Modification and promotion of quantum random walk with phase interference and verifying the Parrondo paradox on IBMQ to emerge quantum advantage

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This research mainly discusses the difference between a quantum walk and a classical walk. The previous papers had proved that in a quantum walk, due to the destructive phase of the wave function and constructive interference, the one-dimensional quantum random walk will walk randomly from left to right and finally ends on both sides. The first part discussed the impact of different coins on the system and designed the quantum gate described in the paper, but we found that the state distribution of the quantum circuit designed by this quantum gate is asymmetrical, and there are other problems like Offset in a particular direction. As for the result, we successfully designed a spherical quantum random walk algorithm and observed the characteristics of its state distribution. Additionally, we propose a one-dimensional quantum random walk with excellent symmetry, which still carries this property into two-dimensional space.. In the end, we use the quantum random walk to realize the Parrondo paradox. We also found that because the diffusion speed of quantum random walk is faster than the classical one, the expected value of Parrondo paradox converges faster than the classical one, which verifies the quantum advantage.

I. Introduction

A. Background

Quantum walking is a new research topic. Although some authors use the name "quantum random walk" to refer to quantum phenomena, it is generally believed that the first paper on quantum random walks was published in 1993 by Aharonov et al.[1], Quantum random walk is described in terms of the magnitude of probability. By combining each possible step with another degree of freedom (such as spin), the actual detection process is incorporated into the theory, and it can be used as a quantum coin:

The measurement of this observable object will select the actual transition experienced when there is a considerable overlap between the probability amplitudes of moving left or right.

In this case, the average displacement of the particles may far exceed the maximum displacement allowed by the classics. All these concepts can easily be extended to multi-dimensional situations. Therefore, the connection between classical random walk and quantum walk and the use of quantum walk in computer science are two new and open research fields. There is a classic random walk theory on finite graphs, although it is still far from perfection. , But it has achieved fruitful results in algorithm development. In order to make full use of quantum walk in computer science, I still need to do more work to compare quantum and classical performance, and put forward new ideas (about performance measurement methods) on how to use quantum walk in algorithm design.

In theory, the distribution speed of quantum random walk is faster than classical, but the asymmetry limits its usability. After solving the phase interference problem of quantum random walk, the application of quantum random walk can be extended to more levels. Physically speaking, We understand the problem of quantum coherence better, and the results of symmetry may be used to predict topological materials, such as Graphene MoS2 and so on.

B. Research Motivations

In the process of learning quantum computing, I also came across the first quantum algorithm I learned-Discrete Quantum Walk (Quantum Walk). Quantum walk is a quantum extension of classical random walk, in which the position of the walker is determined by Probability distribution to describe. The distribution of each position in the classical random walk is replaced by the quantum superposition in the quantum walk. The quantum effect will lead to a wider spread of the final position, which can also be seen as a faster spread of information. In the process of practicing quantum random walk, I found that the H gate that can generate random superposition state originally, the expected distribution curve will spread to both sides, but I can't see this phenomenon in my implementation, but it has a direction. The tendency of walking in a certain direction, so this experiment extends the application of quantum random walking deeper and wider.

With symmetric quantum random walking, we can use the results of this experiment to simulate the actual problems of natural life. Next, I will use quantum random walking to simulate the Brownian motion of three-dimensional behavior, hoping to get the same solution as in actual nature. In bioengineering, quantum random walk can be used to calculate the mapping problem of enzymes to understand the evolution of enzymes when it comes into contact with mutagens. This problem only requires 33 nodes, which can be mapped to 7 qubit circuits. However, we are most interested in designing quantum random walk into a quantum machine learning (QML) algorithm-Quantum Random Forest, which is used to deal with the problem of regression prediction.

C. Aims and Objectives

(1) Explore the influence of different coins on the distribution of quantum walk-confirm. whether the approach in the paper contributes to the symmetry of the distribution

(2) Establish a fair quantum coin and judge-eliminate the influence of quantum phase and get a symmetrical quantum walk

(3) Explore the feasibility of realizing quantum walk in multi-dimensional space-expand to multi-dimensional space to expand the scope of quantum random walking

(4) Analyze the probability standard deviation of the diffusion time of classical walk and quantum walk

(5) Using Jensen-Shannon divergence to analyze the similarity of probability distribution between quantum walk and classical walk

(6) Using one-dimensional quantum random walk on a circle to realize Parrondo's paradox

II. Literature review

A. Classical random walk

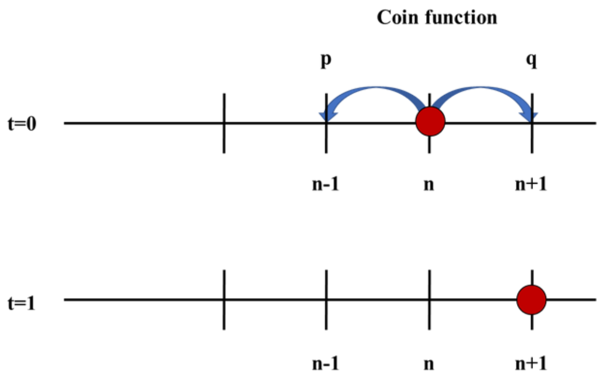


Fig 2.1 :The method of classical random walk

In the paper [1] mentioned that stochastic process is a system that experiences fluctuations of chance over time. A random variable {Xt} can be used to describe such a system, where Xt measures the attributes of the system of interest at time t, and random Walk is a special type of random process, which is related to mathematical entities and many other fields such as physics and computer science. In research, I am interested in discrete random walks (that is, classical random walks with discrete time steps in discrete space). Compare the statistical properties and computational applications of its quantum mechanical counterparts.

B. Quantum random walk on the circle [2]

Classical random walks on graphs are very important to the development of random algorithms. Therefore, quantum walks on graphs have become an active field of quantum computing research. Let G=(V, E) be a d-dimensional graph, Let Hv be the Hilbert space spanned by the state$|\, v >$, Where v∈V. In addition, the coin space H_A is defined as a Hilbert space $\{|\, i > \ |\, i \in \{1\ ,\ldots d\}\}$ with a size of d and a coin operator C to form an auxiliary evolution. As a single transformation in H_A, the edges in each direction are marked with a number between 1 and d, so that the edges in each direction form an arrangement.

Now, define the position shift operator S above so that S $|\, a\ ,\, v >= \ |\, a\ ,\, u >$where u is the ath neighbor of v (because the edge label is permutation, so S is unitary) and finally the quantum step on G is defined as

$$U\ step\ Move\ operator\ S\ \ (Coin\ operator\ C\ \otimes Initial\ operator I$$

Just like when studying quantum walking, if$|\psi >_0$ is the initial state of quantum walking, then the quantum travel on graph G can be defined as

$$|\psi\rangle_t = \hat{U}^t|\psi\rangle_0$$

Before introducing the concept before the quantum phase correction problem, provide an example of quantum walk on the graph: a discrete quantum walk on a loop

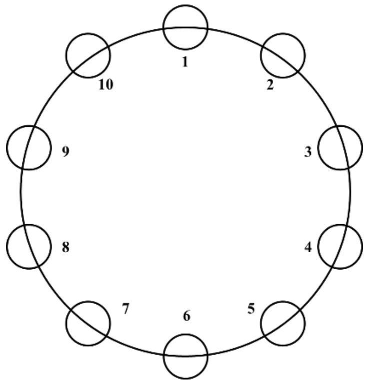


Fig 2.2: Quantum walking in the loop ,the loop shown in this figure has 10 vertices

Example: Discrete quantum walk on the loop:

(1) The pedestrian is located at a specific point n in the N-dimensional space (N = 1, and the walker walks along a line from point n)

(2) Generate a random coin with a custom probability to determine which direction the walker will walk in.

(3) The walker moves to the nearest point (n-1) or (n + 1) on the map based on the random coin generated in step 2 (for example, up and down or 0, 1), by adjusting different coins and Converter, looking for fair coins with more symmetrical distribution.

$$P_t(v|\psi_0) = \sum_{i\in\{1,\ldots,d\}} |\langle i, v|\psi\rangle_t|^2$$

If the probability distributions P_0 and P_1 at time 0 and 1 are different, it can be proved that the above formula Pt does not converge, and P_t = the following formula

$$\bar{P}_t(v|\psi_0) = \frac{1}{T}\sum_{t=0}^{T-1} P_t(v|\psi_0)$$

C. **The difference between quantum and classical**

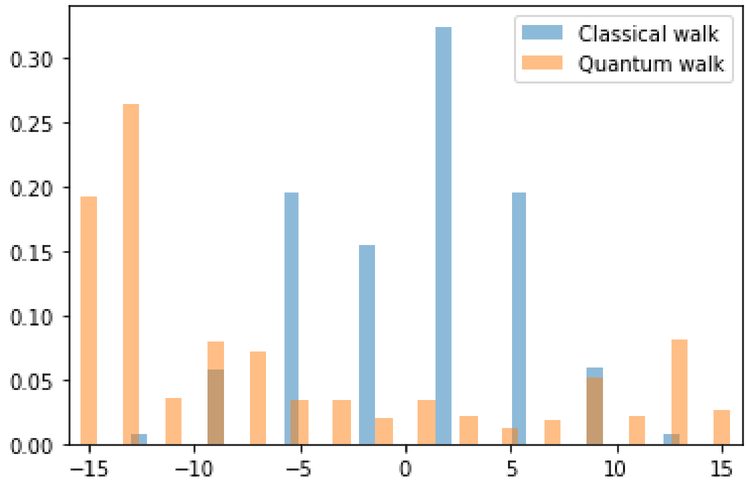


FIG 2.3 :It can be seen that the probability of classical random walking is a bell-shaped distribution, while quantum random walking has a tendency to spread to both sides

D. Parrondo paradox

In game theory, the argument about being self-reported by Parrondo paradox

It happens that only two failed strategies are dependent on each other, combined in a certain way (with the conditions that caused them to fail), can Parrondo's theory be realized.

