

University of Toronto  
Faculty of Applied Science and Engineering  
**Department of Electrical and Computer Engineering**

ECE110S – Electrical Fundamentals  
Term Test 2 – March 20, 2018, 1:00 – 2:30 p.m.

$$(e = 1.6 \times 10^{-19} \text{ C}, \epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}, \mu_0 = 4\pi \times 10^{-7} \text{ H/m}, g = 9.81 \text{ N/kg})$$

ANSWER ALL QUESTIONS ON THESE SHEETS, USING THE BACK SIDE IF NECESSARY.

1. Non-programmable calculators (Casio FX-991MS & Sharp EL-520X) are allowed.
2. You are allowed a one page (8.5" x 11") double-sided aid sheet.
3. For full marks, you must show methods, state UNITS and compute numerical answers when requested.
4. Write in PEN. Otherwise, no remarking request will be accepted.
5. There is one extra blank page at the end for rough work.

Last Name: Full

First Name: Solutions

Student Number: \_\_\_\_\_

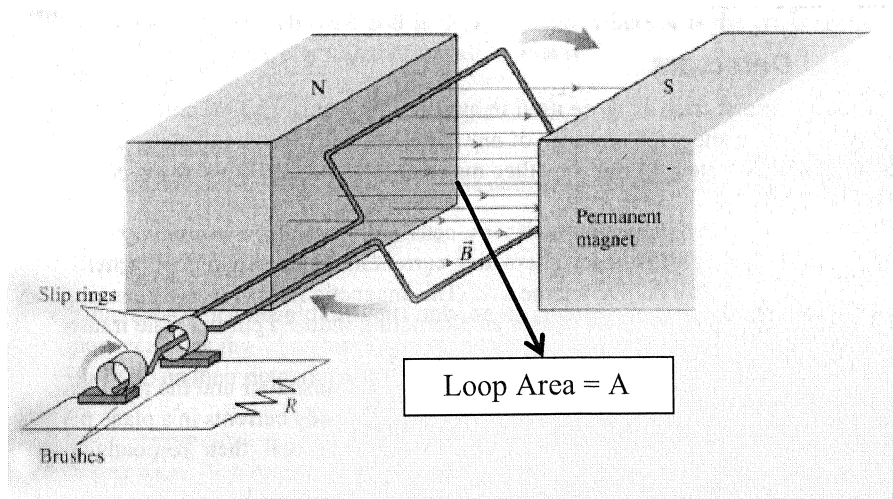
Tutorial Section:

(YOU LOSE ONE MARK FOR NOT MARKING YOUR TUTORIAL SECTION CORRECTLY)

- ☐ 01 WB342 Wed. 4-6 p.m.
- ☐ 02 HA403 Mon. 12-2 p.m.
- ☐ 03 HA403 Thurs. 12-2 p.m.
- ☐ 04 HA403 Fri. 4-6 p.m.
- ☐ 05 GB405 Wed. 12-2 p.m.
- ☐ 06 GB404 Tues. 3-5 p.m.
- ☐ 07 BA1240 Fri. 4-6 p.m.
- ☐ 08 SF3202 Fri. 4-6 p.m.
- ☐ 09 BA1200 Mon. 4-6 p.m.
- ☐ 10 LM162 Thurs. 1-3 p.m.
- ☐ 11 SS2135 Thurs. 1-3 p.m.
- ☐ 12 WB119 Fri. 12-2 p.m.

Question	Mark
1	
2	
3	
TOTAL	

## Q1 [10 marks]



The figure above shows a single metallic loop of constant area  $A$ , immersed inside a uniform magnetic field  $B$  (device is actually a generator). The angle between the loop and magnetic field changes with time according to  $\cos \omega t$  where  $\omega$  is a constant known as angular frequency [rad/s] and  $t$  is time [s].

(a) Give the expression for the magnitude of the induced current through the resistor  $R$ . (6 marks)

$$\theta = \omega t \quad \Phi_B = \iint \vec{B} \cdot d\vec{A} = \iint B dA \cos \omega t = BA \cos \omega t$$

$$|\mathcal{E}| = \left| -\frac{\partial}{\partial t} \Phi_B \right| = \left| -\frac{\partial}{\partial t} BA \cos \omega t \right| = BA \omega \sin \omega t$$

$$|I| = \frac{|\mathcal{E}|}{R} = \frac{BA \omega \sin \omega t}{R}$$

OR

$$\theta = \cos \omega t \quad \Phi_B = BA \cos(\cos \omega t)$$

$$|\mathcal{E}| = BA \omega \sin(\omega t) \sin(\cos \omega t)$$

$$|I| = \frac{|\mathcal{E}|}{R} = \frac{BA \omega \sin(\omega t) \sin(\cos \omega t)}{R}$$

