Modification and promotion of quantum random walk with phase interference and implementation of Parrondo’s paradox on IBMQ

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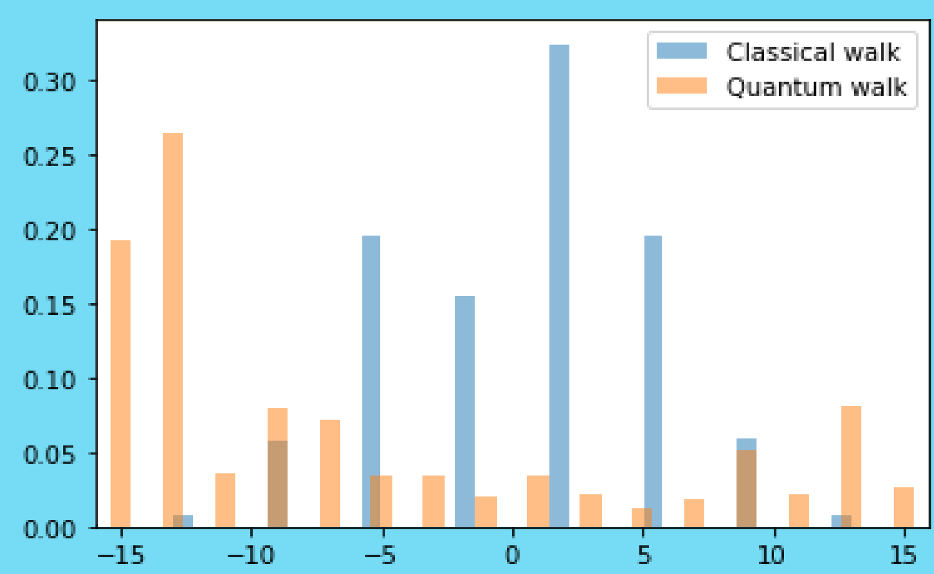
Abstract

This research mainly studies the difference between quantum random walk and classical random walk. The previous papers proved that in quantum evolution, due to the constructive and destructive wave function, one-dimensional quantum random walk will walk to the left or randomly. Finally, the first part will discuss Different coin-to-system quantum gates were designed for different coin-to-systems, and we discovered the state of the quantum gate designed for this quantum gate. The characteristics of quantum random walk. Finally, we use quantum random walk to realize Parando's observation research theory. We also found that because of the characteristics of quantum random walk. The propagation speed is faster than the classical one, which has created the value of Palondo's discussion faster than the classical one, and verified the quantum advantage.

Introduction

Aims and Objectives

1. The distribution of quantum walks with different coins
2. Establish a fair quantum coin
3. Two-dimensional quantum walk
4. Using QW to realize Parrondo's paradox



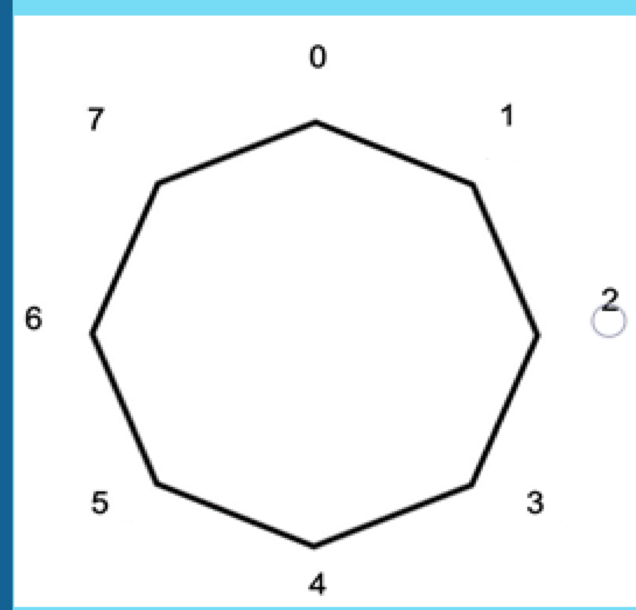
Research Motivation

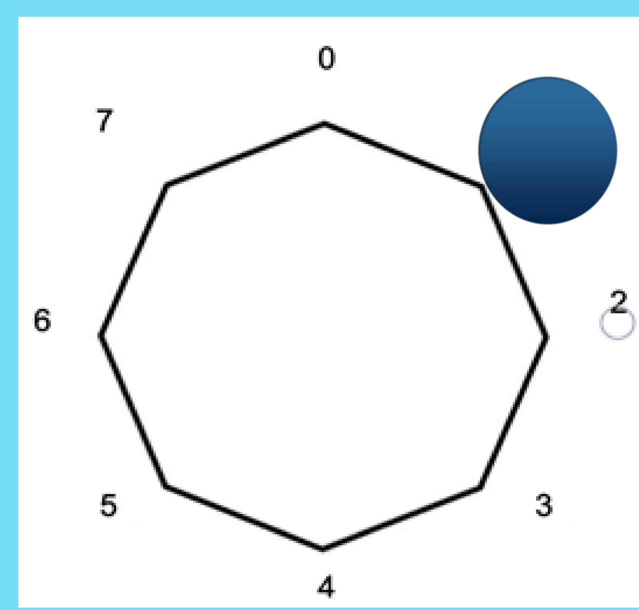
In the process of learning quantum computing, discrete quantum random walk (Quantum Random Walk) is our first contact algorithm. Quantum random walk is extended by classical random walk. Walker's position is described by probability distribution, quantum The superposition state replaces the distribution of each position in the classical random walk. The characteristics of quantum will cause Walker to go farther (close to the extreme value). It can also be seen as information spreading faster. In the process of practicing quantum random walk, we It was originally expected that the distribution curve of the H gate that can produce a uniform superposition state would diffuse to both sides, but our experiments found that it is not, but has a tendency to bias in a certain direction, so this experiment extends the application of quantum random walk deeper and deeper wide.

With symmetrical quantum random walking, we can use the results of this experiment to simulate some phenomena in nature. We hope to simulate three-dimensional Brownian motion with quantum random walking, hoping to get the same trend as in actual nature. In bioengineering, quantum random walk can be used to calculate the mapping problem of enzymes to understand the evolution process of enzymes and mutagens. This problem only requires 33 nodes and can be mapped to 7 qubit circuits. However, we The most interesting is the design of quantum random walk as a quantum machine learning (QML) algorithm—Quantum Random Forest, which is used to deal with regression prediction problems.

