Exploring Zero Noise Extrapolation (ZNE) for Circuit Cutting

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Abstract

Circuit cutting has been proposed as a method to partition a circuit into multiple smaller subcircuits or fragments. Each fragment can be executed independent of the other, and the final probability distribution of the uncut circuit is computed via classical postprocessing over the individual probability distributions of the smaller fragments. The goal of quantum circuit cutting is to fit a large quantum algorithm onto a smaller, more accessible quantum computer. With reduced circuit width and depth for each subcircuit, circuit cutting alone is expected to be a method to lower noise in the quantum systems. However, this is not the case for every type of noise. In this project we aim to study the effect of Zero Noise Extrapolation (ZNE) on circuit cutting for different noise.

1 Background

Quantum circuit cutting is a technique for executing large quantum circuits on smaller quantum devices by dividing them into subcircuits. These can be run separately on the same or different devices. The outputs are then combined using classical post-processing. This method addresses issues common in Noisy Intermediate Scale Quantum (NISQ) devices, limited qubits. It also enables distributed quantum computing across multiple devices [2, 4, 3, 1].

Quantum circuit cutting breaks down complex quantum circuits into simpler subcircuits. A qubit is symbolized by a 2×2 density matrix ρ, and can be expressed in the Pauli matrices basis as [2]:

$$\rho = Tr(\rho I)I + Tr(\rho X)X + Tr(\rho Y)Y + Tr(\rho Z)Z/2 \tag{1}$$

We denote the Pauli matrices as P_i, their eigenprojectors as S_i, and the corresponding eigenvalues as $2\lambda_i$. The process depends on the resolution of the identity channel, represented by the mapping $\Phi_i(\rho) = Tr(\rho P_i)S_i$. The resolution is given by:

$$\Phi(\rho) = \sum_{i=1}^{8} \lambda_i Tr(\rho P_i) S_i \tag{2}$$

Circuit cutting involves performing measurements on two subcircuits for each cut made on the channel. The original result is reconstructed by computing the sum of eight pairs of Kronecker products.

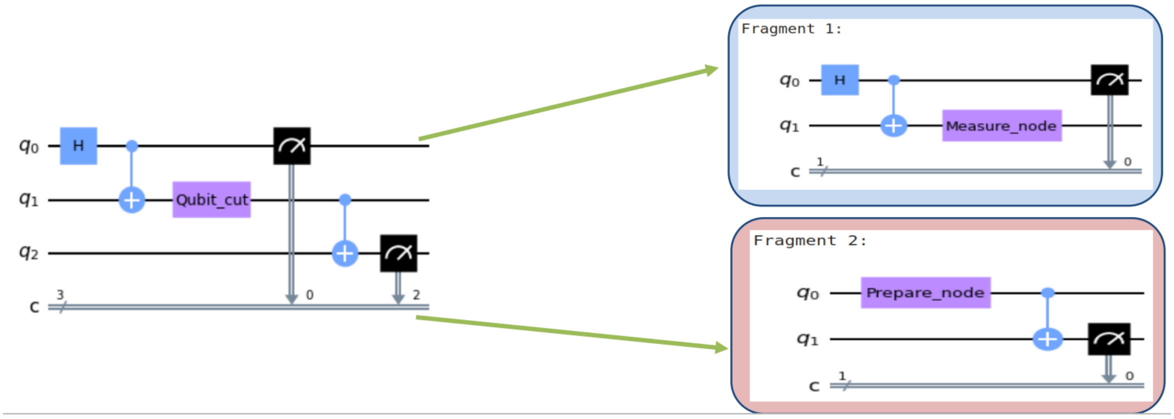

Figure 1: Example of circuit cutting for 3-qubits GHZ circuuit. Applying cut on q_1 resulted in two subcircuits or fragments

Noise remains a major hurdle in today's quantum hardware devices, impacting the viability of quantum algorithms on these devices. ZNE, a promising quantum error mitigation technique, estimates the noise-free result of quantum computation by executing the computation at varying noise levels. As shown in Fig. 2, ZNE is a biased error mitigation technique which computes the expectation value of an observable at different noise scales and extrapolates it to determine the expectation value of the observable at zero noise limit. This project seeks to assess the potential benefits of applying ZNE in conjunction with quantum circuit cutting to tackle the noise issues plaguing quantum computations.

