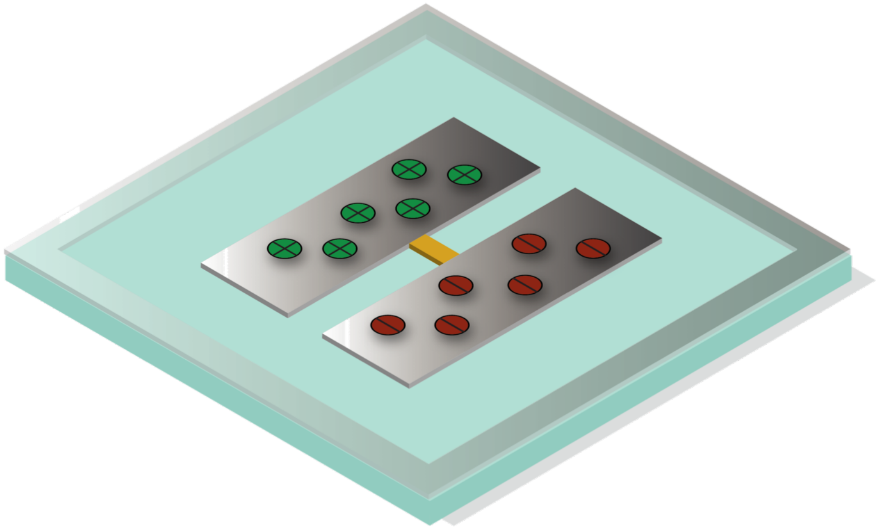
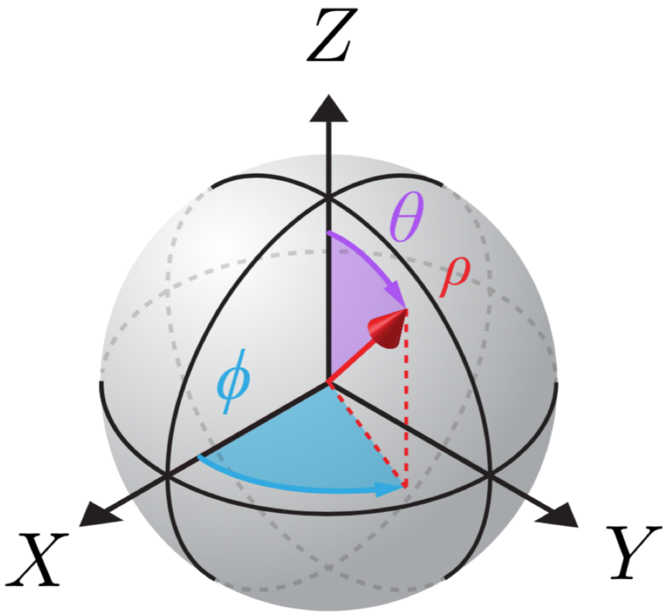
Superconducting Qubits II

Transmon & Measurements



Introduction



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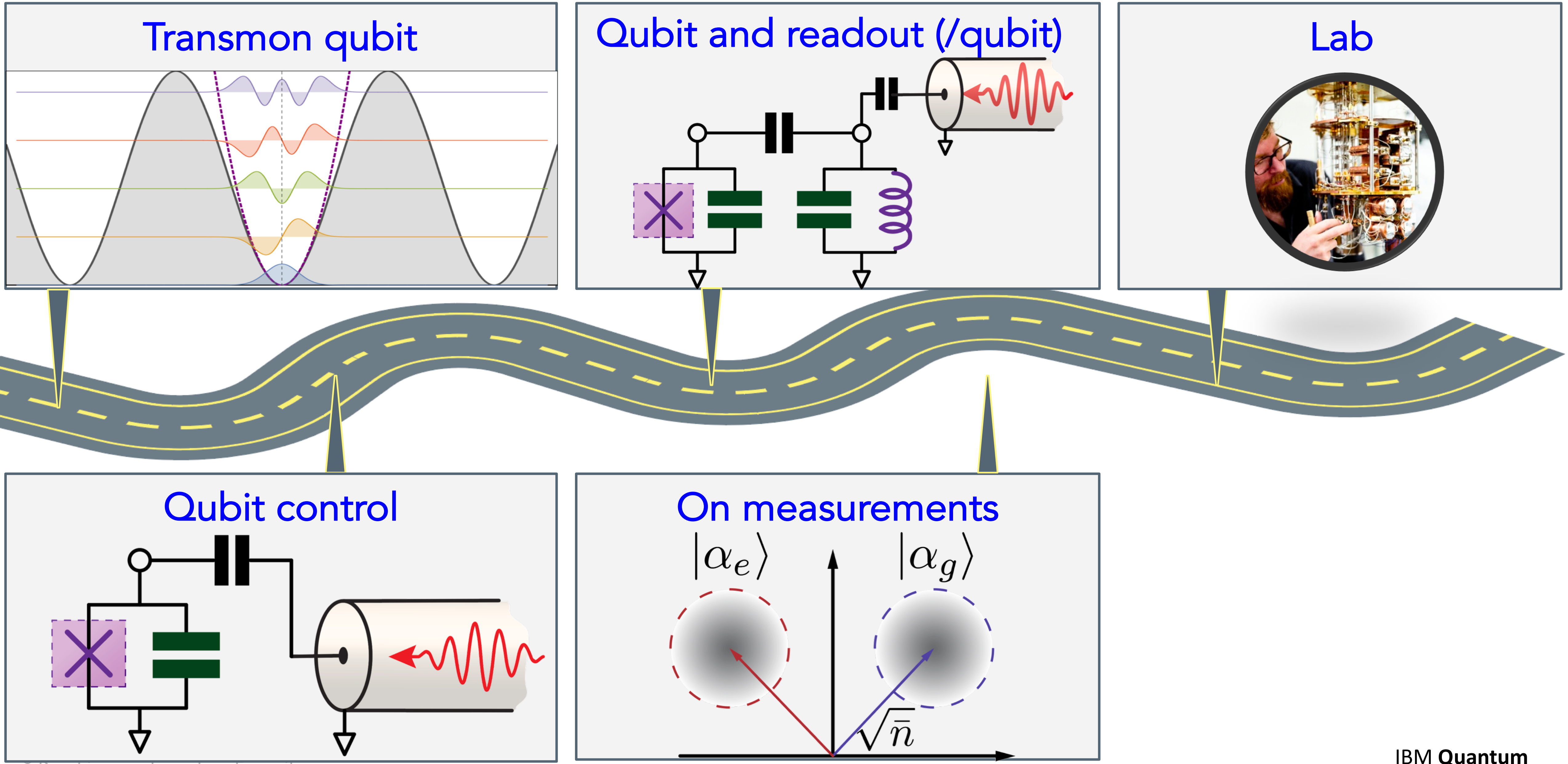
The road ahead



* Road image: based on Freepik

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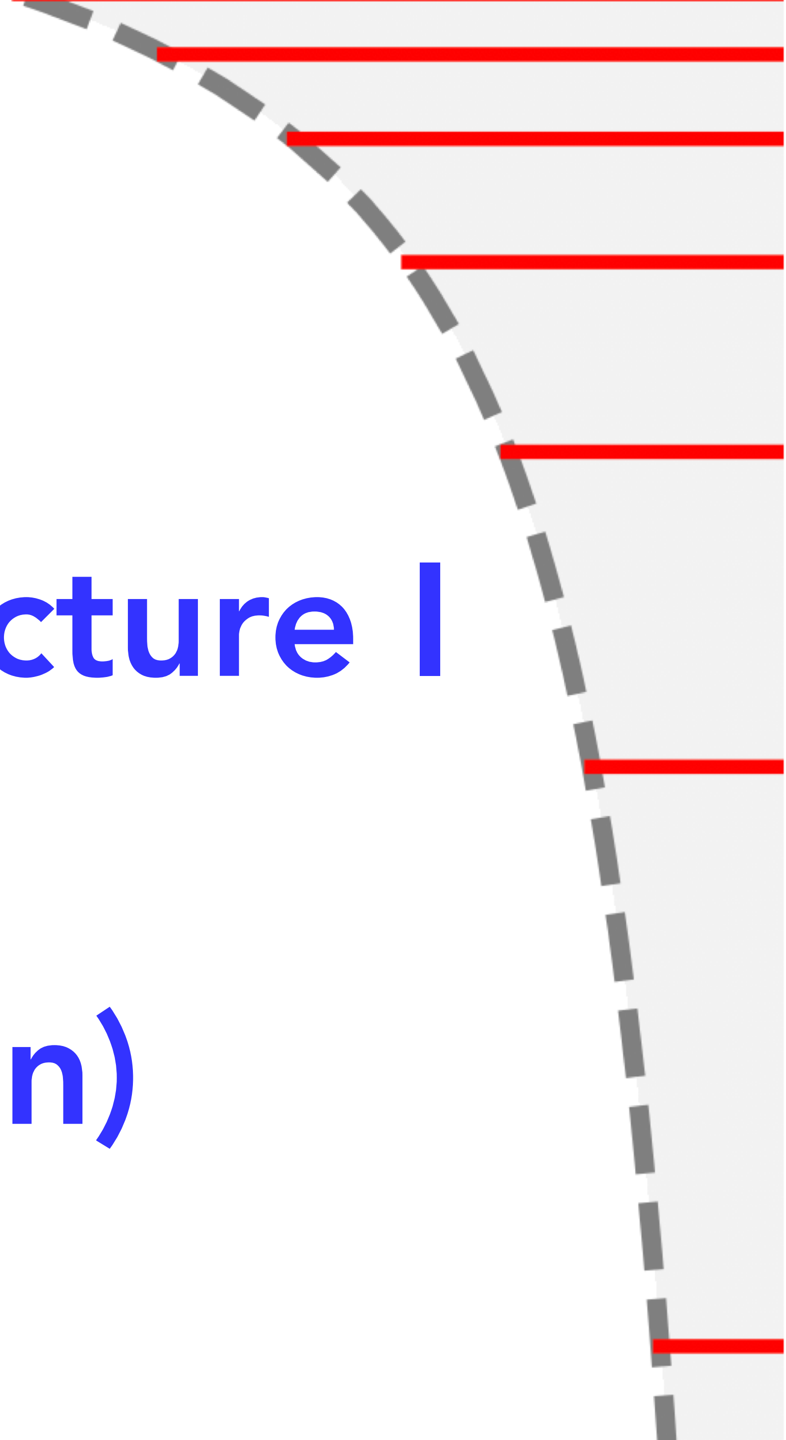
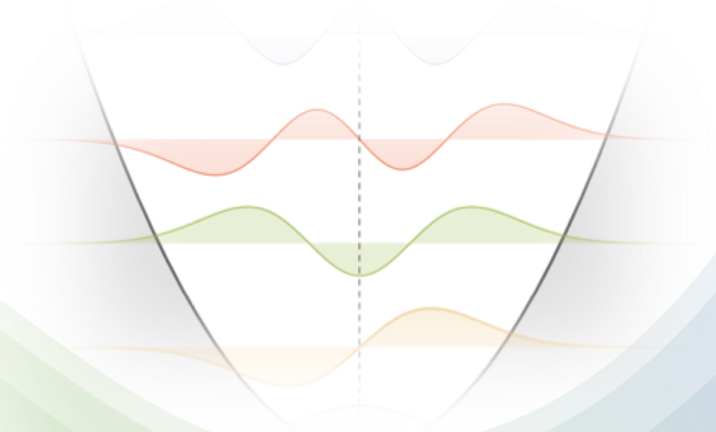
Takeaways from Lecture I

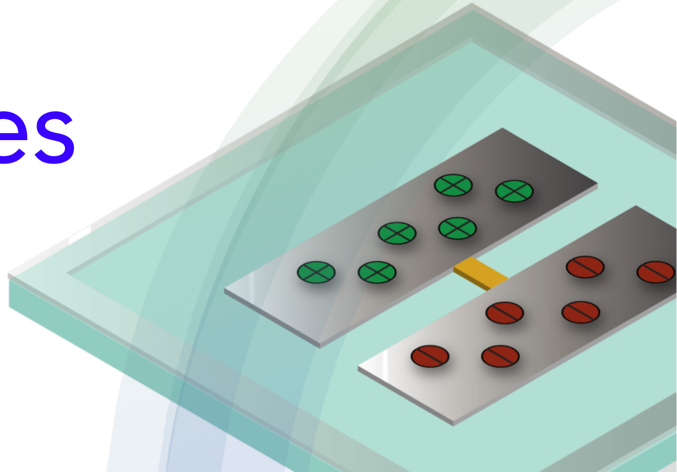
(and building on)



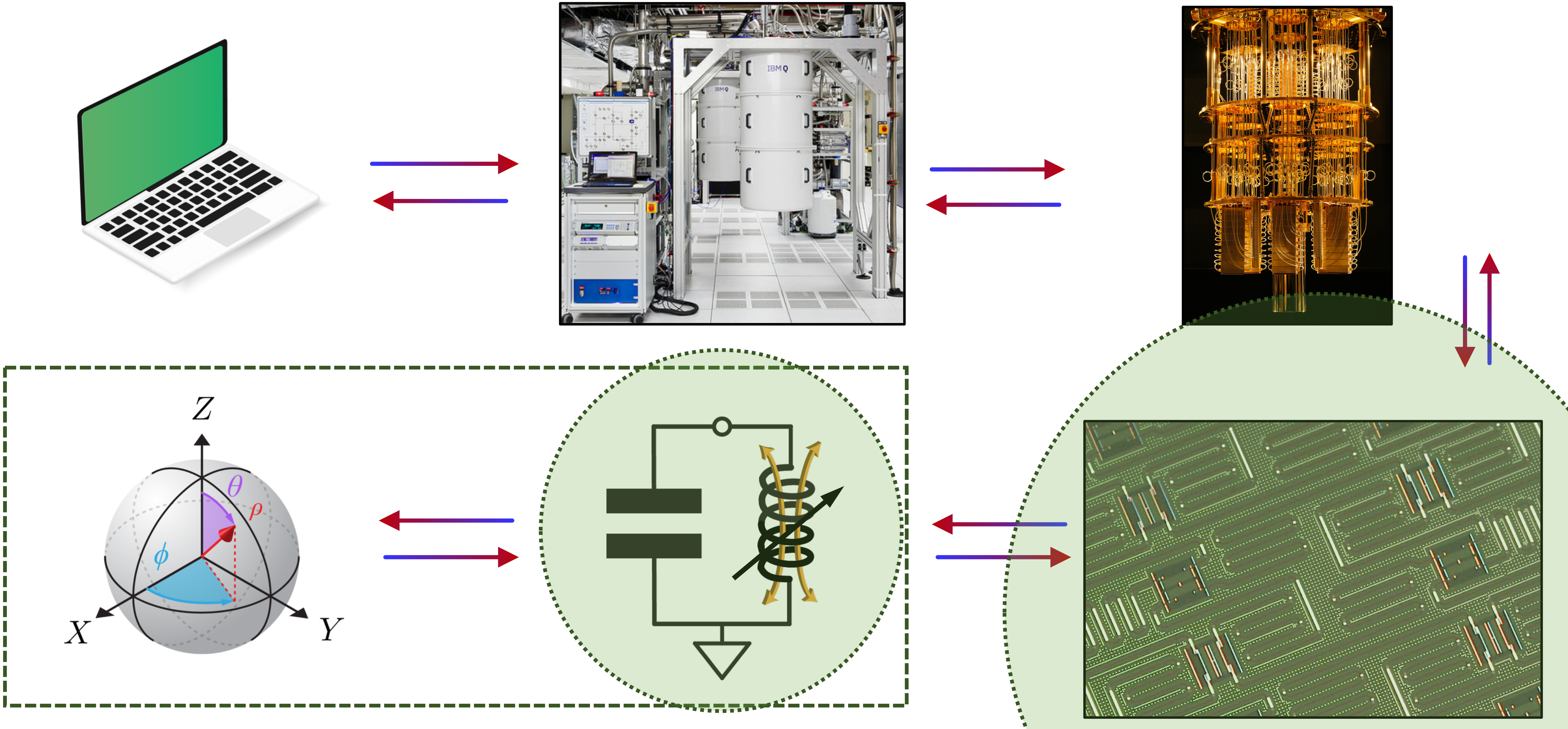


"Quantum phenomena do *not* occur in a Hilbert space, they occur in a laboratory."

Asher Peres

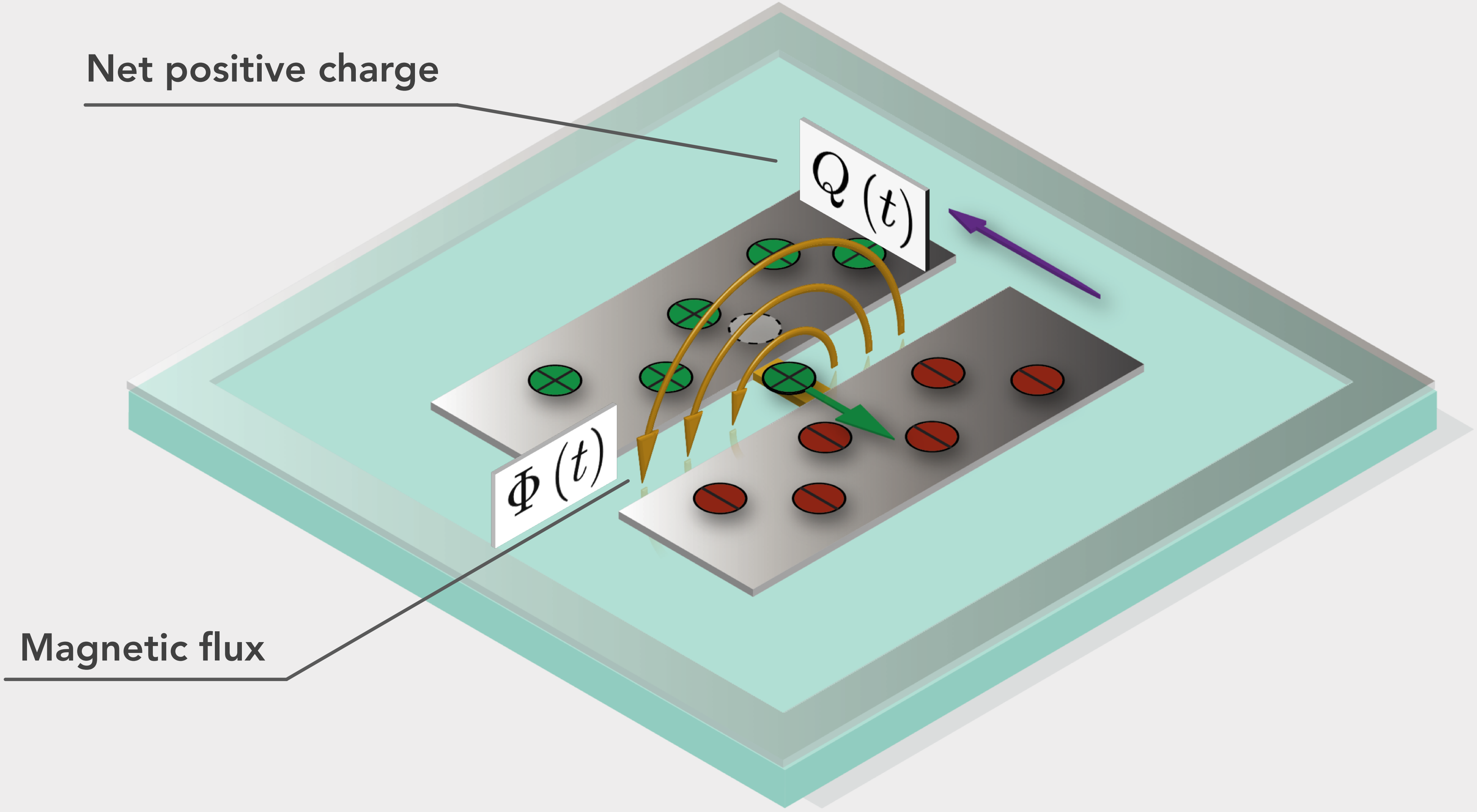


Qubit in the cloud

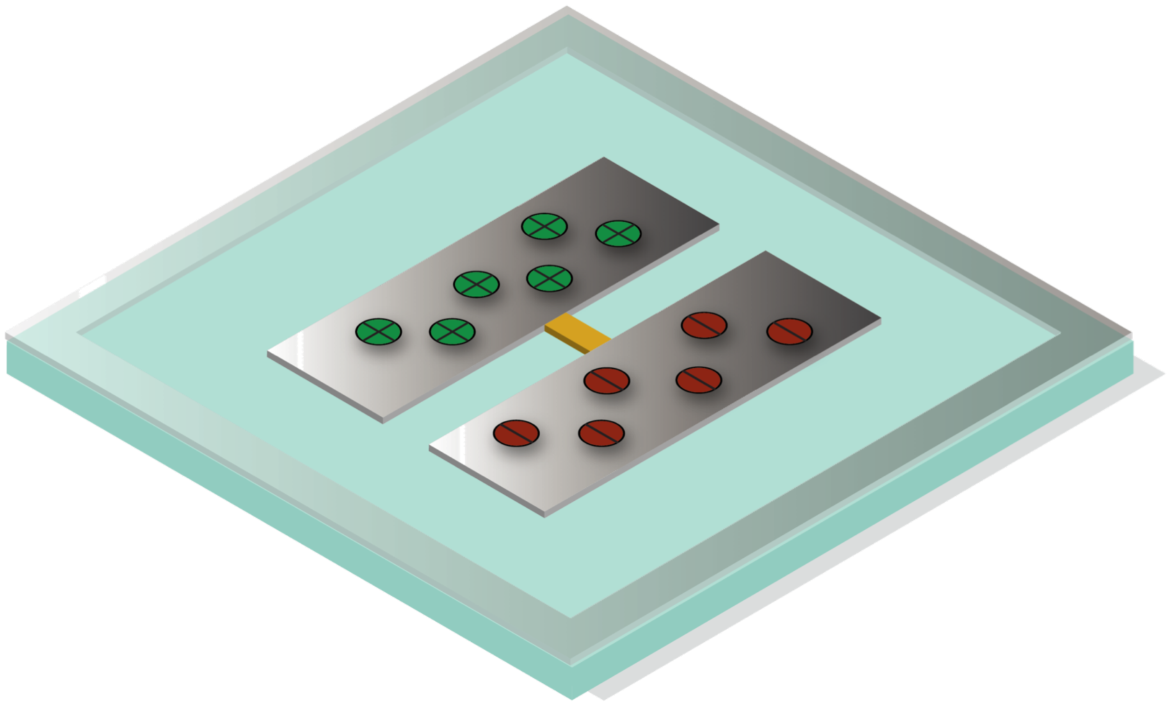


* Laptop image: rawpixel.com; Photos: IBM

Transmon qubit: physical picture

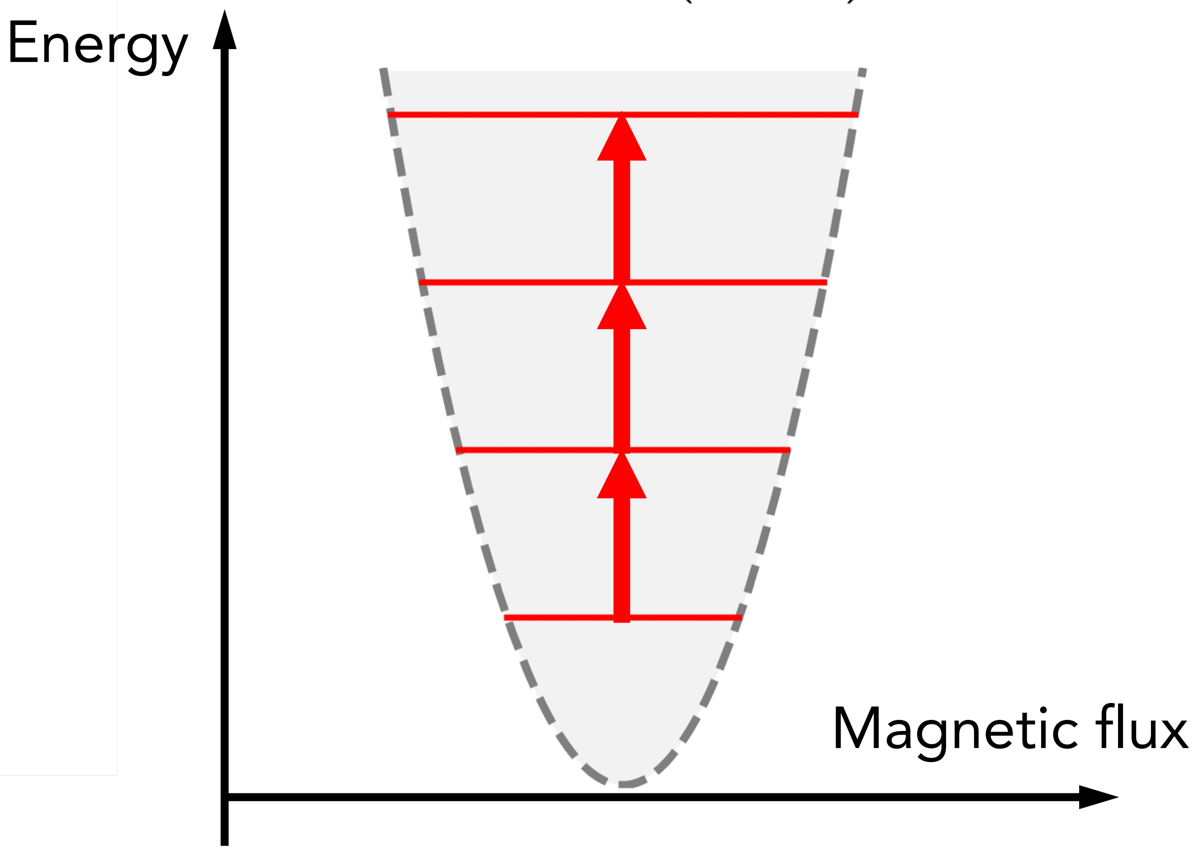


A first approximation



Harmonic oscillator

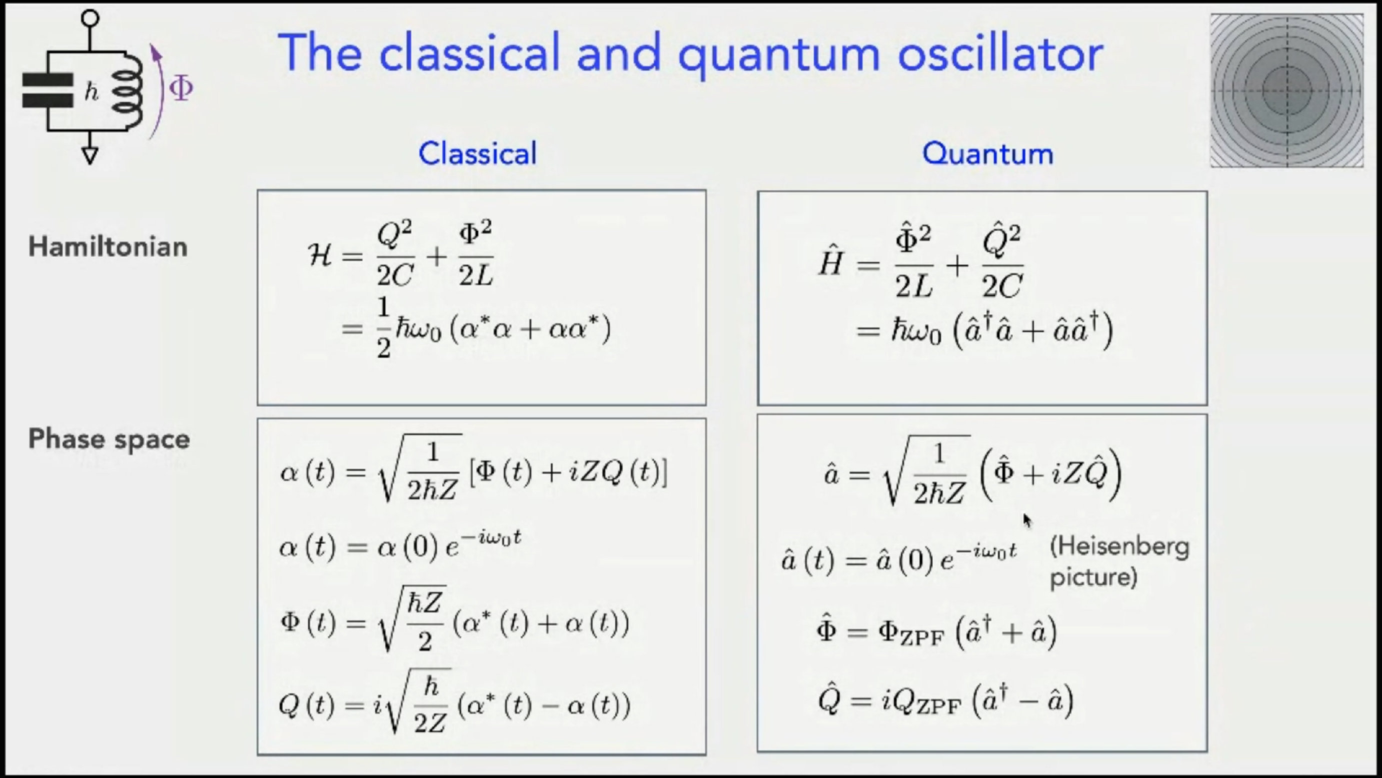
$$\hat{H}\left|n\right\rangle = \hbar\omega_0\left(n+\frac{1}{2}\right)\left|n\right\rangle$$



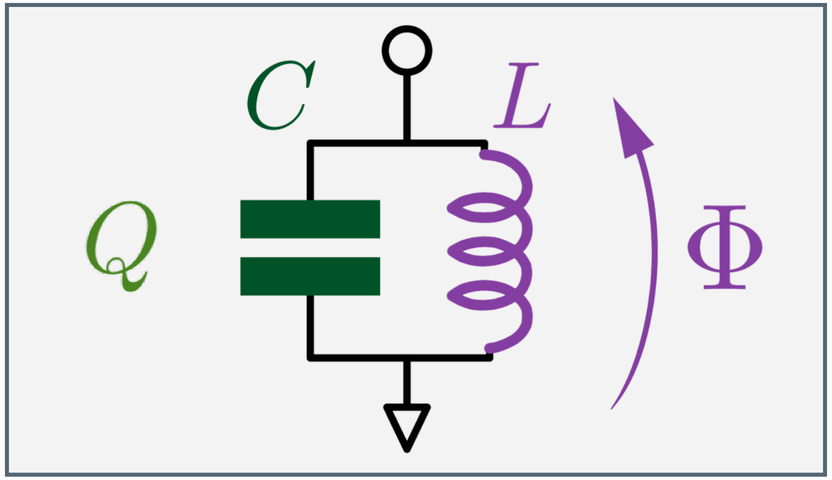
Classical to quantum bridge

βruno @brunormzg · 15h

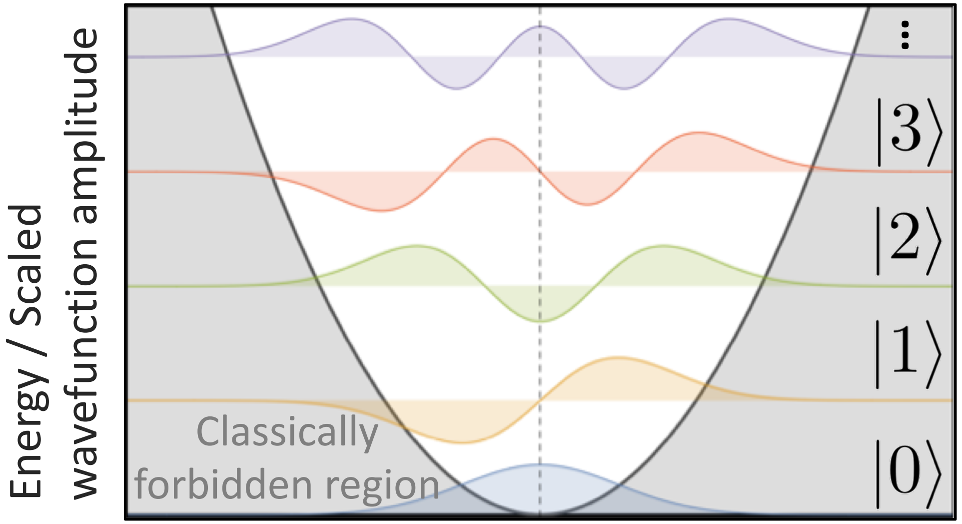
This is perhaps my favorite of many "aha!" moments that I've had during the #qgss. I didn't quite grasp the origin of the creation/annihilation operators, but that is no more. Thanks a lot, looking forward to tomorrow's lecture!! @zlatko_minev @qiskit @IBM



