Name: ______________________________

Student number: ____________________

MAT 186 Quiz 5

1. <u>Using Leibniz notation</u>, find the derivative of $y = \tan^2(x^3 + 2x)$.

CS6

$$\frac{d}{dx}\tan^2(x^3+2x) = \frac{d\tan^2(x^3+2x)}{d\tan(x^3+2x)}\cdot\frac{d\tan(x^3+2x)}{d\,(x^3+2x)}\cdot\frac{d\,(x^3+2x)}{dx}$$
$$= 2\tan(x^3+2x)\cdot\sec^2(x^3+2x)\cdot(3x^2+2)$$

1. Evaluate $\lim_{x\to 2}\frac{|3x^2-5x-2|}{x^2-4x+4}$.

AB4	CS3

We need to look at the numerator as x approaches 2. Notice that the polynomial has two roots and so we can be more precise about where it is positive and where it is negative over the entire number line, but all we actually need is what happens *near* $x = 2$.

$$|3x^2-5x-2| = |(x-2)(3x+1)|$$
$$= \begin{cases} (x-2)(3x+1), & x > 2 \; by\ a\ bit \\ -(x-2)(3x+1), & x < 2 \; by\ a\ bit \end{cases}$$

Therefore, we can break up the limit into one-sided limits:

$$\lim_{x\to 2^+}\frac{|3x^2-5x-2|}{x^2-4x+4} = \lim_{x\to 2^+}\frac{(x-2)(3x+1)}{(x-2)^2} = \lim_{x\to 2^+}\frac{3x+1}{x-2} = \left[\frac{7}{0^+}\right] = +\infty$$

$$\lim_{x\to 2^-}\frac{|3x^2-5x-2|}{x^2-4x+4} = \lim_{x\to 2^-}\frac{-(x-2)(3x+1)}{(x-2)^2} = \lim_{x\to 2^-}-\frac{3x+1}{x-2} = \left[-\frac{7}{0^-}\right] = +\infty$$

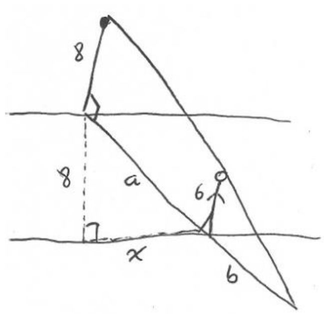
The two sides are equal, so the two-sided limit is equal to $+\infty$.

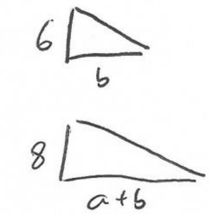
Continued on back.

3. Find the tangent line to $x^2y + \ln^2[(x+2y)^3] = 3 + 3x + y$ at $(-1,1)$.

4. A lamp of 8 foot height is standing on one side of a narrow street. On the opposite side, a 6 foot tall person is walking at a rate of 3 feet per second. If the street is 8 feet across, how fast is the person's shadow lengthening two seconds after walking directly across from the lamp?

CF9	CF10	CF11	CS7	MW3





The diagram above shows the situation – note that it is three-dimensional, so we need to take a couple of cross-sections.

First, we have the $xy-$ plane (the street itself): there is a right-angle triangle on the road, with a height (the width of the road) of 8, the distance between the person and the point opposite the lamppost (a variable x), and the distance between the post and the man, a. We will evaluate these with the data that $\frac{dx}{dt} = 3$ and $x = 3\frac{ft}{sec} \cdot 2\,sec = \;6\,ft$.

So, $8^2 + x^2 = a^2$. We can take a derivative **with respect to time, t,** to get:

$$0 + 2x\frac{dx}{dt} = 2a\frac{da}{dt}$$
$$\frac{da}{dt} = \left(\frac{x}{a}\right) \cdot \frac{dx}{dt}$$

At two seconds, we have $a^2 = 64 + 36 = 100$, so $a = 10$ and $\frac{da}{dt} = \frac{6}{10} \cdot 3 = \frac{9}{5}$.

Meanwhile, we also have the vertical cross-section that includes the post and the person. This plane has two similar triangles (poorly drawn above and to the right). One is the triangle whose height is the person and base is the shadow; the other has the lamppost as its height, and with a base equal to the shadow plus the distance between the person and the post (i.e., $a + b$). We have:

$$\frac{a+b}{8} = \frac{b}{6}$$
$$b = 3a$$
$$\text{TDBS"}t\text{"}$$
$$\frac{db}{dt} = 3\frac{da}{dt}$$
$$\left.\frac{db}{dt}\right|_{t=2} = 3 \cdot \frac{9}{5} = \frac{27}{5}\frac{ft}{sec}$$