National University of Singapore
Mechanical Engineering Department

ME 5405 Computer Vision

2016/2017 Exercise Set 1

Thresholding and Mathematical Morphology

Thresholding

1. In a given gray level image, the gray level distributions of the pixels can be assumed to be represented by the following expression:

$$p(z)=\begin{cases}\dfrac{\pi}{4a}\cos\dfrac{(z-z_o)\pi}{2a} & \text{for} \quad z-z_o \le z \le z+z_o \\ 0 & \text{otherwise}\end{cases}$$

For the object, $z_o = 1$, $a = 1$, and for the background, $z_o = 3$, $a = 2$. From the histogram of the whole image, we can assume that object pixels occupy one-third of the total number of the pixels of the image.

i) Sketch the Probability Density Functions for the object and the background

ii) Dertermine the Optimal Threshold, based on minimum classification approach, of the image.

iii) Calculate the fraction of mis-classified object pixels based on the Optimal Threshold value you have obtained in (ii) above.

[ME5405 Nov04 – Q1a]

2.

In a given gray level image, the gray level of the object pixels and the background pixels are distributed according to the following probability density functions, p_o and p_b, respectively:

$$p_o = \frac{1}{2\sigma_o}\exp\left(-\frac{|x-\mu_o|}{\sigma_o}\right)$$

$$p_b = \frac{1}{2\sigma_b}\exp\left(-\frac{|x-\mu_b|}{\sigma_b}\right)$$

Where μ_o and μ_b are respectively the means of the object and background distributions; and σ_o and σ_b are the standard deviations of the object and background distributions, respectively.

(a) If T is the optimal threshold and λ is the fraction of object pixels in the image, prove the following expression. You may use the conditions for Optimal Thresholding derived in the lecture notes without proving them.

$$\frac{|T-\mu_b|}{\sigma_b} - \frac{|T-\mu_o|}{\sigma_o} = \ln\left\{\frac{\sigma_o(1-\lambda)}{\sigma_b\lambda}\right\}$$

(b) If $\mu_o = 60$, $\mu_b = 40$, $\sigma_o = 10$, $\sigma_b = 5$ and the object occupies two-thirds of the image ($\lambda = 2/3$), determine the optimal threshold value(s). [Hints, there might be multiple possible values of thresholds due to the presence of ABSOLUTE operator $|\bullet|$].

(c) The two probability density functions are plotted as shown in Fig. 2. In your opinion, what would be the reasonable value of the optimal threshold of the image? Explain your reason. Compare, with detailed explanation, your value with those obtained in Part (i) above.