Literature review

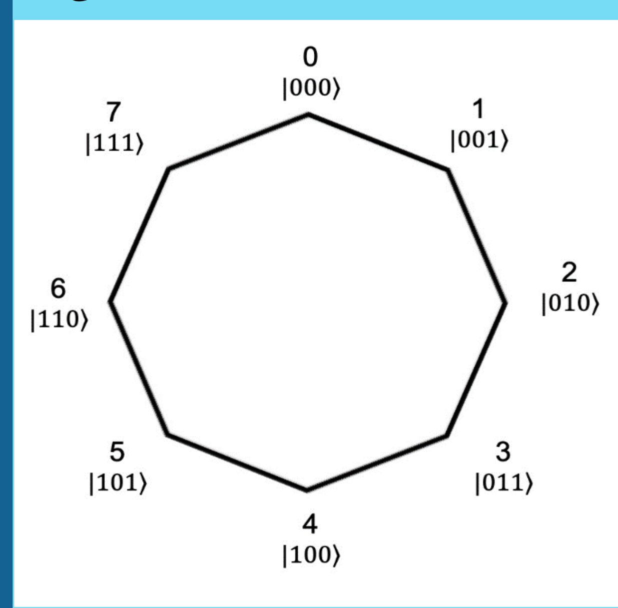
Classical random walk

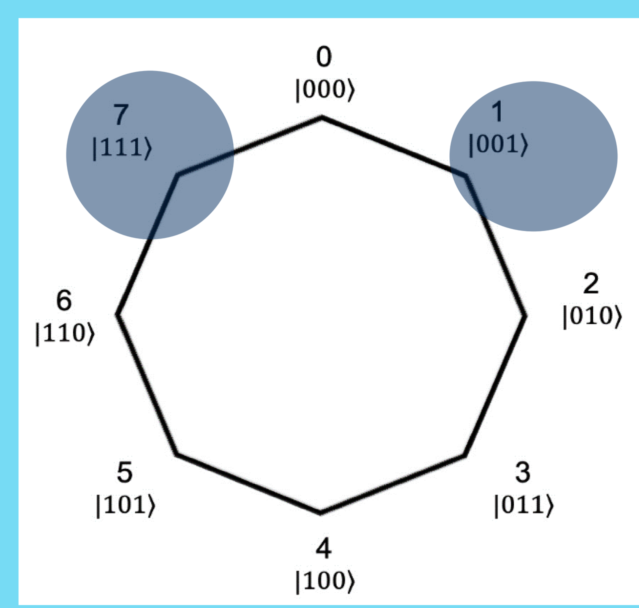


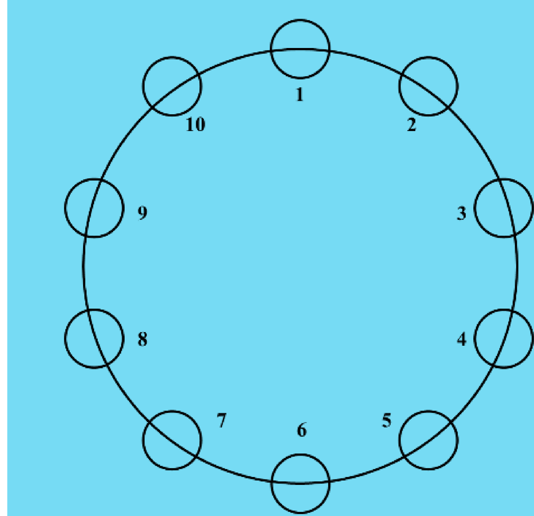


a stochastic process is a system that experiences fluctuations of chance over time. and random Walk is a special type of random process, which is related to mathematical entities and many other fields such as physics and computer science.

Quantum random walk on the circle

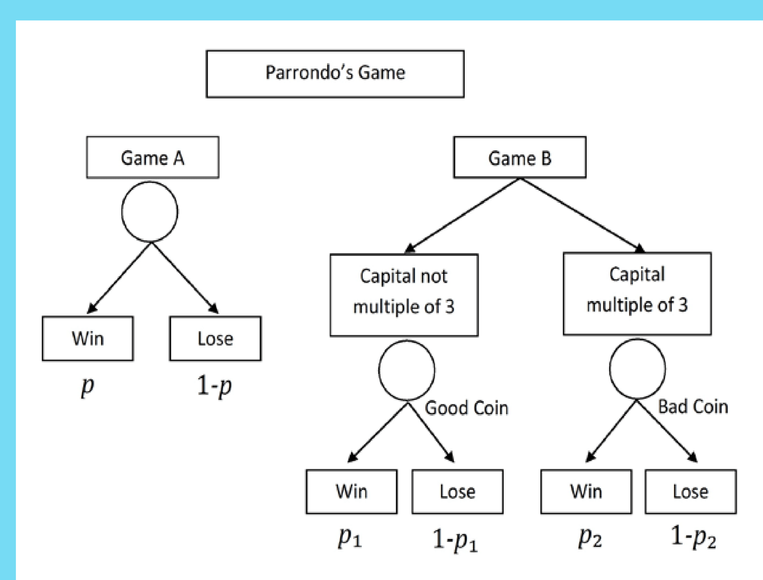






U *step Move operator* S (*Coin operator* $C \otimes$ *Initial operator*)

(1)The pedestrian is located at a specific point n in the N-dimensional space (N = 1, and the walker walks along a line from point n)
(2)Generate a random coin with a custom probability to determine which direction the walker will walk in.
(3)The walker moves to the nearest point (n-1) or (n + 1) on the map



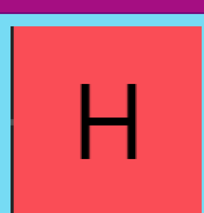
In game theory, the expected winning values of games A and B are negative, but if we combine the two games to play like ABA or BAB, the expected winning value will be positive.

- Game A: it is a simple quantum random walk
- Game B: It is more complicated. We will judge whether the remaining amount of the player is 3 at first, if it is 3, then execute bad coin with a winning rate of only 10%, if not, execute good coin with a winning rate of 75%.

