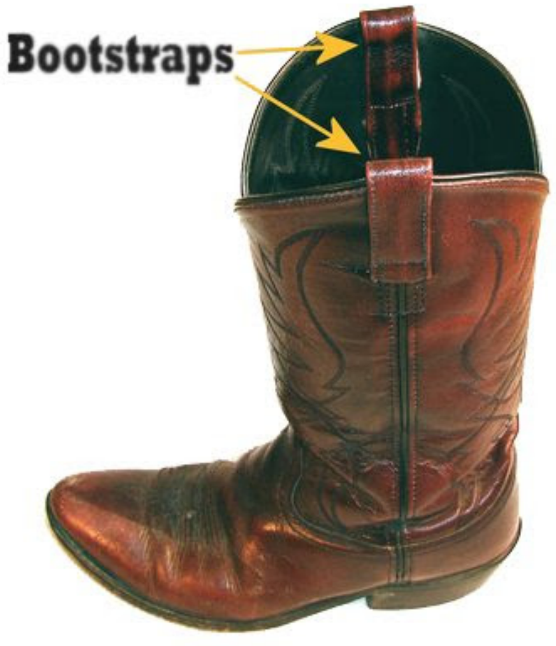
Confidence intervals and the bootstrap



Point Estimate

We use the statistics from a sample as a **point estimate** for a population parameter

- $\overline{x}$ is a point estimate for…?

44% of American approve of Trump's job performance

- Gallup poll from October 14th

Symbols:

π: Trump's approval for all voters

$\hat{p}$: Trump's approval for those voters in our sample

Interval estimate based on a margin of error

An **interval estimate** give a range of plausible values for a population parameter

One common form of an interval estimate is:

Point estimate ± margin of error

Where the **margin of error** is a number that reflects the precision of the sample statistic as a point estimate for this parameter

Example: Fox news poll

44% of American approve of Trump’s job performance, plus or minus 3%

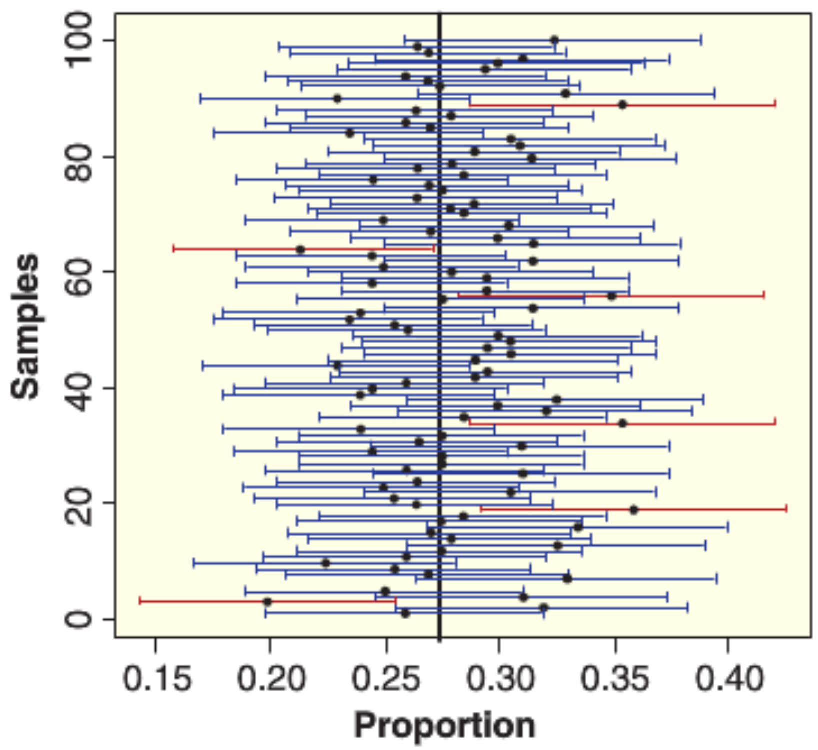
How do we interpret this?

