Morgan State University

Quantum Laboratory Sequence

Quantum Lab #1: Polarization Entanglement – Violating Bell's Inequality

Reference: Textbook Section

Background

The definition of an entangled state in quantum physics can be rather simple: It says two quantum objects cannot be described separately anymore, only together. For example, the polarization-entangled state of two particles given by below state representation cannot be factored (H = horizontal polarization; V = vertical polarization):

$$|\psi\rangle=\frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2+|V\rangle_1|H\rangle_2) \qquad (1)$$

In Eq. (1), the subscript 1 indicates the first particle (which can be a photon) and subscript 2 indicates the second particle. Note that the overall combination of terms in the parentheses cannot be factored (e.g. we cannot factor out $|H\rangle_1$ from the parentheses). Therefore, the state represented by Eq. (1) is entangled, such that particles 1 and 2 are always connected to each other.

The Bell Inequality was derived as a proof to show that "action at a distance", faster than the speed of light, is possible and real in quantum mechanics, contrary to Einstein's theory that nothing can travel faster than the speed of light, including information.

Bell used directionality of measurements to provide a test that entanglement and action-at-a-distance is possible. Assume particle 1 is to be measured at two angles α and α', while particle 2 is to be measured at angles β and β', relative to a common reference direction (e.g. the +z direction). Define the *normalized* measurement done on particle 1 at angle α as $|\bar{A}(\alpha)|\le\frac{1}{2}$ and the *normalized* measurement on particle 2 at angle β as $|\bar{B}(\beta)|\le\frac{1}{2}$. In other words, the measurements are such that the normalized value of a measurement must be equal or less than 1/2. Now, define the normalized product of the two measurements *taken simultaneously* as their *correlation*

$$E(\alpha,\beta)=\langle\bar{A}(\alpha)\cdot\bar{B}(\beta)\rangle\le\frac{1}{4} \qquad (2)$$

where the term in brackets mean "the average after many, many measurements". Bell showed mathematically that for action-at-a-distance to be false, then the result of these using the all four measurement angles must be less than or equal to 2, i.e.

$$S=|E(\alpha,\beta)-E(\alpha,\beta')+E(\alpha',\beta)+E(\alpha',\beta')|\le 2 \qquad (3)$$

Therefore, to prove that action-at-a-distance is a real, quantum effect, we must violate this inequality, meaning we must devise an experiment where we can show that S is greater than 2.

The experiment devised by Clauser, Horne, Shimony and Holt, known as the CHSH-Bell inequality test, uses the linear polarization of entangled photons measured at angles α, α' (photon 1), β and β'

(photon 2) to violate Eq. 3. One can show that for linearly polarized photons
such that Eq. (3) becomes

$$S=\cos(2|\alpha-\beta|)-\cos(2|\alpha-\beta'|)+\cos(2|\alpha'-\beta|)+\cos(2|\alpha'-\beta'|)\leq 2 \tag{4}$$

The maximum value of S from Eq. (4) occurs when $(\beta-\alpha)=(\alpha'-\beta')=(\beta'-\alpha')=22.5^{\circ}$ and $(\beta'-\alpha)=67.5^{\circ}$, which is achieved for α = 0º, α' = 45°, β = 22.5º and β' = 67.5°. Entering these values into Eq. (4) results in the maximum violation of $S=2\sqrt{2}\approx 2.828$. Thus, for this choice of angles, Bell's inequality is certainly violated and would prove that quantum action-at-a-distance is real.

Experimental

Per Eq. (3), we need the *normalized* correlations $E(\alpha,\beta)$ for four different angle combinations, namely α = 0º, α' = 45°, β = 22.5º and β' = 67.5°. Our setup provides coincidence counts due to a pulsed source of entangled photons traveling along different equal-length paths, and being absorbed by respective Avalanche Photodiodes (APDs). These APDs are triggered by the absorption of a photon, which is then amplified into a signal readable by the electronic instrumentation. To discriminate photon absorption events as "coincident", a very short time frame is allowed within which both APDs must trigger (~ 40 nanoseconds). If the separate APDs trigger at times too far apart, then the photons are not considered coincident.

