ME5405 Machine Vision

Sample Solution to Exercise 1: Fundamentals of Digital Imaging

There can be more than one answer to the question. Email me at mpecck@nus.edu.sg if you have found any errors in the solution. – CK Chui

Question 1: Suppose that the resolution of a high-definition TV system is 1125 lines in the vertical direction. The width-to-height aspect ratio of the image is 16:9. The system displays a full color image every 1/30 sec for each of the red, green, and blue component images. Each pixel in the color image has 24 bits of intensity resolution, 8 bits each for a red, a green, and a blue image. How many bits are required to store a 120-minute movie?

Answer

Since the resolution in vertical direction is 1125 lines, there are 1125 pixels in the vertical direction.
It is given that the width-to-height ratio is 16/9, the resolution in the horizontal direction is (1125) x (16/9) = 2000 pixels per line.
There are 120 x 60 = 7200 seconds in 2 hours.
Total digital data generated in this time interval is:
1125 x 2000 x 30 x 8 x 3 x 7200 = 1.166×10^{13} bits or 1.458×10^{12} bytes.

Question 2: Suppose that a flat area with center at (x_0, y_0) is illuminated by a light source with intensity distribution:

$$i(x, y) = Ke^{-[(x - x0)2 + (y - y0)2]}.$$

Assume for simplicity that the reflectance of the area is constant and equal to 1.0, and let $K = 255$. The resulting image is digitalized with k bits of intensity resolution.

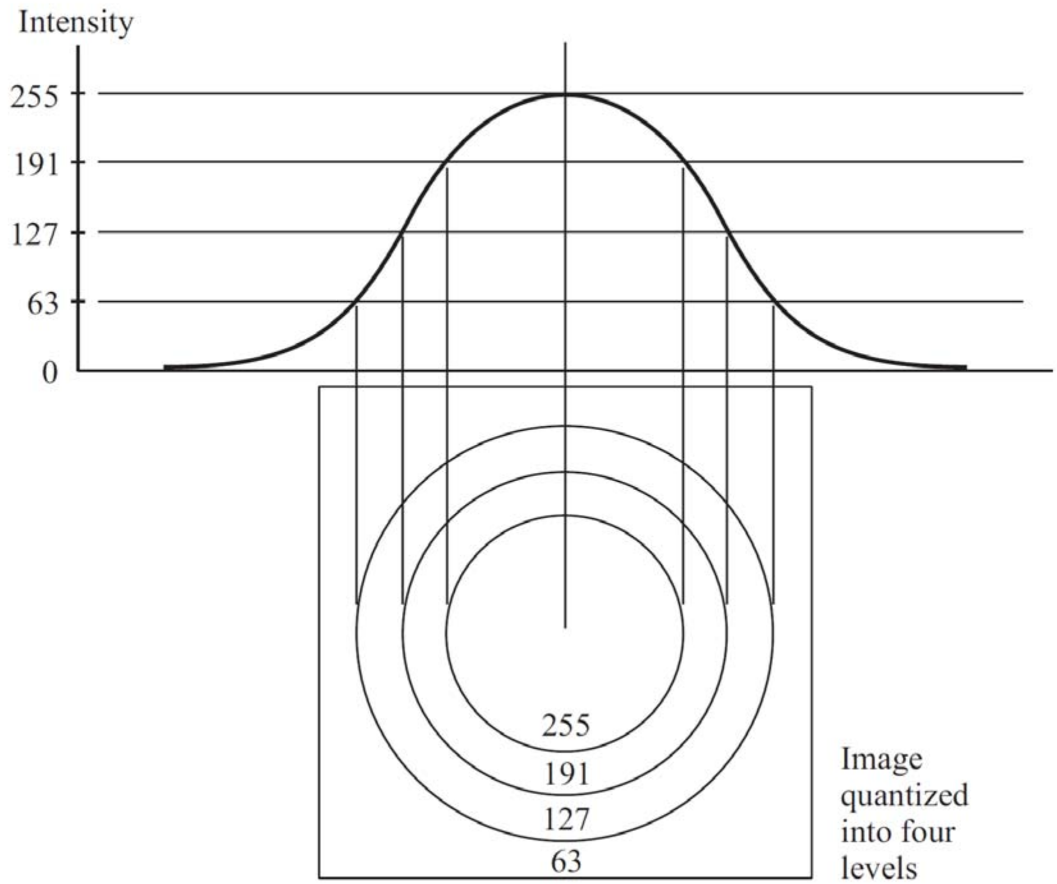
Sketch the image if the image is quantized into 4 intensity levels.

