

# Computational Statistics Midterm: the simulation of mixed-type data using PoisBinOrd package

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## 1 Methodology

The real-world data is sometimes hard to obtain, due in part to the restricted budget and time. Even in the case that the real data is available, the sample size of the data can be small, causing a reduction in the power, which, in turn, limits the statistical inference. To resolve such issues, the simulation studies can be performed, in which the real data can be mimicked satisfactorily. The format of the simulated data follows mixed data that is commonly seen in real world examples; mixed data usually consists of various types of variables, such as count, binary, ordinal, or continuous variables.

There are several advantages of using the simulated data. A particular one is that it allows for large sample size, aiding researcher to construct models for their research problems. After model selection, the true population parameters and their accuracy and the precision can be well estimated. In these circumstances, it is expected to have a better evaluation of validity, reliability, generalization, and plausibility of research.

The package **PoisBinOrd** is used to generate the mixed data that contains the count, binary, and ordinal variables simultaneously. Using the RNG algorithm proposed by Demirtas (2017), together with the marginal distribution specifications and the corresponding association structure, the stochastic simulated dataset is produced. The following procedure describes the algorithm for mixed data generation using the package **PoisBinOrd**.

*Step 1. Define the sample size and the number of variables*

The count, binary, and ordinal variables are denoted by  $P$ ,  $B$ , and  $O$ . The sample size of the mixed data is denoted by  $n$ . The number of binary variable, and the number of ordinal variable are  $n.P$ ,  $n.B$ , and  $n.O$ . The values of  $n$ ,  $n.P$ ,  $n.B$ , and  $n.O$  need to be given in the function `gen.PoisBinOrd`. With those being calculated, the number of the variables in the mixed data can be determined, which is  $N = n.P + n.B + n.O$ .

*Step 2. The specification of the marginal distributions*

In the function `gen.PoisBinOrd`, the variable for the count data is assumed to follow a Poisson distribution with its rate parameter  $\lambda$ . The variable for the binary data is assumed to follow the binomial distribution with its proportion parameter  $p$ . For the ordinal data, the cumulative distribution is described by its thresholds  $t_s, s = 1, 2, 3, \dots$

*Step 3. The specification of feasible association structure for the mixed data*

The Pearson correlation matrix,  $\Sigma$ , with the dimension  $N \times N$ , is used to describe the association structure for the mixed data. Prior to its use, the feasibility of the correlation structure needs to be verified. In particular,  $\Sigma$  has to satisfy the criteria of: 1) being a symmetric positive definite matrix, 2) having all the diagonal entries being one, and 3) having all elements in the correlation matrix within the correlation bounds. To check the last criterion, the sorting algorithm proposed by Demirtas and Hedeker (2011) is used.

*Step. 4. Generate the mixed data*

Using the function `overall.corr.mat` developed by Demirtas (2017), the correlation matrix  $\Sigma$  can be transformed into a positive definite, intermediate correlation matrix  $\Sigma^*$ . If such an intermediate matrix is not available, a nearest positive matrix will be used. After finding  $\Sigma^*$ , the count variables are produced via the

inverse transform sampling. For the binary and ordinal variables, they are obtained by dichotomization and ordinalization in line with the thresholds of the underlying distributions. Recall the underlying assumption of the mixed data generation algorithm, which assumes that all the variables are produced from a multivariate normal distribution (MVN) with mean vector of 0 and the correlation structure  $\Sigma^*$ . With  $\Sigma^*$  being known, the latent MVN can then be employed to generate the mixed data. This completes the entire process.

## 2 Simulation setting

A simulation study is performed by the package **PoisBinOrd** considering 16 scenarios (see Tables 1 and 2). Three aspects are considered to define a scenario. The first is the sample size. Following the common practice, small sample size is given as 100 and large sample size is 10,000. The second is the correlation structure. Two different correlation structure are included to confirm the stability of this algorithm. The first correlation structure is called random correlation. Each element in the correlation matrix is random sampling uniformly from its correlation boundary. The second correlation structure is called weak correlation. Each element in the correlation matrix is randomly sampling from uniform distribution with range (-0.20, +0.20). The last one criteria is the effect of a rare event. To do so, a total of four parameter settings are considered, including large Poisson rate/balanced binary and ordinal data (scenarios 1-4), large Poisson rate/imbalanced binary and ordinal data (scenarios 5-8), small Poisson rate/balanced binary and ordinal data (scenarios 9-12), small Poisson rate/imbalanced binary and ordinal data (scenarios 13-16). Note, small Poisson rate is defined as the one having the rate parameter below one. Large Poisson rate is defined as the one having the rate parameter large than 1.

Table 1: The 16 scenarios considered in the simulations

scenario			1-4	5-8	9-12	13-16
Poisson	$P_1$	$\lambda_1$	2.00	2.00	0.20	0.20
	$P_2$	$\lambda_2$	7.00	7.00	0.70	0.70
Binary	$B_1$	$p_1$	0.45	0.80	0.45	0.80
	$B_2$	$p_2$	0.50	0.90	0.50	0.90
Ordianl	$O_1$	$t_1$	0.25	0.65	0.25	0.65
		$t_2$	0.50	0.80	0.50	0.80
		$t_3$	0.75	0.90	0.75	0.90
	$O_2$	$t_1$	0.20	0.50	0.20	0.50
		$t_2$	0.45	0.80	0.45	0.80
		$t_3$	0.75	0.90	0.75	0.90

Table 2: The correlation structures of the 16 scenarios

scenario	1-2	5-6	9-10	13-14	3-4, 7-8, 11-12, 15-16
$\rho P_1 P_2$	0.1473	0.4397	0.3712	0.4811	0.0172
$\rho P_1 B_1$	-0.1217	-0.0824	-0.2252	-0.3434	-0.0619
$\rho P_1 B_2$	-0.3644	-0.2739	0.4275	-0.1249	0.0460
$\rho P_1 O_1$	0.0965	0.5459	0.3816	0.5349	-0.0962
$\rho P_1 O_2$	-0.1741	-0.0994	0.0918	-0.1313	-0.1050
$\rho P_2 B_1$	0.5454	-0.3534	-0.3767	-0.4763	-0.0405
$\rho P_2 B_2$	-0.5916	0.0635	-0.0998	0.0496	-0.0672
$\rho P_2 O_1$	0.1183	0.1516	0.0924	0.2603	0.1549
$\rho P_2 O_2$	0.4566	0.0831	0.5705	0.1111	0.1468
$\rho B_1 B_2$	-0.0132	0.3847	-0.3303	0.3770	0.0897
$\rho B_1 O_1$	0.2617	-0.4778	-0.3954	-0.3716	0.0209
$\rho B_1 O_2$	0.3623	0.2440	-0.5631	0.2440	-0.1080
$\rho B_2 O_1$	-0.0074	-0.0941	-0.1481	-0.0941	0.0681
$\rho B_2 O_2$	-0.0234	0.2622	0.0354	0.2622	0.1476
$\rho O_1 O_2$	-0.1309	0.2451	0.3750	0.2451	-0.0526

### 3 Results and Discussions

In each scenario, 1,000 simulated datasets are produced to evaluate the performance of the R package **PoisBinOrd**, with particular attention to the accuracy and the precision of the simulation replications. Table 3-10 reveal the point estimate, the accuracy, and the precision of the parameters. The true values (TV) mean the true parameters. The average estimates (AE) and the standard deviations of the estimates (SD) were calculated across the 10000 replications.

