

# Scheduling of independent tasks

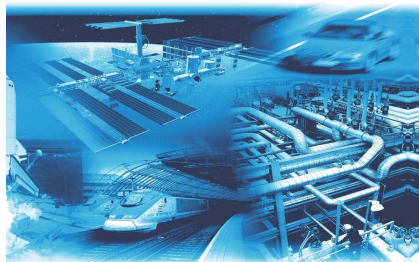
**Kizito NKURIKIYEYU, Ph.D.**

# Readings

- Read Chapter 2, section 2.1 (pages 23-33) of Cottet et al. (2002). Scheduling in Real-Time Systems. Skip other sections!
- Topics
  - rate monotonic
  - inverse deadline
  - earliest deadline first
  - least laxity first
  - On-line scheduling

## SCHEDULING IN REAL-TIME SYSTEMS

Francis Cottet | Joëlle Delacroix | Claude Kaiser | Zoubir Mammeri



<sup>1</sup> Readings are based on Cottet, F., Delacroix, J., Mammeri, Z., & Kaiser, C. (2002). Scheduling in Real-Time Systems. Wiley.

# Review

# Recurrent Task Models

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# Real-time task model

- $r_i$ , task release time, i.e. the execution request time.

$$\tau(r_0, C, D, T)$$

with  $0 \leq C \leq D \leq T$

$r_0$ : release time of the 1st request of task  
 $C$ : worst-case computation time  
 $D$ : relative deadline  
 $T$ : period  
 $r_k$ : release time of  $k+1$ th request of task  
 $r_k = r_0 + kT$  is represented by  $\uparrow$   
 $d_k$ : absolute deadline of  $k+1$ th request of task  
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Note: for periodic task with  $D = T$  (deadline equal to period)  
 deadline at next release time is represented by  $\updownarrow$

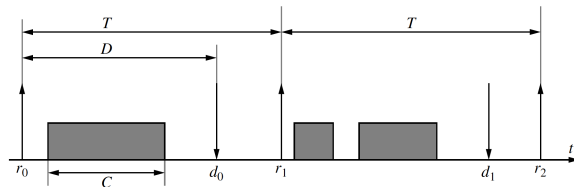


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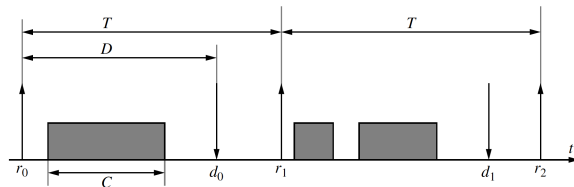


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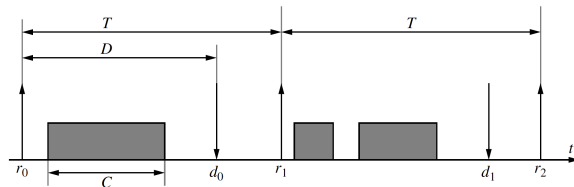


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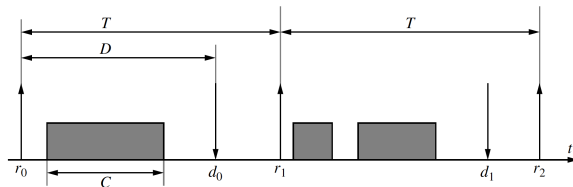
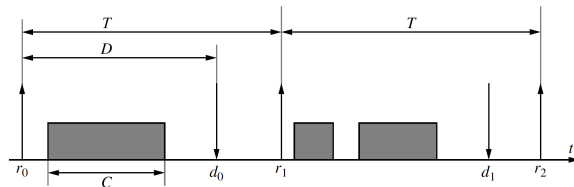
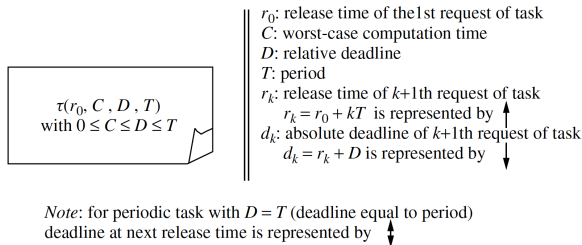


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- $T_i$ , task period (valid only for periodic tasks).
- Absolute deadline  
 $d_i = r_i + D_i$ —transgression of the absolute deadline causes a timing fault.



