Physica A 467 (2017) 240–248

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Competitive seeds-selection in complex networks

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HIGHLIGHTS

- We study the seeds-selection problem in a competitive diffusion model from a new perspective.
- We propose five seeds-selection strategies based on commonly used network centralities.
- Degree centrality and Betweenness centrality are proper indicators for influential seeds in proposed competitive diffusion model for all networks concerned.
- The priority of Betweenness centrality strategy decreases, as the heterogeneity of the network decreases.
- Our findings shed light on how to choose seeds in the competitive diffusion processes in applications.

ARTICLE INFO

Article history:
Received 5 August 2016
Received in revised form 28 September 2016
Available online 12 October 2016

Keywords:
Complex networks
Competitive diffusion process
Centrality measures

ABSTRACT

This paper investigates a competitive diffusion model where two competitors simultaneously select a set of nodes (seeds) in the network to influence. We focus on the problem of how to select these seeds such that, when the diffusion process terminates, a competitor can obtain more supports than its opponent. Instead of studying this problem in the game-theoretic framework as in the existing work, in this paper we design several heuristic seed-selection strategies inspired by commonly used centrality measures—Betweenness Centrality (BC), Closeness Centrality (CC), Degree Centrality (DC), Eigenvector Centrality (EC), and K-shell Centrality (KS). We mainly compare three centrality-based strategies, which have better performances in competing with the random selection strategy, through simulations on both real and artificial networks. Even though network structure varies across different networks, we find certain common trend appearing in all of these networks. Roughly speaking, BC-based strategy and DC-based strategy are better than CC-based strategy. Moreover, if a competitor adopts CC-based strategy, then BC-based strategy is a better strategy than DC-based strategy for his opponent, and the superiority of BC-based strategy decreases as the heterogeneity of the network decreases.

1. Introduction

Diffusion over social networks is a quite common phenomenon, like spread of rumors, viral marketing of new products, viruses propagation, public opinion formation, etc. [1–3]. For the convenience of description, here we say an individual is ‘infected’ if he or she believes a rumor, buys a product, catches a kind of disease, etc. One of the focal research directions related to this dynamic behavior is to study how to choose a set of individuals (here we call ‘seeds’) to start diffusion such

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http://dx.doi.org/10.1016/j.physa.2016.10.011
0378-4371/

that, when the diffusion process terminates, the number of infected individuals in the network can be maximized [4–7]. This problem, usually called *influence maximization*, is practically meaningful, since it might shed light on how to advertise new products for attracting more customers, how to conduct political campaign for obtaining more supporters, how to inject vaccine for limiting the spread of diseases, etc.

The influence maximization problem can be naturally extended to a competitive situation when we consider two or more competitors (here refer to firms, political leaders, cross-immune pathogens, etc.), each of which aims to maximize its influence taking into account of the existence of its opponents [8–11]. In the literature, two classes of competitive diffusion models are mainly investigated, which are the epidemic models (e.g., SIR model) [12,13] and the word of mouth models [14–16]. The former class of models is usually formulated as dynamical systems and one popular research focus is studying the conditions under which one competitor survives or two competitors coexist. The latter class of models is usually investigated from the game theory perspective, and the focus is on finding the Nash equilibrium solution of the game.