We can use a simple example to illustrate the content of Parrondo paradox

Assuming that you have an initial capital of 100 dollars, you can choose to play the following two games in any combination:

• Game A: Every time you play, you will lose money or the probability of losing money.

• Game B: If the remaining amount is an even number, 3 dollars will be left; if the remaining amount is odd, 5 dollars will be lost.

III. Method and Approach

A. Pre-test experiment: production of quantum random walking circuit modelClassical random walk

Define circular walk:

1. Classical random walk

(a) **(b)**

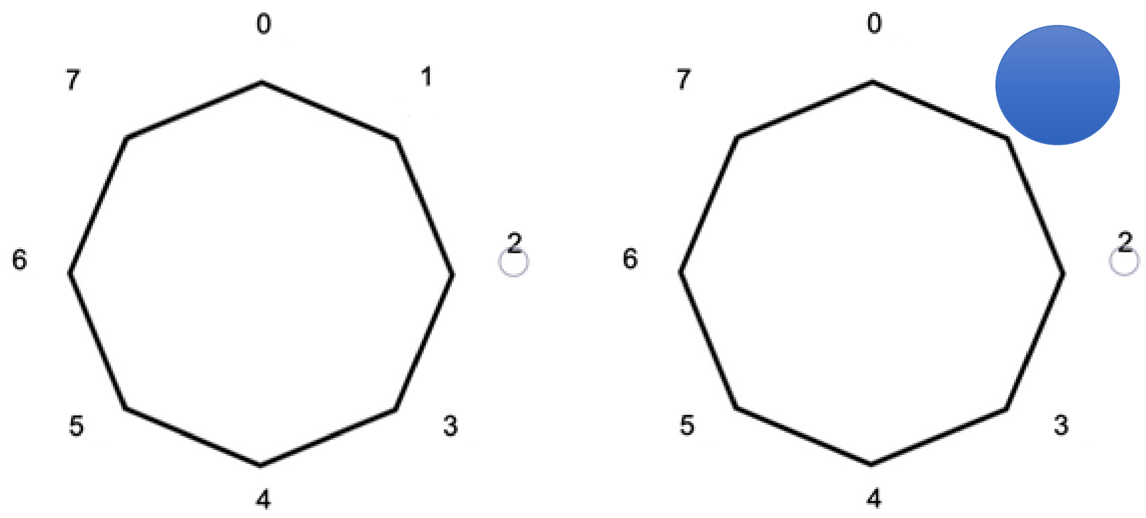


Fig 3.1 (a)Schematic diagram of circular walk (b) Dice on the circle and go right after the first dice

Define the initial position as 0, as shown in Figure3 . If you get 0 after one die, go to 7 to the left, and go to 1 when you die.

2. Quantum Random Walk Appendix [1]

(a) **(b)**

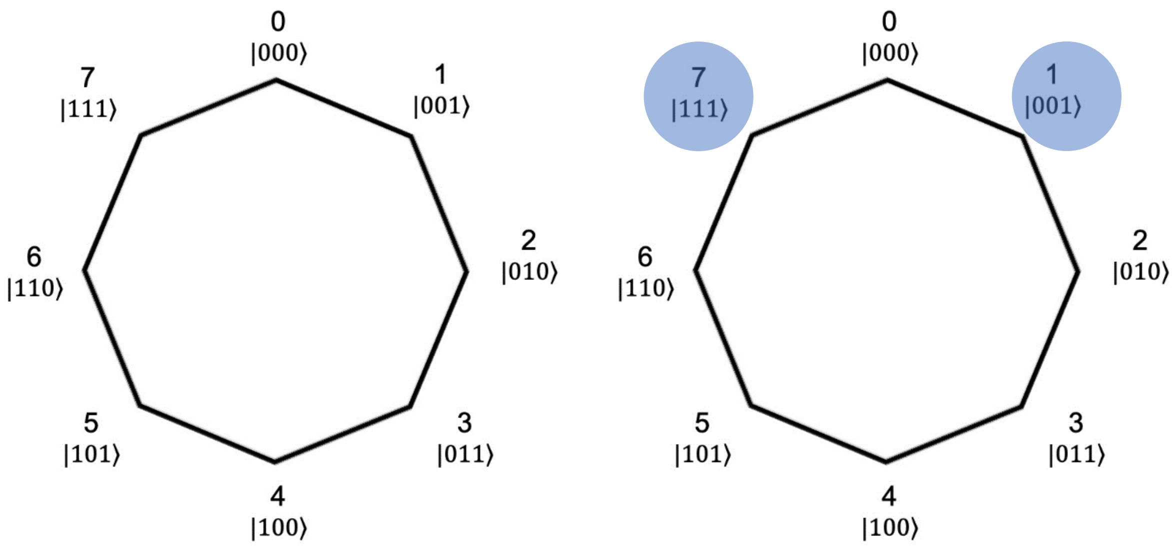


Fig 3.2 (a) Schematic diagram of circular walk (b) after the first dice on the circle

Define the initial position as 0, which is the quantum state | 000 >, as shown in Fig4.(a)

After you roll the dice once, you will go left and right at the same time. If you go to the left, you will reach the position of 7, which is the quantum state | 111 >. Similarly, if you go to the right, you will reach the position of 1, which is the quantum state | 001> The location is shown in Fig4.(b)

B. Design quantum random walk of different coins[3][4]

1. Experimental design:

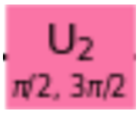
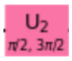
H	Y	Ry	H(Correct the initial value)
			 
$\left\lvert\psi\right\rangle = \frac{1}{\sqrt{2}}\left\lvert 0\right\rangle + \frac{1}{\sqrt{2}}\lvert 1\rangle$	$\left\lvert\psi\right\rangle = \frac{1}{\sqrt{2}}\left\lvert 0\right\rangle + \frac{1}{\sqrt{2}}i\lvert 1\rangle$	$\lvert\psi\rangle = \sqrt{0.85}\lvert 0\rangle - \sqrt{0.15}i\lvert 1\rangle$	$\left\lvert\frac{1}{\sqrt{2}}\right\lvert 0\rangle + \frac{1}{\sqrt{2}}\lvert 1\rangle \otimes \left\lvert\frac{1}{\sqrt{2}}\right\lvert 0\rangle + \frac{1}{\sqrt{2}}i\lvert 1\rangle$

Table3.1: All experimental coins

Expected walking distribution:

In this experiment, the distribution curve of the four gates is expected to be a symmetrical curve.

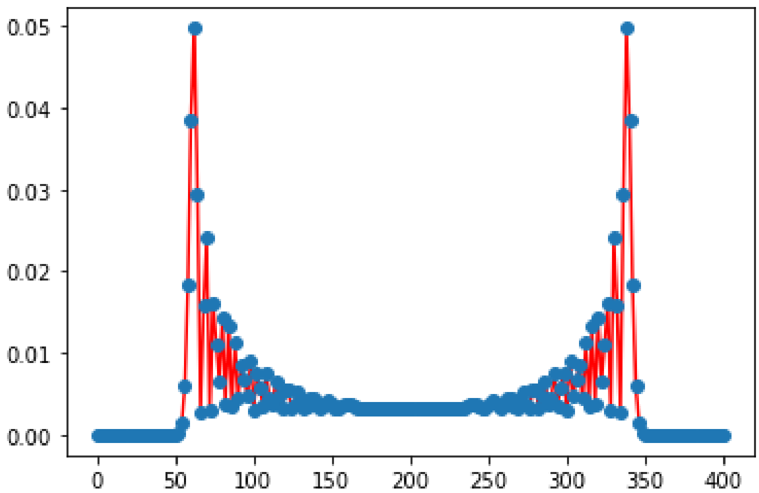


FIG 3.3 Expected quantum random walk

2. The purpose of the experiment: Use the mathematical formula provided in the paper to design a circuit for quantum walk

(1) Experiment 1: H gate

Expected walking distribution:

In view of the fact that the H gate can produce 50%, 50% of 0, 1 which is a superposition state, this experiment speculates that if the H gate is designed to judge whether to go left or right, a two-sided die.

(2) Experiment 2: Y gate

Design reason

The paper[1] points out that the H gate is not a fair dice, and the reason may be that the phase has constructive or destructive interference when moving. Therefore, a new gate Y is designed according to the conditions given in the paper[1].

(3) Experiment 3: Ry gate

Design reason: In addition to the Y gate in the thesis, I think there is still room for improvement in the dice, so in this experiment a new gate Ry is designed.

(4) Experiment 4 Modify the initial value

Design reason: because the dice proposed in the paper[1] is because of the phase interference, so I want to correct the phase of the direct H gate.

C. Using Python to simulate quantum random walking

Expected distribution

1. Experimental design: matrix operations and self-made coins with half states 0 and 1 to simulate quantum random walking
2. Experimental purpose: compare the difference between pure logic operation and physical pulse control

D. Multi-dimensional quantum random walk distribution

1. Experimental design: Extend the applicable scope of quantum walk to two-dimensional space and three-dimensional space
2. The purpose of the experiment: Find out the multi-dimensional randomness of space by circular quantum walk

(1) 2D quantum random walk

First, we must first define the Qubits representing the X-axis or Y-axis directions. Therefore, this experiment defines X as the information access location in the X direction and Y as the access location in the Y direction.

