Linear Model Project

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Question 1

In this problem, Let $\mathbf{y}$ denote stiffness (lb/in^2) and $\mathbf{X}$ denote the design matrix. Since here we have 30 observations, the dimension of $\mathbf{y}$ is 30×1. Then we have:

$$\mathbf{y} = \begin{pmatrix} 2622 \\ 22148 \\ 26751 \\ 18036 \\ 96305 \\ \vdots \\ 49499 \\ 25312 \end{pmatrix}_{30\times 1}$$

\

$$\mathbf{X} = \begin{pmatrix} 1 & 15.0 \\ 1 & 14.5 \\ 1 & 14.8 \\ 1 & 13.6 \\ 1 & 25.6 \\ \vdots & \\ 1 & 16.7 \\ 1 & 15.4 \end{pmatrix}_{30\times 2}$$

. Before running into questions, we firstly want to show that

$$\mathbf{X}^{'}\mathbf{X} = \begin{bmatrix} 30 & 464.1 \\ 464.1 & 8166.29 \end{bmatrix}$$

,

$$(\mathbf{X}^{'}\mathbf{X})^{-1} = \begin{bmatrix} 0.2758892 & -0.0156791 \\ -0.0156791 & 0.001013517 \end{bmatrix}$$

,

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# Import data
x = c(rep(1, 30), 15.0, 14.5, 14.8, 13.6, 25.6, 23.4, 24.4, 23.3, 19.5, 21.2, 22.8, 21.7, 19.8, 21.3, 9
      8.4, 9.8, 11.0, 8.3, 9.9, 8.6, 6.4, 7.0, 8.2, 17.4, 15.0, 15.2, 16.4, 16.7, 15.4)
y = c(2622, 22148, 26751, 18036, 96305, 104170, 72594, 49512, 32207, 48218, 70453, 47661, 38138, 54045,
      17502, 14007, 19443, 7573, 14191, 9714, 8076, 5304, 10728, 43243, 25319, 28028, 41792, 49499, 253

x.matrix = matrix(x, ncol = 2, nrow = 30, byrow = F)
t(x.matrix) %*% x.matrix
```

```
##       [,1]    [,2]
## [1,]  30.0  464.10
## [2,] 464.1 8166.29
```