Confidence Intervals

A **confidence interval** is an interval computed by a method that will contain the ***parameter*** a specified percent of times

- i.e., if the estimation were repeated many times, the interval will have the parameter x% of the time

The **confidence level** is the percent of all intervals that contain the parameter



Think ring toss…

Parameter exists in the ideal world

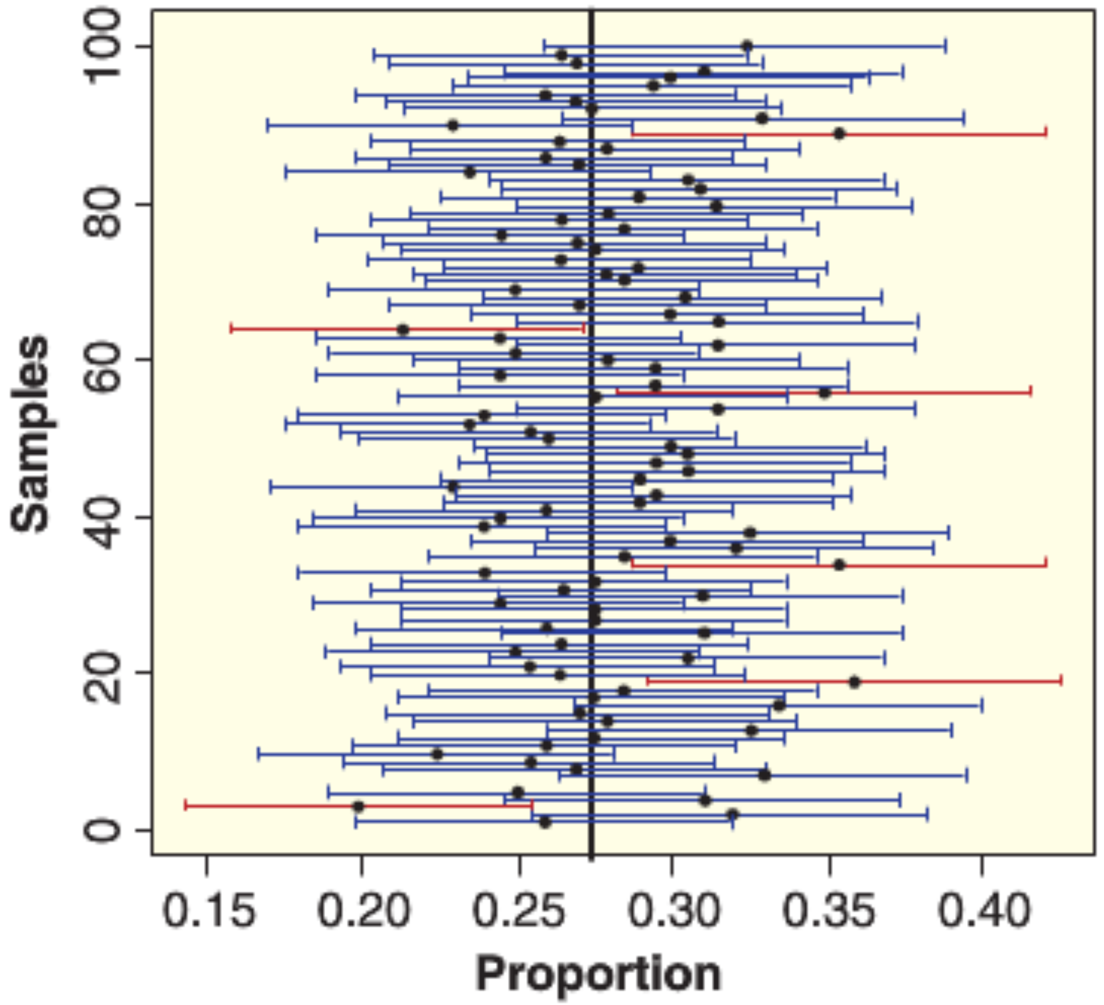
We toss intervals at it

95% of those intervals capture the parameter

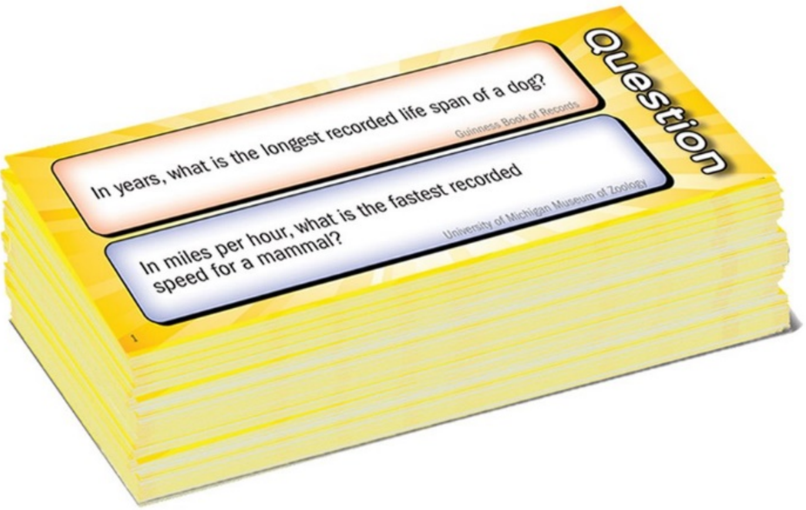
Confidence Intervals

For a **confidence level** of 95%...

95% of the **confidence intervals** will have the parameter in them



Wits and Wagers…



Wits and Wagers…

Question 1: In feet and inches, how tall was the tallest human in recorded history?

Question 2: How many floors does the leaning tower of Pisa have?