In this paper, we revisit a competitive diffusion model originally proposed in Ref. [17]. At the beginning, each of the competitors chooses seeding nodes to infect. The overlapping nodes which are chosen by multiple competitors are immune to the infection. During the diffusion process, if an uninfected node has neighbors infected by only one competitor, say competitor A, then it will be also infected by A; if it has some neighbors infected by one competitor and other neighbors infected by other competitors, then it will be immune to the infection. The diffusion process terminates when there is no further infection to occur in the network. The utility of each competitor is captured by the number of nodes it infects at the steady state. In the original paper, Alon et al. provided some conditions for existence of (pure strategy) Nash equilibrium [17]. Takehara et al. further studied the special case with two competitors and obtained a more precise necessary and sufficient condition for existence of Nash equilibrium [18]. Small and Mason considered this competitive diffusion process on trees [19]. They found that if the underlying social network is a tree, there always exists a Nash equilibrium. Most recently, Etesami and Basar studied the same game model on more general networks, and they showed that in general undirected networks the decision process on the existence of Nash equilibrium is an NP-hard problem [20]. For two special but well-studied classes of networks, namely the lattice and the hypercube, they provided necessary and sufficient condition for the existence of Nash equilibrium. For general networks they also presented some necessary conditions. All the work mentioned above considered diffusion process over undirected networks. Tzoumas et al. studied a class of competitive diffusion games over directed networks [21] and showed that these games do not always possess a Nash equilibrium, and deciding whether an equilibrium exists is co-NP-hard. They also studied the Price of Anarchy of the games, and found that under certain conditions, Nash equilibrium strategies (when it exists) might lead to low utility for both competitors.

As highlighted by the work of Refs. [17–21], deciding whether the competitive diffusion game admits a Nash equilibrium is quite difficult, let alone finding the exact solution. Therefore, the results in the work mentioned above can hardly tell us how to choose the initial seeds in the diffusion process. In this paper, we consider an alternative line of research of the competitive diffusion model. Based on classic centrality measures for networks, we try to construct several heuristic seed-selection strategies. Here we choose five of the most commonly used measures—Betweenness Centrality, Closeness Centrality, Degree Centrality, Eigenvector Centrality, and K-shell Centrality [22,23]. Each competitor can heuristically choose a given number of seeds according to a given measure. We compare these heuristic strategies through simulations on both real and artificial networks. Even though the network structure varies a lot across different networks, we find some interesting common trend by comparing the proposed seeds-selection methods. We hope these intuitive results can shed light on how to choose seeds in the competitive diffusion processes in applications.

2. The competitive diffusion model

We first introduce the competitive diffusion model which is originally proposed in Ref. [17].

2.1. Social network

We consider a social network as an undirected graph $G = (V, E)$, where $V = \{1, 2, \dots, n\}$ is the set of nodes representing individuals in the social network, and $E = \{(i, j)|i, j \in V\}$ is the set of edges between nodes with $(i, j) \in E$ representing that i and j are neighbors and they can influence each other. Therefore, the set of neighbors of i can be denoted by $N_i = \{j|(i, j) \in E\}$.

2.2. Competitive diffusion process

In this paper we only consider two competitors, denoted by A and B. The competitive diffusion process unfolds at discrete time steps.

At the initial time step $t = 0$, each of the competitors chooses a set of seeds to start diffusion. If one node is chosen simultaneously by A and B, then it becomes a gray node, which means it is immune to the infection and it does not further infect its neighbors in all future steps during the following process. Therefore, after the initial time step, any node can only be in one of four states: uninfected, infected by A, infected by B, and becoming gray. As the example shown in Fig. 1, competitor A selects nodes 1 and 2 as its seeds, and competitor A selects nodes 2 and 3 as its seeds. Then after the initial time step, node 1 is infected by A, node 3 is infected by B, node 2 becomes gray, and nodes 4, 5, 6, and 7 are uninfected.

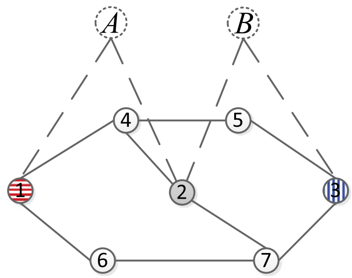


Fig. 1. An example of the four states. Two virtual nodes in dotted circles represent competitor *A* and *B*, respectively. Node 1 with red horizontal line is infected by competitor *A*. Node 3 with blue vertical line is infected by competitor *B*. Since node 2 is selected by both *A* and *B*, it becomes gray. The rest nodes with white color are uninfected at current step. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

