National University of Singapore
Mechanical Engineering Department

<u>ME 5405 Computer Vision</u>

<u>2016/2017 Exercise Set 2</u>

<u>Image Enhancement</u>

1. Fig. 1 (a) shows an image composed of white circles with black inner spots. This is an image which is the result from thresholding into two levels a scene containing polished ball bearings. The dark spots inside the circles are the results of reflections. Device an image processing procedure to eliminate the reflections for the two following cases, i.e. to detect and turn the dark spots within the spheres to bright.

   (i) The circles are not touching each others as shown in Fig. 1(a).

   (ii) Some of the circles are touching each other thus forming dark regions between them as shown in Fig. 1(b).

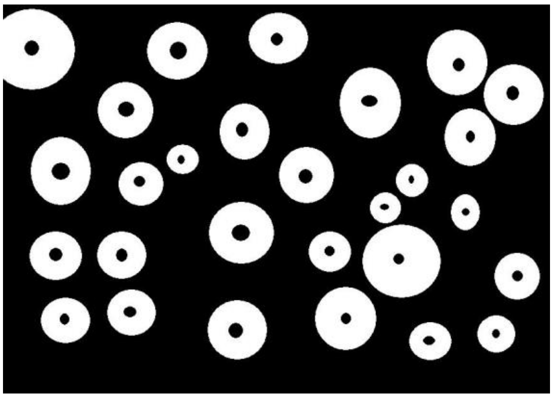

Fig. 1(a)

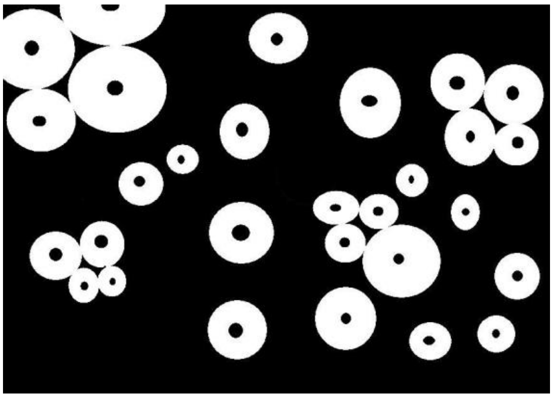

Fig. 1(b)

[ME4243-Apr03-Q2(a)]

2. $p_r(r)$ is the probability density function of a given image. It is of poor dynamic range. We wish to transform the grey levels of this image so that they will have a probability density function $p_z(z)$ given below. In addition, in the new image, the gray levels are limited to lie between the range $z_{\max}$ and $z_{\min}$. Find the necessary transformation (i.e. find z in terms of $p_r(r)$, $z_{\max}$ and $z_{\min}$).

$$p_z(z) = \frac{1}{3}\frac{z^{-\frac{2}{3}}}{z_{\max}^{\frac{1}{3}} - z_{\min}^{\frac{1}{3}}}$$

[ME5405-Nov 09-Q3(a)]

3. (a) An 8-gray level image is shown in Figure 3(a) below; perform histogram equalization of the image.

$$\begin{bmatrix} 2 & 2 & 2 & 3 & 1 \\ 2 & 4 & 7 & 4 & 2 \\ 3 & 6 & 7 & 6 & 3 \\ 3 & 5 & 7 & 5 & 4 \\ 4 & 4 & 4 & 4 & 4 \end{bmatrix}$$

Figure 3(a)

In your result, show the mapping of gray levels between the original ones with respect to that of the equalized image.

Why discrete histogram equalization does not produce a perfectly flat histogram?

(b) (i) An image with the associated histograms are shown in Figure3(b)(i) and Figure 3(b)(ii), respectively. Comment on the quality of the image shown.

Note that the top and bottom plots in Figures 3(b)(ii) are the Cumulative Gray Level distribution and the histogram, respectively.

Figure 3 (b)(i)

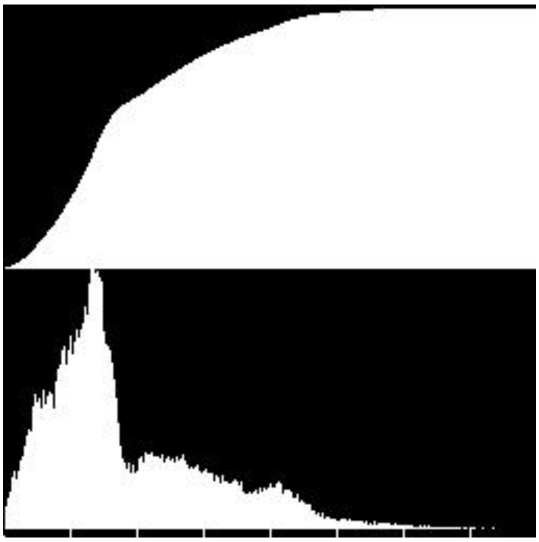

Figure 3(b)(ii)

(ii) Suggest an image processing that enhances the image of Figure 3(b)(i) to produce the one shown in Figure 3(b)(iii), and sketch its histogram..

