

# Assignment 0

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Qishi Intermediate Machine Learning

**Problem 1.** Read the following:

- a) [https://www.ycc.idv.tw/deep-dl\\_1.html](https://www.ycc.idv.tw/deep-dl_1.html)
- b) <https://www.cs.ubc.ca/~schmidtm/Courses/440-W22/L22b.pdf>
- c) [https://www.ycc.idv.tw/deep-dl\\_3.html](https://www.ycc.idv.tw/deep-dl_3.html)

**Problem 2.**  $A \in \mathbb{R}^{m \times n}$ . Prove the following:

- a)  $N(A^T A) = N(A)$
- b)  $C(A^T A) = C(A^T)$
- c)  $r(A) = r(A^T A) = r(A A^T)$

Then we know that  $A^T A$  is invertible iff  $A$  has full column rank,  $A A^T$  is invertible iff  $A$  has full row rank.

**Problem 3.** When  $A \in \mathbb{R}^{n \times n}$  is symmetric positive definite, prove:

- a)  $A$  is non-singular
- b) All eigenvalues of  $A$  are positive.
- c)  $\exists$  full column rank matrix  $R$  s.t.  $A = R^T R$

**Problem 4.** Prove

$$\lambda_{\max} = \sup_x \frac{x^T A x}{x^T x} = \sup_{\|x\|_2=1} x^T A x$$

$$\lambda_{\min} = \inf_x \frac{x^T A x}{x^T x} = \inf_{\|x\|_2=1} x^T A x$$

**Problem 5.** Read I.8 of Gilbert Strang's book: *Linear Algebra and Learning from Data*

Remark: You can find this book in OneDrive/Books.