Method and Approach

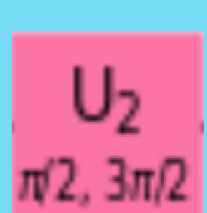
1. DESIGN QUANTUM RANDOM WALK OF DIFFERENT COINS

Experiment 1: H gate



$|\psi\rangle=\frac{1}{\sqrt{2}}|0\rangle+\frac{1}{\sqrt{2}}|1\rangle$

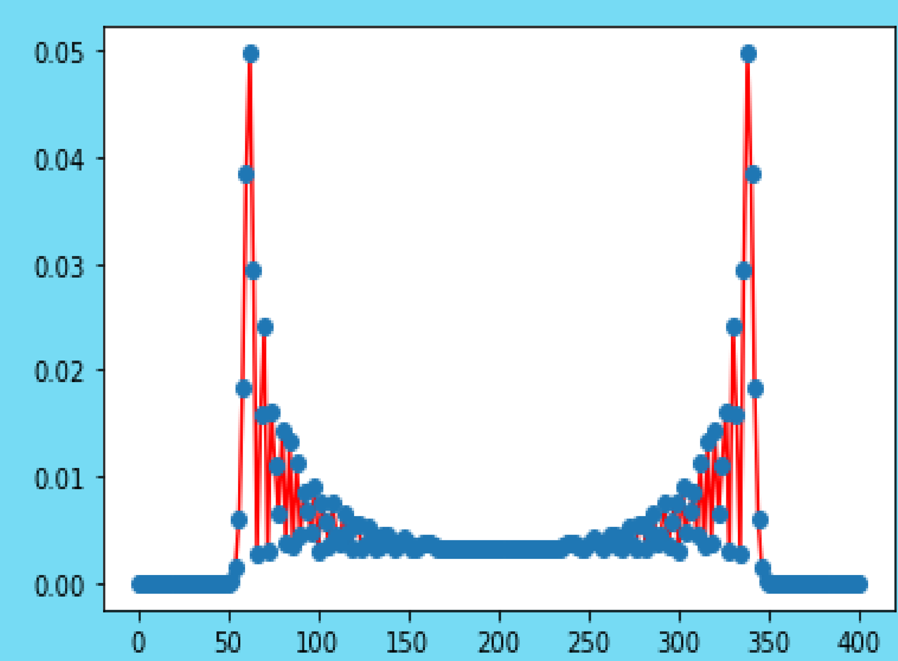
Experiment 2: Y gate



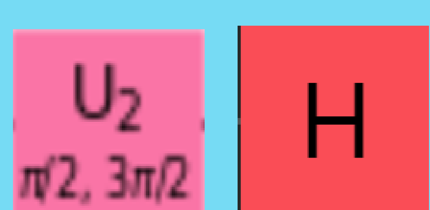
$|\psi\rangle=\frac{1}{\sqrt{2}}|0\rangle+\frac{1}{\sqrt{2}}i1|\quad\rangle$

Experiment 3: Ry gate

$|\psi\rangle=\sqrt{0.85}\,|0\rangle-\sqrt{0.15}\,i|1\rangle$



Experiment 4 Modify the initial value

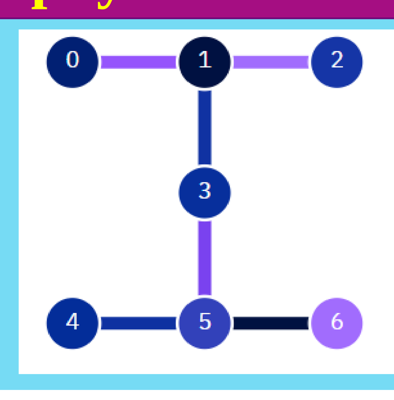


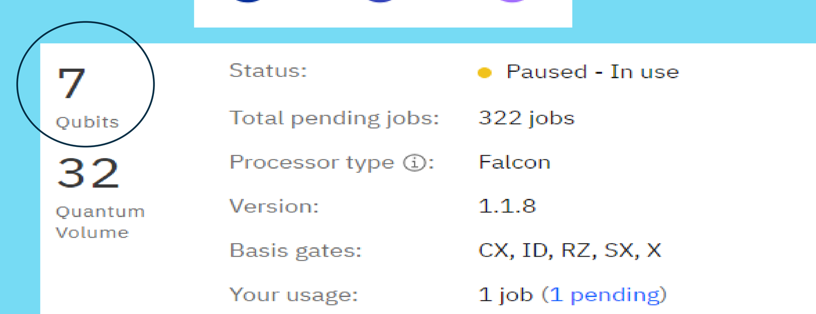
$|\psi\rangle=\left(\frac{1}{\sqrt{2}}|0\rangle+\frac{1}{\sqrt{2}}|1\rangle\right)\times\left(\frac{1}{\sqrt{2}}|0\rangle+\frac{1}{\sqrt{2}}i|1\rangle\right)$

2. Using Python to simulate quantum random walk

Experimental purpose: compare the difference between pure logic operation and physical pulse control

3.The physical machine "ibmq_casablanca"