The LC quantum harmonic oscillator



$$\mathcal{E}_{\text{ind}}(\Phi) = \frac{\Phi^2}{2L} \qquad \psi_n(\Phi) \equiv \langle \Phi | n \rangle$$



Graph: Minev; Spring: Svjo

Position: $\Phi \mapsto x$ Momentum: $Q \mapsto p$

Mass: $C \mapsto m$ Spring constant: $L^{-1} \mapsto k$

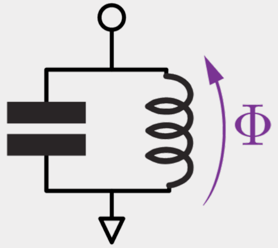
$$\hat{H}\left(\hat{\Phi}, \hat{Q}\right) = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C} = \hbar\omega_0\left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right) \qquad \omega_0^2 = \frac{1}{LC}$$

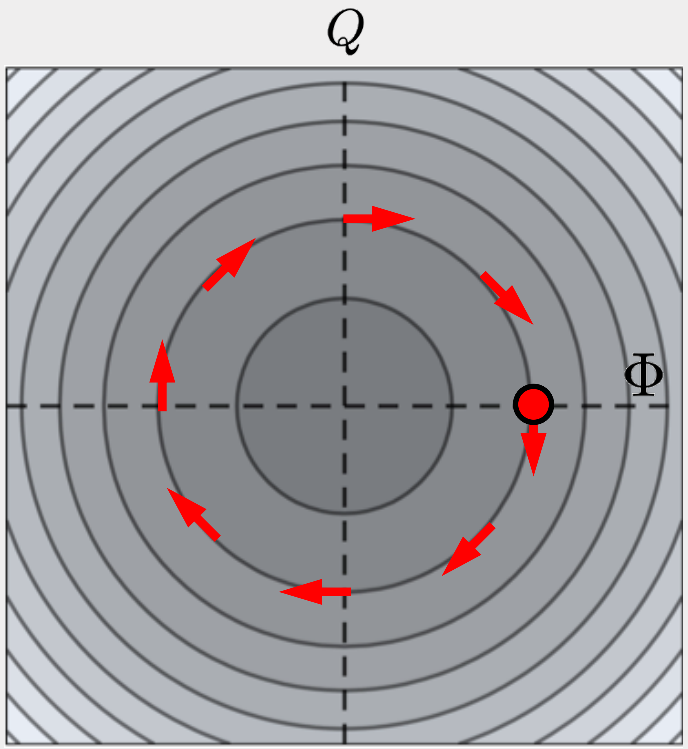
$$\hat{H}\left|n\right\rangle = \hbar\omega_0\left(n + \frac{1}{2}\right)\left|n\right\rangle \qquad Z_0 = \frac{L}{C}$$

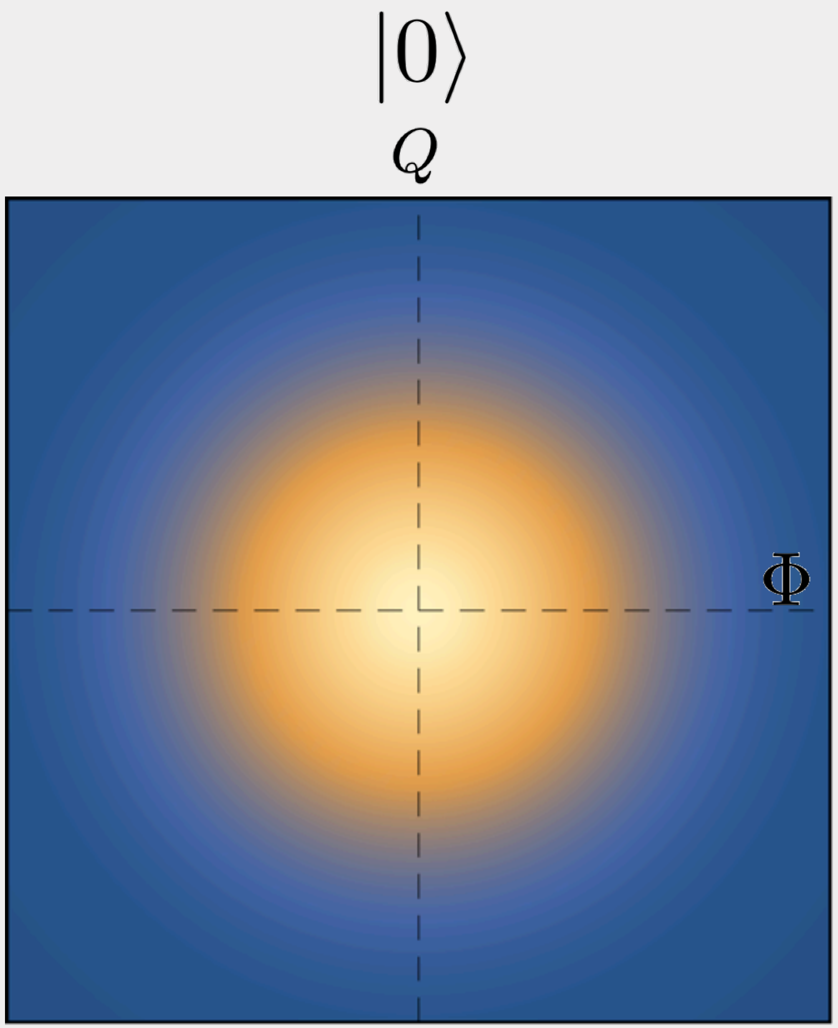
$$\hat{\Phi} = \Phi_{\text{ZPF}}\left(\hat{a}^\dagger + \hat{a}\right) \qquad \Phi_{\text{ZPF}} = \sqrt{\frac{\hbar}{2} Z_0} \qquad \left\langle 0 \middle| \hat{\Phi}^2 \middle| 0 \right\rangle = \Phi_{\text{ZPF}}^2$$

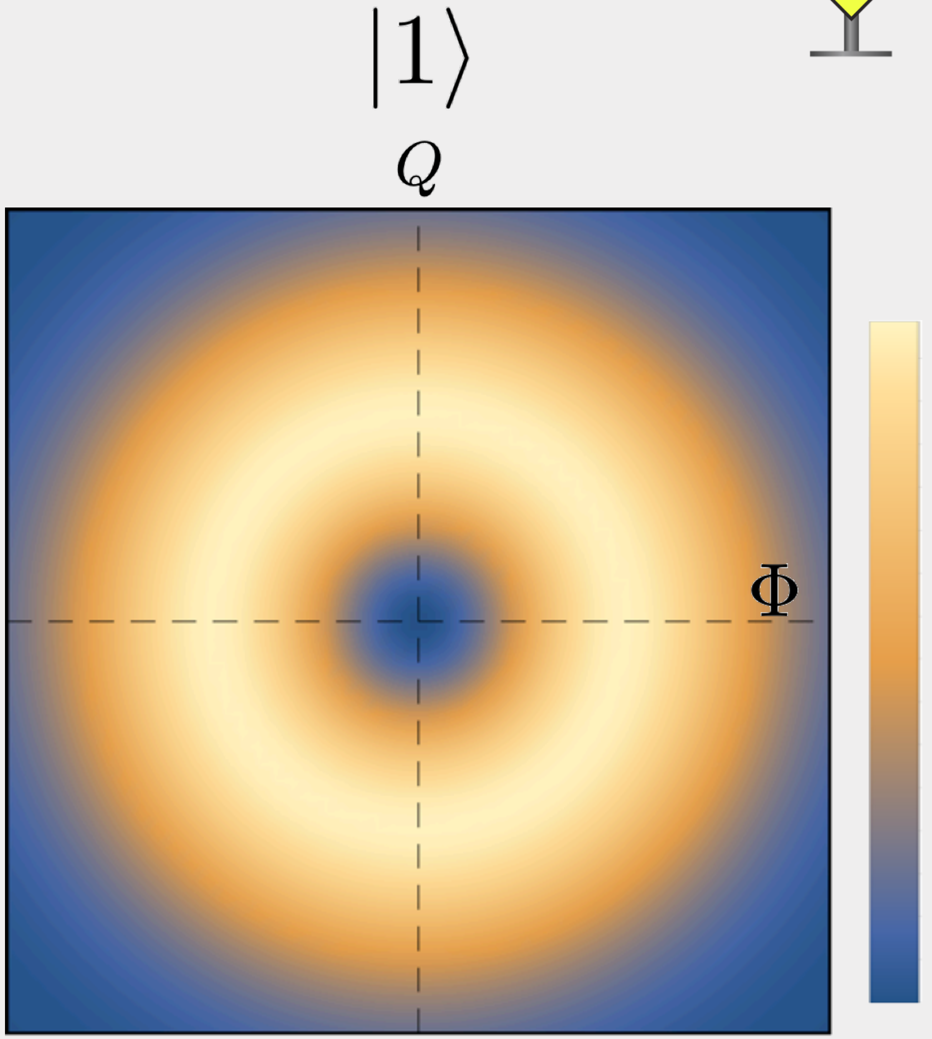
$$\hat{Q} = iQ_{\text{ZPF}}\left(\hat{a}^\dagger - \hat{a}\right) \qquad Q_{\text{ZPF}} = \sqrt{\frac{\hbar}{2} Z_0^{-1}} \qquad \left\langle 0 \middle| \hat{Q}^2 \middle| 0 \right\rangle = Q_{\text{ZPF}}^2$$

Phase-space (Husimi Q function)

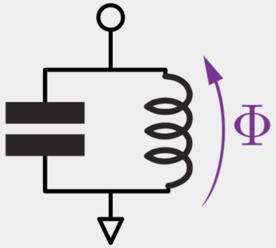




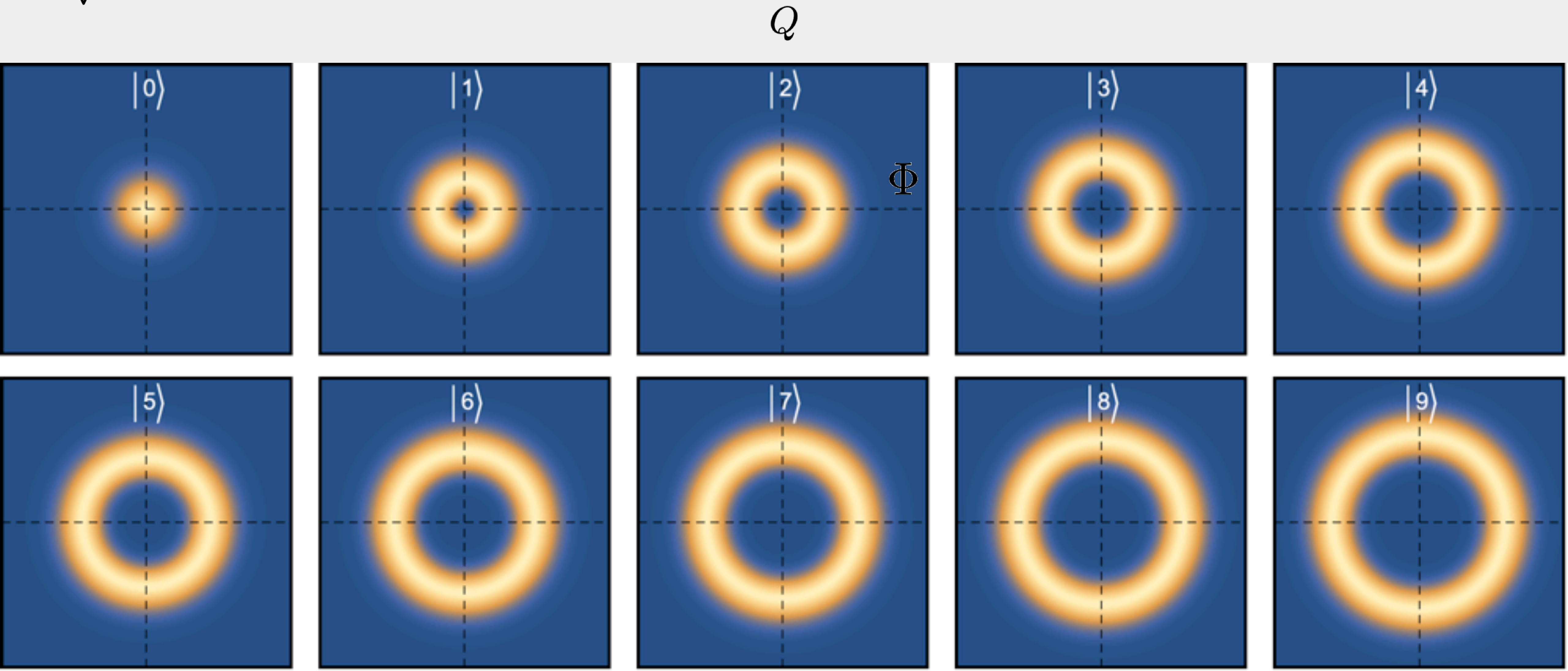




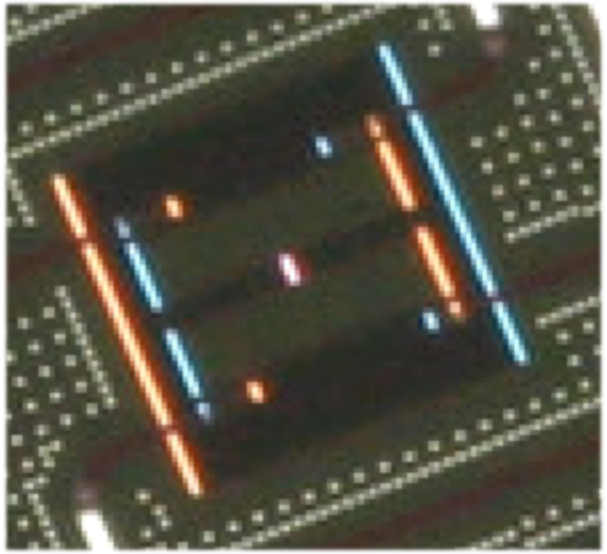
$$Q(\alpha) = \frac{1}{\pi}\langle\alpha|\hat{\rho}|\alpha\rangle$$

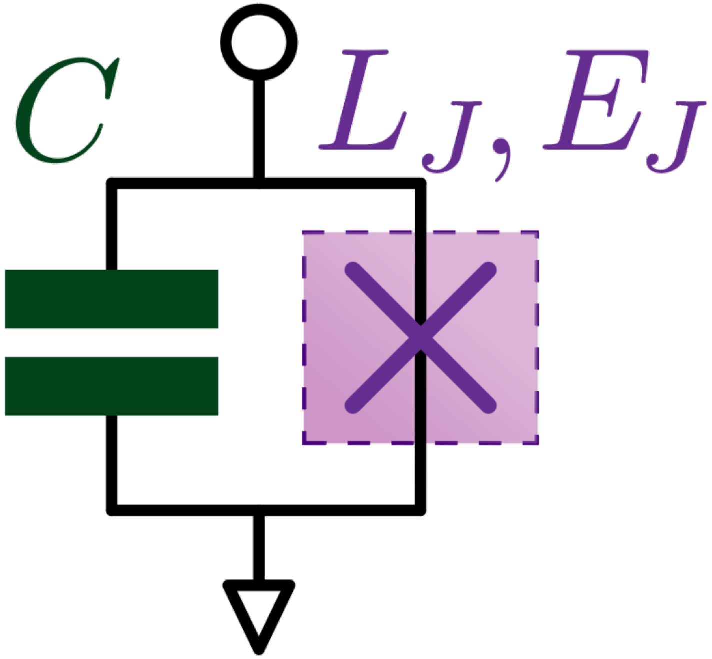


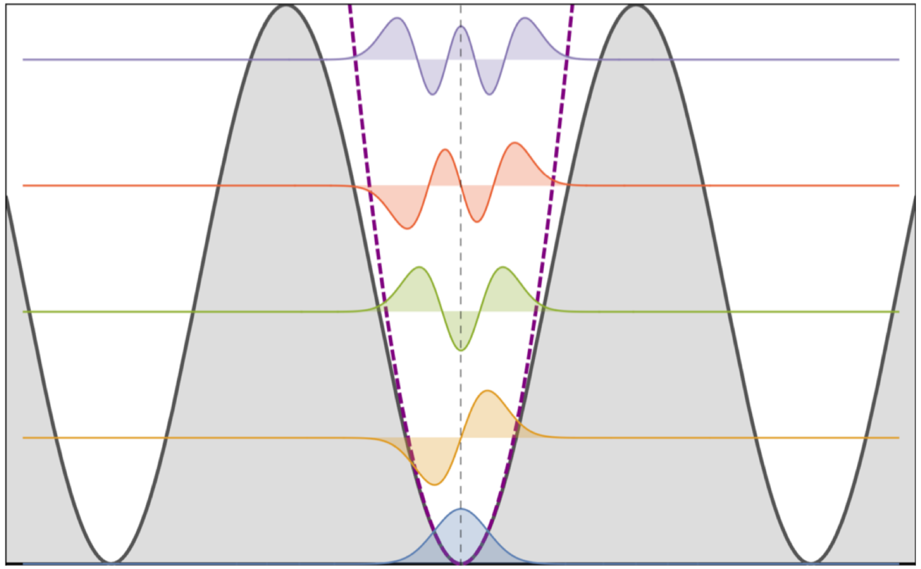
The cloud of quantum uncertainty



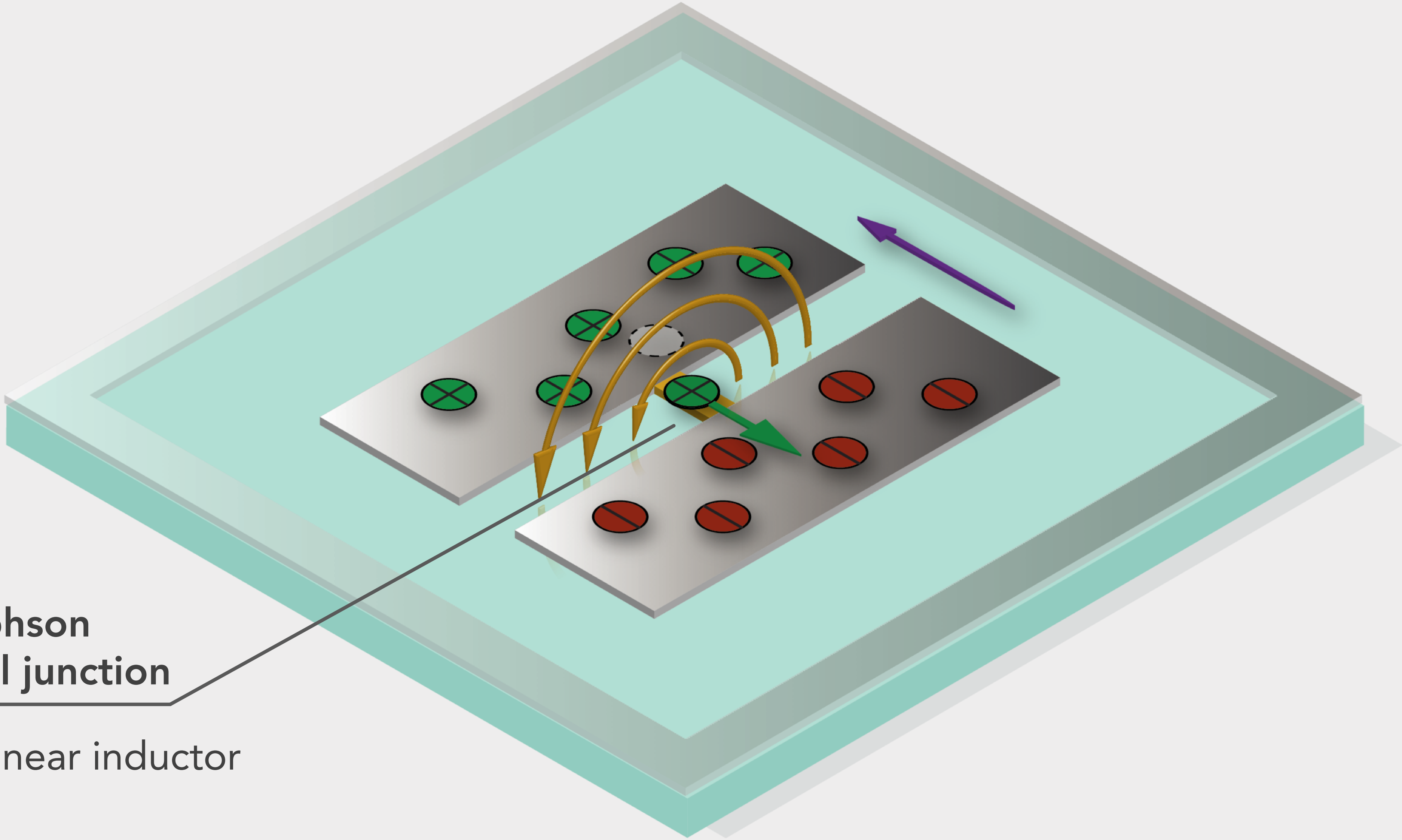
Transmon Qubit







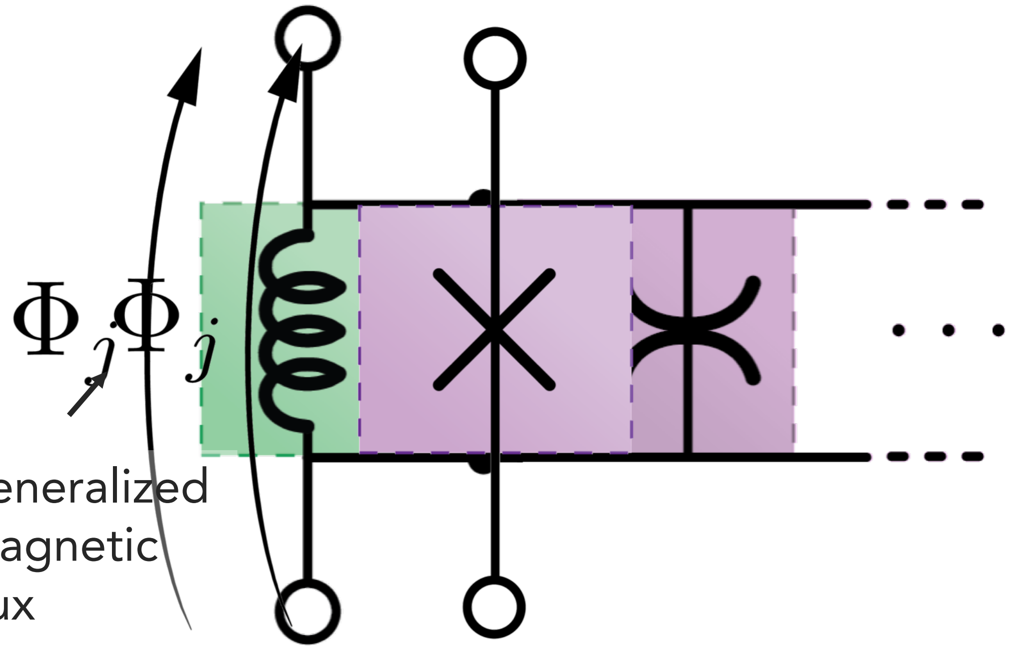
The transmon as a non-linear oscillator

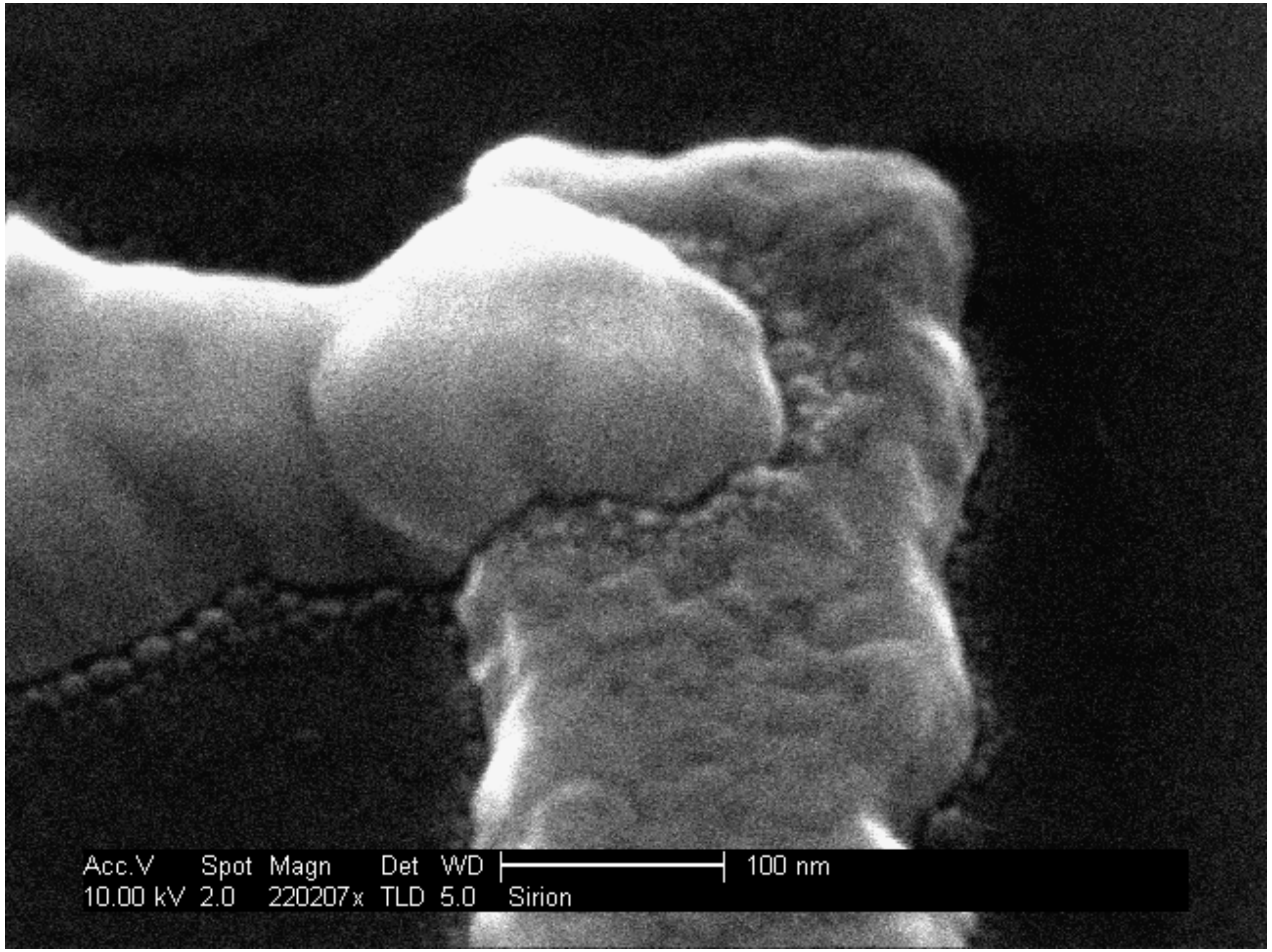


Josephson tunnel junction

Non-linear inductor

Josephson tunnel junction





SEM image: L. Frunzio

$$\mathcal{E}_j\left(\Phi_j\right) = E_j\left(1 - \cos\left(\Phi_j/\phi_0\right)\right)$$

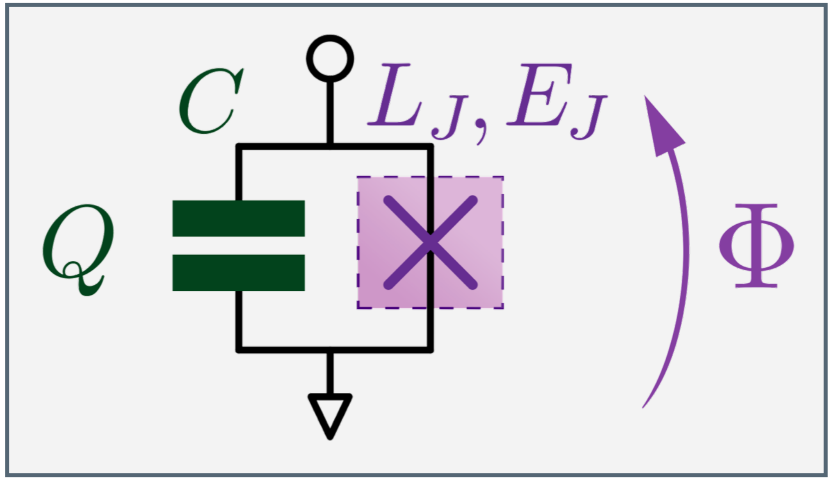
$$= \mathcal{E}_j^{\text{lin}}\left(\Phi_j\right) + \mathcal{E}_j^{\text{nl}}\left(\Phi_j\right)$$

$$\phi_0 \equiv \hbar/2e$$

$$= \frac{E_j}{2}\left(\frac{\Phi_j}{\phi_0}\right)^2 - \frac{E_j}{4!}\left(\frac{\Phi_j}{\phi_0}\right)^4 + \mathcal{O}\left(\Phi_j^6\right)$$

Circuit image: Minev et al., EPR to appear (2020)

The Transmon qubit



$$E_J = \frac{\phi_0^2}{L_J}$$

$$\phi_0 = \frac{\hbar}{2e}$$
$$\approx 3.3 \times 10^{-16}\ \mathrm{Wb}$$

$$\mathcal{E}_J(\Phi) = -E_J \cos(\Phi/\phi_0)$$

$$\mathcal{E}_J^{(2)}(\Phi) \approx \frac{\Phi^2}{2L_J} \qquad \mathcal{E}_{\mathrm{cap}}(\dot\Phi) \approx \frac{1}{2}C\dot\Phi^2$$

