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Social learning with bounded confidence and heterogeneous agents

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A B S T R A C T

This paper investigates an opinion formation model in social networks with bounded confidence and heterogeneous agents. The network topologies are shaped by the homophily of beliefs, which means any pair of agents are neighbors only if their belief difference is not larger than a positive constant called the bound of confidence. We consider a model with both informed agents and uninformed agents, the essential difference between which is the informed agents have access to outside signals which are function of the underlying true state of the social event concerned. More precisely, the informed agents update their beliefs by combining the Bayesian posterior beliefs based on their private observations and weighted averages of the beliefs of their neighbors. The uninformed agents update their beliefs simply by linearly combining the beliefs of their neighbors. We find that the whole group can learn the true state only if the bound of confidence is larger than a positive threshold which is related to the population density. Furthermore, simulations show that the proportion of informed agents required for collective learning decreases as the population density increases. By tuning the learning speed of informed agents, we find the following: the higher the speed, the shorter the time needed for the whole group to achieve a steady state, and on the other hand, the higher the speed, the lower the proportion of agents with successful learning — there is a trade-off.

1. Introduction

Our actions and decisions in daily life are shaped by and reflect our beliefs and opinions to certain extent. Examples to support this statement are prevalent all around us. The kind of clothes we prefer to buy is shaped by our opinions. In a voting situation, our choice to support a certain candidate is decided by our ideological beliefs. The importance of beliefs to our life can never be overstated. Therefore, the study of how beliefs are formed and how the structure of the social network impacts the formation of beliefs has attracted significant attention and gradually developed into a relatively independent research field — social learning theory.

Social learning models can be classified into two categories: Bayesian and non-Bayesian. Bayesian models are those in which fully rational individuals use Bayes' rule to form the best mathematical estimate of the relevant unknowns given their priors and understanding of the world [1]. Each individual updates her belief by performing complex conjectures on others' actions, in which case the strategies of others must be taken into account and the analysis is quite challenging even for simple networks [2–4]. Therefore, it is not realistic to expect individuals to adopt Bayesian learning in the real world. Moreover, a series of experiments suggests that people may process information in a less rational way [5,6], which emphasizes the necessity of investigating non-Bayesian learning approaches. Non-Bayesian models are those in which

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individuals are boundedly rational and build up beliefs in naive ways, such as imitation and replication [7–10]. In most non-Bayesian models, the new belief of an individual is the weighted average of the beliefs of her neighbors. With abundant mathematical tools, these models are convenient for analysis, and indeed, lots of meaningful results are presented. One problem facing non-Bayesian models, however, is that the consensus belief of the whole group (if it exists) is only a mixture of initial beliefs, which generally cannot reveal the underlying true state.

Motivated by the problems facing both sorts of models, most recently, Jadbabaie et al. [11] presented a non-Bayesian learning model, in which each individual updates her belief by combining Bayesian posterior belief based on her private observation and the weighted average of the beliefs of her neighbors. Although Bayesian inference is involved in the model, the observed signals are generated by a process which is a function of the underlying true state of the world rather than being obtained from other individuals, which is an important feature that distinguishes Jadbabaie's model from traditional Bayesian models. Since fully rational deduction is avoided, the model sharply decreases the complexity of analysis and computing.

In Ref. [11], Jadbabaie et al. investigate the social learning model in an exogenous network, where the network topology is assumed to be fixed and strongly connected. However, the social network structures, i.e., neighboring relationships and trust levels toward other individuals, are usually time-varying in the real world. Moreover, existing research suggests that the similarity of beliefs leads to attraction and interaction (see e.g., Ref. [12]), which inspires a class of social learning models with bounded confidence. A common assumption in these models is that any pair of agents can communicate with each other only if their belief difference is not larger than a threshold called the bound of confidence [13–17]. The structure of the bounded-confidence network is time-varying as a result of the evolution of beliefs.

