EVENTS & PROBABILITY

$$P(A \cup B) = P(A) + P(B) - P(AB)$$

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{1 \le i < j \le n} P(A_i A_j) + \sum_{1 \le i < j \le k \le n} P(A_i A_j A_k) + \cdots + (-1)^{n-1} P(A_1 A_2 \cdots A_n)$$

$$P\left(\bigcup_{n=1}^{\infty} A_n \mid B\right) = \sum_{n=1}^{\infty} P(A_n \mid B) \qquad P(A \mid B) = \frac{P(AB)}{P(B)}$$

$$P(A_1 \cdots A_n) = P(A_1) P(A_2 \mid A_1) P(A_3 \mid A_1 A_2) \cdots P(A_n \mid A_1 \cdots A_{n-1})$$

$$P(A) = \sum_{i=1}^{n} P(B_i) P(A \mid B_i) \qquad \bigcup_{i=1}^{n} B_i = \Omega$$

$$P(B_i \mid A) = \frac{P(B_i A)}{P(A)} = \frac{P(B_i) P(A \mid B_i)}{\sum_{j=1}^{n} P(B_j) P(A \mid B_j)}$$

RANDOM VARIABLES

Discrete !

distribution $\leftarrow F(x) = \sum_{i:\, x_i \le x} \underset{\text{mass function}}{f(x_i)}$ $\qquad E(X) = \sum_{i=1}^{n} x_i f(x_i) \qquad Var(X) = E(X^2) - E(X)^2$

bin (n, p) <u>Binomial distribution</u> $\quad f(k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad E(X) = np, \quad Var(X) = np(1-p)$

$P(\lambda)$ <u>Poisson distribution</u> $\quad f(k) = \frac{\lambda^k}{k!} e^{-\lambda}, \; k = 0, 1, 2, \cdots, \; E(X) = \lambda, \quad Var(X) = \lambda$

let $Y \sim bin(n, p)$, let $n \to \infty$, $p \to 0$, then $\lambda = np$.

<u>Trinomial distribution</u> $\quad f(r, w, n-r-w) = \frac{n!}{r!\,w!\,(n-r-w)!} p^r q^w (1-p-q)^{n-r-w}$

the possibility of r reds, w whites, $(n-r-w)$ blues

conduct n trials, red occurs with possibility r, white with possibility q, blue with possibility $(1-p-q)$

Ge (p). <u>Geometric distribution</u> $\quad f(k) = p(1-p)^{k-1}, \; k = 1, 2, \cdots \quad E(X) = \frac{1}{p}, \quad Var(X) = \frac{1-p}{p^2}$

W to be the time before 1^{st} success in bin (n, p).

Nb (r, p) <u>Negative binomial distribution</u> $\quad P(W_r = k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}, \; E(X) = \frac{r}{p}, \; Var(X) = \frac{r(1-p)}{p^2}$

W_r to be the waiting time for r^{th} success

Continuous !

distribution $\leftarrow F(x) = \int_{-\infty}^{x} f(u)\,du$

<u>Uniform distribution</u>

$$F(x) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a < x \le b \\ 1 & x > b \end{cases} \qquad f(x) = \begin{cases} 1 & a < x < b \\ 0 & \text{o.w.} \end{cases}$$

<u>Exponential distribution</u> $\quad F(x) = 1 - e^{-\lambda x}, \; x \ge 0, \; f(x) = \lambda e^{-\lambda x}, \; E(X) = \frac{1}{\lambda}, \; Var(X) = \frac{1}{\lambda^2}$

<u>Normal distribution</u> $\quad F(x) = \int_{-\infty}^{x} f(u)\,du, \quad f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \; E(X) = \mu, \; Var(X) = \sigma^2$

MULTIVARIABLE.

$X \sim P(\lambda_1),\ Y \sim P(\lambda_2) \Rightarrow Z = X+Y \sim P(\lambda_1+\lambda_2)$.

$X \sim bin(n,p),\ Y \sim bin(m,p) \Rightarrow Z = X+Y \sim bin(n+m, p)$

$X \sim N(\mu_1, \sigma_1^2),\ Y \sim N(\mu_2, \sigma_2^2) \Rightarrow Z = X+Y \sim N(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$

$Z = X-Y \sim N(\mu_1-\mu_2, \sigma_1^2+\sigma_2^2)$

X, Y independent. $Z = X+Y$

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(z-y) f_Y(y)\,dy = \int_{-\infty}^{\infty} f_X(x) f_Y(z-x)\,dx$$

$$\begin{cases} x = x(u,v) \\ y = y(u,v) \end{cases} \qquad J = \frac{\partial(x,y)}{\partial(u,v)}$$

$$f(x,y) = f(x(u,v), y(u,v))\,|J|$$

X, Y independent. $U = XY$

$$f_U(u) = \int_{-\infty}^{\infty} f_X\left(\frac{u}{v}\right) f_Y(v) \frac{1}{|v|}\,dv$$

$$U = \frac{X}{Y}$$

$$f_U(u) = \int_{-\infty}^{\infty} f_X(uv) f_Y(v)\,|v|\,dv$$

$E(X) = E(E(X|Y))$

$X_1, X_2, \dots,$ i.i.d N independent with $\{X_n\}$

$E(\sum_{i=1}^{N} X_i) = E(X_1)E(N)$.

MOMENTS & CHARACTERISTIC FUNCTION.

Moment generating function $M(t) = E(e^{tX})$

Characteristic function $\phi(t) = E(e^{itX})$

discrete X : $\varphi(t) = \sum_{k=1}^{\infty} e^{itx_k} P(X=k)$

continuous X : $\varphi(t) = \int_{-\infty}^{+\infty} e^{itx} f(x) dx$

bin (n, p) $\quad \varphi(t) = (pe^{it} + q)^n$

$P(\lambda)$ $\quad \varphi(t) = e^{\lambda(e^{it}-1)}$

$U(a, b)$ $\quad \varphi(t) = \frac{e^{ibt} - e^{iat}}{it(b-a)}$

$N(\mu, \sigma^2)$ $\quad \varphi(t) = \exp\left(i\mu t - \frac{\sigma^2 t^2}{2}\right)$

$Exp(\lambda)$ $\quad \varphi(t) = \left(1 - \frac{it}{\lambda}\right)^{-1}$

ESTIMATORS

$\chi^2(n)$ $X_1, \cdots, X_n \sim N(0,1)$ i.i.d $\chi^2 = X_1^2 + \cdots + X_n^2$ $\chi^2 \sim \chi^2(n)$

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ $s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ x_i from $N(\mu_1, \sigma_1^2)$

$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2(n-1)$.

$F(m,n)$ $X_1 \sim \chi^2(m)$, $X_2 \sim \chi^2(n)$ $F = \frac{X_1/m}{X_2/n}$ $F \sim F(m,n)$.

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ y_i from $N(\mu_2, \sigma_2^2)$

$F = \frac{s_x^2/\sigma_1^2}{s_y^2/\sigma_2^2} \sim F(m-1, n-1)$

$t(n)$ $X_1 \sim N(0,1)$ $X_2 \sim \chi^2(n)$ $t = \frac{X_1}{\sqrt{X_2/n}}$ $t \sim t(n)$

x_i from $N(\mu, \sigma^2)$

$t = \frac{\sqrt{n}(\bar{x} - \mu)}{s} \sim t(n-1)$