

# MAT 188 — Linear Algebra



Text: Linear Algebra With Applications, Otto Bretscher, 5th edition

## What is Linear Algebra?

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Linear algebra is the art of using algebra to study flat geometrical shapes, some examples of such geometrical shapes are lines and planes in the two and three dimensional spaces that you are familiar with. Linear algebra tells us how to use the power of algebra to think about higher dimensional flat objects and how to use them to solve a vast range of problems. Linear algebra underlies a great deal of applications in sciences and engineering such as acoustics and sounds, computer graphics, machine learning, and robotics to name a few.

## About the Course

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The purpose of this course is two-fold. On the one hand, you will learn concepts, techniques and algorithms in elementary linear algebra. On the other, you will learn how to identify these concepts in real world problems, model them, and apply technology (MATLAB) and numerical methods to solve them.

We will start by learning how to solve systems of linear equations with the help of matrices. Then we will reinterpret matrices as linear transformations, these are functions that uniformly modify flat spaces. We will use our spacial intuition to guide our understanding of these functions. Linear transformations and their matrix representations will stay at the heart of the course throughout the semester. See the [Course Calendar](#) in this document for a detailed list of topics we discuss in this course.

MAT188 will be taught via research-based teaching, learning, and assessment methods. In lectures and tutorials, students will engage in ideas through an active, student-centered teaching framework called inquiry-based learning. In an inquiry-based course students, TAs and instructors work together to make sense of mathematical definitions, theorems, and techniques to further our understanding of mathematical ideas and learn mathematical habits of mind. Such courses are organized around **student engagement** in exercises in class and outside of class.

## Learning Outcomes

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After taking this course, you will be able to:

- Identify applications of linear algebra in other contexts, and situations.
- Identify tools and employ technology to solve or understand problems.
- Demonstrate computational fluency by hand and in MATLAB.
- Distinguish between numerical evidence and mathematical argument.
- Translate between algebra and geometry to solve problems.
- Approach and apply mathematical knowledge to novel problems.

## Textbook:

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**Text:** Linear Algebra With Applications, Otto Bretscher, 5th edition

- You should have a physical or an electronic copy of the textbook. All readings and homework are from this textbook.

## Teaching Team

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Instructor	Section	Email
Arthur Chan	LEC0102	arthurwh.chan@utoronto.ca
Daniel Johnstone	LEC0107	daniel.johnstone@utoronto.ca
Camelia Karimianpour	coordinator LEC0104 & 0105	admin188@math.toronto.edu ckarimia@math.toronto.edu
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- See Quercus for the list of TA names and contact information
- You can find the time and location of lectures here.
- Question hours and their location for each instructor will be posted on Quercus.
- If you have a question about any aspect of the course, please follow the *Asking Questions* section in this document.

## Course Website

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- The official course page is <https://q.utoronto.ca/courses/282151>

Quercus is our main communication platform. All course material and information, including announcements, tutorial information, TA and instructor question hours (office hours), homework and solutions, pre-class essential readings and in-class reflection surveys, exam details, and all the relevant deadlines and announcements are done via Quercus. Quercus will get updated frequently as the semester progresses. It is **your responsibility** to check it regularly, at least every 2-3 days.

- Find our **Piazza** class page here. Piazza is also accessible directly from the course page on Quercus.

We will be using Piazza for class discussions. If you have a math question or a logistic question you should ask it in Piazza (and **not** over email). See the *Asking Questions* section in this document for more instructions. The earlier you start using Piazza, the happier you will be!

## Assessments

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Assessment	Weight
Reflections	2%
Tutorials	5%
Online Homework (WeBWorK)	5%
Pre-Class Essentials	5%
Written Homework	8%
MATLAB Practicals	15%
Midterms	25%
Final Exam	35%

