### NodeFormer: A Scalable Graph Structure Learning Transformer for Node Classification











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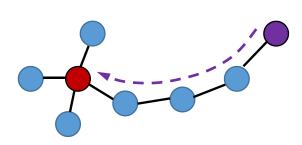


## Pitfalls of Graph Neural Networks

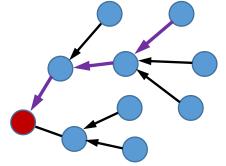
### □ The designs of GNN models:

- Locally aggregate neighbored nodes' features in each layer
- Use other nodes' information for prediction on the target node

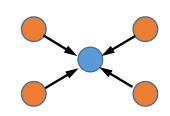
#### □ Common scenarios GNNs show deficient power:



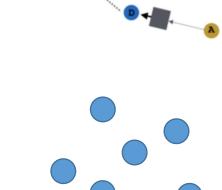
hard to capture longrange dependence [Dai et al., 2018]



distance signals are overly squashed [Alon et al., 2021]



dissimilar linked nodes propagate wrong signals [Zhu et al., 2020]



Layer-2

Laver-

 $\mathbf{X}_A$ 

fail to work without input graphs

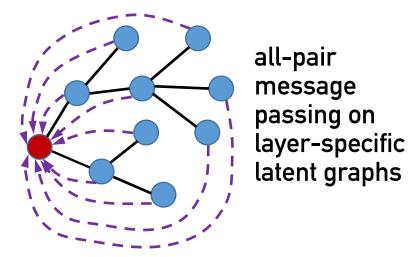
## Message Passing Beyond Input Graphs

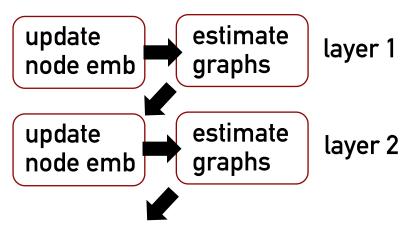
#### □ Basic idea:

- Learn optimal graphs for message passing
- Use node embeddings to learn latent graphs, and use latent graphs for updating node embeddings

### □ Key challenges:

- Scalability: quadratic complexity of all-pair message passing that requires  $\mathcal{O}(N^2)$  complexity (N for #node)
- *Differentiability:* learning discrete structures introduces non-differentiability for gradient-based optimization





## Comparison with Existing Works

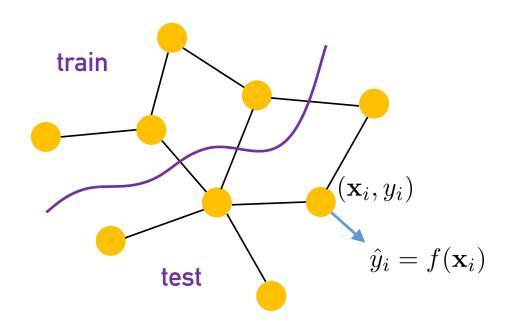
### □ Graph structure learning

- Need quadratic complexity and hard to scale to large graphs (e.g., 10K)
  - Our model has linear complexity with largest demonstration on 2M nodes
- Only learn a single graph shared by all propagation layers
  - Our model considers layer-wise graph learning for adaptive propagation
- Use complicated optimization algorithms, e.g. bi-level optimization
  - Our model enables efficient end-to-end training

### □ Transformer models on Graphs

- Current methods focus on graph classification (a dataset of small graphs)
  - Our model aims at node classification (a dataset of inter-connected nodes)

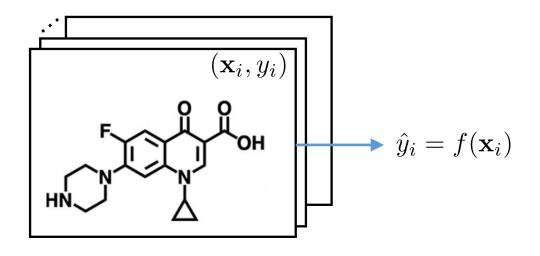
## Two Problems on Graph Data



#### Node-Level Prediction/Classification (our focus)

- > Each node is an instance with a label
- > Train/test on a dataset of nodes in a graph
- > The graph is often large (1K-100M nodes)

scalability issue



#### **Graph-Level Prediction/Classification**

- > Each graph is an instance with a label
- > Train/test on a dataset of graphs
- > The graphs are often small (e.g., 10-100 nodes)

