Implementation of Generalized Subspace Expansion (GSE)

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Purification-based method

Subspace Expansion
[McClean et al., Nature Communication, 2020]

Exponential Error Suppression
[Koczor, PRX, 2021] /
Virtual Distillation
[Huggins et al, PRX, 2021]

Zero-noise Extrapolation
[Temme et al., PRL, 2017]

QEM for coherent error

QEM for stochastic error

Generalized Subspace Expansion [Yoshioka et al., PRL, 2022]

- Generalization of error-agnostic QEM methods → wide applications
- QEM for both coherent/algorithmic error and stochastic error

A strong QEM tool for near-term quantum computation!!

Goal of This Project

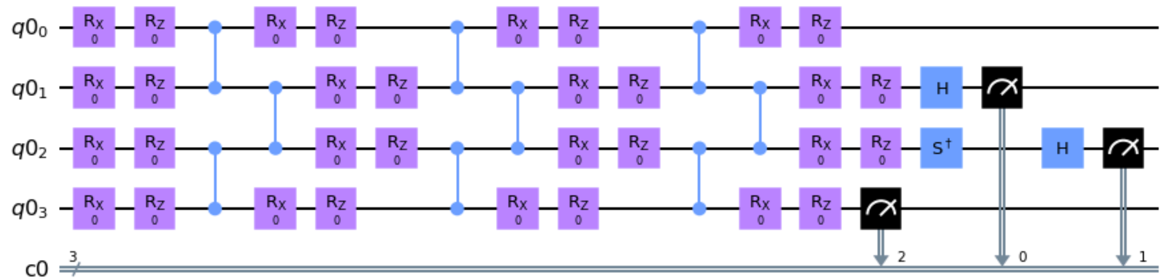
- Provide a GSE library in Qiskit
- Design a hardware efficient GSE circuit for IBM Quantum devices
- Check the performance of GSE on IBM Quantum devices

Checked the performance of GSE on noisy simulator with

- the expectation value of GHZ state (Checkpoint 1, 2)
- **the ground state of transverse-field Ising model (Checkpoint 3)**

Problem Setting

4-sites transverse-field Ising model $H = \sum_{(i,j)\in E} Z_i Z_j + h \sum_i X_i$



Variational Quantum Circuit

- Layers: 4
- Optimizer: BFGS in scipy.optimize.minimize
- Iteration steps: 100
- Backend: aer_simulator_density_matrix

To exclude the shot noise

Ran noisy simulation with different depolarizing levels

Generalized Subspace Expansion [Yoshioka et al., 2022, PRL]

→ Recover the mitigated expectation as a linear combination of expectation values by purification-based QEM

Given the original ansatz A, define the QEM ansatz ρ_{GSE} as

$$\rho_{GSE} = \frac{PAP^\dagger}{\mathrm{Tr}[PAP^\dagger]} \quad \text{where} \quad P(\vec{c}) = \sum_i c_i \sigma_i\,, \qquad \sigma_i = \sum_k \beta_k^{(i)} \prod_{l=1}^{L_k} U_{lk}^{(i)} \rho_{lk}^{(i)} V_{lk}^{(i)}$$

Condition of ρ_{GSE}:

- $\delta_{\vec{c}}\mathrm{Tr}[H\rho_{GSE}] = 0$
- $\mathrm{Tr}[\rho_{GSE}] = 1$

Lagrange multiplier

Optimal $\vec{c}$ is the solution of

$$\widetilde{H}\vec{c} = E\tilde{S}\vec{c} \quad \text{where} \quad \begin{cases} \widetilde{H}_{ij} = \mathrm{Tr}[H\sigma_i\rho\sigma_j] \\ \tilde{S}_{ij} = \mathrm{Tr}[\sigma_i\rho\sigma_j] \end{cases}$$

GSE can mitigate both coherent errors and stochastic errors!