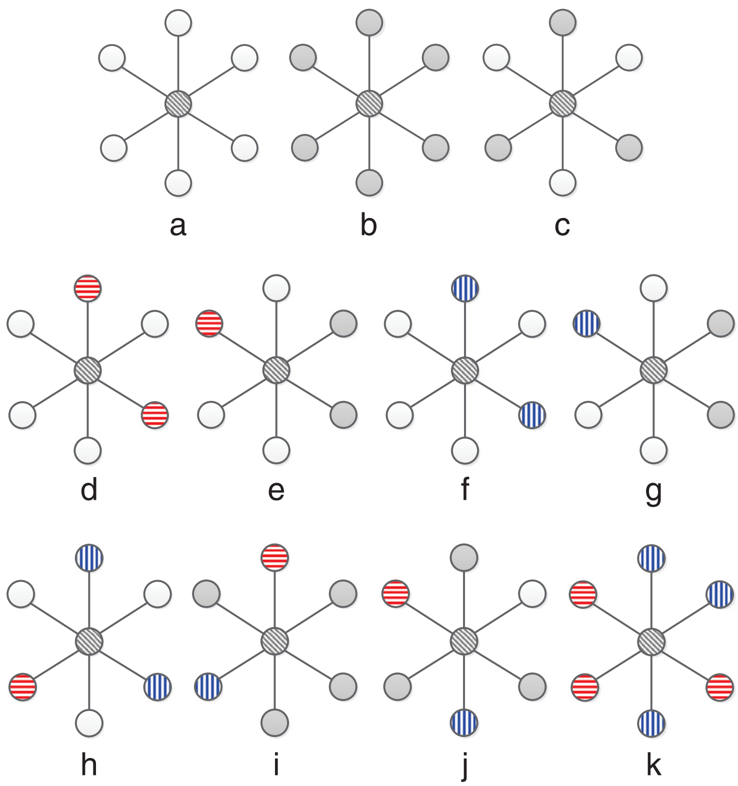


Fig. 2. Illustrations of the diffusion process. The node filled with backslash is the node concerned, say node *i*. The node with red horizontal line is infected by *A*. The node with blue vertical line is infected by *B*. Gray node is immune to the infection. White node is uninfected. In (a)–(c), node *i* stays uninfected in the next step. (d)–(e), node *i* will be infected by competitor *A*. (f)–(g), node *i* will be infected by competitor *B*. (h)–(k), node *i* will be immune to the infection. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Then, the diffusion process unfolds as follows: for any uninfected node i at time t,

- If the neighbors of node i are either uninfected or in gray state, then i will still be uninfected at time step $t+1$ (see, e.g., Fig. 2(a)–(c));
- If node i is neighboring to infected nodes of only one type, i.e., A or B, then it will be infected by that type at $t+1$ (see, e.g., Fig. 2(d)–(g));
- If node i has both type-A neighbors and type-B neighbors, then it will become gray at $t+1$ (see, e.g., Fig. 2(h)–(k)).

Note that the above diffusion process is progressive, i.e., the state of any node does not change once infected or becoming gray.

Remark 1. The work of Refs. [17–21] has studied this competitive diffusion process in the framework of game theory. However, the existing theoretical results do not tell us explicitly how to choose seeds to start diffusion.

Table 1
Basic properties of the connected component of the six real-world networks.

	n	m		n	m
netsci [32]	379	914	infectious [33]	410	2 765
email [34]	1133	5 451	hamsterster-full [35]	2 000	16 098
facebook [36]	4039	88 234	PGP [37]	10 680	24 316

Remark 2. We can compare the competitive diffusion process with the well-studied influence maximization problem which can be viewed as a special competitive diffusion process with only one competitor (for ease of comparison, we continue to call it *competitor* even though there is not competition in the influence maximization problem) [4,5]. Both of the problems focus on the competitor's strategies of choosing seeds such that it has the maximal utility at the end of the diffusion process. Influence maximization problem can be formulated as an optimization problem rather than a game and, generally, finding the optimal solution of influence maximization problem is NP-hard. And when the object function of the optimization problem satisfies certain conditions (e.g. sub-modularity and monotonicity), a properly designed greedy algorithm can be used to search for an acceptable approximate solution. But for large-scale networks, the greedy algorithm has large computational burden.