Let the true parameter be  $\theta$ , and the estimated value be  $\hat{\theta}$ . For checking the accuracy, the relative bias (RB), defined as  $E[\frac{\hat{\theta}-\theta}{\theta}] \times 100\%$ , and the standardized bias (SB), defined as  $E\left[\frac{|\hat{\theta}-\theta|}{SD(\hat{\theta})}\right] \times 100\%$ , are used. Furthermore, to evaluate the hybrid measure of accuracy and precision, two more metrics, including the root mean square error (RMSE) of  $\theta$ , defined as  $\sqrt{E[\hat{\theta}-\theta]^2}$ , and the coverage rate (CR) (which is the percentage of times that  $\theta$  is contained within a 95% confidence interval) are employed. The approximate confidence interval is calculated from the Poisson's square root transformation to reduce the Poisson overdispersion issue. For the ordinal variable, the standard deviation can be calculated from the multinomial distribution. For the sample Pearson correlation, the Fisher z-transformation is performed to convert the sample correlation's skewed distribution into approximately normal.

According to the simulated results, the large sample size can reduce SD, RB, and RMSE. The SD was reduced around by ten times because of the effect of the sample size. The correlation structure seriously affects the parameter estimation results. The scenario with a weak correlation structure all produces stable performances. However, the scenario with a random correlation structure is not relatively stable. The setting of the small Poisson rate does not change the results, but the imbalanced binary/ordinal did produce less accurate results.

Table 3: Scenario 01/02 - random correlation, large Poisson rate and balanced binary/ordinal distribution

Sample Size	Variable	Parameter	TV	AE	SD	RB	SB	RMSE	CR
100	$P_1$	$\lambda_1$	2.0000	2.0090	0.1418	0.4475	6.3102	0.1420	0.9490
	$P_2$	$\lambda_2$	7.0000	7.0024	0.2753	0.0340	0.8645	0.2752	0.9370
	$B_1$	$p_1$	0.4500	0.4488	0.0493	-0.2622	2.3944	0.0493	0.9530
	$B_2$	$p_2$	0.5000	0.4994	0.0518	-0.1100	1.0628	0.0517	0.9360
	$O_1$	$t_1$	0.2500	0.2504	0.0427	0.1760	1.0300	0.0427	0.9440
		$t_2$	0.5000	0.4979	0.0493	-0.4160	4.2203	0.0493	0.9510
		$t_3$	0.7500	0.7488	0.0419	-0.1573	2.8142	0.0419	0.9230
	$O_2$	$t_1$	0.2000	0.2008	0.0407	0.4250	2.0904	0.0407	0.9450
		$t_2$	0.4500	0.4547	0.0482	1.0511	9.8086	0.0484	0.9420
		$t_3$	0.7500	0.7509	0.0433	0.1147	1.9880	0.0432	0.9370
	Correlation	$\rho P_1 P_2$	0.1473	0.1455	0.1002	-1.2250	1.7999	0.1002	0.9410
		$\rho P_1 B_1$	-0.1217	-0.1238	0.0970	1.6859	2.1165	0.0969	0.9470
		$\rho P_1 B_2$	-0.3644	-0.3629	0.0802	-0.4020	1.8264	0.0802	0.9640
		$\rho P_1 O_1$	0.0965	0.0946	0.0968	-1.9369	1.9318	0.0967	0.9520
		$\rho P_1 O_2$	-0.1741	-0.1762	0.0960	1.2066	2.1865	0.0960	0.9500
		$\rho P_2 B_1$	0.5454	0.5421	0.0669	-0.6142	5.0096	0.0669	0.9560
		$\rho P_2 B_2$	-0.5916	-0.5786	0.0582	-2.2035	22.4183	0.0596	0.9770
		$\rho P_2 O_1$	0.1183	0.1161	0.0970	-1.9057	2.3253	0.0970	0.9540
		$\rho P_2 O_2$	0.4566	0.4512	0.0779	-1.1810	6.9203	0.0781	0.9500
		$\rho B_1 B_2$	-0.0132	-0.0259	0.1038	96.7667	12.2864	0.1045	0.9300
		$\rho B_1 O_1$	0.2617	0.2569	0.0888	-1.8404	5.4259	0.0888	0.9620
		$\rho B_1 O_2$	0.3623	0.3637	0.0857	0.3746	1.5835	0.0857	0.9440
		$\rho B_2 O_1$	-0.0074	-0.0073	0.1004	-0.2350	0.0172	0.1003	0.9410
		$\rho B_2 O_2$	-0.0234	-0.0259	0.1013	10.6828	2.4634	0.1013	0.9400
		$\rho O_1 O_2$	-0.1309	-0.1356	0.0959	3.5531	4.8526	0.0959	0.9510
10000	$P_1$	$\lambda_1$	2.0000	1.9998	0.0141	-0.0088	1.2557	0.0141	0.9480
	$P_2$	$\lambda_2$	7.0000	6.9998	0.0259	-0.0026	0.7045	0.0259	0.9500
	$B_1$	$p_1$	0.4500	0.4502	0.0050	0.0402	3.6179	0.0050	0.9510
	$B_2$	$p_2$	0.5000	0.5002	0.0049	0.0326	3.3093	0.0049	0.9630
	$O_1$	$t_1$	0.2500	0.2501	0.0044	0.0372	2.1319	0.0044	0.9490
		$t_2$	0.5000	0.5001	0.0051	0.0172	1.7024	0.0051	0.9710
		$t_3$	0.7500	0.7498	0.0044	-0.0226	3.8345	0.0044	0.9440
	$O_2$	$t_1$	0.2000	0.1998	0.0041	-0.0872	4.2993	0.0041	0.9420
		$t_2$	0.4500	0.4500	0.0047	0.0046	0.4442	0.0047	0.9460
		$t_3$	0.7500	0.7500	0.0043	-0.0056	0.9869	0.0043	0.9550
	Correlation	$\rho P_1 P_2$	0.1473	0.1485	0.0099	0.8037	11.9065	0.0100	0.9510
		$\rho P_1 B_1$	-0.1217	-0.1226	0.0097	0.6649	8.3143	0.0098	0.9580
		$\rho P_1 B_2$	-0.3644	-0.3651	0.0079	0.1908	8.8544	0.0079	0.9660
		$\rho P_1 O_1$	0.0965	0.0968	0.0096	0.3107	3.1371	0.0096	0.9520
		$\rho P_1 O_2$	-0.1741	-0.1748	0.0100	0.4084	7.1434	0.0100	0.9490
		$\rho P_2 B_1$	0.5454	0.5380	0.0064	-1.3562	115.2157	0.0098	0.8540
		$\rho P_2 B_2$	-0.5916	-0.5780	0.0061	-2.3091	225.2070	0.0149	0.4500
		$\rho P_2 O_1$	0.1183	0.1200	0.0099	1.3836	16.5233	0.0100	0.9390
		$\rho P_2 O_2$	0.4566	0.4524	0.0077	-0.9193	54.2631	0.0088	0.9290
		$\rho B_1 B_2$	-0.0132	-0.0198	0.0103	50.2016	63.9364	0.0123	0.8880
		$\rho B_1 O_1$	0.2617	0.2613	0.0099	-0.1444	3.8302	0.0099	0.9280
		$\rho B_1 O_2$	0.3623	0.3646	0.0090	0.6379	25.6899	0.0093	0.9390
		$\rho B_2 O_1$	-0.0074	-0.0070	0.0098	-5.4015	4.0697	0.0098	0.9510
		$\rho B_2 O_2$	-0.0234	-0.0256	0.0101	9.3772	21.6464	0.0104	0.9460
		$\rho O_1 O_2$	-0.1309	-0.1302	0.0097	-0.5854	7.8624	0.0098	0.9520