- **Reflections.** The only way to learn mathematics is by doing mathematics. In this course you are asked to be an active agent in your own learning. **You are expected to be present in all sessions of your lectures, tutorials, and practicals.** During lectures, you will be asked to work on problems with help of your peers, and share your ideas with class. Some lectures will end by a reflection activity. The content and format varies and depends on the activities you have done in class that week.
- **Tutorials.** Tutorials are weekly sessions facilitated by your TAs. During tutorials you will work in groups on a tutorial worksheet. The worksheet covers a variety of standards such as basic computations, core concepts, analysis of ideas, and mathematical language. The specifics for each worksheet will vary. In tutorials, time will be dedicated to working on tutorial worksheets, including giving and receiving feedback from TAs and classmates. At the end of each tutorial, you will submit your work, as a group. Your submission will be graded against the set of course standards. You will receive feedback and a grade on your submission.' Students should expect to submit a report about once every 2 weeks. Groups will be asked to resubmit reports that do not meet the course standards.
- **WeBWorK.** There will be weekly online assignments called WeBWorK due every Wednesday at 11:59 pm. The lowest two WeBWorK grades will be dropped. The first WeBWorK is due on September 15th.
- **Pre-Class Essentials (PCE).** PCEs are assigned readings or videos, followed by a quiz. You are expected to do the PCE before the first lecture of the week. There will be weekly PCEs due every week. See our Quercus page for the due dates. The lowest two PCE grades will be dropped.
- **Written HomeWork.** There will be five written homework assignments. The lowest grade will be dropped. See our Quercus page for the dates.
- **MATLAB Practicals.** There will be weekly assignments during your practicals.
- **Midterms.** There will be two midterms, currently the midterms are scheduled for Oct 4, 1-3 pm and Nov 17th, 7-9 pm. The dates are subject to change.

- **Gateway.** The Gateway exam is an online test which will be open for two weeks. You can take the test as many times as you wish during the time the test is available until you pass it. Failing to pass the test will cost you a third of a letter grade in your final grade for the course (for example from  $B^-$  to  $C^+$ ).

## Missing Evaluations Policy

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The petition policy of the Faculty of Applied Science and Engineering can be found at <http://uoft.me/petitions>. Do not wait to read these rules until an issue (like illness) arises. Read them now so that if things actually happen, you already know how to proceed.

**It is an academic offence to feign illness to skip any coursework.** TAs are not authorized to give any kind of accommodation. Here are course specific policies that are applied on top of the FAES's petition policy.

### Missing PCEs

We are dropping your lowest two PCE marks. Therefore, **you do not need to submit petitions** unless you had a legitimate reason to miss more than two PCEs. **However**, in that case, you need to submit a petition for **all** occasions that you missed. Filing up to two petitions for PCEs will have no effect since the lowest two scores, including zeros, are dropped anyways, with or without petition. If you receive a zero on a PCE due to having committed an academic offence, that zero will not be dropped in any circumstances.

If you miss more than two PCEs and have a valid petition for *all* occasions (including the "first two missed" PCEs), the remaining PCEs will be reweighed accordingly.

### Missing WeBWorK

The same rules as for PCEs apply to WeBWorK.

### Missing Written Homework

The same rules as for PCEs and WeBWorK apply to Written Homework. The only difference is we drop one lowest Written Homework as opposed to two. Therefore, **you do not need to submit petitions** unless you had a legitimate reason to miss more than one Written Homework. In that case, you need to submit a petition for **all** occasions that you missed your Written HomeWork.

### Missing an Exam

If you miss a Test without a valid petition, you will receive a grade of zero.

If you miss a Test and a petition for it was deemed valid, we will redistribute your marking scheme according to the course coordinator's discretion. Regarding the final exam, there are dedicated faculty policies around petitions. Please refer to the faculty rules here: <https://undergrad.engineering.utoronto.ca/petitions/final-exam-petitions/>

## Course Structure

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**Pre-Class Essentials (PCE).** A typical week in our course starts by doing the PCEs for the following week. PCEs are assigned readings, followed by a quiz on those readings. You should do all the assigned readings for the week and submit your PCE quiz on Quercus before your first lecture of that week. You will need to review the textbook before the other two lectures in that week. PCEs make up 5% of your final grade. You should check our Quercus page for the due dates of PCEs.

**Lectures.** You are assigned to a lecture section ranging between LEC0101 to LEC0108. You will have weekly lectures. Lectures assume that you have done the PCEs for that week. You are expected to actively participate in all lectures, and participate in all activities prompted by your instructor. Your active participation in lecture is part of your grade.

**Tutorials.** You will have weekly tutorials. During tutorials, you will work with your classmates in small groups on tutorial worksheets. Tutorial sessions are facilitated by your TAs and are one of the most important components of the course, because they facilitate your communication with your peers. Make the most out of these short sessions by actively explaining concepts to your group mates and ask them to do the same for you. Your active participation in tutorials is part of your participation grade. Tutorials will begin in the week of Sep 19th.

