Quantum Simulating Phase Transition via Partition Function Zeros

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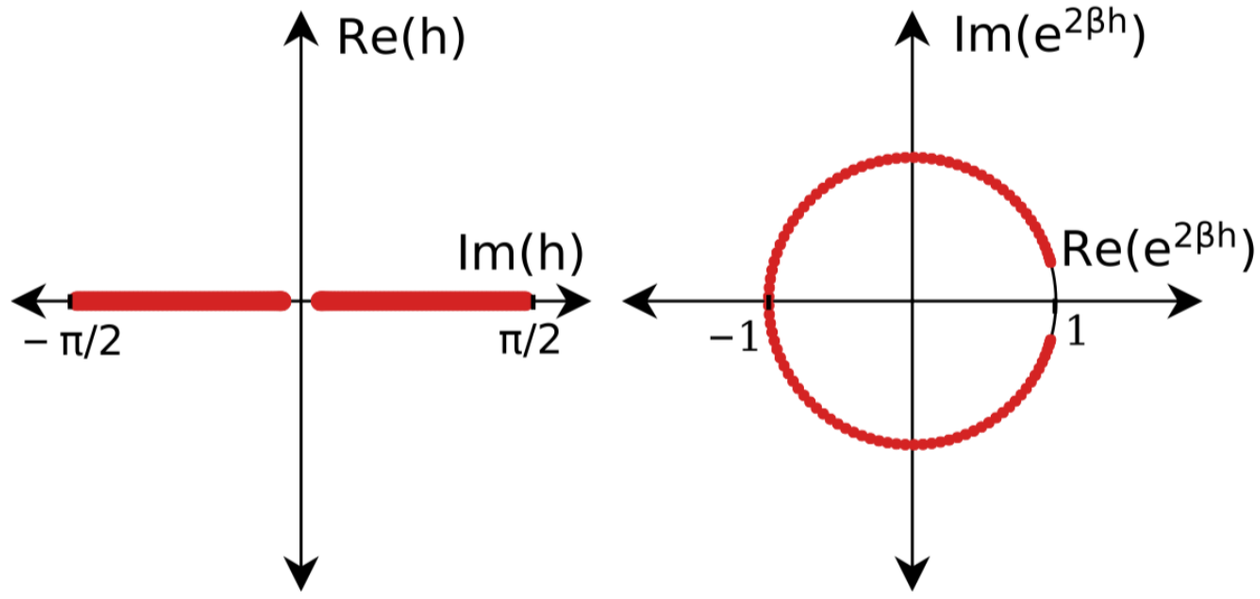
Phase transition from partition function zeros

- In the thermodynamic limit, the free energy of a system becomes singular (non-analytic) at the critical point of a phase transition.

- Lee-Yang zeros: Through analytical continuation of free energy into the *complex* plane of control parameters, Yang and Lee showed that singularities of free energy, given as zeros of the partition function ($F = -k_B T \ln Z$), accumulate exactly at the transition point.

- Classical Ising model:

$$H = -\sum_{i,j} J_{ij} s_i s_j - h \sum_j s_j$$



Unit Circle Theorem (in complex fugacity plane)

Refs:
Yang & Lee, Phys. Rev. 87, 404 (1952);
Yang & Lee, Phys. Rev. 87, 410 (1952);
Fig taken from Francis et al., Sci. Adv. 7, eabf2447 (2021)

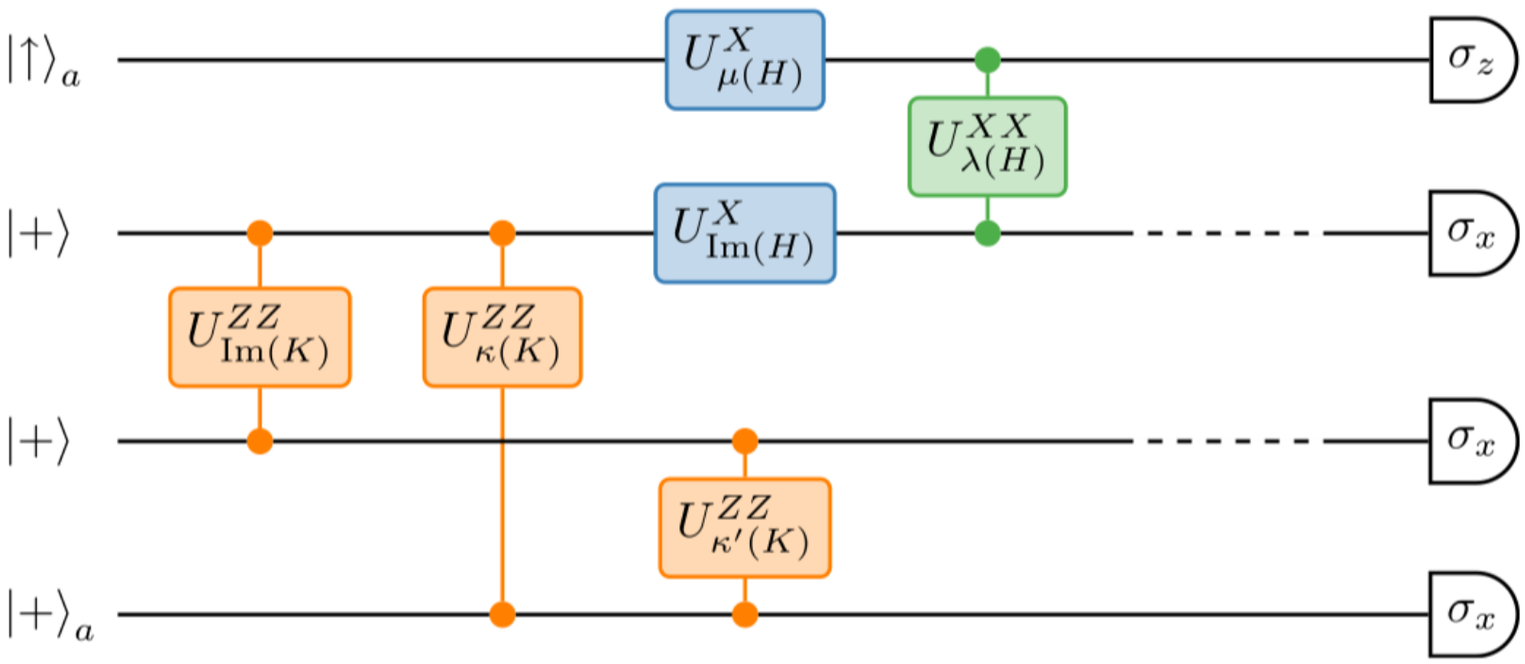
Measuring partition function zeros on quantum computers

- Krishnan et al. proposed a way of measuring the partition function zeros and spin correlation functions for classical Ising model via quantum circuits

Ref: Krishnan et al., PRA 100, 022125 (2019).

$$Z = \sum_{\vec{s}} \exp\left(-\sum_{i,j} K_{ij} s_i s_j - \sum_i H_i s_i \right),$$

$$\left| \langle + | \prod_{i,j} e^{-K_{ij}^R \sigma_i^z \sigma_j^z} e^{-iK_{ij}^I \sigma_i^z \sigma_j^z} e^{-H_i^R \sigma_i^z} e^{-iH_i^I \sigma_i^z} | + \rangle \right|^2 = \frac{|Z|^2}{2^{2N}}$$



Project plan

- Verify existing results in Krishnan et al. (2019)
  - Verify the expression of return probability ✓
  - Construct and run the quantum circuit on Qiskit (simulator) to reproduce some of the plots
- Investigate spin glass models and their phases using this technique on real quantum hardware