Report: QAMP Fall 2022 Checkpoint-2
Issue 15: Building out qiskit-qec (Implementing the XP Formalism in qiskit-qec)
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The goal is to implement the algorithms and functionality developed by Mark Webster in the XPF package in the framework of qiskit-qec. To that end, the first task carried out was implementing the basic structure for the class BaseXPPauli in a format similar to BasePauli class in qiskit-qec. The purpose of this class is to contain functions for algebra of XP Pauli operators, which would later be used by XPPauli and XPPauliList classes. In this task, the dummy class BaseXPPauli was added with placeholders for specific functions. This created the basic software structure for further programming. (PR: https://github.com/qiskit-community/qiskit-qec/pull/259).

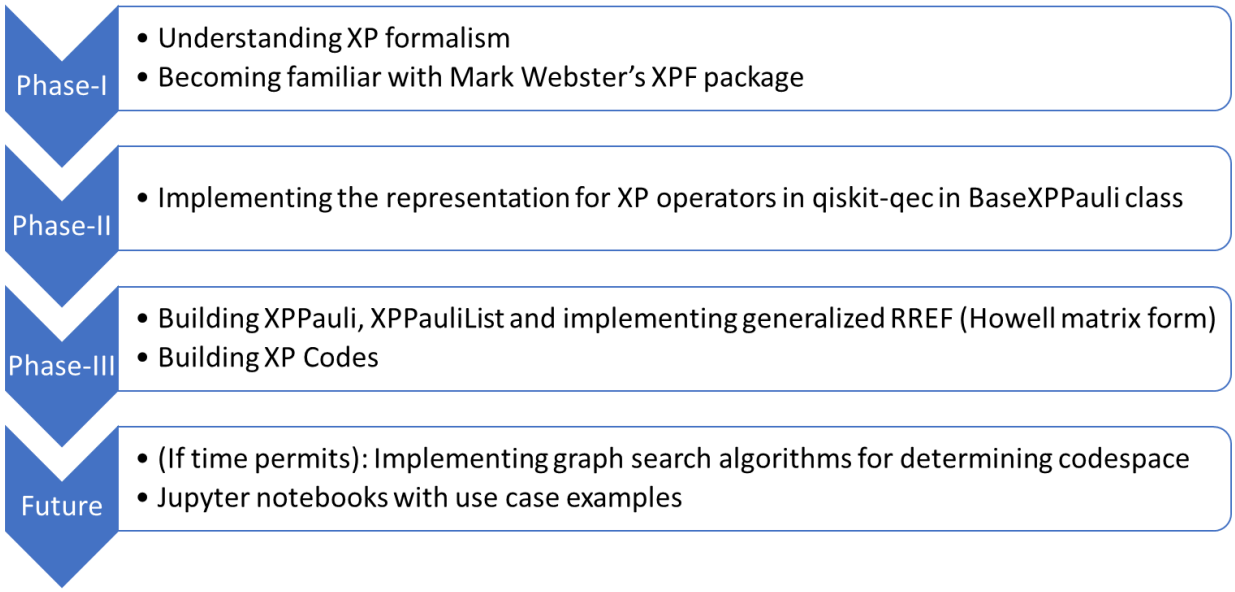
The second task we worked on is divided into two themes.

The first part is implementation of the Howell matrix form, which is the generalization of the reduced row echelon form (RREF) that gives us a canonical basis for the row span of a matrix over a ring. This is central to many algorithms for the XP formalism, including those for determining codespaces and logical operators. We first implemented a handful of basic operations that define the modular arithmetic over rings, such as Gcdex and Stab [1]. Then for an arbitrary matrix over a ring, one can convert it to the Howell matrix form by using the algorithm described in one of Mark Webster's XPF package demos. The function implemented, called howell, takes a matrix M and the modulus N as inputs, and outputs its Howell matrix form H, the transformation matrix U (U @ M = H), and the kernel matrix K (M @ K = 0).
(PR: https://github.com/qiskit-community/qiskit-qec/pull/279)

The other part is the implementation of functionality analogous to Pauli and PauliList classes, in the form of XPPauli and XPPauliList classes. Both these new classes extend from BaseXPPauli and the basic infrastructure for these classes and testing them has been coded. Next is to implement initial functionality of XP operator algebra acting on generalized vector representation of XP operators from Mark's XPF package into BaseXPPauli, so that it can be used by XPPauli and XPPauliList. These are functions to calculate the unique representation of XP operators, calculating quantities like degree and weight, and carrying out algebraic manipulation, such as multiplication. A substantial number of these functions and their tests have been implemented, and this is still in progress.
(PR: https://github.com/qiskit-community/qiskit-qec/pull/281)

The next steps would be adding the remaining functions for XP algebra to BaseXPPauli for both XPPauli and XPPauliList, and then moving on to XP codes. If time allows, we will also implement the graph search algorithm, which is an efficient component (compared with the exhaustive algorithm) in the codewords algorithm for identifying the codespace stabilized by an arbitrary XP code.

Visual:



Project flow for QAMP 15: Implementing XP Formalism in Qiskit-qec

References:

1. Arne Storjohann. Algorithms for Matrix Canonical Forms. PhD thesis, Department of Computer Science, Swiss Federal Institute of Technology – ETH, 2000. URL https: //cs.uwaterloo.ca/~astorjoh/diss2up.pdf.