In this paper, we examine the non-Bayesian learning approach presented in Ref. [11] in bounded-confidence networks and investigate the influence of bounded confidence on the process of belief evolution. Furthermore, we consider heterogeneous agents: only a subset of agents, called *informed agents*, can receive outside signals and perform Bayesian inference. We introduce a parameter into the updating process that makes the learning speed of informed agents adjustable. The rest of the agents, called *uninformed agents*, do not have access to outside signals and update their beliefs simply by averaging the beliefs of their neighbors.

Simulations illustrate that the whole group can learn the true state only if the bound of confidence is larger than a certain threshold. Furthermore, it is shown that for a fixed population density, the proportion of agents learning the true state increases as the proportion of informed agents increases. As the population density increases, the proportion of informed agents required to guide the group to successful learning decreases. By tuning the learning speed of informed agents, we find that there exists a trade-off between the proportion of agents learning the true state and the speed by which agents form steady beliefs.

The remainder of this paper is organized as follows: In Section 2, a social learning model with bounded confidence and heterogeneous agents is presented. In Section 3, simulations and analyses are provided. Section 4 concludes the paper.

2. A social learning model with bounded confidence and heterogeneous agents

2.1. Underlying states and beliefs of individuals

Let θ denote a possible state of the world, representing the quality of a new product, the underlying reason of a phenomenon, and so on. All the possible states compose a finite state set $\Theta = \{\theta_1, \theta_2, \dots, \theta_m\}$, in which the true state is denoted by θ^*. Agent *i*'s belief on θ at time t is denoted by $\mu_{i,t}(\theta)$, which is the probability that she believes θ is the underlying true state. Thus, $\{\mu_{i,t}(\theta), \theta \in \Theta\}$ is a probability distribution over the state set Θ. We say that agent i learns the true state if she assigns a belief of one to the true state and zero to other states. For simplicity, the belief vector $[\mu_{i,t}(\theta_1), \mu_{i,t}(\theta_2), \dots, \mu_{i,t}(\theta_m)]$ is denoted by $\mu_{i,t}$.

2.2. Network structure

The set of agents in a social network is denoted by $V = \{1, 2, \dots, n\}$. All the agents are endowed with bounded confidence, and any pair of them are neighbors only if their belief difference is not larger than a positive constant r called the bound of confidence. The set of neighbors of agent i at time t is $N_i(t) = \{j \in V : \|\mu_{j,t} - \mu_{i,t}\| \le r\}$, where $\|\cdot\|$ is a proper norm for measuring the difference between two beliefs. The neighboring relationship changes with the evolution of beliefs and is symmetrical with respect to any pair of agents.

2.3. Signal structures

Conditional on the true state θ^*, signal vector $s_t \in S$ is generated by the likelihood function $\ell(s_t|\theta^*)$ at each time $t > 0$, where S denotes the signal space. We consider two sorts of agents, which can be classified as *informed agents* and *uninformed agents* according to their learning approaches. Only informed agents have access to outside signals. For each signal observed by the informed agent i at time t, denoted by s_t^i, there exists a corresponding personal signal structure for each θ, denoted by $\ell_i(s_t^i|\theta)$, representing the probability that s_t^i appears if the true state is θ from the view of point of agent

i. These personal signal structures are decided by agents themselves and may be totally different from the truth. We further impose the following essential assumption on signal structures about the true state: the signal structure about the true state θ^*, i.e., $\ell_i(s_t^i|\theta^*)$, is the i-th marginal of $\ell(s_t|\theta^*)$, which means the informed agents have correct knowledge of what will happen if the true state is θ^*.