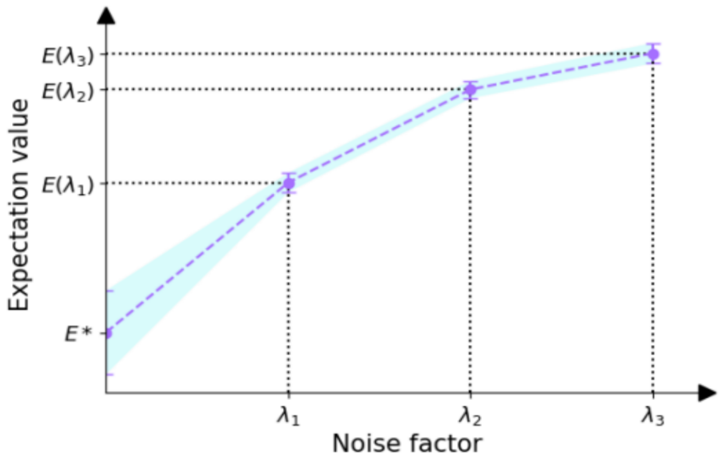

Figure 2: ZNE with three noise factors

2 Implementation and Experimental Results

In this study we applied ZNE on each fragment (see Fig. 1) obtained after circuit cutting, and computed the expectation value of the full circuit at zero noise limit. The aim is to understand the effectiveness and the overhead of applying ZNE on circuit cutting; whether ZNE on each subcircuit can indeed produce expectation values closer to the ideal one than ZNE on the entire circuit.

We tested the implementation on GHZ circuits so that cutting is trivial. First, we applied varying number of cuts to a 20-qubits symmetric (see fig 3) and assymetric GHZ circuits and executed the fragments of the circuits using a simulator backend with depolarizing noise probability of 0.01. Using noise factors of 1, 3, and 5 and setting Local folding amplifier to 2, we applied linear ZNE extrapolator (using IBM's ZNE framework) to each circuit fragment and recombined them to get the uncut expectation values. The results of this simulation is shown in fig. 4 and fig. 5. The results show lower expectation error or improvement in the expectation values of the GHZ circuits when ZNE is applied to the circuit fragments.

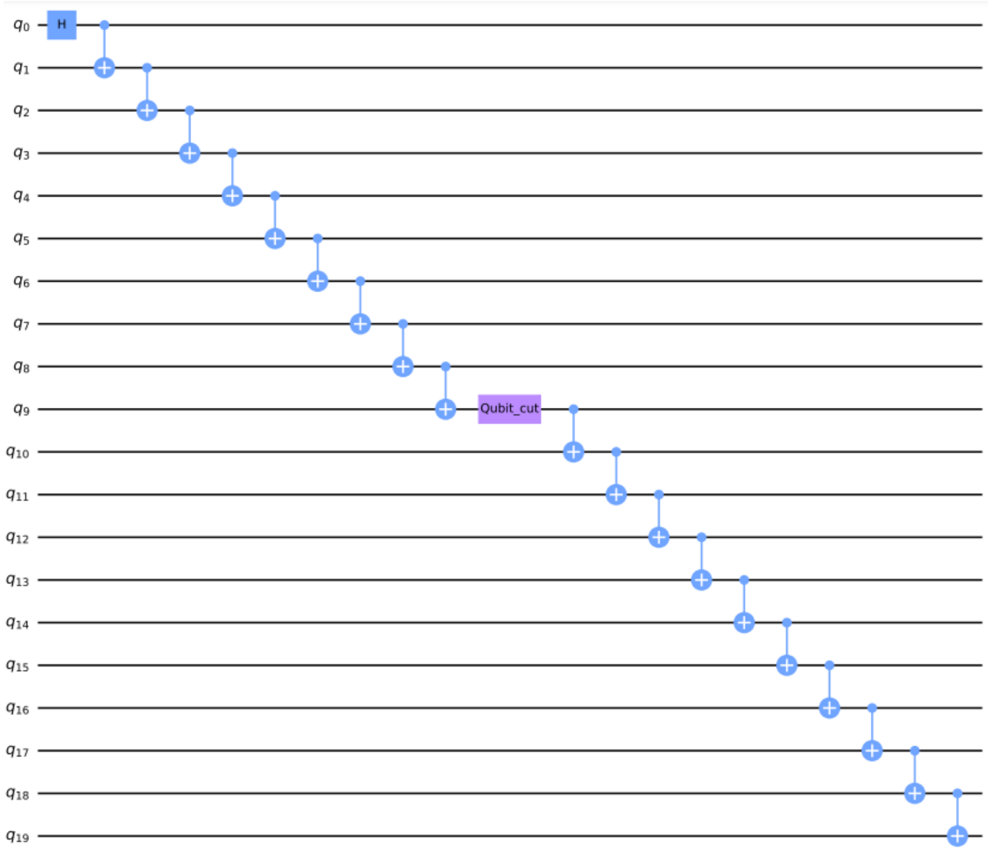

Figure 3: 20-Qubit Symmetric GHZ circuit

Also, we chose GHZ circuits with even number of qubits varying from 4 to 20 qubits. This is to ensure that the expected expectation value is always 1 for ease of comparison. We applied linear ZNE extrapolator of degree 1 to the circuits with depolarizing noise probability of 0.01, and noise factors 1, 3, and 5. Only one cut was applied to the different circuits resulting in two subcircuits. As shown in fig. 6, the reconstructed expectation value of cut circuit with ZNE is better than that without ZNE. However, in the presence of depolarizing noise, there was no significant improvement by applying only circuit cut.

