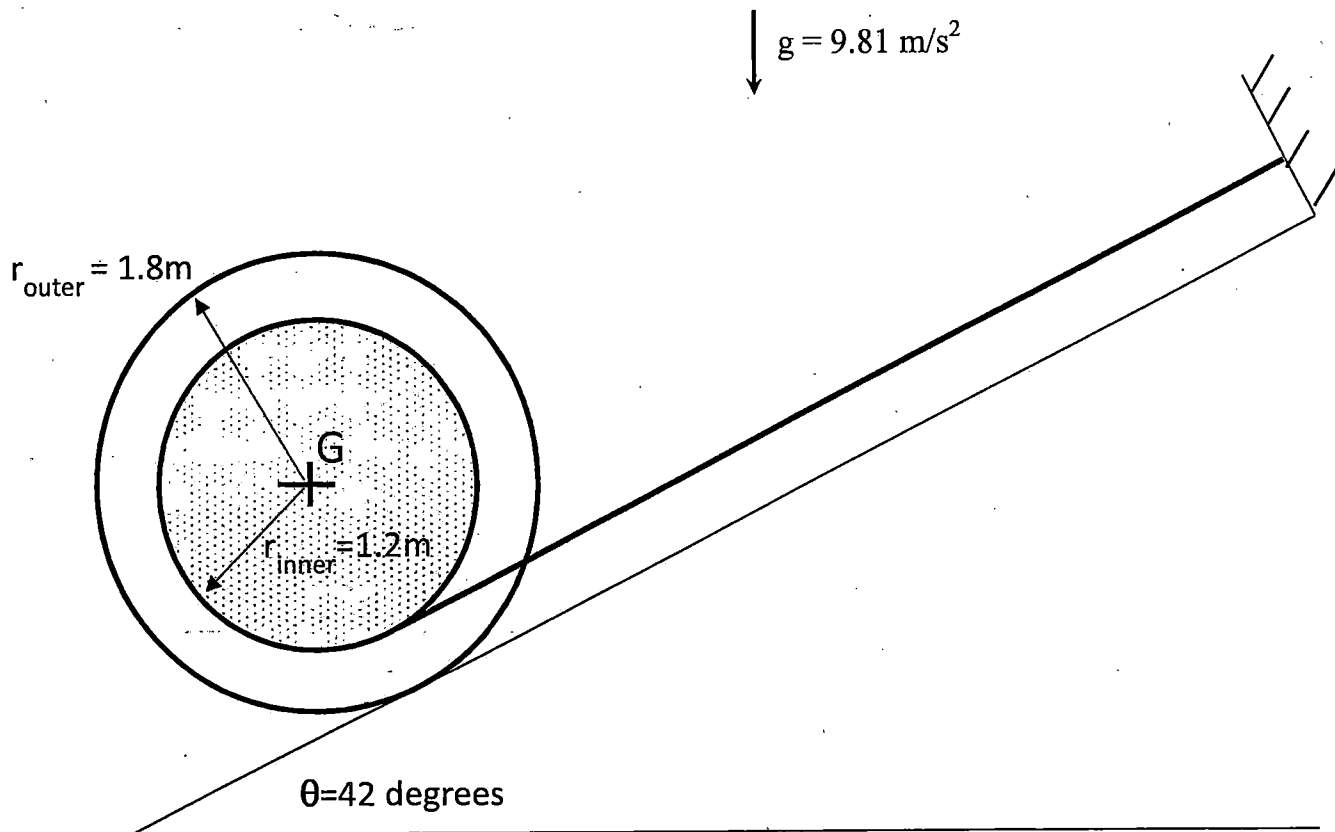
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- b) Assume that the spool starts to move down the incline when it is released. In what direction is the spool rotating? Indicate in a simple diagram the location of the instantaneous centre of zero velocity.

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- c) After the center of the spool has displaced 3m, what is the speed of the centre of mass?

20 marks for question #4



5. A slender rod  $AB$  of length  $L$  and mass  $m$ , with centre of mass located at  $G$ , is suspended from the ceiling and attached to the spring-damper system as shown in the diagram below.  $k_1 = 1000 \text{ N/m}$ ,  $k_2 = 2000 \text{ N/m}$ ,  $L = 6.0 \text{ m}$ ,  $k_3 = 1500 \text{ N/m}$ , and  $m = 40 \text{ kg}$ .

*Assume that the rod  $AB$  oscillates only in the  $y$ -direction. Ignore any rotation.*

- 4 (a) If  $c = 280 \text{ N}\cdot\text{s/m}$ , find the natural frequency of the system,  $\omega_n$ .
- 10 (b) The slender rod is displaced in the  $y$ -direction from its equilibrium position located at  $y=0$ . The initial conditions at  $t=0$  are  $y(0) = 0.12 \text{ m}$  and  $\dot{y}(0) = 0.5 \text{ m/s}$ .  
If  $c = 280 \text{ N}\cdot\text{s/m}$ , find the vertical location  $y(t)$  at  $t = 0.2$  seconds.
- 6 (c) An oscillatory force  $\vec{F} = 200 \sin(5t) \hat{j} \text{ N}$  is applied vertically at point  $G$ , leading to forced vibrations.  
If  $c = 0$  (undamped), **find the amplitude** of the steady-state vibration.

**20 marks for question #5**