Question 3: What year was the parking meter invented?

Wits and Wagers…

Question 4: How many time zones does Russia have?

Question 5: What percentage of US households own a cat?

Question 6: What percent of the world's population lives in the U.S.?

Question 7: On average, what percent of a watermelon's weight comes from water?

Wits and Wagers…

Question 8: How many chemical elements are there on the Periodic Table of the Elements?

Question 9: What percent of the world's surface is water?

Question 10: In what year was an ATM machine first installed in the U.S.?

Note

For any given confidence interval we compute, we don't know whether it has really captured the parameter

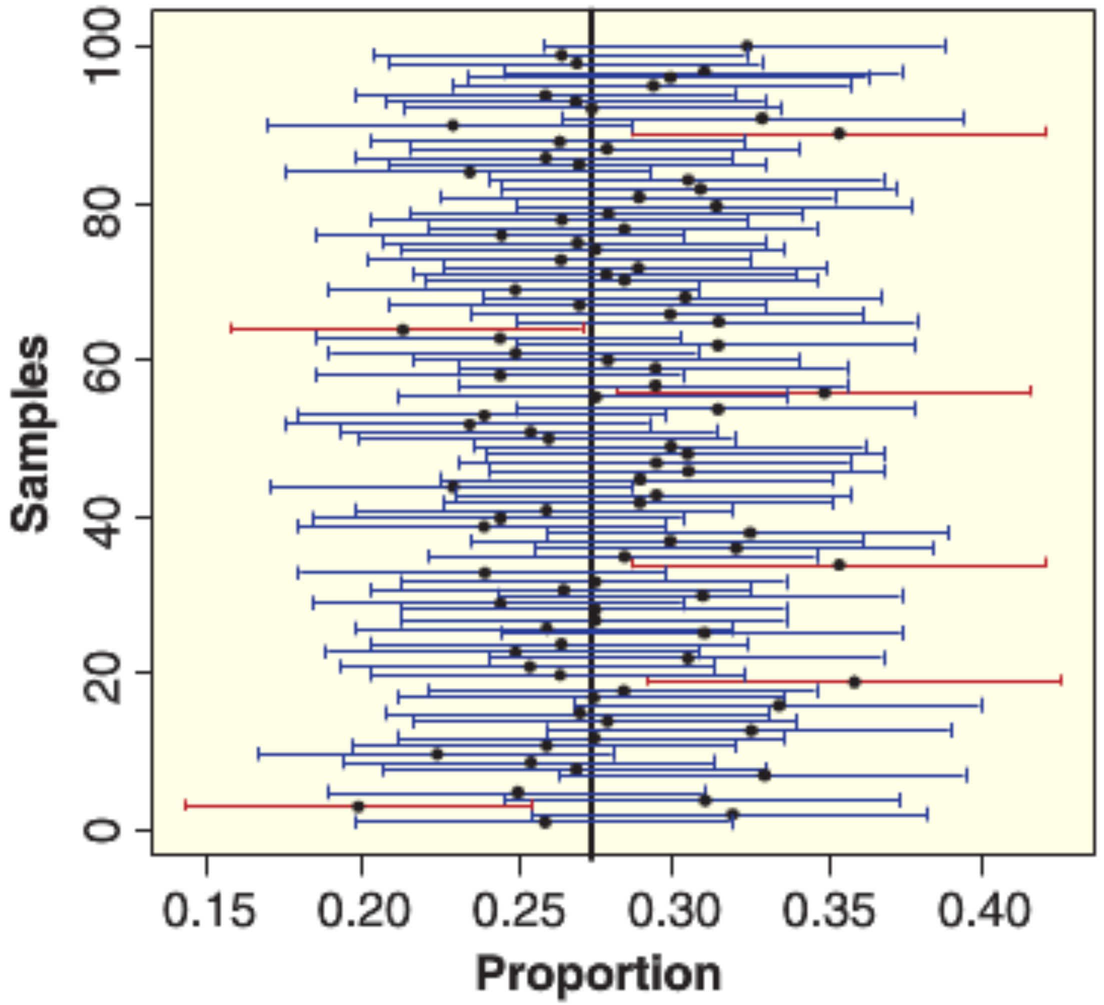
But we do know that if we do this 100 times, 95 of these intervals will have the parameter in it

