BIOL 450 / IBS 534 HW2 Population Coding

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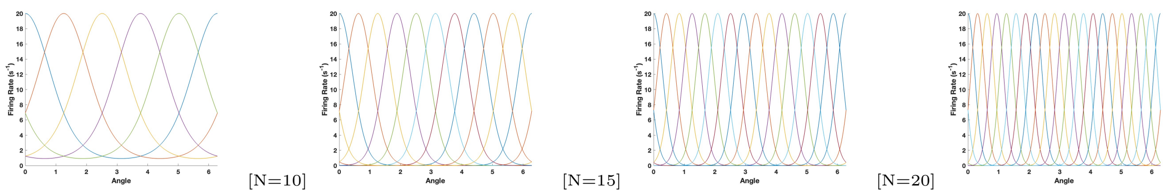
For this homework set, we will use study the coding properties of populations of neuronsthat are arrayed to encode a periodic stimulus (e.g., head-direction cells in the hippocampus). Specifically, we will use receptive fields that can be described via the von Mises distribution ($f(s) = r_{max}e^{kcos(2-\mu_i)}$), which is essentially a gaussian that has periodic boundary conditions on the circle. To make this process a little easier, the code will take the receptive field width (σ) as an input and will numerically calculate the appropriate value of k needed to create the appropriate distribution (or, in some cases, it will use N and assume that $\sigma = \pi/N$.

Add the code from Canvas to your path and use drawReceptiveFields(N) to draw N evenly- space receptive fields. Try performing this for N = 5, 10, 15, and 20 to get a sense of the receptive fields.



[N=5] [N=10] [N=15] [N=20]

Figure 1: Evenly-spaced Receptive Fields

1. If $y = Ax^m$, show that $logy \propto mlogx$.
   given $y = Ax^m$, by applying log on both side we get $logy = log(Ax^m)$, which can be written as $logy = logA + mlogx$, therefore, $logy$ is proportional to $logx$.

2. Use $[outputValues \sim] = histogramMLEstimates(s, N, rMax, deltaT)$ to plot the distribution of maximum likelihood estimates using the population code for s = 1 given N = 10 neurons, a maximum firing rate (r_{max}) of 20, and a measurement time (Δt) of 2. Report the mean ($< \hat{s} >$) and standard deviation ($\sigma_{est}(\hat{s})$) of the distribution. Does the distribution look gaussian? (Note: outputValues is an array of all simulated estimated values. This will come in handy in a moment.)
   mean (μ) = 1.0022 and standard deviation $\sigma = 0.0545$.

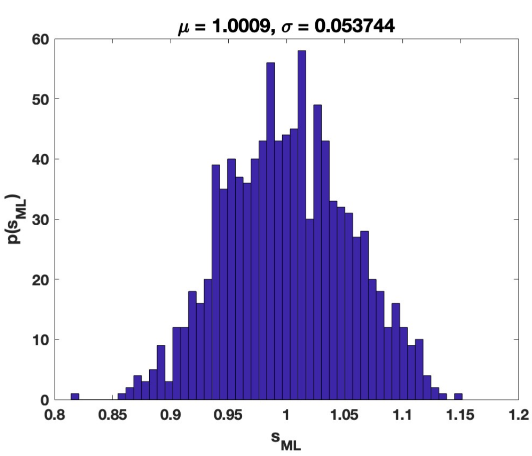


Figure 2: Maximum Likelihood Estimates Distribution

3. What is the estimate's normalized error ($\frac{<\hat{s}>-s}{\sigma_{est}(\hat{s})}$) for your results in the previous problem. Are these results consistent with those of an unbiased estimator?
   the estimate normalized error is $\frac{<\hat{s}>-s}{\sigma_{est}(\hat{s})} = \frac{1.0022-1}{0.0545} = 0.0404$. The result of an unbiased estimator (0) is close to the normalized error.

4. The Cramer-Rao bound for this system is given by $\sigma^2_{est}(s) \geq \frac{\sqrt{2\pi\sigma^2}}{N^2 r_{max}\Delta t}$, where σ is the width of a single tuning curve. Using this information, estimate the smallest possible Δt you would need to have $\sigma_{est}(S) \leq 10^{-4}$ (all other parameters remaining the same). Confirm this finding by plugging this value into the above function (note that $\sigma = \frac{\pi}{N}$.
   The smallest $\Delta(t)$ value is 3.9374 where the $\sigma = 0.0001$.

5. Calculate and plot $(<\hat{s}> -s)$ and $\sigma^2_{est}(\widehat{S})$ as a function of N for N = 5, 10, 15, ..., 50 (using $[outputValues,] = histogramMLEstimates(s, N, rMax, deltaT, [], [], false)$ to suppress plots). Again, use s = 1, $\Delta t = 2$, and $r_{max} = 20$.