2.4. Belief updating rule

The belief updating rule adopted by any informed agent i can be described as, for all $\theta \in \Theta$

$$\mu_{i,t+1}(\theta) = \omega\mu_{i,t}(\theta)\frac{\ell_i(s_{t+1}^i|\theta)}{m_{i,t}(s_{t+1}^i)} + \frac{1-\omega}{|N_i(t)|}\sum_{j\in N_i(t)}\mu_{j,t}(\theta) \tag{1}$$

where $\omega \in (0, 1]$ is a parameter to tune the learning speed of informed agents; $m_{i,t}(s_{t+1}^i) = \sum_{\theta\in\Theta}\ell_i(s_{t+1}^i|\theta)\mu_{i,t}(\theta)$, and $|N_i(t)|$ is the cardinality of the set $N_i(t)$.

The first term on the right hand side of (1) is the Bayesian posterior belief based on signal s_{t+1}^i, and the second term is the average of the beliefs of i's neighbors (including herself). In fact, these two terms represent two approaches of learning in our daily life: learning from our own observation and inference, and learning from communication with other people. By tuning the parameter ω, informed agents can balance these two sorts of belief updating approaches. If $\omega = 1$, the informed agents are not concerned with other individuals, and perform Bayesian learning in personal situations. By Savage's statement in *The Foundations of Statistics* [18], it is well known that an individual becomes almost certain of the truth when the amount of her observation increases indefinitely. With ω decreasing, the agents pay more attention to their neighbors.

For an uninformed agent, she simply adopts the average of the beliefs of her neighbors as her new belief, as shown in (2)

$$\mu_{i,t+1}(\theta) = \frac{1}{|N_i(t)|}\sum_{j\in N_i(t)}\mu_{j,t}(\theta) \tag{2}$$

which is similar to the opinion formation rule used in the Hegselmann–Krause model (the HK model for short) [13–15].

Updating rules similar to (1) have been proposed in existing literature [19,20], where the "truth", or "desire direction", is set to be a constant, and informed individuals balance the influence of the constant and their social interactions. The constant representing the true state is called the "attraction of truth" in Ref. [20], where its interpretation is as follows: *Individuals have access to, or generate, new data (arguments, evidences, test results, etc.) that point in the direction of the truth.* Our model provides a reasonable realization of the "attraction of truth", where informed agents have access to signals generated by the underlying true state, and, in a Bayesian manner, observed signals turn into the "attraction of truth" that leads the beliefs in the desired direction.

Jadbabaie et al. [11] investigated a belief updating rule similar to (1) in a fixed directed network of homogeneous agents. They have shown that the whole group can learn the true state under certain mild conditions, among which the connectivity is an indispensable assumption. By contrast, we focus on the social structure shaped by bounded confidence, which cannot guarantee the connectivity of the whole group. Once a part of uninformed agents compose a cluster, which can be viewed as an isolated sub-network, the dynamic law inside the cluster is similar to that in the HK model, and they will definitely fail to learn the true state unless all agents in the sub-network have already learned the truth when they are separated from others. One more feature that distinguishes our model from Ref. [11] is that we consider heterogeneous agents and the learning speed of informed agents is adjustable, which leads to attractive behavior in our model.

Similar to other social learning models with bounded confidence, there exists a strong coupling between the beliefs of agents and the social structure in our model. More specifically, the beliefs of agents decide the social structure, i.e., whether any pair of agents can communicate with each other, and in turn, the social structure affects the formation of new beliefs. If we capture the social structure by a matrix, as we usually do in social learning literature, the matrix is a function of beliefs. Therefore, the updating rule would be nonlinear, and one cannot even write it explicitly, which makes precise mathematical analysis very difficult. Therefore, we examine the model by computer simulations, and obtain the evolution of beliefs, which could give us a deeper insight to social learning from the qualitative aspect.

