BIO 450/IBS 534 HW #9

Attractor Networks

Due before 11:59pm, Thursday, April 3rd

Problem 1: Ring Attractor Dynamics in *Drosophila*

In class, we studied the ring attractor circuit in the central complex of the *Drosophila* brain, following the model proposed by Kim, Rouault, Druckmann, and Jayaraman (2017). In this exercise, you will implement a simplified version of the ring attractor circuit with sigmoidal activation, local excitation, and a central inhibitory neuron providing global feedback inhibition.

Part A: Model Setup

Network Architecture: The network consists of $N + 1$ neurons:

- Neurons $i = 1, \ldots, N$ are excitatory, arranged in a ring.
- Neuron $i = 0$ is a central inhibitory neuron, not on the ring.

Let $r_i(t)$ denote the activity of neuron i at time t. The inhibitory neuron $r_0(t)$ receives input from all excitatory neurons and sends uniform inhibition back to them.

Excitatory Neuron Dynamics ($i = 1, \ldots, N$):

$$\tau \frac{dr_i}{dt} = -r_i + F\left(\sum_{j=1}^{N} w_{ij} r_j + I_i(t) - w_{\text{inh}} r_0\right)$$

where:

- $F(z) = \frac{1}{1+e^{-\beta(z-\theta)}}$ is a sigmoidal activation function,
- $w_{ij} = A \exp\left(-\frac{(i-j)^2}{2\sigma_w^2}\right)$ defines local excitation on the ring (use modular arithmetic),
- $I_i(t)$ is an external stimulus to neuron i,
- r_0 is the activity of the inhibitory neuron, and w_{inh} controls the strength of global inhibition.

Inhibitory Neuron Dynamics:

$$\tau_0 \frac{dr_0}{dt} = -r_0 + F\left(\sum_{j=1}^{N} w_0 r_j\right)$$

where w_0 is the weight from each excitatory neuron to the inhibitory neuron (typically constant and positive).

Part B: Simulations

Stable Bump Attractor Simulate the network with no input ($I_i(t) = 0$), and initialize $r_i(0)$ with a small localized bump.

- Show that the bump is stable over time.
- Visualize activity as a heatmap or space-time plot.

Stimulus-Driven Bump Formation Apply a transient stimulus $I_i(t)$ centered on a subset of neurons (e.g. a Gaussian pulse).

- Show that the bump forms in the stimulated region and persists after the input is removed.
- Try different stimulus locations and show flexibility of bump location.

Parameter Exploration Explore the effects of varying:

- w_{inh}: strength of global inhibition,
- σ_w: width of the local excitation kernel.

Part C: Analysis and Discussion

- Why is the central inhibitory neuron necessary for stabilizing the bump?
- What happens when $w_{\text{inh}} = 0$? When $w_0 = 0$?
- Can the network support multiple bumps? Why or why not?
- Under what conditions does the bump drift or dissipate?

Problem 2: Transition to Chaos in Random Neural Networks

In class, we discussed how recurrent neural networks with random connectivity can exhibit a transition from fixed-point dynamics to chaotic behavior as the strength of recurrent input increases. This phenomenon was first explored in the seminal work by Sompolinsky, Crisanti, and Sommers (1988). While that paper focused on membrane potential dynamics, we will study this using firing rate dynamics for consistency.

Part A: Model Definition

Network Architecture:

- Simulate a network of N neurons with random recurrent connectivity.
- Each synaptic weight $J_{ij} \sim \mathcal{N}(0, \frac{1}{N})$, independently sampled.
- The state of the network is the firing rate $r_i(t)$ of each neuron.

Dynamics:

$$\tau \frac{dr_i}{dt} = -r_i + F\left(g \sum_{j=1}^{N} J_{ij} r_j\right)$$

where:

- $F(z) = \tanh(z)$ is a smooth, saturating nonlinearity,
- g is a gain parameter scaling recurrent input,
- τ is the time constant (you may set $\tau = 1$).

Part B: Simulations

Implement the Network

- Simulate the dynamics for $N = 500$ neurons over 100–200 time units.
- Initialize $r_i(0)$ randomly near zero (e.g., Gaussian noise).

Explore Behavior as a Function of Gain g

- Run simulations for $g = 0.5, 1.0, 1.5, 2.0$.
- For each g, plot trajectories of representative neurons.
- Compute and plot the population variance:

$$\mathrm{Var}_{\mathrm{pop}}(t) = \frac{1}{N} \sum_{i=1}^{N} \left(r_i(t) - \langle r(t) \rangle\right)^2$$

Observe the Transition

- At low g, expect convergence to a fixed point.
- At higher g, expect chaotic, sustained activity.

Part C: Analysis and Discussion

- Around what value of g does the network transition from stability to chaos?
- How does the time evolution of population variance differ across regimes?
- Why does increasing g destabilize the network?
- How do the dynamics differ qualitatively from the ring attractor network in Problem 1?