Figure 3(b) (iii)

[ME5405-Nov 10-Q3]

4 Figure 4 shows a 128 gray level image. It contains two noise spots of gray levels 127 (Row 2 Column 2) and 22 (Row 2, Column 5). Perform separately, noise cleaning using a 3 by 3 (i) Median Filter and (ii) Averaging Filter on the pixels at Row 2, Columns 2 to 5 only. Comment on your results. Note that the left side of the image is a dark 3 by 3 region, while that on the right side is a bright region.

$$\begin{bmatrix} 18 & 24 & 33 & 100 & 85 & 90 \\ 34 & 127 & 24 & 105 & 22 & 91 \\ 20 & 19 & 32 & 117 & 97 & 87 \end{bmatrix}$$

Figure 4

[ME5405-Nov 10-Q4(a)]

5. (a) An image has the gray level probability density function $p_r(r) = \frac{\pi}{2}\sin(\pi r)$ for $0 \le r \le 1$. We wish to transform the gray levels of the image so that they will have the specified probability density function $p_z(z) = 2z$ for $0 \le z \le 1$. Assume continuous quantities; find the transformation (r as a function of z) that will accomplish this .

(b) Figure 5(a) is a 6 × 6 gray level image. It is filtered separately by three 3 × 3 filters shown in Figures 5(b), 5(c) and 5(d). The corresponding filtered images are shown in Figures 5(e), 5(f) and 5(g), respectively. Note that "?" indicates unknown values. Figure 5(h) shows the overall process schematically.

(i) Determined Filtered Images 1 and 3, after the given image (Figure 5(a)) has been filtered by Filters 1 and 3, respectively.

$$\text{Input image} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 6 & 6 & 2 & 0 \\ 0 & 14 & 10 & 8 & 4 & 0 \\ 0 & 6 & 8 & 8 & 4 & 0 \\ 0 & 4 & 6 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 5(a)

$$\text{Filter 1} = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}; \quad \text{Filter 2} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}; \quad \text{Filter 3} = \frac{1}{2}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Figure 5(b) Figure 5(c) Figure 5(d)

$$\text{Filtered Image 1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & ? & ? & ? & ? & 0 \\ 0 & ? & ? & ? & ? & 0 \\ 0 & ? & ? & ? & ? & 0 \\ 0 & ? & ? & ? & ? & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

Figure 5(e)

$$\text{Filtered Image 2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 12 & 8 & 5 & 0 \\ 0 & 12 & 18 & 14 & 7 & 0 \\ 0 & 13 & 15 & 11 & 7 & 0 \\ 0 & 6 & 7 & 8 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 5(f)

$$\text{Filtered Image 3} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & ? & ? & ? & ? & 0 \\ 0 & ? & ? & ? & ? & 0 \\ 0 & ? & ? & ? & ? & 0 \\ 0 & ? & ? & ? & ? & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

Figure 5(g)

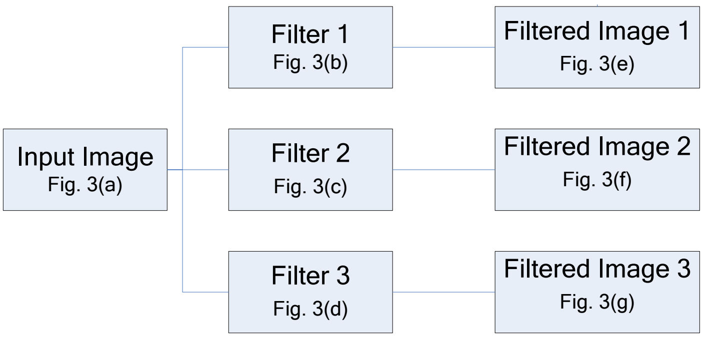

Figure 5(h)

(ii) The image filtered by Filter 2 is shown in Figure 5(f). By comparing it with the results obtained for Filtered Images 1 and 3, find the relationship between the three filtered images.

(iii) From the relationship that you have established in Part (ii) above, deduce what Filter 2 should be.

[ME5405-Nov 11-Q3]

6.