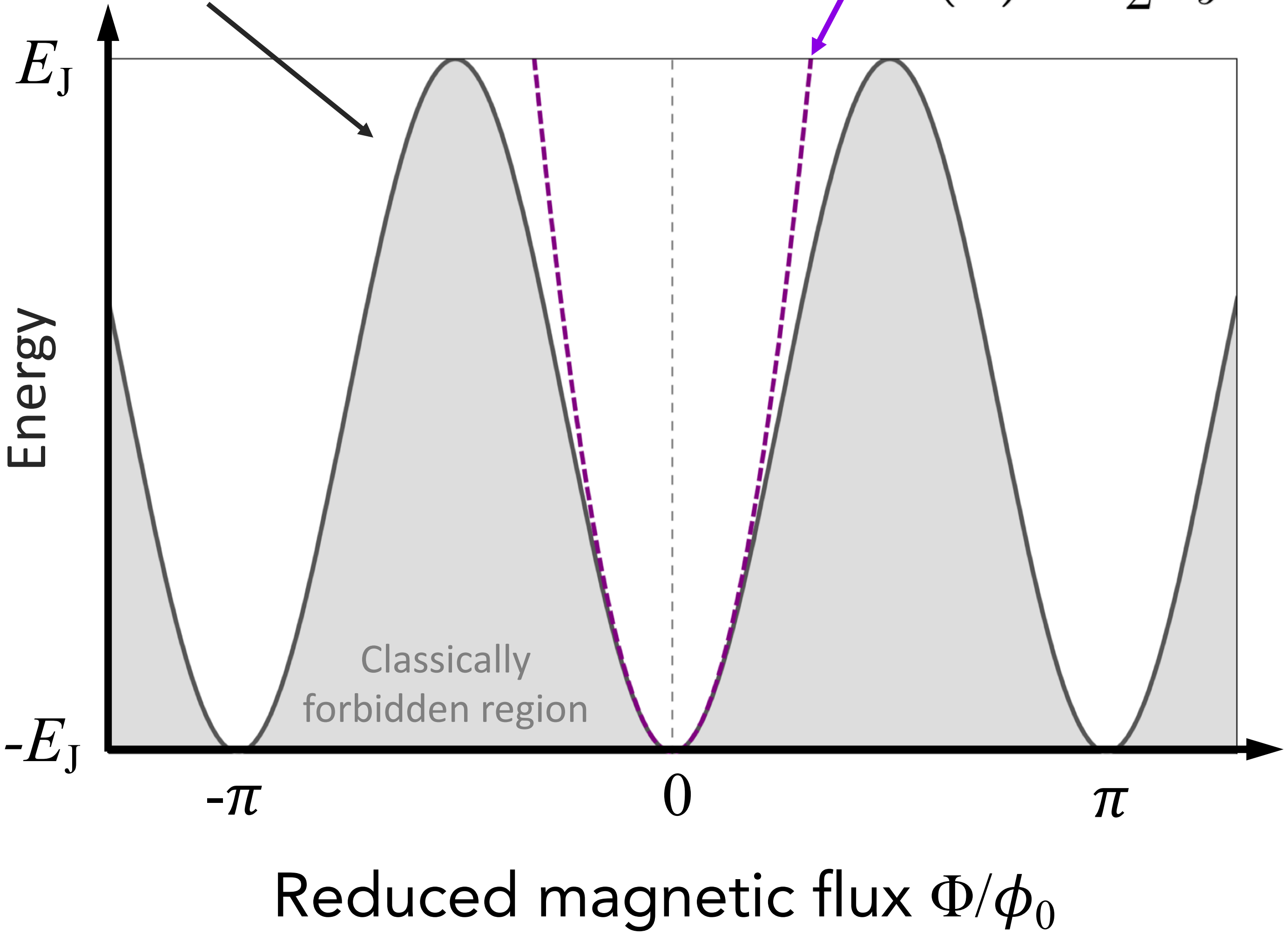
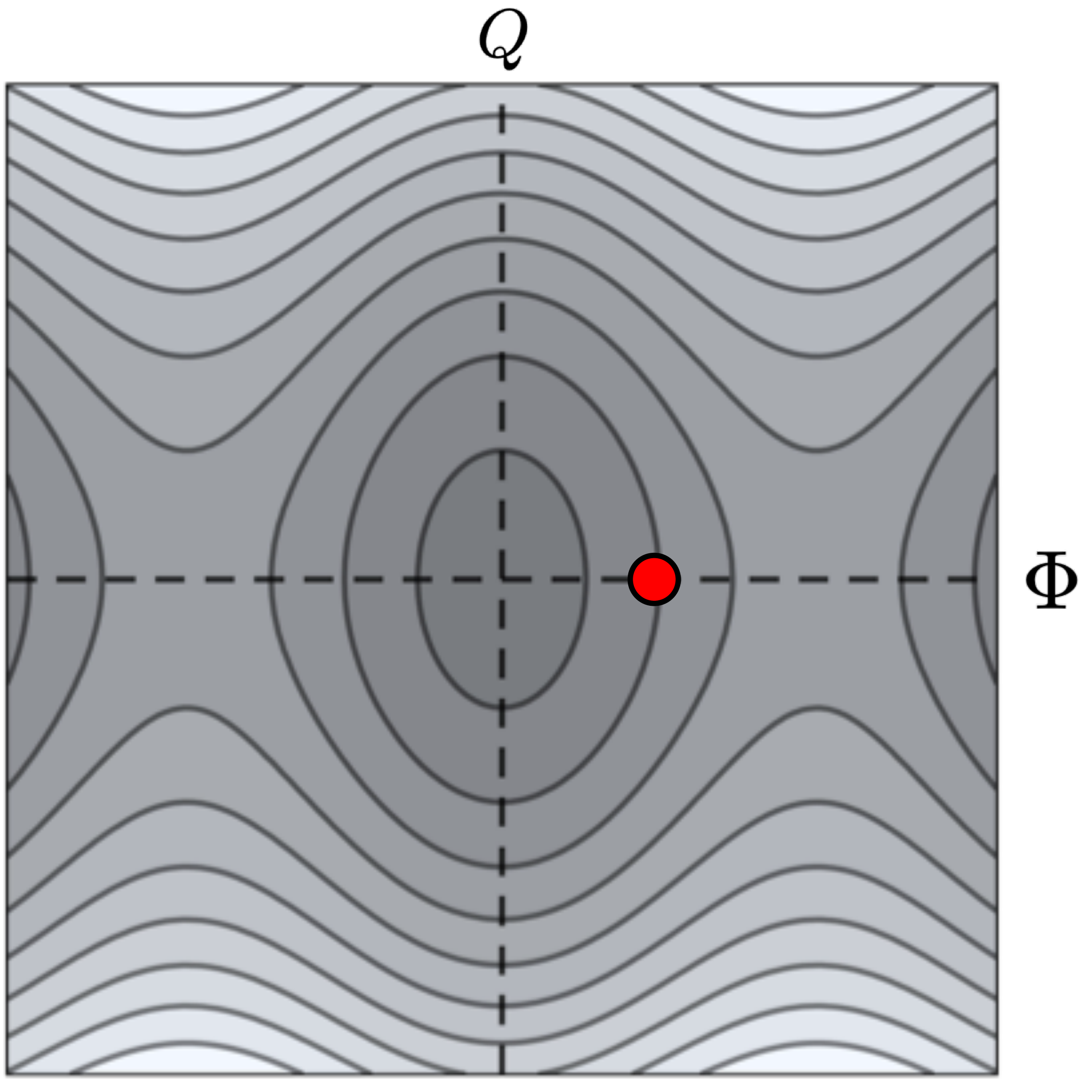


Image: Minev et al., EPR to appear (2020)

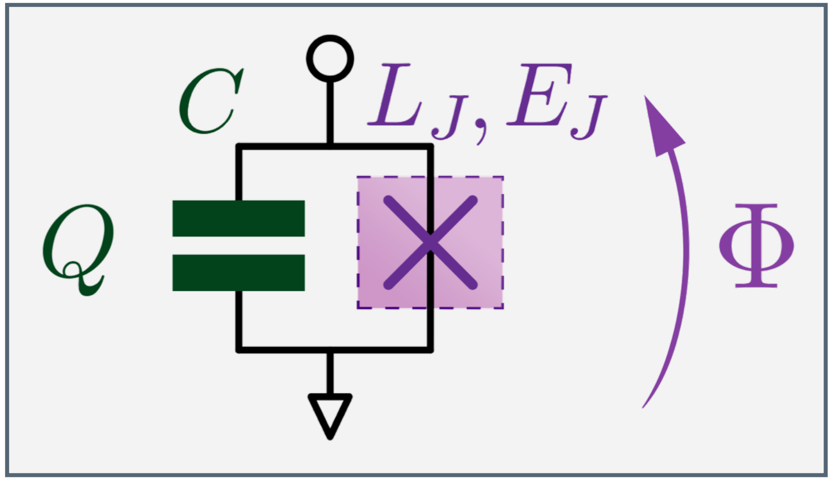
Semi-classical intuition: phase space picture

$$\mathcal{H}(\Phi, Q) = \frac{Q^2}{2C} - E_J \cos(\Phi/\phi_0)$$

$$E = \hbar\omega_0 \left(n + \frac{1}{2}\right)$$
$$= \frac{Q^2}{2C} - E_J \cos(\Phi/\phi_0)$$



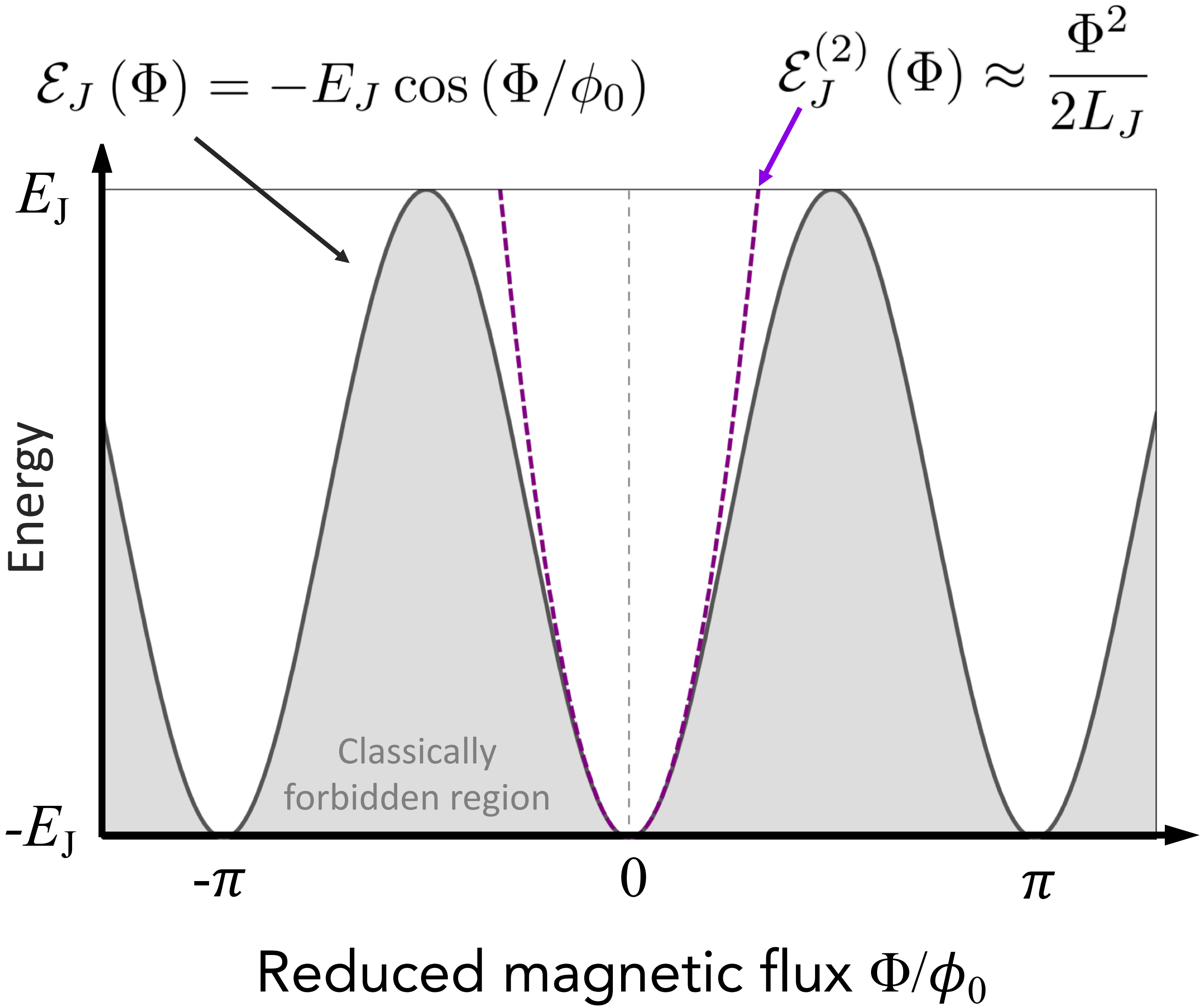
The Transmon qubit



$$\hat{H} = \frac{\hat{Q}^2}{2C} - E_J \cos\left(\hat{\Phi}/\phi_0\right)$$

$$\approx \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L_J} - \frac{E_J}{4!}\left(\frac{\hat{\Phi}}{\phi_0}\right)^4$$

$$= \hbar\omega_0 \hat{a}^\dagger \hat{a} - \frac{E_J \phi_{\mathrm{ZPF}}^4}{4!}\left(\hat{a} + \hat{a}^\dagger\right)^4$$



Hand-written notes

Day 2

$$\boxed{-E_J \phi_{ZPF}^4} \; (\hat{a} + \hat{a}^\dagger)^4$$

Drop hats

$$(a + a^\dagger)(a + a^\dagger)(a + a^\dagger)(a + a^\dagger)$$

$$aaaa + aaaa^\dagger + aaa^\dagger a + \cdots$$

$$aa^\dagger aa$$ 4 wave mixing

$$aa^\dagger aa$$

$$[a, a^\dagger] = 1$$

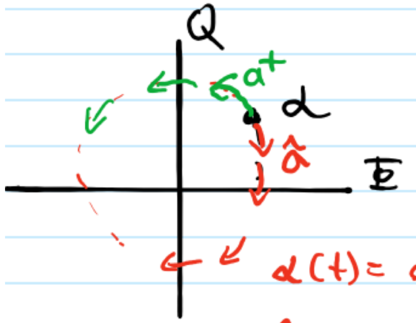
$$aa^\dagger - a^\dagger a = 1$$

$$aa^\dagger = 1 + a^\dagger a$$

$$(a^\dagger a + 1)\, aa$$

$$a^\dagger aaa + aa$$

Hand-written notes



$a^\dagger a^3 + a^2$

($a^\dagger a^3$: 4-wave mixing; a^2: non-classical term)

$\alpha(t) = \alpha(0)\, e^{-i\omega_0 t}$

$\hat{a}(t) = \hat{a}(0)\, e^{-i\omega_0 t}$

$\hat{a}^\dagger(t) = \hat{a}(0)^\dagger\, e^{+i\omega_0 t}$

$$a^\dagger a^3 = a^\dagger e^{+i\omega_0 t} a^3 e^{-3i\omega_0 t} = a^\dagger a^3 e^{-2i\omega_0 t}$$

rotating

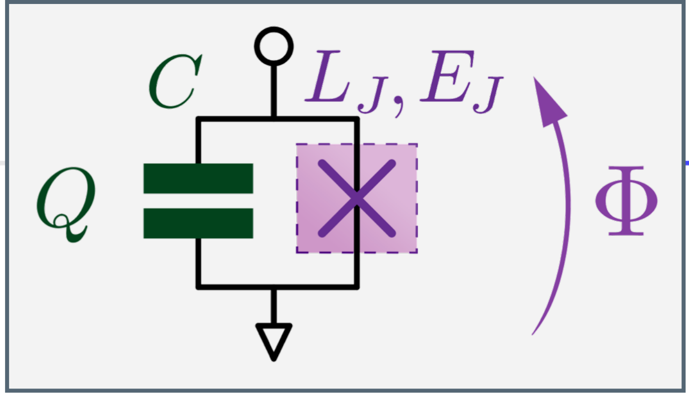
$$\hat{a}^{\dagger 2} \hat{a}^2 = \left(\hat{a}^\dagger e^{+i\omega_0 t}\right)^2 \left(\hat{a} e^{-i\omega_0 t}\right)^2 = (\hat{a}^\dagger)^2 (\hat{a})^2 e^{+2i\omega_0 t} e^{-2i\omega_0 t} = \hat{a}^{\dagger 2} \hat{a}^2$$

non-rotating

$$\hat{H}(t) = \hat{H}_0 + \hat{H}_R(t)$$

1st appr. $\hat{H} \simeq \hat{H}_0$

The Transmon qubit



$$= \hbar\omega_0 \hat{a}^\dagger \hat{a} - \frac{E_J \phi_{\mathrm{ZPF}}^4}{4!} (\cancel{3} + 6\cancel{\hat{a}^2} + 12\hat{a}^\dagger\hat{a} + 6\cancel{\hat{a}^{\dagger 2}}$$

$$+ \cancel{\hat{a}^4} + 4\cancel{\hat{a}^\dagger\hat{a}^3} + 6\hat{a}^{\dagger 2}\hat{a}^2 + 4\cancel{\hat{a}^{\dagger 3}\hat{a}} + \cancel{\hat{a}^{\dagger 4}})$$

RWA or 1st order PT

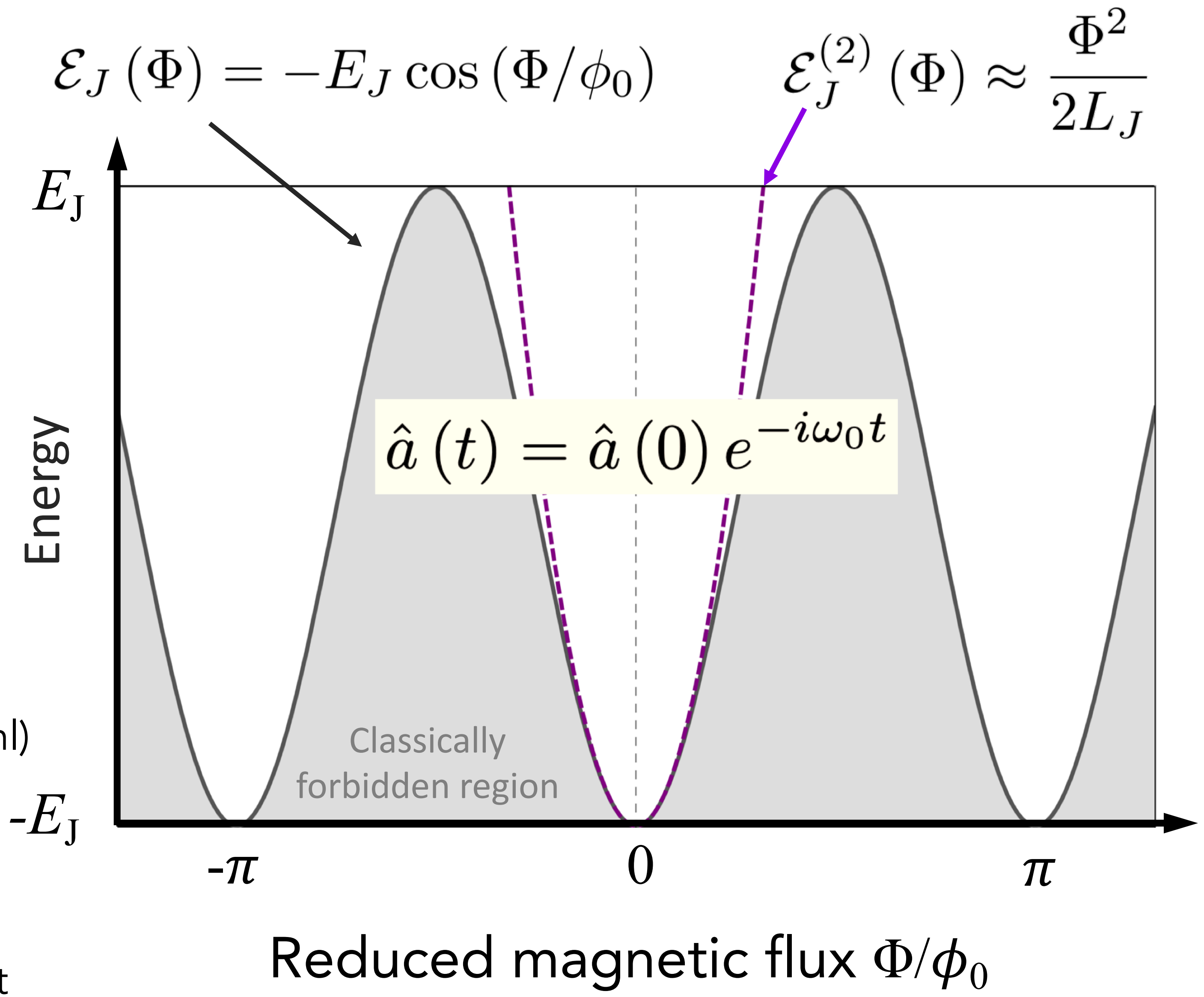
$$\hat{H}_4^{\mathrm{RWA}} = \hbar\omega_0 \hat{a}^\dagger \hat{a} - \frac{E_J}{4!}\phi_a^4 \left(12\hat{a}^\dagger\hat{a} + 6\hat{a}^{\dagger 2}\hat{a}^2 \right)$$

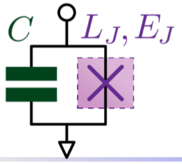
$$= \boxed{\hbar\left(\omega_0 - \Delta_q\right)\hat{a}^\dagger\hat{a} - \frac{\hbar\alpha}{2}\hat{a}^{\dagger 2}\hat{a}^2}$$

Due to nl & commutator — Due to non-linearity (nl)

where $\hbar\Delta_q = \hbar\alpha \equiv \frac{1}{2} E_J \phi_{\mathrm{ZPF}}^4$

"Lamb shift" due to ZPF — Anharmonicity — Zero-point fluctuations





The Kerr Hamiltonian of the transmon

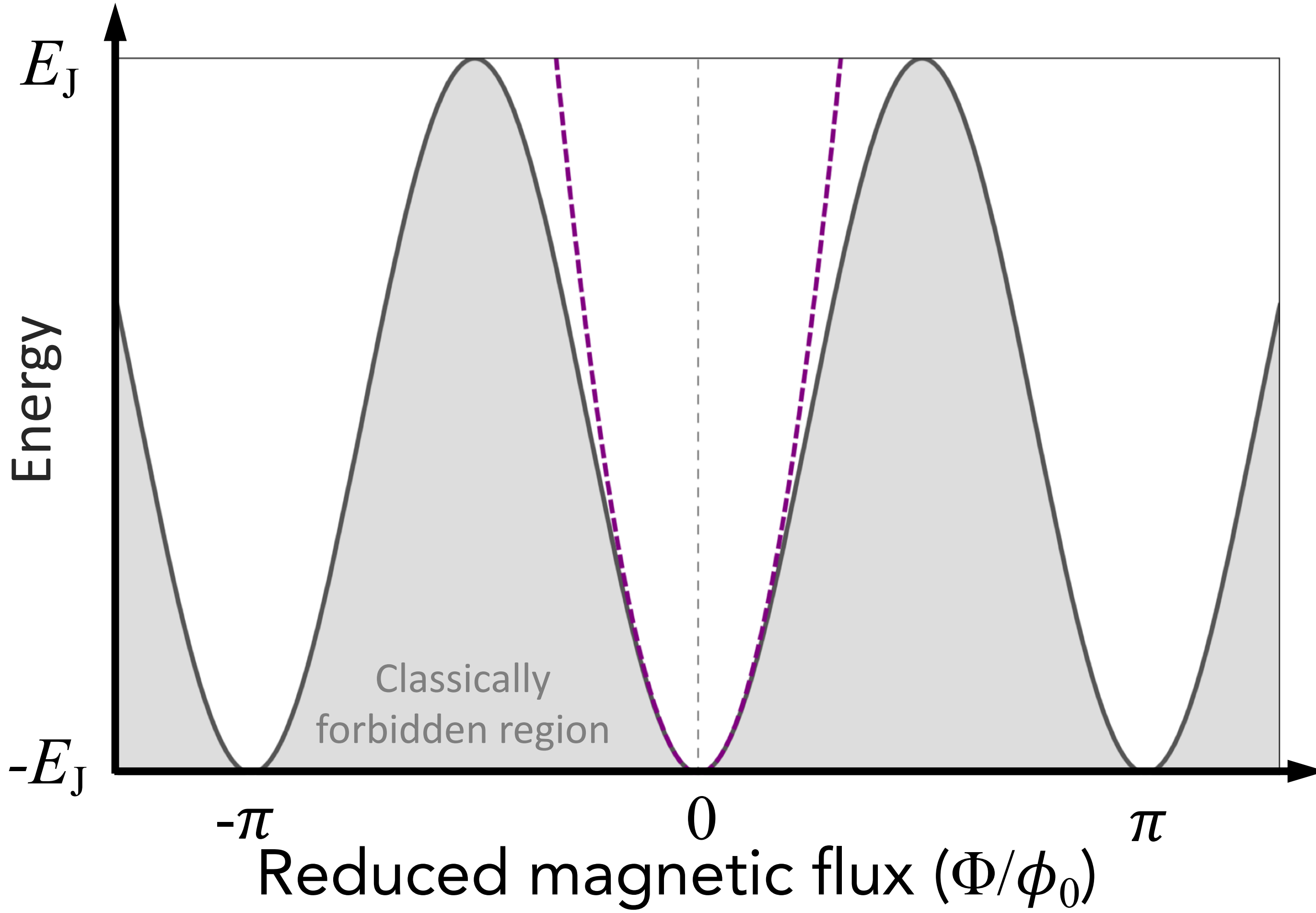
$$\hat{N} \equiv \hat{a}^\dagger \hat{a}$$

$$\hat{H}_4^{\mathrm{RWA}} \approx \underbrace{\hbar\omega_q \hat{N}}_{\hat{H}_{\mathrm{lin}}} \underbrace{- \frac{\hbar\alpha}{2}\hat{N}\left(\hat{N}-1\right)}_{\hat{H}_{\mathrm{nl}}}$$

$\hat{H}_{\mathrm{lin}}$: Solution known (SHO)

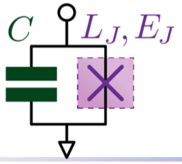
$\hat{H}_{\mathrm{nl}}$: New from nonlinearity

$$\hat{H}_4^{\mathrm{RWA}} \left|n\right\rangle = \hbar\omega_q n \left(1 - \frac{\hbar\alpha_a}{2}(n-1)\right)\left|n\right\rangle$$



1st order correction to energy: $E_n^{(1)} = \left\langle n^{(0)} \right| \hat{H}_{\mathrm{nl}} \left| n^{(0)} \right\rangle$ and to eigenstates: $\left|n^{(1)}\right\rangle = \sum_{k \neq n} \frac{\left\langle k^{(0)}\right| \hat{H}_{\mathrm{nl}} \left|n^{(0)}\right\rangle}{E_n^{(0)} - E_k^{(0)}} \left|k^{(0)}\right\rangle.$

To first order perturbation theory the eigenstates do not change! Only the energy changes. Dispersive.



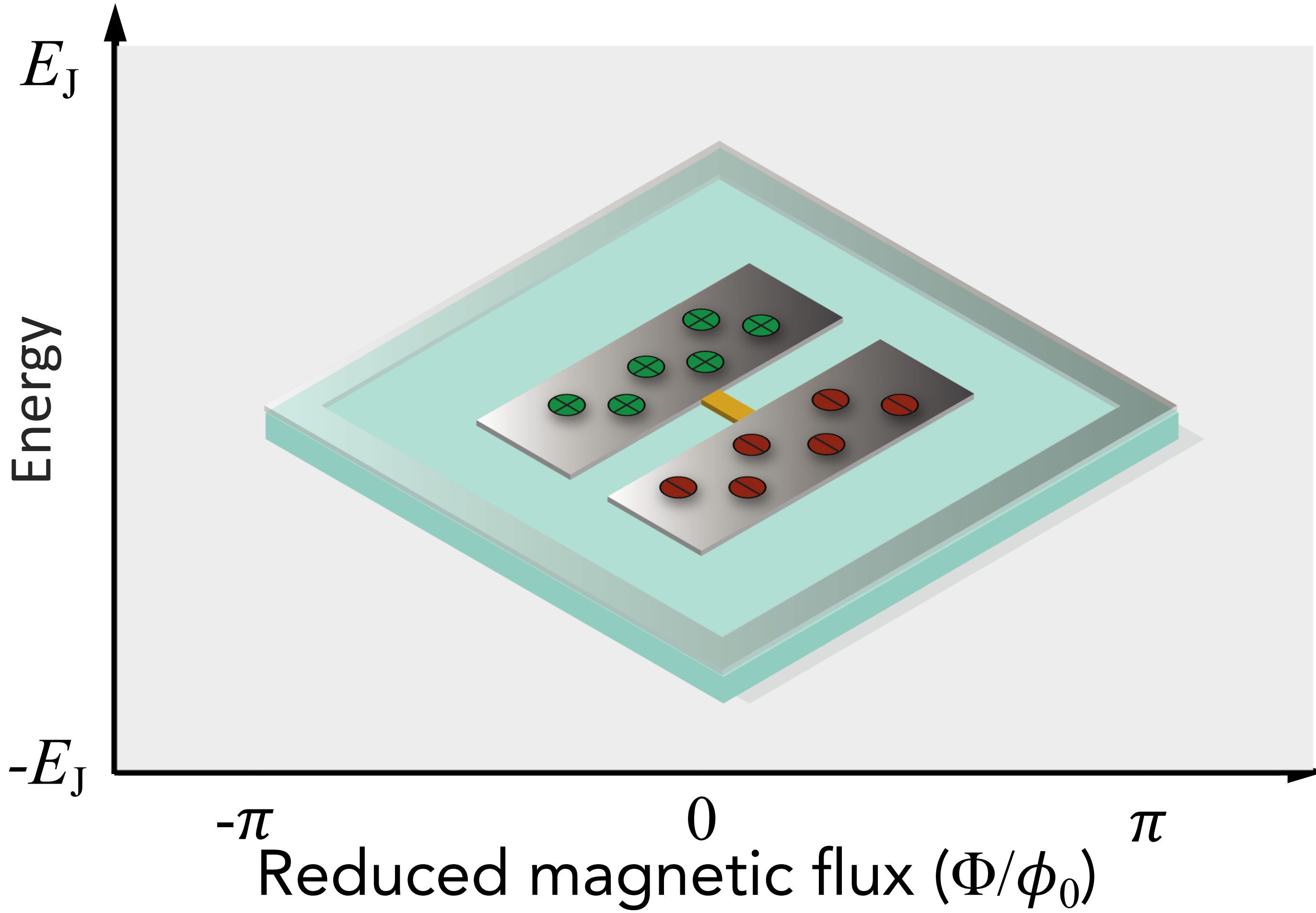
Exploring a real transmon qubit

$$\hat{N} \equiv \hat{a}^\dagger \hat{a}$$

$$\hat{H}_4^{\mathrm{RWA}} \approx \underbrace{\hbar\omega_q \hat{N}}_{\hat{H}_{\mathrm{lin}}} - \underbrace{\frac{\hbar\alpha}{2}\hat{N}\left(\hat{N}-1\right)}_{\hat{H}_{\mathrm{nl}}}$$

Solution known (SHO) → $\hat{H}_{\mathrm{lin}}$

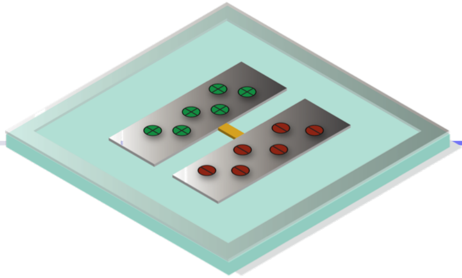
$\hat{H}_{\mathrm{nl}}$ ← New from nonlinearity