(for a 95% confidence interval)

Confidence Intervals

For a **confidence level** of 90%...

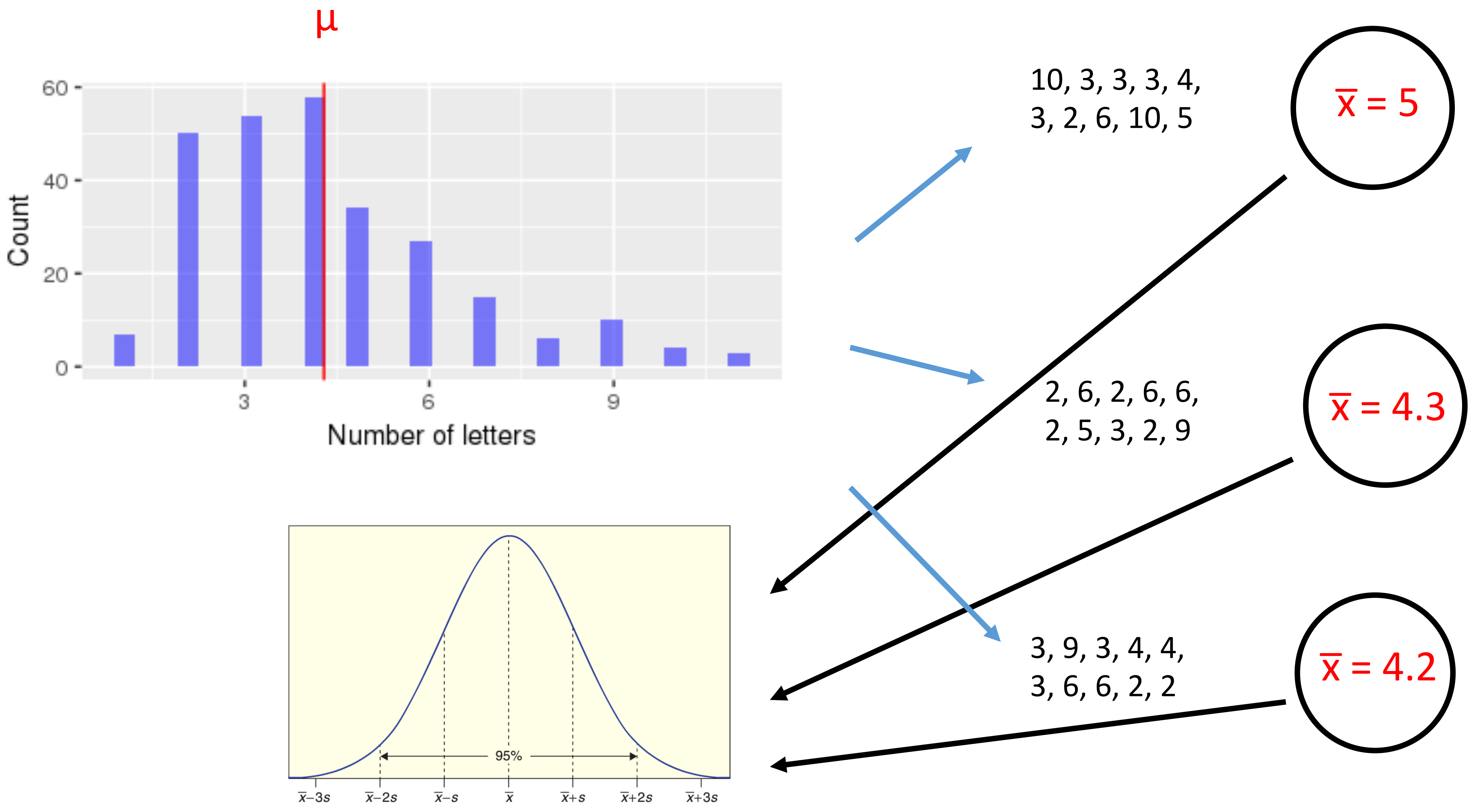
90% of the **confidence intervals** will have the parameter in them