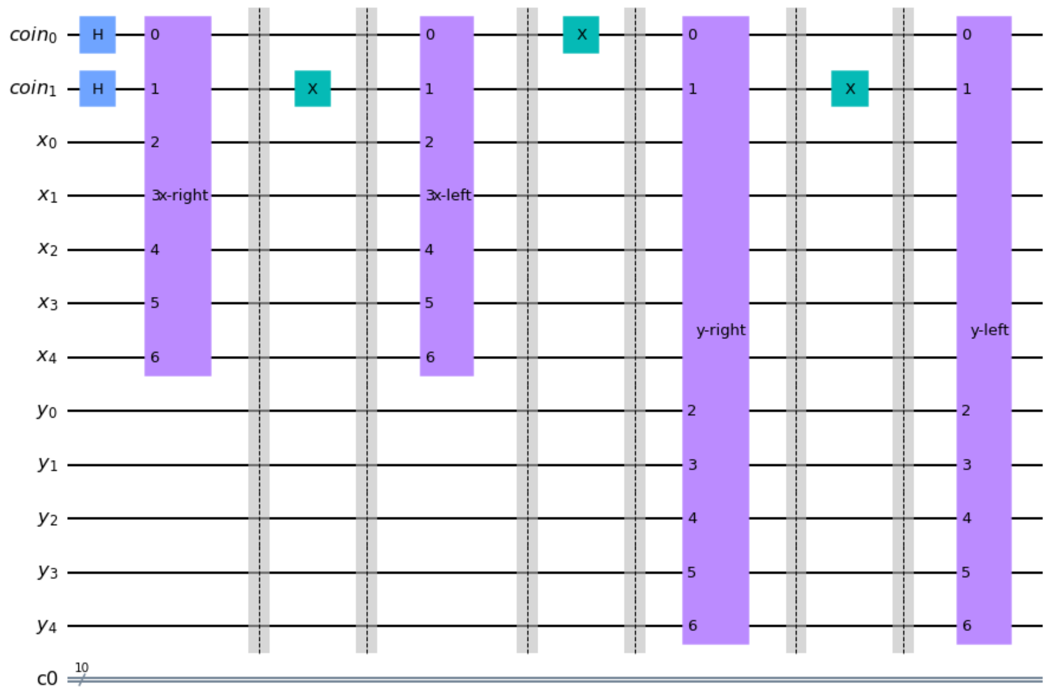


FIG 3.4 2D quantum random walk circuit diagram

(1) coin0 is the dimension judge. When the 0th bit is 0, the walker will decide to go to the Y axis. On the contrary, when the 0th bit is 1, the walker will decide to go to the X axis.

(2) Coin1 is the direction judge. After coin0's judgment, the next step is to decide which way to go, so coin1 is used to decide whether to walk left or right.

(3) X Qubits is the position accessor of X axis

(4) Y Qubits is the position accessor of Y axis

E. Perform quantum random walk on the physical machine "ibmq_casablanca"

Forecast distribution:

Because IBM's superconducting quantum computer still has a large error value, it is speculated that there may be some walking characteristics.

Kit description:

QISKIT is currently one of the few platforms that are open to the public to use real quantum computers, so we can actually send our experiment remotely to a foreign quantum computer and then return it to our own computer. Therefore, in this experiment we use QISKIT to actually perform quantum randomness Walk and expect similar results to the simulator.

F. Design Parrondo paper circuit

Palondo said it was a feast of gambling, with one dollar for winning and one dollar for losing and it is composed of two games, Game A and Game B

We designed the quantum logic circuits of Game A and Game B by ourselves, and simulated the quantum random walk of Game A and Game B in one dimension on the circle, and used the probability of walking to judge whether to win or lose.

1. Game A

First design the coin of Game A,

According to Parrondo's paradox, the probability of A coin should be 1/2

Based on the previous experimental results, the probability distribution of the H gate will not be symmetrical after multiple H gates, which also means that the A coin is unfair. Therefore, we use the H gate with the modified initial value as our coin. The reason is the probability part will not evolve over time.

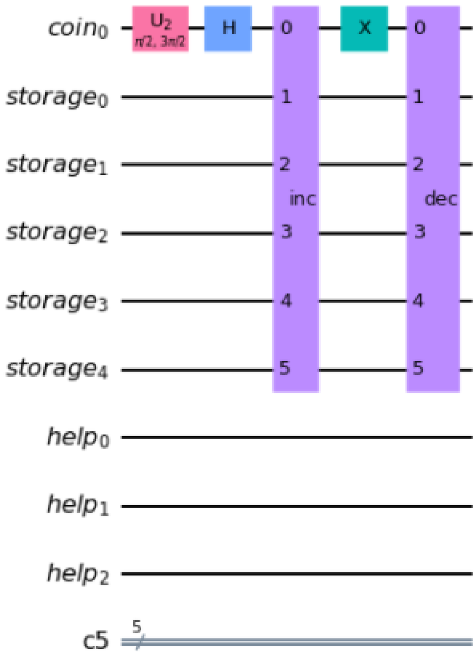


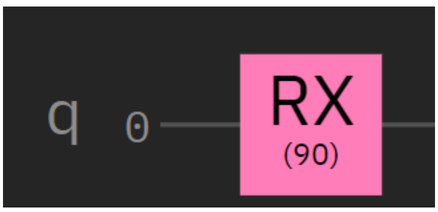
FIG 3.5 the way we design the circuit of Game A is that we make this circuit based on the concept of Parrondo's paradox that Game A will lose money or win money every time it is played, but the probability of losing money is greater.

2. Game B

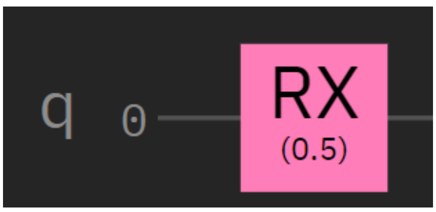
Design Game B's coin

Based on Parrondo's paradox, we designed two coins, good coin and bad coin

Good coin



Bad coin



The probability is 0.724. The probability is 0.061

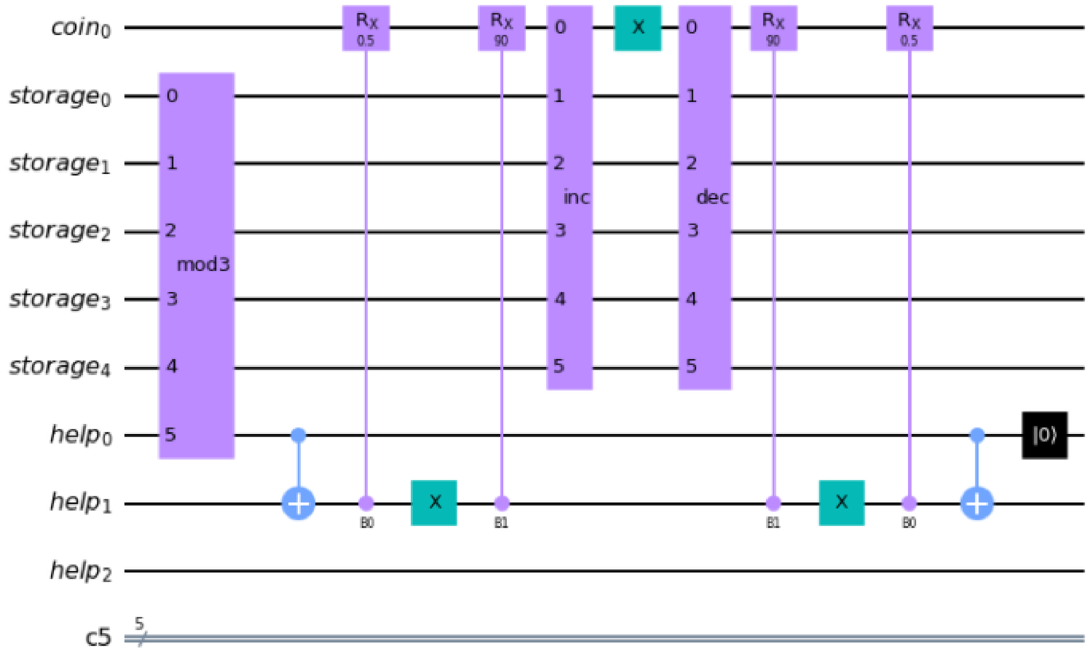


FIG 3.6 game B depends on the amount of accumulated capital. Capital is determined by the number of qubits. The number of qubits in our experiment is 5, so the capital is 31 dollars. If his accumulated capital is a multiple of 3, the player throws Bad coin, otherwise, throws Good coin.

IV. RESULT

A. Pre-test: the walker circuit design

1. Walking mechanism

According to the reference [5], this experiment designs coins (qubit 0) and walkers (qubit 1, 2). The shift_circ gate, which is a custom gate, is used to represent how the walker walks.

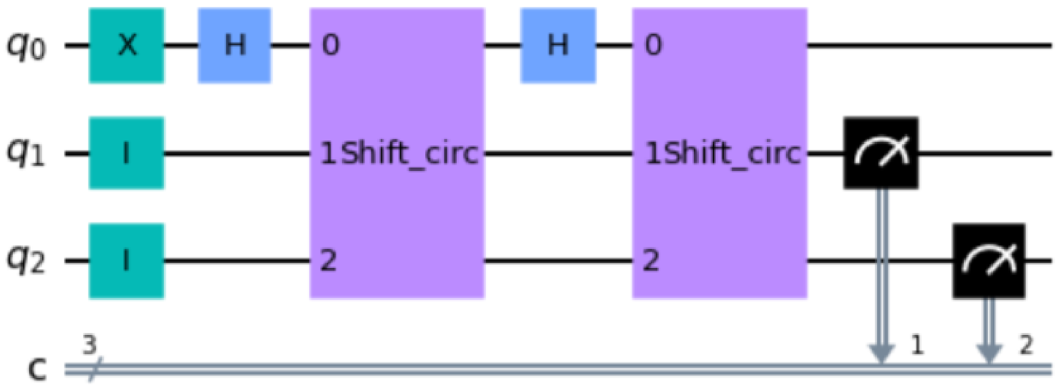


FIG4.1: fundamental model of quantum walk circuit

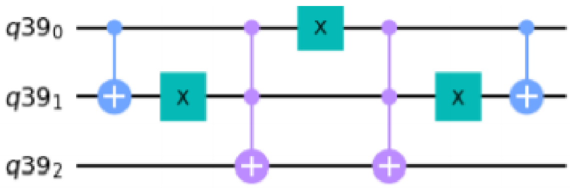


FIG4.2: shift_circ represents in circuit.This circuit shows how the walker walks in binary when controlled by H gate.