Table 4: Scenario 03/04 - weak correlation, large Poisson rate and balanced binary/ordinal distribution

Sample Size	Variable	Parameter	TV	AE	SD	RB	SB	RMSE	CR
100	$P_1$	$\lambda_1$	2.0000	1.9988	0.1380	-0.0595	0.8624	0.1379	0.9580
	$P_2$	$\lambda_2$	7.0000	6.9979	0.2739	-0.0296	0.7556	0.2738	0.9260
	$B_1$	$p_1$	0.4500	0.4504	0.0505	0.0822	0.7326	0.0505	0.9330
	$B_2$	$p_2$	0.5000	0.4976	0.0525	-0.4700	4.4735	0.0526	0.9240
	$O_1$	$t_1$	0.2500	0.2504	0.0433	0.1800	1.0392	0.0433	0.9490
		$t_2$	0.5000	0.5008	0.0498	0.1700	1.7074	0.0498	0.9480
		$t_3$	0.7500	0.7517	0.0428	0.2227	3.9052	0.0428	0.9280
	$O_2$	$t_1$	0.2000	0.1996	0.0406	-0.2100	1.0344	0.0406	0.9500
		$t_2$	0.4500	0.4492	0.0491	-0.1733	1.5871	0.0491	0.9550
		$t_3$	0.7500	0.7502	0.0425	0.0227	0.3997	0.0425	0.9450
	Correlation	$\rho P_1 P_2$	0.0172	0.0177	0.1021	2.9406	0.4959	0.1021	0.9420
		$\rho P_1 B_1$	-0.0619	-0.0608	0.0990	-1.6395	1.0248	0.0989	0.9550
		$\rho P_1 B_2$	0.0460	0.0427	0.0962	-7.1895	3.4363	0.0963	0.9560
		$\rho P_1 O_1$	-0.0962	-0.0950	0.0965	-1.2477	1.2429	0.0965	0.9550
		$\rho P_1 O_2$	-0.1050	-0.1015	0.1003	-3.3185	3.4752	0.1003	0.9400
		$\rho P_2 B_1$	-0.0405	-0.0407	0.0990	0.4266	0.1748	0.0989	0.9450
		$\rho P_2 B_2$	-0.0672	-0.0669	0.1021	-0.4419	0.2911	0.1020	0.9400
		$\rho P_2 O_1$	0.1549	0.1605	0.0991	3.5906	5.6119	0.0992	0.9400
		$\rho P_2 O_2$	0.1468	0.1424	0.1004	-3.0004	4.3875	0.1005	0.9420
		$\rho B_1 B_2$	0.0897	0.0907	0.0966	1.1309	1.0498	0.0966	0.9510
		$\rho B_1 O_1$	0.0209	0.0189	0.1008	-9.3592	1.9405	0.1007	0.9430
		$\rho B_1 O_2$	-0.1080	-0.1107	0.0955	2.4979	2.8238	0.0955	0.9560
		$\rho B_2 O_1$	0.0681	0.0689	0.0985	1.2663	0.8748	0.0985	0.9480
		$\rho B_2 O_2$	0.1476	0.1500	0.0969	1.6017	2.4390	0.0969	0.9420
		$\rho O_1 O_2$	-0.0526	-0.0494	0.0970	-6.0600	3.2878	0.0970	0.9600
10000	$P_1$	$\lambda_1$	2.0000	2.0000	0.0143	-0.0017	0.2337	0.0143	0.9410
	$P_2$	$\lambda_2$	7.0000	6.9992	0.0265	-0.0118	3.1271	0.0265	0.9550
	$B_1$	$p_1$	0.4500	0.4496	0.0050	-0.0848	7.6435	0.0050	0.9460
	$B_2$	$p_2$	0.5000	0.5001	0.0050	0.0106	1.0594	0.0050	0.9490
	$O_1$	$t_1$	0.2500	0.2502	0.0043	0.0942	5.5233	0.0043	0.9570
		$t_2$	0.5000	0.5002	0.0049	0.0326	3.3310	0.0049	0.9510
		$t_3$	0.7500	0.7500	0.0045	0.0058	0.9617	0.0045	0.9540
	$O_2$	$t_1$	0.2000	0.1997	0.0039	-0.1278	6.4889	0.0039	0.9360
		$t_2$	0.4500	0.4497	0.0048	-0.0581	5.4616	0.0048	0.9480
		$t_3$	0.7500	0.7500	0.0043	-0.0061	1.0474	0.0043	0.9430
	Correlation	$\rho P_1 P_2$	0.0172	0.0171	0.0099	-0.9153	1.5889	0.0099	0.9490
		$\rho P_1 B_1$	-0.0619	-0.0621	0.0097	0.4678	2.9869	0.0097	0.9550
		$\rho P_1 B_2$	0.0460	0.0465	0.0099	1.1459	5.3300	0.0099	0.9530
		$\rho P_1 O_1$	-0.0962	-0.0961	0.0102	-0.0280	0.2642	0.0102	0.9380
		$\rho P_1 O_2$	-0.1050	-0.1049	0.0100	-0.1259	1.3154	0.0100	0.9490
		$\rho P_2 B_1$	-0.0405	-0.0409	0.0099	0.8270	3.3707	0.0099	0.9440
		$\rho P_2 B_2$	-0.0672	-0.0672	0.0101	0.0090	0.0603	0.0101	0.9450
		$\rho P_2 O_1$	0.1549	0.1548	0.0096	-0.1035	1.6654	0.0096	0.9580
		$\rho P_2 O_2$	0.1468	0.1467	0.0094	-0.0796	1.2392	0.0094	0.9620
		$\rho B_1 B_2$	0.0897	0.0899	0.0099	0.2498	2.2610	0.0099	0.9550
		$\rho B_1 O_1$	0.0209	0.0210	0.0096	0.3741	0.8131	0.0096	0.9530
		$\rho B_1 O_2$	-0.1080	-0.1081	0.0101	0.1271	1.3557	0.0101	0.9460
		$\rho B_2 O_1$	0.0681	0.0678	0.0102	-0.3375	2.2604	0.0102	0.9450
		$\rho B_2 O_2$	0.1476	0.1469	0.0097	-0.4828	7.3258	0.0097	0.9510
		$\rho O_1 O_2$	-0.0526	-0.0525	0.0101	-0.1716	0.8922	0.0101	0.9380

Table 5: Scenario 05/06 - random correlation, large Poisson rate and imbalanced binary/ordinal distribution

