Homework 2 Yipeng Zhao

3.32 (a) $X(z) = \dfrac{1}{(1+\frac{1}{2}z^{-1})^2(1-2z^{-1})(1-3z^{-1})}$

$x[n]$ stable $\Rightarrow |z|=1$ in ROC

$\Rightarrow$ ROC is $\frac{1}{2}<|z|<2$

$$= \frac{1/35}{(1+\frac{1}{2}z^{-1})^2} + \frac{58/1225}{1+\frac{1}{2}z^{-1}} + \frac{1568/1225}{(1-2z^{-1})} + \frac{2700/1225}{(1-3z^{-1})}$$

$$= \frac{1}{35}\cdot\frac{-\frac{1}{2}z^{-1}}{(1+\frac{1}{2}z^{-1})^2}\cdot(-2z) + \frac{58/1225}{1+\frac{1}{2}z^{-1}} + \frac{1568/1225}{(1-2z^{-1})} + \frac{2700/1225}{(1-3z^{-1})}$$

$$\Rightarrow x[n] = \frac{-2}{35}\left(n\cdot(-\tfrac{1}{2})^n u[n]\right) * \left((-2)\,\delta[n+1]\right) + \frac{58}{1225}(-\tfrac{1}{2})^n u[n]$$

$$+ \frac{1568}{1225}\, 2^n u[-n-1] - \frac{2700}{1225}\, 3^n u[-n-1]$$

$$= \frac{-2}{35}(n+1)(-\tfrac{1}{2})^{n+1}u[n+1] + \frac{58}{1225}(-\tfrac{1}{2})^n u[n] + \frac{1568}{1225}2^n u[-n-1]$$

$$- \frac{2700}{1225}3^n u[-n-1]$$

(b) $X(z) = e^{z^{-1}} = \sum_{n=0}^{\infty} \frac{z^{-n}}{n!} \Rightarrow x[n] = \frac{1}{n!}u[n]$

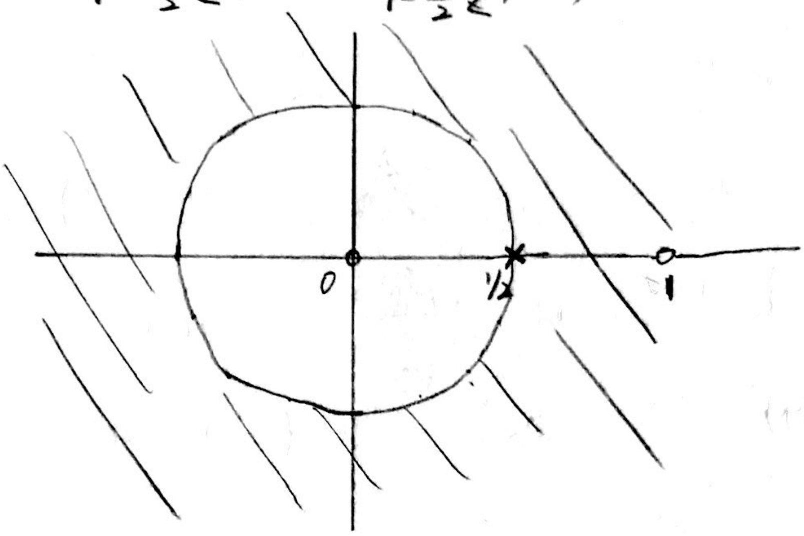
(c) $X(z) = \dfrac{z^3-2z}{z-2} = z^2+2z+\dfrac{2}{1-2z^{-1}}$ $x[n]$ is left-sided $\Rightarrow$ ROC is $|z|<2$

$x[n] = \delta[n+2] + 2\delta[n+1] - 2^{n+1}u[-n-1]$

3.40 (a) $X(z) = \frac{1}{1-z^{-1}} \quad (|z|>1)$

$y[n] = (\frac{1}{2})^{n-1} u[n+1] = 4(\frac{1}{2})^{n+1} u[n+1] \Rightarrow Y(z) = 4 \cdot \frac{z}{1-\frac{1}{2}z^{-1}} \quad (|z| > \frac{1}{2})$

