1) Toxicity:

$$mean = ANETS = \xi$$
 Let  $\xi$  be the ANETS. 
$$variance = \xi \cdot \frac{(1 - \xi)}{N} = \sigma^2$$

Under the setting of EWOUC-NETS, we have

$$\xi | x_i = F(\beta_0 + \beta_1 x_i) = \frac{\exp(\beta_0 + \beta_1 x_i)}{1 + \exp(\beta_0 + \beta_1 x_i)}$$
$$\xi | x_i = inverse \ logit \ (\beta_0 + \beta_1 x_i)$$

Re-parameterization:

$$\beta_0 = \frac{X_{min}logit(\tilde{\theta}) - \gamma_T logit(\rho_0)}{X_{min} - \gamma_T}$$
 
$$\beta_1 = \frac{logit(\rho_0) - logit(\tilde{\theta})}{X_{min} - \gamma_T}$$
 
$$logit(\xi|x_i) = \frac{(\gamma_T - x_i) \cdot logit(\rho_0) - (x_i - x_{min}) \cdot logit(\tilde{\theta})}{\gamma_T - x_{min}}$$
 
$$where \ \gamma_T = maximum \ tolerated \ dose$$
 
$$\rho_0 = NETS \ at \ the \ starting \ dose \ x_{min}$$
 
$$\tilde{\theta} = target \ normalized \ equivalent \ toxicity \ score$$

For each dose level, we have one  $\xi | x_i$ .

Marginal density function for toxicity:

$$\pi_{T} = \frac{1}{\sqrt{\xi |x_{i}(1 - \xi |x_{i})/N}} \frac{\Phi(\frac{x_{i} - \xi |x_{i}}{\sqrt{\xi |x_{i}(1 - \xi |x_{i})}})}{\Phi(\frac{1 - \xi |x_{i}}{\sqrt{\xi |x_{i}(1 - \xi |x_{i})/N}})} \Phi(\frac{-\xi |x_{i}}{\sqrt{\xi |x_{i}(1 - \xi |x_{i})/N}})$$

2) Efficacy

$$\pi_E = f(y_E|x) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left\{-\frac{(y_E - \mu_i)^2}{2\sigma_i^2}\right\}$$

Where 
$$\mu_i = 1 - \frac{1}{1 + (x/\beta_2)^{\beta_3}}$$
 and  $\sigma_i^2 = \sigma^2 x^{\lambda}$ 

Comment: here the  $\mu_i$  has been normalized and range from 0 to 1.

3) Joint distribution of  $(S, Y_E)$ :

$$f(y_E, S) = f(S)f(y_E|S)$$

Given S, we assumed that the distribution of  $y_E$  is normal.

$$f(y_E|S) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left[-\frac{\{y_E - \mu_i - \tau(S - \xi|x_i)\}^2}{2\sigma_i^2}\right]$$

Where  $\tau$  is the parameter for the regression of  $y_E$  on S. Large absolute values of  $\tau$  indicate a strong correlation between the two outcomes. When  $\tau = 0$ , the two outcomes are independent given the dose level of agent A in the model. The correlation based on this model is

$$\rho(y_E, S|x) = \frac{\tau}{\sqrt{\tau^2 + \frac{{\sigma_i}^2}{\xi(1-\xi)}}}$$

- 1) Selling point: continuous toxicity endpoint as NETS ranging from 0 to 1, instead of 0 and 1 (binary).
- 2) We used EWOUC (over- and underdose control)
- 3) Efficacy is truncated normal distributed in simulation (more meaning).
- 4) Reduced the parameters and we standardized the efficacy within 0 and 1.