### □ Kernelized softmax message passing

$$\tilde{a}_{uv}^{(l)} = \frac{\exp((W_Q^{(l)} \mathbf{z}_u^{(l)})^\top (W_K^{(l)} \mathbf{z}_v^{(l)}))}{\sum_{w=1}^N \exp((W_Q^{(l)} \mathbf{z}_u^{(l)})^\top (W_K^{(l)} \mathbf{z}_w^{(l)}))}, \quad \mathbf{z}_u^{(l+1)} = \sum_{v=1}^N \tilde{a}_{uv}^{(l)} \cdot (W_V^{(l)} \mathbf{z}_v^{(l)})$$
$$\mathbf{z}_u^{(l+1)} = \sum_{v=1}^N \frac{\kappa(W_Q^{(l)} \mathbf{z}_u^{(l)}, W_K^{(l)} \mathbf{z}_v^{(l)})}{\sum_{w=1}^N \kappa(W_Q^{(l)} \mathbf{z}_u^{(l)}, W_K^{(l)} \mathbf{z}_w^{(l)})} \cdot (W_V^{(l)} \mathbf{z}_v^{(l)})$$

 $\kappa(\cdot,\cdot):\mathbb{R}^d imes\mathbb{R}^d o\mathbb{R}$  is a positive-definite kernel

[Mercer's theorem]  $\kappa(\mathbf{a}, \mathbf{b}) = \langle \Phi(\mathbf{a}), \Phi(\mathbf{b}) \rangle_{\mathcal{V}} \approx \phi(\mathbf{a})^{\top} \phi(\mathbf{b})$  $\phi(\cdot) : \mathbb{R}^d \to \mathbb{R}^m$  is a random feature map

$$\mathbf{z}_{u}^{(l+1)} = \sum_{v=1}^{N} \frac{\phi(\mathbf{q}_{u})^{\top} \phi(\mathbf{k}_{v})}{\sum_{w=1}^{N} \phi(\mathbf{q}_{u})^{\top} \phi(\mathbf{k}_{w})} \cdot \mathbf{v}_{v} = \frac{\phi(\mathbf{q}_{u})^{\top} \sum_{v=1}^{N} \phi(\mathbf{k}_{v}) \cdot \mathbf{v}_{v}}{\phi(\mathbf{q}_{u})^{\top} \sum_{w=1}^{N} \phi(\mathbf{k}_{w})}$$

only require O(N) compute the sum at once

#### □ Kernelized Gumbel-Softmax

$$\mathbf{z}_{u}^{(l+1)} = \sum_{v=1}^{N} \frac{\exp((\mathbf{q}_{u}^{\top} \mathbf{k}_{u} + g_{v})/\tau)}{\sum_{w=1}^{N} \exp((\mathbf{q}_{u}^{\top} \mathbf{k}_{w} + g_{w})/\tau)} \cdot \mathbf{v}_{u}$$

$$= \sum_{v=1}^{N} \frac{\kappa(\mathbf{q}_{u}/\sqrt{\tau}, \mathbf{k}_{v}/\sqrt{\tau})e^{g_{v}/\tau}}{\sum_{w=1}^{N} \kappa(\mathbf{q}_{u}/\sqrt{\tau}, \mathbf{k}_{w}/\sqrt{\tau})e^{g_{w}/\tau}} \cdot \mathbf{v}_{v}$$

$$\approx \sum_{v=1}^{N} \frac{\phi(\mathbf{q}_{u}/\sqrt{\tau})^{\top} \phi(\mathbf{k}_{v}/\sqrt{\tau})e^{g_{v}/\tau}}{\sum_{w=1}^{N} \phi(\mathbf{q}_{u}/\sqrt{\tau})^{\top} \phi(\mathbf{k}_{w}/\sqrt{\tau})e^{g_{w}/\tau}} \cdot \mathbf{v}_{v}$$

$$= \frac{\phi(\mathbf{q}_{u}/\sqrt{\tau})^{\top} \sum_{v=1}^{N} e^{g_{v}/\tau} \phi(\mathbf{k}_{v}/\sqrt{\tau}) \cdot \mathbf{v}_{v}}{\phi(\mathbf{q}_{u}/\sqrt{\tau})^{\top} \sum_{w=1}^{N} e^{g_{w}/\tau} \phi(\mathbf{k}_{w}/\sqrt{\tau})}$$

approximate sampling discrete edges from a potential, large graph that connects all nodes

