

Project 33: Comprehensive Tutorial on Two Qubit Decomposition (KAK + Weyl Chamber)

Final Showcase

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Paper #1: Constructive Quantum Shannon Decomposition from Cartan Involutions

- Two qubit decomposition using Cartan KAK decomposition
- Base case for more general compilation routine

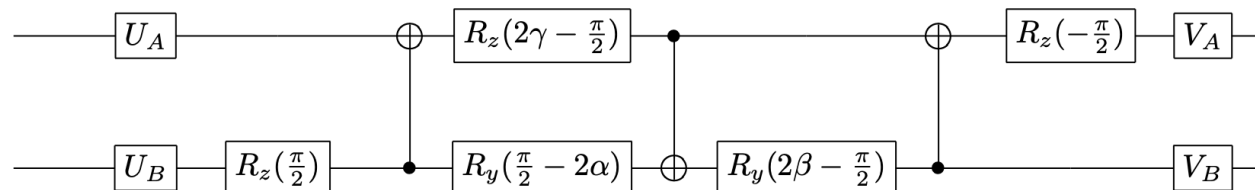


Figure 1. The CNOT optimized universal two qubit circuit; U_A , U_B , V_A , and V_B may be decomposed into 3 single qubit rotations each by the Euler angle decomposition given above, and V_A and U_B may absorb the z -rotations preceding and following them respectively yielding a circuit consisting of 3 CNOT gates and 15 single qubit rotations.

What is KAK Decomposition?

- KAK is a method to decompose two-qubit gates into entangling gates (such as CNOT) and single qubit gates. Using Cartan decomposition from Lie Algebra, KAK has the form:

$$U(4) = e^{i\phi} (A_0 \otimes A_1) e^{i(k_1\sigma_X^2 + k_2\sigma_Y^2 + k_3\sigma_Z^2)} (B_0 \otimes B_1)$$

where

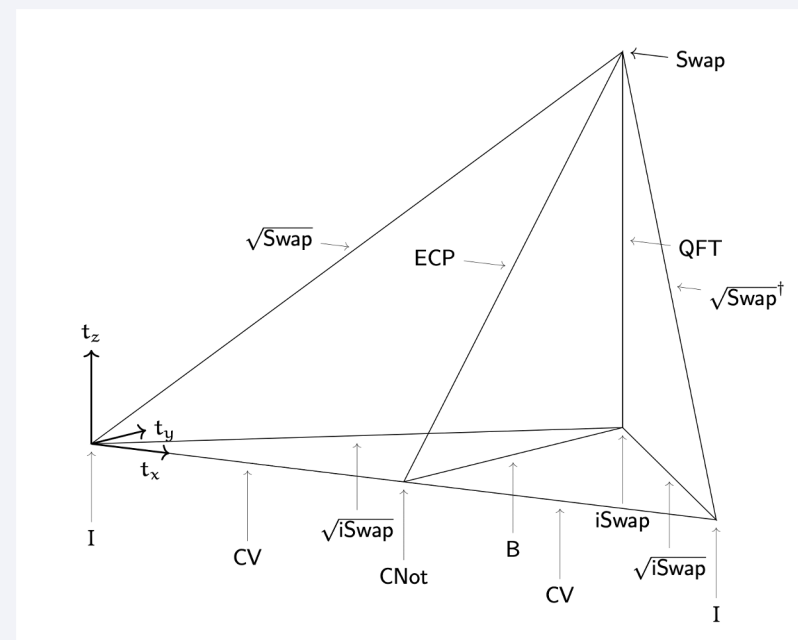
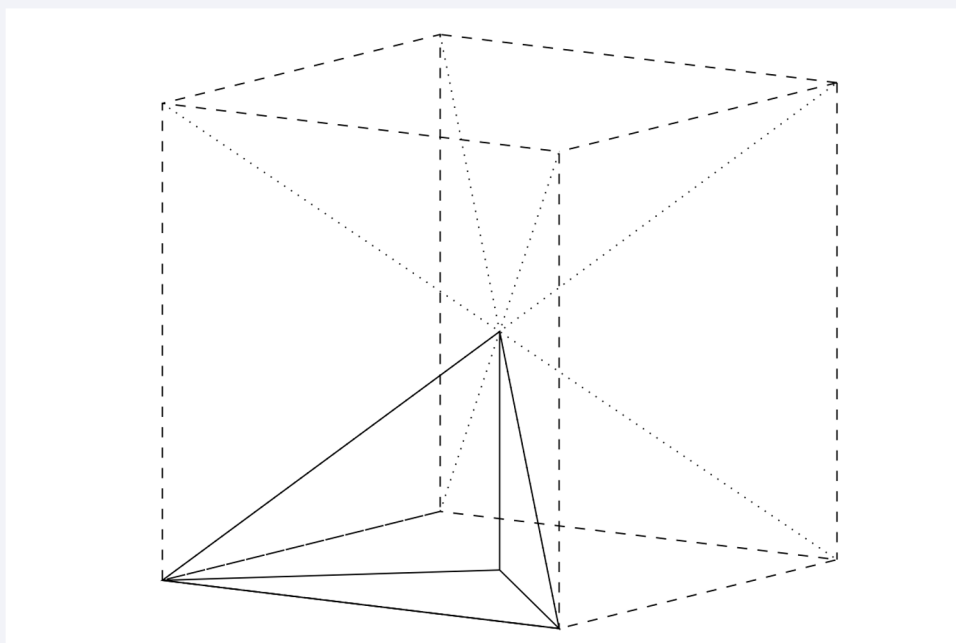
- $U(4)$ is the two-qubit unitary matrix to be decomposed
- Φ is the global phase
- A_0, A_1, B_0, B_1 are single qubit gates
- k_1, k_2, k_3 are “canonical” parameters