An alternative research direction is to design heuristic seed-selection algorithms, which is mainly based on standard measures of networks and new network centralities created for specific models [24–26]. Inspired by the research in influence maximization problem, in the following section we design some heuristic seed-selection strategies based on network centrality measures and see which one is suitable for indicating the seeds in our competitive diffusion model.

3. Heuristic seeds-selection strategies and simulations

3.1. Several heuristic seeds-selection strategies

In the literature, various measures of networks have been proposed, among which Betweenness Centrality, Closeness Centrality, Degree Centrality, Eigenvector Centrality and K-shell Centrality are commonly used. Here we say a competitor choosing nodes according to Betweenness Centrality means that it chooses nodes with the largest Betweenness Centrality values. For ease of reference, here we briefly introduce these five measures. More detailed information can be found in the network science literature (see, e.g., Refs. [22,23] and the references therein).

. Betweenness Centrality (BC): Let g_{jk} be the number of shortest paths between node j and node k, and $g_{jk,i}$ the number of shortest paths between node j and node k that contain node i. The BC of node i is denoted by $\mathrm{BC}_i = \sum_{j\neq k} \frac{g_{jk,i}}{g_{jk}}$.
. Closeness Centrality (CC): Let d_{ij} be the shortest distance between node i and node j. The CC of node i is denoted by $\mathrm{CC}_i = \sum_{j\neq i} \frac{1}{d_{ij}}$.
. Degree Centrality (DC): The DC of node i (also known as its degree) is denoted by the cardinality of its set of neighbors, i.e., $\mathrm{DC}_i = |N_i|$.
. Eigenvector Centrality (EC): The EC is a natural extension of DC by considering both the number and the importance of those nodes that a node could directly influence. The EC of a network is equal to the eigenvector corresponding to the largest eigenvalue of the coupling matrix.
. K-shell Centrality (KS): Nodes are assigned to different shells according to their remaining degrees, which is obtained by successive pruning of nodes with degree smaller than the current k-shell value. We start by removing all nodes with degree less than or equal to 1, until that all nodes left are with degree greater than 1. The removed nodes, along with the corresponding links, form an k-shell with index KS = 1. In a similar fashion, we iteratively remove the next k-shell. As a result, each node is associated with one KS index.

It is worth noting that, all these network measures can reflect the centrality of a node only in certain proper applications. For instance, BC is believed useful in determining who has more 'interpersonal influence' on others [27]; a node with large CC is usually located in the geodesic center of the network, and therefore, has a good vision of information flow [28]; a node with large DC plays a vital role in maintaining the network's connectivity [29]; EC is a suitable measure of social influence in the opinion formation process [30]; KS can predict the efficient spreaders in SIR epidemic models [31]. In the literature, these network measures have been used to indicate influential nodes in various diffusion models, but yet no one applies them in competitive diffusion processes. Our work is a first attempt in this research direction.

3.2. Test on real-world social networks

We test the proposed heuristic seeds-selection strategies through simulations on six undirected real-world social networks. The numbers of nodes (n) and edges (m) of the largest connected component of the given networks are listed in Table 1. More detailed information about the networks can be found in the corresponding references.