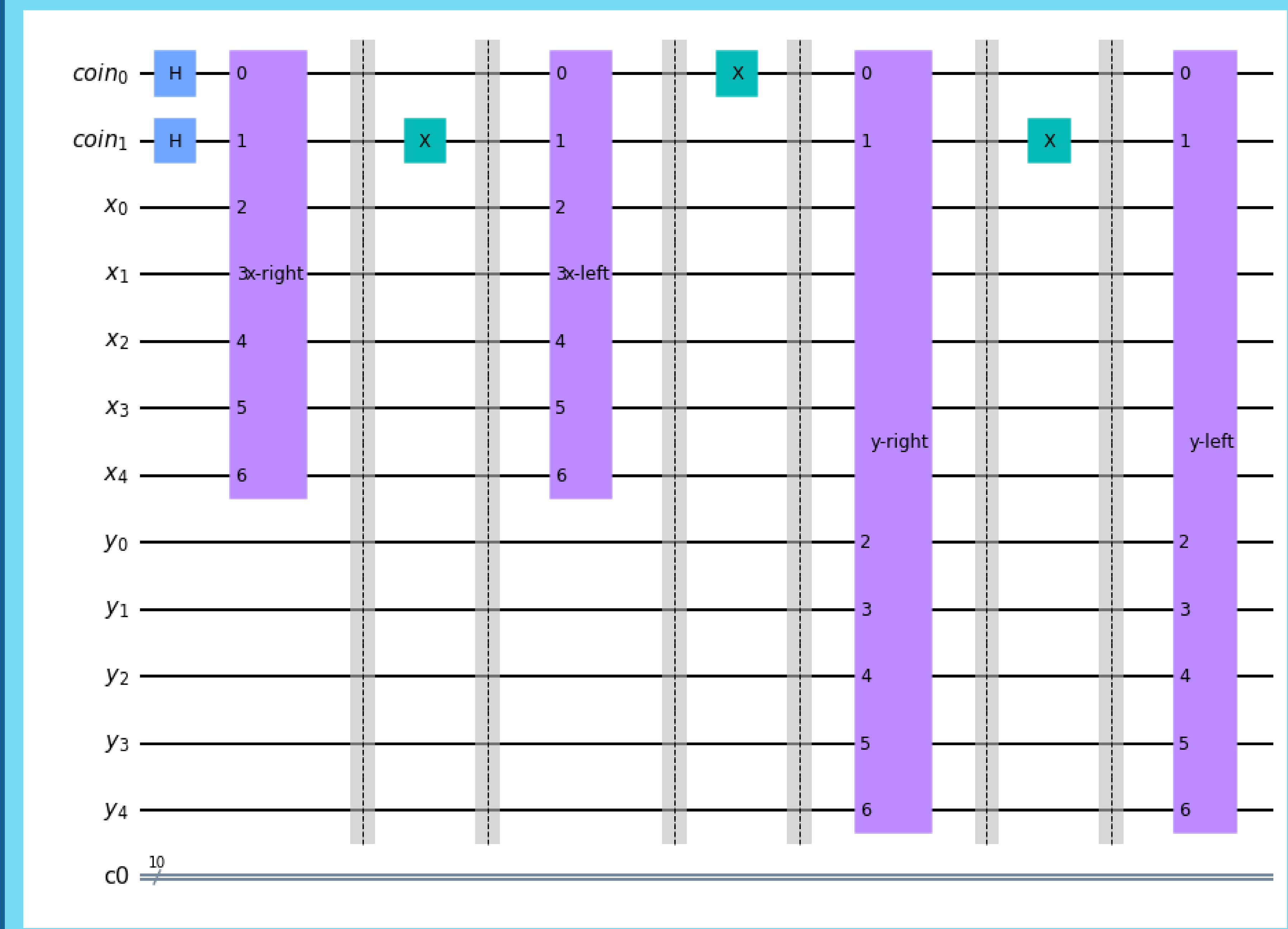


Because IBM's superconducting quantum computer still has a large error value, it is speculated that there may be some walking characteristics.

Method and Approach

4. Two-dimensional quantum random walk distribution

Also , we want to find out the multi-dimensional randomness of space by circular quantum walk. First, we must first define the Qubits representing the X-axis or Y-axis directions. Therefore, this experiment defines X as the information access location in the X direction and Y as the access location (counterpart/that) in the Y direction.

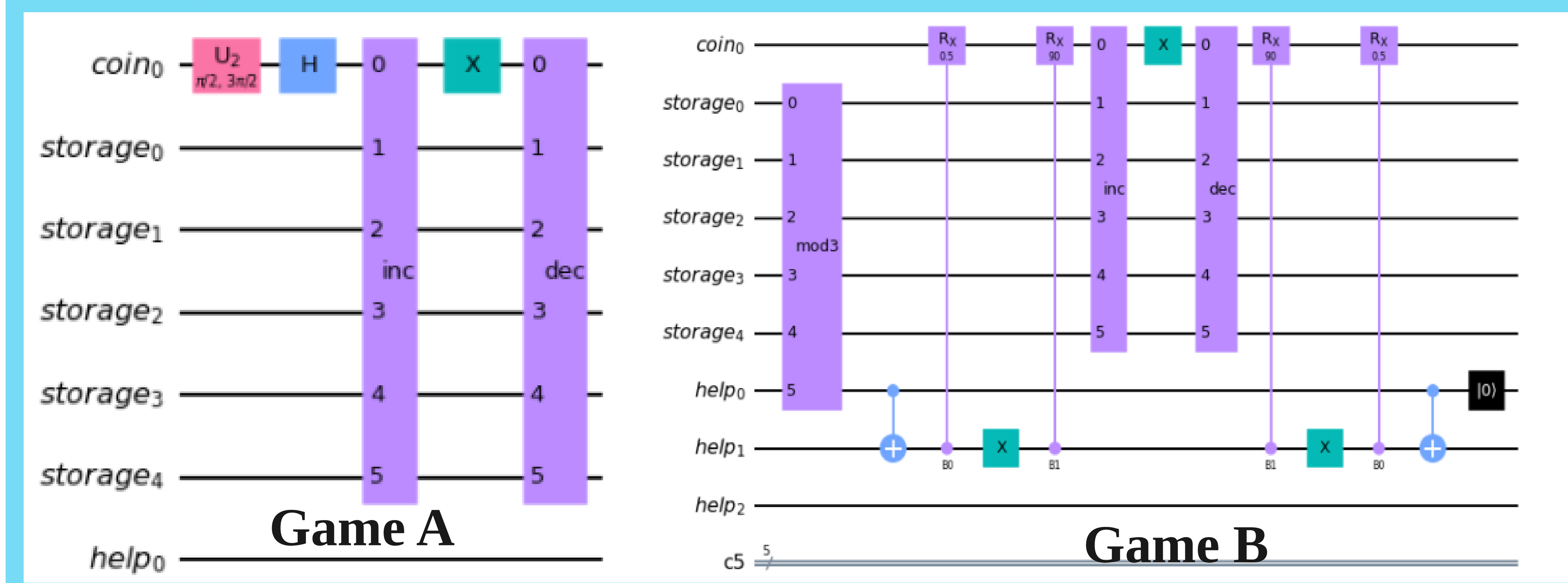


a. coin0 is the dimension judge. When the 0th bit is 0, the walker will decide to go to the Y axis. On the contrary, when the 0th bit is 1, the walker will decide to go to the X axis.
b. Coin1 is the direction judge. After coin0's judgment, the next step is to decide which way to go, so coin1 is used to decide whether to walk left or right.
c. X Qubits is the position accessor of X axis
d. Y Qubits is the position accessor of Y axis

5. Parrondo's Paradox under Quantum Walk

Palondo said it was a feast of gambling, with one dollar for winning and one dollar for losing and it is composed of two games, Game A and Game B

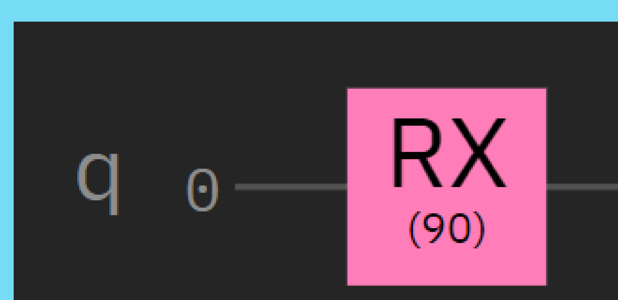
We designed the quantum logic circuits of Game A and Game B by ourselves, and simulated the quantum random walk of Game A and Game B in one dimension on the circle, and used the probability of walking to judge whether to win or lose.