3. Simulations and analyses

3.1. Simulation environment

We should point out that simulations for Bayesian inference involve complicated choices of parameters. However, it is arguable that different choices would generally not affect simulations to such an extent that they change their qualitative features. Since informed agents in our model conduct Bayesian inference (only the first term on the right hand side of (1)) in the same way as one standard Bayesian agent does in a personal situation, we can apply the statement in Ref. [18] to our case to support our argument. Firstly, for the number of possible states, two (one true and one false) and countably infinite have no essential difference. Considering the true state θ^* and any one of false states $\overline{\theta}$, we always have $\mu_{i,t}(\theta^*)/\mu_{i,t}(\overline{\theta}) \to \infty$, as the amount of observation increases for standard Bayesian agent *i*, which means all the beliefs on false states asymptotically converge to zero. Secondly, the choices of signal structures do not affect the beliefs on true state converging to one, as long as no state is observationally indistinguishable from the true state, i.e., there exists no false state $\overline{\theta}$ satisfying $\ell_i(s|\overline{\theta}) = \ell_i(s|\theta^*)$ for all observed signals *s*. Given the above considerations, we only need to keep the simulation environment as simple as possible, and focus our attention on the performance of the model with different choices of the key parameters, such as the bound of confidence, the population density, and the learning speed of informed agents.

Simulations are performed on two possible states, i.e., $\Theta = \{\theta_1, \theta_2\}$, and agent *i*'s belief vector at time *t* is $\mu_{i,t} = [\mu_{i,t}(\theta_1), \mu_{i,t}(\theta_2)] \in R^2$. Since $\{\mu_{i,t}(\theta_1), \mu_{i,t}(\theta_2)\}$ is a proper probability distribution, we always have $\mu_{i,t}(\theta_1)+\mu_{i,t}(\theta_2) = 1$. The initial beliefs assigned to θ_1 are uniformly distributed in the interval [0,1]. Here we use maximum norm to measure the difference between two beliefs. More precisely, the difference between $\mu_{i,t}$ and $\mu_{j,t}$ can be denoted by $|\mu_{i,t}(\theta_1) - \mu_{j,t}(\theta_1)|$, or equally, $|\mu_{i,t}(\theta_2) - \mu_{j,t}(\theta_2)|$. The state θ_1 is set to be the true state, and we say agent *i* learns the true state if $\mu_{i,t}(\theta_1) \to 1$ as $t \to 0$. The signal set generated by the true state is $S = \{H, T\}$. We assume that signal *H* appears with the possibility of 80%, and *T* with 20%, which implies the private signal structures about θ_1 of any informed agent *i* are $\ell_i(H|\theta_1) = 0.8$ and $\ell_i(T|\theta_1) = 0.2$. Furthermore, we set the signal structures about θ_2 to be $\ell_i(H|\theta_2) = 0.2$ and $\ell_i(T|\theta_2) = 0.8$. In fact, other values can be chosen as long as the existence of an observationally equivalent state can be avoided. Informed agents are selected randomly according to given proportions. We next discuss how the key parameters affect the learning performance of the whole group.

3.2. Bound of confidence

The belief evolution with different bounds of confidence is shown in Fig. 1. In the case of $r = 0.3$ (the top row in Fig. 1), each agent has a relatively large neighborhood, and social interactions are dominant at the initial periods when the cumulation of outside signals is relatively scarce. Extensive communication leads the whole group to an agreement on the belief, the value of which then moves towards the truth as the observed information increases. The consensus value eventually reaches the belief vector $[\mu_{i,t}(\theta_1), \mu_{i,t}(\theta_2)] = [1, 0]$ for all $i \in V$, which means the whole group learn the true state collectively. In the case of a smaller bound of confidence $r = 0.1$ (the middle row in Fig. 1), clusters of beliefs emerge at the beginning because of the difference in initial beliefs and the relatively small neighborhood. However, as time goes on, or to be more specifically, as the amount of observed information grows, the separate groups merge together in the process of discovering the truth, and all agents learn the true state eventually. In the case of an extremely small bound of confidence $r = 0.03$ (the bottom row in Fig. 1), only a subset of agents can still learn the true state while the rest of the agents hold their false beliefs forever. For $r = 0.1$ and $r = 0.03$, clusters appear in both cases. However, an essential difference exists between these two cases: all the clusters merge into a single one which asymptotically reaches the correct belief in the former case, while certain clusters in the latter case stay away from the correct belief forever. The underlying reason for the difference is that a relatively small bound of confidence narrows agents' neighborhood, and once a cluster only consists of uninformed agents, it has no opportunity to receive outside information. The final belief of the cluster is simply the mixture of the beliefs of the agents inside the cluster. To show the reason more intuitively, we enlarge the sub-figure in the lower-left corner in Fig. 1, i.e., beliefs on θ_1 with $r = 0.03$, and mark the beliefs of informed agents by solid lines and uninformed ones by dashed lines. Therefore, it is easy to figure out whether a cluster consists of informed agents or not. One can see in Fig. 2 that once a cluster only consists of uninformed agents (e.g., any one of the seven clusters located in the middle-lower part of the figure denoted by dashed lines), its belief evolution is the same as in the HK model, and therefore, the final belief cannot reflect the true state.