Experimental parameters

$$L_J = 14\ \mathrm{nH} \qquad E_J = \frac{\phi_0^2}{L_J} = 12\ \mathrm{GHz}$$

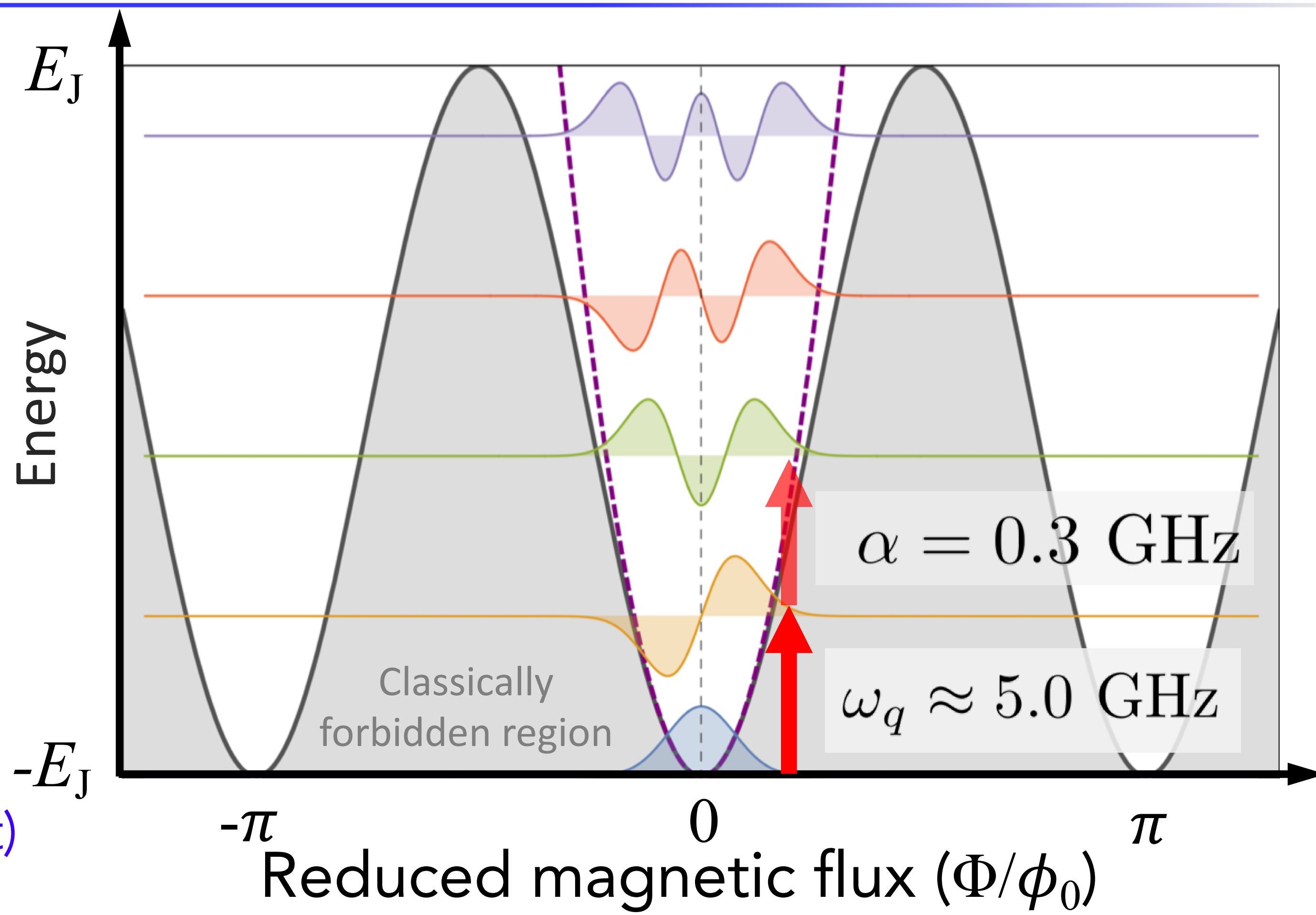
$$C_J = 65\ \mathrm{fF} \qquad E_C = \frac{e^2}{2C} = 0.3\ \mathrm{GHz}$$



Exploring a real transmon qubit

$$\hat{H}_4^{\mathrm{RWA}} \approx \hbar\omega_q \hat{N} - \frac{\hbar\alpha}{2}\hat{N}\left(\hat{N}-1\right)$$

Dispersive, states didn't change to 1st order
"Lamb shift" due to ZPF



Parameters used in figure (of a measured qubit)

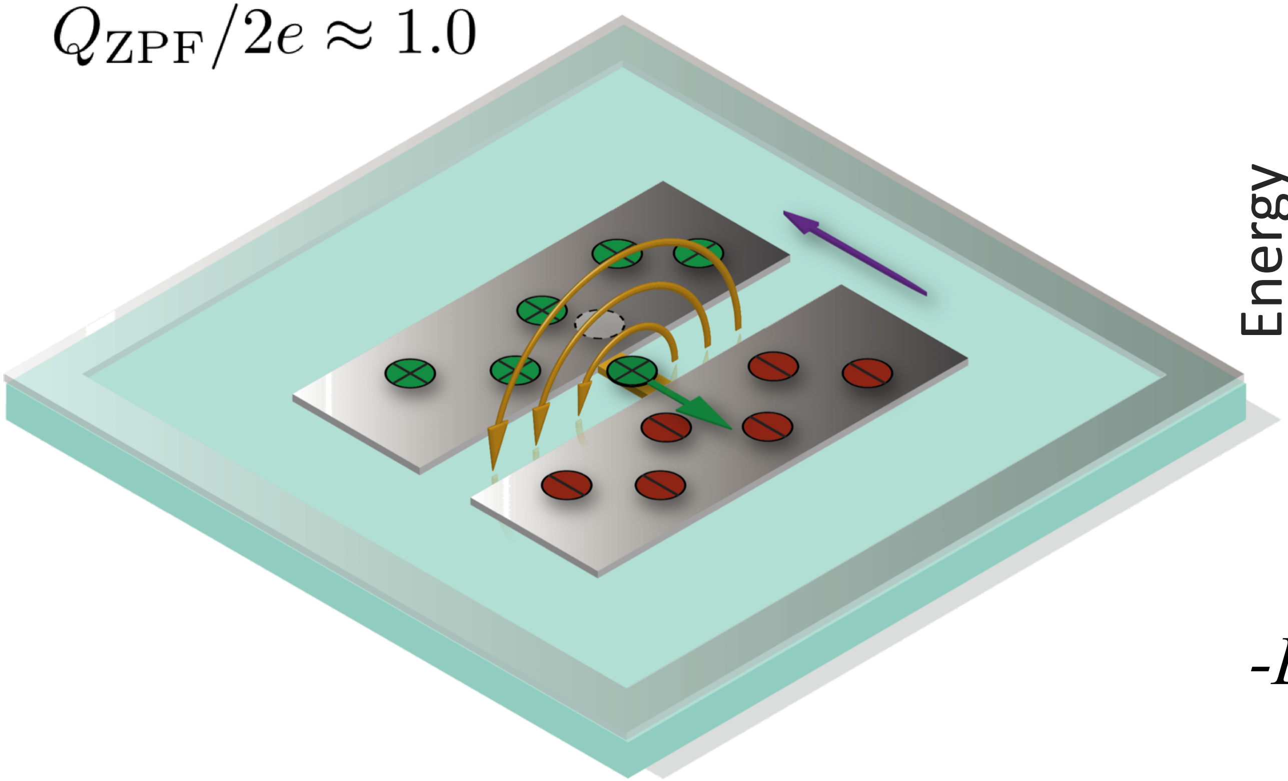
$$L_J = 14 \text{ nH} \qquad E_J = \frac{\phi_0^2}{L_J} = 12 \text{ GHz} \qquad \omega_0 = \sqrt{\frac{1}{LC}} = 5.3 \text{ GHz}$$

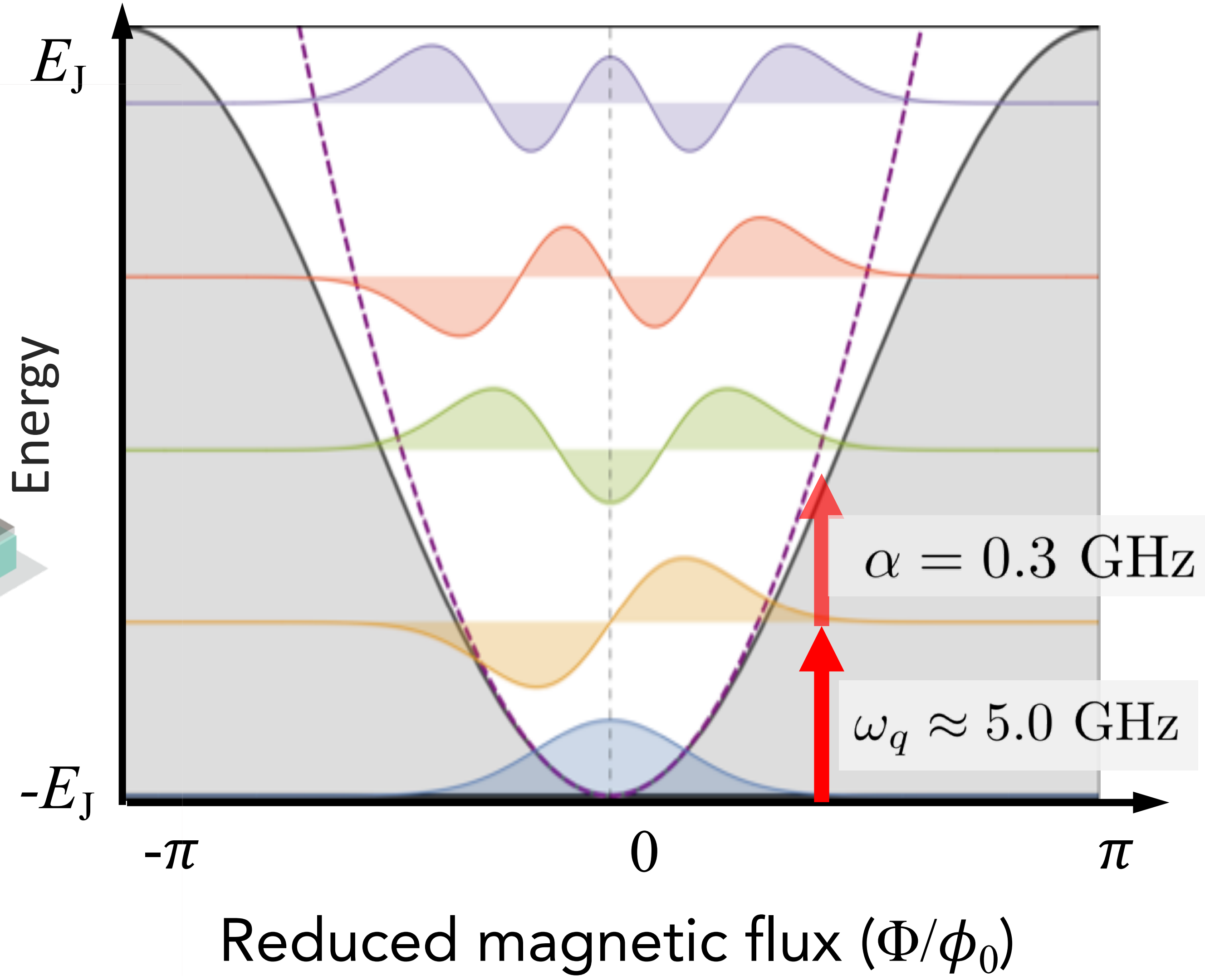
$$C_J = 65 \text{ fF} \qquad E_C = \frac{e^2}{2C} = 0.3 \text{ GHz} \qquad Z = \sqrt{\frac{L}{C}} \approx 450\ \Omega$$

Quantum fluctuations of the transmon qubit

$\Phi_{\mathrm{ZPF}}/\phi_0 \approx 0.5$

$Q_{\mathrm{ZPF}}/2e \approx 1.0$



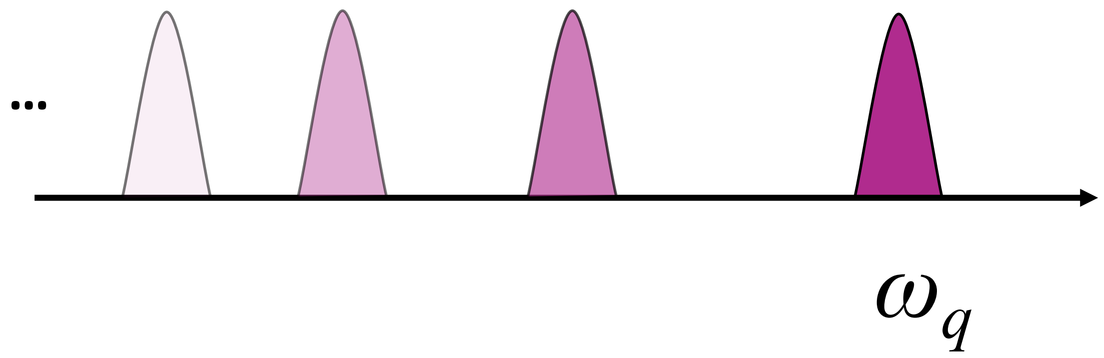


Energy diagram and transition spectrum

Energy levels

$$\hat{H} \approx \omega_0 \hat{a}^\dagger \hat{a} - \frac{\alpha}{2} \hat{a}^{\dagger 2} \hat{a}^2$$

Transition spectrum



Ladder operators and matrix representation

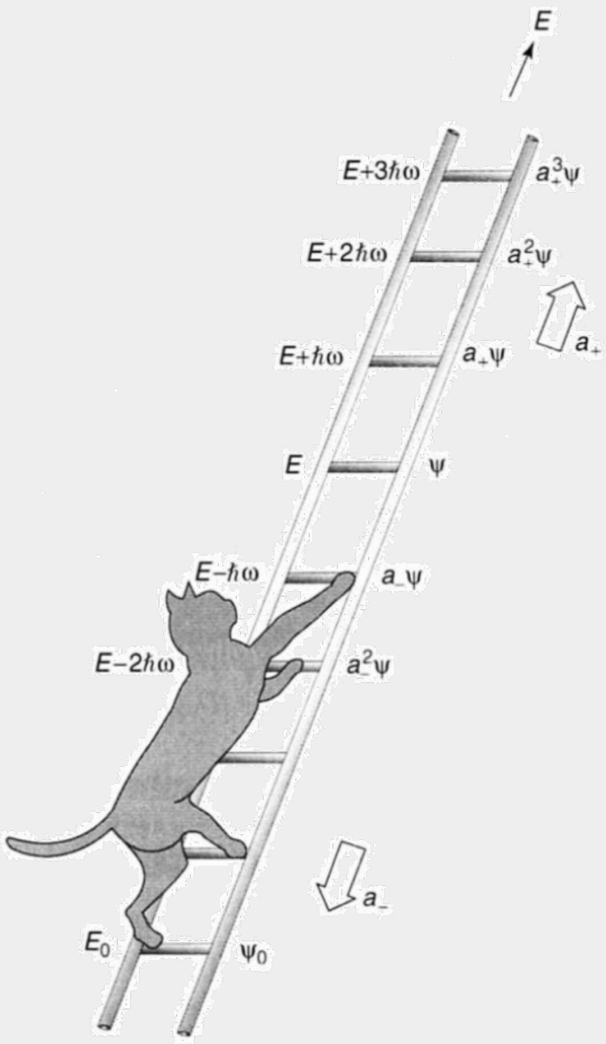


Image: Griffiths, D.J.

annihilation

$$\hat{a}\,|0\rangle = 0$$

$$\hat{a}\,|1\rangle = \sqrt{1}\,|0\rangle$$

creation

$$\hat{a}^\dagger\,|0\rangle = \sqrt{1}\,|1\rangle$$

$$\hat{a}^\dagger\,|1\rangle = \sqrt{2}\,|2\rangle$$

general hopping

$$\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$\hat{a}^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

The Transmon qubit: restricting Hilbert space

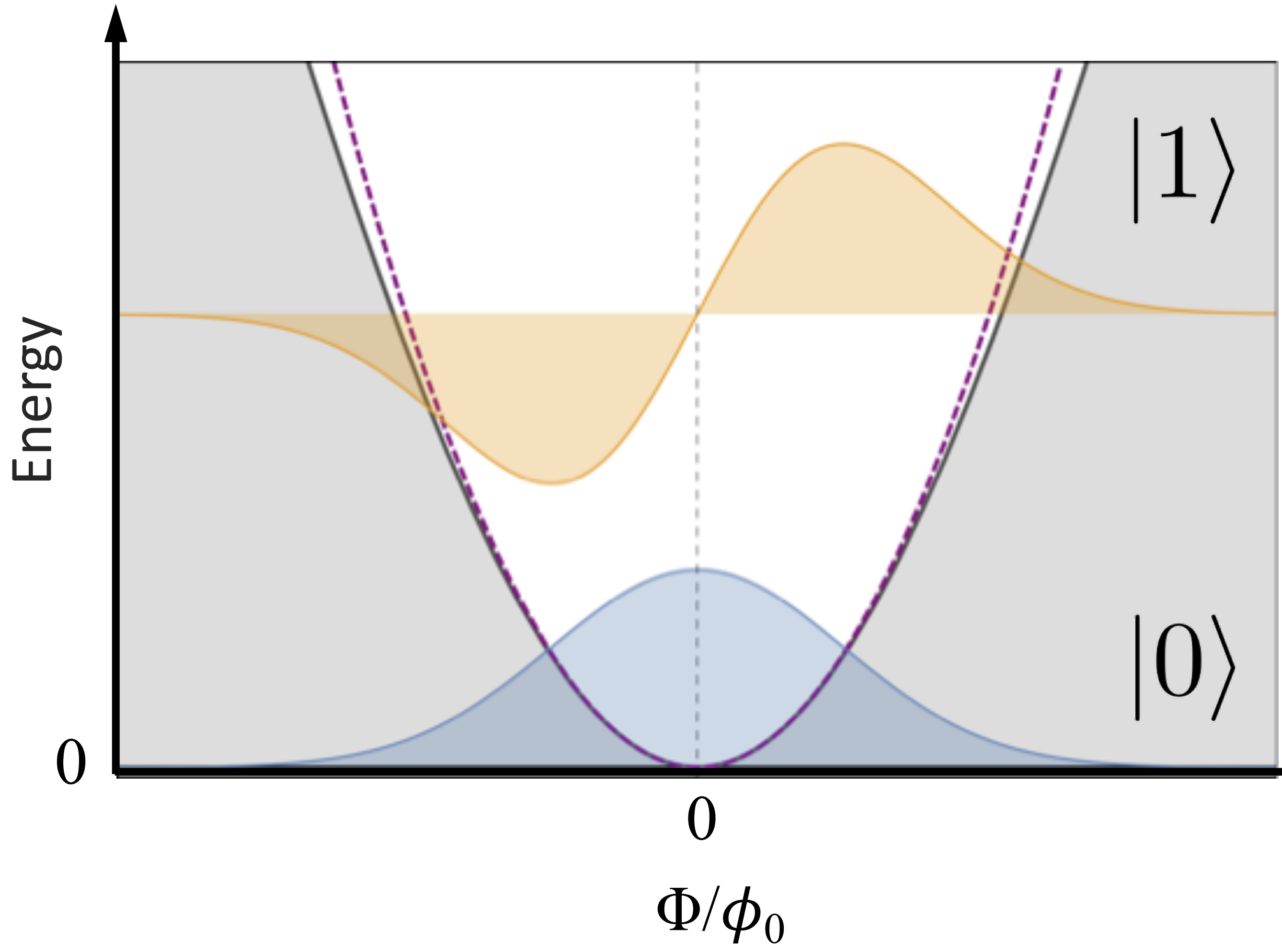
$$\hat{H}_4^{\mathrm{RWA}} \approx \hbar\omega_q \hat{N} - \frac{\hbar\alpha}{2}\hat{N}\left(\hat{N}-1\right)$$

$$\hat{N} \equiv \hat{a}^\dagger \hat{a}$$

Restrict to qubit subspace of |0> and |1>

$$\hat{a}^\dagger \hat{a} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & \ddots \end{pmatrix}$$

$$\hat{a} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & \ddots \end{pmatrix}$$



The Transmon qubit: restricting Hilbert space

$$\hat{H}_4^{\mathrm{RWA}} \approx \hbar\omega_q \hat{N} - \frac{\hbar\alpha}{2}\hat{N}\left(\hat{N}-1\right)$$

$$\hat{N} \equiv \hat{a}^\dagger \hat{a}$$

Restrict to qubit subspace of |0> and |1>

$$\left(\hat{N} - \frac{1}{2}\hat{I}\right) \mapsto -\frac{1}{2}\hat{Z} \qquad \hat{a} \mapsto \hat{\sigma}_- = \frac{1}{2}\left(\hat{X} - i\hat{Y}\right)$$

Fock number operator

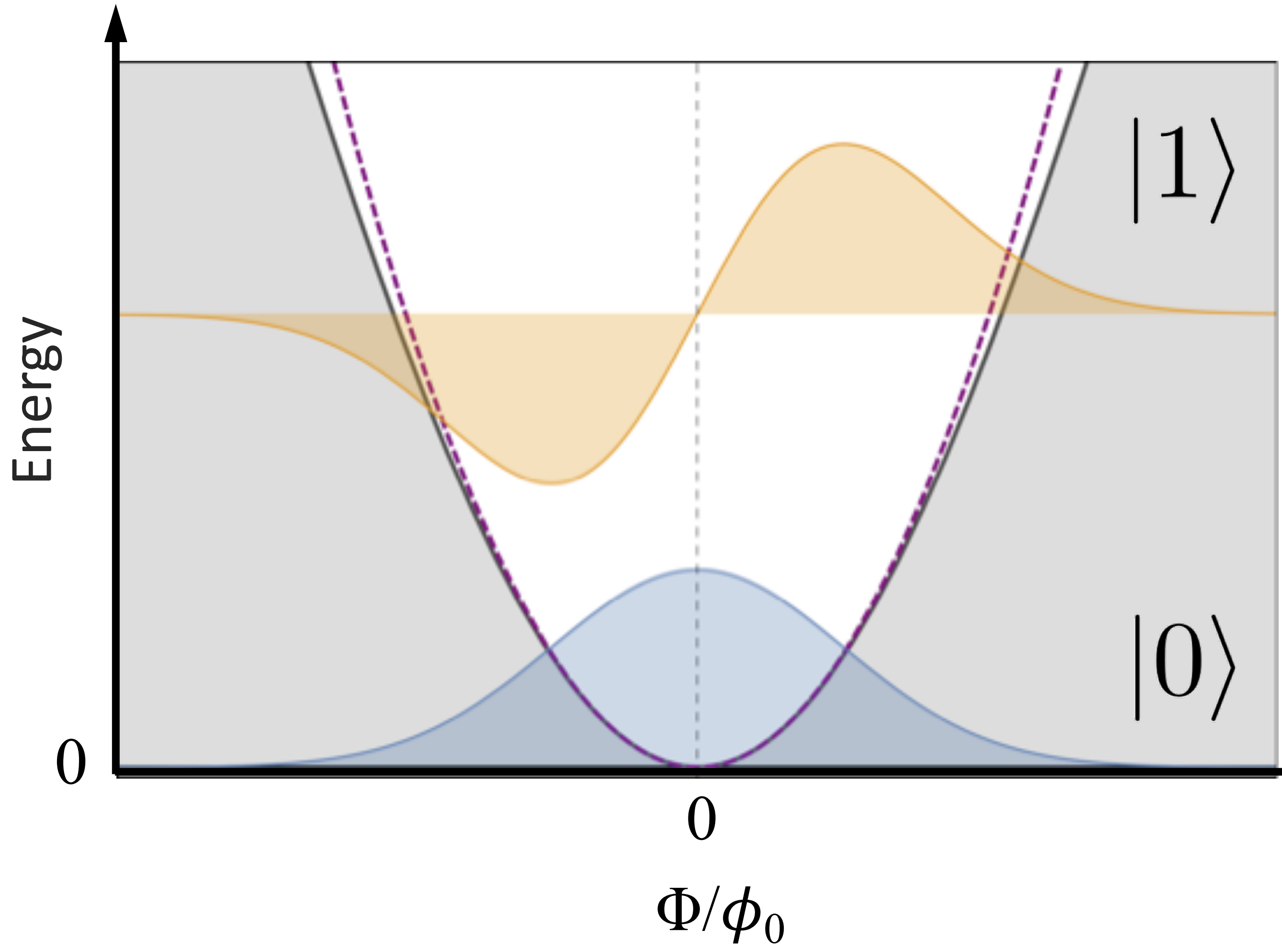
$$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

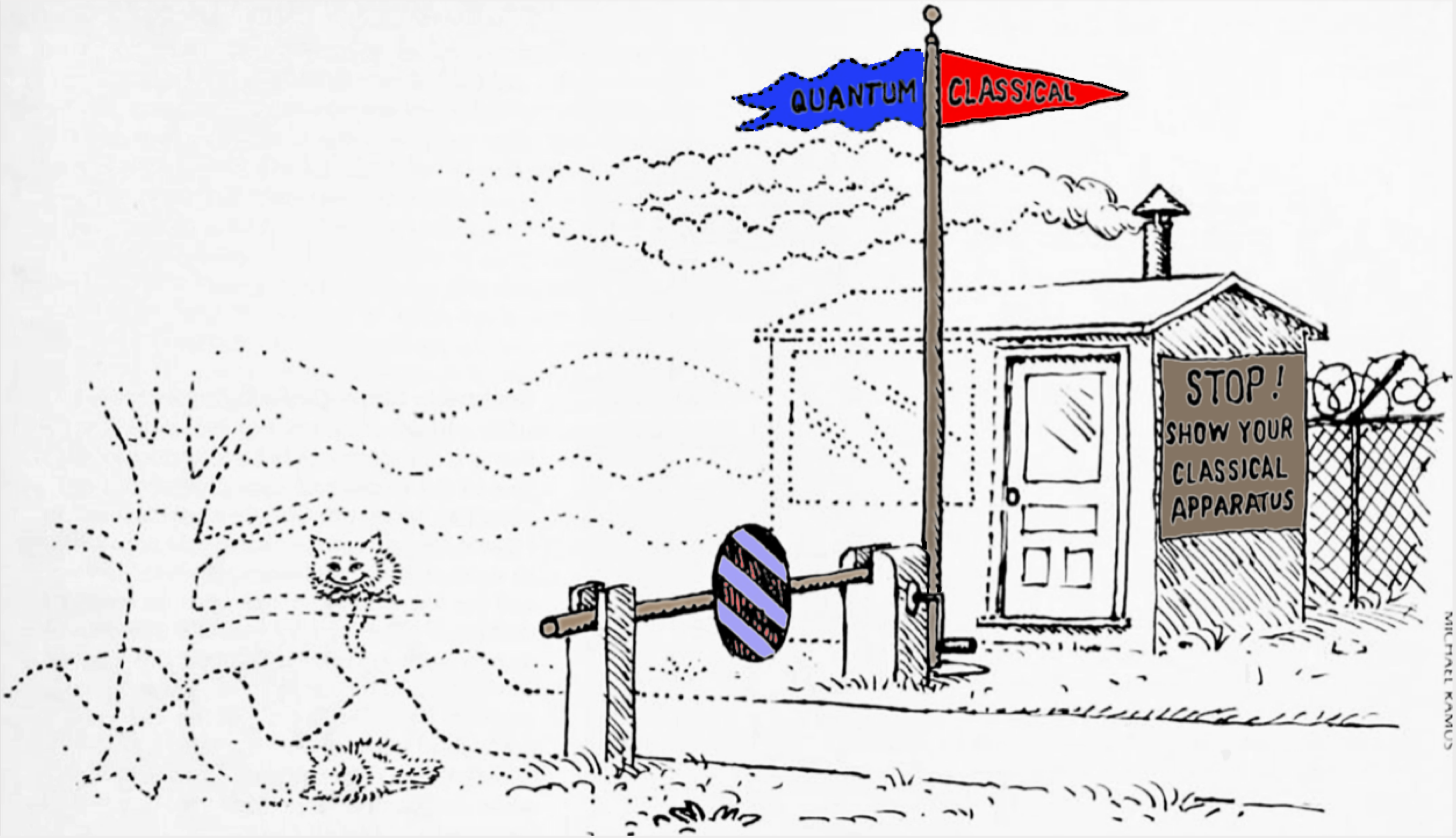
Qubit Pauli Z operator

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Qubit Pauli X and Y operators

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

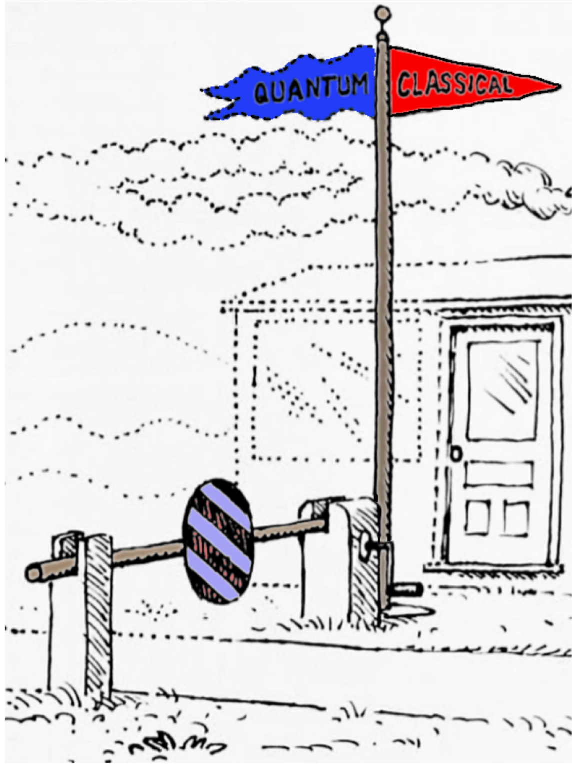




Drawing: Zurek, Physics Today (1991)

Calculating the energy-participation ratio

$$p_m = \frac{\text{Energy stored in junction}}{\text{Inductive energy stored in mode } m}$$



$$0 \leq p_{mj} \leq 1 \,.$$

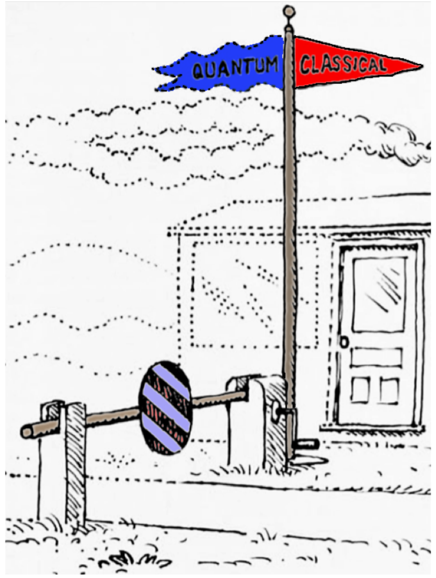
Minev et al., EPR to appear (2020); Dissertation arXiv: 1902.10355

Energy participation-ratio: a bridge

$$\hat{H}_{\text{full}}$$

What fraction of the energy of mode *m* is stored in junction *j*?

$$\frac{1}{\hbar}\phi_{mj}^2 = p_{mj}\frac{\omega_m}{2E_j}$$



for $j>1$, root requires sign bit $s_{mj} = \pm 1$

Drawing: Zurek, Physics Today (1991)

Minev et al., EPR to appear (2020); Dissertation arXiv: 1902.10355

All this explicated in

Minev dissertation Sec. 4.1 (arXiv: 1902.10355)

In detail, to appear soon in the EPR paper:

Energy-participation quantization of Josephson circuits

Zlatko K. Minev,[1] Zaki Leghtas,[1,2] Shantanu O. Mundhada,[1]
Lysander Christakis,[1,3] Ioan M. Pop,[1,4] and Michel H. Devoret[1]

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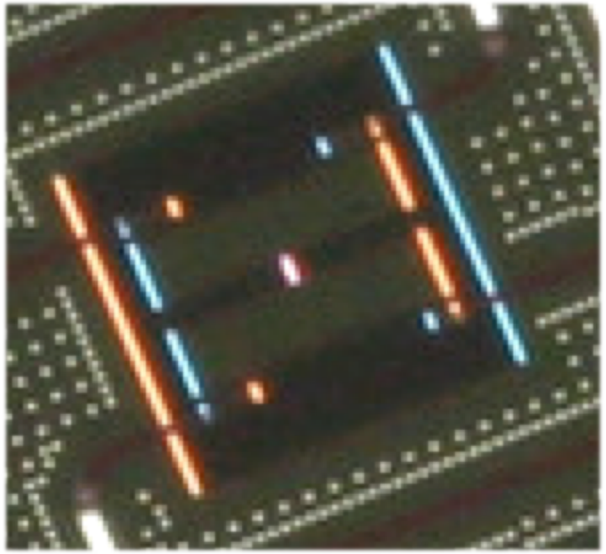
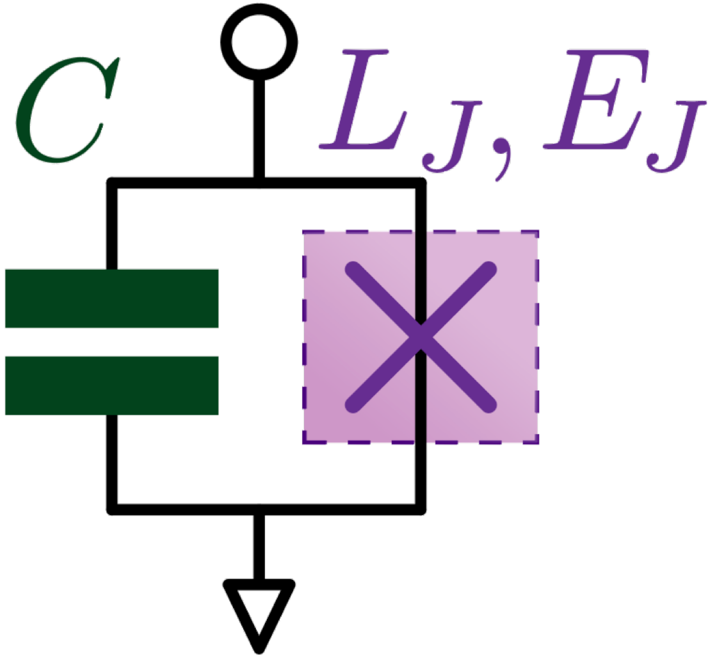
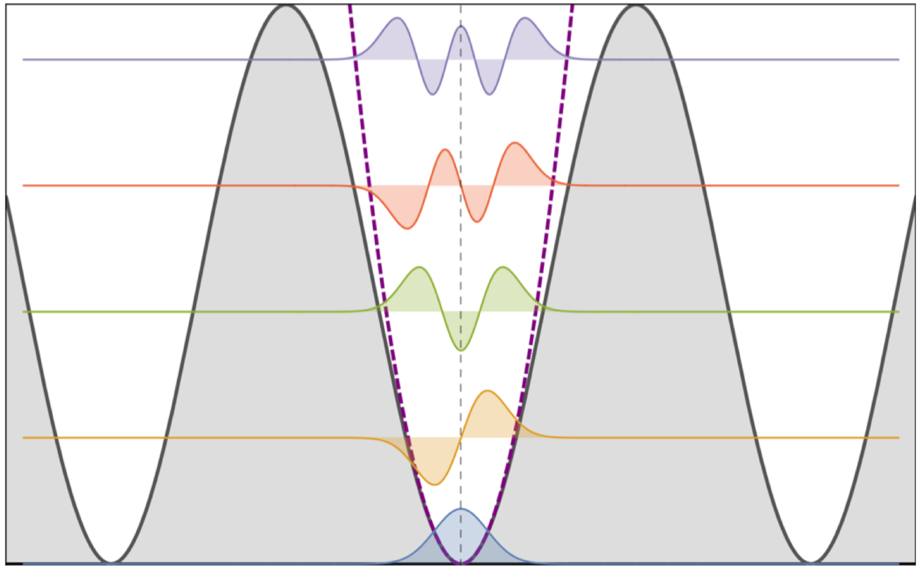
[4]*Physikalisches Institut, Karlsruhe Institute of Technology, 76131 Karlsruhe, Germany*

Superconducting microwave circuits incorporating nonlinear devices, such as Josephson junctions, are an appealing platform for emerging quantum technologies. Further increase of circuit complexity requires efficient numerical methods for the calculation and optimization of the spectrum, nonlinear interactions, and dissipation in multi-mode distributed quantum circuits. Here, we present a method based on a powerful concept—the energy-participation ratio (EPR) of a dissipative or nonlinear element in an electromagnetic mode. The EPR, a number between zero and one, quantifies how much of the energy of a mode is stored in each element. It is calculated from a unique, efficient electromagnetic eigenmode simulation of the linearized system, including lossy elements, and is the key to the modeling of the quantum Hamiltonian of the system. The method provides an intuitive and simple-to-use tool to quantize multi-junction circuits, and is especially well-suited for weakly anharmonic systems, such as transmon qubits coupled to resonators, or Josephson transmission lines. We experimentally tested this method on a variety of Josephson circuits in three-dimensional and flip-chip architectures, and demonstrated agreement within several percents for nonlinear couplings and modal Hamiltonian parameters, spanning five-orders of magnitude in energy, across a dozen samples.