False contouring is quantization noise. It refers to the creation of false edges or outlines not present in the original scene. It is reasonable to assume that human eye can detect an abrupt change of 8 intensity levels between adjacent pixels, what value of k will cause visible false contouring?

Answer

$$f(x, y) = i(x, y)\, r(x, y) = 255\, e^{-[(x - x0)2 + (y - y0)2]} \times 1.0 = 255\, e^{-[(x - x0)2 + (y - y0)2]}$$

If the image is quantized into 4 ($= 2^2$) intensity levels, $\Delta G = (255 + 1)/2^2 = 64$.



Since an abrupt change of 8 intensity levels is assumed to be detectable by a human eye, $\Delta G = 8 = (255 + 1)/2^k = (256)/2^k$, or $k = 5$.

$2^5 = 32$, or fewer, intensity levels will produce visible false contouring.

Therefore, k is less than or equal to 5.

Question 3: Image data can be represented by a list of rectangular spatial cells. The most common method of representing an image space is by a 2D array (or a matrix) of pixels (rectangular cells with fixed size). Quadtree is a generalized form of spatial enumeration in which the disjoint cells are not necessary rectangular or identical.

Compare the memory usage in representing the image of Figure 1(a) using matrix and quadtree. You may assume any type of implementation.

Answer

In matrix representation, the intensity of each pixel can be represented by one integer. The 8 x 8 = 64 pixels can be represented by a matrix of size 8 x 8. Hence, 64 integers are required to represent the image in Figure 1(a).

In quadtree representation, only the intensities of leaf nodes are stored. There are 46 leaf nodes in the quadtree representation of the image. Since each leaf node can be represented by an integer. 46 integers are required.

Theoretically, quadtree representation is an improvement over matrix in terms of memory usage depending on the image homogeneity.

In practice, we need to allocate memory for the intermediate nodes, for example, pointers to the parent and 4 children nodes. The actual memory usage of quadtree representation is also dependent on the implementation.

Question 4: Write an algorithm to determine the histogram of an image represented using quadtree. Compare the performance of this algorithm with that of image representation using matrix.

Answer

Suppose that the height of the quadtree is h. For example, h=3 in the quadtree of Figure 1, and the level of root node is 0.

Step 1: Assign zero values to all elements of the histogram array H.
Step 2: For all leaf nodes of the quadtree
    Determine the intensity f and level l of the leaf node
    Increment H(f) by 4^(h-l)

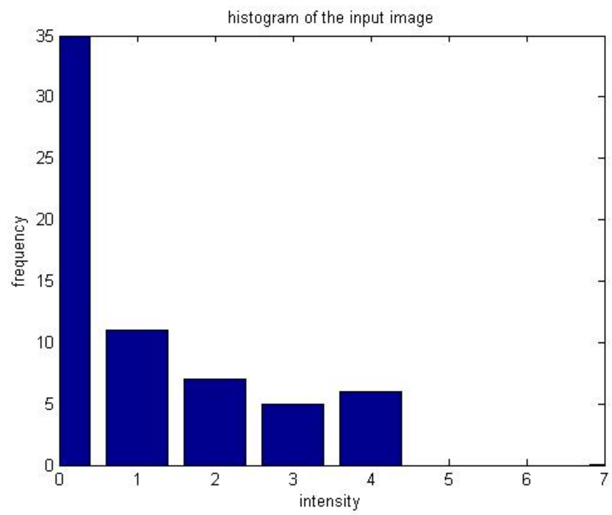
Theoretically, quadtree representation is an improvement over matrix in terms of performance since only leaf nodes are processed. The histogram array element is also incremented as a block of pixels. In practice, there are overheads, for example in traversing the tree.