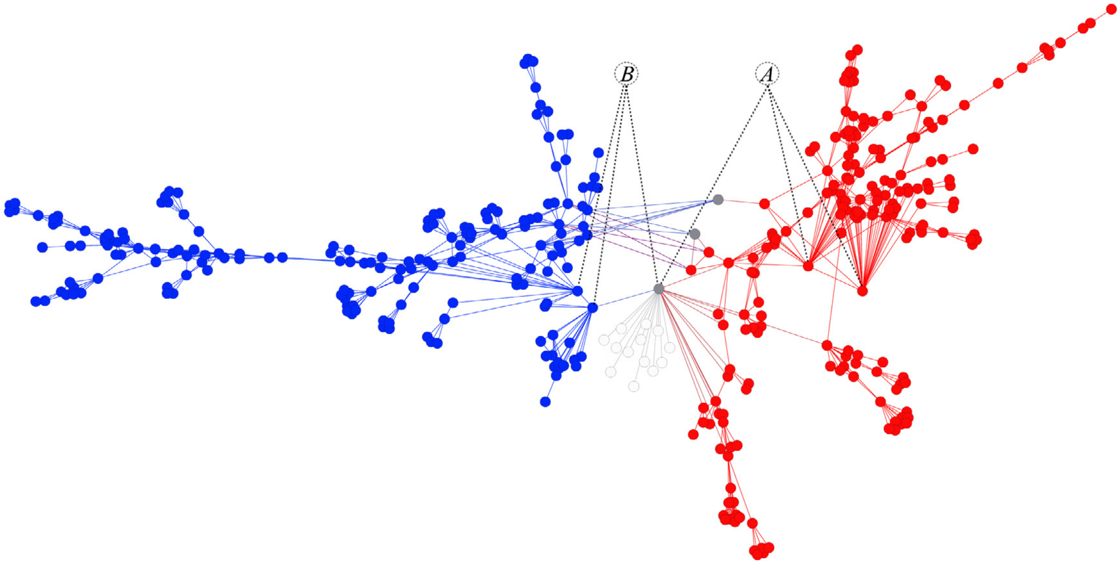
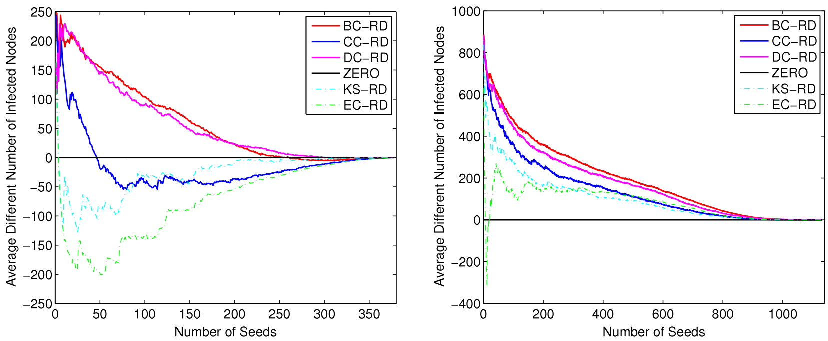


Fig. 3. An example of diffusion result on network *netsci*. The virtual nodes in dotted circles represent competitor A and B, respectively. Nodes filled with red (blue) color mean that they are infected by competitor A (B), and nodes filled with gray (white) color represent immune (uninfected) nodes. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



(a) netsci.