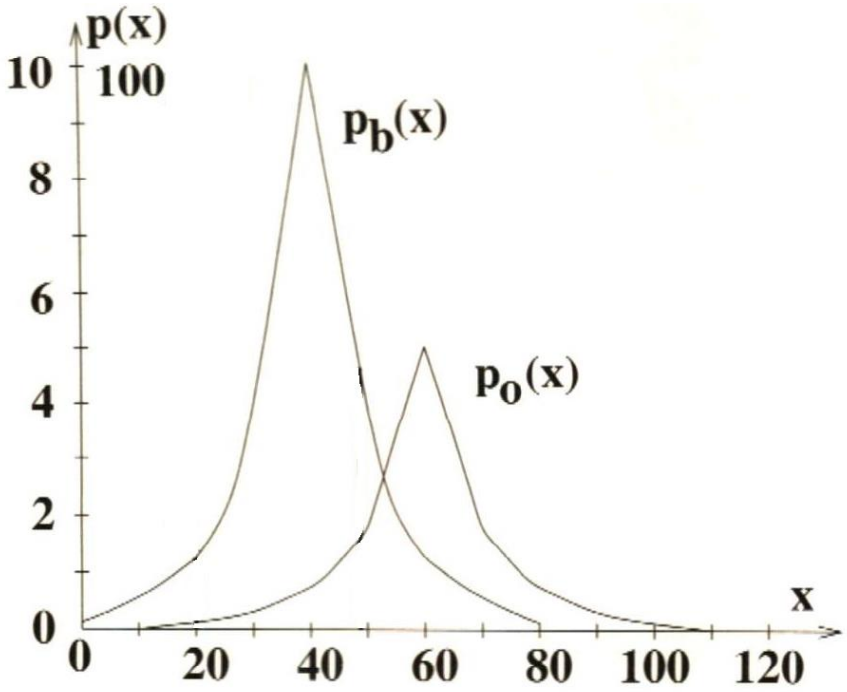


Fig. 2

[ME5405 Nov12 – Q4(b)]

3.

Fig. 3(a) shows a scanned image of a document, and Fig. 3(b) gives the associated histogram.

(i) Comment on the quality of the image shown in Fig. 3(a).
(ii) Looking at the histogram Fig. 3(b), if you wish to pick a value to threshold the image, discuss whether it is appropriate to choose the value at the lowest point of the valley or the value midway between the peak locations.
(iii) If we wish to improve the scanned image Fig. 3(a), to obtain a better picture as shown in Fig. 3(c), the histogram of which is shown in Fig. 3(d), propose an image processing technique to achieve this.

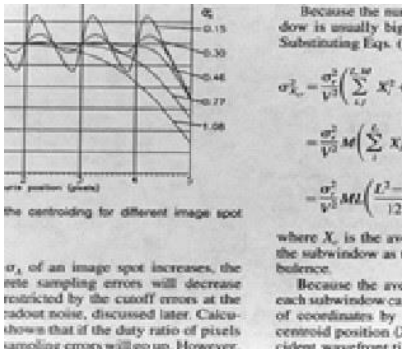


Fig. 3(a)

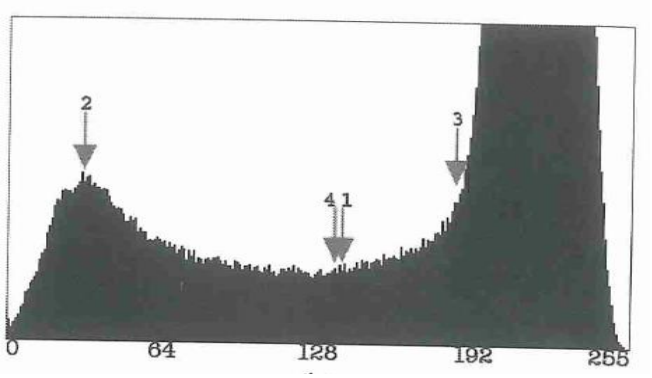


Fig. 3(b)

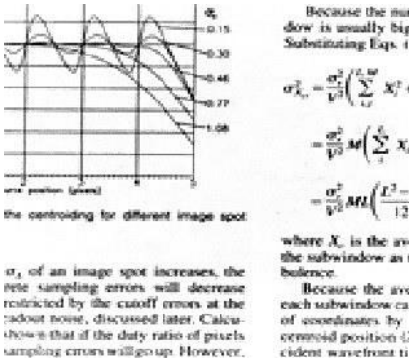


Fig. 3(c)

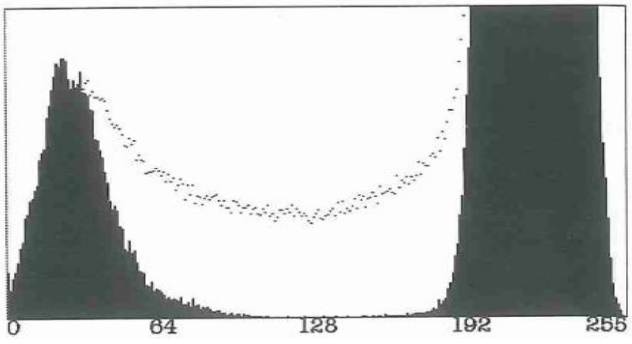


Fig. 3(d)