Setting of GSE Ansatz and Subspaces $\rho_{GSE} = \frac{PAP^\dagger}{\mathrm{Tr}[PAP^\dagger]}$

Power subspace $\{\boldsymbol{\rho}, \boldsymbol{H\rho}\}$ $P(\vec{c}) = c_0\rho + c_1 H\rho$

GSE ansatz: $\rho_{GSE} = c_0c_0^*\rho + c_0c_1^*\rho^2 + c_1c_0^*\rho^2 + c_1c_1^*\rho^3$

$$\begin{bmatrix} \mathrm{Tr}[H\rho\rho] & \mathrm{Tr}[H\rho\rho H] \\ \mathrm{Tr}[HH\rho\rho] & \mathrm{Tr}[HH\rho\rho H] \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \boldsymbol{E} \begin{bmatrix} \mathrm{Tr}[\rho\rho] & \mathrm{Tr}[\rho\rho H] \\ \mathrm{Tr}[H\rho\rho] & \mathrm{Tr}[H\rho\rho H] \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

Fault subspace $\{\boldsymbol{\rho(\epsilon)}, \boldsymbol{\rho(2\epsilon)}, \boldsymbol{(3\epsilon)}\}$ $P(\vec{c}) = c_0\rho(\epsilon), +c_1\rho(2\epsilon) + c_2\rho(3\epsilon)$

GSE ansatz: $\rho_{GSE} = c_0c_0^*\rho + c_0c_1^*\rho^2 + c_1c_0^*\rho^2 + c_1c_1^*\rho^3$