## Approximation Error and Concentration

#### Theorem 1 (Approximation Error for Softmax-Kernel)

Assume  $\|\mathbf{q}_u\|_2$  and  $\|\mathbf{k}_v\|_2$  are bounded by r, and  $\phi$  the Positive Random Features, then with probability at least  $1-\epsilon$ , the approximation error gap will be bounded by

$$\Delta = \left| \phi(\mathbf{q}_u / \sqrt{\tau})^\top \phi(\mathbf{k}_v / \sqrt{\tau}) - \kappa(\mathbf{q}_u / \sqrt{\tau}, \mathbf{k}_v / \sqrt{\tau}) \right| \leq \left[ \mathcal{O}\left(\sqrt{\frac{\exp(6r/\tau)}{m\epsilon}}\right) \right]$$

m for random feature dimension,  $\boldsymbol{\tau}$  for temperature the error is independent of node number N

#### Theorem 2 (Concentration of Kernelized Gumbel-Softmax Random Variables)

Suppose the random feature dimension m is sufficiently large, we have the convergence property for the kernelized Gumbel-Softmax operator

$$\lim_{\tau \to 0} \mathbb{P}(c_{uv} > c_{uv'}, \forall v' \neq v) = \frac{\exp(\mathbf{q}_u^\top \mathbf{k}_v)}{\sum_{w=1}^N \exp(\mathbf{q}_u^\top \mathbf{k}_w)}, \quad \lim_{\tau \to 0} \mathbb{P}(c_{uv} = 1) = \frac{\exp(\mathbf{q}_u^\top \mathbf{k}_v)}{\sum_{w=1}^N \exp(\mathbf{q}_u^\top \mathbf{k}_w)}$$

The sampled results converge to the ones induced by the Softmax categorical distribution

**Algorithm 1:** Scalable All-Pair Message Passing on Latent Graphs with Linear Complexity  $(\mathcal{O}(N))$  or  $\mathcal{O}(N+E)$ 

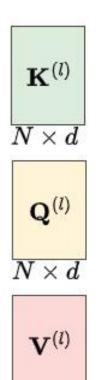
```
Input: Node features \mathbf{Z}^{(0)}=\mathbf{X}, input adjacency \mathbf{A}.

1 for l=0\ldots,L-1 do

2 \mathbf{Q}^{(l)}\leftarrow W_Q^{(l)}\mathbf{Z}^{(l)},\mathbf{K}^{(l)}\leftarrow W_K^{(l)}\mathbf{Z}^{(l)},\mathbf{V}^{(l)}\leftarrow W_V^{(l)}\mathbf{Z}^{(l)};

3 4
5 6
7 8
```

**Output:** Predict node labels  $\hat{\mathbf{Y}} = \text{MLP}(\{\mathbf{Z}^{(l)}\}_{l=0}^{L})$ .



```
Algorithm 1: Scalable All-Pair Message Passing on Latent
   Graphs with Linear Complexity (\mathcal{O}(N) \text{ or } \mathcal{O}(N+E))
    Input: Node features \mathbf{Z}^{(0)} = \mathbf{X}, input adjacency \mathbf{A}.
1 for l = 0..., L-1 do
           \mathbf{Q}^{(l)} \leftarrow W_O^{(l)} \mathbf{Z}^{(l)}, \mathbf{K}^{(l)} \leftarrow W_K^{(l)} \mathbf{Z}^{(l)}, \mathbf{V}^{(l)} \leftarrow W_V^{(l)} \mathbf{Z}^{(l)};
                                                                                                                                N 	imes d
                                                                                                                                                            N \times m
          for k = 1, 2, ..., K do G_k = \{e^{g_{ku}/\tau}\}_{u=1}^N, \ g_{ku} \sim Gumbel(0, 1);
                 \tilde{G}_k = G_k.unsqueeze(1).repeat(1, m);
                                                                                                                                   \mathbf{Q}^{(l)}
                 	ilde{\mathbf{K}}_k^{(l)} = 	ilde{G}_k \odot \phi(\mathbf{K}^{(l)}/\sqrt{	au}), \, 	ilde{\mathbf{Q}}_k^{(l)} = 	ilde{G}_k \odot \phi(\mathbf{Q}^{(l)}/\sqrt{	au});
                                                                                                                                N 	imes d
                                                                                                                                                            N \times m
8
                                                                                                                                   \mathbf{V}^{(l)}
9
    Output: Predict node labels \hat{\mathbf{Y}} = \text{MLP}(\{\mathbf{Z}^{(l)}\}_{l=0}^L).
```

