Changes based on report for revised version JPAA QIN

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I would like to thank the referee for his patience with and continued support of my paper. His comments have greatly improved the paper and I am very grateful for that.

To (hopefully) address the main complain, I have added full proofs to all the results from Druel's paper. I have also made other changes based on the suggestions of the report. Please see below for details.

1 Previous comments and questions

1. The revised version now uses ss(s) for semistable(stable).

2. The paper now uses $\mathcal{O}_C(1)$to denote the very ample line bundle corresponding to the embedding of the conic. It uses θ and $\theta(1)$ to denote the two generators of $\mathrm{Pic}(C)$.

3. Improved the proof, in particular, added a reference showing the six elements listed in the proof generates a subring that is a polynomial ring ([Do], Corollary 5.1(i)) and coincides with the ring of semi-invariants ([Do], Corollary 5.1(ii)). This provides a complete proof that the moduli is $\mathbb{P}^5$.

4. We now give full proof for Lemma 3.1, Proposition 3.7, Lemma 3.8, Theorem 3.10, adapting arguments in [D] to V_5. I would like to mention the proofs made use of Lemma 2.1,3.2,3.3 in [D]. Those lemmas are general statements about subvariety or sheaves in projective space, which do not depend on the threefold one works on, so I only refer the reader to those results instead of proving them.

5. See previous entry.

2 New comments and questions

1. Changed all k to $\mathbb{C}$.

2. The assumption $\dim(X) \geq 3$ is not necessary and removed.

   Added a reference for this statement (https://arxiv.org/pdf/math/0610015.pdf). The statement here is a special case of Theorem 1 on page 3 in the reference. As mentioned on the first page of the reference: *However, although this construction (already known as Hartshorne-Serre correspondence) is very well-known and thoroughly used, it is very difficult to provide a good reference of it. Indeed the general result is only in Vogelaar's PhD thesis, which is not published elsewhere, and hence it is usually embarrassing to use as a reference.* The best published version of this result is the special case when $X = \mathbb{P}^n$ by Hartshorne.

3. It is stated in (now) Lemma 2.11 that we are working on V_5.

4. Delete the sentence *'we will try to get an contradiction'*. (It was left over of a failed proof, I apologize.)

5. Changed all related C to c.

6. We now give full proof, following [D].

7. Changed 'the theorem' to 'Theorem 5.7'

8. For the last few comments regarding the reference, I changed according to the comments as well as improved the consistency of the reference.

References

[Do] M. Domokos, *Poincaré series of semi-invariants of* 2×2 *matrices*, Linear Algebra and its Applications, 310 (2000):183—194.

[D] S. Druel, *Espace des modules des faisceaux de rang* 2 *semistables de classes de Chern* $c_1 = 0, c_2 = 2$ *et* $c_3 = 0$ *sur la cubique de* $\mathbb{P}^4$, Internat. Math. Res. Notices, 19 (2000):985–1004.