Fault:

If you use mathematical methods to verify, you can find that an input state must be executed after running all shift _ circ to know the result. This operation mixes left and right directions which are not intuitive for beginners.

2. Walking mechanism B [4][5]

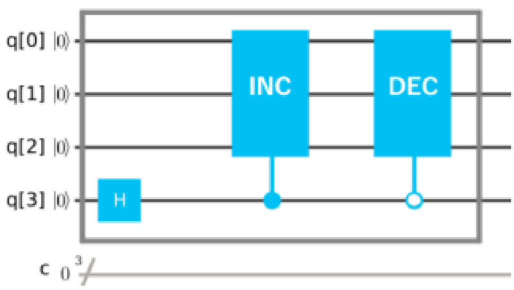


Fig4.3 Intuitive quantum walking circuit

This experiment designs another more intuitive circuit. If the result of qubit 3 is 1, INC will be executed. Otherwise, if qubit 3 is 0, DEC will be executed. In this way, we can achieve the goal of going to both sides.

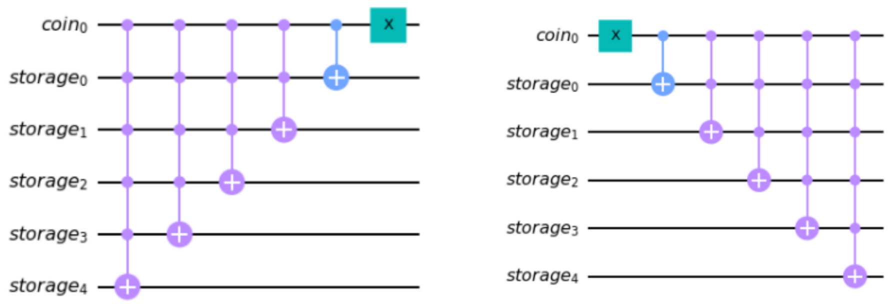


Fig4.4: INC(left) and DEC(right) represent in circuit.

B. Design quantum random walks for different coins

1. Experience 1: Hadamard (H) gate

Result:

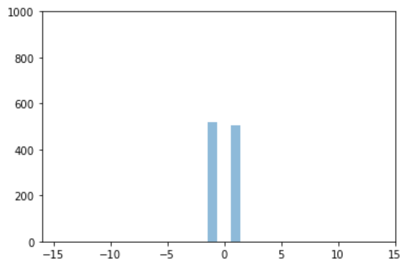
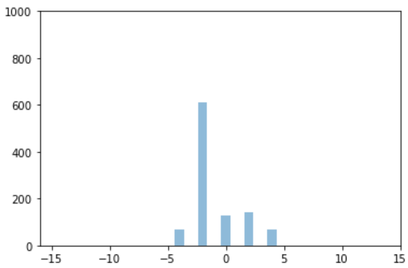
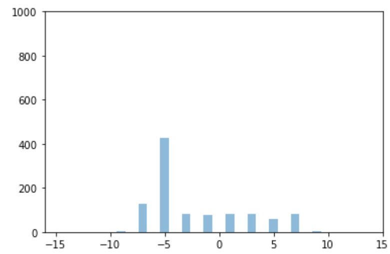
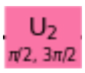
Step = 1	**Step = 5**	**Step = 10**
		

Table4.1: Distribution of walkers whose coin is H gate

In order to consider Error Bar, we repeat this experiment 10 times. It shows that this distribution after using H gate as the coin is not symmetrical with diffusion to both sides as expected by the experiment.

2. Experience 2: Y gate



Result:

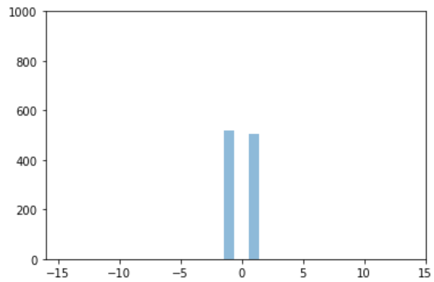
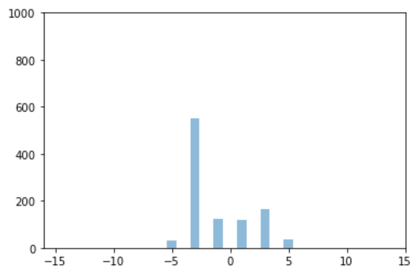
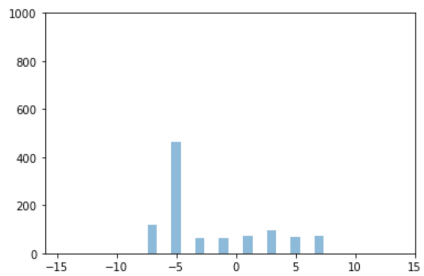
Step = 1	**Step = 5**	**Step = 10**
		

Table4.2: Distribution of walkers whose coin is Y gate

After considering the Error Bar, the two-sided diffusion equilibrium is not reached with using the Y gate given in the paper,

3. **Experience 3: Ry gate**

Result:

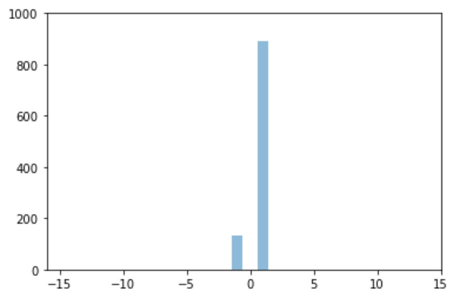
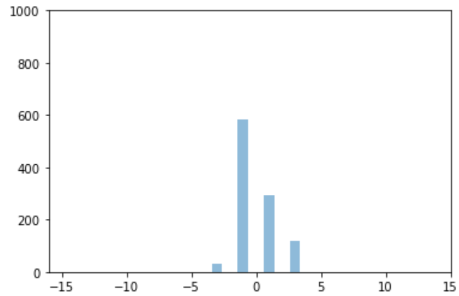
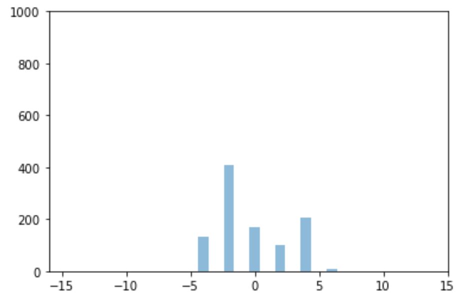
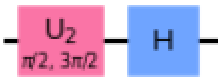
Step = 1	**Step = 5**	**Step = 10**
		

Table4.3: Distribution of walkers whose coin is Ry gate

After considering the Error Bar, the two-sided diffusion equilibrium is not reached with using the Ry gate given in the paper.

4. **Experience 4: Modify initial value**



Result:

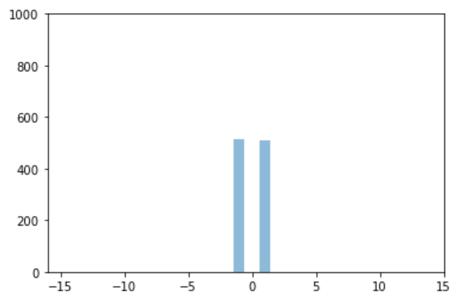
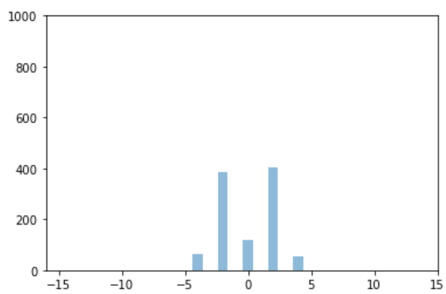
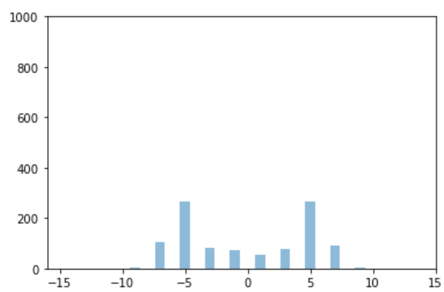
Step = 1	**Step = 5**	**Step = 10**
		

Table4.4: Distribution of walkers whose coin is modify-H gate

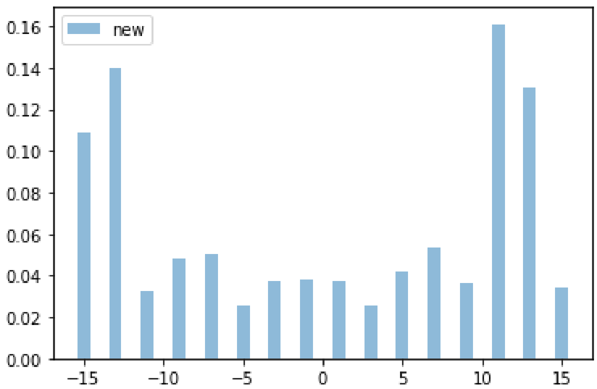


FIG4.5: Distribution of walkers whose coin is modify-H gate for 20 steps

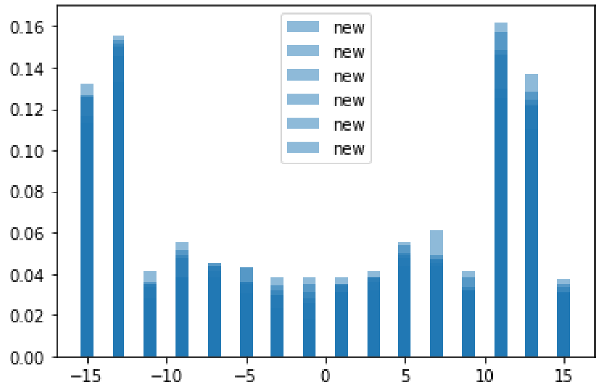


FIG4.6: Distribution of walkers whose coin is modify-H gate for 20 steps but execute for 6 times. We can see that the average of figure3.6 show the two-sided diffusion equilibrium better.