[ME5405 Nov15 – Q1(a)]

Mathematical Morphology

1. (a) A, B and X are sets in 2-dimensional integer space $\boldsymbol{Z^2}$ and $A = B \oplus Y$ for some Y in $\boldsymbol{Z^2}$ show that

$$X \bullet A \supseteq X \bullet B$$

and

$$(X \bullet A) \bullet B = (X \bullet B) \bullet A = X \bullet A$$

(b) Fig. 1(a) and Fig. 1(b) show a digital image (A) and a structure element (B), respectively. Determine the digital image after the following morphological operations:

(i) $(A \oplus B) - A$

(ii) $(A \oplus B) - (A \eth B)$

[Note: Submit your results in the pages with squares provided]

(iii) Describe the functions of each of these operations.

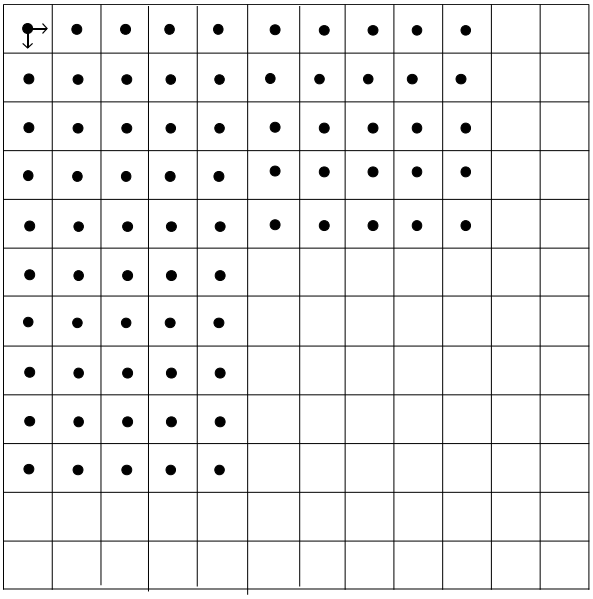


Fig. 1(a)

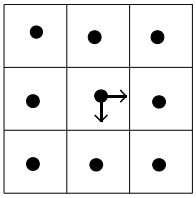


Fig. 1(b)

(c) Fig. 1(c) and Fig. 1(d) show a digital image (C) and a structure element (D), respectively. respectively. Determine the result of the 'Open' operation $C \circ D$. Is the result expected? Why?

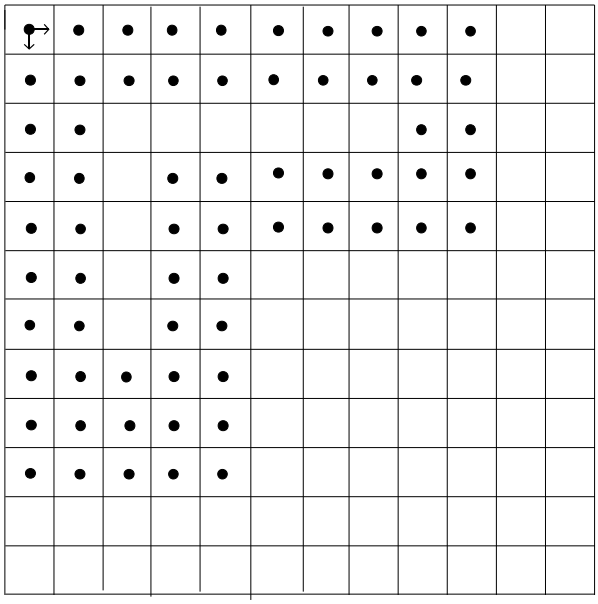


Fig. 1(c)

