# Computing Rigid Body Rotations with Qiskit

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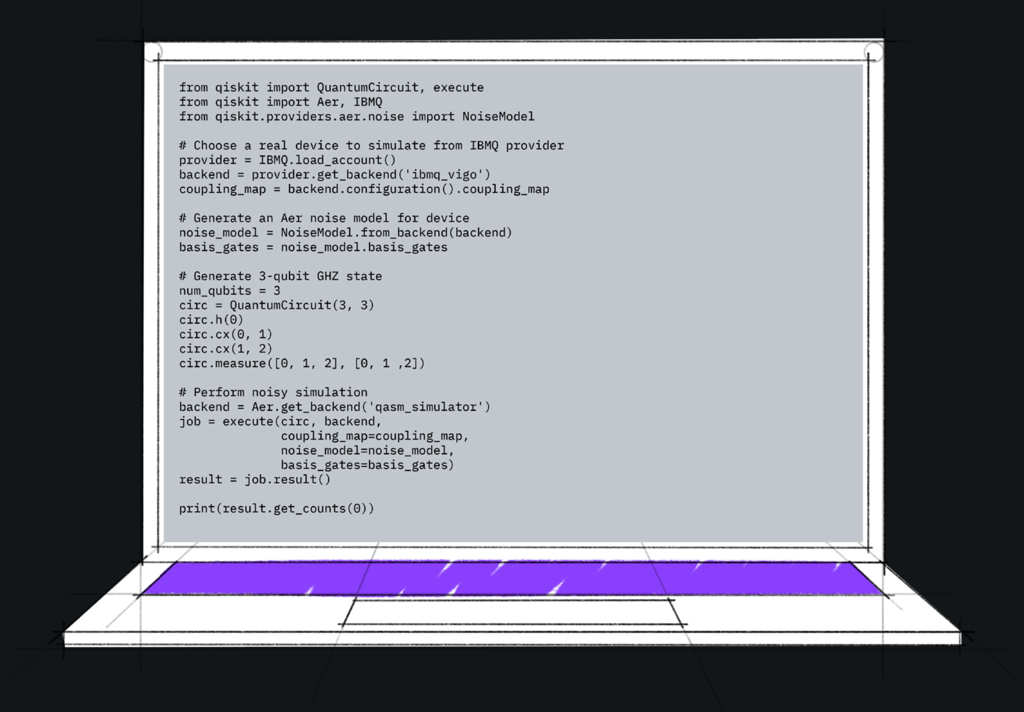
# Problem Definition

Qiskit

This project aims to study the feasibility of using current Quantum Computing devices to implement a method to compute the Euler-Rodrigues Parameters (ERP). These parameters allow for defining a transformation matrix to calculate rigid body rotations



# Team

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# Euler-Rodrigues Formula

This formula was first discovered by Euler and later rediscovered independently by Rodrigues, hence the terms Euler parameters for the parameters a, b, c, d and Euler–Rodrigues formula for the rotation matrix in terms of a, b, c, d [1].

$$\begin{bmatrix} a^2+b^2-c^2-d^2 & -2ad+2bc & 2ac+2bd \\ 2ad+2bc & a^2-b^2+c^2-d^2 & -2ab+2cd \\ -2ac+2bd & 2ab+2cd & a^2-b^2-c^2+d^2 \end{bmatrix}$$