**Practicals.** You will have weekly practicals. Practical are computer labs. During the practicals you will learn to use MATLAB software to operate numerical computations related to the concepts you learn in lectures and tutorials. Practicals give you a hands-on opportunity to see the application of linear algebra in different disciplines and to practice problem solving through theory and computation. During practicals you will work individually or in pairs on your Lab assignments and projects. Lab assignments and exercises make up 15% of your final mark. Your active participation in practicals is part of your participation grade, measured by your answer to reflection questions in lab assignments. Practicals will begin in the week of Sep 13th.

**Homework.** You will have four types of homework: weekly **PCEs**, weekly **WeBWork** assignments, five sets of **Written Homework**, and weekly **Lab assignments**. See the **Assessments** section in this document for details. You should regularly check Quercus for information regarding homework.

## How to Succeed

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Here are a few tips to help you succeed in this course. You will find more on our course webpage.

- Most of your learning in this course will happen when you work with your peers in tutorials, or on homework and suggested problems outside of class, and when you bring your questions to one of our many question hours. Reading the textbook and lectures only initiate the learning process. They are crucial to your learning, but are nowhere near enough.
- Form study groups and work with your peers as much as possible. Explain the concepts to your peers and listen to them explaining the concepts to you. You know you understand a concept if you can explain it well. Do not do all your study in isolation. Math is learned best in interaction with others.
- Get comfortable with feeling challenged. Learning is hard and challenging. If you don't

feel challenged you are not learning. Indeed, learning feels quite uncomfortable for the most part.

- Never skip readings before class. You should always come to class with questions in mind. Do not expect everything to be explained in lectures. Lectures always assume your readings. Lectures are there to give you a guideline on how to approach new concepts and to address your questions.

## Asking Questions

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Our goal is to have all of your questions answered in a timely manner and to encourage you to ask more math questions. If your situation is urgent, talk to your professor after class or during their question hours (office hours), or come to your course coordinator's question hours (office hours). You can find details about question hours on Quercus.

In all other cases, if you have a question please follow the following procedure:

- All mathematical and administrative questions about the course should be asked in Piazza. Your question and the responses will be visible to all students and the teaching team. You should choose the most relevant folder in Piazza for your question.
- If you do not get an answer to your posted question on Piazza or, if your question contains personal information that cannot be shared with the rest of the class (for instance, if you have a question regarding your grade), then ask your question directly by emailing your TA. Your TA will either respond directly or will forward your question to another member of the teaching team who can address it.
- If, after emailing your TA, you still do not have an answer to your question, forward the email you sent to your TA to [admin188@math.toronto.edu](mailto:admin188@math.toronto.edu), or directly to your professor. Your course coordinator will check [admin188@math.toronto.edu](mailto:admin188@math.toronto.edu) on a weekly basis.
- Once again, if your situation is urgent, talk to your professor after class or in their question hours (office hours), or come to your course coordinator's question hours (office hours).

### Emailing Etiquette:

- You should always use your utoronto email address for all university related communication, including MAT188.
- Put MAT188 in the subject line followed by the subject of your email. For example "MAT188-Midterm 1 regrade request".
- Include your full name and your utorid in your email.
- Always check the course syllabus, and Quercus and Piazza for the answer to your question. If your question is answered on the syllabus, Quercus, or Piazza we may not respond to your email.
- If your email is addressed to your professor start your email by "Dear professor..." and end with "Best" or "Thank you". Please use an appropriate tone and level of formality in your emails.
- Note that we will not answer any math questions over email. All such questions should be publicly asked on Piazza or during question hours.

## Course Calendar

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Here is a tentative order in which we will cover material and is only meant to give you an overview. This order is subject to change.

	Topic	Otto Bretscher
Week 0 (Sep8-Sep9)	motivation, intro to linear systems	1.1
Week 1 (Sep 12-Sep 16)	sets, vectors in $\mathbb{R}^n$ , lines and planes,	appendix A
Week 2 (Sep 19-Sep 23)	solving a system, Matrix form of a system	1.2, 1.3
Week 3 (Sep 26-Sep 30)	linear transformations, matrix algebra	2.1,2.2
Week 4 (Oct 3-Oct 7)	composition of maps, matrix multiplication, inverse	2.3, 2.4
Week 5 (Oct 11-Oct 14)	determinant and properties	6.1-6.2
Week 6 (Oct 17-Oct 21)	image, kernel, subspaces, linear independence	3.1, 3.2
Week 7 (Oct 24-Oct 28)	dimension, coordinates	3.3, 3.4
Week 8 (Oct 31-Oct 4)	orthogonal projection, Grahm Schmidt	5.1, 5.2
Week 9 (Nov 14-Nov 18)	orthogonal LT and matrices, least square	5.3, 5.4
Week 10 (Nov 21-Nov 25)	diagonalization, eigenvalues and vectors	7.1,7.2
Week 11 (Nov 28-Dec 2)	more eigenvalues, applications	7.3
Week 12 (Dec5-Dec 8)	symmetric matrices, spectral theorem	8.1, 8.2