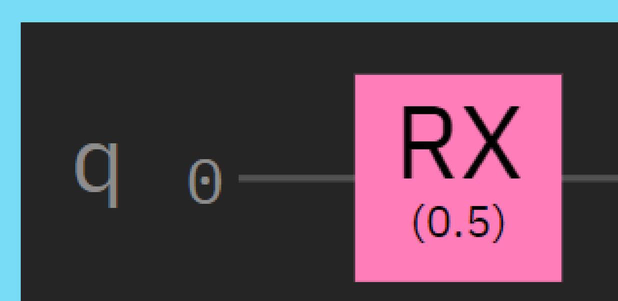
a. Game A
First design the coin of Game A,
According to Parrondo's paradox, the probability of A coin should be 1/2
Based on the previous experimental results, the probability distribution of the H gate will not be symmetrical after multiple H gates, which also means that the A coin is unfair. Therefore, we use the H gate with the modified initial value as our coin. The reason is the probability part will not evolve over

•Game B
Design Game B's coin
Based on Parrondo's paradox, we designed two coins, good coin and bad coin

Good coin



Bad coin

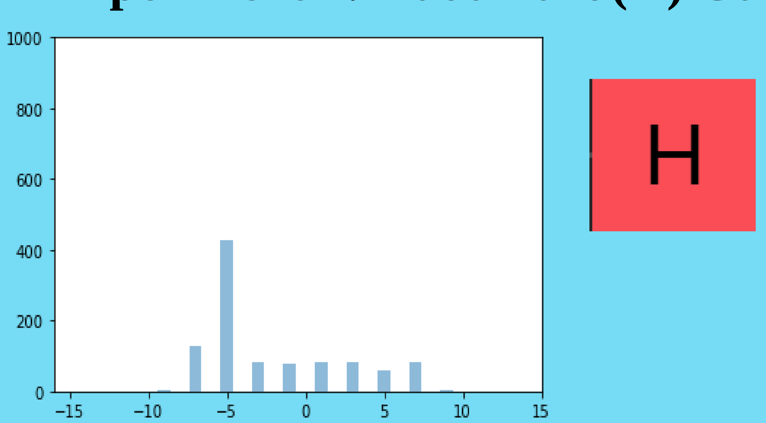


Game B depends on the amount of accumulated capital. Capital is determined by the number of qubits. The number of qubits in our experiment is 5, so the capital is 31 dollars. Ifhis accumulated capital is a multiple of 3, the player throws Bad coin, otherwise, throws Good coin.

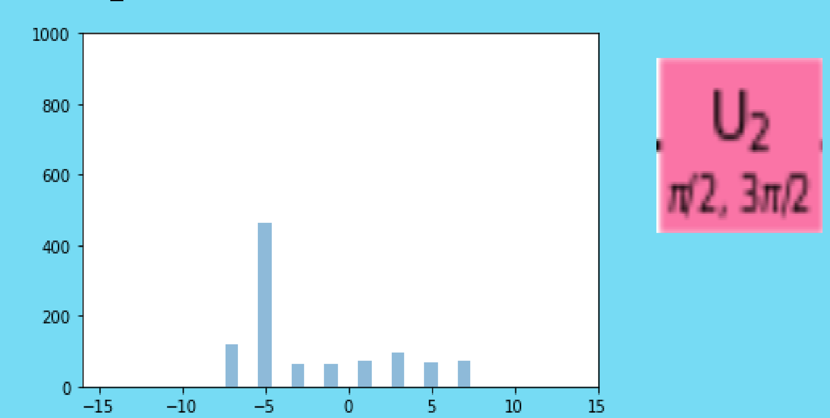
Result

1.Design quantum random walks for different coins

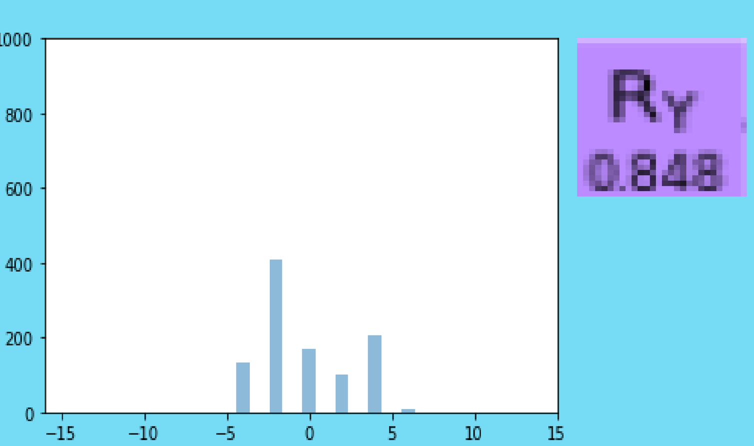
Experiment 1: Hadamard(H) Gate.



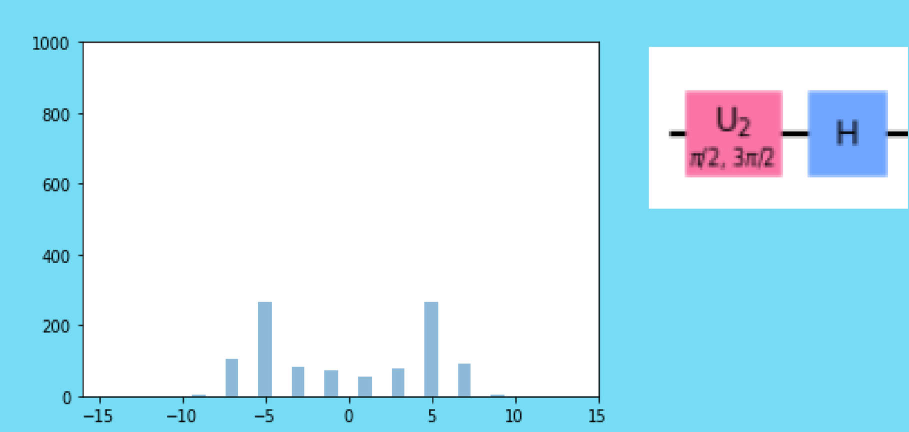
Experiment 2: Y Gate



Experiment 3: Ry gate.