Sample Size	Variable	Parameter	TV	AE	SD	RB	SB	RMSE	CR
100	$P_1$	$\lambda_1$	2.0000	2.0024	0.1451	0.1195	1.6475	0.1450	0.9480
	$P_2$	$\lambda_2$	7.0000	6.9876	0.2662	-0.1779	4.6777	0.2663	0.9430
	$B_1$	$p_1$	0.8000	0.8003	0.0403	0.0400	0.7941	0.0403	0.9310
	$B_2$	$p_2$	0.9000	0.8993	0.0308	-0.0767	2.2406	0.0308	0.9300
	$O_1$	$t_1$	0.6500	0.6498	0.0483	-0.0231	0.3106	0.0483	0.9420
		$t_2$	0.8000	0.8000	0.0418	0.0037	0.0717	0.0418	0.9220
		$t_3$	0.9000	0.8999	0.0303	-0.0078	0.2311	0.0303	0.9400
	$O_2$	$t_1$	0.5000	0.5009	0.0503	0.1800	1.7876	0.0503	0.9310
		$t_2$	0.8000	0.7991	0.0405	-0.1088	2.1500	0.0405	0.9200
		$t_3$	0.9000	0.8987	0.0305	-0.1400	4.1363	0.0305	0.9320
	Correlation	$\rho P_1 P_2$	0.4397	0.3833	0.0876	-12.8259	64.3422	0.1042	0.8860
		$\rho P_1 B_1$	-0.0824	-0.1566	0.1017	89.9311	72.8939	0.1258	0.8750
		$\rho P_1 B_2$	-0.2739	-	-	-	-	-	-
		$\rho P_1 O_1$	0.5459	0.5066	0.0815	-7.2028	48.2666	0.0904	0.9020
		$\rho P_1 O_2$	-0.0994	-0.0666	0.0988	-32.9879	33.1658	0.1041	0.9420
		$\rho P_2 B_1$	-0.3534	-0.3026	0.0900	-14.3527	56.3684	0.1032	0.9200
		$\rho P_2 B_2$	0.0635	-	-	-	-	-	-
		$\rho P_2 O_1$	0.1516	0.2073	0.0966	36.7273	57.6322	0.1115	0.9090
		$\rho P_2 O_2$	0.0831	0.0637	0.1031	-23.3217	18.8065	0.1048	0.9290
		$\rho B_1 B_2$	0.3847	-	-	-	-	-	-
		$\rho B_1 O_1$	-0.4778	-0.3777	0.1091	-20.9526	91.7527	0.1481	0.7180
		$\rho B_1 O_2$	0.2440	0.2228	0.0747	-8.6826	28.3599	0.0776	0.9700
		$\rho B_2 O_1$	-0.0941	-	-	-	-	-	-
		$\rho B_2 O_2$	0.2622	-	-	-	-	-	-
		$\rho O_1 O_2$	0.2451	0.2105	0.1084	-14.0946	31.8664	0.1137	0.8940
10000	$P_1$	$\lambda_1$	2.0000	1.9999	0.0137	-0.0054	0.7921	0.0137	0.9470
	$P_2$	$\lambda_2$	7.0000	6.9989	0.0280	-0.0159	3.9718	0.0280	0.9250
	$B_1$	$p_1$	0.8000	0.7999	0.0040	-0.0165	3.2905	0.0040	0.9570
	$B_2$	$p_2$	0.9000	0.8998	0.0030	-0.0174	5.2802	0.0030	0.9650
	$O_1$	$t_1$	0.6500	0.6501	0.0048	0.0082	1.1158	0.0048	0.9490
		$t_2$	0.8000	0.8001	0.0039	0.0163	3.3326	0.0039	0.9430
		$t_3$	0.9000	0.8999	0.0030	-0.0081	2.4489	0.0030	0.9490
	$O_2$	$t_1$	0.5000	0.5002	0.0050	0.0413	4.1673	0.0050	0.9530
		$t_2$	0.8000	0.8001	0.0041	0.0169	3.2888	0.0041	0.9520
		$t_3$	0.9000	0.9000	0.0030	0.0024	0.7273	0.0030	0.9450
	Correlation	$\rho P_1 P_2$	0.4397	0.3824	0.0088	-13.0319	648.6600	0.0580	0.0000
		$\rho P_1 B_1$	-0.0824	-0.1557	0.0104	88.8932	704.4716	0.0740	0.0000
		$\rho P_1 B_2$	-0.2739	-0.2640	0.0108	-3.6008	91.3872	0.0146	0.7730
		$\rho P_1 O_1$	0.5459	0.5063	0.0081	-7.2532	488.1481	0.0404	0.0020
		$\rho P_1 O_2$	-0.0994	-0.0685	0.0099	-31.0470	312.3768	0.0324	0.1210
		$\rho P_2 B_1$	-0.3534	-0.3071	0.0092	-13.0832	505.2346	0.0471	0.0000
		$\rho P_2 B_2$	0.0635	0.0406	0.0101	-36.0898	226.6982	0.0250	0.3690
		$\rho P_2 O_1$	0.1516	0.2074	0.0100	36.7494	556.9621	0.0566	0.0000
		$\rho P_2 O_2$	0.0831	0.0602	0.0101	-27.5517	227.3882	0.0250	0.3880
		$\rho B_1 B_2$	0.3847	0.3350	0.0125	-12.9168	398.9775	0.0512	0.0050
		$\rho B_1 O_1$	-0.4778	-0.3784	0.0108	-20.8074	919.1400	0.1000	0.0000
		$\rho B_1 O_2$	0.2440	0.2219	0.0076	-9.0691	292.2057	0.0234	0.3140
		$\rho B_2 O_1$	-0.0941	-0.1262	0.0116	34.0819	275.6993	0.0341	0.1300
		$\rho B_2 O_2$	0.2622	0.2650	0.0044	1.0886	65.2029	0.0052	0.9990
		$\rho O_1 O_2$	0.2451	0.2133	0.0107	-12.9856	296.5106	0.0336	0.0970

Table 6: Scenario 07/08 - weak correlation, large Poisson rate and imbalanced binary/ordinal distribution