Algorithm 1: Scalable All-Pair Message Passing on Latent Graphs with Linear Complexity  $(\mathcal{O}(N))$  or  $\mathcal{O}(N+E)$ 

```
Input: Node features \mathbf{Z}^{(0)} = \mathbf{X}, input adjacency \mathbf{A}.

1 for l = 0 \dots, L - 1 do

2 \mathbf{Q}^{(l)} \leftarrow W_Q^{(l)} \mathbf{Z}^{(l)}, \mathbf{K}^{(l)} \leftarrow W_K^{(l)} \mathbf{Z}^{(l)}, \mathbf{V}^{(l)} \leftarrow W_V^{(l)} \mathbf{Z}^{(l)};

3 for k = 1, 2, \dots, K do

4 G_k = \{e^{g_{ku}/\tau}\}_{u=1}^N, \ g_{ku} \sim Gumbel(0, 1);

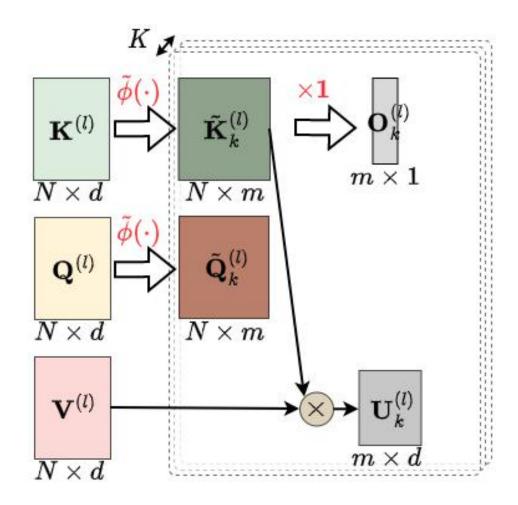
5 \tilde{G}_k = G_k.unsqueeze(1).repeat(1, m);

6 \tilde{\mathbf{K}}_k^{(l)} = \tilde{G}_k \odot \phi(\mathbf{K}^{(l)}/\sqrt{\tau}), \tilde{\mathbf{Q}}_k^{(l)} = \tilde{G}_k \odot \phi(\mathbf{Q}^{(l)}/\sqrt{\tau});

7 \mathbf{U}_k^{(l)} \leftarrow (\tilde{\mathbf{K}}_k^{(l)})^{\top} \mathbf{V}^{(l)}, \mathbf{O}_k^{(l)} \leftarrow (\tilde{\mathbf{K}}_k^{(l)})^{\top} \mathbf{1}_{N \times 1};

8 9
```