Q1 continued

(b) If instead of a single loop, N number of loops were used, how does the result in part (a) change? (2 marks)

$$\theta = \omega t \quad |I| = \frac{NBA\omega \sin \omega t}{R}$$

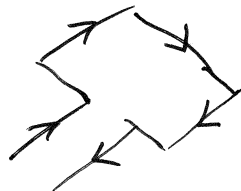
$$\theta = \cos \omega t \quad |I| = \frac{NBA\omega \sin(\omega t) \sin(\cos \omega t)}{R}$$

(c) During the period of  $0 < t < \frac{\pi}{2\omega}$ , what is the direction of the induced current in the metallic loop? Draw the current arrow on the loop indicating the direction of the current. (2 marks)

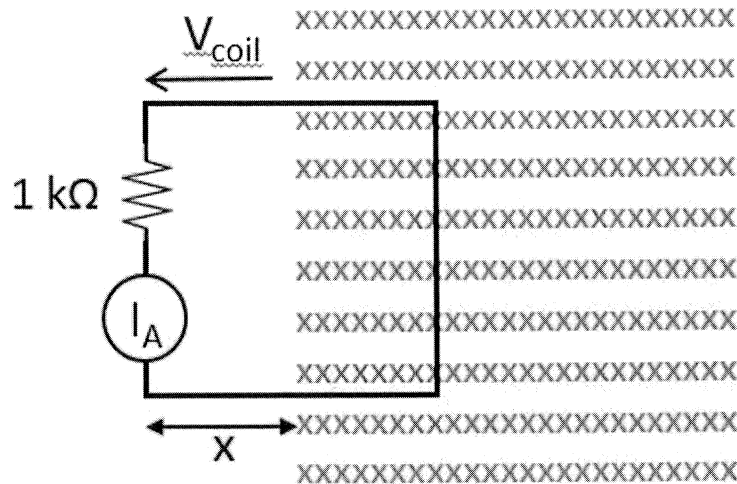
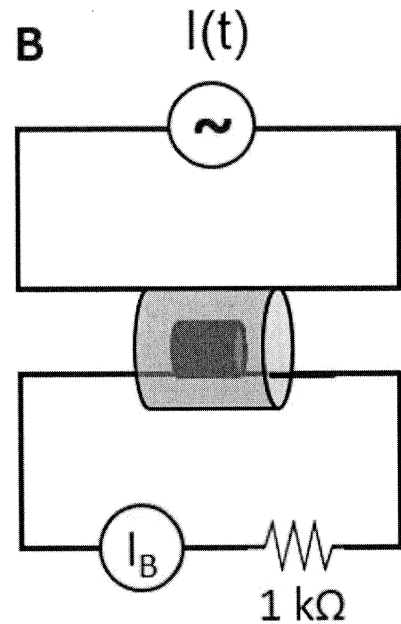
$$\theta = \omega t$$



$$\theta = \cos \omega t$$



## Q2 [10 marks]

**A****B**

Consider the following two examples above in which we can magnetically induce currents through a simple resistive circuit. In both cases, the induced current is measured by an ammeter connected in series within each respective resistive circuit. We assume that the ammeter does not affect any of the circuits.

- In example A, a square coil (10 cm x 10 cm) is pulled through a uniform magnetic field (2.5 T, directed into page) at a constant velocity,  $V_{coil} = 5 \text{ m/s}$ .
- In example B, there is an inner coil centered within an outer solenoid. The outer solenoid has 10,000 turns, radius of 4.5 cm, and length of 20 cm; whereas the inner coil has 40 turns and a diameter of 1.2 cm.