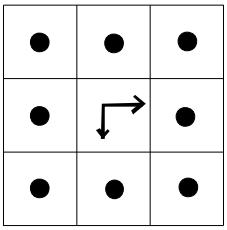
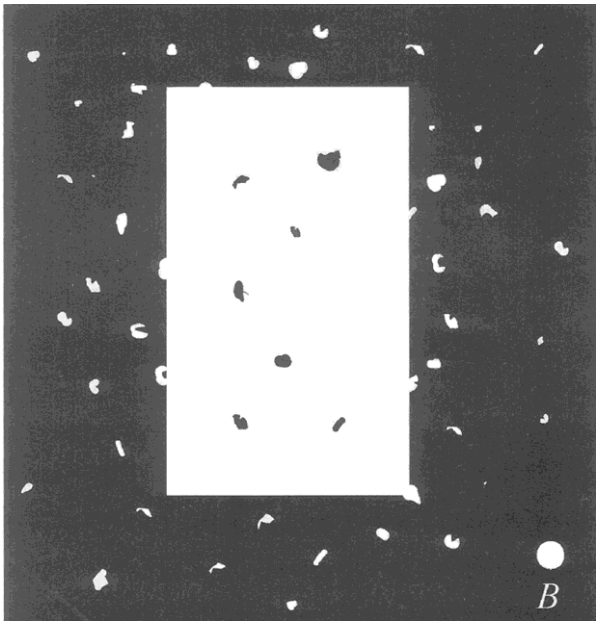


Fig. 1(d)

[ME5405 Apr 01 – Q 3]

2 Fig.2 shows a binary image, ***A***, with a structure element ***B.*** The initial image ***A*** consists of all the image components in white (except ***B***), which is the structure element. Assuming ***B*** is just large enough to enclose each of the noise components, sketch what the sets ***C***, ***D***, ***E*** and ***F*** would look like in the following sequence of morphological operations.



$C = A \ominus B$

$D = C \oplus B$

$E = D \oplus B$

$F = E \ominus B$

Fig. 2

Based on your results of ***F***, discuss the purpose of the abovementioned sequence of operations

[ME5405 Nov 08 – Q4(b)]

3 (i) Without using mathematical proof, explain which of the following morphological expressions is correct.

$$(1)\quad (A \eth B) \eth C = A \eth (B \eth C)$$
$$(2)\quad (A \eth B) \eth C = A \eth (B \oplus C)$$

(ii) Figure 3(a) shows a binary image ***A***, and Figures 3(b) and 3(c) give two structural elements ***B*** and ***C***. "X" indicates the origin of the image and structure elements.

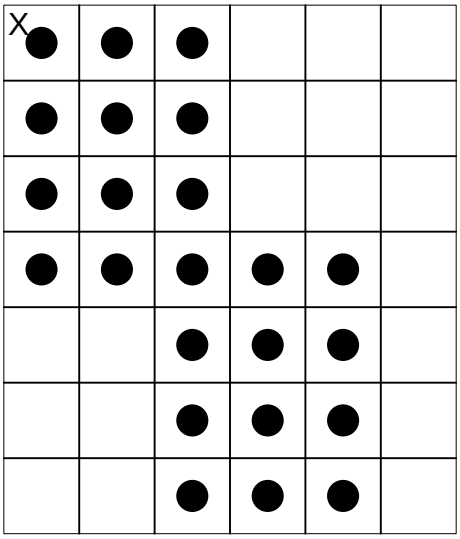


Figure 3(a) – Image **A**

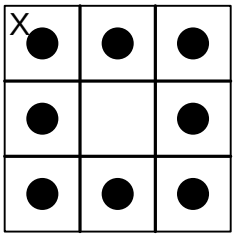


Figure 3(b) **B**

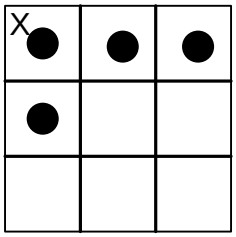


Figure 3(c) **C**

Note that $C \subseteq B$. Show the resulting images after the following morphological operations.

$$(1)\quad A \circ B$$
$$(2)\quad A \circ C$$

What meaningful conclusion can you derive from the resulting images?