**Output:** Predict node labels  $\hat{\mathbf{Y}} = \text{MLP}(\{\mathbf{Z}^{(l)}\}_{l=0}^L)$ .



**Algorithm 1:** Scalable All-Pair Message Passing on Latent Graphs with Linear Complexity  $(\mathcal{O}(N))$  or  $\mathcal{O}(N+E)$ 

```
Input: Node features \mathbf{Z}^{(0)} = \mathbf{X}, input adjacency \mathbf{A}.

1 for l = 0 \dots, L - 1 do

2 \mathbf{Q}^{(l)} \leftarrow W_Q^{(l)} \mathbf{Z}^{(l)}, \mathbf{K}^{(l)} \leftarrow W_K^{(l)} \mathbf{Z}^{(l)}, \mathbf{V}^{(l)} \leftarrow W_Q^{(l)} \mathbf{Z}^{(l)};

3 for k = 1, 2, \dots, K do

4 G_k = \{e^{g_{ku}/\tau}\}_{u=1}^N, \ g_{ku} \sim Gumbel(0, 1);

5 \tilde{G}_k = G_k.unsqueeze(1).repeat(1, m);

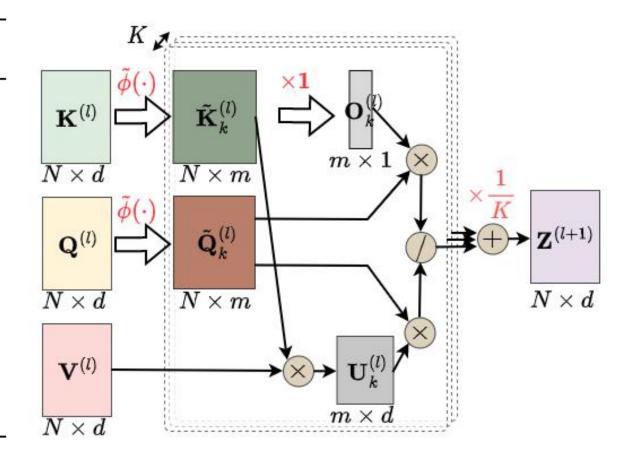
6 \tilde{\mathbf{K}}_k^{(l)} = \tilde{G}_k \odot \phi(\mathbf{K}^{(l)}/\sqrt{\tau}), \ \tilde{\mathbf{Q}}_k^{(l)} = \tilde{G}_k \odot \phi(\mathbf{Q}^{(l)}/\sqrt{\tau});

7 \mathbf{U}_k^{(l)} \leftarrow (\tilde{\mathbf{K}}_k^{(l)})^{\top} \mathbf{V}^{(l)}, \mathbf{O}_k^{(l)} \leftarrow (\tilde{\mathbf{K}}_k^{(l)})^{\top} \mathbf{1}_{N \times 1};

8 \mathbf{Z}^{(l+1)} \leftarrow \frac{1}{K} \sum_{k=1}^K \frac{\tilde{\mathbf{Q}}_k^{(l)} \mathbf{U}_k^{(l)}}{\tilde{\mathbf{Q}}_k^{(l)} \mathbf{O}_k^{(l)}}; \% \text{ average } \mathbf{K} \text{ samples}

9
```

**Output:** Predict node labels  $\hat{\mathbf{Y}} = \text{MLP}(\{\mathbf{Z}^{(l)}\}_{l=0}^L)$ .



**Algorithm 1:** Scalable All-Pair Message Passing on Latent Graphs with Linear Complexity  $(\mathcal{O}(N) \text{ or } \mathcal{O}(N+E))$ **Input:** Node features  $\mathbf{Z}^{(0)} = \mathbf{X}$ , input adjacency  $\mathbf{A}$ .  $ilde{\mathbf{K}}_k^{(l)}$ 1 **for** l = 0..., L-1 **do**  $\mathbf{Q}^{(l)} \leftarrow W_{Q}^{(l)} \mathbf{Z}^{(l)}, \mathbf{K}^{(l)} \leftarrow W_{K}^{(l)} \mathbf{Z}^{(l)}, \mathbf{V}^{(l)} \leftarrow W_{V}^{(l)} \mathbf{Z}^{(l)};$ for k = 1, 2, ..., K do  $| G_k = \{e^{g_{ku}/\tau}\}_{u=1}^N, g_{ku} \sim Gumbel(0, 1);$  $\overline{N imes d}$  $\overline{N imes m}$  $\tilde{G}_k = G_k.unsqueeze(1).repeat(1, m);$  $ilde{\mathbf{Q}}_k^{(l)}$  $\mathbf{Q}^{(l)}$  $ilde{\mathbf{K}}_k^{(l)} = ilde{G}_k \odot \phi(\mathbf{K}^{(l)}/\sqrt{ au}), \, ilde{\mathbf{Q}}_k^{(l)} = ilde{G}_k \odot \phi(\mathbf{Q}^{(l)}/\sqrt{ au});$  $\mathbf{U}_{k}^{(t)} \leftarrow (\mathbf{K}_{k}^{(t)})^{\top} \mathbf{V}^{(t)}, \mathbf{O}_{k}^{(t)} \leftarrow (\mathbf{K}_{k}^{(t)})^{\top} \mathbf{1}_{N \times 1};$  $\overline{N imes d}$ N imes dN imes m $\mathbf{Z}^{(l+1)} \leftarrow \frac{1}{K} \sum_{k=1}^{K} \frac{\tilde{\mathbf{Q}}_{k}^{(l)} \mathbf{U}_{k}^{(l)}}{\tilde{\mathbf{Q}}^{(l)} \mathbf{Q}^{(l)}}; \% \text{ average K samples}$ 8  $\mathbf{V}^{(l)}$  $\mathbf{Z}^{(l+1)} \leftarrow \mathbf{Z}^{(l+1)} + \sigma(b^{(l)}) \cdot \mathbf{A} \mathbf{Z}^{(l+1)}$ ; % add relational bias **Output:** Predict node labels  $\hat{\mathbf{Y}} = \text{MLP}(\{\mathbf{Z}^{(l)}\}_{l=0}^L)$ .