Transmon Qubit

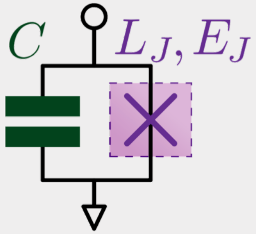


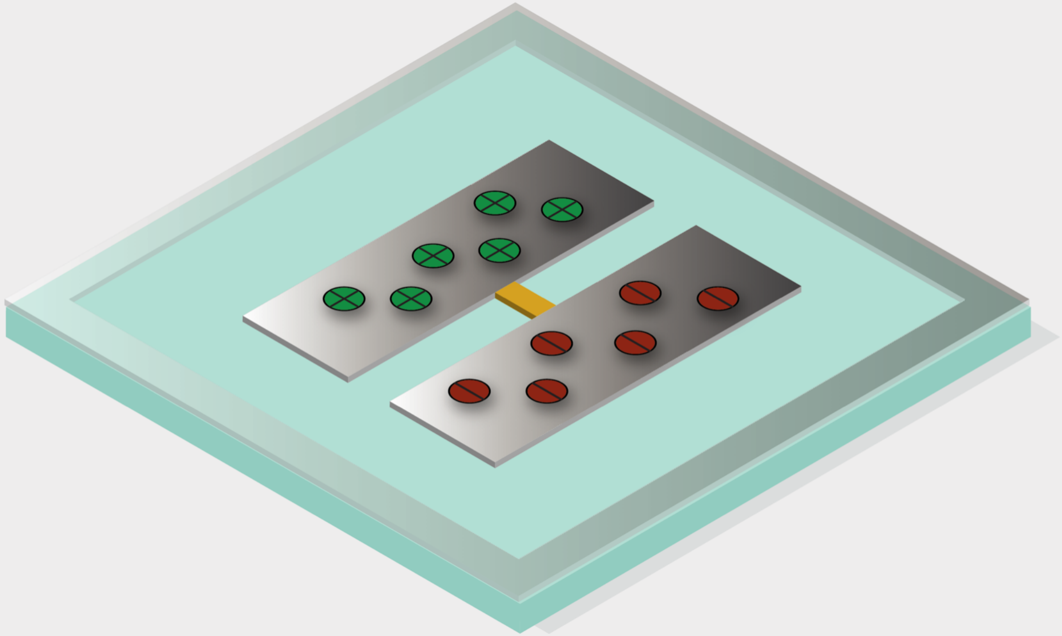


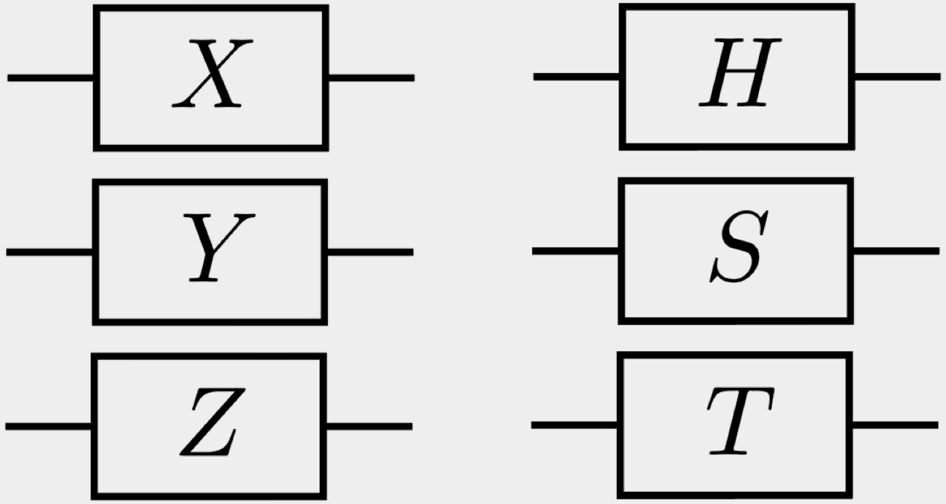


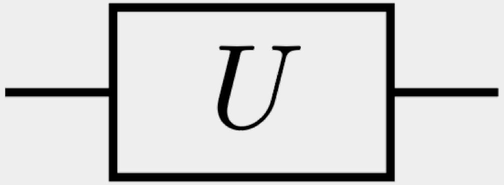
Control and beyond

Single-qubit quantum gates



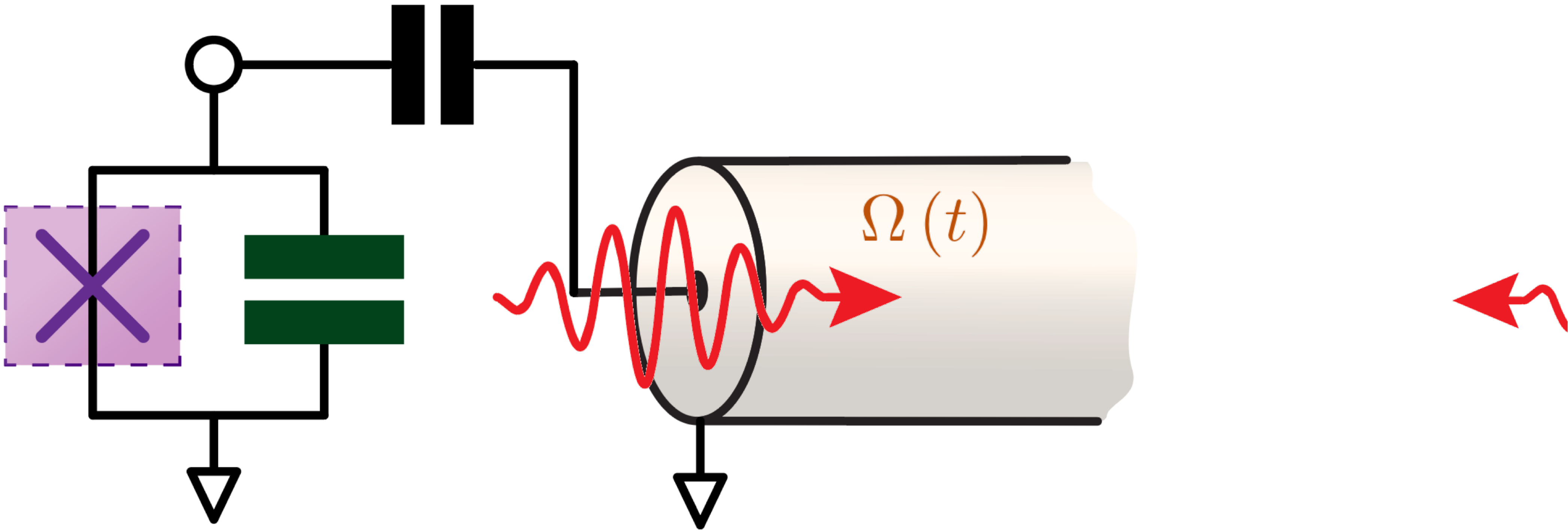






Qubit control

$$\hat{H}_{\mathrm{drive}} = -i\frac{1}{2}\Omega(t)\left(\hat{a}^{\dagger} - \hat{a}\right) \quad \longmapsto \quad \frac{1}{2}\Omega(t)\,\hat{Y}$$



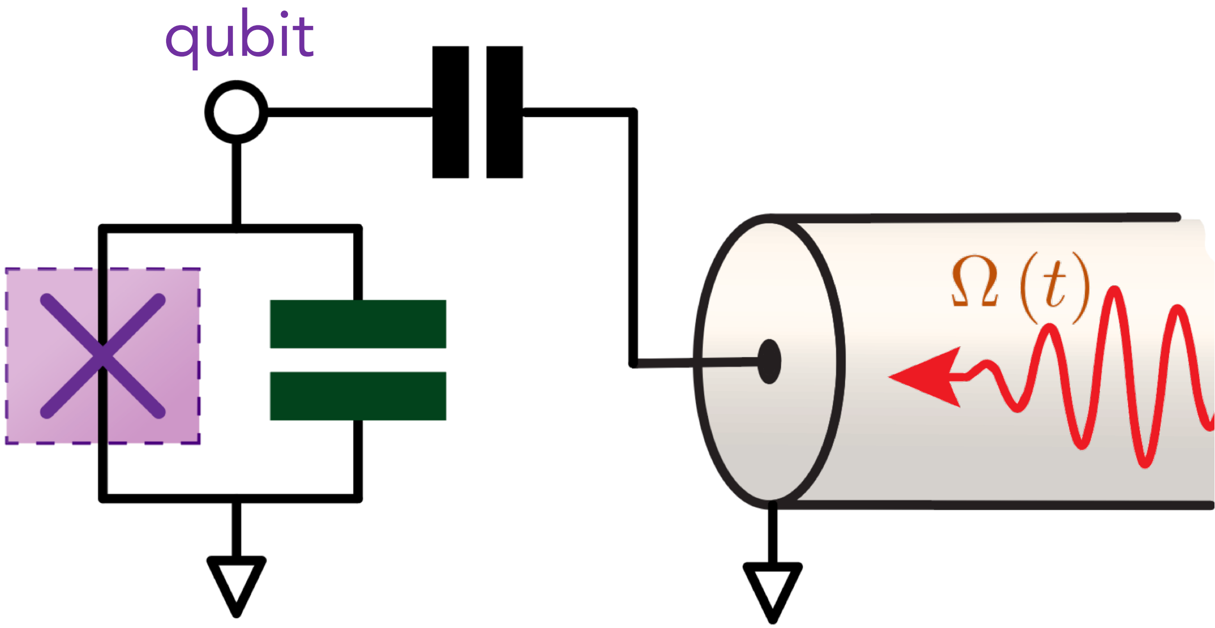
qubit

input-output line
(control, shaped signals, but also environment)

Qubit control: overview

$$\hat{H}_{\text{int}} = -\frac{i}{2}\Omega\left(t\right)\left(\hat{a}^{\dagger}\left(t\right) - \hat{a}^{\dagger}\left(t\right)\right)$$

$$\mapsto -\frac{i}{2}\Omega\left(t\right)\left(\hat{\sigma}^{\dagger} - \hat{\sigma}^{\dagger}\right)$$



$$\Omega\left(t\right) = \Omega_0 \sin\left(\omega_d t + \theta_d\right)$$

$$\sin\left(\omega_d t + \theta_d\right) = \frac{i}{2}\left(e^{-i(\omega_d t+\theta)} - e^{+i(\omega_d t+\theta)}\right)$$

$$\hat{\sigma} \mapsto \hat{\sigma}e^{-i\omega_d t}$$

Qubit control: overview

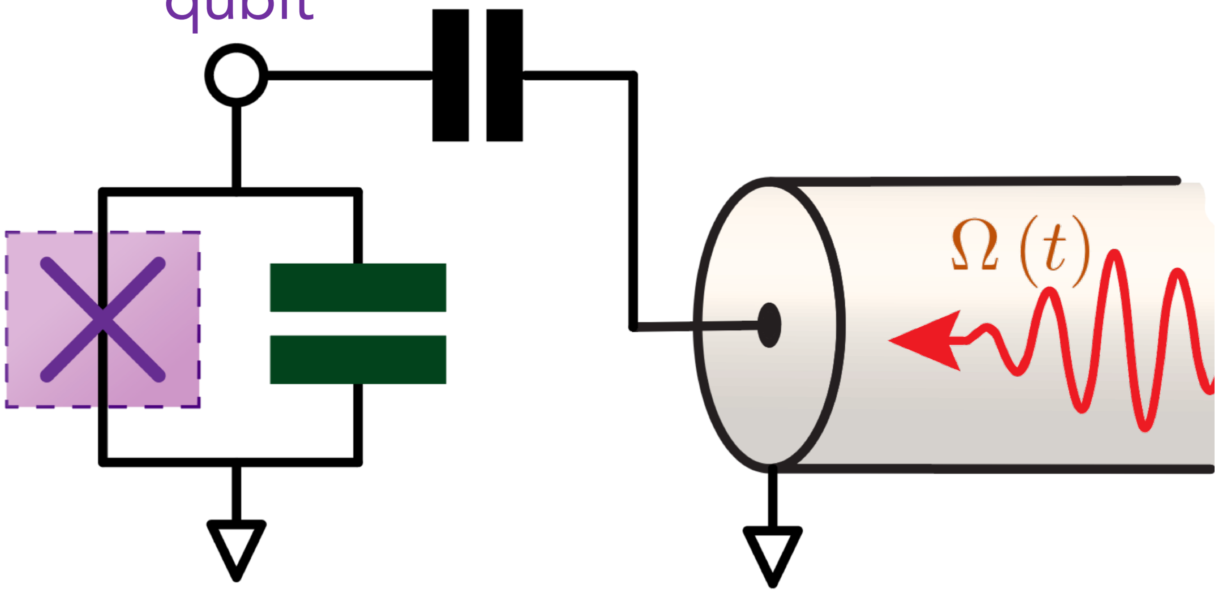
$$\hat{H}_{\mathrm{q}} = -\frac{\hbar\omega_0}{2}\hat{Z}$$

$$\hat{U}(t) = \exp\left[\hat{u}(t)/i\hbar\right]$$

$$\hat{u}(t) = +\frac{\hbar\omega_d}{2}\hat{Z}$$

$$\hat{H}_{\mathrm{R}} = U\hat{H}U^{\dagger} + \frac{\mathrm{d}}{\mathrm{d}t}\hat{u}(t)$$

$$= \frac{\hbar}{2}\Delta\hat{Z} \quad \Delta := \omega_d - \omega_0$$



$|1\rangle$

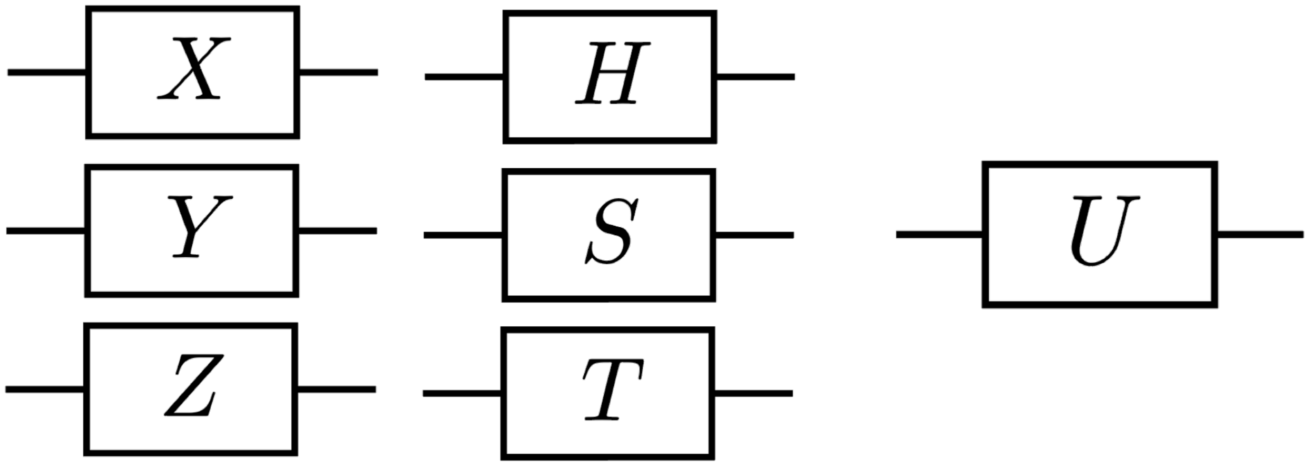
$|0\rangle$

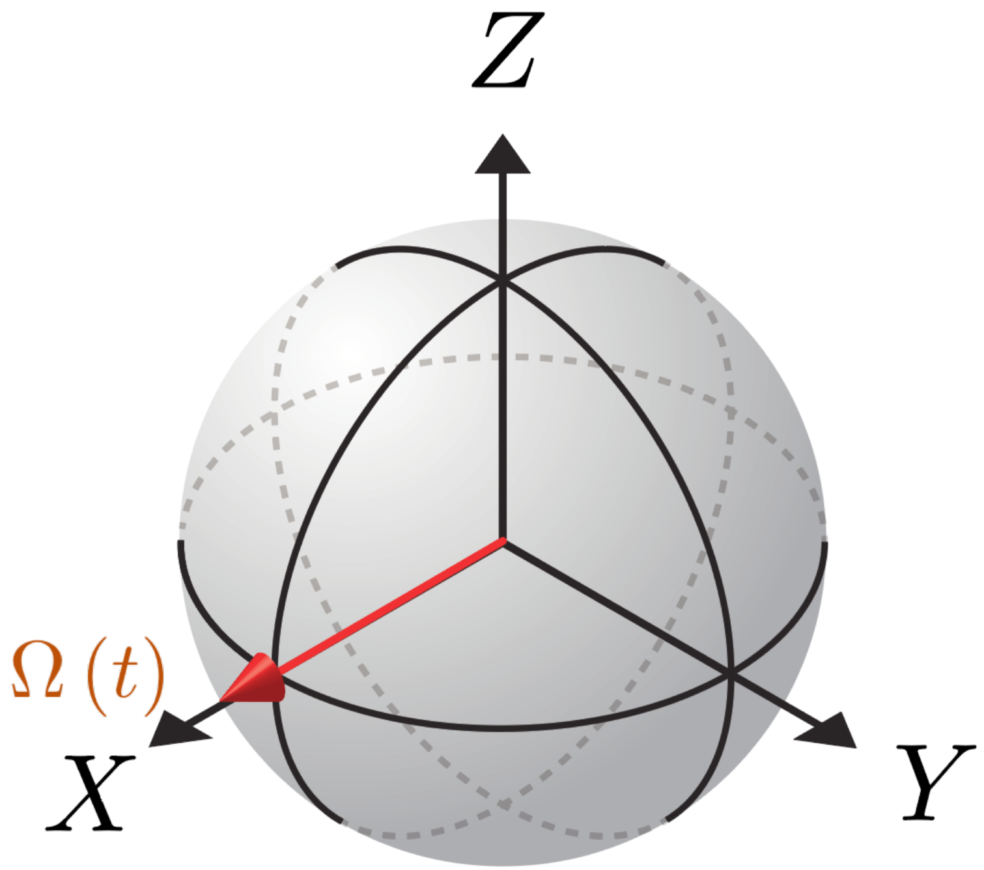
Qubit control: overview

$$\hat{H}_R = \frac{\hbar}{2}\Delta\hat{Z} + \frac{\hbar}{2}\Omega\left(e^{-i\theta}\hat{\sigma} + e^{+i\theta}\hat{\sigma}^\dagger\right)$$

$$\hat{X} = \frac{1}{2}\left(\hat{\sigma}^\dagger - \hat{\sigma}\right)$$

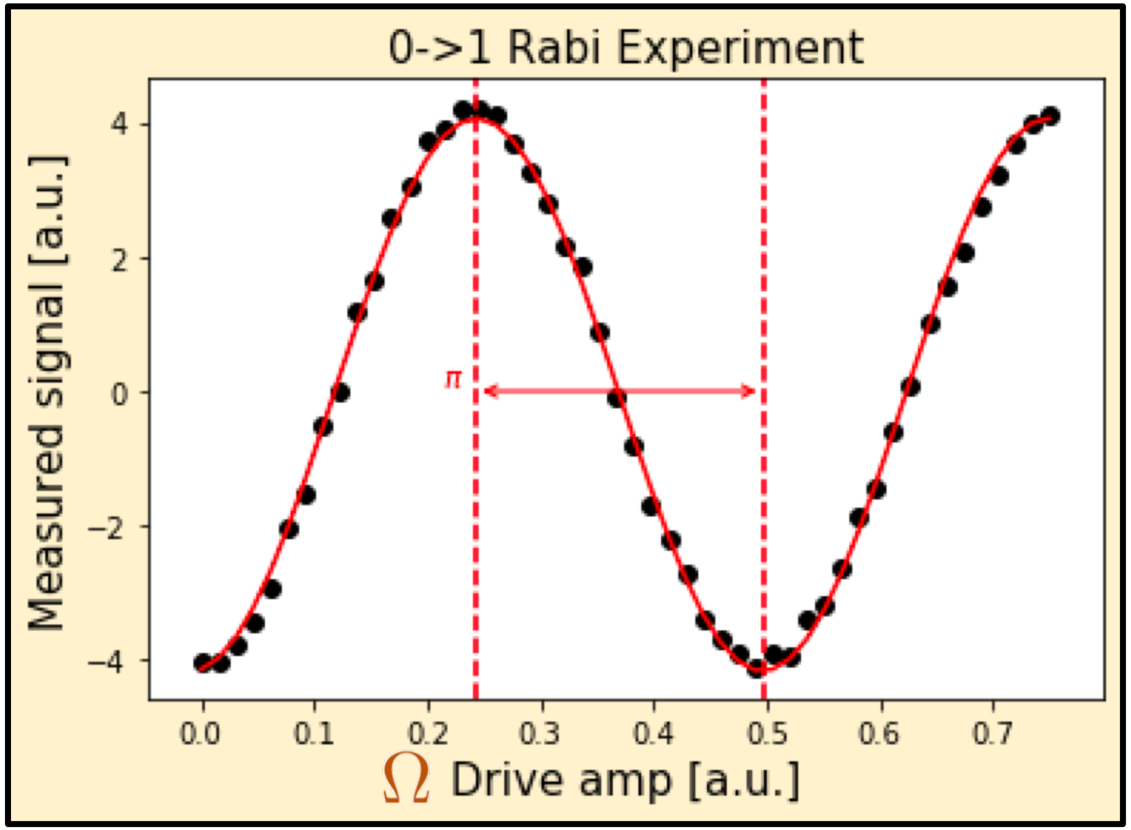
$$\hat{Y} = i\frac{1}{2}\left(\hat{\sigma}^\dagger + \hat{\sigma}\right)$$

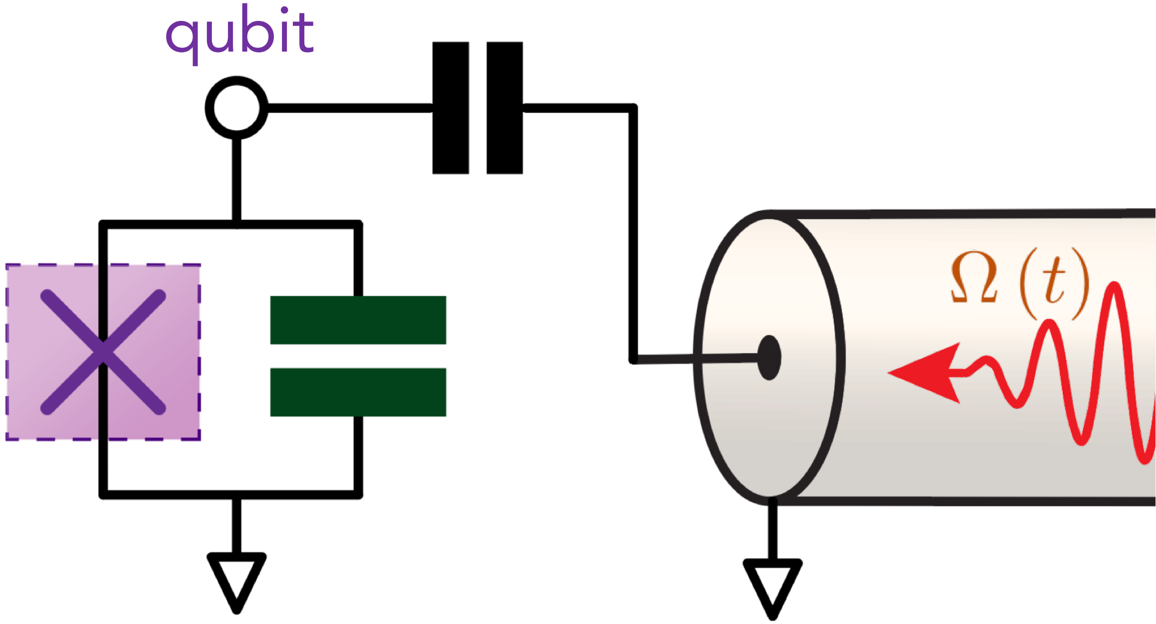




Qubit control: Covered in Lab 1 by Nick Bronn & Co.

$$\hat{H}_{\mathrm{drive}} = -i\frac{1}{2}\Omega\left(t\right)\left(\hat{a}^{\dagger} - \hat{a}\right) \quad \longmapsto \quad \frac{1}{2}\Omega\left(t\right)\hat{Y}$$





input-output line
(control, shaped signals, but also environment)

Control, noise, and dissipation go hand-in-hand

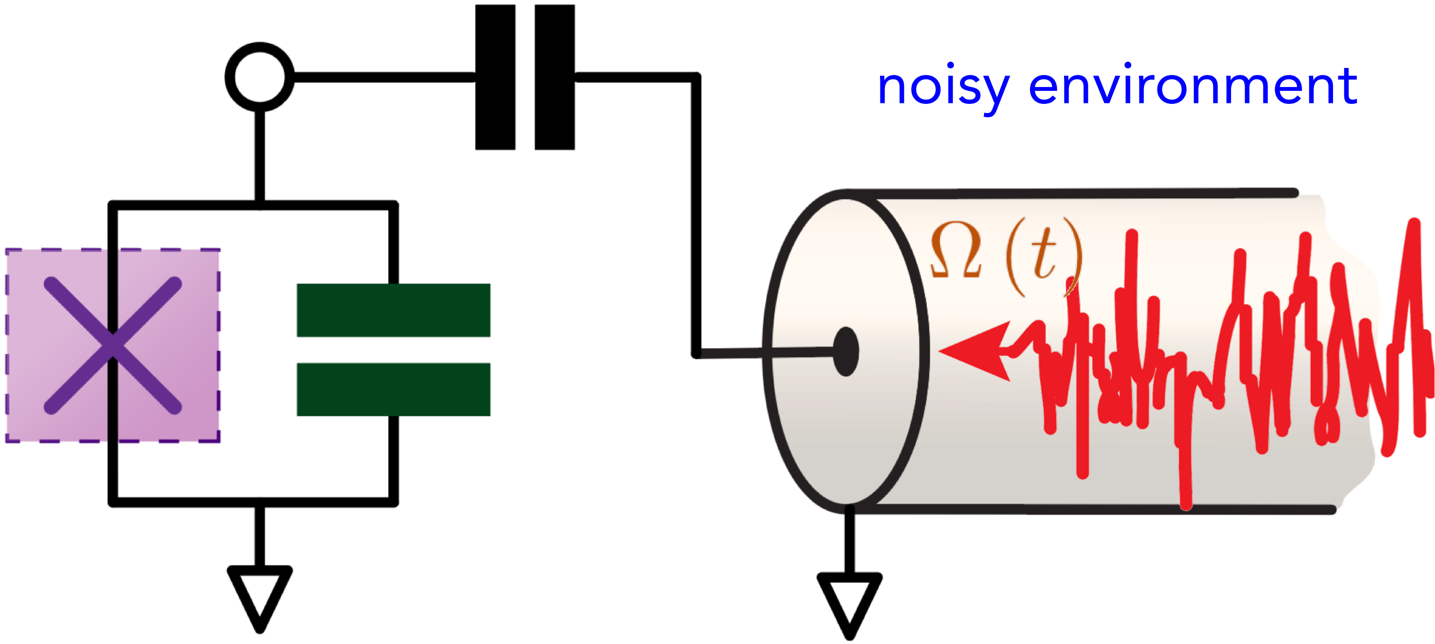
Can lead to uncontrolled, random bit and phase flip.

Fluctuation-dissipation theorem*

susceptibility, noise, and dissipation always go hand-in-hand

Intrinsic and *I-O* channels, lead to relaxation times:

T_1 : energy
T_2 : (coherence) transverse



Noisy environment, and always zero-point quantum fluctuations

* This is a major topic in condensed matter physics; we will only touch on it.