Sample Size	Variable	Parameter	TV	AE	SD	RB	SB	RMSE	CR
100	$P_1$	$\lambda_1$	2.0000	2.0031	0.1400	0.1575	2.2506	0.1399	0.9550
	$P_2$	$\lambda_2$	7.0000	6.9920	0.2627	-0.1140	3.0375	0.2627	0.9430
	$B_1$	$p_1$	0.8000	0.7992	0.0391	-0.0988	2.0230	0.0390	0.9340
	$B_2$	$p_2$	0.9000	0.8986	0.0307	-0.1522	4.4619	0.0307	0.9280
	$O_1$	$t_1$	0.6500	0.6504	0.0462	0.0662	0.9304	0.0462	0.9540
		$t_2$	0.8000	0.8000	0.0399	0.0062	0.1253	0.0399	0.9290
		$t_3$	0.9000	0.8996	0.0297	-0.0400	1.2135	0.0297	0.9380
	$O_2$	$t_1$	0.5000	0.5013	0.0509	0.2660	2.6108	0.0509	0.9350
		$t_2$	0.8000	0.8008	0.0408	0.1000	1.9600	0.0408	0.9420
		$t_3$	0.9000	0.9006	0.0310	0.0700	2.0292	0.0310	0.9150
	Correlation	$\rho P_1 P_2$	0.0172	0.0160	0.1004	-6.9688	1.1954	0.1003	0.9440
		$\rho P_1 B_1$	-0.0619	-0.0632	0.1073	2.1596	1.2452	0.1072	0.9280
		$\rho P_1 B_2$	0.0460	0.0473	0.0986	2.9042	1.3552	0.0985	0.9500
		$\rho P_1 O_1$	-0.0962	-0.0950	0.0971	-1.1919	1.1799	0.0971	0.9540
		$\rho P_1 O_2$	-0.1050	-0.1034	0.0946	-1.4833	1.6473	0.0945	0.9620
		$\rho P_2 B_1$	-0.0405	-0.0385	0.0971	-4.9782	2.0790	0.0971	0.9470
		$\rho P_2 B_2$	-0.0672	-0.0702	0.1008	4.3457	2.8995	0.1008	0.9420
		$\rho P_2 O_1$	0.1549	0.1521	0.0996	-1.8303	2.8463	0.0996	0.9390
		$\rho P_2 O_2$	0.1468	0.1424	0.1018	-2.9973	4.3226	0.1019	0.9460
		$\rho B_1 B_2$	0.0897	0.0803	0.1147	-10.4572	8.1773	0.1150	0.9050
		$\rho B_1 O_1$	0.0209	0.0175	0.1007	-16.3906	3.4006	0.1007	0.9480
		$\rho B_1 O_2$	-0.1080	-0.1061	0.1081	-1.7358	1.7338	0.1081	0.9200
		$\rho B_2 O_1$	0.0681	0.0671	0.0888	-1.4635	1.1216	0.0888	0.9720
		$\rho B_2 O_2$	0.1476	0.1518	0.0719	2.8645	5.8767	0.0720	0.9920
		$\rho O_1 O_2$	-0.0526	-0.0526	0.1012	0.0605	0.0315	0.1011	0.9470
10000	$P_1$	$\lambda_1$	2.0000	1.9995	0.0143	-0.0263	3.6637	0.0143	0.9420
	$P_2$	$\lambda_2$	7.0000	6.9999	0.0258	-0.0020	0.5327	0.0258	0.9560
	$B_1$	$p_1$	0.8000	0.8001	0.0039	0.0183	3.7198	0.0039	0.9520
	$B_2$	$p_2$	0.9000	0.8999	0.0031	-0.0064	1.8729	0.0031	0.9420
	$O_1$	$t_1$	0.6500	0.6499	0.0047	-0.0088	1.2048	0.0047	0.9530
		$t_2$	0.8000	0.8000	0.0040	-0.0047	0.9360	0.0040	0.9600
		$t_3$	0.9000	0.9000	0.0031	-0.0012	0.3436	0.0031	0.9550
	$O_2$	$t_1$	0.5000	0.5000	0.0050	-0.0044	0.4401	0.0050	0.9420
		$t_2$	0.8000	0.8001	0.0039	0.0083	1.6814	0.0039	0.9540
		$t_3$	0.9000	0.9000	0.0030	0.0007	0.2112	0.0030	0.9490
	Correlation	$\rho P_1 P_2$	0.0172	0.0177	0.0099	2.8101	4.9052	0.0099	0.9470
		$\rho P_1 B_1$	-0.0619	-0.0627	0.0104	1.3465	7.9766	0.0105	0.9320
		$\rho P_1 B_2$	0.0460	0.0452	0.0099	-1.7359	8.0515	0.0099	0.9450
		$\rho P_1 O_1$	-0.0962	-0.0945	0.0095	-1.6856	17.1194	0.0096	0.9560
		$\rho P_1 O_2$	-0.1050	-0.1035	0.0096	-1.4060	15.3555	0.0097	0.9600
		$\rho P_2 B_1$	-0.0405	-0.0405	0.0099	-0.1261	0.5188	0.0099	0.9400
		$\rho P_2 B_2$	-0.0672	-0.0684	0.0105	1.7352	11.0966	0.0106	0.9330
		$\rho P_2 O_1$	0.1549	0.1562	0.0098	0.8458	13.4212	0.0098	0.9400
		$\rho P_2 O_2$	0.1468	0.1472	0.0102	0.2822	4.0702	0.0102	0.9430
		$\rho B_1 B_2$	0.0897	0.0893	0.0114	-0.3827	3.0228	0.0114	0.9170
		$\rho B_1 O_1$	0.0209	0.0213	0.0097	1.8356	3.9361	0.0097	0.9550
		$\rho B_1 O_2$	-0.1080	-0.1081	0.0105	0.0704	0.7274	0.0105	0.9340
		$\rho B_2 O_1$	0.0681	0.0683	0.0089	0.3796	2.8931	0.0089	0.9740
		$\rho B_2 O_2$	0.1476	0.1475	0.0077	-0.0732	1.3938	0.0077	0.9900
		$\rho O_1 O_2$	-0.0526	-0.0533	0.0099	1.2760	6.7619	0.0099	0.9550

Table 7: Scenario 09/10 - random correlation, small Poisson rate and balanced binary/ordinal distribution