leverage the input graph as relational bias (reinforce the weights for observed edges)

## Training Objective

### □ Supervised classification loss

$$\mathcal{L}_s(\mathbf{Y}, \hat{\mathbf{Y}}) = -\frac{1}{N} \sum_{v=1}^{N} \sum_{c=1}^{C} \mathbb{I}[y_u = c] \log \hat{y}_{u,c}$$

### □ Edge-level regularization loss

$$\mathcal{L}_e(\mathbf{A}, \tilde{\mathbf{A}}) = -\frac{1}{NL} \sum_{l=1}^{L} \sum_{(u,v) \in \mathcal{E}} \frac{1}{d_u} \log \pi_{uv}^{(l)}$$

$$\pi_{uv}^{(l)} = \frac{\phi(W_Q^{(l)} \mathbf{z}_u^{(l)})^\top \phi(W_K^{(l)} \mathbf{z}_v^{(l)})}{\phi(W_Q^{(l)} \mathbf{z}_u^{(l)})^\top \sum_{w=1}^N \phi(W_K^{(l)} \mathbf{z}_w^{(l)})}$$

#### □ Final loss function

$$\mathcal{L} = \mathcal{L}_s + \lambda \mathcal{L}_e$$

Key observation:

# labeled nodes  $< N << N^2 =$  # node pairs

The log-likelihood of observed edges, if assuming data distribution as

$$p_0(v|u) = \left\{ egin{array}{l} rac{1}{d_u}, & a_{uv} = 1 \\ 0, & otherwise. \end{array} 
ight.$$

#### only require O(E)

Since we only need to query the probability for each observed edges, where the complexity of each query is  $\mathcal{O}(1)$ 

## Dissecting the Rationale of New Objective

### □ A variational perspective look at the training objective

#### Key insights:

Treat the latent structure estimation as a variational distribution  $p(\mathbf{Y}|\tilde{\mathbf{A}},\mathbf{X},\mathbf{A})$ 

The all-pair message passing module induces a predictive distribution  $q(\tilde{\mathbf{A}}|\mathbf{X},\mathbf{A})$ 

$$\mathcal{L}_e(\mathbf{A}, \tilde{\mathbf{A}}) = -\frac{1}{NL} \sum_{l=1}^{L} \sum_{(u,v) \in \mathcal{E}} \frac{1}{d_u} \log \pi_{uv}^{(l)}$$

$$\mathcal{L}_s(\mathbf{Y}, \hat{\mathbf{Y}}) = -\frac{1}{N} \sum_{v=1}^{N} \sum_{c=1}^{C} \mathbb{I}[y_u = c] \log \hat{y}_{u,c}$$

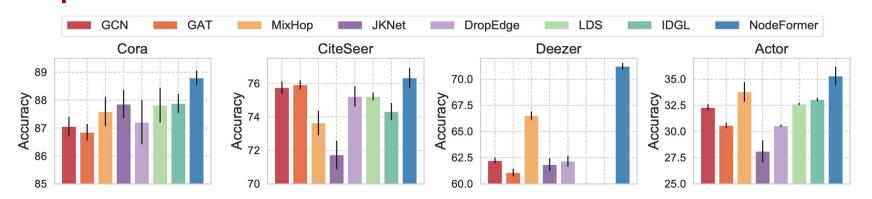
$$p^*, q^* = \arg\min_{p,q} \underbrace{-\mathbb{E}_q[\log p(\mathbf{Y}|\tilde{\mathbf{A}}, \mathbf{X}, \mathbf{A})]}_{\mathcal{L}_s} + \underbrace{\mathcal{D}(q(\tilde{\mathbf{A}}|\mathbf{X}, \mathbf{A})||p_0(\tilde{\mathbf{A}}|\mathbf{X}, \mathbf{A}))}_{\mathcal{L}_e}$$