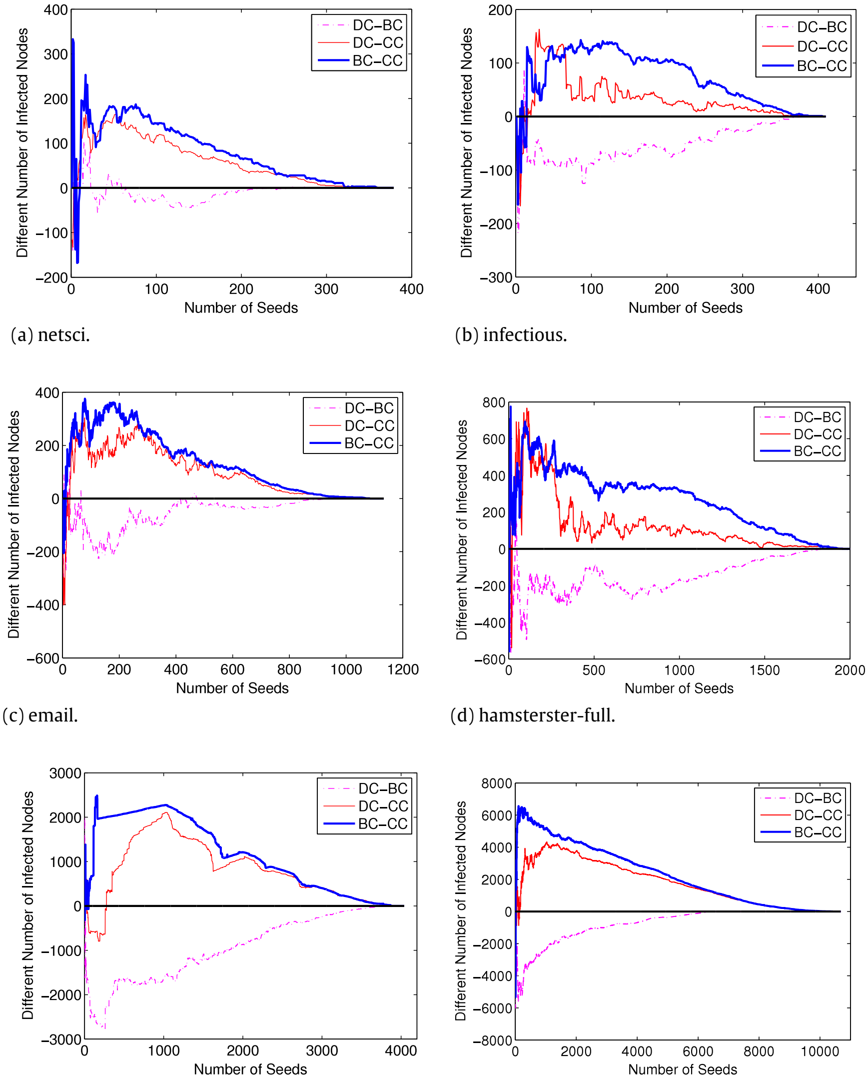
(b) email.

Fig. 4. Comparisons of centrality-based strategies with random-selection strategy.

The work of Refs. [17–20] focuses on the case of selecting one single seed for each competitor because of the complexity of analysis. In our simulations, we vary the number of seeds from 1 to the maximal value (i.e., the number of all nodes in the network). We provide an example of diffusion result on network *netsci* (see Fig. 3). In this example, we choose 3 nodes with the largest DC values as the seeds for competitor A and 3 nodes with the largest CC values as the seeds for competitor B. When the diffusion process terminates, 188 (175) nodes are infected by competitor A (B), 3 nodes become immune, and 13 nodes stay uninfected. The competition result can be shown by the difference of the numbers of infectors of two competitors, denoted by 'DC–CC'. Hence, in this example competitor A wins.

We first compare these five centrality-based strategies with random-selection strategy. Take BC-based strategy versus random-selection strategy for example. For any given number of seeds m, let competitor A choose m seeds according to BC-based strategy, and competitor B randomly choose m seeds. Then we get the difference of the numbers of the infected nodes of two competitors as the competition result of this trial. We repeat the trial for 300 times, and regard the average as the competition result of these two strategies, denoted by 'BC-RD'. Since we need to repeat the trial for random-selection strategy, we only test it on one small network *netsci* and one mesoscale network *email*. As shown in Fig. 4, when compared with random-selection strategy, the priority of these five centrality-based strategies keeps the same in the two tested networks, which is as follows: BC $>$ DC $>$ CC $>$ KS $>$ EC. And BC-based strategy and DC-based strategy are always better than random-selection strategy on both networks (above the horizontal zero line). The performance of KS-based strategy and EC-based strategy are even worse than random-selection strategy (under the horizontal zero line) on the network *netsci*.

Next, we show the comparisons among centrality-based strategies. It is worth noting that, if we have L strategies, the number of comparisons is $\binom{L}{2}$, which grows fast as the number of strategies increases. To keep it simple in this article, and according to their performances in comparing with random-selection strategy, we choose the top three centrality-based



(a) netsci. (b) infectious. (c) email. (d) hamsterster-full. (e) facebook. (f) PGP.

Fig. 5. Comparison of seed-selection strategies on six real social networks.