**FIG 1.** Task model

# Relative Deadline vs Period

When we have a task set, we say that the task set is with

- **implicit deadline** when the relative deadline  $D_i$  is equal to the period  $T_i$ , i.e.,  $D_i = T_i$  for every task  $\tau_i$  **constrained deadline** when the relative deadline  $D_i$  is no more than the period  $T_i$ , i.e.,  $D_i \leq T_i$ , for every task  $\tau_i$

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- **arbitrary deadline** when the relative deadline  $D_i$  could be larger than the period  $T_i$  for some task  $\tau_i$

# Sporadic and Periodic Tasks

- For **periodic tasks**

- **Synchronous system**—each task  $\tau_i$  has a phase of 0, i.e.,  $\phi_i = 0$

- Hyperperiod: Least common multiple (LCM) of  $T_i$

- Task utilization of task

$$u_i = \frac{C_i}{T_i} \quad (1)$$

- Total system utilization

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  - **Asynchronous system**—the phase are arbitrary
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# **Scheduling of independent tasks**

# On-line algorithms for periodic tasks

**simple rule**—that assigns priorities according to temporal parameters of tasks.

- **static**—the priority is fixed. the priorities are assigned to tasks before execution and do not change over time. For example:
  - rate monotonic (Liu and Layland, 1973)<sup>1</sup>

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$$U = \sum_{i=1}^n U_i = \sum_{i=1}^n \frac{C_i}{T_i} \leq \cdot n \left( 2^{\frac{1}{n}} - 1 \right) \quad (3)$$

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- $n$ —number of tasks to be scheduled.

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- For two tasks (i.e.,  $n = 2$ ), the upper bounds on utilization is (Equation (4))

$$n \left( 2^{\frac{1}{2}} - 1 \right) = 2 \left( \sqrt{2} - 1 \right) = 0.828 \quad (4)$$

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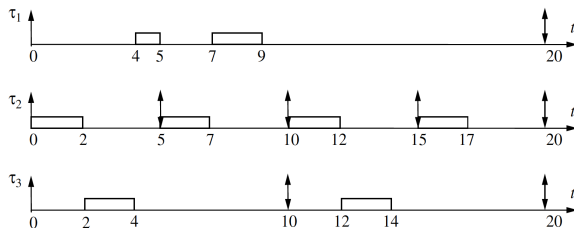
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- As a general rule, when  $n > 10$ , the RMS can meet its deadlines if  $U < 70\%$

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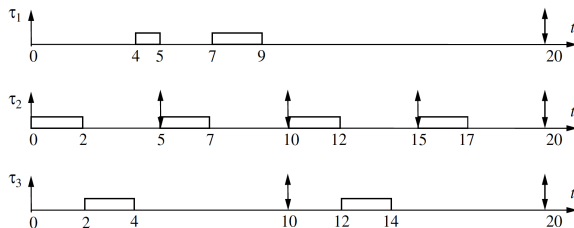
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**FIG 2.** Example of a rate monotonic schedule with three periodic tasks:  $\tau_1(0, 3, 20, 20)$ ,  $\tau_2(0, 2, 5, 5)$  and  $\tau_3(0, 2, 10, 10)$

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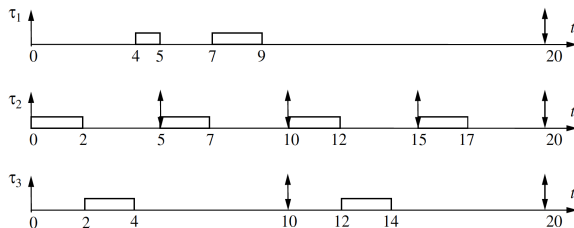
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- At  $t = 2$   $\tau_3$  (intermediate priority) executes second until  $t = 4$