Results of Experiment 4 show that the distribution after executing 10 steps is close to symmetrical.

5. **Comprehensive results**

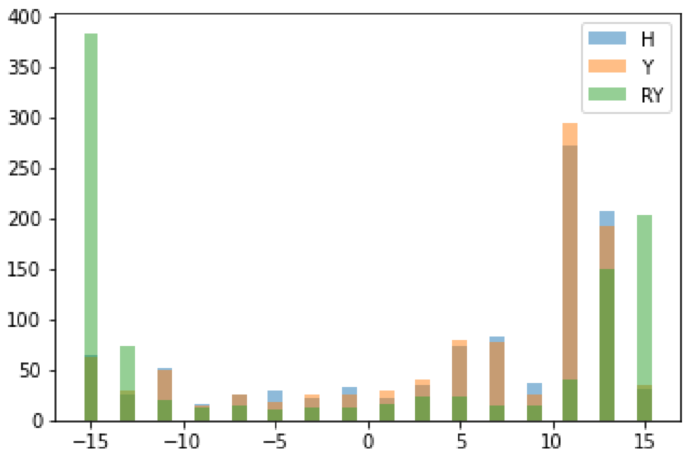


FIG:4.7 :comparison of one-dimensional quantum random walk

The experiment shows that changing the initial value of the coin operator of quantum random walk can effectively improve the symmetry.

C. Using Python to simulate quantum random walking

Result:

1. **Step=100**

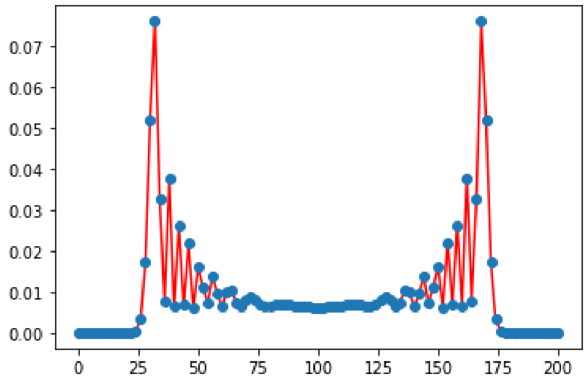


FIG4.8: Quantum walk calculated by matrix calculation, Step = 100

2. Step=200

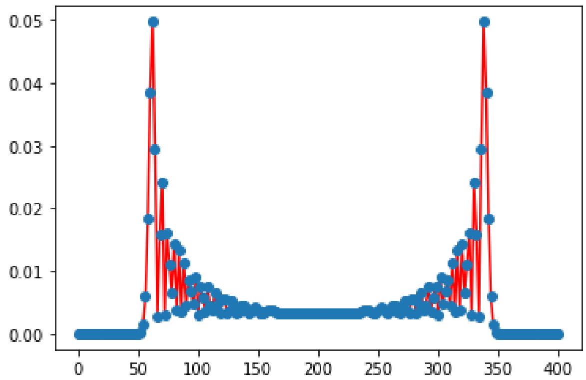
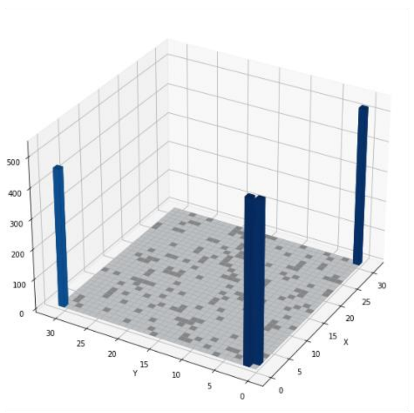
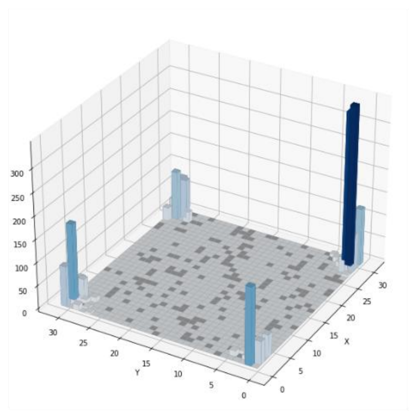


FIG4.9: Quantum walk calculated by matrix calculation, Step = 200

This experiment shows that if it is a pure matrix operation, the effect is diffusion and symmetry on both sides. It also proves that the discrete quantum random walk is indeed symmetrical, rather than inclined to a certain side.

D. Multi-dimensional quantum random walk distribution

1. **Experiment 5:H gate**

Step = 1	**Step = 5**
	
Step = 10	**Step = 20**

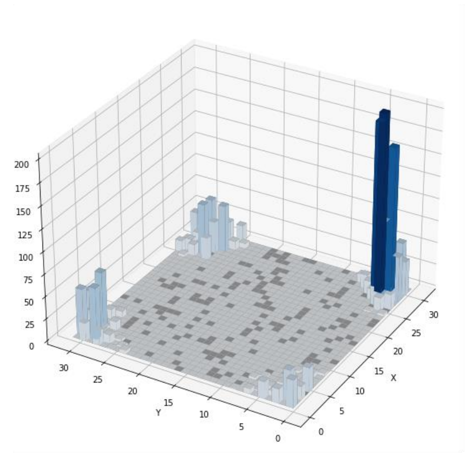
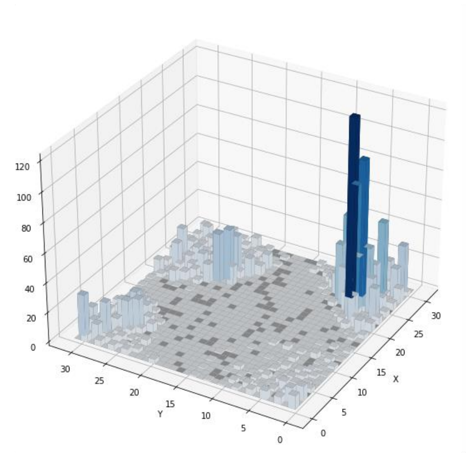
	

Table 4.5 : Two-dimensional quantum random whose coin is H gate

The tables show that the coin made into a two-dimensional space using the H gate are also not symmetrical.

1. Experiment 5: Modify initial state

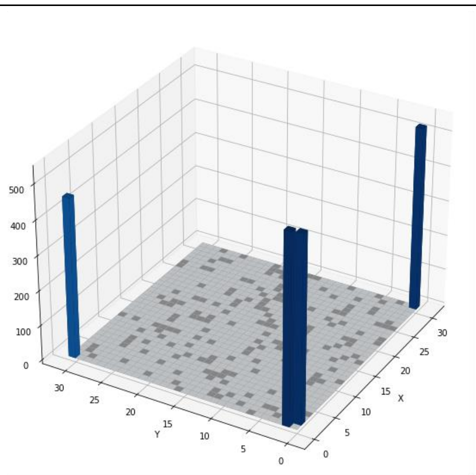
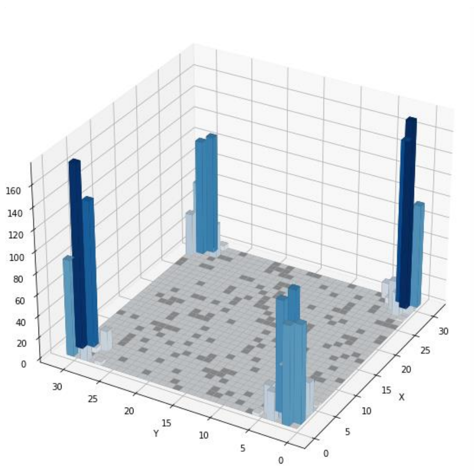
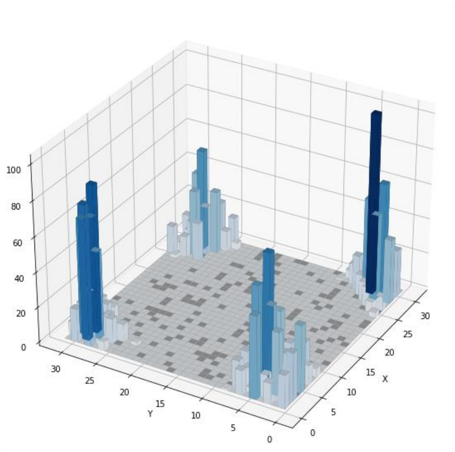
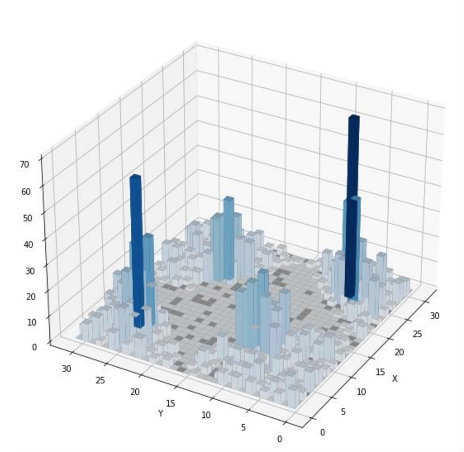
Step = 1	**Step = 5**
	
Step = 10	**Step = 20**
	

Table 4.6:Two-dimensional quantum random walk with modified initial value of H coin

The tables show that the coin made into a two-dimensional space by modifying the initial value of the H gate is symmetry.