To summarize, a relatively large bound of confidence will be helpful to expand social interactions, and therefore, informed individuals have more opportunities to affect other people directly or indirectly. As a result the whole group is lead to successful learning. For a relatively small bound of confidence, even if the whole group is divided into clusters, any cluster consisting of informed agents can still learn the true state. This demonstrates the importance of wide social interactions, especially keeping in touch with informed ones. Once a cluster only consists of uninformed agents, and cuts off any possible communication with informed ones, it will fail in learning the truth.

3.3. Population density

Through numerous simulations, we find that the population density has a certain effect on the performance of social learning. By population density, we mean the number of agents in a group. Fig. 3 shows the threshold of the bound of confidence for the whole group to learn the true state as a decreasing function of the population density, in which the data are the results of averaging over 100 realizations. An explanation of the decreasing curve is that the increase in the population density results in the increase in the number of neighboring relationships. For two agents, *i* and *j* for instance, if they hold beliefs $\mu_{i,t}(\theta_1) = 0.3$ and $\mu_{j,t}(\theta_1) = 0.7$, and if the bound of confidence is set to be $r = 0.3$, they cannot communicate with each other. If another agent *k* is added into the group, who holds belief $\mu_{k,t}(\theta_1) = 0.5$, then both *i* and *j* are neighbors of *k*, and they may communicate indirectly through *k*. Another description of Fig. 3 is that, for a fixed bound of confidence, larger population density helps more agents to learn the true state.