The correlations between the states of two particles, whether entangled or not, can be calculated using coincidence counting for different combinations of the known states for particle 1 and particle 2. As shown in Fig. 1, the photon pair source, which might be producing polarization-entangled photons depending on the setup, sends each photon into separate detectors. Each detector has a defined polarizer in front, so only those photons having the same polarization as the polarizer will transit at 100% (photons with polarization not aligned with the polarizer will arrive at the detector less than 100%). Photons with linear polarization not aligned with the polarizer will transmit at less than 100% (i.e. cosine of the angle). The coincidence instrumentation counts how many times both photons arrive simultaneously over a rather long measurement averaging time (typically 1-2 seconds for this experimental setup).

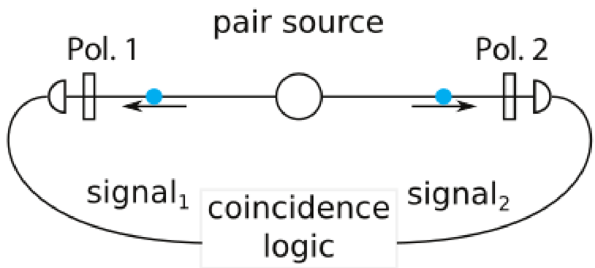

Figure 1. A photon pair source producing photons with certain polarizations can be measured with separate detectors that have defined polarizer orientations in front. The coincidence instrumentation counts how many photons arrive simultaneously for a given setup.

Define the coincident counts for the two APDs with the polarizers for photons 1 and 2 set to angles α and β as

$$C(\alpha,\beta)=\langle A(\alpha)\cdot B(\beta)\rangle \tag{5}$$

Note that this is a raw coincidence number, but we need the *normalized* correlation $E(\alpha,\beta)$. The normalized correlation can be calculated using the measured coincidence counts using

$$E(\alpha,\beta)=\frac{C(\alpha,\beta)-C(\alpha,\beta_\perp)-C(\alpha_p,\beta)+C(\alpha_\perp,\beta_\perp)}{C(\alpha,\beta)+C(\alpha,\beta_\perp)+C(\alpha_\perp,\beta)+C(\alpha_\perp,\beta_\perp)} \tag{6}$$

where $\alpha' = \alpha_\perp$ and $\beta' = \beta_\perp$, meaning the second setting for each polarizer is 90° from the original setting. Let's assume the vertical direction defines 0° polarization. The maximum polarization angle is then 180° (any larger angles are redundant). Thus, horizontal polarization means setting a polarizer to 90°. Equation (6) shows that to calculate each correlation $E(\alpha,\beta)$, four coincidence counting measurements are needed, and because we need four correlations at different angles to calculate the Bell violation S of Eq, (3), a total of 16 coincidence measurements are needed. The angle settings that provide the highest violation is shown in Table 1.

Table 1. Polarizer settings for each beam path to achieve maximum violation of Bell's Inequality