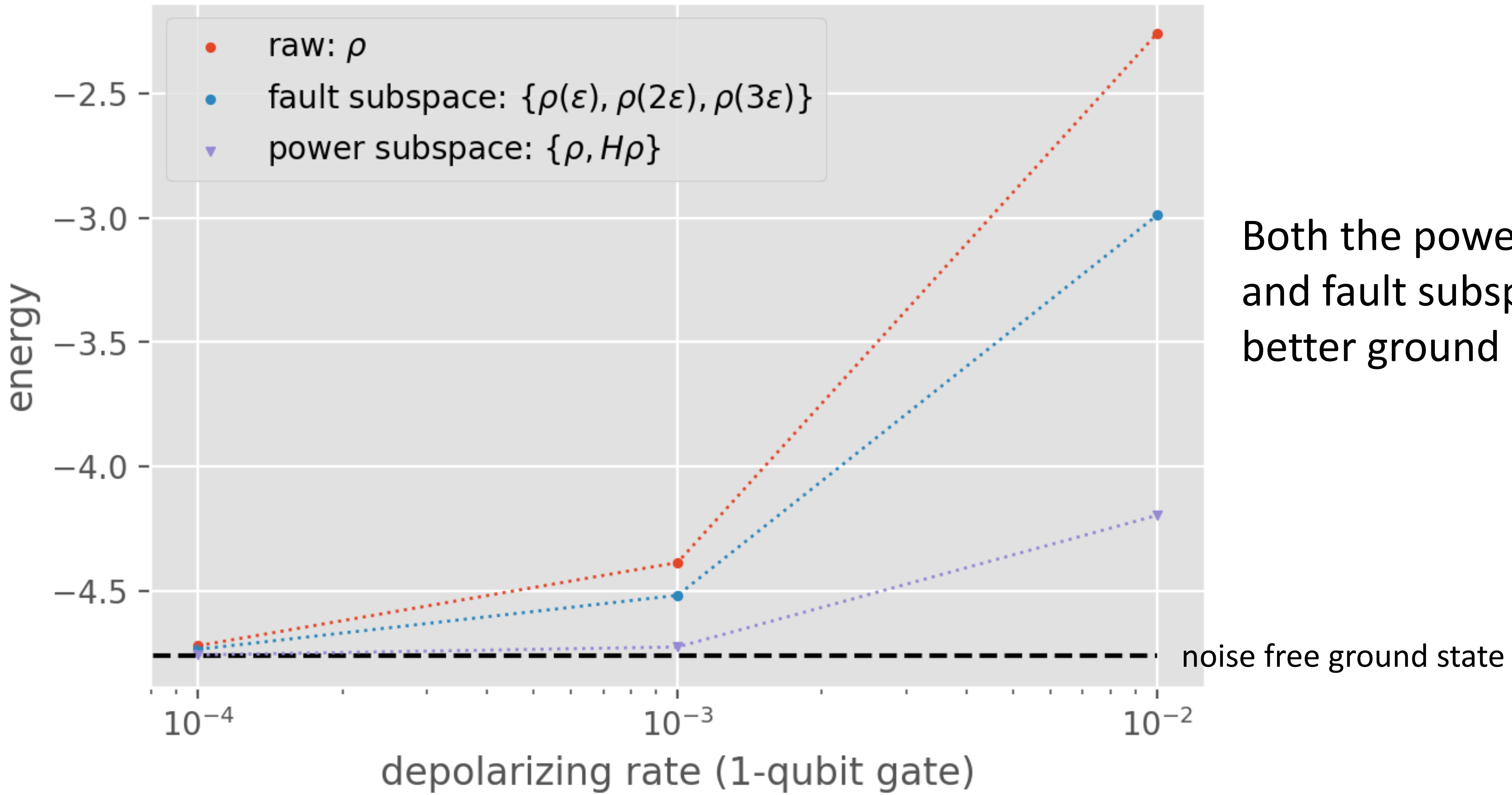
$$\begin{bmatrix} \mathrm{Tr}[H\rho(\epsilon)\rho(\epsilon)] & \mathrm{Tr}[H\rho(\epsilon)\rho(2\epsilon)] & \mathrm{Tr}[H\rho(\epsilon)\rho(3\epsilon)] \\ \mathrm{Tr}[H\rho(2\epsilon)\rho(\epsilon)] & \mathrm{Tr}[H\rho(2\epsilon)\rho(2\epsilon)] & \mathrm{Tr}[H\rho(2\epsilon)\rho(3\epsilon)] \\ \mathrm{Tr}[H\rho(3\epsilon)\rho(\epsilon)] & \mathrm{Tr}[H\rho(3\epsilon)\rho(2\epsilon)] & \mathrm{Tr}[H\rho(3\epsilon)\rho(3\epsilon)] \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \boldsymbol{E} \begin{bmatrix} \mathrm{Tr}[\rho(\epsilon)\rho(\epsilon)] & \mathrm{Tr}[\rho(\epsilon)\rho(2\epsilon)] & \mathrm{Tr}[\rho(\epsilon)\rho(3\epsilon)] \\ \mathrm{Tr}[\rho(2\epsilon)\rho(\epsilon)] & \mathrm{Tr}[\rho(2\epsilon)\rho(2\epsilon)] & \mathrm{Tr}[\rho(2\epsilon)\rho(3\epsilon)] \\ \mathrm{Tr}[\rho(3\epsilon)\rho(\epsilon)] & \mathrm{Tr}[\rho(3\epsilon)\rho(2\epsilon)] & \mathrm{Tr}[\rho(3\epsilon)\rho(3\epsilon)] \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

Results

Depolarizing noise level

1-qubit gates: $\{0.0001, 0.001, 0.01\}$

2-qubit gates: 10 times larger than 1-qubit gates



Both the power subspace and fault subspace gave better ground state energies.

Remaining Tasks

Run GSE on the real devices and write a research report

Next...

1. Run the programs on the fake backends
2. Run the programs on the real backends

TODO

- Rewrite the VQE/GSE process with Qiskit Runtime (as an OSS product)
- Run the simulation with finite shot counts
- Further optimize the circuit under the real device constraints
- Implement general ansatz and subspaces

We will continue to work on this project after QAMP!

Many thanks for this opportunity and the warmest support!