[ME5405 Nov 09 – Q4(b)]

4. Figure 4(a) shows a 7 ×7 binary image, and Figure 4(b) is a 3 × 3 structure element. Note that the pixel marked with an "×" is the origin.

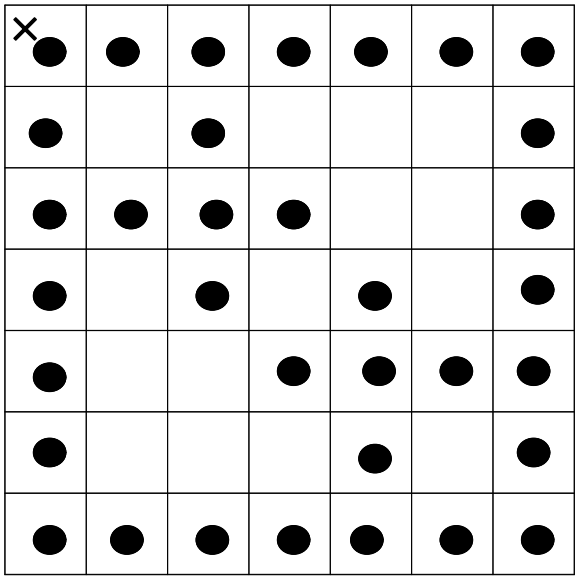


Figure. 4(a)

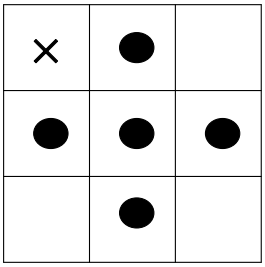


Figure. 4(b)

(i) Perform an Erosion operation on the given image. and show the resulting image.

(ii) Perform an Opening operation of the given image and show the resulting image

(iii) Figure 4(c) is a given image with noise spots and small protrusions. Design a structure element and using appropriate morphological operation to clean up the image. The resultant image should look like the image shown in Figure 4(d).

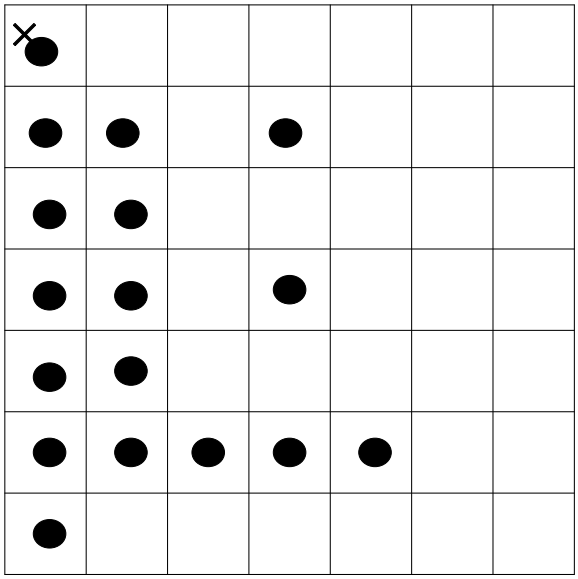


Figure 4(c)