Coherence in superconducting circuits

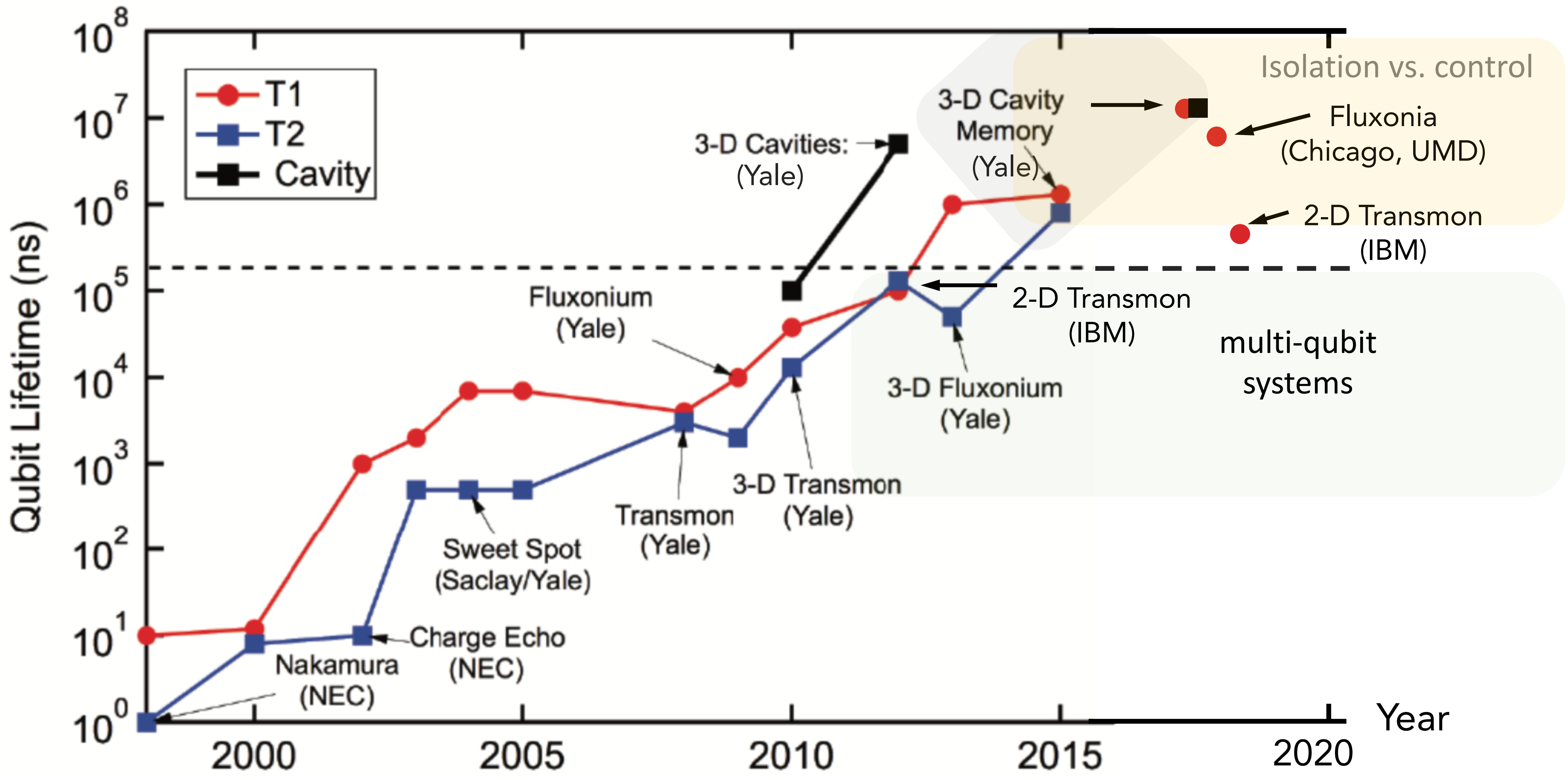
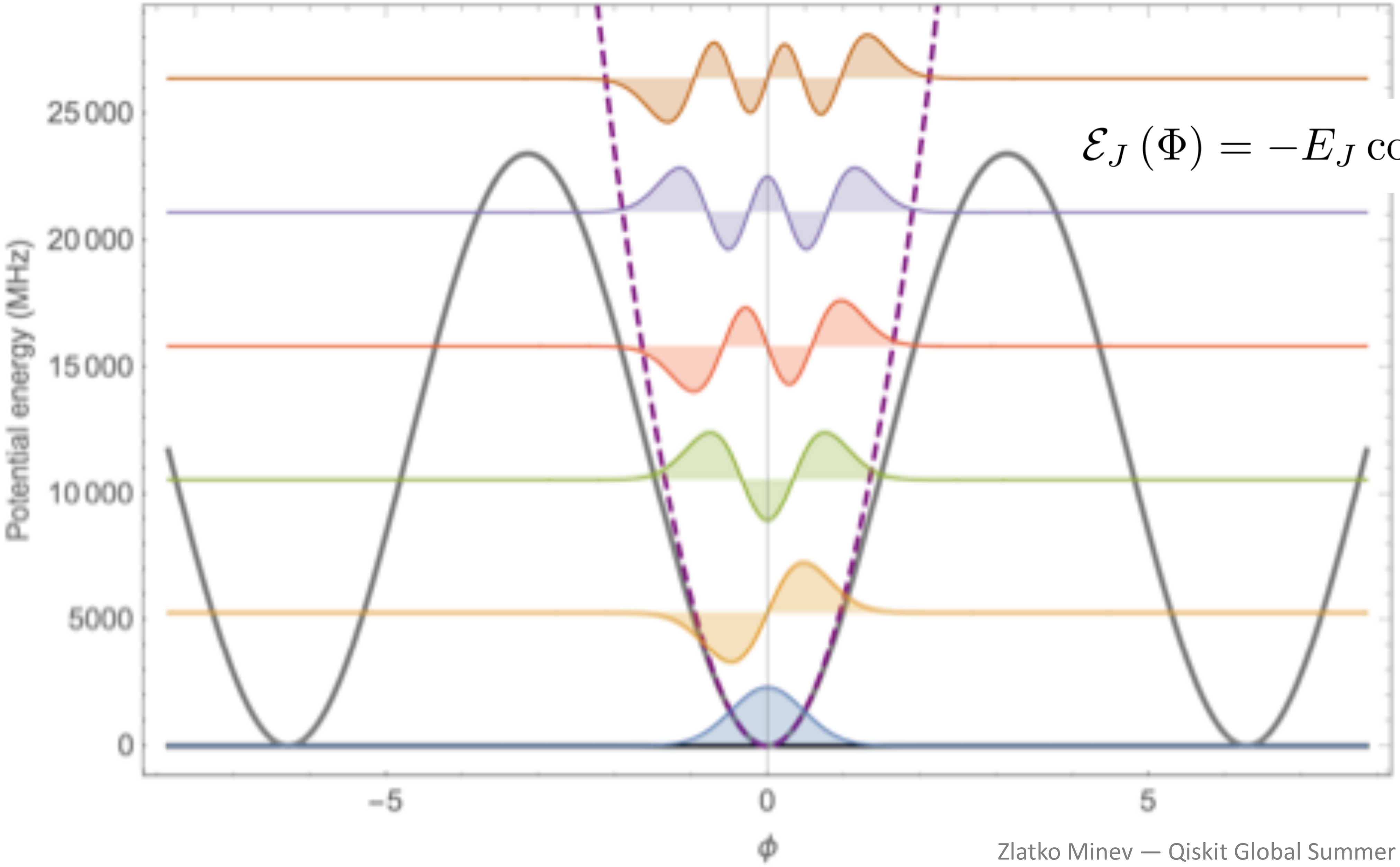


Image reproduced Reagor (2015), an update of Devoret and Schoelkopf (2013), and updated

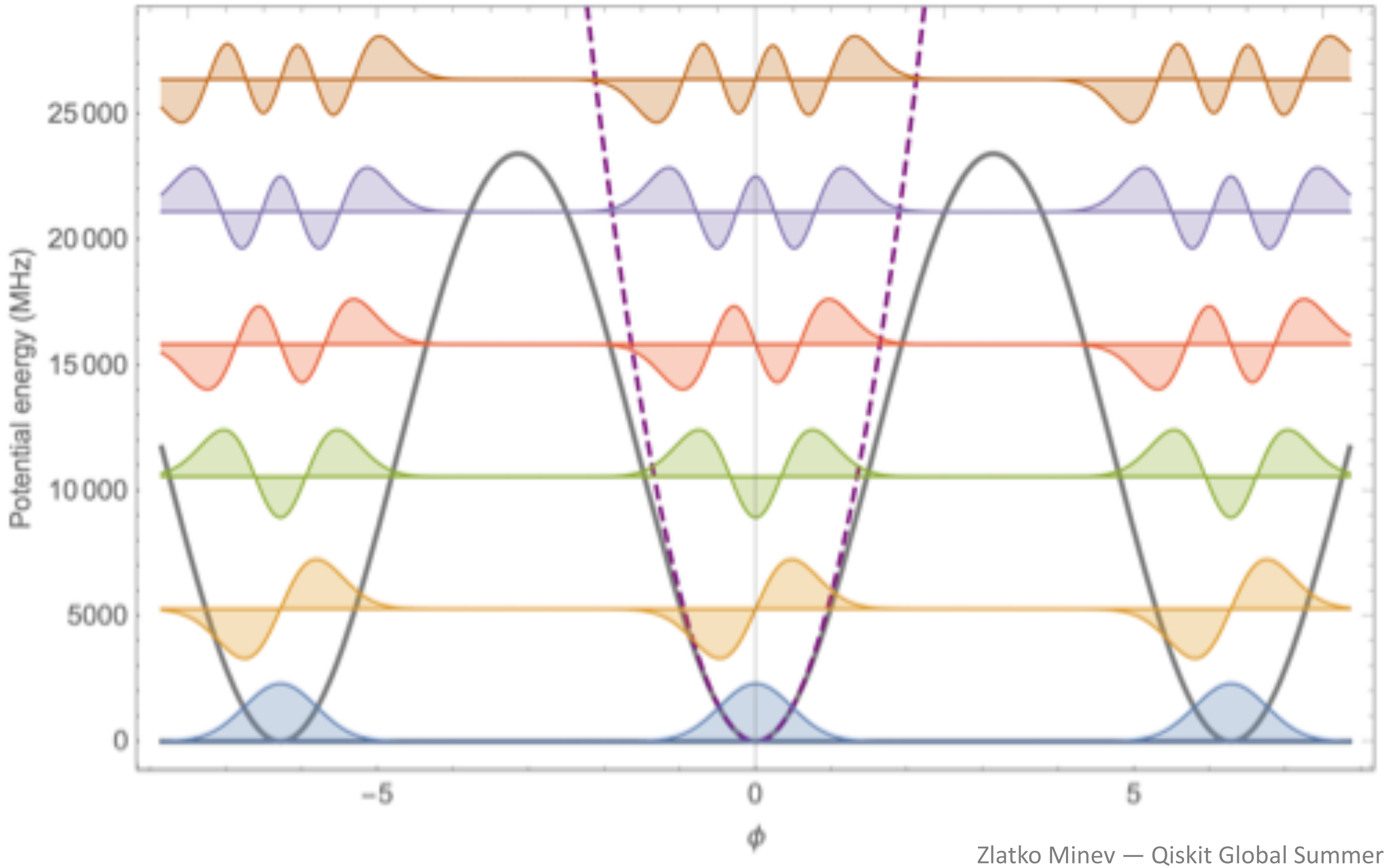
Is the junction phase/flux compact or not?

Harmonic Approximation



$$\mathcal{E}_J\left(\Phi\right) = -E_J \cos\left(\Phi/\phi_0\right)$$

Tight-binding model



Quantum measurement:

a very brief primer

Principal element of sensing: the measurement

A classical measurement example

Are there fumes in the oil barrel?

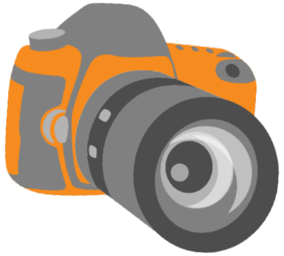
Example based on Wiseman and Milburn (2010)

Barrel image:freepik.com

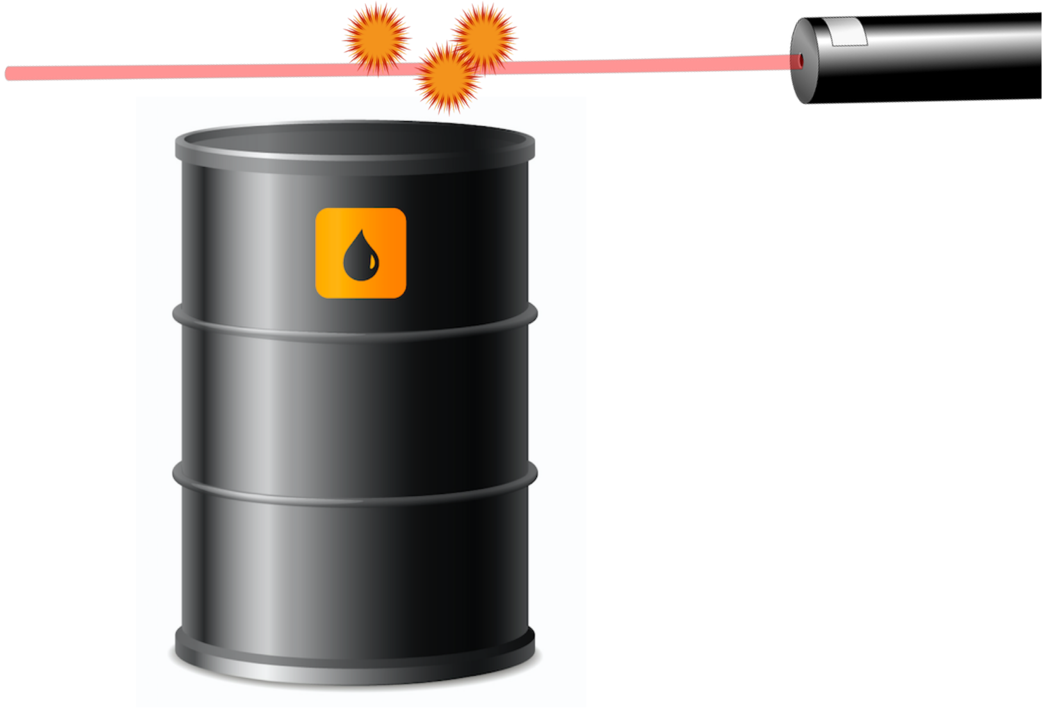
Two basic classes of measurements

Demolition

Caution: Classical measurements can also be disturbing!



Non-demolition



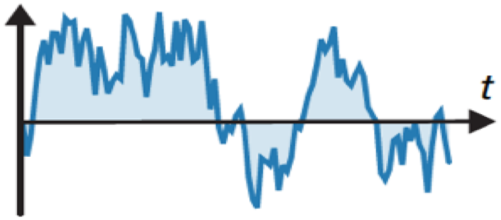
e.g., photon absorption

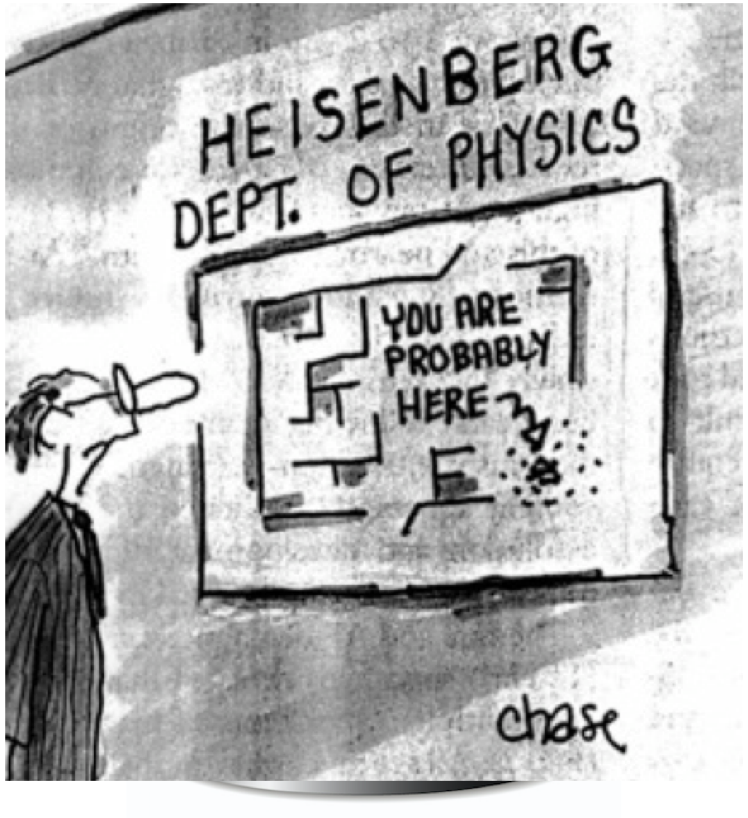
e.g., dispersive cavity

Both accessible in circuits, more on this later

Takeaways: Basic character of quantum measurements

Necessarily *disturb* system (back-action)





non-commuting

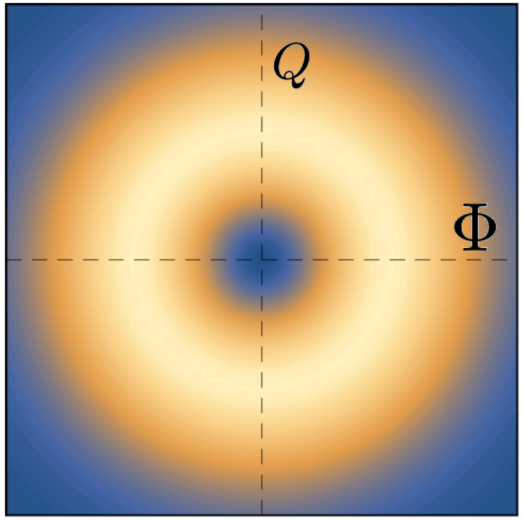
Heisenberg uncertainty

fundamental limits to precision, e.g., SQL, …

no joint probability distribution (over x, p)

quasi-probabilities (Wigner, Q)

no classical Fisher information



entropy-increasing

contextuality

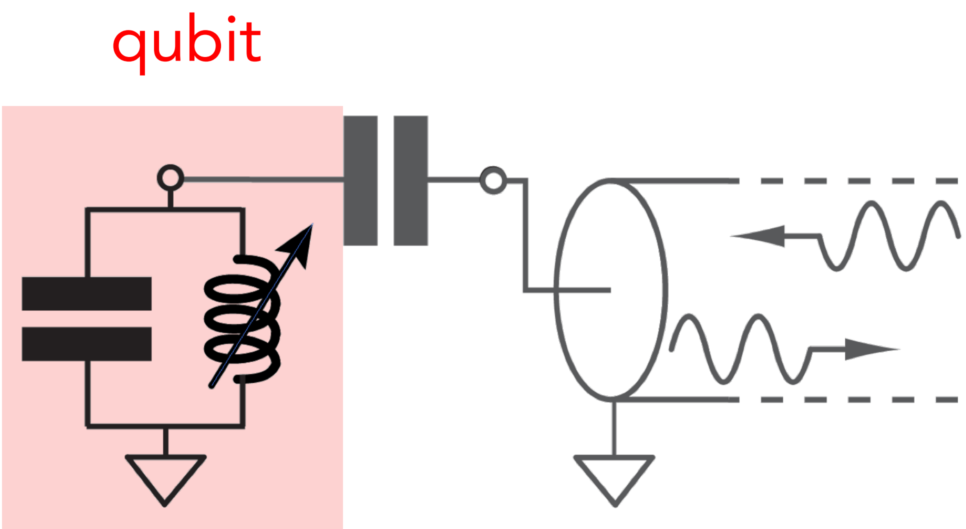
….

In quantum physics, you don't see what you get, you get what you see.

A.N. Korotkov
Private communication

Qubit measurement with circuits

Direct monitoring

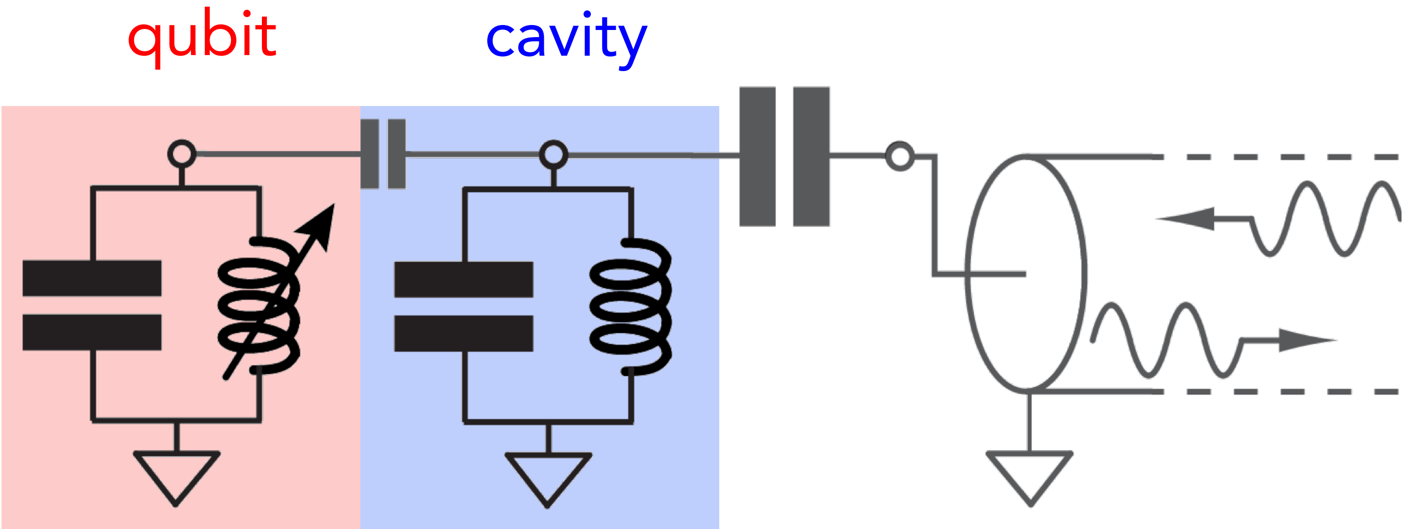


Demolition

Spontaneous emission

Clarke & Wilhelm, Nature (2008)

cQED dispersive

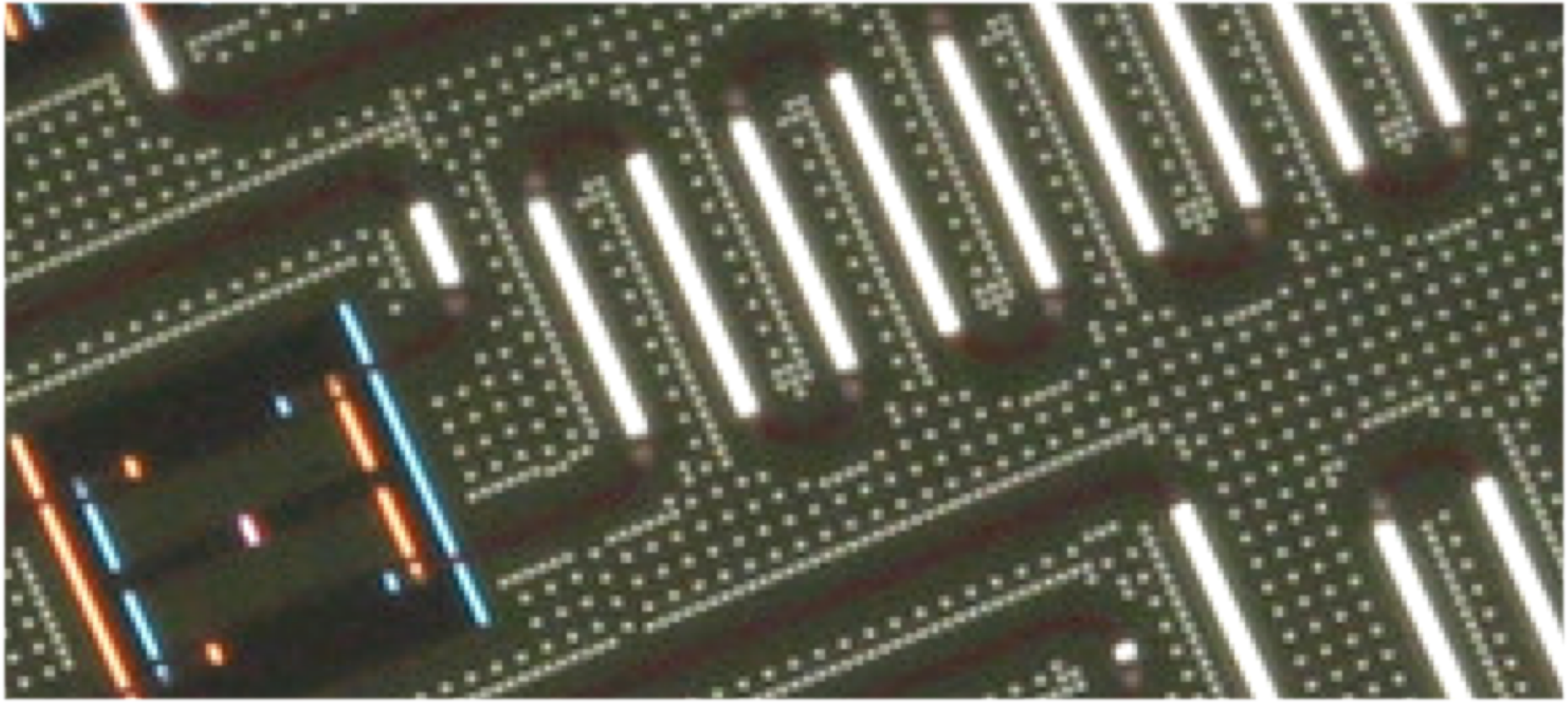


Quantum non-demolition* (QND)

Inhibited spontaneous emission

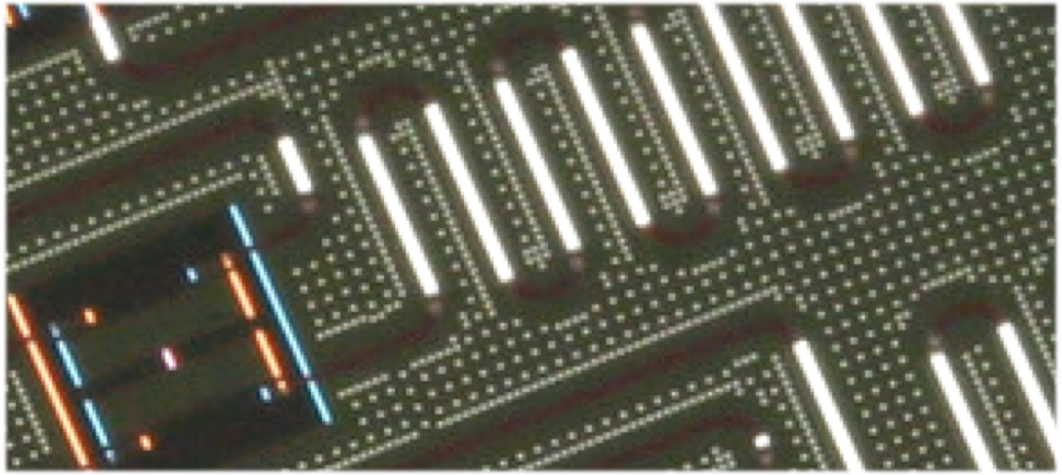
Blais *et al.*, PRA (2004)

