BSTT 565- Computational Statistics-Homework 5

November 09, 2020

Due: December 02, 2020

1) Solve Problem 5.3 on Page 148.

2) Solve Problem 5.4 on Page 148. Assume that $a = e$.

3) Solve Problem 5.5 on Page 149.

4) Solve Problem 5.6 on Page 149.

HINTS

1) Write down $f(\bar{x}|\mu)$ (which is a function of $\bar{x}$ and μ) and $f(\mu)$ (which is a function of μ only). First one is normal, second one is Cauchy. Posterior will be proportional to their product. It is unnormalized, i.e. we know it up to a proportionality constant. Your task is to find this number. You need to solve $\int f(\bar{x}|\mu)f(\mu)d\mu$ where μ ranges from $-\infty$ to $+\infty$. The reciprocal of this number will be the proportionality constant. You can easily compute $\bar{x}$ given the data. One complication is that limits of the integral are not finite. I suggest that you use $\mu = log\left(\frac{y}{1-y}\right)$ transformation. You may also use another transformation if you wish to. Then use your favorite numerical integration technique (Monte Carlo, Simpson, trapezoidal, Romberg, etc.). The answer is 0.127447. Part b is easy after doing part a. This time limits of the integration are 2 and 8. You proceed as before. I suggest that you write very general functions for trapezoidal, Simpson, Romberg and Reimann integration so that you don't have to write another function every time. I find it silly to double the number of intervals at each iteration and stop when convergence criterion is reached. Take a large enough n such as 50, and if you get close enough to the truth, stop. When the truth is not known (which is not the case here), repeat it for $n = 100$ and compare what you have found with $n = 50$. Part c is very similar, you may remove the singularity at 1 (under above mentioned transformation) by making one of the limits 0.99999999999. Part d is based on another transformation, if you got this far, it must be easy to do.

2) Solve Problem 5.4 on Page 148. Assume that $a = e$.

You must have a function for Romberg integration from the previous question. I don't care about triangular array. Just show me the right answer which is $log(e) = 1$.

3) Nodes and weights are given for Gauss-Legendre quadrature. You'll need two write a very simple function with at most three lines [(sum(weight vector*f(node vector)))]. If you didn't think it is a piece of cake, you must be doing something wrong.

4) For part a, use the recursive formula on Page 143 and table on page 144. Get $H_5(x)$ and $H_6(x)$ (you'll need this in part d). In part b, orthonormalization needs to be done. You'll need to integrate $H_5(x)^2 exp(-x^2/2)$. I hope you understand why.In part c, you need a root finding algorithm. See the solutions for HW2. There are multiple roots, so graphing the function will be useful for you to select the appropriate starting values. Also, could there be an R function that finds the roots automatically? You are PhD students and should be able to find it, if there is such a thing. In part d, answer is somewhere on page 143. In part e, you should first find the normalizing constant in the posterior. Write down the posterior which is known up to a proportionality constant and use the nodes and weights you have found before. (You are effectively converting an integral to a sum with five elements). After this, to find the posterior variance you need to integrate your posterior twice (one for $E[\mu|\bar{x}]$ and one for $E[\mu^2|\bar{x}]$). This is a hard question, so I'd understand if your solution is less than perfect.