$H(z) = \frac{Y(z)}{X(z)} = \frac{4z(1-z^{-1})}{1-\frac{1}{2}z^{-1}} = \frac{4z-4}{1-\frac{1}{2}z^{-1}}, \quad |z| > 1/2$



(b) Inverse z-transform of $H(z)$

$H(z) = \frac{4z}{1-\frac{1}{2}z^{-1}} - \frac{4}{1-\frac{1}{2}z^{-1}}$

$= 4 \cdot \frac{1}{1-\frac{1}{2}z^{-1}} \cdot z - \frac{4}{1-\frac{1}{2}z^{-1}}, \quad |z| > 1/2$

$\Rightarrow h[n] = 4((\frac{1}{2})^n u[n]) * (\delta[n+1]) - 4(\frac{1}{2})^n u[n]$

$= 4(\frac{1}{2})^{n+1} u[n+1] - 4(\frac{1}{2})^n u[n]$

(c) $|z|=1$ is included in $|z|>1/2 \Rightarrow$ stable.

(d) $h[-1] \neq 0, \Rightarrow$ not causal

3.42 (a)

$$\left.\begin{aligned} V(z) &= X(z) + W(z) \\ V(z)H(z) + E(z) &= W(z) \end{aligned}\right\rangle \Rightarrow W(z) = \frac{H(z)}{1+H(z)} X(z) + \frac{1}{1+H(z)} E(z)$$

(b)

$$H_1(z) = \frac{\frac{z^{-1}}{1-z^{-1}}}{1+\frac{z^{-1}}{1-z^{-1}}} = z^{-1}, \quad H_2(z) = \frac{1}{1+\frac{z^{-1}}{1-z^{-1}}} = 1 - z^{-1}$$

(c) For general cases, can't determine whether $H(z)$. $H_1(z)$. $H_2(z)$ is stable or not.

For the special case in (b), $H(z)$ is not stable, since it has a pole at $z=1$. $H_1(z)$ and $H_2(z)$ are stable.

3.48 (a) $y[n]$ is stable $\Rightarrow$ $|z|=1$ is in the ROC $\Rightarrow$ $\frac{1}{2}<|z|<2$

(b) Two-sided. ROC $\frac{1}{2}<|z|<2$ is a ring

(c) $x[n]$ is stable $\Rightarrow$ $|z|=1$ in ROC $\Rightarrow$ $|z|>\frac{3}{4}$

(d) $X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n} < \infty$ at $n=\infty$, so $x[n]=0$ when $n<0$,

So $x[n]$ is causal

(e) $X(z)=\sum_{n=0}^{\infty} x[n]z^{-n}$,

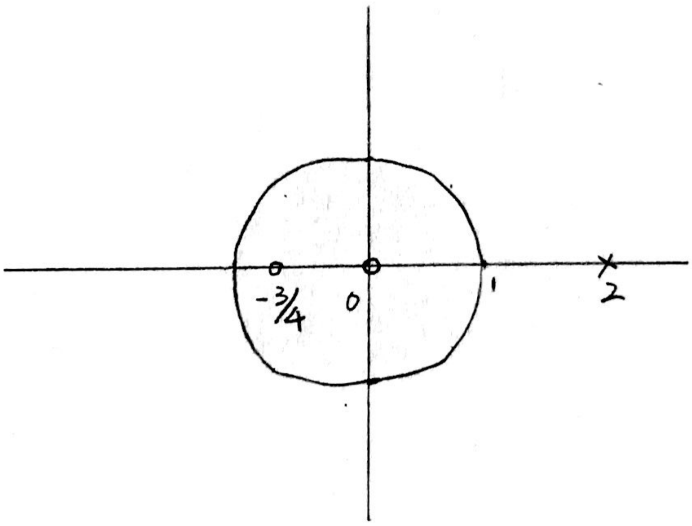
$$X(z\to\infty)=\sum_{n=0}^{\infty} x[n]\lim_{z\to\infty} z^{-n} = x[0].$$

So $x[0]=X(z\to\infty)=\lim_{z\to\infty}\frac{A(1+\frac{3}{4}z^{-1})(1-\frac{1}{2}z^{-1})}{(1+\frac{1}{4}z^{-1})}=0$

where A is constant

$$H(z)=\frac{Y(z)}{X(z)}$$

(f)



ROC is $|z|<2$.