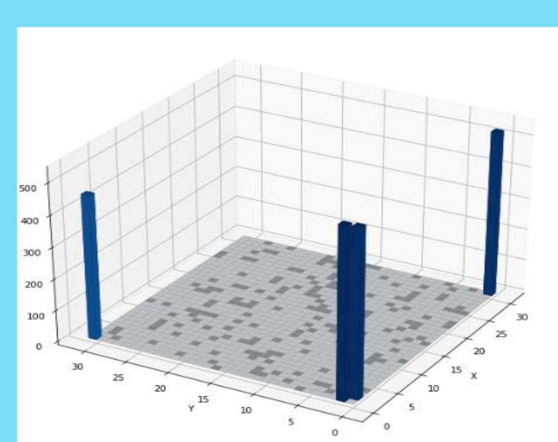
Experiment 4 :Modify the initial value



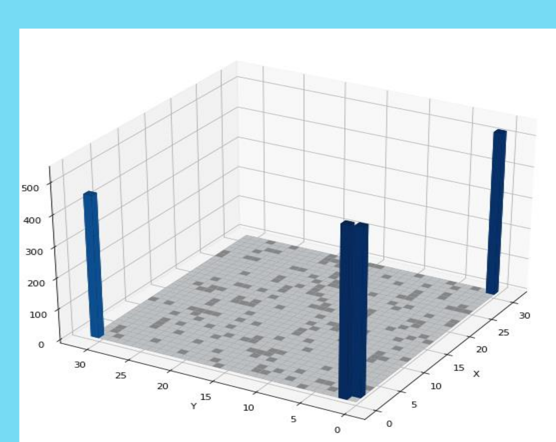
From the above table, it can be seen that after using the H,U2,Ry gate given in the paper, it is completely impossible to achieve uniform diffusion on both sides. The distribution is obviously to the right at 1 step. As the number of steps increases, it gradually tends to balance, but there is still no symmetry. In the case which has modified the initial state value.We can see it has the symmetrical distribution .

Two-dimensional quantum random walk distribution

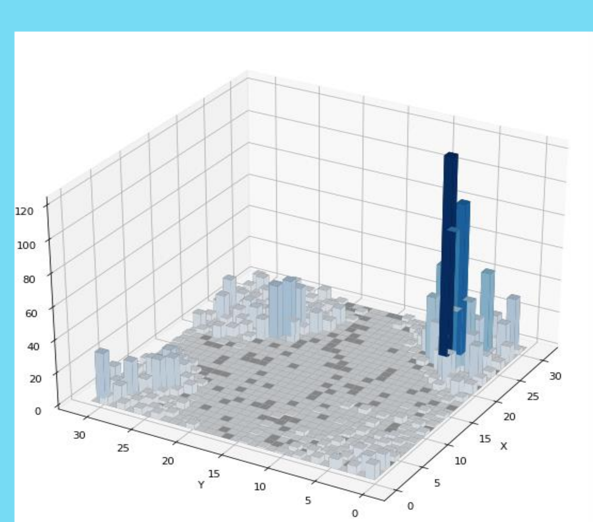
H gate for 1 step



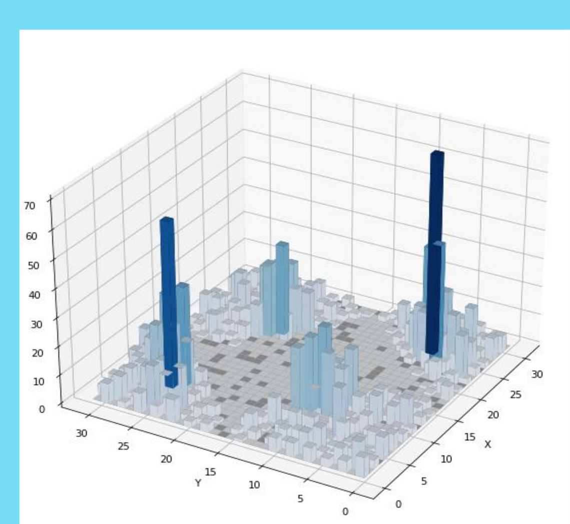
Modify initial state for 1 step



H gate for 20 step



Modify initial state for 20 step

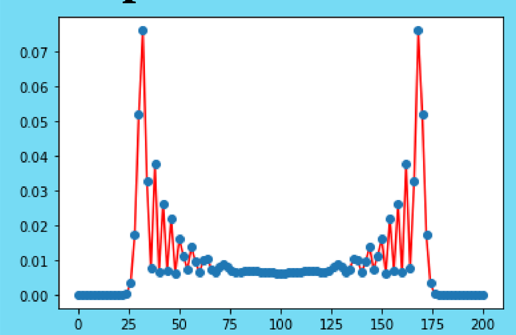


Look at the two figures above , we find that H gate has the same distribution as the one which has modified initial state at 1 step, but as the number of steps increases. We find that H gate loses symmetry, while b has a tendency to diffuse toward the extreme value.

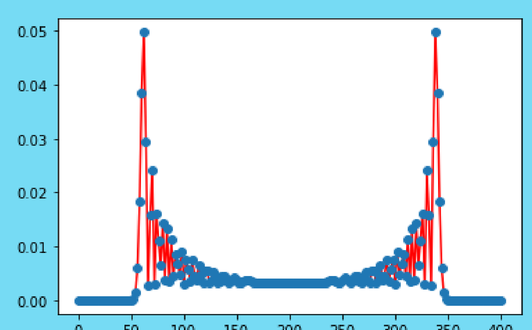
Result

3.PYTHON

Step100

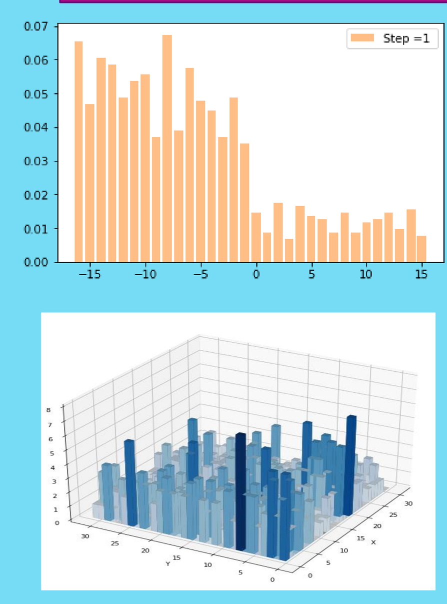


Step200



Execute math model in python.If it is a pure matrix operation, the effect is indeed diffusion symmetric on both sides, which also proves that the discrete quantum random walk is indeed symmetric, rather than inclined to one side.