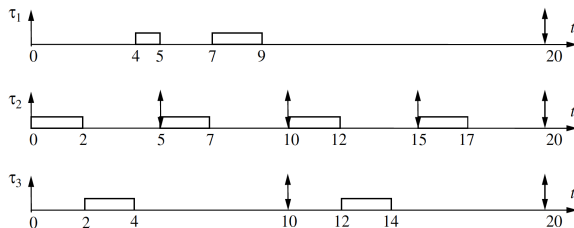


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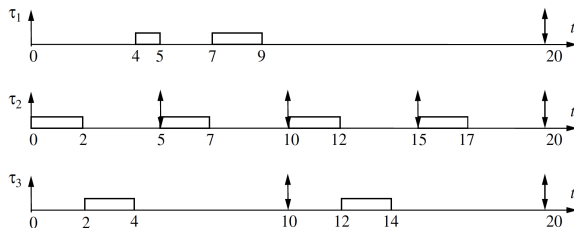
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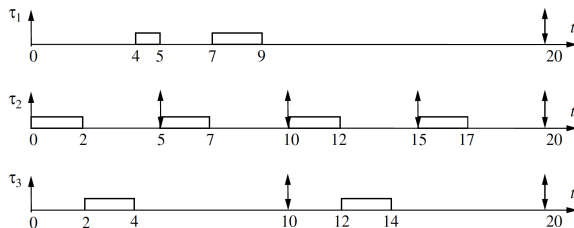
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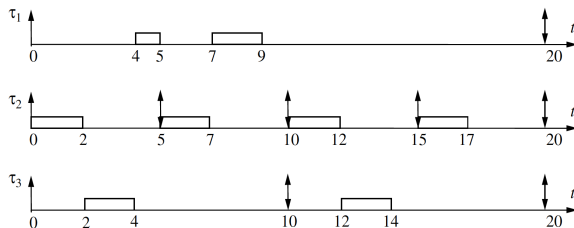
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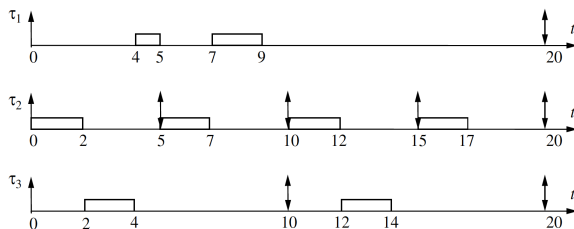
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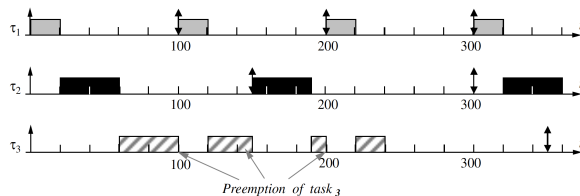
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The three tasks meet their deadline since the utilization factors

$$U = \frac{3}{20} + \frac{2}{5} + \frac{2}{10} = 0.75 \leq 3(2^{\frac{1}{3}} - 1) = 0.779 \quad (6)$$

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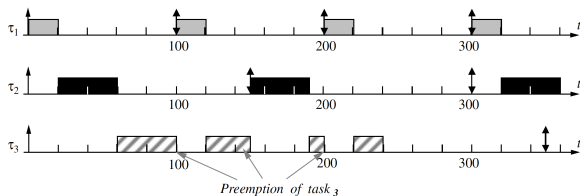
- In this example, we have a set of three periodic tasks for which the relative deadline is equal to the period



**FIG 4.** Example of a rate monotonic schedule with three periodic tasks:  $\tau_1$  (0, 20, 100, 100),  $\tau_2$ (0, 40, 150, 150) and  $\tau_3$ (0, 100, 350, 350)