(a) Calculate the current ( $I_A$ ) measured in the resistive circuit in example A. (3 marks)

$$I_A = \frac{\mathcal{E}}{R} = \frac{BLv}{R} = 1.25 \text{ mA}$$

Q2 continued

(b) In example A, determine the direction and magnitude of the external force that must be applied to the square coil. (3 marks)

$$F = iLB = I_A LB = 0.31 \text{ mN} \left[ \leftarrow \right] \quad \swarrow \text{direction}$$

(c) Determine the input current  $[I(t)]$  in example B, such that  $I_A = I_B$  (4 marks)

$$I_A = I_B = \frac{\mathcal{E}}{R} \rightarrow \mathcal{E} = 1.25 \text{ V}$$

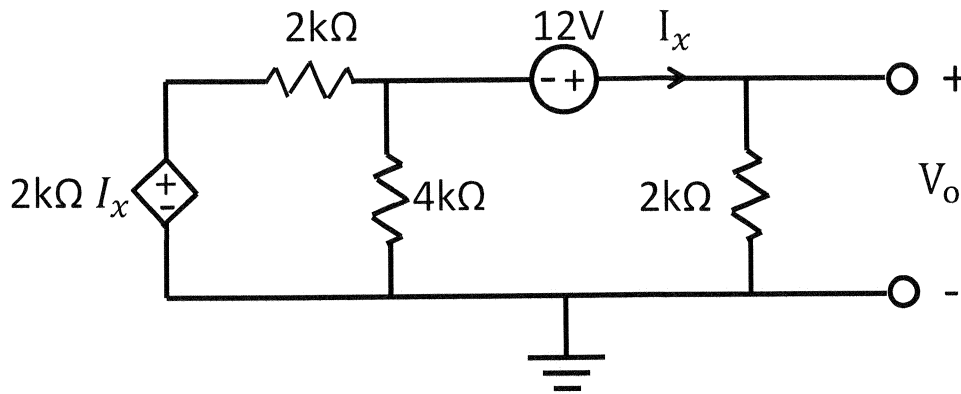
$$\mathcal{E} = N \frac{\partial}{\partial t} \phi_B$$

$$= N \frac{\partial}{\partial t} (\mu_0 i n A)$$

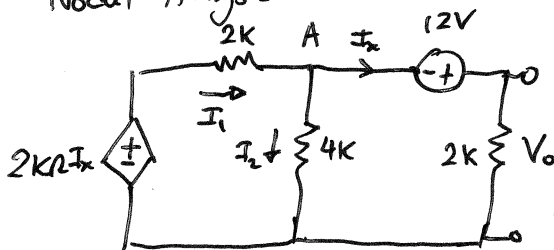
$$\int \frac{di}{dt} = \int 4397.6$$

$$I(t) = 4397.6t \text{ A} \quad \nwarrow \text{unit}$$

Q3 [10 marks]

(a) Use either nodal or mesh analysis to find  $V_o$  in the circuit. (6 marks)

Nodal Analysis



KCL @ A

$$\frac{2kI_x - V_A}{2k} = \frac{V_A}{4k} + I_x$$

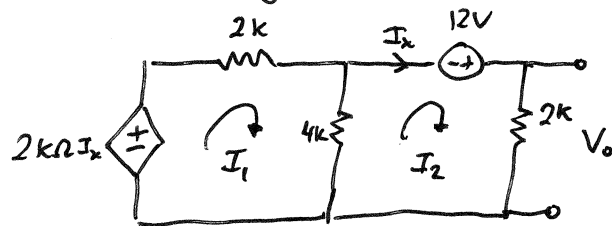
$$V_o = V_A + 12$$

$$I_x = \frac{V_o}{2k}$$

$$V_o = 12V$$

$$V_A = 0V$$

Mesh Analysis



KVL @ loop 1

$$-2kI_x + 2kI_1 + 4k(I_1 - I_2) = 0$$

KVL @ loop 2

$$-12 + 2kI_2 + 4k(I_2 - I_1) = 0$$

$$I_x = I_2$$

$$I_1 = I_2 = 6mA \quad V_o = 2kI_2 = 12V$$

Q3 continued

(b) Compute power in the  $4k\Omega$  resistor. (1 mark)

$$\begin{aligned} P &= VI \\ &= 0 \times I \\ &= 0 \end{aligned}$$

(c) Compute power in the dependent source. (1 mark)

$$\begin{aligned} P &= VI \\ &= (2k\Omega \frac{V_o}{2k\Omega}) \cdot - \left( \frac{2k\Omega \frac{V_o}{2k\Omega} - V_A}{2k\Omega} \right) \\ &= -72mW \end{aligned}$$

(d) Compute power in the independent source. (1 mark)

$$\begin{aligned} P &= VI \\ &= 12 \cdot -I_x \\ &= 12 \cdot -\frac{V_o}{2k} \\ &= -72mW \end{aligned}$$

(e) Compute the power in the two  $2k\Omega$  resistors. (1 mark)

$$P = VI \quad \text{or} \quad \text{sum of all power in circuit must} = 0$$

$$P = 144mW$$