(g) $H(z)=\sum_{n=-\infty}^{\infty} h[n]z^{-n}$ $\quad H(0)=0<\infty \Rightarrow h[0]=0$ for $n>0$.

so $h[n]$ is anti-causal.

2. Matlab problem 1

(a) $R(e^{j\omega}) = \sum_{k=-\infty}^{\infty} r[k] e^{-j\omega k} = \sum_{k=0}^{L-1} e^{-j\omega k} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}}$

(d) 12 zero crossings for $L=12$

Peak height is 12, at $\omega=0$

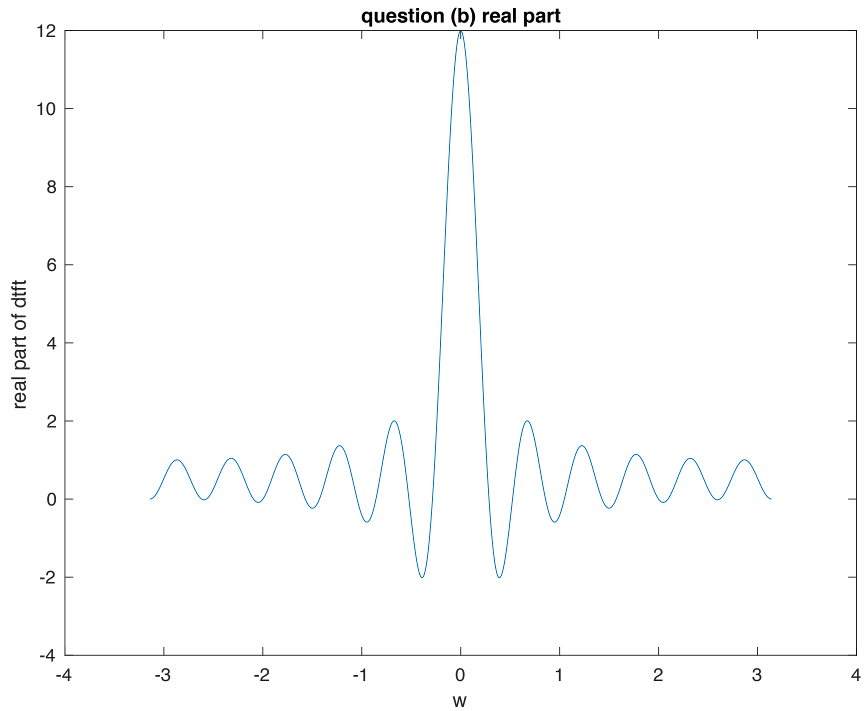
(e) 14 zero crossings. peak height is 15, at $\omega=0$

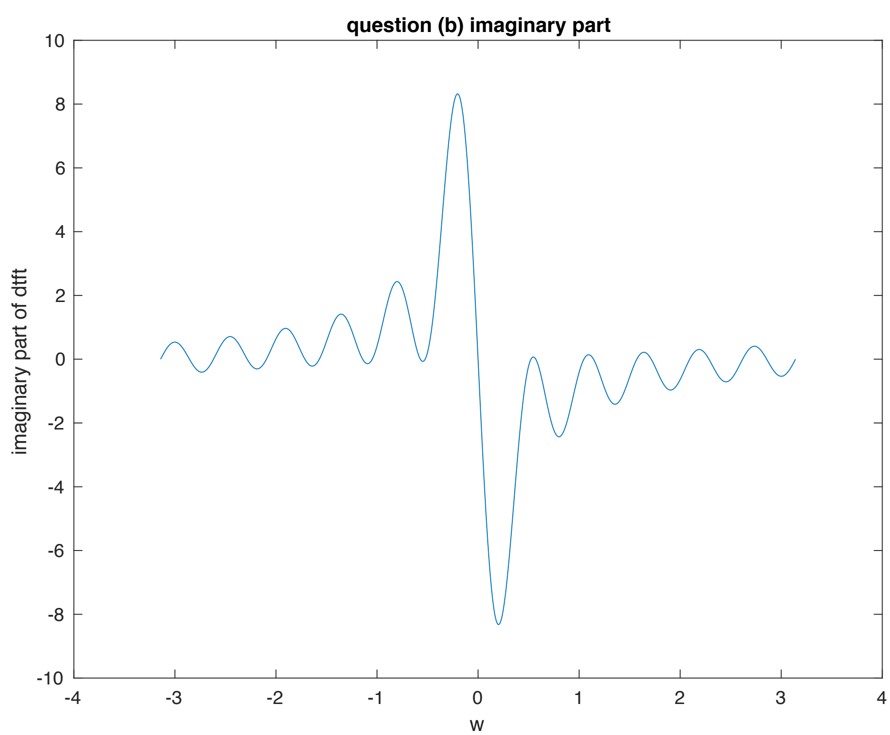
(f) zero crossings at $\omega_{crossing} = \pm\frac{2\pi k}{L}$, where $k=1,2,\cdots,\text{floor}(\frac{L}{2})$