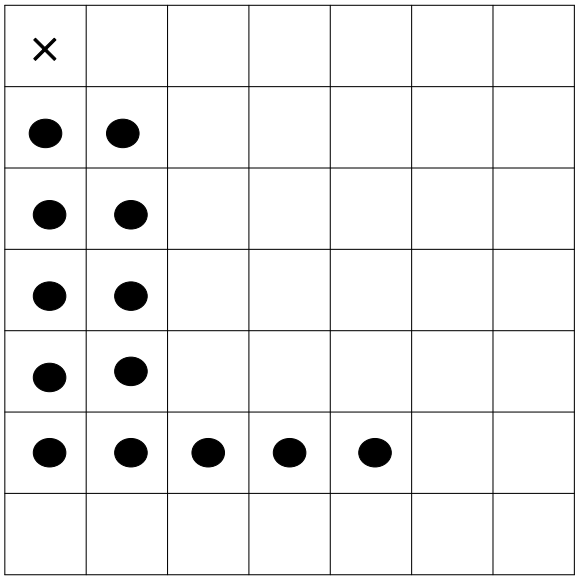


Figure. 4(d)

[ME5405 Nov 11 – Q4(b)]

5.

Fig. 5(a) shows a binary image ***A***. Fig. 5(b) and Fig. 5(c) are two structure elements ***B*** and ***C***.

(a) Perform the following morphological operations $B \cup C$, $A\eth B$ and $A\eth C$.

(b) From the results obtained in Part (b) (i) above, complete the following morphological expression:

$$A\eth (B \cup C) = ?$$

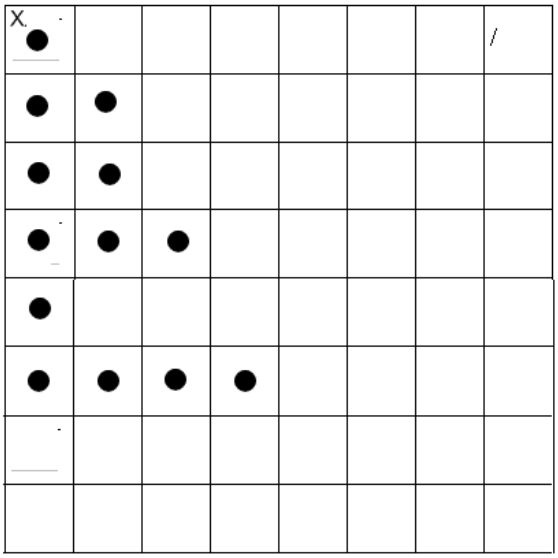


Fig. 5(a) Image ***A***

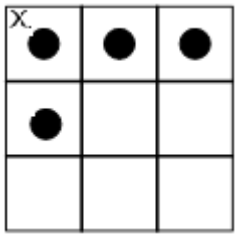


Fig. 5(b) ***B***

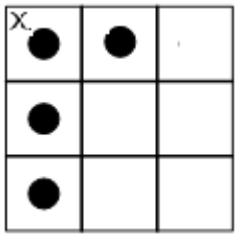


Fig. 5(c) *C*

[ME5405 Nov 12 – Q3(b)]

6.

Design a Structure Element and the associated morphological operations to produce a one-pixel thick exterior outline of the image shown in Figure 6(a). The final image should look like Figure 6(b)

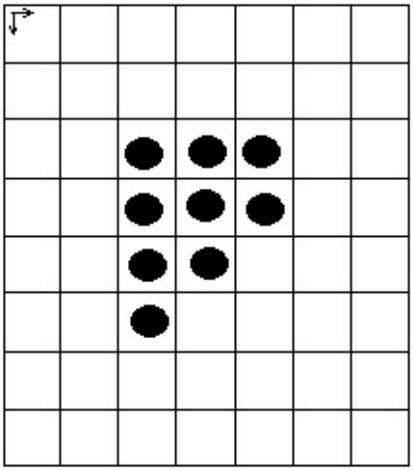


Fig. 6(a)

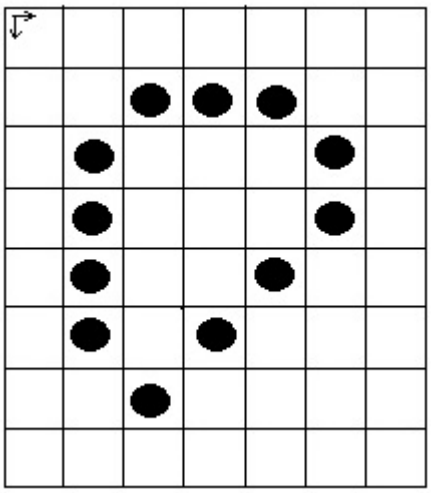


Fig. 6(b)

[ME5405 Nov 13 – Q3(b)]

7.