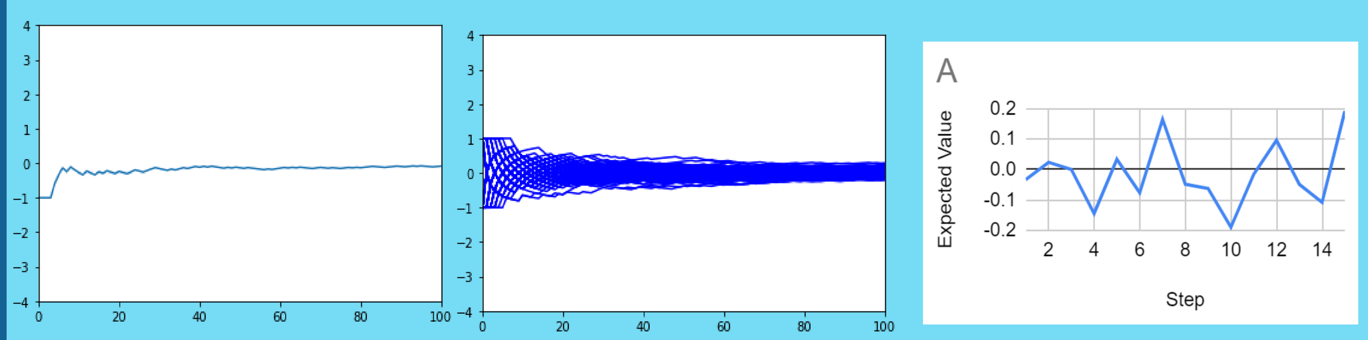
4.EXECUTE IN REAL QUANTUM COMPUTER



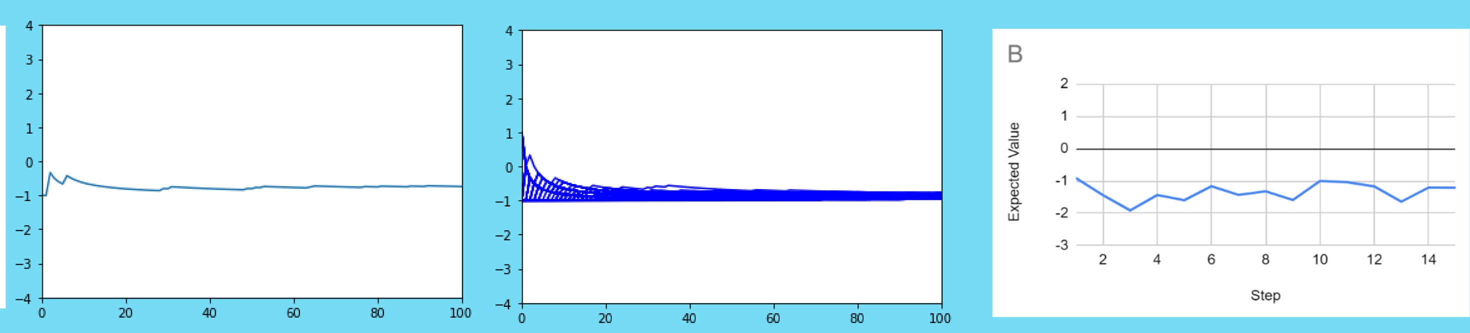
Modify initial state run in quantum computer for 1 step.We can see that even if it is modify one, the effect of symmetry cannot be achieved on the actual quantum computer. Because the actual quantum computer has errors, we can see that the distribution is still random at all positions when there is only one step.