Computing confidence intervals

Let’s now discuss how we can compute confidence intervals…

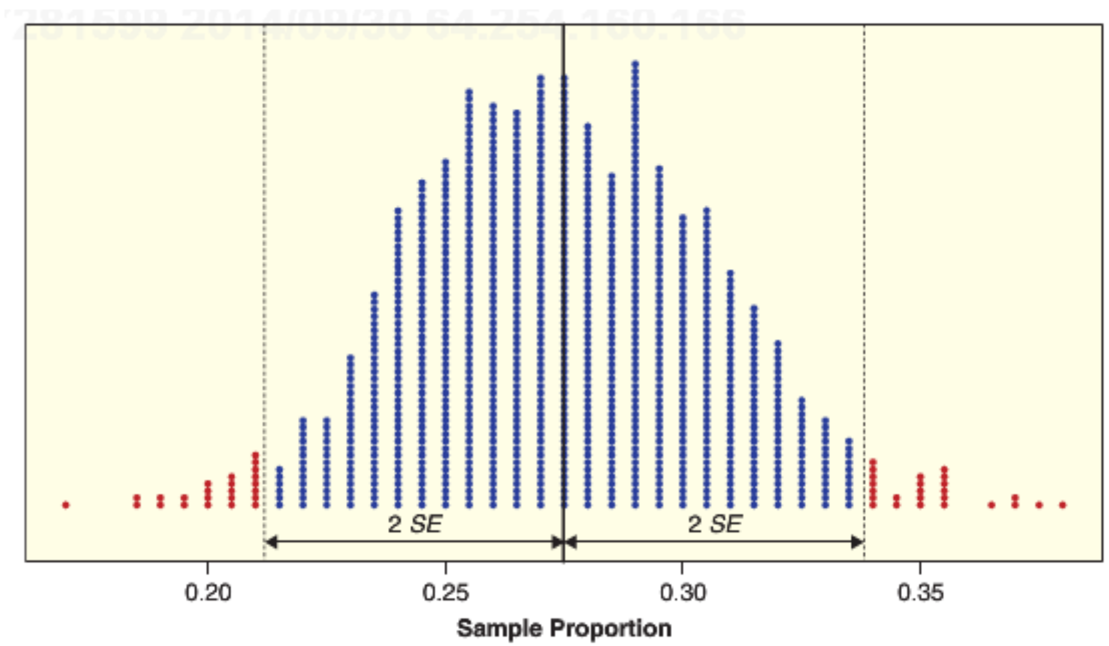
Review: sampling distribution illustration



Sampling distribution!

Sampling distributions

Q: For a sampling distribution that is a normal distribution, what percentage of ***statistics*** lie within 2 standard deviations (SE) for the population mean?

