Linear Model Project

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Question 1

In this problem, Let $\mathbf{y}$ denote stiffness (lb/in^2) and $\mathbf{X}$ denote the design matrix. We want to investigate the relationship between stiffness and density(lb/ft^3). Since here we have 30 observations, and we only have one predictor and one outcome, the dimension of $\mathbf{y}$ is 30×1 and the dimension of $\mathbf{X}$ is 30×2. Then we have:

$$\mathbf{y} = \begin{pmatrix} 2622 \\ 22148 \\ 26751 \\ 18036 \\ 96305 \\ \vdots \\ 49499 \\ 25312 \end{pmatrix}_{30\times 1}$$

$$\mathbf{X} = \begin{pmatrix} 1 & 15.0 \\ 1 & 14.5 \\ 1 & 14.8 \\ 1 & 13.6 \\ 1 & 25.6 \\ \vdots & \vdots \\ 1 & 16.7 \\ 1 & 15.4 \end{pmatrix}_{30\times 2}$$

Here the first column in the design matrix allows estimation of the y-intercept and the second column contains the values of our variable (density).
Before running into questions, we firstly want to show

$$result1 : \mathbf{X}^{'}\mathbf{X} = \begin{bmatrix} 30 & 464.1 \\ 464.1 & 8166.29 \end{bmatrix}$$

$$result2 : (\mathbf{X}^{'}\mathbf{X})^{-1} = \begin{bmatrix} 0.2758892 & -0.0156791 \\ -0.0156791 & 0.001013517 \end{bmatrix}$$

$$result3 : \mathbf{X}^{'}\mathbf{y} = \begin{bmatrix} 1017405 \\ 19589339 \end{bmatrix}$$

, and

$$result4 : s^2 = 165242295.59$$

Below is my code to show these four results.

```r
# Import data
x = c(rep(1, 30), 15.0, 14.5, 14.8, 13.6, 25.6, 23.4, 24.4, 23.3, 19.5, 21.2, 22.8, 21.7, 19.8, 21.3, 9
      8.4, 9.8, 11.0, 8.3, 9.9, 8.6, 6.4, 7.0, 8.2, 17.4, 15.0, 15.2, 16.4, 16.7, 15.4)
y = c(2622, 22148, 26751, 18036, 96305, 104170, 72594, 49512, 32207, 48218, 70453, 47661, 38138, 54045,
      17502, 14007, 19443, 7573, 14191, 9714, 8076, 5304, 10728, 43243, 25319, 28028, 41792, 49499, 253

# Create design matrix
x.matrix = matrix(x, ncol = 2, nrow = 30, byrow = F)

# Result 1
result1 = t(x.matrix) %*% x.matrix
result1
```

```
##       [,1]    [,2]
## [1,]  30.0  464.10
## [2,] 464.1 8166.29
```

```r
# Result 2
solve(result1)
```

```
##             [,1]         [,2]
## [1,]  0.27588920 -0.015679112
## [2,] -0.01567911  0.001013517
```

```r
# Result 3
t(x.matrix) %*% y
```

```
##          [,1]
## [1,]  1017405
## [2,] 19589339
```

```r
# Result 4
density = c(15.0, 14.5, 14.8, 13.6, 25.6, 23.4, 24.4, 23.3, 19.5, 21.2, 22.8, 21.7, 19.8, 21.3, 9.5,
            8.4, 9.8, 11.0, 8.3, 9.9, 8.6, 6.4, 7.0, 8.2, 17.4, 15.0, 15.2, 16.4, 16.7, 15.4)
s2 = var(x)
```