Figure 7 shows a binary image *A*, and two structure elements ***B*** and ***C.*** Determine the results of the two following morphological operations:

(i) $A \eth (B \oplus C)$

(ii) $(A \eth B) \eth C$

(iii) What conclusions can be drawn from the results of parts (i) and (ii).

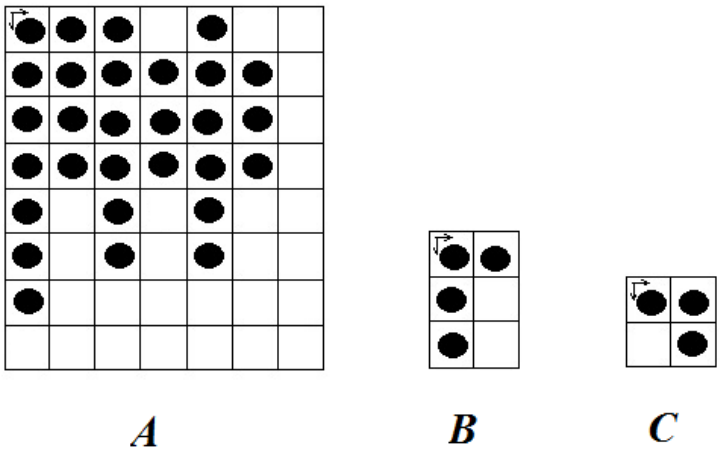
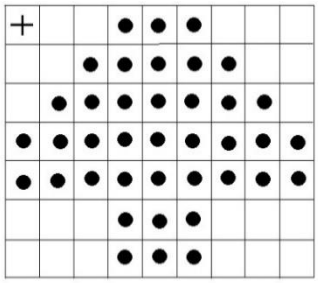


Figure 7

[ME5405 Nov 14 – Q3(a)]

8.

(i) Fig. 8(a) shows a binary image A. Design a structure element and the associated morphological operation(s) to obtain the final image B shown in Fig. 8(b). Note that the pixel marked "+" is the origin of the image.



8(a) Image A

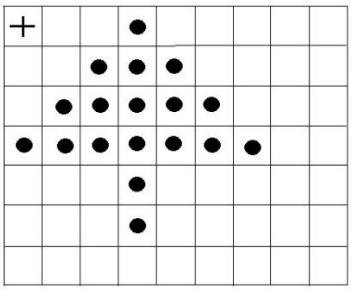


Fig. 8(b) Image B

(ii) Figure 8(c) is a binary image C, and Fig. 8(d) is a structure element. Show the result of opening the Image C by the structure element X. Note that the pixel marked "+" is the origin of the image.

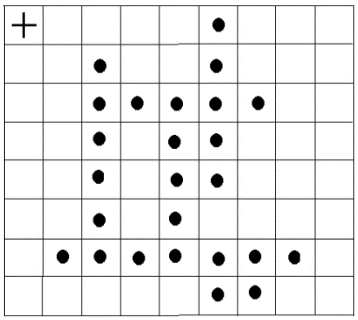


Fig. 8(c) Image C

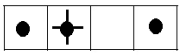


Fig. 8(d) Structure element X

[ME5405 Nov 15 – Q4(b)]

9
.
(a)

(i) Fig. 9(a) shows a binary image A.

(1) Design a structure element and the associated morphological operation(s) to remove the "leg" of the figure as shown in Fig. 9(b).