peak height $= L$

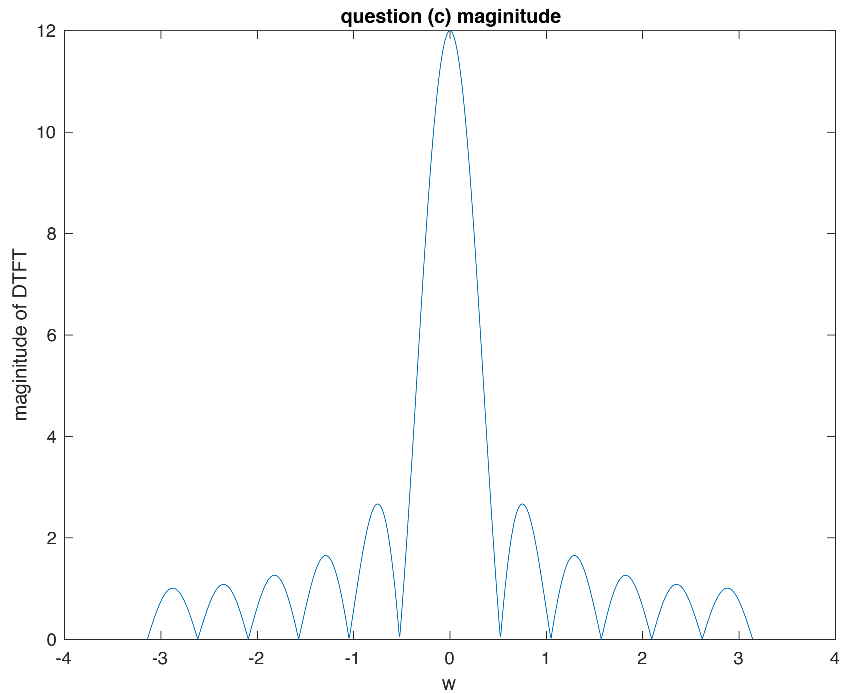
2, Matlab problem 1

(b)