In matrix representation, all pixels have to be examined. There are 64 pixels, and 64 increment operations have to be formed, excluding the initialization of the array H. Initialization of H is also required in quadtree representation (Step 1).

Question 5: Given that the intensities of the image in Figure 1(a) is as follows (Figure 2). Perform histogram equalization on the image.

<u>Answer</u>
Suppose that the number of gray levels is 8. (Depending on the number of gray levels assumed, the answer will vary.)

The histogram for this image is:



Cumulative distribution function:

Gray Level	0	1	2	3	4	5	6	7
Number of pixels	35	46	53	58	64	64	64	64

The transformation T is given by:

R	0	1	2	3	4	5	6	7
S	4	5	6	6	7	7	7	7

The equalized image is as follows:

5	4	4	4	4	4	4	5
4	6	6	6	6	6	6	4
4	4	6	7	7	6	4	5
4	4	4	7	7	4	4	4
4	5	4	7	7	4	5	4
4	4	4	6	6	4	4	4
5	4	5	6	6	4	5	4
4	4	4	5	5	4	4	5

Following Matlab codes (not required in examination) could compute the histogram, cumulative histogram, transformation or look up table as well as the equalized image.

```
im = [  1 0 0 0 0 0 0 1 ; ...
        0 2 2 3 3 3 2 0 ; ...
        0 0 2 4 4 2 0 1 ; ...
        0 0 0 4 4 0 0 0 ; ...
        0 1 0 4 4 0 1 0 ; ...
        0 0 0 3 3 0 0 0 ; ...
        1 0 1 2 2 0 1 0 ; ...
        0 0 0 1 1 0 0 1 ];

% to use the hist equalization method from text book. The intensities are [0,levels-1]
% but Matlab starts indexes at 1. Doing the +1 shift only once saves some computation time.
levels = 8;
imp = im+1;

% Compute the histogram of the input image first.
H = zeros(1,levels);
% scan all pixels ...
for i=1:size(im,1)
  for j=1:size(im,2)
    % pixel intensity indexes the accumulator
    H(imp(i,j)) = H(imp(i,j)) + 1;
  end
end

% Form the cumulative image histogram Hc.
Hc = zeros(size(H));
Hc(1) = H(1);
for i=2:size(Hc,2)
  Hc(i) = Hc(i-1)+H(i);
end

% Create the look-up table
% normalizing the cumulative histogram to have integer values between 0--(levels-1).
T = round( (levels-1)/(size(im,1)*size(im,2)) * Hc );

% Apply the look-up table to each level in the input image.
% Formally, rescan the image and write an output image with gray levels g_q,
% setting g_q=T[g_p].
im_out = zeros(size(im)); % memory allocation for the new image
im_out = T(imp);

im % input image
H % histogram
Hc % cumulative image histogram
im_out % output image
```

Question 6: Determine the histograms of Image A and the negative of Image A. Can gray level transforms be determined to bring these two histograms into each other? Briefly explain your answer.

Answer

Histogram of Image A:

Gray Level	Number of pixels
0	35
1	9
2	0
3	0
4	0
5	2
6	0
7	2
8	7
9	9
10	0
11	0
12	0
13	0
14	0
15	0

Histogram of negative of Image A:

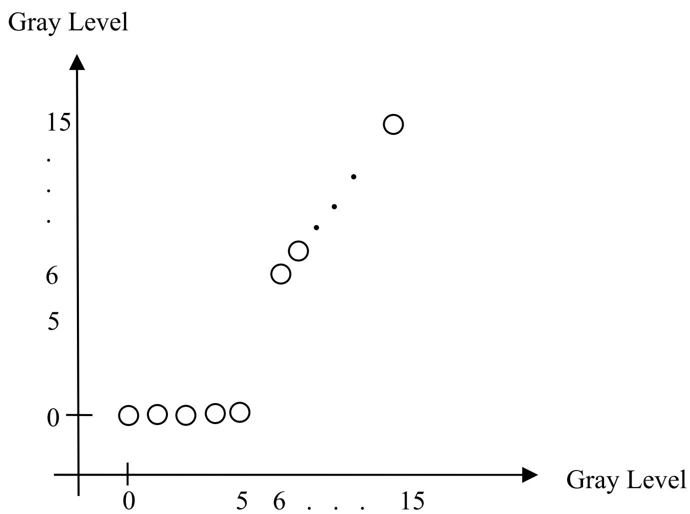
Gray Level	Number of pixels
0	0
1	0
2	0
3	0
4	0
5	0
6	9
7	7
8	2
9	0
10	2
11	0
12	0
13	0
14	9
15	35

Yes. Suppose that the two density functions are p(r) and p(z). With $r = 15 - z$, $s(r) = s(z)$. Gray level transforms can therefore be determined to bring these two histograms into each other.