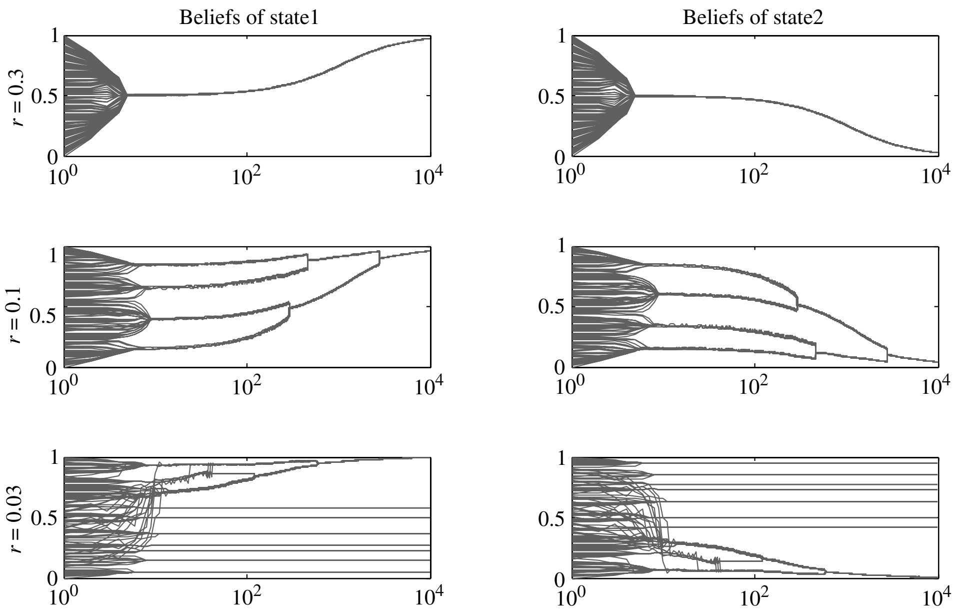

Fig. 1. The belief evolution of the whole group with different bounds of confidence (100 agents, 20% informed, $\omega = 0.5$).

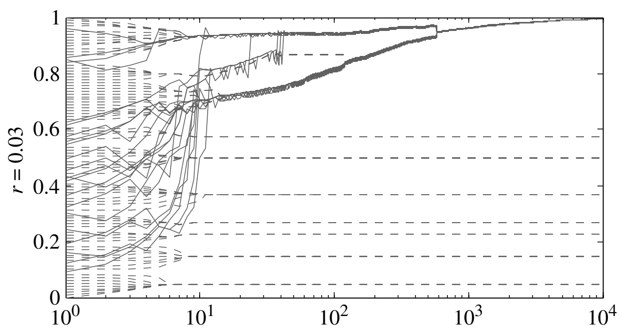

Fig. 2. Beliefs of state 1 with $r = 0.03$.

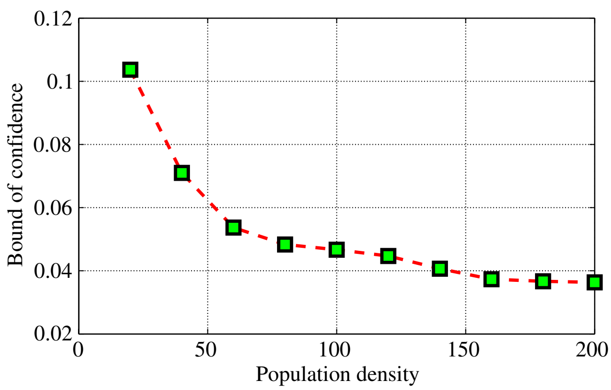

Fig. 3. The threshold of the bound of confidence for learning the true state with different population densities (20% informed, $\omega = 0.5$).

3.4. Proportion of informed agents

In our model, learning of the whole group is guided by informed agents. The proportion of agents learning the true state increases as the proportion of informed agents increases, which is confirmed by Fig. 4. The informed agents may have various representatives in different contexts. They may have access to more information sources about the truth, or they may be

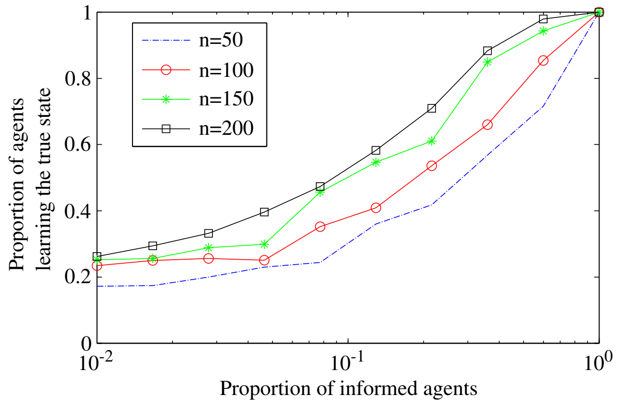

Fig. 4. Proportion of agents learning the true state as an increasing function of the proportion of informed agents and the population density ($r = 0.03$, $\omega = 0.5$).