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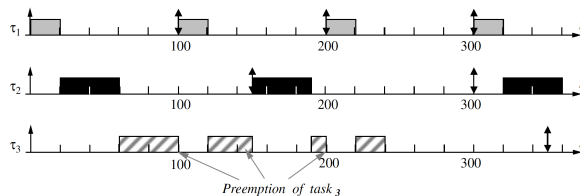
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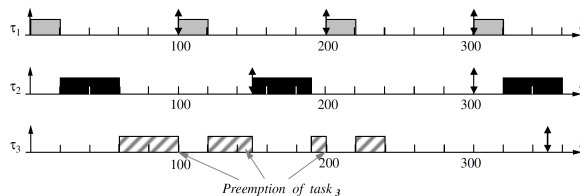


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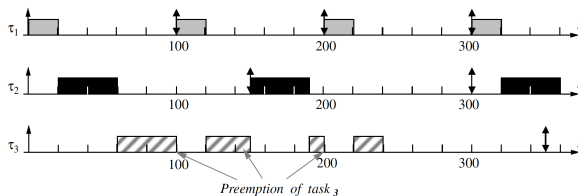
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The processor utilization factor is:

$$U = \frac{20}{100} + \frac{40}{150} + \frac{100}{350} = 0.75 < 3 \cdot (\sqrt[3]{2} - 1) = 0.779 \quad (7)$$

So this task set is schedulable. All the three tasks meet their deadlines.



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- For smaller values of  $n$  or in cases where  $U$  is close to this estimate, the calculated utilization bound should be used.

# **Inverse (monotonic) deadline algorithm**



# Deadline-monotonic scheduling

**summary**—Deadline-monotonic priority assignment is a priority assignment policy used with fixed-priority pre-emptive scheduling<sup>5</sup>

- Allows a weakening of the condition which requires equality between periods and deadlines in static-priority schemes.

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- Allows a weakening of the condition which requires equality between periods and deadlines in static-priority schemes.
- The task with the shortest relative deadline is assigned the highest priority<sup>6</sup>
- For an arbitrary set of  $n$  tasks with deadlines shorter than periods, a sufficient condition is given in **Equation** (8)

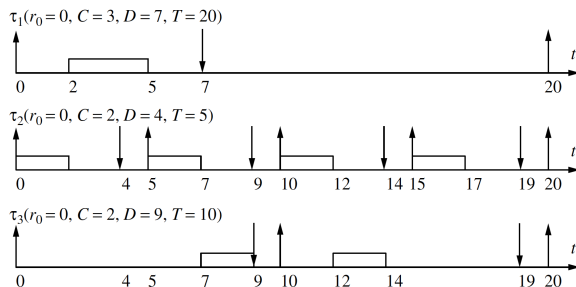
$$U = \sum_{i=1}^n \frac{C_i}{D_i} \leq n \cdot \left(2^{\frac{1}{n}} - 1\right) \quad (8)$$

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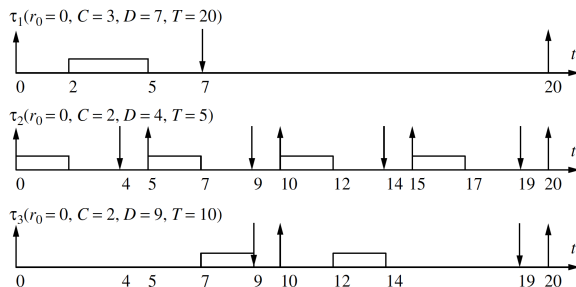


**FIG 5.** Inverse deadline schedule for a set of three periodic tasks  $\tau_1$  ( $r_0 = 0, C = 3, D = 7, T = 20$ ),  $\tau_2$  ( $r_0 = 0, C = 2, D = 4, T = 5$ ) and  $\tau_3$  ( $r_0 = 0, C = 2, D = 9, T = 10$ )

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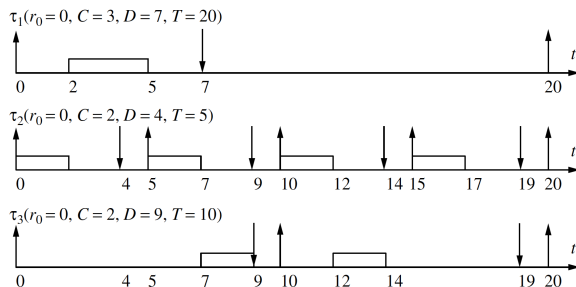


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- However, the task set is schedulable because the schedule is given within the major cycle of the task set.