Sample Size	Variable	Parameter	TV	AE	SD	RB	SB	RMSE	CR
100	$P_1$	$\lambda_1$	0.2000	0.2014	0.0442	0.7150	3.2327	0.0442	0.9420
	$P_2$	$\lambda_2$	0.7000	0.7054	0.0851	0.7771	6.3958	0.0852	0.9530
	$B_1$	$p_1$	0.4500	0.4469	0.0514	-0.6911	6.0548	0.0514	0.9350
	$B_2$	$p_2$	0.5000	0.5001	0.0503	0.0260	0.2585	0.0503	0.9430
	$O_1$	$t_1$	0.2500	0.2509	0.0435	0.3720	2.1356	0.0435	0.9480
		$t_2$	0.5000	0.4992	0.0488	-0.1600	1.6379	0.0488	0.9370
		$t_3$	0.7500	0.7495	0.0433	-0.0720	1.2459	0.0433	0.9160
	$O_2$	$t_1$	0.2000	0.1980	0.0418	-1.0250	4.9035	0.0418	0.9460
		$t_2$	0.4500	0.4482	0.0504	-0.4022	3.5908	0.0504	0.9510
		$t_3$	0.7500	0.7482	0.0438	-0.2427	4.1572	0.0438	0.9440
	Correlation	$\rho P_1 P_2$	0.3712	0.3036	0.1144	-18.1924	59.0313	0.1328	0.8090
		$\rho P_1 B_1$	-0.2252	-0.2489	0.0783	10.5374	30.2901	0.0818	0.9720
		$\rho P_1 B_2$	0.4275	0.3351	0.0751	-21.6005	122.9551	0.1190	0.8500
		$\rho P_1 O_1$	0.3816	0.3009	0.0818	-21.1568	98.7318	0.1149	0.8830
		$\rho P_1 O_2$	0.0918	0.1180	0.0959	28.4337	27.2292	0.0993	0.9340
		$\rho P_2 B_1$	-0.3767	-0.3348	0.0821	-11.1213	51.0185	0.0921	0.9400
		$\rho P_2 B_2$	-0.0998	-0.0300	0.0977	-69.9505	71.4608	0.1200	0.8960
		$\rho P_2 O_1$	0.0924	0.1551	0.0961	67.9575	65.2882	0.1148	0.9020
		$\rho P_2 O_2$	0.5705	0.5382	0.0645	-5.6620	50.0909	0.0721	0.9490
		$\rho B_1 B_2$	-0.3303	-0.2869	0.0990	-13.1336	43.8056	0.1081	0.9010
		$\rho B_1 O_1$	-0.3954	-0.3559	0.0884	-9.9791	44.6256	0.0968	0.9170
		$\rho B_1 O_2$	-0.5631	-0.5795	0.0669	2.9159	24.5563	0.0688	0.9250
		$\rho B_2 O_1$	-0.1481	-0.0733	0.1004	-50.5181	74.5146	0.1252	0.8910
		$\rho B_2 O_2$	0.0354	0.0010	0.0989	-97.0898	34.7104	0.1047	0.9410
		$\rho O_1 O_2$	0.3750	0.3458	0.0890	-7.7909	32.8234	0.0936	0.9370
10000	$P_1$	$\lambda_1$	0.2000	0.1998	0.0046	-0.0879	3.8004	0.0046	0.9380
	$P_2$	$\lambda_2$	0.7000	0.6997	0.0086	-0.0462	3.7689	0.0086	0.9470
	$B_1$	$p_1$	0.4500	0.4501	0.0050	0.0163	1.4711	0.0050	0.9480
	$B_2$	$p_2$	0.5000	0.5002	0.0050	0.0448	4.4852	0.0050	0.9490
	$O_1$	$t_1$	0.2500	0.2500	0.0044	0.0079	0.4448	0.0044	0.9480
		$t_2$	0.5000	0.5001	0.0051	0.0141	1.3938	0.0051	0.9490
		$t_3$	0.7500	0.7499	0.0044	-0.0160	2.7522	0.0044	0.9540
	$O_2$	$t_1$	0.2000	0.2000	0.0040	-0.0121	0.6026	0.0040	0.9520
		$t_2$	0.4500	0.4501	0.0052	0.0206	1.7964	0.0052	0.9470
		$t_3$	0.7500	0.7501	0.0045	0.0106	1.7777	0.0045	0.9460
	Correlation	$\rho P_1 P_2$	0.3712	0.3064	0.0110	-17.4409	588.9485	0.0657	0.0000
		$\rho P_1 B_1$	-0.2252	-0.2468	0.0079	9.6060	273.2154	0.0230	0.3340
		$\rho P_1 B_2$	0.4275	0.3362	0.0073	-21.3427	1254.7885	0.0915	0.0000
		$\rho P_1 O_1$	0.3816	0.3042	0.0083	-20.2882	932.4564	0.0779	0.0000
		$\rho P_1 O_2$	0.0918	0.1193	0.0094	29.8656	292.6293	0.0290	0.2020
		$\rho P_2 B_1$	-0.3767	-0.3385	0.0081	-10.1403	470.7302	0.0390	0.0050
		$\rho P_2 B_2$	-0.0998	-0.0278	0.0097	-72.1006	743.9126	0.0726	0.0000
		$\rho P_2 O_1$	0.0924	0.1595	0.0091	72.6383	734.2651	0.0677	0.0000
		$\rho P_2 O_2$	0.5705	0.5397	0.0060	-5.3960	509.0141	0.0314	0.0010
		$\rho B_1 B_2$	-0.3303	-0.2868	0.0100	-13.1745	436.3842	0.0446	0.0070
		$\rho B_1 O_1$	-0.3954	-0.3567	0.0091	-9.7845	423.6002	0.0397	0.0020
		$\rho B_1 O_2$	-0.5631	-0.5801	0.0069	3.0293	245.8231	0.0184	0.2820
		$\rho B_2 O_1$	-0.1481	-0.0713	0.0101	-51.8298	757.9108	0.0774	0.0000
		$\rho B_2 O_2$	0.0354	0.0048	0.0100	-86.4372	305.1975	0.0322	0.1330
		$\rho O_1 O_2$	0.3750	0.3456	0.0091	-7.8511	323.3223	0.0308	0.0840



Table 8: Scenario 11/12 - weak correlation, small Poisson rate and balanced binary/ordinal distribution

Sample Size	Variable	Parameter	TV	AE	SD	RB	SB	RMSE	CR
100	$P_1$	$\lambda_1$	0.2000	0.2005	0.0442	0.2500	1.1316	0.0442	0.9470
	$P_2$	$\lambda_2$	0.7000	0.6978	0.0827	-0.3129	2.6482	0.0827	0.9520
	$B_1$	$p_1$	0.4500	0.4496	0.0491	-0.0844	0.7735	0.0491	0.9420
	$B_2$	$p_2$	0.5000	0.4990	0.0502	-0.2100	2.0927	0.0502	0.9370
	$O_1$	$t_1$	0.2500	0.2493	0.0442	-0.2800	1.5821	0.0442	0.9400
		$t_2$	0.5000	0.4998	0.0510	-0.0460	0.4511	0.0510	0.9360
		$t_3$	0.7500	0.7500	0.0435	0.0000	0.0000	0.0434	0.9250
	$O_2$	$t_1$	0.2000	0.1968	0.0399	-1.5850	7.9408	0.0400	0.9460
		$t_2$	0.4500	0.4505	0.0508	0.1022	0.9062	0.0507	0.9290
		$t_3$	0.7500	0.7487	0.0456	-0.1733	2.8506	0.0456	0.9320
	Correlation	$\rho P_1 P_2$	0.0172	0.0149	0.0995	-13.7388	2.3786	0.0994	0.9540
		$\rho P_1 B_1$	-0.0619	-0.0628	0.0994	1.5404	0.9588	0.0993	0.9470
		$\rho P_1 B_2$	0.0460	0.0485	0.0978	5.5041	2.5877	0.0978	0.9530
		$\rho P_1 O_1$	-0.0962	-0.0955	0.0950	-0.7321	0.7410	0.0950	0.9600
		$\rho P_1 O_2$	-0.1050	-0.1002	0.0971	-4.5392	4.9074	0.0972	0.9540
		$\rho P_2 B_1$	-0.0405	-0.0417	0.0983	2.9416	1.2135	0.0983	0.9590
		$\rho P_2 B_2$	-0.0672	-0.0638	0.1009	-5.1396	3.4248	0.1009	0.9460
		$\rho P_2 O_1$	0.1549	0.1551	0.1003	0.0874	0.1350	0.1003	0.9410
		$\rho P_2 O_2$	0.1468	0.1483	0.0975	0.9772	1.4717	0.0975	0.9530
		$\rho B_1 B_2$	0.0897	0.0921	0.1026	2.6919	2.3528	0.1026	0.9430
		$\rho B_1 O_1$	0.0209	0.0179	0.0994	-14.4660	3.0401	0.0994	0.9510
		$\rho B_1 O_2$	-0.1080	-0.1052	0.0964	-2.5911	2.9033	0.0964	0.9560
		$\rho B_2 O_1$	0.0681	0.0701	0.1007	3.0470	2.0592	0.1007	0.9480
		$\rho B_2 O_2$	0.1476	0.1439	0.0981	-2.5112	3.7770	0.0982	0.9440
		$\rho O_1 O_2$	-0.0526	-0.0520	0.1001	-1.0517	0.5527	0.1000	0.9470
10000	$P_1$	$\lambda_1$	0.2000	0.2001	0.0044	0.0662	3.0201	0.0044	0.9550
	$P_2$	$\lambda_2$	0.7000	0.6996	0.0086	-0.0541	4.4001	0.0086	0.9490
	$B_1$	$p_1$	0.4500	0.4501	0.0050	0.0138	1.2351	0.0050	0.9460
	$B_2$	$p_2$	0.5000	0.4997	0.0049	-0.0610	6.2412	0.0049	0.9570
	$O_1$	$t_1$	0.2500	0.2501	0.0045	0.0203	1.1352	0.0045	0.9450
		$t_2$	0.5000	0.5000	0.0051	-0.0057	0.5580	0.0051	0.9400
		$t_3$	0.7500	0.7501	0.0041	0.0138	2.4967	0.0041	0.9550
	$O_2$	$t_1$	0.2000	0.2001	0.0040	0.0465	2.3025	0.0040	0.9620
		$t_2$	0.4500	0.4501	0.0050	0.0161	1.4407	0.0050	0.9450
		$t_3$	0.7500	0.7503	0.0043	0.0437	7.6743	0.0043	0.9560
	Correlation	$\rho P_1 P_2$	0.0172	0.0197	0.0101	14.5791	24.8092	0.0104	0.9450
		$\rho P_1 B_1$	-0.0619	-0.0615	0.0092	-0.5137	3.4550	0.0092	0.9660
		$\rho P_1 B_2$	0.0460	0.0460	0.0102	-0.1012	0.4545	0.0102	0.9410
		$\rho P_1 O_1$	-0.0962	-0.0954	0.0100	-0.8153	7.8785	0.0100	0.9570
		$\rho P_1 O_2$	-0.1050	-0.1053	0.0098	0.3148	3.3805	0.0098	0.9530
		$\rho P_2 B_1$	-0.0405	-0.0402	0.0098	-0.7422	3.0615	0.0098	0.9590
		$\rho P_2 B_2$	-0.0672	-0.0670	0.0100	-0.2784	1.8779	0.0100	0.9480
		$\rho P_2 O_1$	0.1549	0.1550	0.0097	0.0345	0.5497	0.0097	0.9460
		$\rho P_2 O_2$	0.1468	0.1459	0.0095	-0.6182	9.5337	0.0096	0.9520
		$\rho B_1 B_2$	0.0897	0.0891	0.0101	-0.6855	6.1165	0.0101	0.9420
		$\rho B_1 O_1$	0.0209	0.0212	0.0097	1.5166	3.2714	0.0097	0.9550
		$\rho B_1 O_2$	-0.1080	-0.1085	0.0098	0.4387	4.8277	0.0098	0.9610
		$\rho B_2 O_1$	0.0681	0.0686	0.0100	0.8376	5.7179	0.0100	0.9390
		$\rho B_2 O_2$	0.1476	0.1473	0.0102	-0.2288	3.3156	0.0102	0.9410
		$\rho O_1 O_2$	-0.0526	-0.0526	0.0095	0.0789	0.4347	0.0095	0.9640