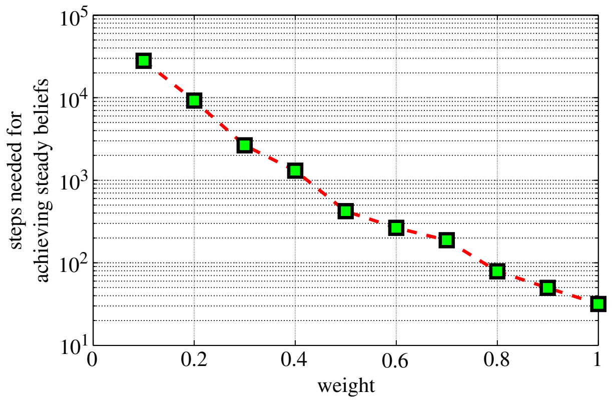

Fig. 5. The number of steps needed to achieve steady beliefs as a function of different learning speeds of informed individuals (50 agents, 10% informed, $r = 0.05$).

highly educated and have more reasonable deduction and deeper understanding of the social event concerned. A larger number of these informed individuals would be helpful for the whole group to learn the truth. It is also shown that the larger the population density, the smaller the proportion of informed agents required to leads a certain proportion of agents to successful learning.

3.5. *Learning speed of informed agents*

As we know, Bayesian inference is the essential ingredient in leading the informed agents, even the whole group, learn the true state. The weight ω assigned to the posterior belief affects the learning speed of the informed agents. Fig. 5 shows the number of steps needed to achieve steady beliefs as a function of ω. As the learning speed of informed agents increases, the number of steps needed for the informed agents to learn the true state (also the number of steps for the whole group to achieve steady beliefs) decrease. This relationship can be described approximatively by a straight line in a semi-log coordinate system.

It is desired to learn the true state as quickly as possible. However, Fig. 6 illustrates that it is not the case that the larger the learning speed the better. As the learning speed of the informed agents increases, the proportion of agents learning the true state decreases. High learning speed helps a minor part of the agents learn the true state quickly, while leaving a major part of the agents unable to learn the true state. Therefore, there exists a trade off between a large fraction of agents learning the true state collectively and a small fraction learning the state as quickly as possible.

4. Conclusions

In this paper, we present a social learning model with bounded confidence and heterogeneous agents, where the structure of the social network is time-varying and belief-dependent. We consider both informed agents and uninformed agents. Informed agents update their beliefs by combining Bayesian posterior beliefs based on their private observations and the weighted averages of beliefs of their neighbors. The Bayesian inference helps informed agents to learn the true state, while

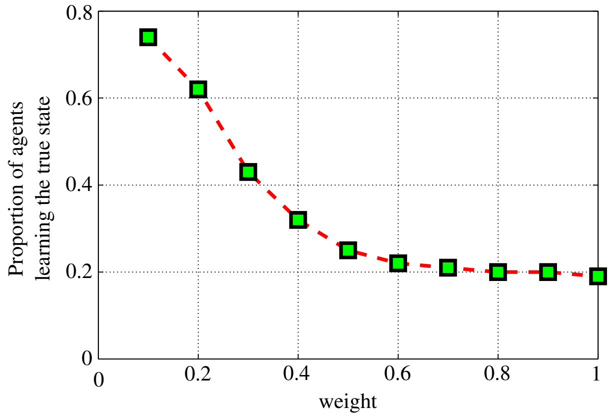

Fig. 6. Proportion of agents learning the true state with different learning speed of informed individuals (50 agents, 10% informed, $r = 0.05$).

the averages help them to maintain agreement with their neighbors. We also introduce a parameter for the informed agents to balance these two parts of the update. The uninformed agents update their beliefs simply by adopting averages of neighbors' beliefs. Simulations show that the whole group can learn the true state only if the bound of confidence is larger than a positive threshold. Simulations also show that the proportion of informed agents required to guide the group to successful learning decreases as the population density increases. By tuning the learning speed of informed agents, we find that there exists a trade-off between the proportion of agents learning the true state and the speed by which agents form steady beliefs.

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