(2) To obtain Fig. 9 (c) from Fig. 9(a), design the structure elements and the necessary morphological operation(s) to achieve the objective [*Hint: You will need two morphological operations with different structure elements*].

Note that the pixel marked "+" is the origin of the image.

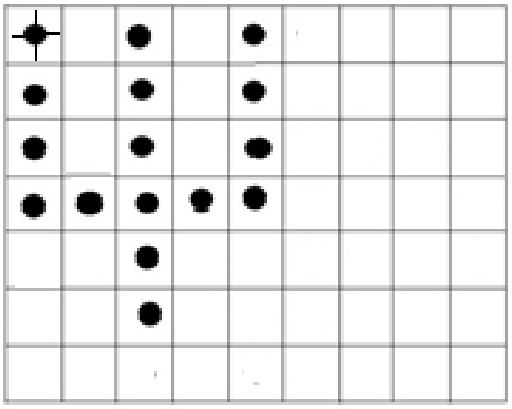


Fig. 9(a) Image A

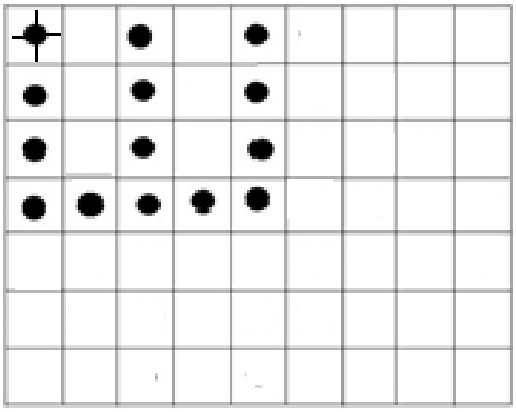


Fig. 9(b) Image B

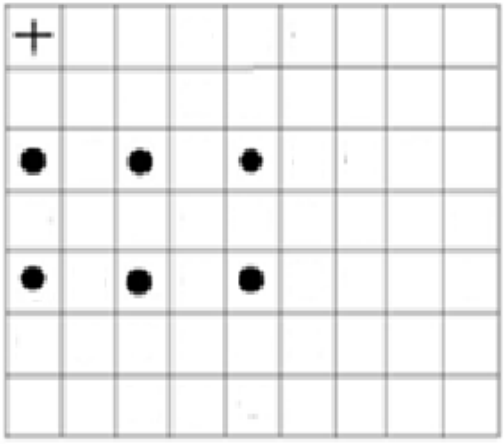


Fig. 9(c) Image C

(ii) Fig. 9(d) shows a binary image D. Figures Fig 9(e) and 9(f) are two different structure elements E and F, respectively. Determine the result of the following morphological operations. Note that the pixel marked "+" is the origin of the image.

(I) $G = D \eth E$

(II) $G \oplus F$

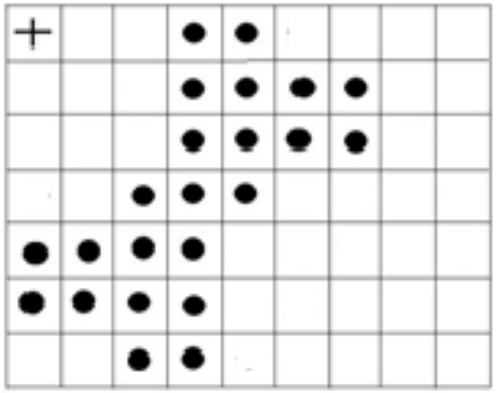


Fig 9(d) Image D

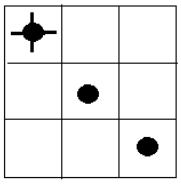


Fig 9(e) Structure Element E

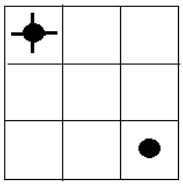


Fig 9(f) Structure Element F

[ME5405 Nov 16 – Q4(a)]