Sources:

[1] Drury, B., & Love, P. (2008). Constructive quantum Shannon decomposition from Cartan involutions. *Journal of Physics A: Mathematical and Theoretical*, 41(39), 395305. <https://doi.org/10.1088/1751-8113/41/39/395305>

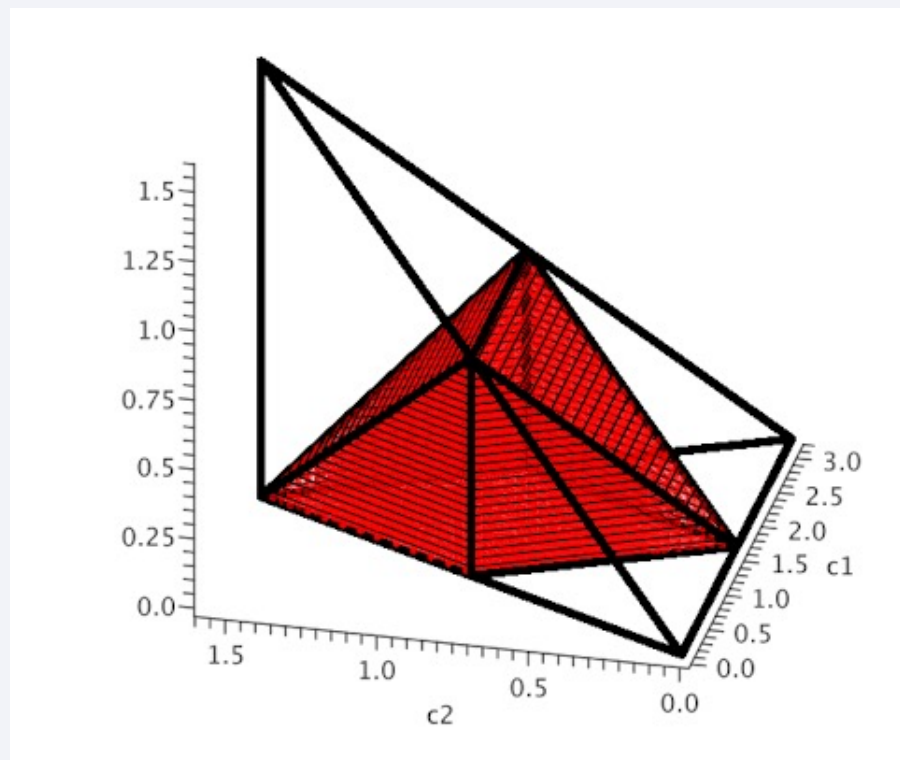
Paper #2: A geometric theory of non-local two-qubit operations

- The Weyl chamber is a geometric tools to “clean up” the KAK decomposition. Formally, it reduces the symmetry symmetry of the “canonical” parameters k_1, k_2, k_3 and allows for “almost unique KAK decomposition.