Question 7. Design a point processing transform to convert pixels with gray level less than 6 to black, and leave the pixels within the object unchanged. Determine the histogram of this transformed image.

Answer

The point processing transformation is given as follows.



Alternatively,

$$s(r) = \begin{cases} 0 & r < 6 \\ r & r \geq 6 \end{cases}$$

Histogram of transformed image:

Gray Level	Number of pixels
0	46
1	0
2	0
3	0
4	0
5	0
6	0
7	2
8	7
9	9
10	0
11	0
12	0
13	0
14	0
15	0

Question 8. Design a linear point processing transform to stretch the gray levels in the object to cover the whole available range. Has the contrast of the image improved? Briefly explain your answer.

Answer

Given a point processing transform
$s(r) = ar + b$
In order to cover the whole available range, $s(0) = 0$ and $s(9) = 15$
Using linear system of equations,
$s(r) = \text{round}\ (15/9)\ r$
where round(x) is a function that returns the nearest integer of the input parameter x.

The contrast has improved since while the black (0) pixels remain, the object has become more white-ish. The most grayish pixels (with intensity = 9) is now white. The different between black and white is more apparent.

Question 9. The following matlab codes will display Image A as a gray level image with gray level between 0 (black) and 255 (white). Briefly explain how this could affect the contrast of the original image with gray level between 0 and 15.

Answer

The number of gray levels has increased from 0-15 (4-bit) to 0-255 (8-bit). It does not necessary imply that the contrast has improved. With a linear transformation, the utilization of the gray scale remains the same, with possibly no change in the shape of the histogram.

Question 10. Determine the two gray level transforms that can bring the histograms in Figure 4(a) and Figure 4(b) into each other.

<u>Answer</u>

$$s(r) = \int_0^r \frac{\pi}{2}\sin(\pi r) \quad dr = \frac{1}{2}(1-\cos(\pi r))$$

$$s(z) = \int_0^z 2z \quad dz = z^2$$

$$s(r) = s(z) \rightarrow \frac{1}{2}(1-\cos(\pi r)) = z^2$$

$$z(r) = z = \sqrt{\frac{1}{2}(1-\cos(\pi r))}$$

$$r(z) = r = \frac{1}{\pi} ar\cos(1-2z^2)$$

Question 11. The histograms in Figure 4(b) can also be converted to the histogram in Figure 4(c). Design the point processing transform that can achieve that.

<u>Answer</u>

For Figure 4(b), $p(z) = 2z \rightarrow s(z) = z^2$

Figure 4(c) can be represented using the following transformation

$$T(r) = ar + b \quad ,a = 1, b = 0.5$$

$$T(r) = r + 0.5$$

$$s(r) = \int T(r)dr = (r+1)\frac{r}{2}$$

$$s(z) = s(r) \rightarrow z = \sqrt{(r+1)\frac{r}{2}}$$

Question 12. The following photo has 4 different shades indicated as (a), (b), (c) and (d) in Figure 5. Suppose that the segment indicated as (b) has the right exposure. Explain how by transforming the pixel intensities of the image, we can obtained the shades indicated in segments (a), (c) and (d).

<u>Answer</u>
Given that the segment in (b) has the right exposure.

The image in (a) is over expose. The image is white-ish. This can be created by having the intensities of pixels concentrated near the white intensity as shown in the following histogram.
The image in (c) is slightly under expose. This is characterized by a shift of the peak of the histogram towards the black (0) intensity.
The image in (d) is under expose. The image is too dark. The pixels have intensities concentrated near the black intensity.