#### **Proposition (Underlying Effect for Learning Optimal Structures)**

Assume q can exploit arbitrary distributions over  $\tilde{\mathbf{A}}$ . When the objective achieves the optimum, we have 1)  $\mathcal{D}(q(\tilde{\mathbf{A}}|\mathbf{X},\mathbf{A})||p(\tilde{\mathbf{A}}|\mathbf{Y},\mathbf{X},\mathbf{A})) = 0$ , and 2)  $\log p(\mathbf{Y}|\mathbf{X},\mathbf{A})$  is maximized.

## **Comparative Experiments**

#### □ Experiment on small node classification benchmarks



LDS [Franceschi et al., 2020]
IDGL [Chen et al., 2021]

### □ Experiment on large-scale datasets OGB-Proteins and Amazon2M

Method	Accuracy (%)	Train Mem
MLP GCN		1.4 GB 5.7 GB
SGC	$81.21 \pm 0.12$	1.7 GB
GraphSAINT-GCN GraphSAINT-GAT	$83.84 \pm 0.42 \\ 85.17 \pm 0.32$	2.1 GB 2.2 GB
NodeFormer NodeFormer-dt NodeFormer-tp	$87.85 \pm 0.24$ $87.02 \pm 0.75$ $87.55 \pm 0.11$	4.0 GB 2.9 GB 4.0 GB

NodeFormer successfully scales to graphs with 2M nodes

NodeFormer using batch size 0.1M only requires 4GB memory and hours for training on a single GPU

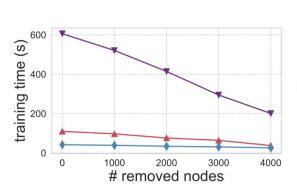
## **Comparative Experiments**

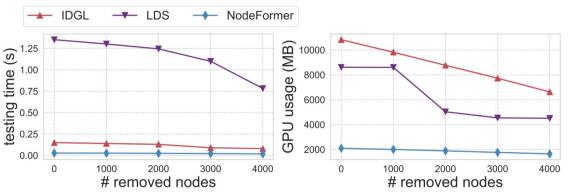
### □ Experiment on image/text classification (no input graph)

Method	Mini-ImageNet			20News-Group				
	k = 5	k = 10	k = 15	k = 20	k = 5	k = 10	k=15	k = 20
GCN	84.86 ± 0.42	$85.61 \pm 0.40$	$85.93 \pm 0.59$	$85.96 \pm 0.66$	$65.98 \pm 0.68$	$64.13 \pm 0.88$	$62.95 \pm 0.70$	$62.59 \pm 0.62$
GAT	$84.70 \pm 0.48$	$85.24 \pm 0.42$	$85.41 \pm 0.43$	$85.37 \pm 0.51$	$64.06 \pm 0.44$	$62.51 \pm 0.71$	$61.38 \pm \textbf{0.88}$	$60.80 \pm 0.59$
DropEdge	$83.91 \pm 0.24$	$85.35 \pm 0.44$	$85.25 \pm 0.63$	$85.81 \pm \textbf{0.65}$	$64.46 \pm 0.43$	$64.01 \pm 0.42$	$62.46 \pm 0.51$	$62.68 \pm 0.71$
IDGL	$83.63 \pm 0.32$	$84.41 \pm 0.35$	$85.50 \pm 0.24$	$85.66 \pm 0.42$	$65.09 \pm 1.23$	$63.41 \pm 1.26$	$61.57 \pm 0.52$	$62.21 \pm 0.79$
LDS	OOM	OOM	OOM	OOM	<b>66.15</b> ± 0.36	$64.70 \pm 1.07$	$63.51 \pm 0.64$	$63.51 \pm 1.75$
NodeFormer	<b>86.77</b> ± 0.45	<b>86.74</b> ± 0.23	<b>86.87</b> ± 0.41	<b>86.64</b> ± 0.42	66.01 ± 1.18	<b>65.21</b> ± 1.14	<b>64.69</b> ± 1.31	<b>64.55</b> ± 0.97
NODEFORMER w/o graph	<b>87.46</b> ± 0.36			<b>64.71</b> ± 1.33				