where $a^2+b^2+c^2+d^2=1$

Alternatively,

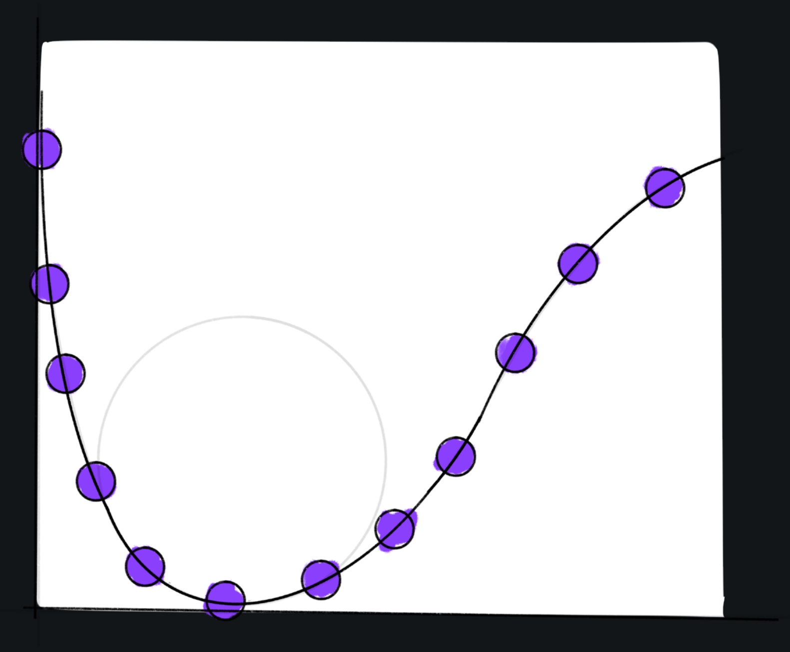
$$\bar{x}_{rot} = \bar{x}\cdot\cos(\theta) + \left(\bar{k}\times\bar{x}\right)\sin(\theta) + \bar{k}\left(\bar{k}\cdot\bar{x}\right)(1-\cos(\theta))$$

where $\bar{k}$ rotation axis and $\theta$ the rotation angle.

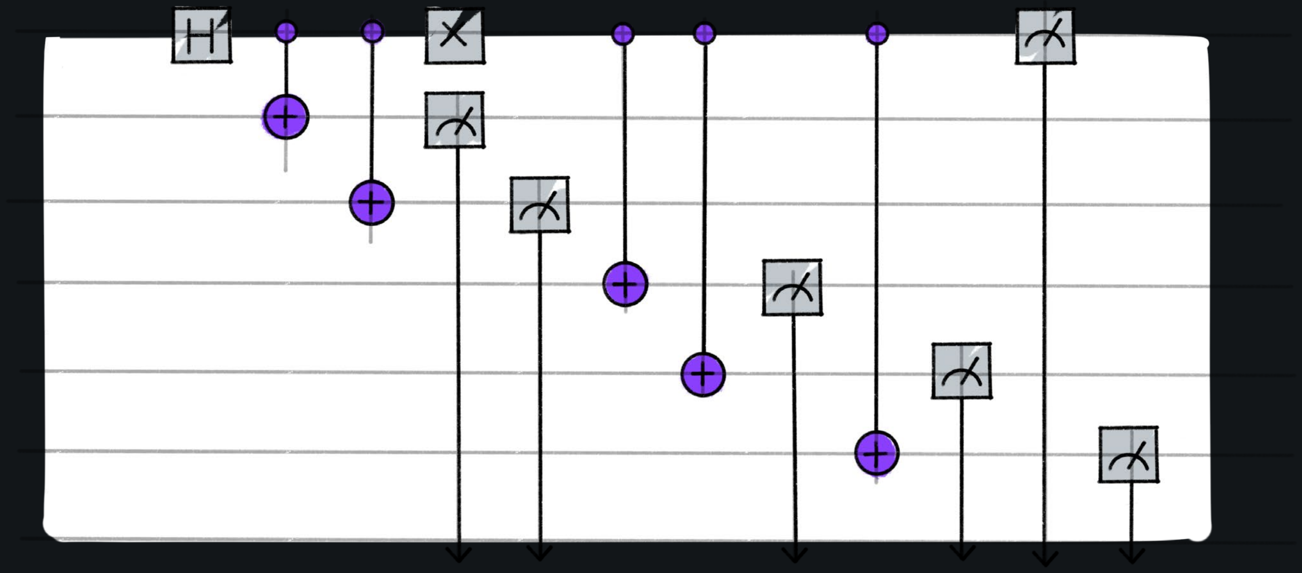
*[1] Mebius, J. E. (2007). Derivation of the Euler-Rodrigues formula for three-dimensional rotations from the general formula for four-dimensional rotations. arXiv preprint math/0701759.*

# Proposed Solutions

## A Variational approach



## A Circuit approach



# A Variational approach

Hybrid Quantum-Classical Neural Networks [3] will be used for rotating the  initial state(initial position) to the final state, after rotation through the euler angles $(\alpha, \beta, \gamma)$.