E. Perform quantum random walk on physical machine ibmq_casablanca

1. The H gate that can produce a uniform superposition state is used as a coin to perform one-dimensional quantum random walk

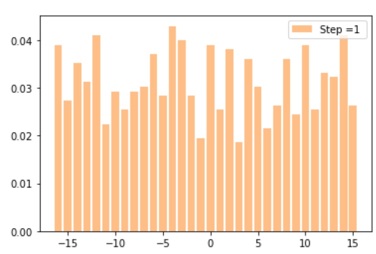
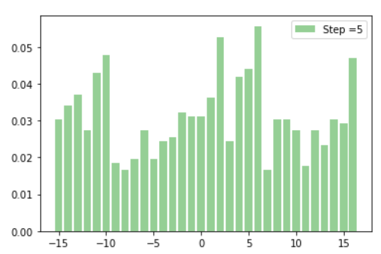
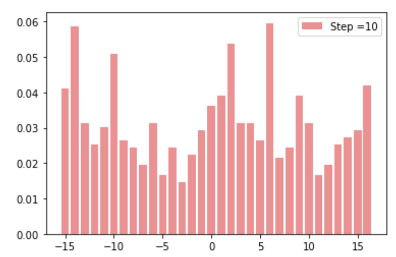
Step = 1	**Step = 5**	**Step = 10**
		

Table 4.7: Simulating the quantum random walk of H coin in a real quantum computer (one-dimensional)

We found that in real quantum computers the distribution is not symmetrical, no matter how state it is.

2. Use modified initial value as a coin to perform a one-dimensional quantum random walk

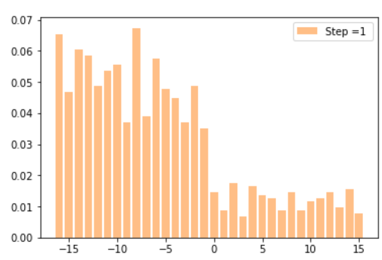
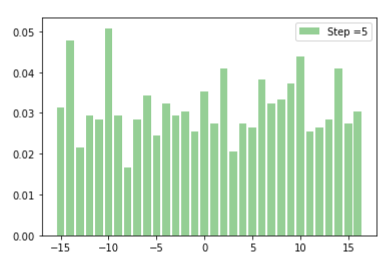
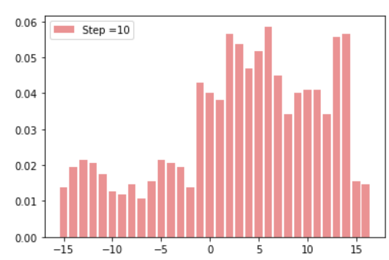
Step = 1	**Step = 5**	**Step = 10**
		

Table 4.8: Simulating the quantum random walk of correcting the initial value of H coin in a real quantum computer

3. The H gate that can produce a uniform superposition state is used as a coin to perform a two-dimensional quantum random walk

Step = 1	Step = 5

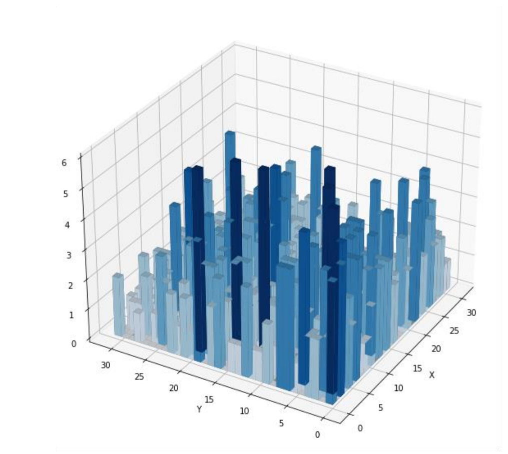
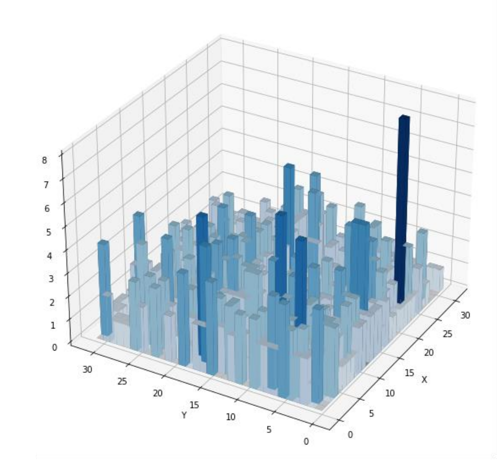
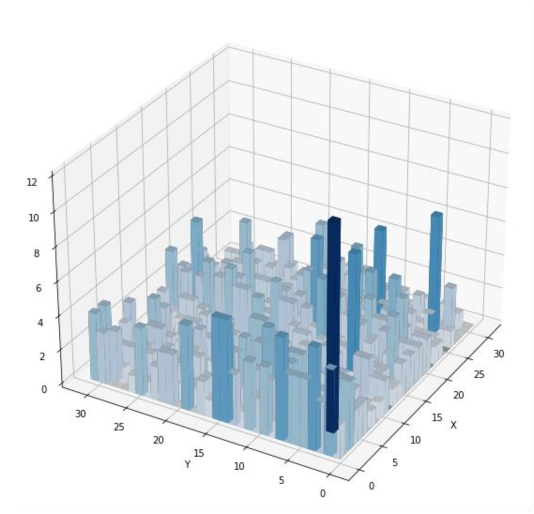
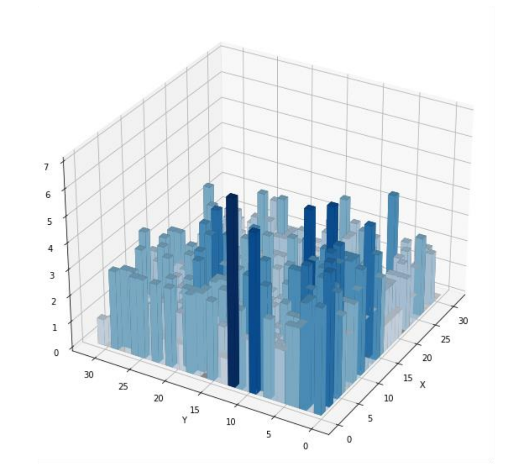
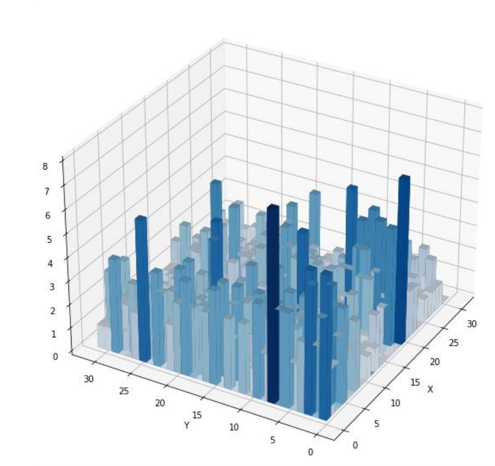
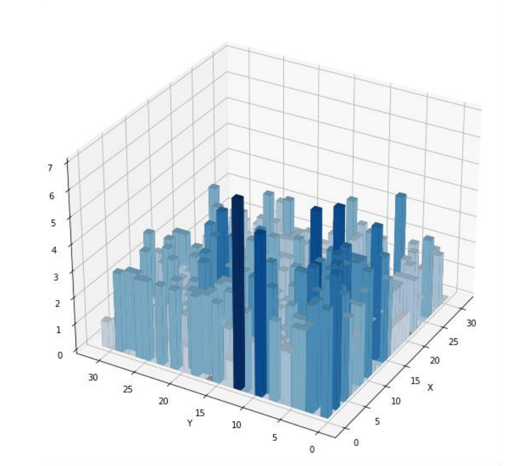
	
Step = 10	Step = 20
	

Table 4.9: Performing a two-dimensional quantum random walk using H coin in a real quantum computer

4. Use modified initial value as a coin to perform a two-dimensional quantum random walk

Step = 1	Step = 5
	
Step = 10	Step = 20

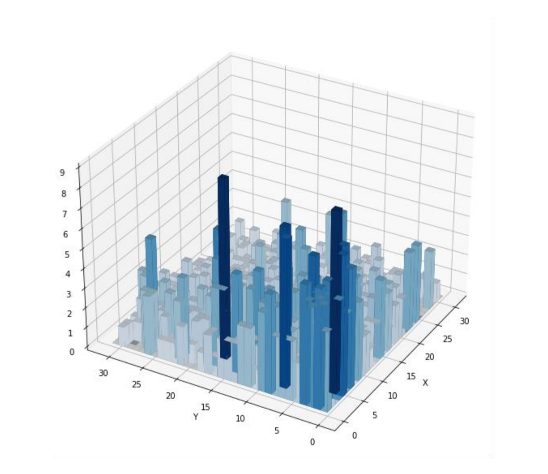
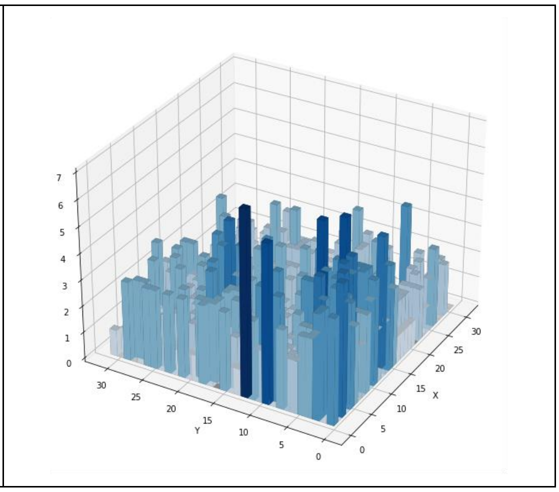
	

Table 4.10: Simulating the two-dimensional quantum random walk of correcting the initial value of H coin in a real quantum computer

F. The result of classical Parrondo

We define classical data will convergent when the difference of expectation values is less than 1.5, then we simulate the classical Parrondo by python for Game A,B and design different Game period ABA,ABB and BAAAA. From the expectation values corresponding to the convergent, we identify the results of the classical Parrondo.