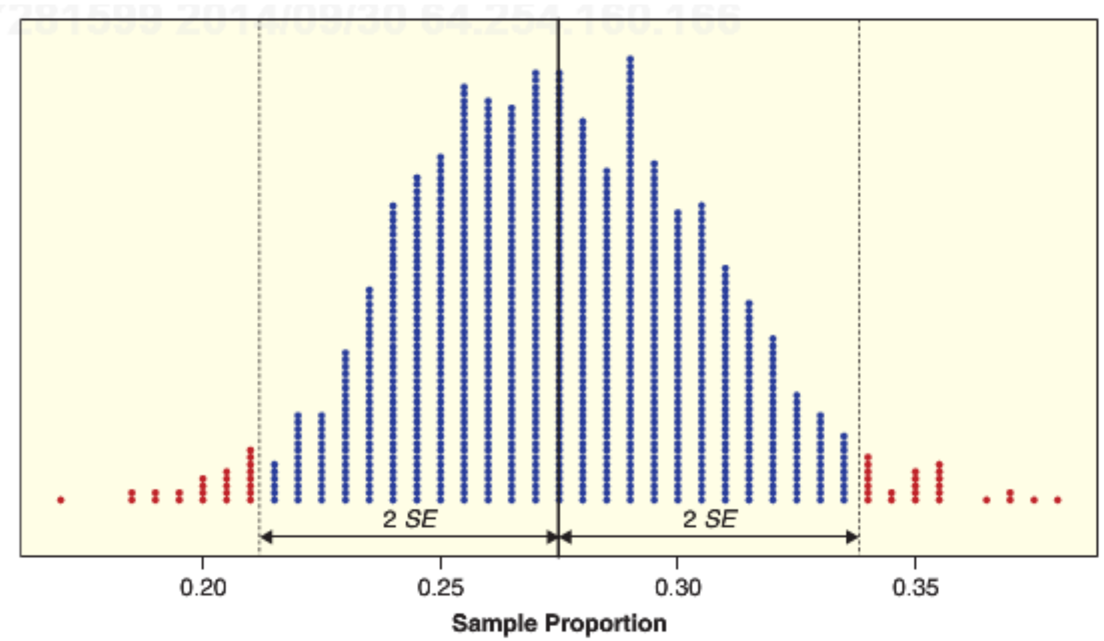


Sampling distributions

Q: If we had:

- A statistics value
- The SE

Could we compute a 95% confidence interval?



Sampling distributions

Q: Could we repeat the sampling process many times to create a sampling distribution and then calculate the SE?

Sampling distributions

Q: If we can't calculate the sampling distribution, what's else could we do?

Plug-in principle

Suppose we get a sample from a population of size *n*

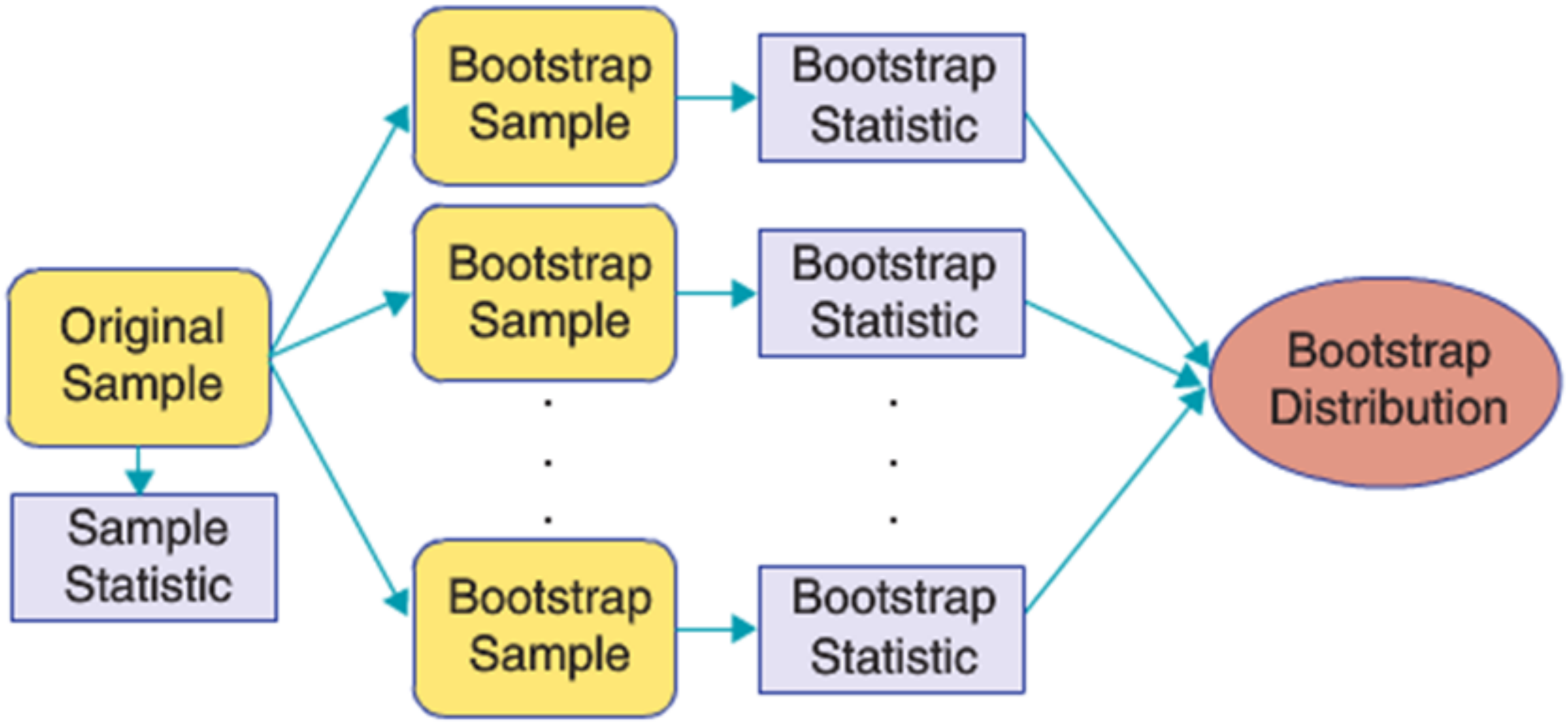
We pretend that *the sample is the population* (plug-in principle)

