Name: ______________________

Student number: ______________

MAT 186
Quiz 9 Redux

Write with confidence.

1. Evaluate

CF16	CF22

$$\int_0^5 (x-1)\sqrt{x+4}\,dx$$

Two good solutions exist. We can substitute for $x+4$ (that is the solution shown here), but it would actually be much better to use $\sqrt{x+4}$. Try that one yourselves.

Let $u = x+4$, so $du = dx$.

$$\int_0^5 (x-1)\sqrt{x+4}\,dx = \int_4^9 (u-5)\sqrt{u}\,du$$
$$= \int_4^9 u^{3/2} - 5u^{1/2}\,du$$
$$= \left[\frac{2}{5}u^{5/2} - \frac{10}{3}u^{3/2}\right]_4^9$$
$$= \left(\frac{2}{5}3^5 - \frac{10}{3}3^3\right) - \left(\frac{2}{5}2^5 - \frac{10}{3}2^3\right)$$
$$= ugh, or\ 316/15$$

2. Find the area of the region between $y = |x-2|$ and $x + 2y = 5$.

AB4	CF16	CF25

We need to take cases:

$$|x-2| = \begin{cases} 2-x, & x < 2 \\ x-2, & x \geq 2 \end{cases}$$

These will intersect at $(-1,3)$ and $(3,1)$. So, we need to find the area using two integrals (we could use symmetry and just double one of them):

$$A = \int_{-1}^{2} \left(\frac{5-x}{2}\right) - (2-x)\ dx + \int_2^3 \left(\frac{5-x}{2}\right) - (x-2)\,dx$$

Continued on back.

$$= \int_{-1}^{2} \frac{x}{2} + \frac{1}{2}\, dx + \int_{2}^{3} -\frac{3x}{2} + \frac{9}{2}\, dx$$
$$= \left(\frac{x^2}{4} + \frac{x}{2}\right)_{-1}^{2} + \left(-\frac{3x^2}{4} + \frac{9x}{2}\right)_{2}^{3}$$
$$= 3$$

3. A spherical balloon is being filled so that the rate of growth of its radius is constant. Show that its volume is increasing at a rate that is proportional to its surface area.

CF10

We are given that $\frac{dr}{dt} = k$, for some k. So:

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$
$$= 4\pi r^2 \cdot \frac{dr}{dt}$$
$$= SA \cdot k$$

[This one is optional. Solve it if you need better marks in the main attribute tested here. If you try this problem, it will not lower your mark.]

4. Find $\lim_{\theta \to 0^+} (\sin\theta)^{\tan\theta}$.

AB3	AB5	CF6	CS13	CS14

This limit is of the form 0^0, so L'Hopital's rule applies.

$$L = \lim_{\theta \to 0^+} (\sin\theta)^{\tan\theta}$$
$$\mathbf{\ln} \boldsymbol{L} = \lim_{\theta \to 0^+} \tan\theta \ln(\sin\theta)$$
$$= \lim_{\theta \to 0^+} \frac{\ln(\sin\theta)}{\left(\frac{1}{\tan\theta}\right)}$$
$$\overset{L'H}{=} \lim_{\theta \to 0^+} \frac{\left(\frac{1}{\sin\theta} \cdot \cos\theta\right)}{(-\csc^2\theta)}$$
$$= \lim_{\theta \to 0^+} -\sin\theta \cdot \cos\theta = 0$$
$$\therefore L = \boldsymbol{e^0} = 1$$