Qubit coupled to resonator

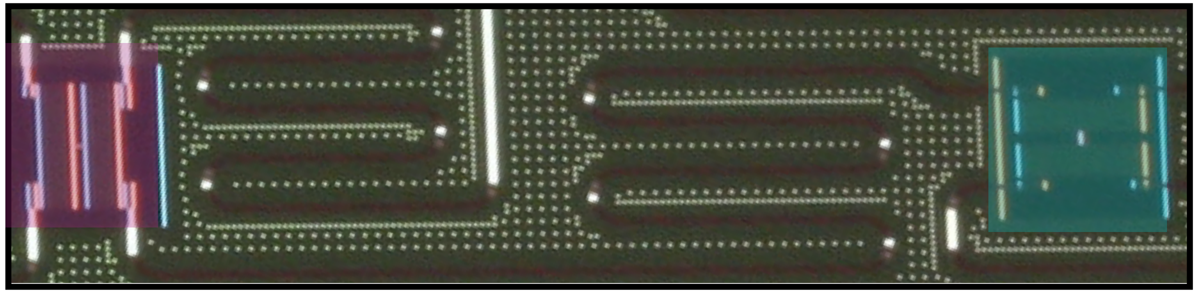


Qubit coupled to resonator

Qubit coupled to qubit

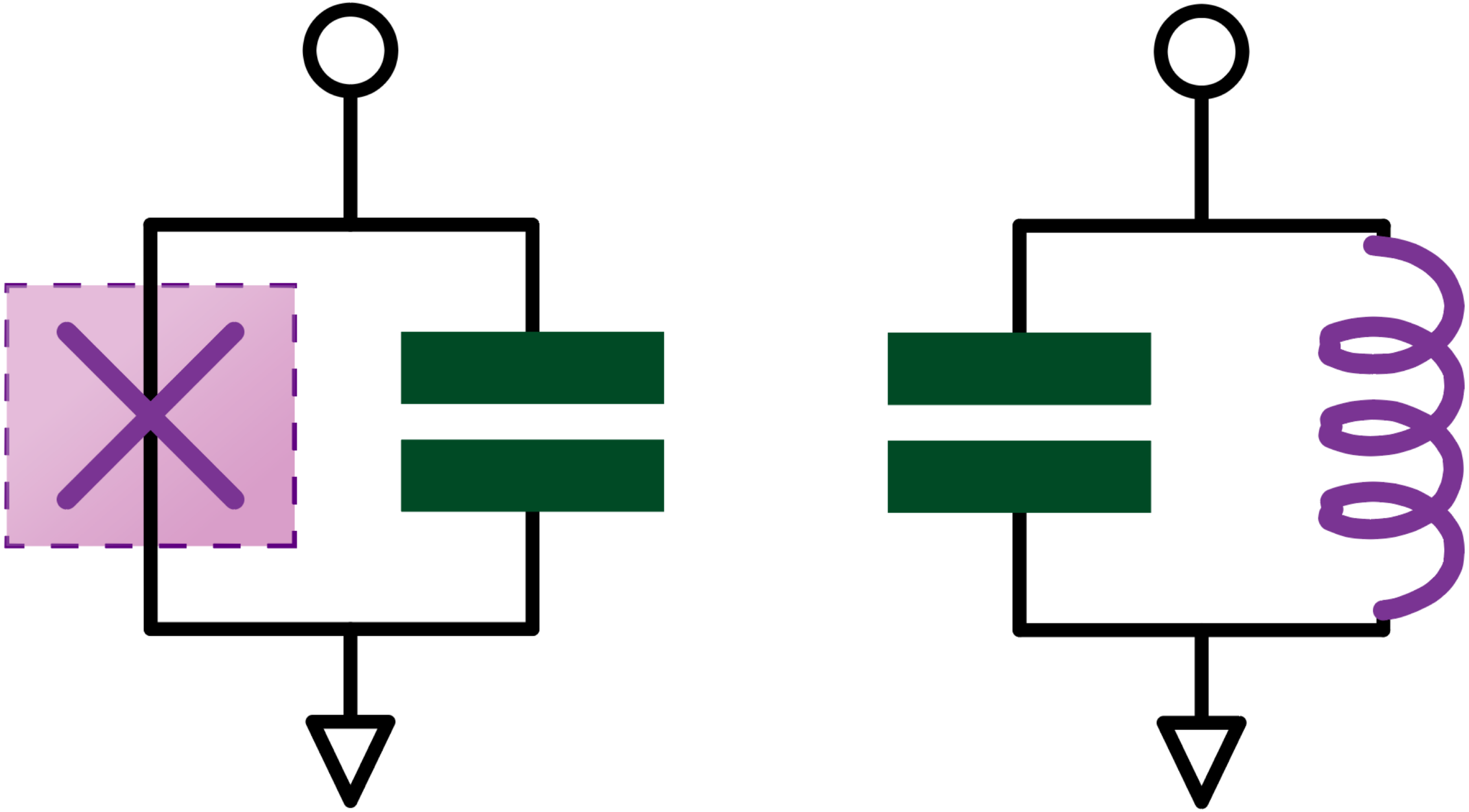


Readout

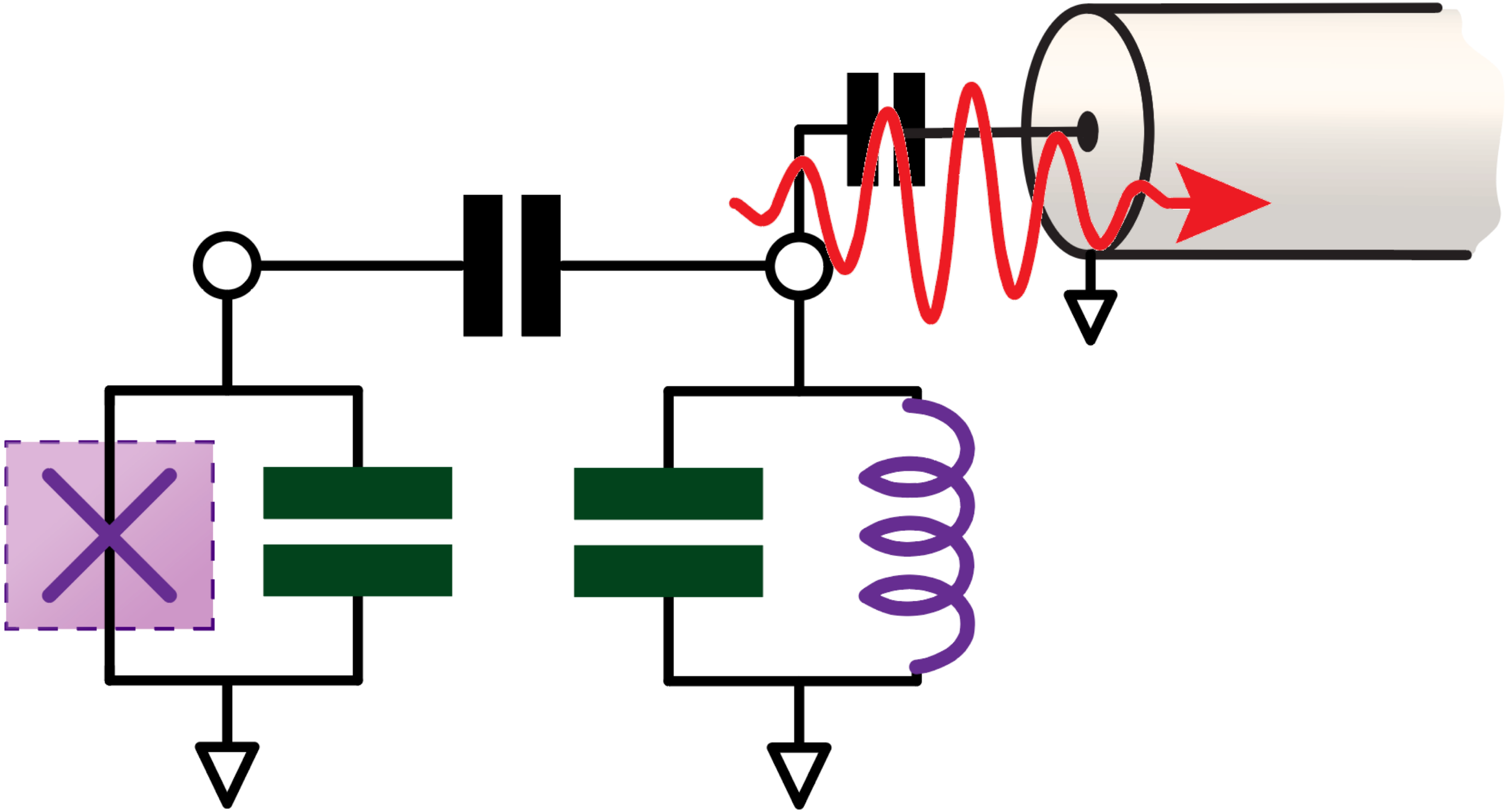


Two-qubit gates

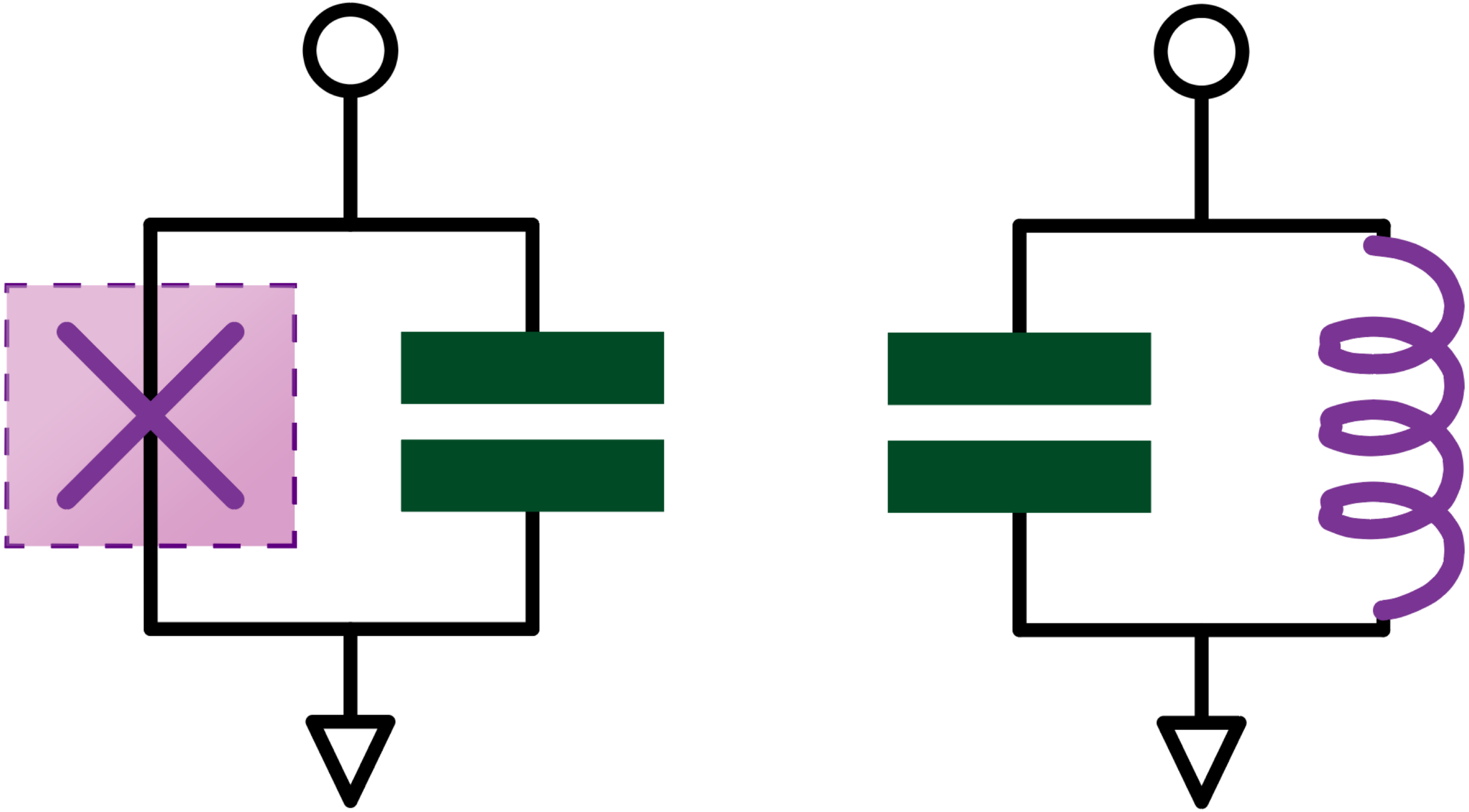
Qubit and oscillator



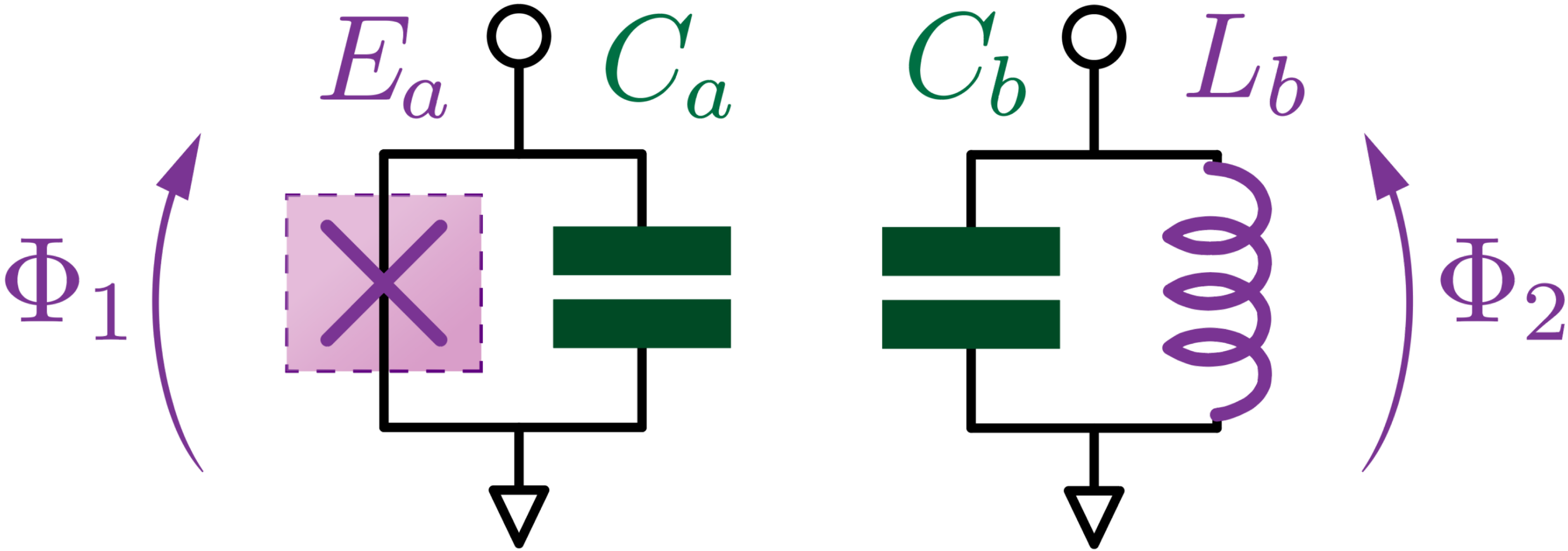
Qubit measurement with circuits



Qubit and oscillator

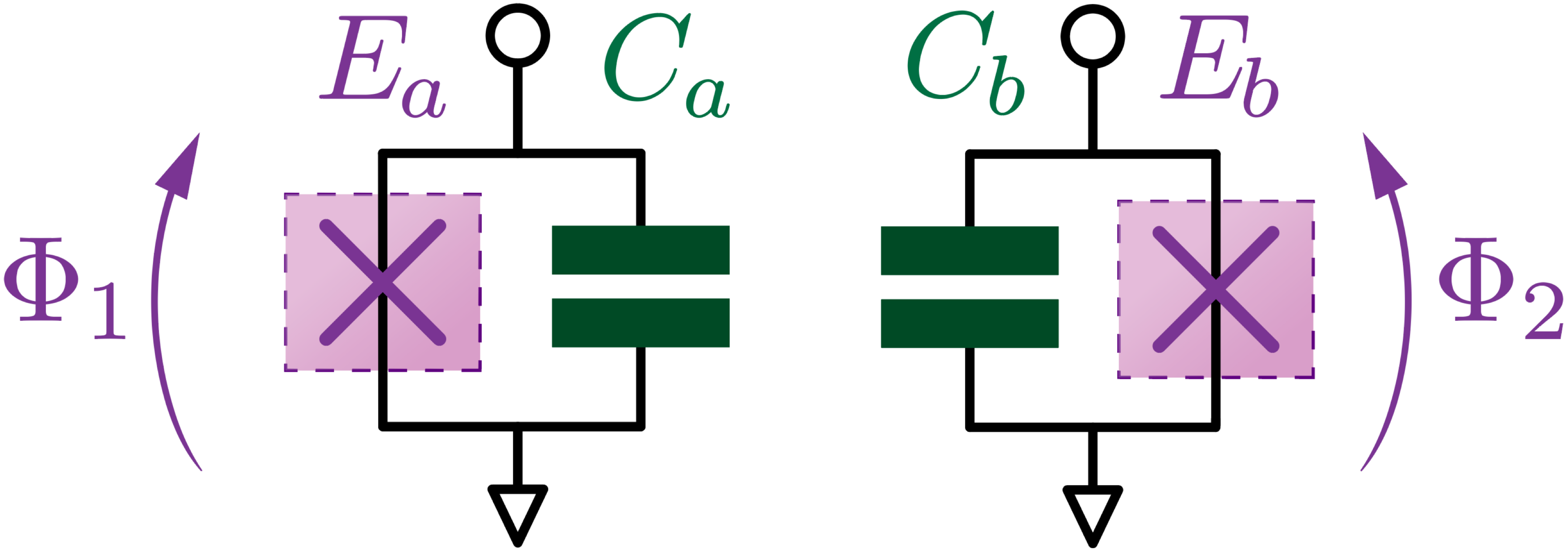


Qubit and oscillator



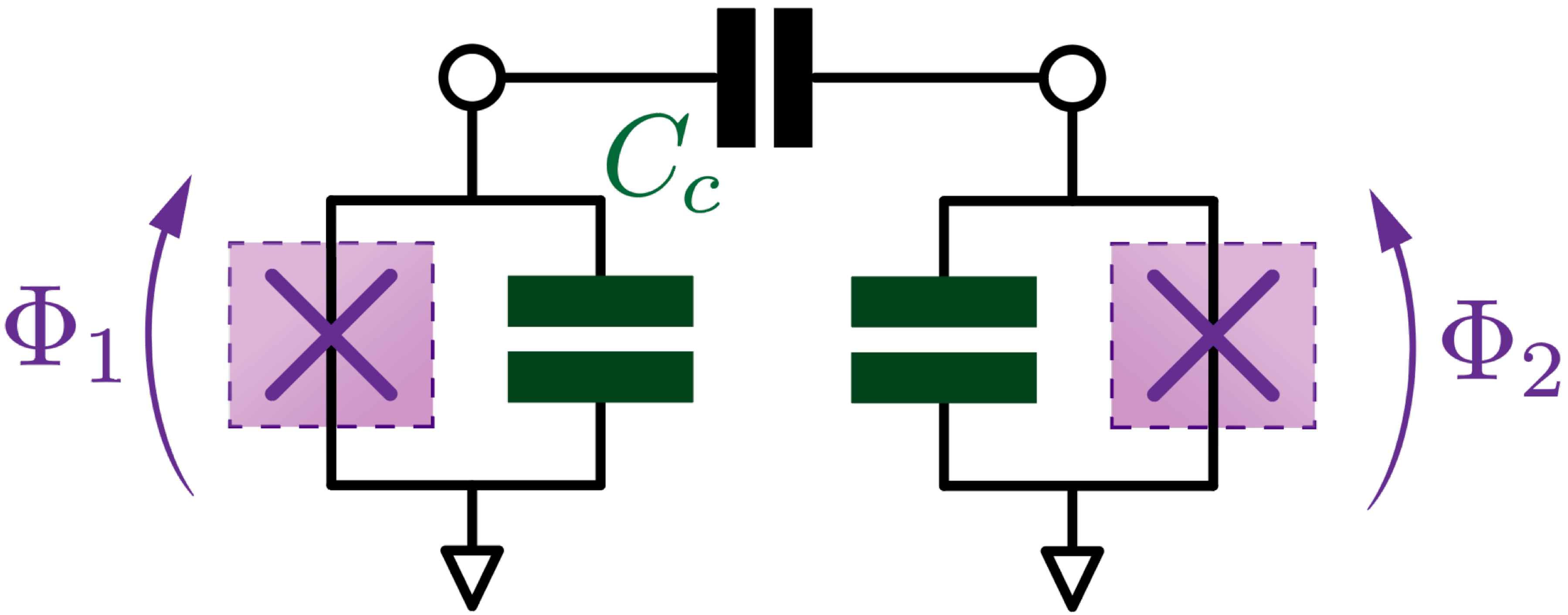
$$\hat{H}_1 = \frac{\hat{Q}_1^2}{2C_a} - E_a \cos\left(\hat{\Phi}_1/\phi_0\right) \qquad \hat{H}_2 = \frac{\hat{Q}_2^2}{2C_b} + \frac{\hat{\Phi}_1^2}{2L_b}$$

Qubit and oscillator



$$\hat{H}_1 = \frac{\hat{Q}_1^2}{2C_a} - E_a \cos\left(\hat{\Phi}_1/\phi_0\right) \qquad \hat{H}_2 = \frac{\hat{Q}_2^2}{2C_b} - E_b \cos\left(\hat{\Phi}_2/\phi_0\right)$$

Two coupled transmon qubits



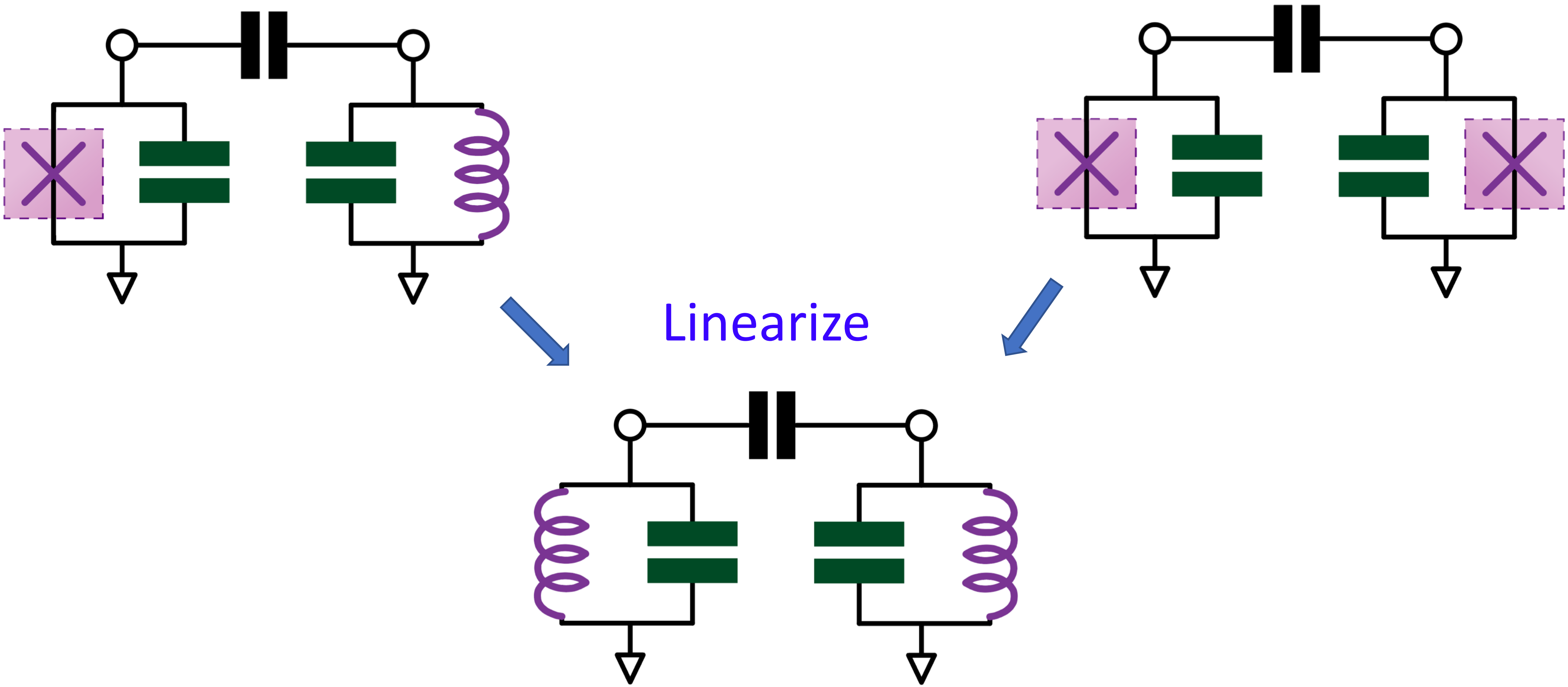
Canonical charge is modified by coupling capacitor

Can use EPR to avoid issues

$$\hat{H}_1 = \frac{\hat{Q}_1^2}{2C_a} - E_a \cos\left(\hat{\Phi}_1/\phi_0\right) \qquad \hat{H}_2 = \frac{\hat{Q}_2^2}{2C_b} - E_b \cos\left(\hat{\Phi}_2/\phi_0\right)$$

Minev et al., EPR to appear (2020); Dissertation arXiv: 1902.10355

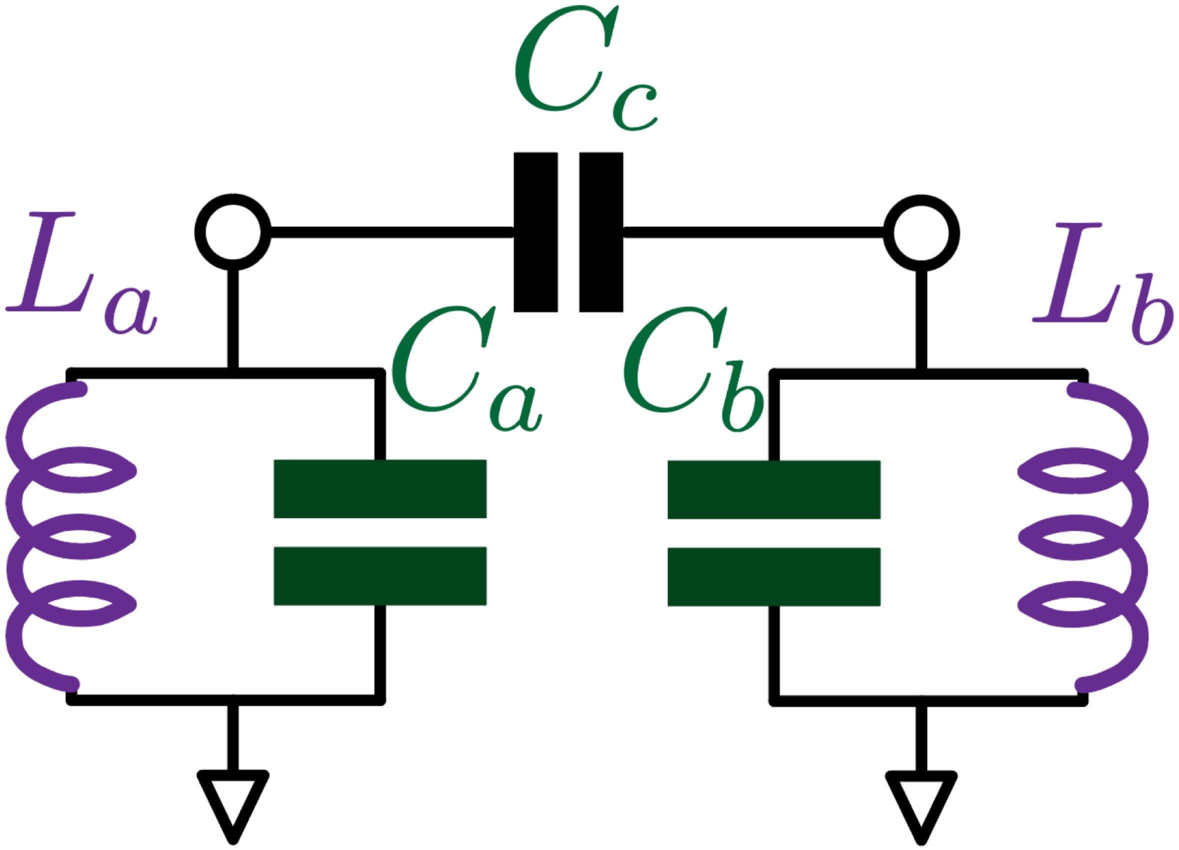
Energy-participation eigenmode approach



Minev et al., EPR to appear (2020); Dissertation arXiv: 1902.10355

Linearize

$$\hat{H}_{\mathrm{full}} = \boxed{\hat{H}_{\mathrm{lin}}} + \boxed{\hat{H}_{\mathrm{nl}}}$$



All linear

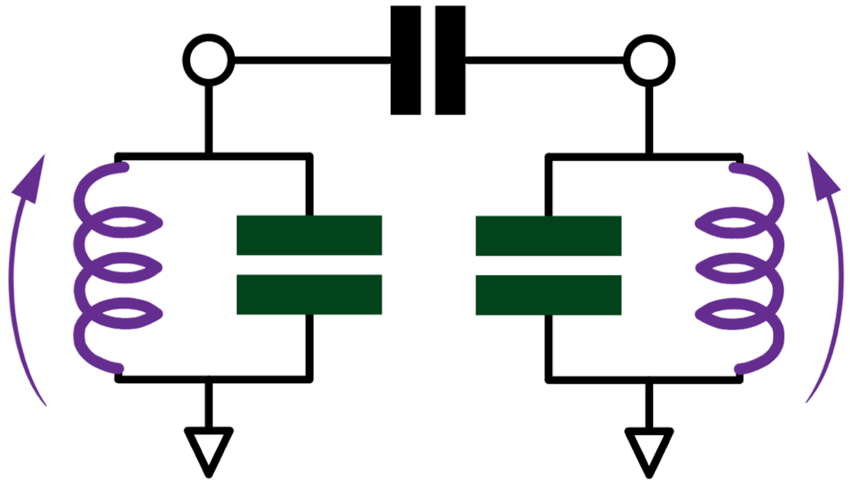
Easy to solve, classical problem

Eigenmode solutions

Minev et al., EPR to appear (2020); Dissertation arXiv: 1902.10355

Linearize

$$\hat{H}_{\mathrm{full}} = \hat{H}_{\mathrm{lin}} + \hat{H}_{\mathrm{nl}}$$



Recall

$$\mathcal{E}_J(\Phi) = -E_J \cos(\Phi/\phi_0) \approx \frac{E_J}{2!}\left(\hat{\Phi}/\phi_0\right)^2 - \frac{E_J}{4!}\left(\hat{\Phi}/\phi_0\right)^4 + \mathcal{O}\left(\hat{\Phi}^6\right)$$

Solutions are eigenmodes (normal mode), just more SHO. Their Eigenfrequencies are

$$\omega_a, \omega_b$$

The Hamiltonian of two SHO is

$$\hat{H}_{\mathrm{lin}} = \hbar\omega_a \hat{a}^\dagger \hat{a} + \hbar\omega_a \hat{b}^\dagger \hat{b}$$

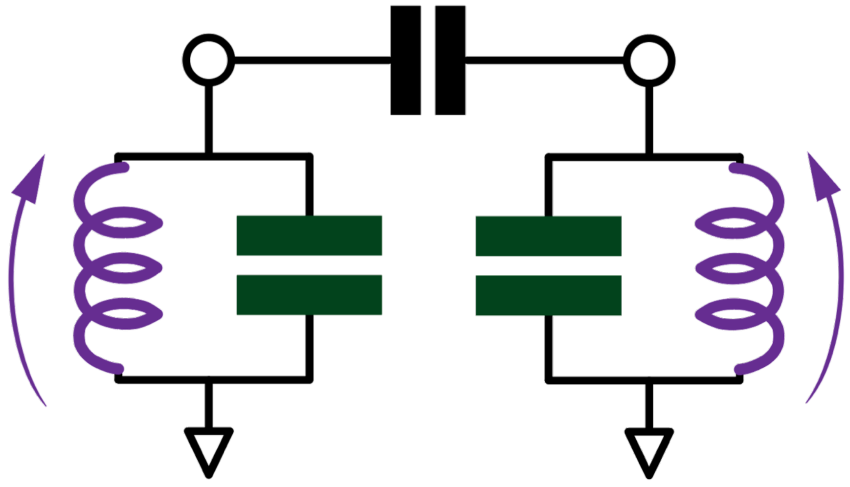
The non-linear part of the Hamiltonian is

$$\hat{H}_{\mathrm{nl}} \approx -\frac{E_a}{4!}\left(\hat{\Phi}_1/\phi_0\right)^4 - \frac{E_b}{4!}\left(\hat{\Phi}_2/\phi_0\right)^4$$

The coupling capacitor C_C effect is accounted for in the definition of the eigenmode *a* and *b* operators.

We need the junction flux in mode operators.

Eigen-decomposition of junction flux $\hat{H}_{\text{full}} = \hat{H}_{\text{lin}} + \hat{H}_{\text{nl}}$



Eigen (dressed) coordinates

"Bare" coordinates

$$\Phi_1 = e_{1a}\Phi_a + e_{1b}\Phi_b$$

$$\Phi_2 = e_{2a}\Phi_a + e_{2b}\Phi_b$$

Canonical transformation eigenvectors (found classically)

Use in

$$\hat{H}_{\text{nl}} \approx -\frac{E_a}{4!}\left(\hat{\Phi}_1/\phi_0\right)^4$$

It follows

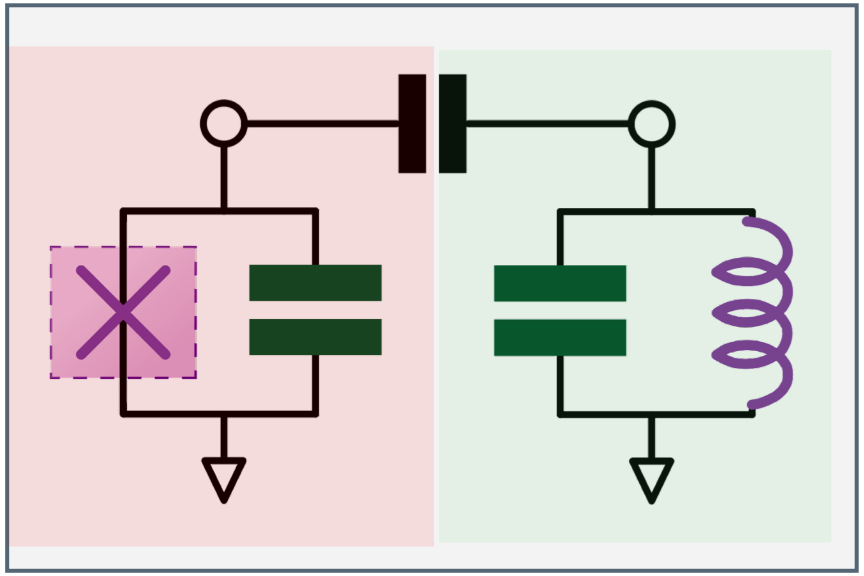
$$\Phi_1 = \Phi_{1a}^{\text{ZPF}}\left(\hat{a}^\dagger + \hat{a}\right) + \Phi_{1b}^{\text{ZPF}}\left(\hat{b}^\dagger + \hat{b}\right)$$

and, similarly, for the second junction flux

(second quantization in eigen basis of linearized circuit)

See Minev et al., EPR to appear (2020); Dissertation arXiv: 1902.10355

Partitioning the Hamiltonian



$$\hat{H}_{\text{full}} = \hat{H}_{\text{lin}} + \hat{H}_{\text{nl}}$$

$$\hbar\omega_c \hat{a}_c^\dagger \hat{a}_c + \hbar\omega_q \hat{a}_q^\dagger \hat{a}_q$$

$$-\frac{E_a}{4!}\left[\phi_{1a}^{\text{ZPF}}\left(\hat{a}^\dagger + \hat{a}\right) + \phi_{1b}^{\text{ZPF}}\left(\hat{b}^\dagger + \hat{b}\right)\right]^4$$

What fraction of the energy of a mode *m* is stored in the junction?