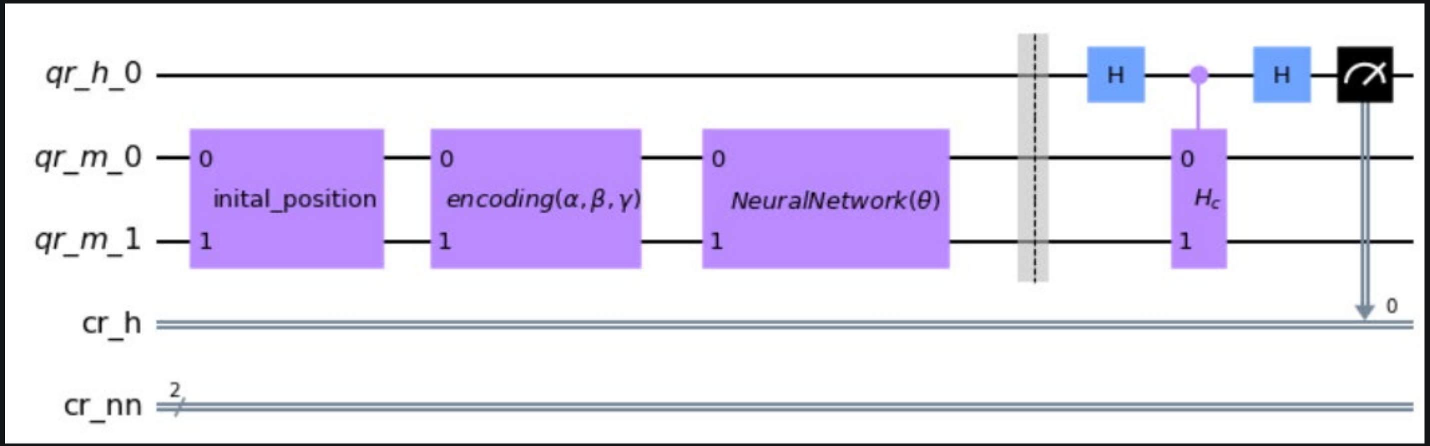
Variational encoding will be used to encode the euler angles.

The cost hamiltonian, will be used to give a measure of how far the state generated from our circuit is to the original true state. It is defined as:

$$H_c = I - |gt\rangle\langle gt|$$

where $|gt\rangle$ is the final true state.

Hadamard test will be used to measure the expectation value of our cost Hamiltonian





[3] Xia, Rongxin, and Kais, Sabre. Hybrid Quantum-Classical Neural Network for Calculating Ground State Energies of Molecules. United States: N. p., 2020. Web. https://doi.org/10.3390/e22080828.

[4] Bravo-Prieto, Carlos et al. "Variational Quantum Linear Solver: A Hybrid Algorithm for Linear Systems." Bulletin of the American Physical Society (2020)

# A Circuit Approach

Qiskit

We are looking for an efficient way of solving the cross product that defines the rotation matrix, which could be done by defining the matrix in a block encoding way such as described in [2]. A reformulation of the problem could be finding the parameters that satisfy the quaternion definition for the same rotation, which could be more efficiently implementable by using the group SU(2) to represent three-dimensional rotations in a 2 by 2 matrix.

$$U = \begin{pmatrix} a+di & b+ci \\ -b+ci & a-di \end{pmatrix} = a\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + b\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + c\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + d\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

$$= aI + ic\sigma_x + ib\sigma_y + id\sigma_z$$

*[2] Chakraborty, S., Gilyén, A., & Jeffery, S. (2018). The power of block-encoded matrix powers: improved regression techniques via faster Hamiltonian simulation. arXiv preprint arXiv:1804.01973.*