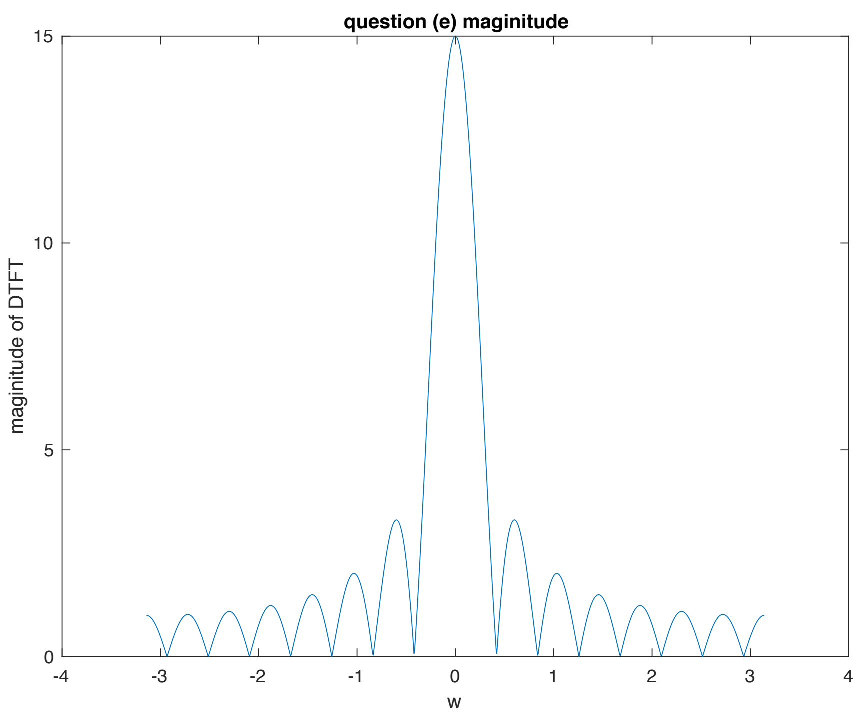




(c)


question (c) maginitude
maginitude of DTFT
w
12
10
8
6
4
2
0
-4
-3
-2
-1
0
1
2
3
4

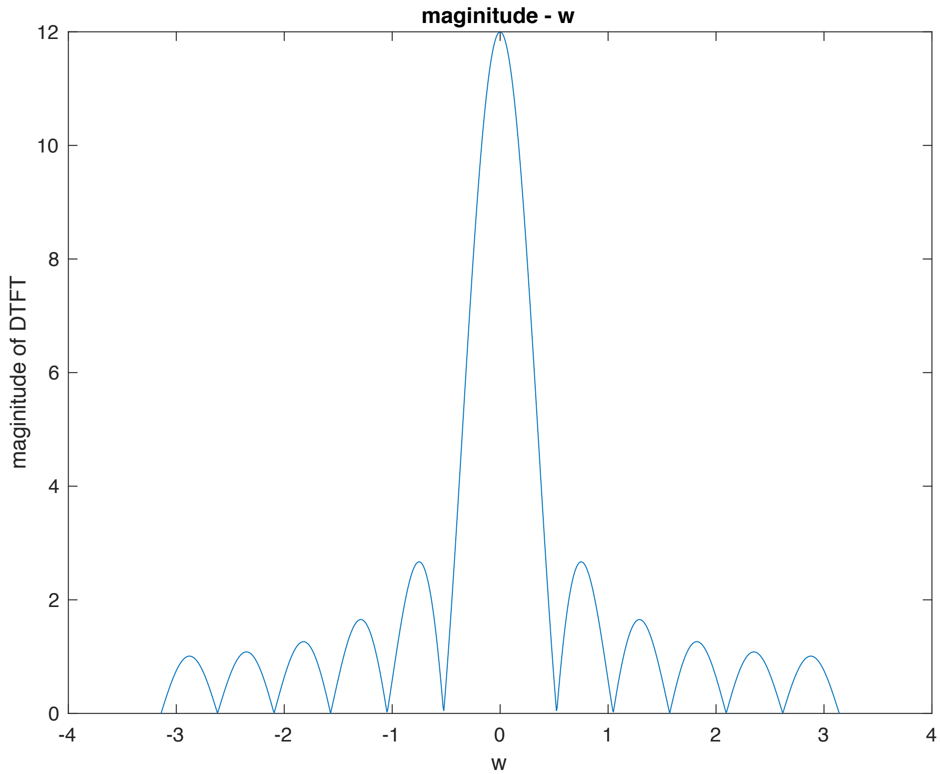
(e)