1. We then sample *n* points *with replacement* from our sample, and compute our statistic of interest

2. We repeat this process 1000's of times and get a ***bootstrap sample distribution***

3. The standard deviation of this bootstrap distribution (SE* bootstrap) is a good approximate for standard error SE from the real sampling distribution

Bootstrap process



95% Confidence Intervals

When a bootstrap distribution for a sample statistic is approximately normal, we can estimate a 95% confidence interval using:

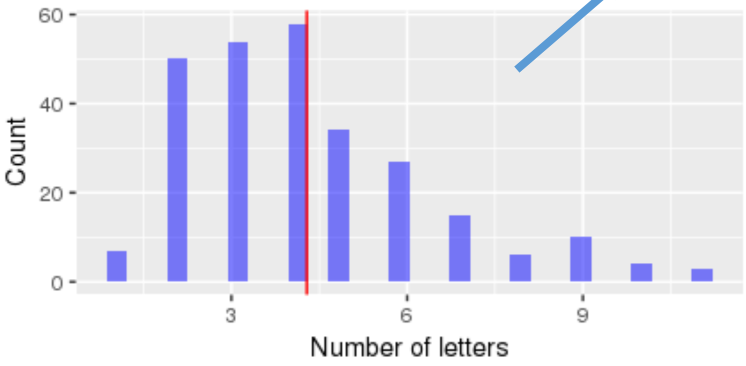
$$Statistic \pm 2 \cdot SE^*$$

Where SE* is the standard error estimated using the bootstrap

Bootstrap distribution illustration

The sample (n = 10)
10, 3, 3, 3, 4, 3, 2, 6, 4, 5

μ



3, 3, 3, 5, 3,
4, 5, 2, 2, 10

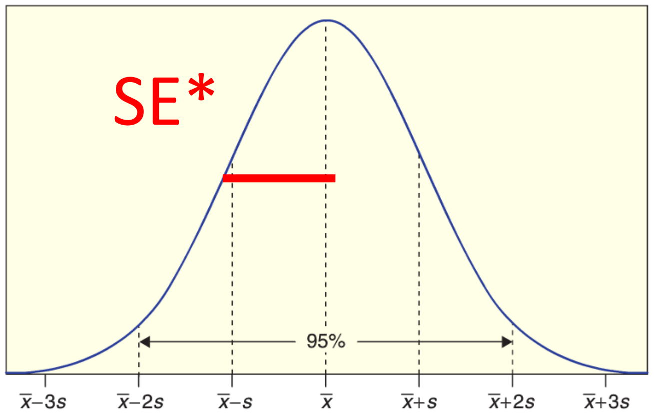
$\bar{x}^* = 4$

3, 3, 2, 3, 6,
4, 6, 5, 3, 6

$\bar{x}^* = 4.1$

5, 3, 2, 3, 3,
3, 10, 3, 4, 3

$\bar{x}^* = 3.9$



Bootstrap distribution!

Notice there is no 9’s in the bootstrap samples

Let’s try it in R…