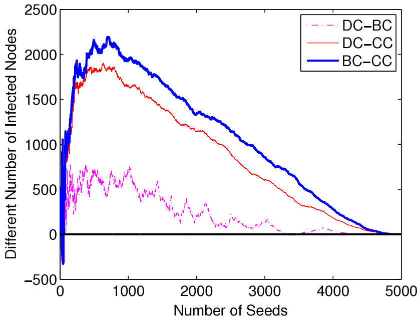
strategies, i.e., BC-based strategy, DC-based strategy and CC-based strategy. The results of comparison among these three centrality-based strategies on different real networks are shown in Fig. 5, in which if a curve goes above the horizontal zero line, we conclude that the first seed-selection strategy is better than the second one when they compete in the proposed diffusion process.

From the figure we have the following observations:

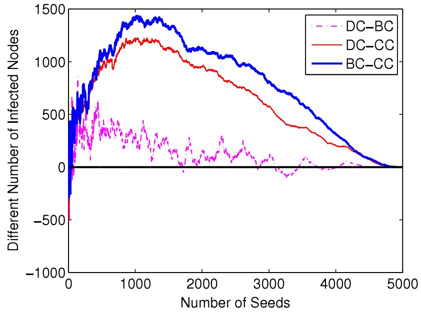
Observation 1. DC-based strategy and BC-based strategy are better than CC-based strategy, since the BC–CC curve and the DC–CC curve are generally above the zero line.

Observation 2. BC-based strategy is a better choice than DC-based strategy for a competitor, if his opponent uses CC-based strategy, since the BC–CC curve is generally higher than the DC–CC curve.

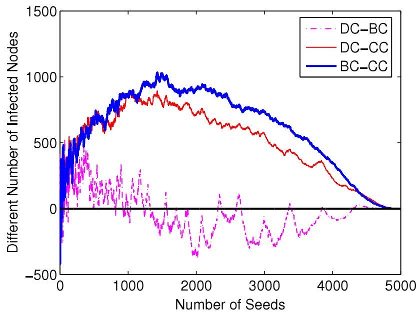
Note that the poor performance of CC-based strategy is counter-intuitive to some extent. In fact, a node with larger CC has shorter distance to other nodes in a statistic sense, and therefore, it might spread its influence to other nodes in a smaller number of steps. However, when more seeds are chosen to spread the information, seeds chosen according to CC are all in the center of the network and overlap one another in the information spreading process, which results in a relatively small infected region. In contrast, the nodes of large BC and DC values might be far away from one another, and therefore, infect more nodes scattered over the network. This might be the underlying reason for Observation 1. From the definition of BC, we can find that nodes with larger BC value are those who are more likely to lie on shortest paths connecting two nodes and



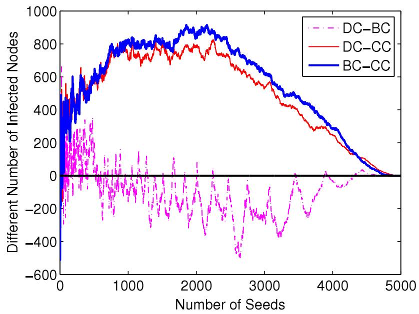
(a) Scale-Free $\gamma = 2.5$.



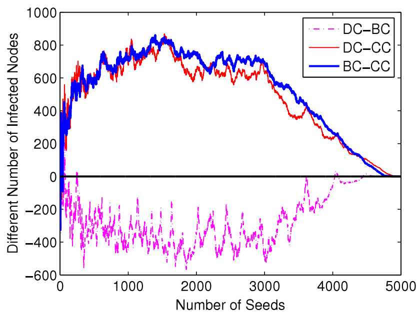
(b) Scale-Free $\gamma = 3$.



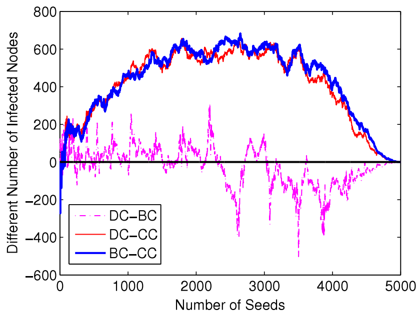
(c) Scale-Free $\gamma = 4$.