An image is given in Fig. 6(a) below:

$$\begin{bmatrix} 0 & 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 1 & 1 & 4 & 5 \\ 0 & 2 & 0 & 4 & 5 & 4 \\ 0 & 0 & 5 & 4 & 6 & 6 \\ 0 & 0 & 6 & 6 & 5 & 6 \\ 5 & 4 & 6 & 5 & 4 & 5 \end{bmatrix}$$

Fig 6(a)

(a) Use the following masks shown in Fig. 6(b) and Fig. 6(c) to estimate the magnitude and orientation of the local gradient at all pixel positions of the image, except the boundary pixels.

$$\begin{bmatrix} -1 & 0 & 1 \\ -3 & 0 & 3 \\ -1 & 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} -1 & -3 & -1 \\ 0 & 0 & 0 \\ 1 & 3 & 1 \end{bmatrix}$$

Fig 6(b) Fig 6(c)

(b) We are only interested the gradient orientations to vertical and horizontal ones only, we shall therefore make the following approximations for the gradient orientation *θ:*

$$\theta = 0^o \text{ when } 0^o \le |\theta| \le 45^o \text{ and } \theta = 90^o \text{ when } 45^o < |\theta| \le 90^o$$

Indicate with explanation, which pixels represent horizontal edge pixels and which represent vertical.

(c) Show how the masks in Fig 6(b) and Fig 6(c) can each be separated into two one-dimensional filters. Explain how we can make use of this property of a 2-D mask to reduce the computational cost of convolution (or filtering process).

[ME5405-Nov 12-Q3(a)]

7.

The histogram of a given image is given by: $p_r(r) = \frac{3}{2}\sqrt{r}$. Determine the transformation which should be applied in order to change the histogram into $p_z = \frac{\pi}{2}\sin(\pi z)$.

[ME5405-Nov 12-Q4(a)]

8.

The gray level probability density function of an image is given by $p(r) = 8\exp(-8r);\quad 0 \le r \le 1$. Sketch *p(r)* and describe the image represented by this probability density function.
Which of the transformations: (i) $s = r^2$ or $s = \sqrt{r}$ would produce a better image? Explain.

[ME5405-Nov 14-Q4(d)]

9.

(a) Consider the 5 × 5 image shown in Fig. 9 below. The pixel gray level values are given in the cell.

(i) Apply a 3 × 3 median filter to the image shown in Fig. 9.
(ii) Use the Sobel Operator to calculate the gradient of the image.

Note that to avoid problems at the edge, you only need to determine the required value for the pixels in the central 3 × 3 region in both cases.

3	2	1	2	4
2	1	3	200	3
6	7	8	7	9
8	100	6	6	7
7	9	6	8	8

Fig. 9

[ME5405 Nov 15 – Q 4(a)]

10.

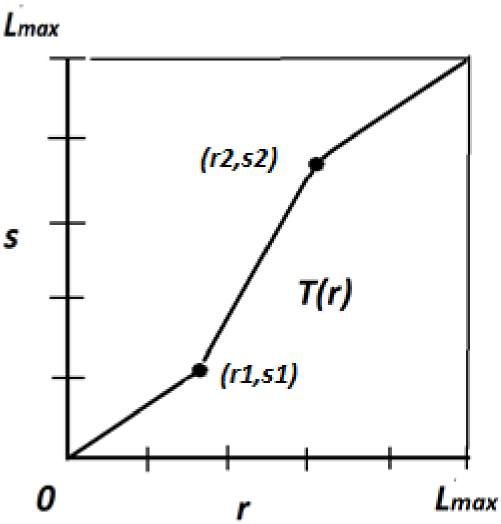

Fig. 10

In the global enhancement of an image by point transformation, a Transfer Function *T(r)* is designed to transform the gray level of the original image (*r*) to new gray level (*s*) of the transformed image with the desired enhancement effects. *T(r)* usually consists of line segments joined at the point where transformation of a certain effect begins. Fig 10 shows an example of a $s = T(r)$. In the figure *Lmax* is the maximum gray level value available.

(i) Given a gray-level image with gray level *r* lying in the interval [0,240]. We wish to compress the gray level intervals [0,80] and [160,240] by a factor of 2, and expand the gray level in the interval [80,160] by a factor of 2. Find the transfer function *T(r)* required to achieve the transformation. Sketch the transfer function and label the points (r_1, s_1) and (r_2, s_2) .

(ii) Suppose now we have an image with gray level *r* ranges in the interval [0,30]. Determine the transfer function *T(r)* for the following transformation.

- Stretches the gray level *r* in interval [0,10] to *s* in the interval [0,15];
- Shift the gray level *r* in the interval [10,20] to *s* in the interval [15,25]; and
- Compresses the gray level *r* in the interval [20,30] to *s* in the gray level [25,30].

Sketch the transfer function and label the points (r_1, s_1) and (r_2, s_2).

[ME5405 Nov 16 – Q4(b)]