$E(\alpha,\beta)$	$C(\alpha,\beta)$	Polarization Angle 1	Polarization Angle 2
$E(0^\circ,22.5^\circ)$	$C(0^\circ,22.5^\circ)$	$\alpha = 0^o$	$\beta = 22.5^o$
	$C(0^\circ,112.5^\circ)$	$\alpha = 0^o$	$\beta_\perp = 112.5^o$
	$C(90^\circ,22.5^\circ)$	$\alpha_\perp = 90^o$	$\beta = 22.5^o$
	$C(90^\circ,112.5^\circ)$	$\alpha_\perp = 90^o$	$\beta_\perp = 112.5^o$
$E(45^\circ,22.5^\circ)$	$C(45^\circ,22.5^\circ)$	$\alpha = 45^o$	$\beta = 22.5^o$
	$C(45^\circ,112.5^\circ)$	$\alpha = 45^o$	$\beta_\perp = 112.5^o$
	$C(135^\circ,22.5^\circ)$	$\alpha_\perp = 135^o$	$\beta = 22.5^o$
	$C(135^\circ,112.5^\circ)$	$\alpha_\perp = 135^o$	$\beta_\perp = 112.5^o$
$E(0^\circ,67.5^\circ)$	$C(0^\circ,67.5^\circ)$	$\alpha = 0^o$	$\beta = 67.5^o$
	$C(0^\circ,157.5^\circ)$	$\alpha = 0^o$	$\beta_\perp = 157.5^o$
	$C(90^\circ,67.5^\circ)$	$\alpha_\perp = 90^o$	$\beta = 67.5^o$
	$C(90^\circ,157.5^\circ)$	$\alpha_\perp = 90^o$	$\beta_\perp = 157.5^o$
$E(45^\circ,67.5^\circ)$	$C(45^\circ,67.5^\circ)$	$\alpha = 45^o$	$\beta = 67.5^o$
	$C(45^\circ,157.5^\circ)$	$\alpha = 45^o$	$\beta_\perp = 157.5^o$
	$C(135^\circ,67.5^\circ)$	$\alpha_\perp = 135^o$	$\beta = 67.5^o$
	$C(135^\circ,157.5^\circ)$	$\alpha_\perp = 135^o$	$\beta_\perp = 157.5^o$

We would also like to know the error of the measurements to have confidence that Bell's inequality is actually violated. It can be shown that the uncertainty in correlations, $\Delta E(\alpha,\beta)$, can be calculated using the same coincidence count results, as

$$\Delta E(\alpha,\beta)=2\frac{[C(\alpha,\beta)+C(\alpha_{\perp},\beta_{\perp})][C(\alpha,\beta_{\perp})+C(\alpha_{\perp},\beta)]}{[C(\alpha,\beta)+C(\alpha_{\perp},\beta_{\perp})+C(\alpha,\beta_{\perp})+C(\alpha_{\perp},\beta)]^2}\sqrt{\frac{1}{C(\alpha,\beta)+C(\alpha_{\perp},\beta_{\perp})}+\frac{1}{C(\alpha,\beta_{\perp})+C(\alpha_{\perp},\beta)}} \tag{7}$$

The corresponding uncertainty in Bell's inequality is then the square root of the sum of squares of the correlation uncertainties, as

$$\Delta S=\sqrt{\sum_{\alpha,\beta}\Delta E(\alpha,\beta)^2} \tag{8}$$

The number of standard deviations that of uncertainty that fill the difference in Bell's inequality is given by

$$n_{\sigma}=\frac{S-2}{\Delta S} \tag{9}$$

A large value of n_{σ} (> 3) indicates a strong confidence that Bell's inequality has indeed been violated.

Lab Setup

Connect the fiber optics to the Avalanche Photodiode (APD) ports #1 and #2 of the quED Control & Readout (quCR) unit as shown in Figure 2.