(d) Scale-Free $\gamma = 5$.



(e) Scale-Free $\gamma = 10$.



(f) Erdos–Renyi.

Fig. 6. Comparison of seed-selection strategies on six artificial networks.

also to bridge different communities. Either property allows the nodes to spread information fast to other nodes far away over the network. The nodes of large DC values might have a large number of initial infections, but lose the competition with large BC nodes in infecting distant nodes. This explains Observation 2.

3.3. *Test on artificial networks*

Simulations on the above networks have shown that the observed trends are common. There still exist delicate differences between the carves in these networks. For example, though BC–CC curve is generally higher than the DC–CC curve in all these networks, in some networks these two carves are close to each other, but in other networks they are far apart. To compare the strategies on a more controllable test platform, we resort to Barabasi–Albert Scale-Free networks in which the power-law exponent is adjustable. We investigate the influence of network heterogeneity on the comparison of the proposed strategies. We construct 5 Scale-Free networks with power-law exponent $\gamma = 2.5, 3, 4, 5, 10$, and one Erdos–Renyi network. We fix the size of these six networks to $n = 5000$ and the number of edges to $m = 49\,955$. The heterogeneity is the only difference of these networks.

As shown in Fig. 6, the above two trends still exist in our artificial networks. Moreover, curves of BC–CC and DC–CC are both unimodal in all these networks. As the network's heterogeneity decreases, the peak value shifts to the right half of the figure and the curves of BC–CC and DC–CC become closer and, as an extreme situation, in the Erdos–Renyi network BC-based and DC-based strategies almost have the same superiority when competing with CC-based strategy. This phenomenon means that, as the network's heterogeneity decreases the superiority of BC-based strategy over DC-based strategy decreases, when competing with the CC-based strategy. This is because when the network is more heterogeneous, the larger degree agents are more likely to distribute in the center of the network, for which an extreme example is the star network. As the network becomes more and more homogeneous, the large degree nodes can emerge in anywhere of the network and hence become more scattered, which leads to the result that the superiority of BC-based strategy over DC-based strategy decreases, when competing with CC-based strategy.

BC-based strategy being better than DC-based strategy when competing with CC-based strategy does not imply that BC-based strategy will defeat DC-based strategy, when they are competed with each other. In fact, when two groups of scattered seeds competing with each other, we can see from Fig. 6 that the DC–BC curve frequently goes across the zero line under different power-law exponents and numbers of seeds. We could not observe a common trend like that mentioned in Observations 1 and 2, but could only conclude the following:

Observation 3. The priority of DC-based strategy when competing with BC-based strategy depends on the specific network structure and the number of seeds.

4. Concluding remarks

In this article, we have revisited a competitive diffusion model, which is mainly studied from the game theory perspective in the literature. Here, we consider this problem from a new perspective. To obtain a practical strategy of choosing seeds to start the diffusion process, we have proposed five heuristic seed-selection strategies based on commonly used network centralities and compared their performances on real-world and man-made networks. The results show that DC-based strategy and BC-based strategy are both better than CC-based strategy. Moreover, if one competitor's opponent adopts CC-based strategy, BC-based strategy is better than DC-based strategy for this competitor. And the superiority of BC-based strategy over DC-based strategy decreases, when competing with CC-based strategy, as the heterogeneity of the network decreases.

In future research, we plan to extend the work at least in three directions: firstly, we will test the proposed heuristic seed-selection strategies on more networks to further examine their performances; secondly, we will design more strategies based on other network measures; finally, we will investigate some other competitive diffusion processes with more comparisons.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant Nos. 61374176, 61473189, and 61503207, the Natural Science Foundation of Shandong Province (ZR2015PF003), the Project Funded by China Postdoctoral Science Foundation (2015M571996), the Qingdao Postdoctoral Application Research Project (2015123) and the Science Fund for Creative Research Groups of the National Natural Science Foundation of China (No. 61221003).

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