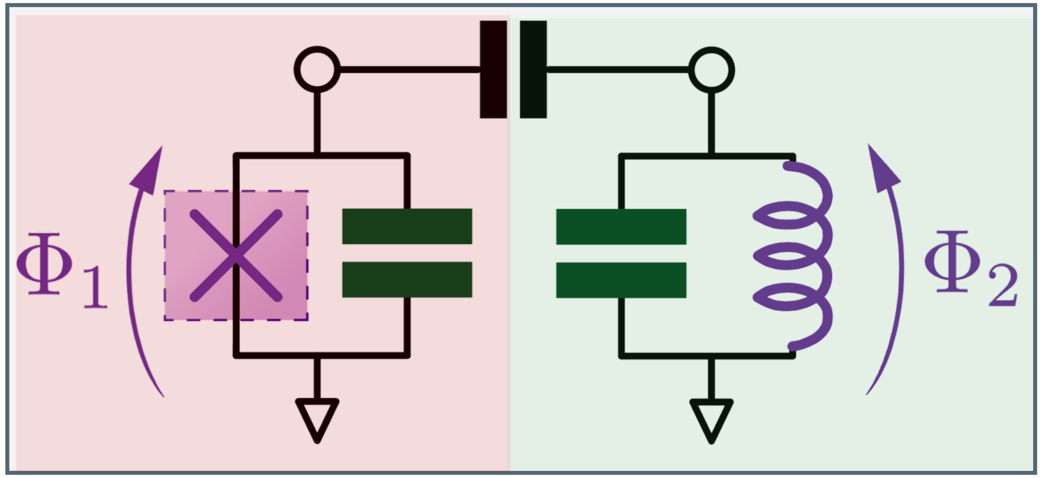
$$\frac{1}{\hbar}\phi_{mj}^2 = p_{mj}\frac{\omega_m}{2E_j}$$

where $\phi_{1a}^{\text{ZPF}} = \Phi_{1a}^{\text{ZPF}}/\phi_0$, and so on

and for weak coupling

$$\phi_{1a}^{\text{ZPF}} \gg \phi_{1b}^{\text{ZPF}}$$

Breakdown of the Hamiltonian



RWA. Notation: dropping ZPF and index for 1

$$\overline{\left[\phi_a^{\mathrm{ZPF}}\left(\hat{a}^\dagger+\hat{a}\right)+\phi_b^{\mathrm{ZPF}}\left(\hat{b}^\dagger+\hat{b}\right)\right]^4}$$

$$=12\left(\phi_a^4+\phi_a^2\phi_b^2\right)\hat{a}^\dagger\hat{a}+6\phi_a^4\hat{a}^{\dagger 2}\hat{a}^2+$$

...

$$24\phi_a^2\phi_b^2\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}$$

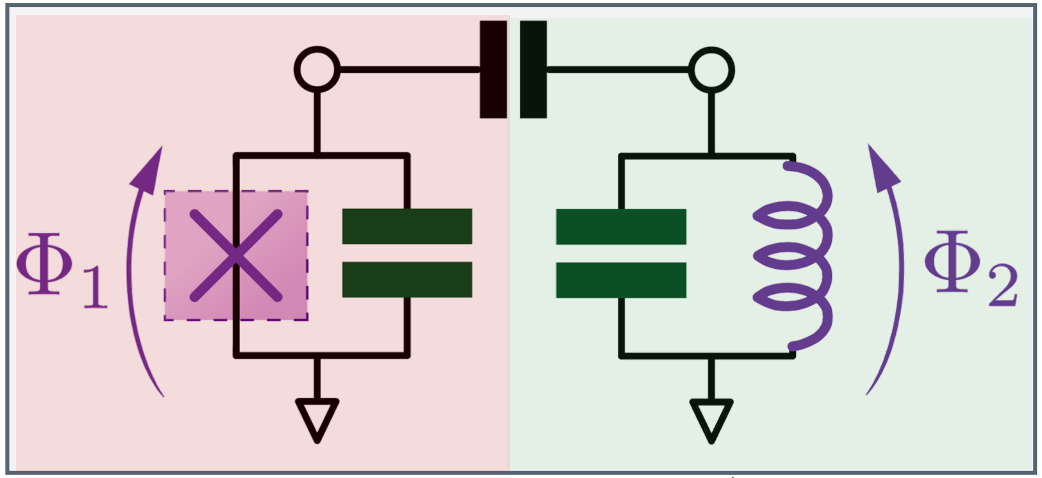
Terms like those of isolated transmon

Drop small terms (cross-participation)

$$\phi_b \ll \phi_a$$

Only new addition is this non-linear, photon-number dependent coupling, called the cross-Kerr.

Linearize system and find eigenmodes



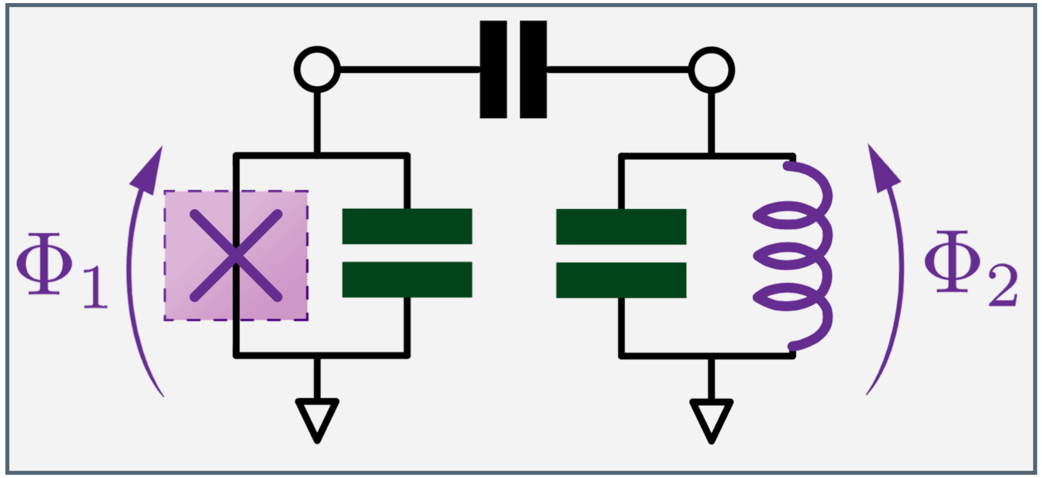
Review of steps

1. Linearize and find eigenmodes
2. Taylor series of nl. potential to 4th order
3. RWA and normal order
4. Dispersive approximation

$$\hat{H}_{\text{eff}} = (\omega_q - \Delta_q)\hat{n}_q + (\omega_c - \Delta_c)\hat{n}_c - \chi_{qc}\hat{n}_q\hat{n}_c - \frac{1}{2}\alpha_q\hat{n}_q\left(\hat{n}_q - \hat{1}\right)$$

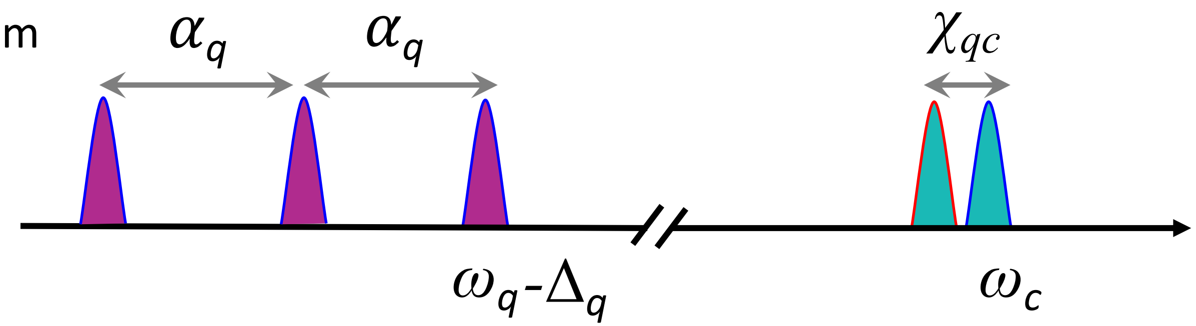
Using labels to *q* for qubit eigenmode and *c* for cavity eigenmode, and $\hat{n}_q = \hat{a}^\dagger\hat{a} \quad \hat{n}_c = \hat{b}^\dagger\hat{b}$

Joint spectrum



$$\hat{H}_{\mathrm{eff}} = (\omega_q - \Delta_q)\,\hat{n}_q + (\omega_c - \Delta_c)\,\hat{n}_c - \chi_{qc}\hat{n}_q\hat{n}_c - \frac{1}{2}\alpha_q\hat{n}_q\left(\hat{n}_q - \hat{1}\right) - \frac{1}{2}\alpha_c\hat{n}_c\left(\hat{n}_c - \hat{1}\right),$$

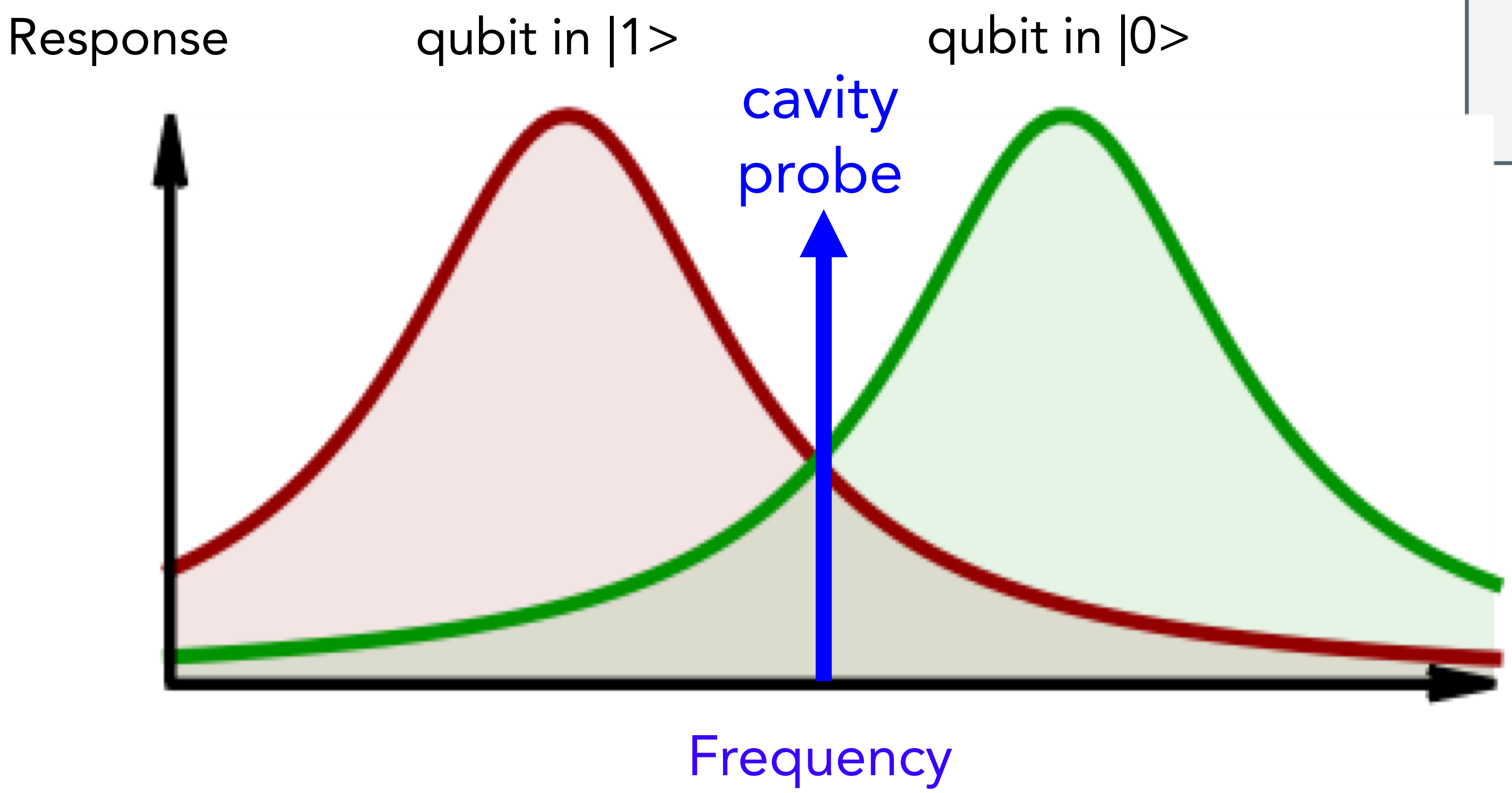
$$\hat{H}^{\mathrm{eff}}_{\mathrm{cav}} = \hbar\,\underbrace{\left(\omega_c - \chi\hat{a}^\dagger\hat{a}\right)}_{\text{cavity frequency}}\,\hat{b}^\dagger\hat{b}$$

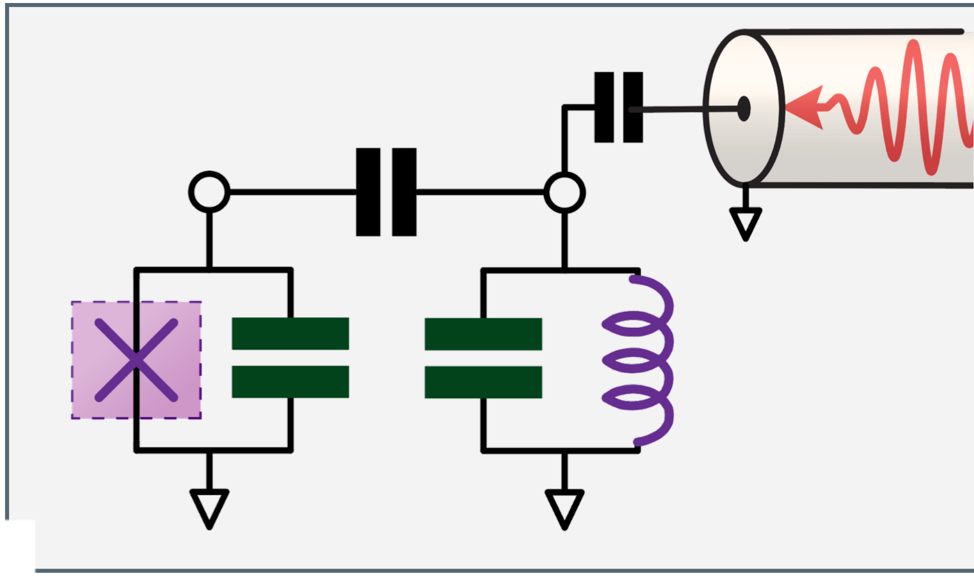


Recall that harmonic oscillator has evenly spaced levels

Conditional cavity spectrum

$$\hat{H}_{\text{cav}}^{\text{eff}} = \hbar \underbrace{\left(\omega_c - \chi \hat{a}^\dagger \hat{a}\right)}_{\text{cavity frequency}} \hat{b}^\dagger \hat{b}$$

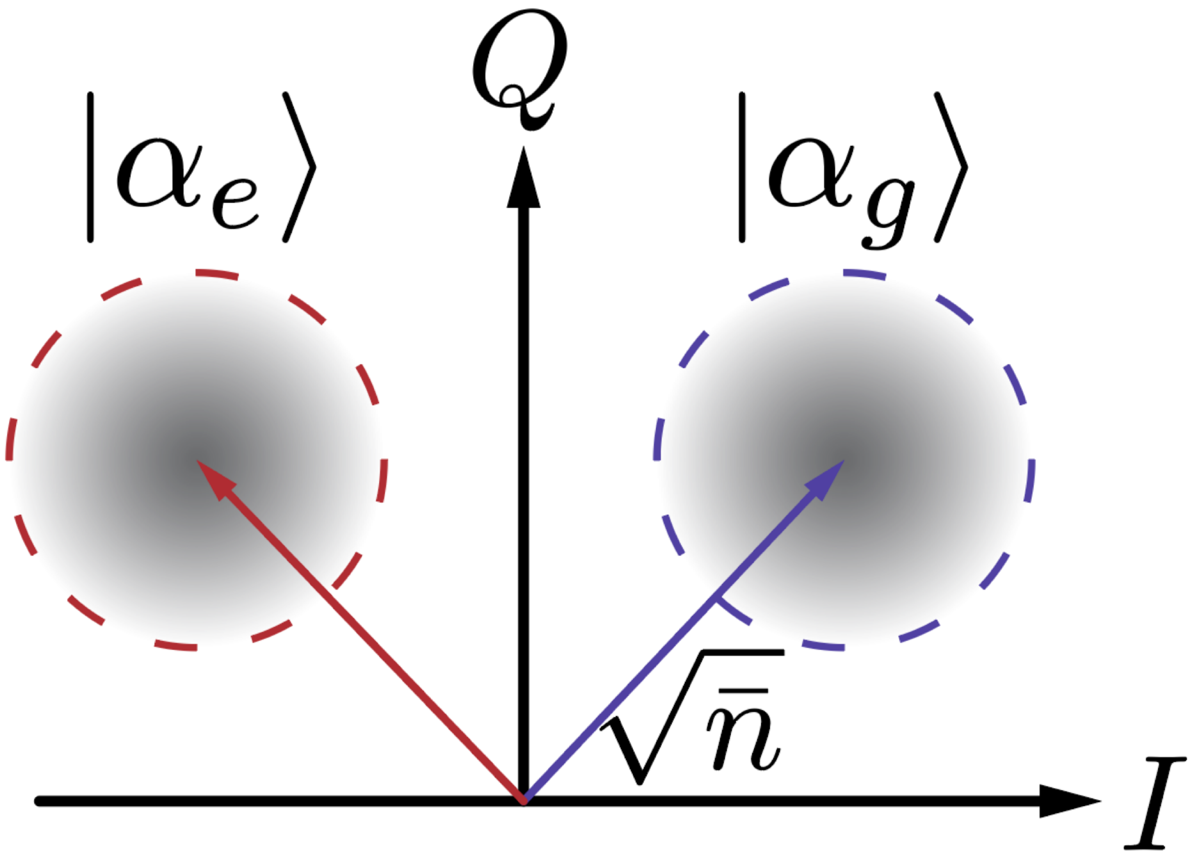


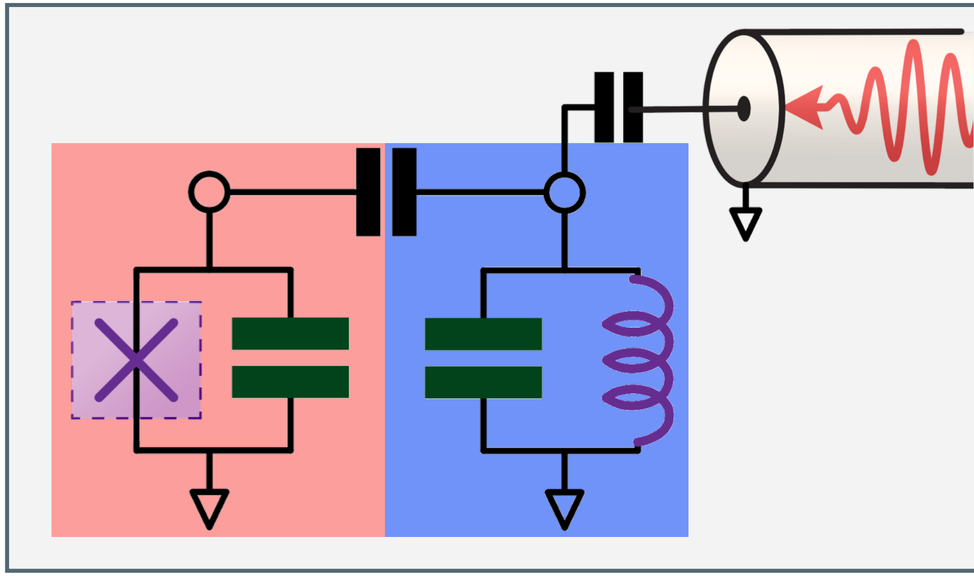


Lorentzian peak
(response of cavity on irradiation)

cQED dispersive readout: measuring Z

$$\hat{H}_{\text{int}} = \hbar\chi \hat{a}_q^\dagger \hat{a}_q \hat{a}_c^\dagger \hat{a}_c$$

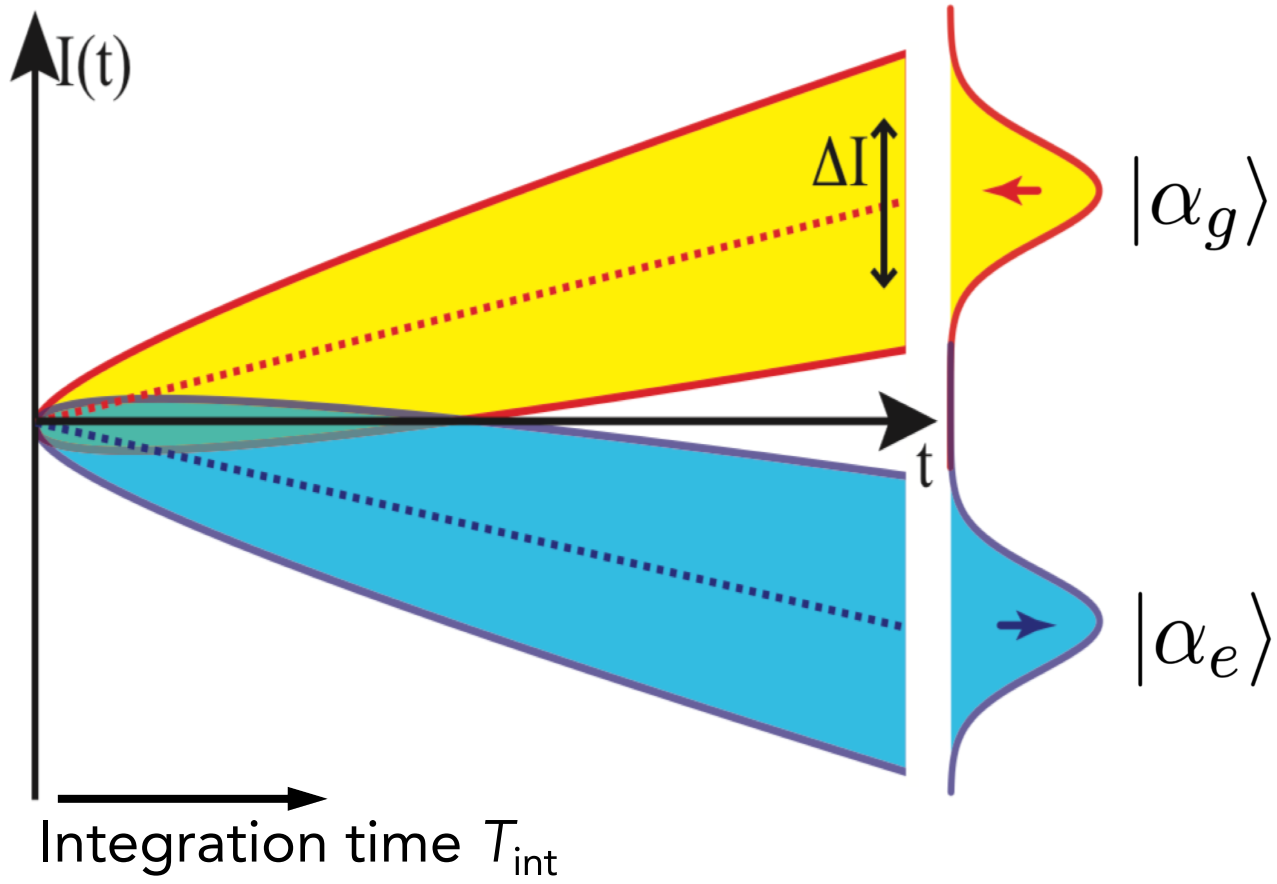


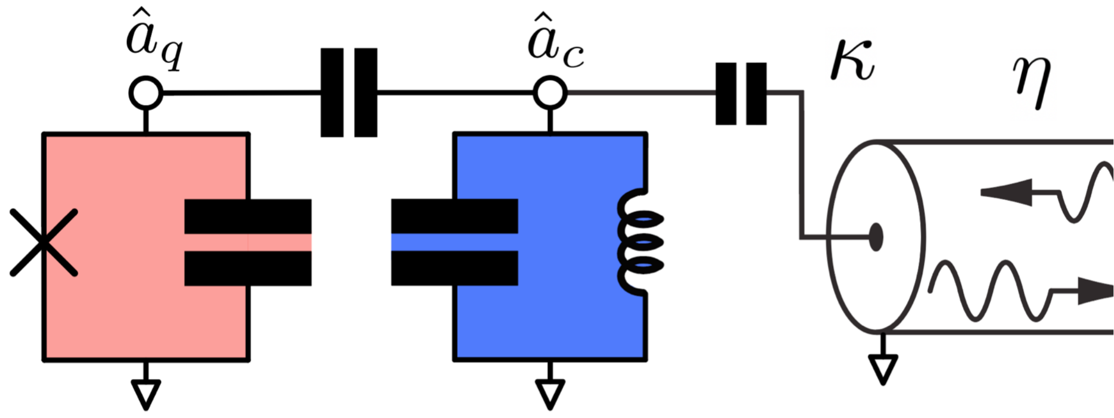


Pointer states of measurement apparatus (cavity) corresponding to the qubit in the ground (*g*) and excited (*e*) states

$\hat{I} \equiv \hat{c} + \hat{c}^\dagger$ and $\hat{Q} \equiv -i\left(\hat{c} - \hat{c}^\dagger\right)$

cQED dispersive readout: measuring Z





$$\mathrm{SNR} = \frac{1}{2}\eta\kappa T_{\mathrm{int}}\left|\alpha_e - \alpha_g\right|^2$$

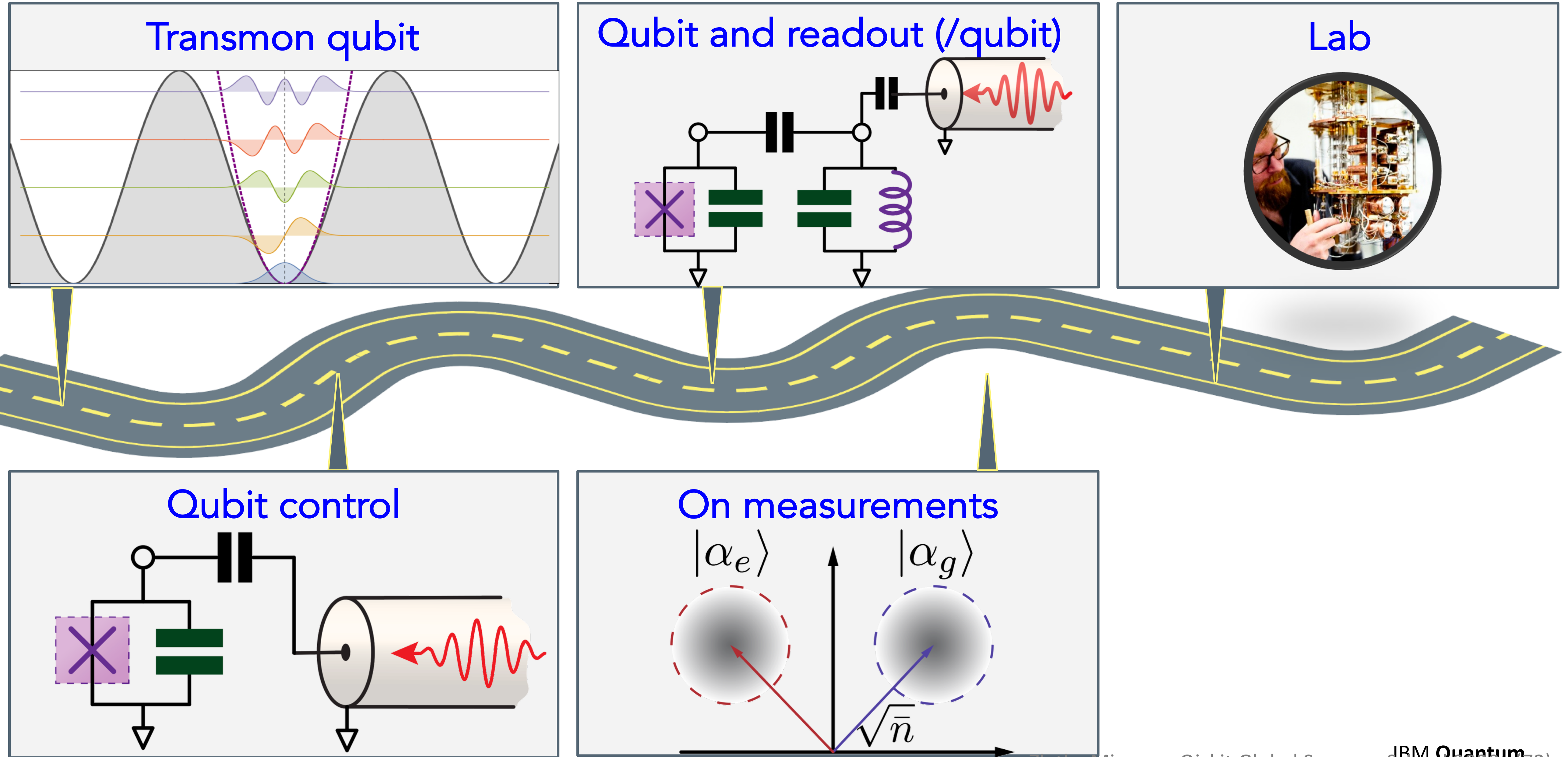
distinguishability

$$\eta_{\mathrm{disc}} = \frac{1}{2}\,\mathrm{erfc}\left[-\sqrt{\frac{\mathrm{SNR}}{2}}\right]$$

efficient, fast measurements

image: Clerk *et al.* (2010)

The road behind



* Road image: based on Freepik

Next steps

Tightly integrated lab work with Dr. Nick Bronn and Co.!

Run experiments on real devices

Check out references, problems given in the lecture, dangerous bends

Break away from the rules of today

Thank you!

Zlatko K. Minev

The important thing is not to stop questioning. Curiosity has its own reason for existence.

One cannot help but be in awe when he contemplates the mysteries of eternity, of life, of the marvelous structure of reality.

It is enough if one tries merely to comprehend a little of this mystery each day.

Albert Einstein

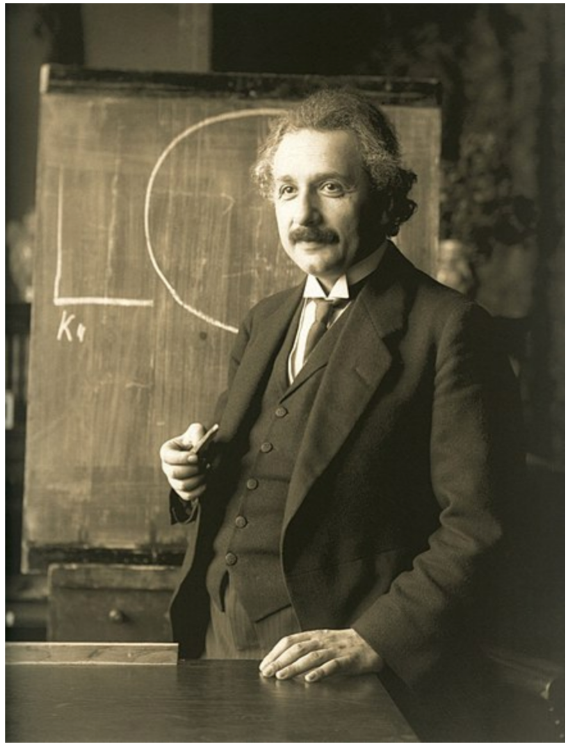


Photo: F. Schmutzer