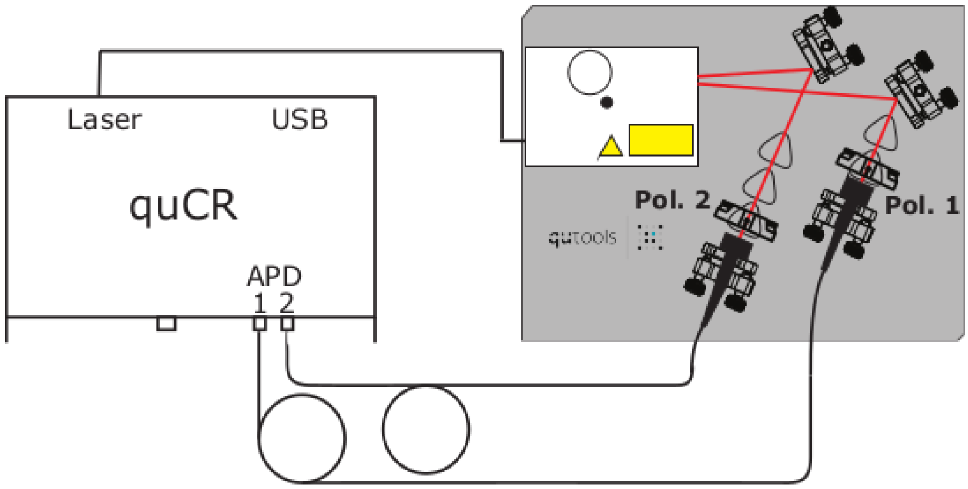

Figure 2. Setup for the Violating Bell's Inequality experiment

1. The quED should be aligned for best entanglement quality, see the quED manual for specifics.

2. The half waveplate should be inserted into the source in the usual direction (round black hole points towards the pump laser).

3. Mount the two polarizers into the intended slots as shown in Fig. 2.

Measurements and Calculations

1. On the quCR main instrument set the integration time to be 10 seconds

2. Measure the coincidence counts with 10 s integration time for all of the 16 measurement settings

3. Fill out the lab worksheet of Table 2 for each of the 16 measurements by writing in the observed coincident counts.

4. Using the worksheet of coincident counts in Table 2, calculate each normalized correlation $E(\alpha,\beta)$ using Eq. (6) and write the answer in the corresponding cell in column 1 of Table 2.

5. Calculate Bell's inequality using Eq. (3). Is it > 2?

6. Calculate the uncertainty of each of the four correlations using Eq. (7), and use these to calculate the uncertainty in the Bell inequality measurement using Eq. (8)

7. Finally, use Eq. (9) to calculate the number of standard deviations of confidence you obtained for this experiment (note your S must be greater than 2 for this to be valid)

Discussion

Table 2. Coincident count worksheet to fill out for each polarization setting.

$E(\alpha,\beta)$	$C(\alpha,\beta)$	**Polarization Angle 1**	**Polarization Angle 2**	**Counts**
$E(0^\circ,22.5^\circ)$	$C(0^\circ,22.5^\circ)$	$\alpha = 0^\circ$	$\beta = 22.5^\circ$	
	$C(0^\circ,112.5^\circ)$	$\alpha = 0^\circ$	$\beta_\perp = 112.5^\circ$	
	$C(90^\circ,22.5^\circ)$	$\alpha_\perp = 90^\circ$	$\beta = 22.5^\circ$	
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$E(45^\circ,22.5^\circ$	$C(45^\circ,22.5^\circ)$	$\alpha = 45^\circ$	$\beta = 22.5^\circ$	
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	$C(135^\circ,22.5^\circ)$	$\alpha_\perp = 135^\circ$	$\beta = 22.5^\circ$	
	$C(135^\circ,112.5^\circ$	$\alpha_\perp = 135^\circ$	$\beta_\perp = 112.5^\circ$	
$E(0^\circ,67.5^\circ)$	$C(0^\circ,67.5^\circ)$	$\alpha = 0^\circ$	$\beta = 67.5^\circ$	
	$C(0^\circ,157.5^\circ)$	$\alpha = 0^\circ$	$\beta_\perp = 157.5^\circ$	
	$C(90^\circ,67.5^\circ)$	$\alpha_\perp = 90^\circ$	$\beta = 67.5^\circ$	
	$C(90^\circ,157.5^\circ)$	$\alpha_\perp = 90^\circ$	$\beta_\perp = 157.5^\circ$	
$E(45^\circ,67.5^\circ$	$C(45^\circ,67.5^\circ)$	$\alpha = 45^\circ$	$\beta = 67.5^\circ$	
	$C(45^\circ,157.5^\circ)$	$\alpha = 45^\circ$	$\beta_\perp = 157.5^\circ$	
	$C(135^\circ,67.5^\circ)$	$\alpha_\perp = 135^\circ$	$\beta = 67.5^\circ$	
	$C(135^\circ,157.5^\circ$	$\alpha_\perp = 135^\circ$	$\beta_\perp = 157.5^\circ$	