question (e) maginitude
maginitude of DTFT
w
15
10
5
0
-4
-3
-2
-1
0
1
2
3
4

Code of problem 2, matlab problem 1:

```
% Homework 2, problem 2
%% question (b)
% L = 12
dtft = @(w, L)(1 - exp(-1i*w*L))./(1-exp(-1i*w));
w = -pi:2*pi/1000:pi;
DTFT_12 = dtft(w, 12);
figure;
plot(w, real(DTFT_12));
title('question (b) real part')
xlabel('w');
ylabel('real part of dtft');
figure;
plot(w, imag(DTFT_12));
title('question (b) imaginary part')
xlabel('w');
ylabel('imaginary part of dtft');
%% question (c)
magnitude_12 = abs(DTFT_12);
figure;
plot(w, magnitude_12);
title('question (c) maginitude');
xlabel('w');
ylabel('maginitude of DTFT');
%% question (e)
DTFT_15 = dtft(w, 15);
maginitude_15 = abs(DTFT_15);
figure;
plot(w, maginitude_15);
title('question (e) maginitude');
xlabel('w');
ylabel('maginitude of DTFT');
```

3, Matlab problem 2



The magnitude-w plot is almost the same as the one in problem 2.

Function psinc:

```
% function psinc
function y=psinc(w, L)
    y = (w==0).*L + (w~=0).*(sin(1/2*w*L)./sin(1/2*w));
end
```

Code of plotting:

```
% Homework 2, problem 3
% L = 12, DTFT = exp(-1/2*j*w*(L-1))*psinc(w, L)
w = -pi:2*pi/1000:pi;
L = 12;
DTFT = exp(-1/2*1i*w*(L-1)).*psinc(w, L);
magnitude = abs(DTFT);
figure;
plot(w, magnitude);
title('maginitude - w');
xlabel('w');
ylabel('maginitude of DTFT');
```

4. Matlab problem 3

(a) A DTFT $X(e^{jw})$ has the property

$$X(e^{-jw}) = \sum_{k=-\infty}^{\infty} x[k]e^{jwk} = \sum_{k=-\infty}^{\infty} x[k](e^{-jwk})^*$$

$$= \sum_{k=-\infty}^{\infty} (x[k]e^{-jwk})^*$$

$$= (X(e^{jw}))^*$$

So the magnitude is even about w. and the phase is odd.

(b) $$X(e^{jw}) = \sum_{k=0}^{\infty} (0.9)^k e^{-jwk} = \frac{1}{1-0.9e^{-jw}}$$

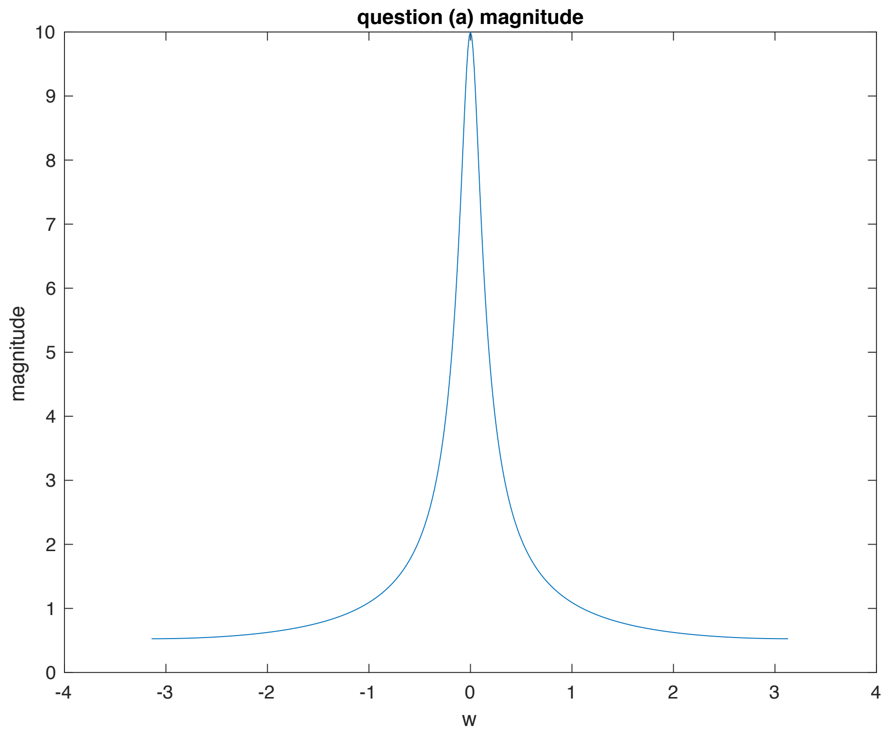
$$= \frac{1-0.9\cos w}{1.81-1.8\cos w} + j\frac{-0.9\sin w}{1.81-1.8\cos w}$$