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**Earliest deadline first algorithm**

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**summary**—the **earliest deadline first (EDF)** algorithm assigns priority to tasks according to their absolute deadline: the task with the earliest deadline will be executed as the highest priority.

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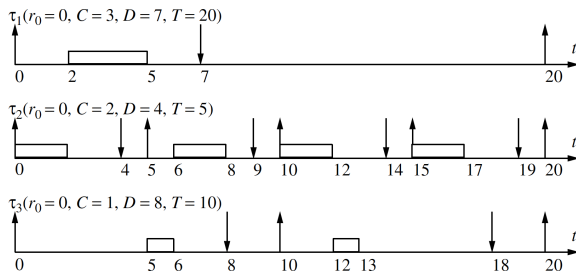
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- A hybrid task set is schedulable with the EDF algorithm if (**Equation** (11)):

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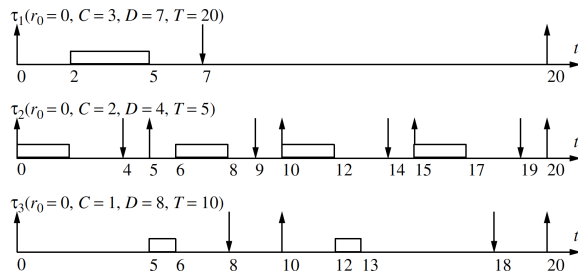
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**FIG 6.** EDF EDF schedule for a set of three periodic tasks  $\tau_1(r_0 = 0, C = 3, D = 7, 20 = T)$ ,  $\tau_2(r_0 = 0, C = 2, D = 4, T = 5)$ ,  $\tau_3(r_0 = 0, C = 1, D = 8, T = 10)$

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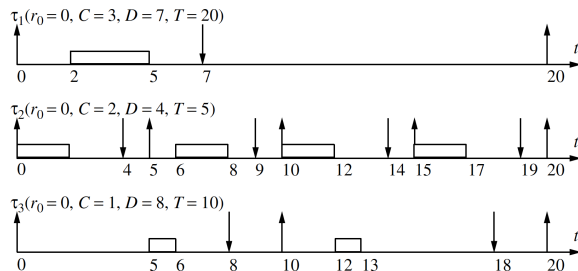
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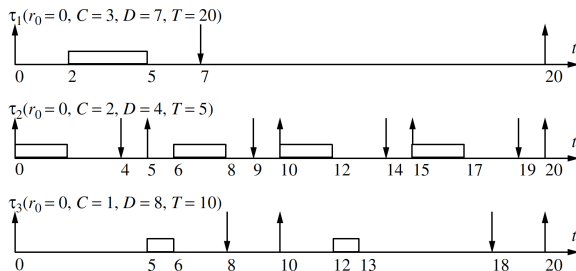
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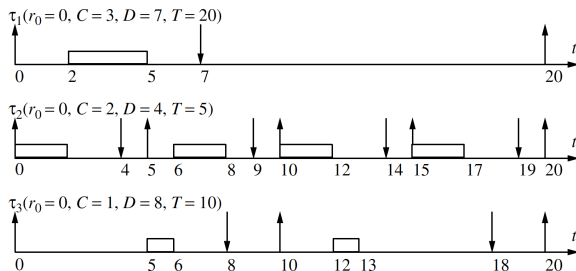
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# **Least laxity first algorithm**

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- Please take a closer look at example Figure 2.9 on page 32 of the textbook

**The end**