Table 4.11(a)From this data graph, the expectation value converges to approximately step 20, and the corresponding expectation value is approximately -0.016648168701442843. It can be seen that the result of Game A is a loss. Table 4.11(b) In order to see more clearly that the range of convergence, we did 100 times for Game A Step 100, and the expectation value is below 0, so the result is a loss.

Table 4.11: Expected vale for game A in each step

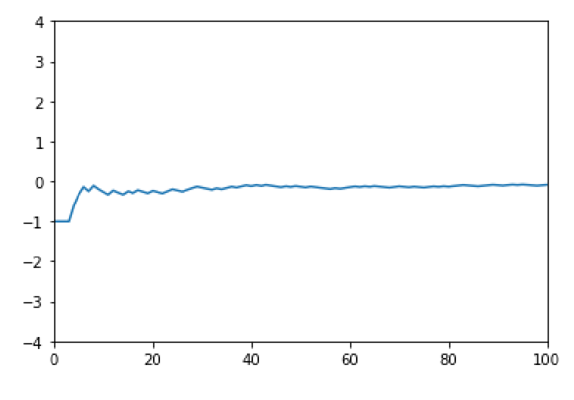
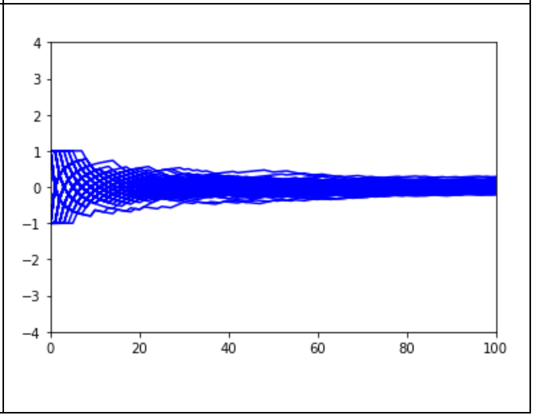
(a) Game A one time for Step100	(b) Game A a hundred times for Step100
	

Table 4-12(a) From this data graph, the expected value is approximately converged at step 25,and the corresponding expected value is approximately -0.8002219755826859 the expected value is less than 0. It can be seen that the result of Game A is a loss.Table 4-12(b) The range of convergence is about the 25th step and the expected value result is also below 0, so we can know that the result is a loss

Table 4.12: Expected vale for game B in each step

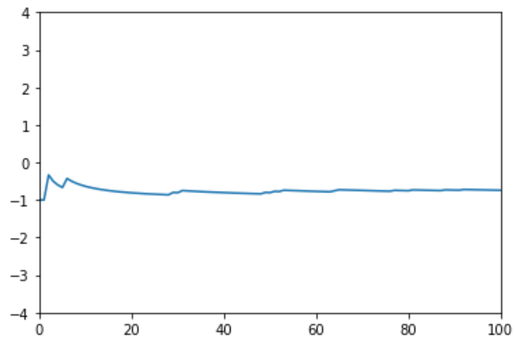
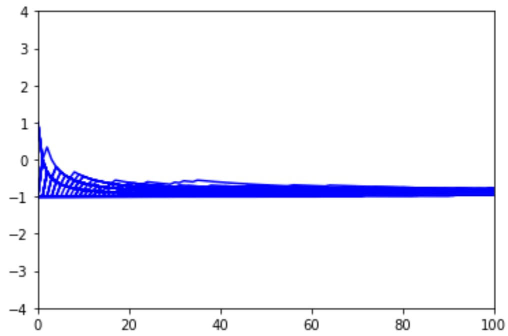
(a) Game B one time for Step100	(b) Game B a hundred times for Step100
	

Table 4-13(a)From this data graph, the expectation value converges to approximately step 60, and the corresponding expectation value is approximately 0.2785793562708102. It can be seen that the result of Game ABA is a win. Table 4-13(b)The range of convergence is about step 60 and the result of expectation value is above 0, so we can know that the result is a win.

Table 4.13: Expected vale for game ABA in each step

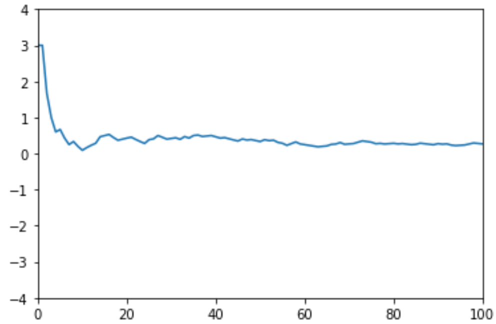
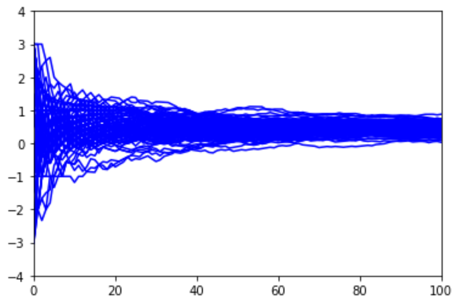
(a) Game ABA one time for Step100	(b) Game ABA a hundred times for Step100
	

Table 4-14(a)From this data graph, the expectation value converges to approximately step 50, and the corresponding expectation value is approximately 0.169811320. It can be seen that the result of Game ABB is a win. Table 4-14(b)The range of convergence is about step 50 and the result of expectation value is above 0, so we can know that the result is a win.

Table 4.14: Expected vale for game ABA in each step

(a)Game ABB one time for Step100	(b)Game ABB a hundred times for Step100

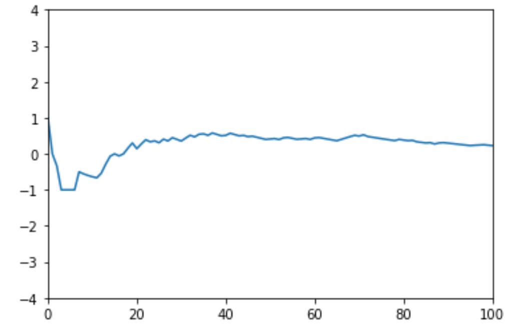
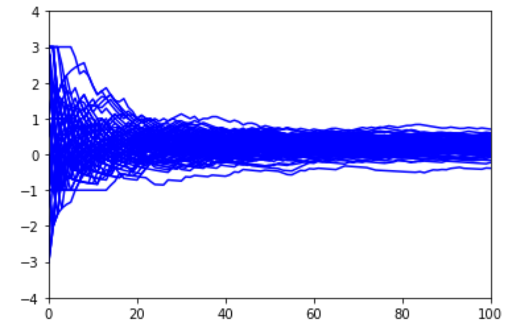
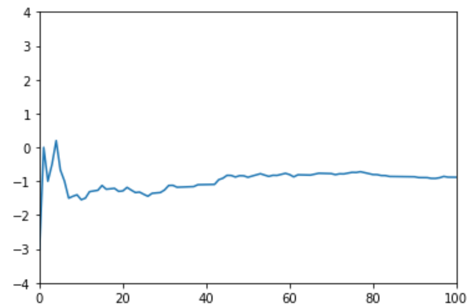
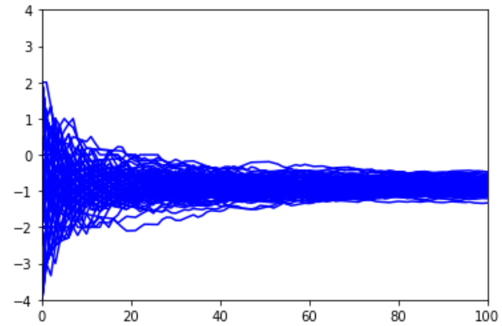
	

Table 4-15(a)From this data graph, the expectation value converges to approximately step 20, and the corresponding expectation value is approximately -0.871396895, the expectation value is less than 0. This shows that the result of Game BAAAA is a loss. Table 4-15(b)The range of convergence is about step 20 and the result of expectation value is less than 0, so we can know that the result is a loss.

Table 4.15: Expected vale for game BAAAA in each step

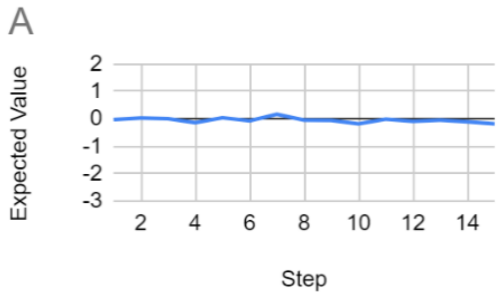
(a) Game BAAAA one time for Step100	(b)Game BAAAA a hundred times for Step100
	

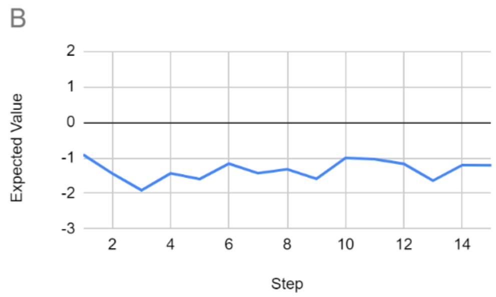
G. Expected value for games in quantum

Table 4.16 show the expected value for each game and how they evolution. Since the winning rate of game A is close to 0.5. The expected value is close to 0 but naggitive.Game A have already converged in step 1.For game sequence ABA , we can see the expected value is close to one but as the number of steps increases, the expected value converged. The convergent value is close to 0 but is positive.For game sequence ABB, we can see the expected value is close to one but as the number of steps increases, the expected value converged. The convergent value is close to 1. Game ABB successfully win the with the positive expected value.For game sequence BAAAA, we lose the game in the beginning but the value soon converge to positive in step 5.