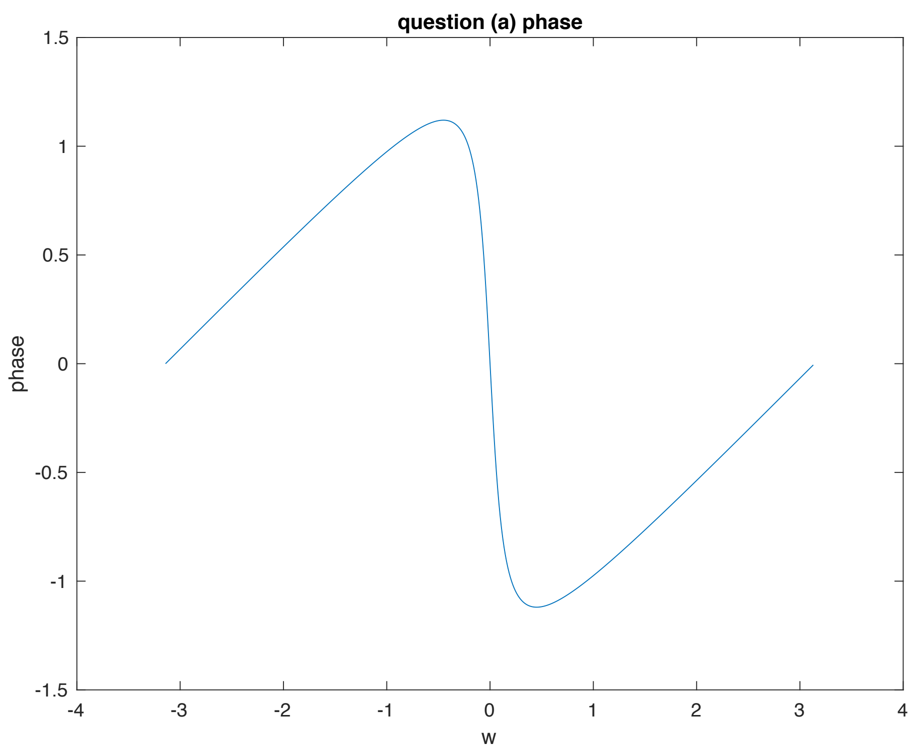
$$\text{magnitude} = \frac{\sqrt{(1-0.9\cos w)^2+(0.9\sin w)^2}}{1.81-1.8\cos w} = \frac{1}{\sqrt{1.81-1.8\cos w}}$$