## Mental Health and Wellness

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As a university student, you may experience a range of issues regarding, for example your physical or mental health, your financial situation or your friends and family. These may result in significant barriers to achieving your personal and academic goals. The University of Toronto offers a wide range of free and confidential services and programs that may be able to assist you. We encourage you to seek out these resources early and often. You can find resources at <http://studentlife.utoronto.ca/hwc>. If, at some point during the year, you find yourself in need of immediate support, visit the **Feeling Distressed Webpage**: <http://www.studentlife.utoronto.ca/feeling-distressed>, for more campus resources.

Immediate help is available 24/7 through U of T's My Student Support Program in 35 languages. You can call via phone or wifi or even chat using the app. Details can be found at <https://studentlife.utoronto.ca/service/myssp/>

All students in the Faculty of Engineering have an Academic Advisor who can advise on academic and personal matters. You can find your department's Academic Advisor here: <http://undergrad.engineering.utoronto.ca/advising-support-services/academic-advising/>

## Academic Integrity: Why It Matters

We at U of T want you to feel proud of what you accomplish as a student. Please respect all of the hard work you're doing this term by making sure that the work you do is your own.

We don't expect you to score perfectly on the assessments and there will be some things that you may not know. Using an unauthorized resource or asking someone else for the answer robs you of the chance to later feel proud of how well you did because you'll know that it wasn't really your work that got you there.

Success in university isn't about getting a certain mark, it's about becoming the very best person you can by enriching yourself with knowledge, strengthening yourself with skills, and building a healthy self-esteem based on how much you've grown and achieved. No one assessment captures that but your conscience will stay with you forever. Make yourself and your loved ones proud of the student that you are by conducting yourself honestly at all times.

We know that the vast majority of students are honest. If you are one of them, we want to say that we appreciate this and thank you for your commitment to learning! Honouring the achievements of hardworking students like you is one of the major reasons why we implement the policies below. Honesty always pays off in the long run! **If you are experiencing personal challenges that are having an impact on your academic work, please do NOT commit an academic offence. Instead, talk to us or your academic advisor. We are very understanding and happy to help if you are facing any issues. There is never a valid reason for committing an academic offence. Please talk to us!**

## Academic Integrity: The Fine Print

Familiarize yourself with the University of Toronto's Code of Behaviour on Academic Matters, available at <https://www.academicintegrity.utoronto.ca/>.

You are expected to know the rules. Not being aware of a rule is not an acceptable excuse for not having followed it (just as in *real life*).

If you have any questions about what is or is not permitted in this course, please do not hesitate to contact us.

As specified under **Assessments** for some of the course components the lowest, or the two lowest mark are dropped. This does **not** apply for an assessment in which an academic offense was committed and a sanction of "zero on the assessment" was imposed. Potential offenses include, but are not limited to:

- Posting assessment questions on any kind of website/online forum to solicit help,
- Using someone else's ideas or words without appropriate acknowledgement,
- Looking at someone else's answers during an exam or test,
- Misrepresenting your identity,
- Obtaining or providing unauthorized assistance on any assignment, and

- Falsifying or altering any documentation required by the University, including (but not limited to) forms related to a petition.

**It is an Academic Offence both to receive or to provide unauthorized assistance.** It does not matter if you “helped” or “were helped”. The University of Toronto treats cases of academic misconduct very seriously. All suspected cases of academic dishonesty will be investigated following the procedures outlined in the Code. The consequences for academic misconduct can be severe, including a failure in the course and a notation on your transcript. Every year, students get expelled from the University of Toronto for academic offences. Remember that you enrolled in University to learn something.

## Land Acknowledgment

We wish to acknowledge this land on which the University of Toronto operates. For thousands of years it has been the traditional land of the Huron-Wendat, the Seneca, and most recently, the Mississaugas of the Credit. Today, this meeting place is still the home to many Indigenous people from across Turtle Island and we are grateful to have the opportunity to work on this land.