Table 9: Scenario 13/14 - random correlation, small Poisson rate and imbalanced binary/ordinal distribution

Sample Size	Variable	Parameter	TV	AE	SD	RB	SB	RMSE	CR
100	$P_1$	$\lambda_1$	0.2000	0.1999	0.0451	-0.0550	0.2437	0.0451	0.9330
	$P_2$	$\lambda_2$	0.7000	0.6998	0.0814	-0.0257	0.2211	0.0814	0.9560
	$B_1$	$p_1$	0.8000	0.8009	0.0399	0.1162	2.3318	0.0399	0.9330
	$B_2$	$p_2$	0.9000	0.9007	0.0292	0.0744	2.2961	0.0292	0.9400
	$O_1$	$t_1$	0.6500	0.6492	0.0469	-0.1262	1.7493	0.0469	0.9400
		$t_2$	0.8000	0.8002	0.0383	0.0287	0.5998	0.0383	0.9340
		$t_3$	0.9000	0.8997	0.0290	-0.0356	1.1035	0.0290	0.9400
	$O_2$	$t_1$	0.5000	0.5021	0.0505	0.4140	4.1026	0.0505	0.9420
		$t_2$	0.8000	0.7995	0.0398	-0.0675	1.3556	0.0398	0.9470
		$t_3$	0.9000	0.8992	0.0305	-0.0944	2.7844	0.0305	0.9320
	Correlation	$\rho P_1 P_2$	0.4811	0.4606	0.0992	-4.2562	20.6378	0.1013	0.8730
		$\rho P_1 B_1$	-0.3434	-0.5032	0.1051	46.5539	152.0598	0.1913	0.4740
		$\rho P_1 B_2$	-0.1249	-0.1562	0.1323	25.1122	23.7090	0.1359	0.8330
		$\rho P_1 O_1$	0.5349	0.6658	0.0803	24.4635	162.8941	0.1535	0.4280
		$\rho P_1 O_2$	-0.1313	-0.0884	0.0915	-32.6478	46.8207	0.1010	0.9420
		$\rho P_2 B_1$	-0.4763	-0.5258	0.0824	10.4102	60.1415	0.0962	0.8410
		$\rho P_2 B_2$	0.0496	0.0339	0.0941	-31.6626	16.7053	0.0953	0.9550
		$\rho P_2 O_1$	0.2603	0.3297	0.1029	26.6784	67.4691	0.1241	0.8330
		$\rho P_2 O_2$	0.1111	0.0817	0.1047	-26.4997	28.1400	0.1087	0.9150
		$\rho B_1 B_2$	0.3770	0.3547	0.1227	-5.9325	18.2229	0.1247	0.8260
		$\rho B_1 O_1$	-0.3716	-0.3397	0.1116	-8.5819	28.5833	0.1160	0.8610
		$\rho B_1 O_2$	0.2440	0.2265	0.0766	-7.1995	22.9279	0.0786	0.9770
		$\rho B_2 O_1$	-0.0941	-0.0912	0.1125	-3.0621	2.5618	0.1125	0.9090
		$\rho B_2 O_2$	0.2622	0.2573	0.0457	-1.8551	10.6453	0.0459	0.9990
		$\rho O_1 O_2$	0.2451	0.1896	0.1054	-22.6314	52.6055	0.1191	0.8950
10000	$P_1$	$\lambda_1$	0.2000	0.2000	0.0047	0.0049	0.2085	0.0046	0.9380
	$P_2$	$\lambda_2$	0.7000	0.7001	0.0087	0.0109	0.8809	0.0086	0.9440
	$B_1$	$p_1$	0.8000	0.8000	0.0041	0.0031	0.6049	0.0041	0.9510
	$B_2$	$p_2$	0.9000	0.8999	0.0029	-0.0069	2.1405	0.0029	0.9570
	$O_1$	$t_1$	0.6500	0.6502	0.0049	0.0316	4.2338	0.0049	0.9430
		$t_2$	0.8000	0.8002	0.0040	0.0247	4.9075	0.0040	0.9470
		$t_3$	0.9000	0.9002	0.0030	0.0167	5.0135	0.0030	0.9400
	$O_2$	$t_1$	0.5000	0.5001	0.0051	0.0220	2.1500	0.0051	0.9470
		$t_2$	0.8000	0.7999	0.0041	-0.0076	1.4758	0.0041	0.9510
		$t_3$	0.9000	0.9000	0.0032	-0.0048	1.3706	0.0032	0.9500
	Correlation	$\rho P_1 P_2$	0.4811	0.4614	0.0106	-4.0913	186.5482	0.0223	0.3190
		$\rho P_1 B_1$	-0.3434	-0.5018	0.0103	46.1429	1534.0222	0.1588	0.0000
		$\rho P_1 B_2$	-0.1249	-0.1579	0.0130	26.4766	253.8010	0.0355	0.1420
		$\rho P_1 O_1$	0.5349	0.6701	0.0075	25.2706	1798.6662	0.1354	0.0000
		$\rho P_1 O_2$	-0.1313	-0.0873	0.0091	-33.5133	480.9068	0.0449	0.0010
		$\rho P_2 B_1$	-0.4763	-0.5229	0.0084	9.7997	555.4842	0.0474	0.0000
		$\rho P_2 B_2$	0.0496	0.0396	0.0094	-20.2819	106.7437	0.0138	0.8480
		$\rho P_2 O_1$	0.2603	0.3280	0.0101	26.0471	669.4777	0.0685	0.0000
		$\rho P_2 O_2$	0.1111	0.0833	0.0104	-24.9997	268.1195	0.0297	0.2060
		$\rho B_1 B_2$	0.3770	0.3559	0.0125	-5.5989	169.2863	0.0245	0.3540
		$\rho B_1 O_1$	-0.3716	-0.3367	0.0107	-9.4019	325.4799	0.0365	0.0460
		$\rho B_1 O_2$	0.2440	0.2304	0.0075	-5.5832	182.0283	0.0155	0.7490
		$\rho B_2 O_1$	-0.0941	-0.0903	0.0110	-4.0532	34.8265	0.0116	0.9090
		$\rho B_2 O_2$	0.2622	0.2614	0.0044	-0.3000	17.6946	0.0045	1.0000
		$\rho O_1 O_2$	0.2451	0.1910	0.0104	-22.0612	517.6334	0.0551	0.0000