$$\text{phase} = \text{atan}\left(\frac{-0.9\sin w}{1-0.9\cos w}\right)$$

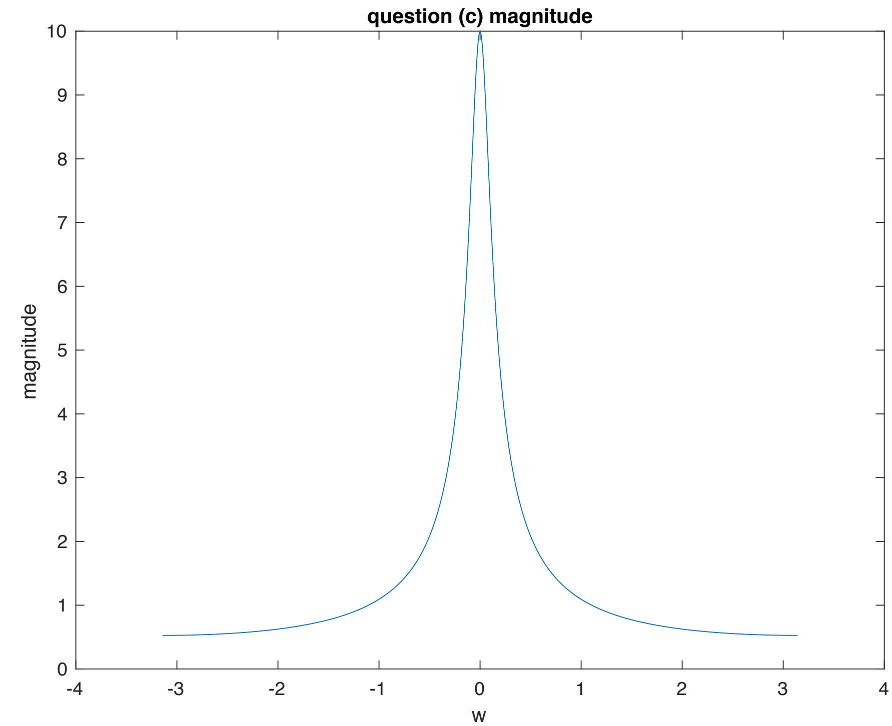
4, Matlab problem 3

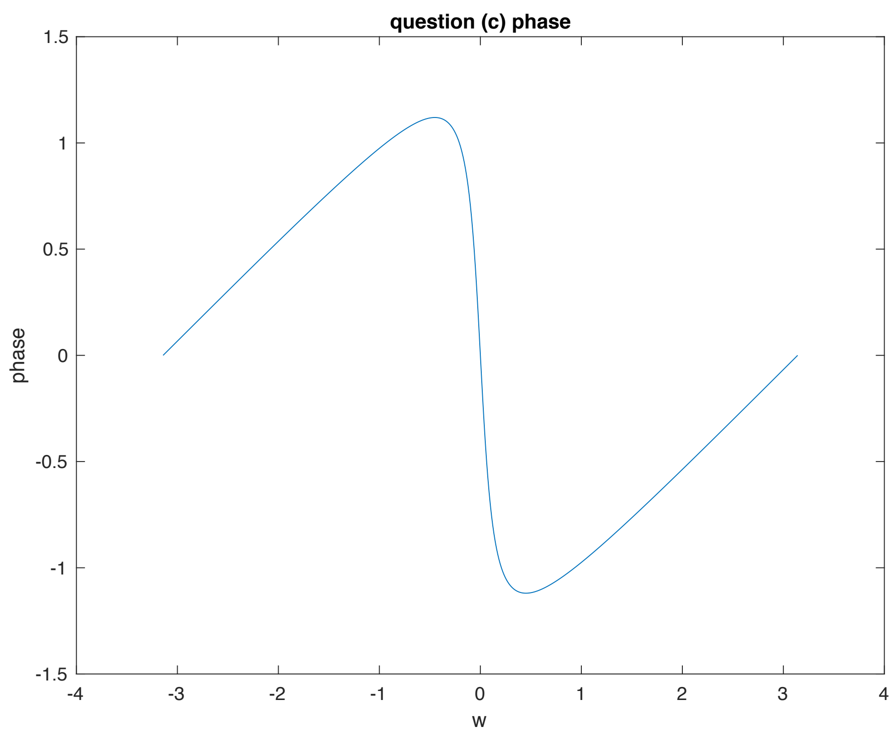
(a)





(c)





They are almost the same.

Code of problem 4, matlab problem 3

```
% Homework 2, problem 4
% After calculation, A=[1, -0.9], B=[1];
%% question (a)
A = [1, -0.9];
B = 1;
[X, W] = freqz(B, A, 512, 'whole');
X = [X(257:end); X(1:256)];
figure;
plot(W - pi, abs(X));
plot(W - pi, abs(X));
title('question (a) magnitude');
xlabel('w');
ylabel('magnitude');
figure
plot(W - pi, angle(X));
title('question (a) phase');
xlabel('w');
ylabel('phase');
%% question (c)
w = -pi:2*pi/512:pi;
magnitude = 1./sqrt(1.81 - 1.8*cos(w));
phase = atan(-0.9*sin(w)./(1 - 0.9*cos(w)));
figure;
plot(w, magnitude);
title('question (c) magnitude');
xlabel('w');
ylabel('magnitude');
figure;
plot(w, phase);
title('question (c) phase');
xlabel('w');
ylabel('phase');
```