NodeFormer also works with no input graph

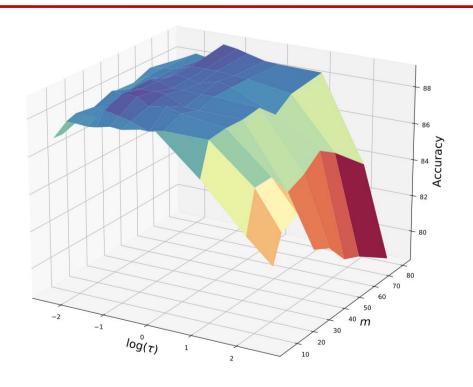
### □ Scalability analysis on time/space costs





NodeFormer reduces training time by 93.1%

## Ablation Study and Hyper-parameters



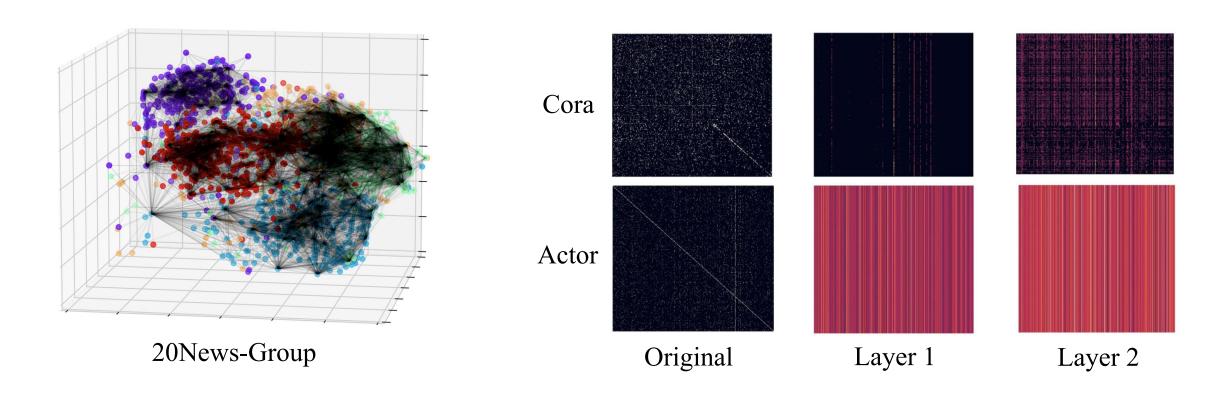
Larger random feature dimension m allows better approximation

Moderate temperature (tau=0.25) yields stably good performance

Dataset	NodeFormer	NodeFormer w/o reg	NodeFormer w/o rb
Cora	<b>88.69</b> ± 0.46	$81.98 \pm 0.46$	$88.06 \pm 0.59$
Citeseer	$76.33 \pm 0.59$	$70.60 \pm 1.20$	$74.12 \pm 0.64$
Deezer	$71.24 \pm 0.32$	$71.22 \pm 0.32$	$71.10 \pm 0.36$
Actor	<b>35.31</b> ± 1.29	$35.15 \pm 1.32$	$34.60 \pm 1.32$

Ablation study on edge regularization loss and relational bias

### Visualization of Learned Structures



The latent structures produced by NodeFormer tend to connect nodes within the same class and increase the overall connectivity of the whole graph

### Conclusion

#### □ Contributions of this work:

- Propose a kernelized Gumbel-Softmax operator that achieves all-pair message passing with linear complexity.
- Introduce NodeFormer as a new class of graph networks with layer-wise message passing as operated over latent graphs potentially connecting all nodes.
- Verify the efficacy of NodeFormer for handling long-range dependence,
   heterophily, large graphs, graph incompleteness and the absence of graphs.

Code will be available at: https://github.com/qitianwu/NodeFormer

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