Table 10: Scenario 15/16 - weak correlation, small Poisson rate and imbalanced binary/ordinal distribution

Sample Size	Variable	Parameter	TV	AE	SD	RB	SB	RMSE	CR
100	$P_1$	$\lambda_1$	0.2000	0.2011	0.0470	0.5550	2.3602	0.0470	0.9220
	$P_2$	$\lambda_2$	0.7000	0.7020	0.0825	0.2814	2.3883	0.0825	0.9590
	$B_1$	$p_1$	0.8000	0.7975	0.0396	-0.3150	6.3692	0.0396	0.9370
	$B_2$	$p_2$	0.9000	0.8997	0.0304	-0.0300	0.8874	0.0304	0.9340
	$O_1$	$t_1$	0.6500	0.6504	0.0468	0.0662	0.9197	0.0467	0.9480
		$t_2$	0.8000	0.8006	0.0400	0.0712	1.4242	0.0400	0.9370
		$t_3$	0.9000	0.8993	0.0292	-0.0778	2.4002	0.0292	0.9520
	$O_2$	$t_1$	0.5000	0.4994	0.0491	-0.1280	1.3045	0.0490	0.9310
		$t_2$	0.8000	0.7993	0.0404	-0.0825	1.6354	0.0403	0.9320
		$t_3$	0.9000	0.8997	0.0301	-0.0378	1.1283	0.0301	0.9310
	Correlation	$\rho P_1 P_2$	0.0172	0.0251	0.1063	45.4862	7.3680	0.1065	0.9360
		$\rho P_1 B_1$	-0.0619	-0.0628	0.1052	1.5171	0.8924	0.1051	0.9440
		$\rho P_1 B_2$	0.0460	0.0479	0.0868	4.1982	2.2260	0.0867	0.9720
		$\rho P_1 O_1$	-0.0962	-0.0923	0.0884	-4.0359	4.3894	0.0885	0.9720
		$\rho P_1 O_2$	-0.1050	-0.0993	0.0894	-5.4672	6.4216	0.0895	0.9690
		$\rho P_2 B_1$	-0.0405	-0.0471	0.1057	16.2741	6.2461	0.1058	0.9460
		$\rho P_2 B_2$	-0.0672	-0.0756	0.1071	12.4024	7.7895	0.1073	0.9300
		$\rho P_2 O_1$	0.1549	0.1627	0.1087	5.0129	7.1481	0.1089	0.9130
		$\rho P_2 O_2$	0.1468	0.1524	0.1044	3.7728	5.3068	0.1045	0.9240
		$\rho B_1 B_2$	0.0897	0.0870	0.1148	-2.9310	2.2900	0.1148	0.9130
		$\rho B_1 O_1$	0.0209	0.0176	0.0969	-15.7111	3.3865	0.0969	0.9530
		$\rho B_1 O_2$	-0.1080	-0.1067	0.1062	-1.2228	1.2431	0.1062	0.9290
		$\rho B_2 O_1$	0.0681	0.0662	0.0895	-2.7920	2.1246	0.0894	0.9770
		$\rho B_2 O_2$	0.1476	0.1452	0.0713	-1.6152	3.3456	0.0713	0.9920
		$\rho O_1 O_2$	-0.0526	-0.0492	0.0962	-6.4908	3.5478	0.0962	0.9580
10000	$P_1$	$\lambda_1$	0.2000	0.1999	0.0045	-0.0501	2.2245	0.0045	0.9490
	$P_2$	$\lambda_2$	0.7000	0.7001	0.0083	0.0165	1.3951	0.0083	0.9500
	$B_1$	$p_1$	0.8000	0.8002	0.0040	0.0300	5.9610	0.0040	0.9490
	$B_2$	$p_2$	0.9000	0.9001	0.0030	0.0113	3.3572	0.0030	0.9410
	$O_1$	$t_1$	0.6500	0.6499	0.0050	-0.0095	1.2445	0.0050	0.9410
		$t_2$	0.8000	0.7999	0.0041	-0.0092	1.8108	0.0041	0.9620
		$t_3$	0.9000	0.8999	0.0030	-0.0129	3.8432	0.0030	0.9530
	$O_2$	$t_1$	0.5000	0.4999	0.0049	-0.0242	2.4439	0.0049	0.9560
		$t_2$	0.8000	0.7998	0.0039	-0.0191	3.8714	0.0039	0.9400
		$t_3$	0.9000	0.8999	0.0030	-0.0081	2.4519	0.0030	0.9550
	Correlation	$\rho P_1 P_2$	0.0172	0.0193	0.0097	12.1725	21.6611	0.0099	0.9520
		$\rho P_1 B_1$	-0.0619	-0.0650	0.0106	5.1414	29.8670	0.0111	0.9180
		$\rho P_1 B_2$	0.0460	0.0426	0.0088	-7.3484	38.4502	0.0094	0.9590
		$\rho P_1 O_1$	-0.0962	-0.0899	0.0085	-6.5609	74.2276	0.0106	0.9370
		$\rho P_1 O_2$	-0.1050	-0.0993	0.0088	-5.3898	64.1506	0.0105	0.9420
		$\rho P_2 B_1$	-0.0405	-0.0411	0.0102	1.2961	5.1523	0.0102	0.9480
		$\rho P_2 B_2$	-0.0672	-0.0708	0.0105	5.3360	34.0149	0.0111	0.9180
		$\rho P_2 O_1$	0.1549	0.1623	0.0106	4.7332	68.8938	0.0129	0.8580
		$\rho P_2 O_2$	0.1468	0.1508	0.0107	2.7025	37.0181	0.0114	0.9070
		$\rho B_1 B_2$	0.0897	0.0898	0.0113	0.1579	1.2525	0.0113	0.9170
		$\rho B_1 O_1$	0.0209	0.0212	0.0098	1.3036	2.7856	0.0098	0.9490
		$\rho B_1 O_2$	-0.1080	-0.1080	0.0103	-0.0175	0.1832	0.0103	0.9470
		$\rho B_2 O_1$	0.0681	0.0685	0.0082	0.5891	4.8953	0.0082	0.9790
		$\rho B_2 O_2$	0.1476	0.1476	0.0075	-0.0230	0.4497	0.0075	0.9900
		$\rho O_1 O_2$	-0.0526	-0.0521	0.0099	-1.0038	5.3125	0.0099	0.9480