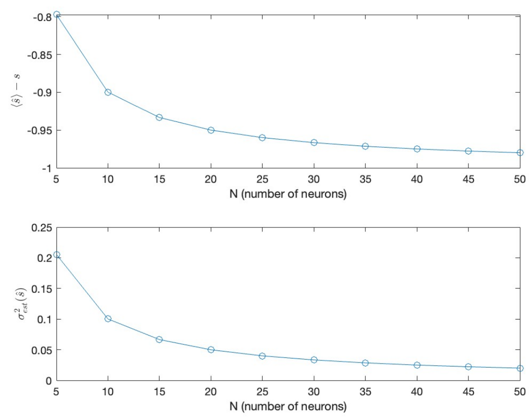


Figure 3: (top) Bias $((<\hat{s}> -s))$ v.s. Number of neuron (N); (bottom) Variance $(\sigma^2_{est}(\widehat{S}))$ v.s. Number of neuron (N)

6. Plot $log\sigma^2_{est}(\hat{s})$ vs. $logN$ from the previous question and calculate the slope of the resulting line. Do these results agree with your intuition from the $Cramer - Rao$ bound? Explain your reasoning.
The slope is -2.001, which align with my intuition. The Cramer-Rao bound for this system is given by $\sigma^2_{est}(s) \geq \frac{\sqrt{2\pi\sigma^2}}{N^2 r_{max}\Delta t}$, treating $\frac{\sqrt{2\pi\sigma^2}}{r_{max}\Delta t}$ as constant C, we know that $\sigma^2_{est}(s) \geq \frac{C}{N^2}$. By applying log to both size, we got $\log\sigma^2_{est}(s) \geq \log C - 2logN$. Therefore, the slope of plot displaying $log\sigma^2_{est}(\hat{s})$ vs. $logN$ should have slope of -2.

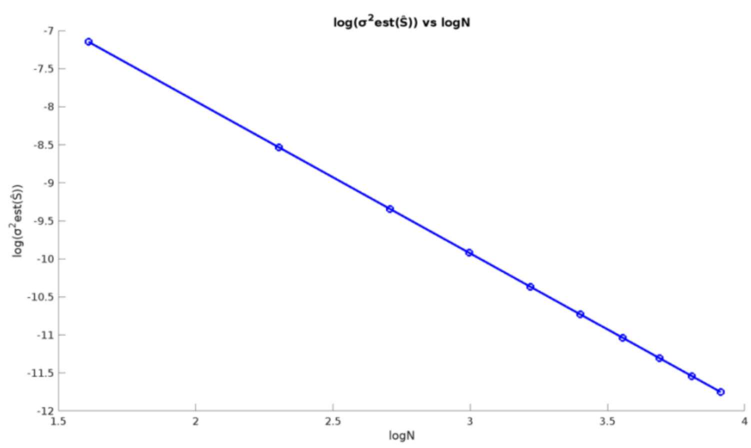


Figure 4: $log\sigma^2_{est}(\hat{s})$ vs. $logN$

7. Now we will probe how altering σ, the tuning curve width, affects σ^2_{est}. Use $[outputValues,] = histogramMLEstimates(s, N, rMax, deltaT, [], sigma, false)$ to find σ^2_{est} for $\sigma = 0.08$, 0.10, 0.12, ..., 0.98, 1.00. Once more, use s = 1, $\Delta t = 2$, and $r_{max} = 20$. Plot σ^2_{est} vs. σ on a semi-log axis (*semilogy* in Matlab). You should find that the curve is non-monotonic. At what value of σ is the function minimized?
When $\sigma = 0.08$, the function is minimized.

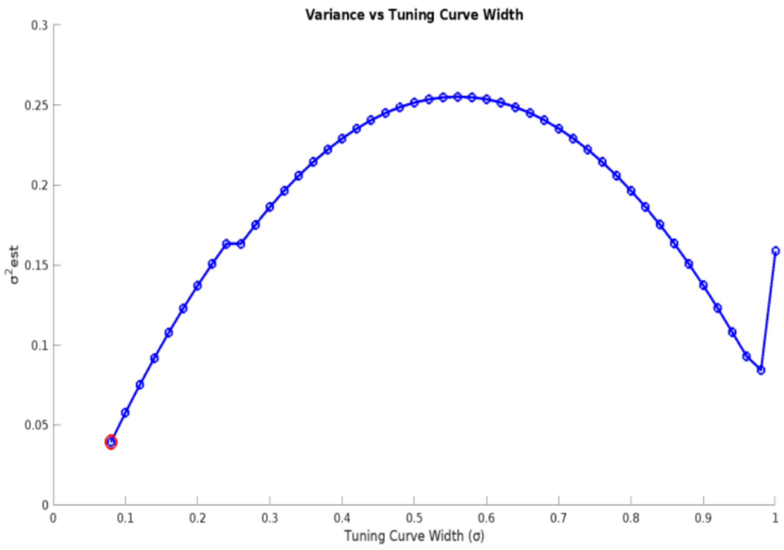


Figure 5: σ^2_{est} vs. σ

8. Use $drawReceptiveFields(N, rMax, sigma)$ to draw the receptive fields for the value of σ that you found in the previous question.

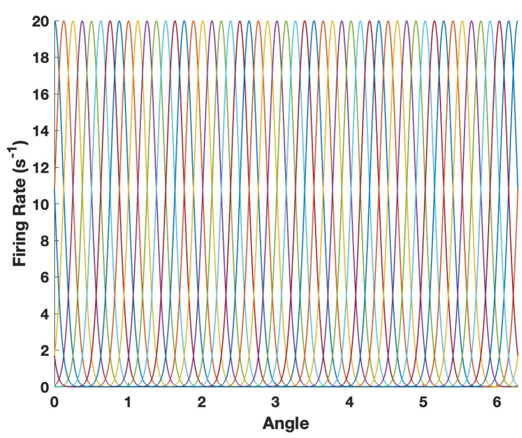


Figure 6: σ^2_{est} vs. σ

9. Give an intuitive explanation (no equations allowed) as to why there should be an optimal value for σ, rather than the optimal value being infinitely large or infinitesimally small.
An optimal value for σ, the receptive field width, because when receptive field is infinitely large or infinitely small the coding could be over-complicated (neuron wouldn't fire for different angles) or too find-tuned, in both way are not energetically efficient way for population coding.