HW1

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Question 1

(a)

$$\int_0^\infty x(1+x^2)^{-2}dx$$

Set $u = (1+x^2)$, then we can get: $\int_0^\infty x(1+x^2)^{-2}dx = \frac{1}{2}\int_1^\infty u^{-2}du = \frac{1}{2}$
In order to transform $\int_0^\infty$ to $\int_0^1$, set $y = \frac{1}{1+x}$.
So $dx = -\frac{1}{y^2}dy$. Then we have $\int_1^0 -\frac{1}{y^2}\frac{(1/y-1)}{(1+(1/y-1)^2)^2}dy$.

```
# set seed
set.seed(1029)

# check theoretical value of the given integral using R:
integral_a = function(x) {
  x*(1+x^{2})^{-2}
}
integrate(integral_a, lower = 0, upper = Inf)$value
```

```
## [1] 0.5
```

```
# simulate using uniform distribution
s = runif(100000)
transform = (1/s^2)*(1/s-1)/(1+(1/s-1)^2)^2
mean(transform)
```

```
## [1] 0.4983382
```

(b)

$\int_{-\infty}^\infty e^{x^2}dx$
Because we already know the probability density function of standard Normal distribution: $f(x) = \frac{-1}{}$