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$$D = \frac{pv_1 t_1 + pv_2 t_2 + \cdots + pv_n t_n}{pv_1 + pv_2 + \cdots + pv_n} = \frac{\sum pv_i t_i}{\sum_1^n pv_i}$$

from one price theory

$$p = \sum_1^n pv_i$$

so

$$D = \frac{\sum_1^n pv_i t_i}{p}$$

for the later use

$$\sum_1^n pv_i t_i = Dp$$

accoding to definition

$$pv_i = \frac{c_i}{(1+r)^{t_i}}$$

so

$$\frac{\partial pv_i}{\partial r} = -\frac{t_i c_i}{(1+r)^{t_i+1}} = -\frac{c_i}{(1+r)^{t_i}} \times \frac{t_i}{1+r} = -\frac{t_i}{1+r} pv_i$$

since $p = \sum_1^n pv_i$

$$\frac{\partial p}{\partial r} = \frac{\partial \sum_1^n pv_i}{\partial r} = \sum_1^n \frac{\partial pv_i}{\partial r} = \sum_1^n -\frac{t_i}{1+r} pv_i = -\frac{1}{1+r} \sum_1^n t_i pv_i$$

and we know that

$$\sum_{1}^{n} pv_i t_i = Dp$$

so

$$\frac{\partial p}{\partial r} = -\frac{1}{1+r}\sum_{1}^{n} t_i pv_i = -\frac{1}{1+r}Dp$$

finally, we get

$$D = -\frac{\frac{\partial p}{p}}{\frac{\partial r}{1+r}} \approx -\frac{\frac{\Delta p}{p}}{\frac{\Delta r}{1+r}}$$