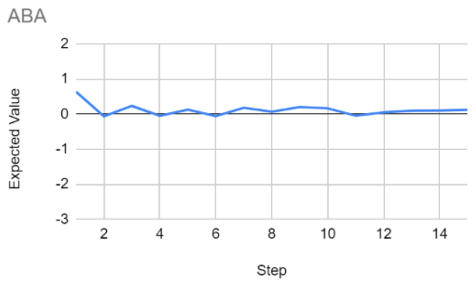
Table 4.16: Expected vale for game A in each step

Game A	Game B

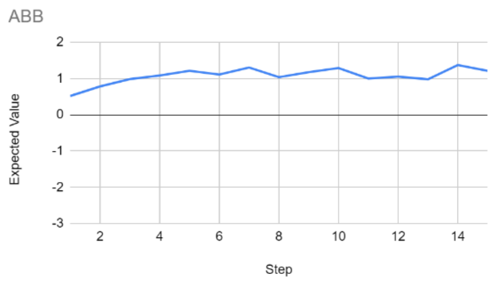




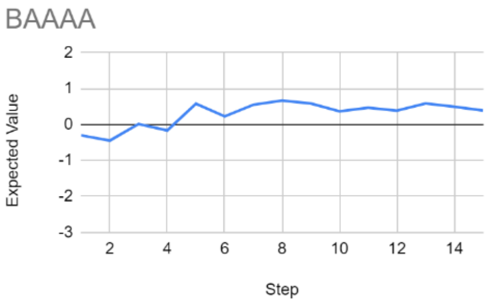
Game ABA



Game ABB



Game BAAAA



V. Discussion

A. Why is there an asymmetry?

The second variable suggested that quantum random walking should be symmetrical walking, but when we apply H gates, which can produce an average superposition state for quantum random walking, we found that the position probability distribution graph is not symmetrical. The reason is that when we consider quantum behavior, particles have volatility when they walk, or more strictly speaking, in quantum mechanics, we use wave functions to describe the motion behavior of particles. Therefore, quantum behavior has phase destructiveness. The detailed derivation of the first three steps is as follows:

$|H> \otimes|0> \rightarrow \frac{1}{\sqrt{2}}(|\uparrow> |1> +|\downarrow> |-1>)$ ($|\uparrow>$ left，$|\downarrow>$ right) ------(a)

$\rightarrow \frac{1}{2}((|\uparrow> |2> +|\downarrow> |0>) + (|\uparrow> |0> -|\downarrow> |-2>))$----------(b)

$\rightarrow \frac{1}{2\sqrt{2}}((|\uparrow> |3> +|\downarrow> |1>) + (|\uparrow> |1> -|\downarrow> |-1>) + (|\uparrow> |1> +|\downarrow> |-1>) - (|\uparrow$

$> |-1> -|\downarrow> |-3>))$------------(c)

$= \frac{1}{2\sqrt{2}}(|\uparrow> |3> +2|\uparrow> |1> +|\downarrow> |1> -|\uparrow> |-1> -|\downarrow> |-3>)$----(d)

Among these equations,
(a) represents the result of moving after the first step (the first quantum dice throwing),
(b) represents the result of the move after the second step (the second quantum dice throwing), and
(c) represents the third step (the first result after throwing the quantum dice three times), (d) is the result of (c). From the formula above (c), we can see that the last position at state 1 will be 2|↑>|1>+|↓>|1>, and the probability at -1 will be -|↑>|-1>. Therefore, the odds of appearing at 1 is 4/8+1/8=5/8, and the odds of appearing at -1 is 1/8. (c) can be the indicator of this outcome since the path to 1 and -1 will be cancel due to the quantum destructiveness. Therefore, to avoid the occurrence of quantum cancellation, the initial value of the H gate coin is changed so that the dice do not have the opposite phase characteristic, thereby eliminating the possibility of quantum signal cancellation, as shown in the following formula:

$$\frac{1}{\sqrt{2}}(|\uparrow> +i|\downarrow>)|H> \otimes|0>$$

B. Two-dimensional walking

This experiment successfully made a two-dimensional random walk and showed its outward diffusion characteristics after drawing. The two-dimensional walk of the H gate does not have symmetry but tends to walk towards (31, 0), so by experiment 4, which has a one-dimensional symmetry in variable 1, at step = 5 has the symmetrical and wide-spreading characteristics, and at step = 20 still retains its balanced property, and its tendency of preference in (31, 0) and (0, 31). Therefore, this experiment successfully represents a symmetrical two-dimensional quantum walk. From Table (6), we can observe the two-dimensional random walk designed in this experiment is a spherical random walk, because when the walker reaches the edge, the next step will be There is a half chance of returning to the starting point of the ball, and then walking randomly from the starting point of the ball.

C. The comparison between the simulator and the actual quantum computer

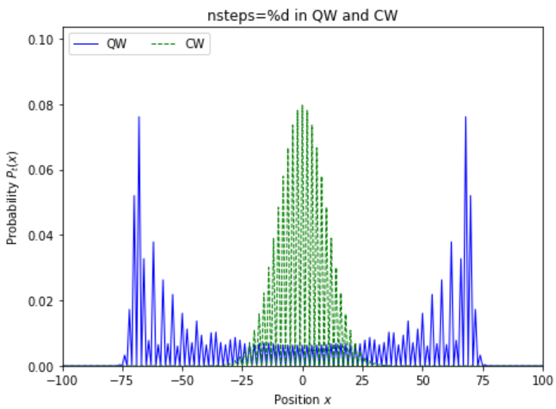
We know from Experiment 4 that although one-dimensional quantum random walk can successfully display the symmetry characteristics of the quantum walk when executed on the simulator, as shown in Table (4). However, after being run on an actual quantum computer, it is not only the initial symmetry change, the value of the H coin (Table (8)), even the basic H gate can not see the distribution characteristics of quantum random walk, as shown in table (7).

The same is true for the two-dimensional quantum random walk. We can indicate from Table (6) that the spherical quantum random walk had a success performing on the simulator, which is the so-called two-dimensional quantum random walk, but when using an actual quantum computer, But it is impossible to see the distribution of quantum random walk from the data, as shown in Table (5, 6),

Therefore, it is hypothesized that the error value of the quantum computer is high (the main error comes from the initial state, each operation, and the measurement method and the mathematical [physical] form is qubit flip (Decoheren quantum decoherence), Quantum phase shift (phase flip) [Dephasing phase decoherence]). Therefore, the results of the quantum algorithm are still unable to be presented, and there will be no good distribution.

D. The probability standard deviation of the diffusion time of classical walk and quantum walk

(a)



(b)

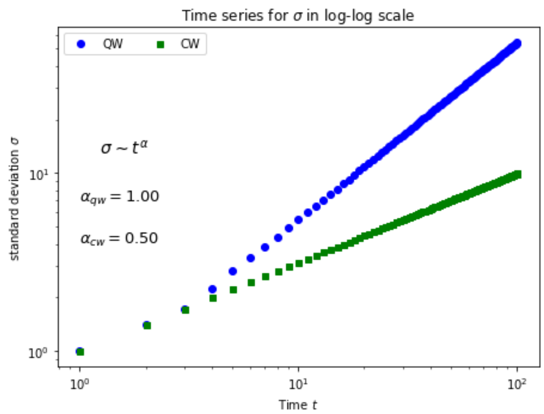


FIG 5.1(a)The distribution of quantum random walk shows a tendency to diffuse to both sides, while the classical random walk shows a bell-shaped distribution.

This is a characteristic that distinguishes quantum from classical physics. Intuitively, this result can consider as: the probability of quantum has coherent superposition, so probability waves can destructively interfere near the origin, resulting in the probability density is distributed in a vast range.

E. Parrondo’s Paradox in Classical and Quantum

We use the quantum gate modified in Experiment 1 to play the game of Quantum Parrondo's Paradox, because we can see the result after the H gate is thrown ten times in Table 4.1, which is not symmetrical, which means that Game A will be played later. It is becoming more and more unfair, but what we want is a fixed probability, so we use the H gate with a modified initial value. With this we can continue to play Game A without worrying that the probability will change due to time.

In the part of the result F, we present the results of the classical and quantum games. We can see that quantum not only has the same trend as classical, but the convergence speed of quantum methods is also faster than classical ones, because when we discuss D, we discuss The diffusion speed to quantum space is the classical square, so it is much faster than classical when the expected value converges. Therefore, we think that quantum Parrondo's Paradox successfully verified the quantum advantage**.**

VI. Conclusion

(1) Destructively interfere near the origin, resulting in a distribution of probability density in a larger range. This experiment successfully designed a quantum random walk with excellent symmetry.

(2) This experiment proves that the H gate that can produce an average superposition state cannot be designed as a quantum dice, because after many dice, quantum coherence will be generated due to the phase problem, which will affect the walking result.

(3) This experiment successfully designed two-dimensional quantum random walking, also known as spherical quantum random walking, and got the same trend as one-dimensional walking.

(4) Fourth, the quantum walk is diffused to both sides, and the classical walk is a bell-shaped distribution.

(5) The current quantum computer has a relatively large error value and still cannot handle multi-bit quantum algorithms

(6) Quantum walking is a unidirectional evolution process of microscopic particles, and the diffusion rate of walkers is increasing in a flat manner compared with the classics. Therefore, the spatial distribution diffusion rate of the quantum method is faster than that of the classics. Quantum walking is a unidirectional evolution process of microscopic particles, and the diffusion rate of walkers is increasing in a flat manner compared with the classics. Therefore, the spatial distribution diffusion rate of the quantum method is faster than that of the classics.

(7) Quantum diffusion is faster, the expected value of quantum Parrondo paradox converges faster than classical.

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