5.PARRONDO PARADOX

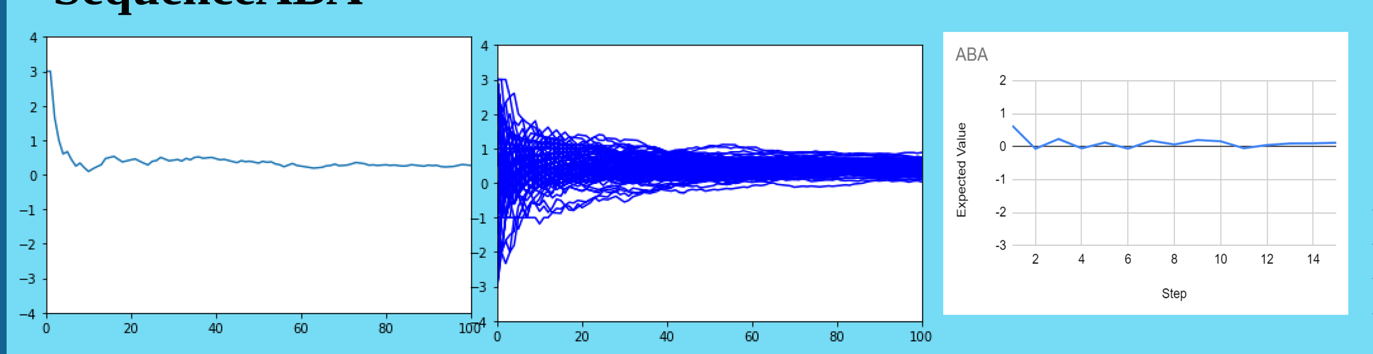
GameA



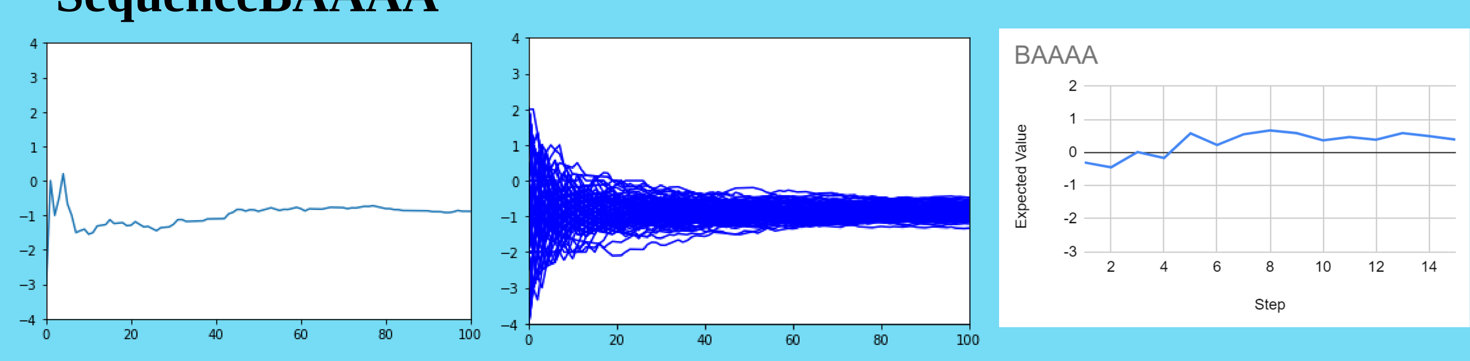
GameB



SequenceABA



SequenceBAAAA



Since the winning rate of the A game is close to 50%, the expected value is close to zero. Game A has already converged when it walks 1 step randomly. For the combination with ABA as the game period, the expected value is close to 1, but as the number of random walking steps increases, the expected value gradually converges.
The convergence value is close to 0 but positive. For the combination with ABB as the game cycle, we can see that the expected value is close to 0.6 when the number of random walking steps is 1, but as the number of steps increases, the expected value converges. The final convergence value is close to 1. The game ABB wins because it has a positive expectation value. For the combination with BAAAA as the game cycle, we lost the game at the beginning, but when the number of random walking steps is 5, the expected value quickly converges to about 0.5

Discussion

1. Why is there an asymmetry

Quantum random walking should be symmetrical random walking, but when using the H gate that can produce an average superposition state for quantum random walking, it can be found that the position probability distribution graph is not symmetrical. The reason is that when we consider quantum behavior, the particles are randomly There is volatility between walking, or more strictly speaking, in quantum mechanics, we use wave functions to describe the motion behavior of particles. Therefore, quantum behavior has phase destructiveness. The detailed derivation of the first three steps is as follows:

|H>⊗|0>→1/√2(|↑>|1>+|↓>|-1>)   (|↑>left，|↓>right) -----(a)

→1/2((|↑>|2>+|↓>|0>)+(|↑>|0>-|↓>|-2>))------------(b)

→1/(2√2)((|↑>|3>+|↓>|1>)+(|↑>|1>-|↓>|-1>)+(|↑>|1>+|↓>|-1>)-(|↑>|-1>-|↓>|-3>))-(c)

=1/(2√2)(|↑>|3>+2|↑>|1>+|↓>|1>-|↑>|-1>-|↓>|-3>)-----(d)

(a)represents the result after the first step.(b) represents the result after the second step and (c) represents the result after the third step.

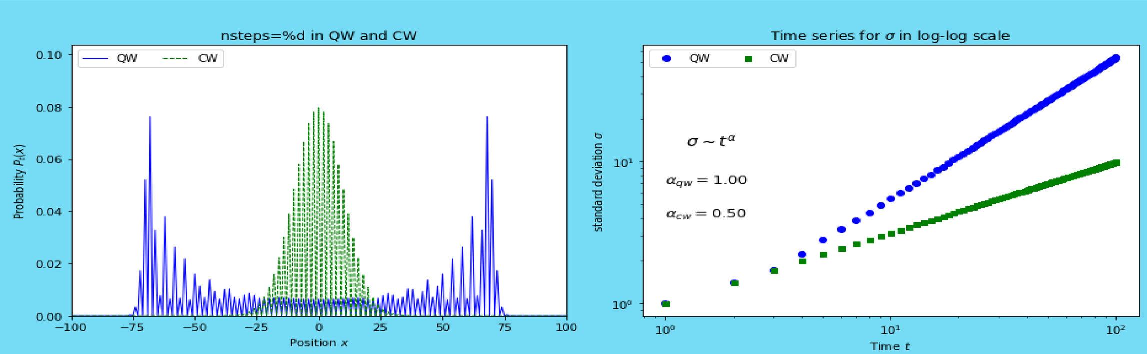
It can be seen from the above formula that the last position at 1 may be 2|↑>|1>+|↓>|1>, and the position at -1 may be -|↑>|-1>, so it appears in The probability of 1 is 4/8+1/8=5/8, and the probability of appearing at -1 is 1/8. These various reasons can be known, because the way to go to 1 and -1 is offset is different, which is called quantum destructiveness.

2.Why must we use the coin which have modify initial state?

$$\frac{1}{\sqrt{2}}(|\uparrow> +i|\downarrow>)|H> \otimes |0>$$

In order to avoid the occurrence of quantum cancellation, the initial value of the H gate coin is changed so that the coin does not have the opposite phase characteristic, thereby eliminating the possibility of quantum signal cancellation, as shown in the above formula

3. Different between classical and quantum result



The distribution of quantum random walk shows a tendency to spread to both sides, while the classical random walk shows a bell-shaped distribution.

Quantum probabilities have coherent superposition, so probability waves can destructively interfere near the origin, resulting in probability density distribution in a large range. In quantum random walking, since coins are quantized particles, they follow the superposition of quantization. Therefore, Walker can walk in two directions at the same time with a certain probability.

4. Comparison between the simulator and the actual quantum computer

Although the one-dimensional quantum random walk can successfully show the symmetrical characteristics of quantum random walk when executed on the simulator, after executing it on the actual quantum computer, not only the initial value of H coin is changed, but the basic H gate does not show quantum randomness. The distribution characteristics of walking are the same for two-dimensional quantum random walking. When using an actual quantum computer, the distribution of quantum random walking cannot be seen from the data.

It is speculated that the error value of the quantum computer is relatively large (the main error comes from the initial state, each operation, and the measurement method, and the main mathematical [physical] form is qubit flip (quantum decoherence、Quantum phase flip、Dephasing phase decoherence). Therefore, the results of the quantum algorithm are still unable to be presented, and there will be no good distribution.

Conclusion

a. Quantum random walk with excellent symmetry.
b. Bacause of the phase problem, H gate cannot be designed as a quantum dice.
c. Two-dimensional quantum random walk can also implement the symmetry distribution.
d. QW is diffused to both sides and the CW is a bell-shaped distribution.
e. Quantum have coherent superposition.probability waves can destructively interfere
f. The diffusion rate of QW is square times of CW
g. We successfully demonstrated quantum advantage

Qiskit

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