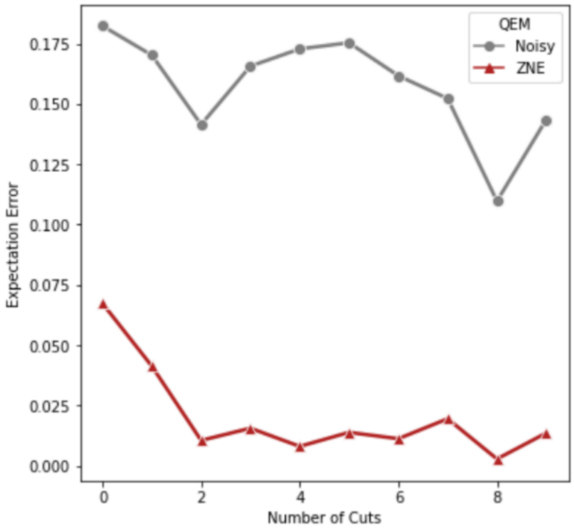

Figure 4: 20-Qubit Symmetric GHZ circuit

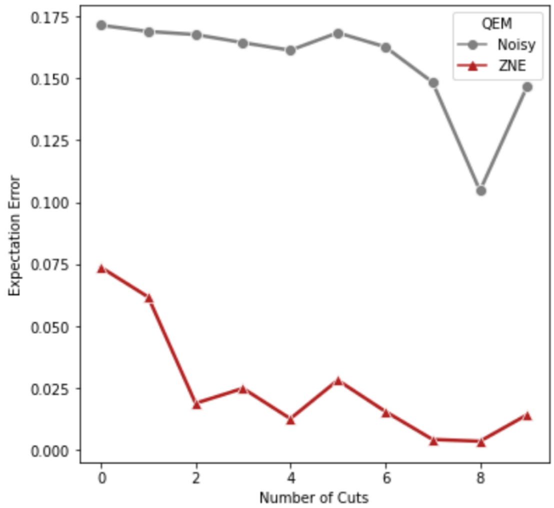

Figure 5: 20-Qubit Asymmetric GHZ circuit

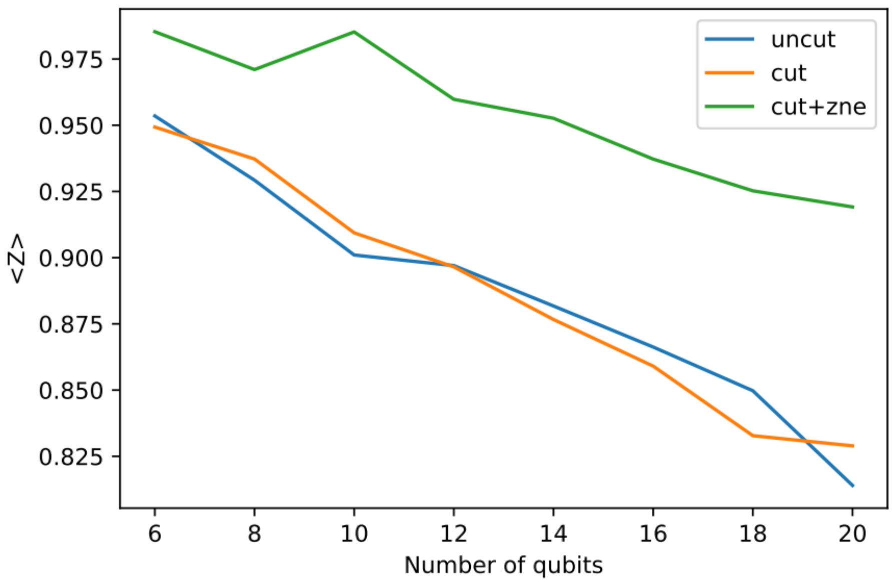

Figure 6: ZNE with three noise factors

3 Outlook

We also attempted applying ZNE to circuits in the presence of coherent noise. However, it was difficult to replicate the improvement in expectation values we observed for depolarizing noise. Ideally we don't expect ZNE to be useful for coherent noise, but we want to understand why this is the case especially with circuit cutting. We want to also understand why ZNE is performing well for circuit cutting in the presence of depolarizing noise.

Our next step will be to investigate how the variance (or rmse) of circuit cutting with zne compares to uncut circuit with zne. We also want to understand how the number of shots affect the mean and the variance. For a given circuit, as the circuit becomes deep, ZNE will start preferring higher degree polynomials as extrapolator. For instance for n qubits, uncut circuit with zne with quadratic/exponential extrapolator is the best option. Can circuit cutting with zne still have linear extrapolator as the best extrapolator? We expect that at some point uncut circuit with zne is not good anymore, since noise starts to overwhelm. At that point, does circuit cutting with zne still have acceptable performance in that regime?

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