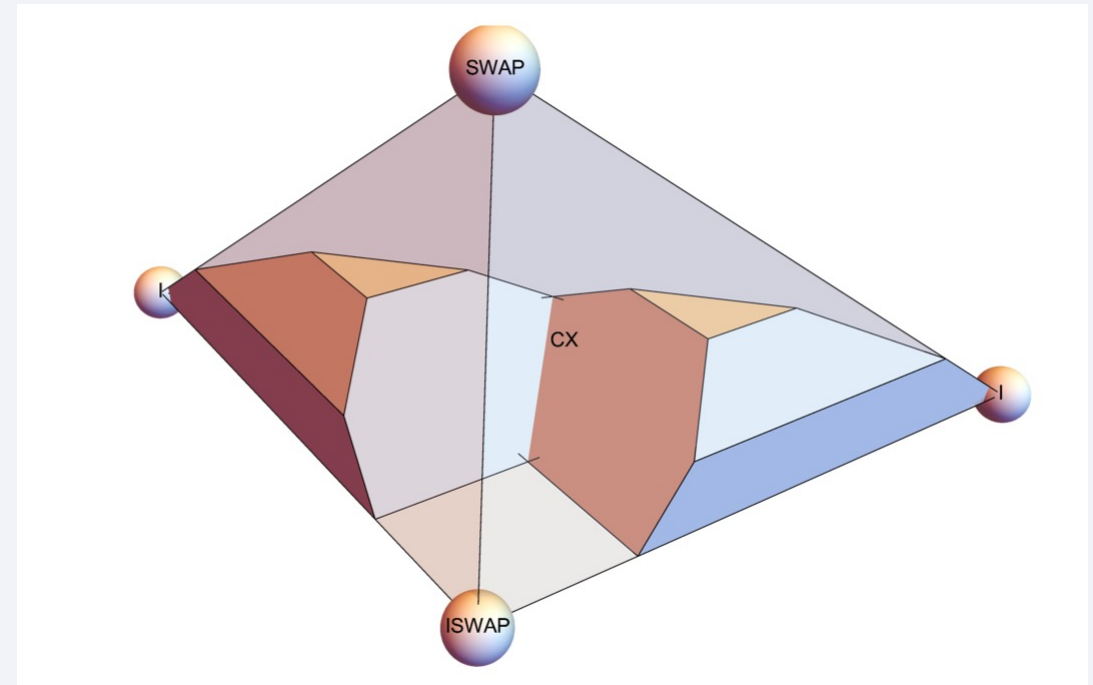
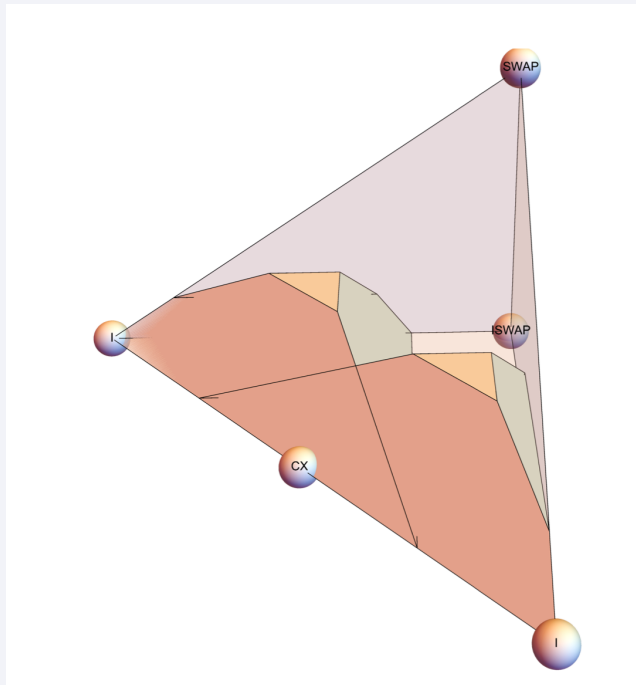
Weyl Chamber

- The Weyl chamber allows Zhang et al. to show that “exactly half the non-local [two-qubit] gates are perfect entanglers”.



Future Directions

- Medium article with high level overview and visualizations
- Explore papers that builds up from KAK and Weyl chamber



Links

- [Notes and Code Implementation](#) (Paper #1)
- [More Background](#) (Paper #1)
- [Notes and Code Implementation](#) (Paper #2)

