

# A multiple model filtering approach to transmission line fault diagnosis



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## ABSTRACT

This paper provides justification and implementation for a multiple model filtering approach to diagnosis of transmission line three-phase short to ground faults in the presence of protection misoperations. This approach utilizes the electric network dynamics and wide area measurements to provide diagnosis outcomes. A second focus of this paper is on the reduction of computational complexity of the diagnosis algorithm. This issue is addressed by a two-step heuristic. The first step designs subsystem models through measurement selection. The second step reduces the dynamic model order. The performance of the diagnosis algorithms are evaluated on a simulated WSCC 9-bus system.

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## 1. Introduction

The authors of this paper recently presented the motivation, the principle, and some preliminary results of a real-time diagnosis scheme for transmission faults implemented with multiple model filters (Wu & Qin, 2015). Here, diagnosis of a transmission fault is defined as detecting a fault in a system and identifying the faulty transmission line, in the presence of relay misoperations. The scheme has three unique features: use of the transmission network dynamics and time-stamped samples of the secondary-side waveforms of instrument transformers, and tolerance to relay misoperations. This paper provides justification and implementation for the multiple model filtering approach. This diagnosis approach is intended to assist the primary protection system, i.e. to isolate a fault correctly and create the opportunity to take corrective actions when the primary protection system fails to trip or falsely trips. In addition, a reduced order approach is proposed. To emphasize the need to maintain tolerance to protection misoperations in diagnosis, this paper starts with a brief review on published works by others related to the subject.

Protection system misoperation is one of the major contributors to cascading failures in power systems (Phadke & Thorp, 1996). North American Electric Reliability Corporation (NERC) (2013) considered it as the top-rated reliability issue in the recent state of reliability report. Protective relays are designed to trip circuit breakers when a fault is believed to be present. While

traditional relays are operating based on local measurements, relay misoperations are likely to occur when the system is under stress or disturbance. Such disturbance could be caused by topological changes due to faults, switching and maintenance, or power flow changes due to significant generation and load variations. The latter can also come from distributed generations. In general, there are two types of relay misoperations: failure to trip and false trip. Majority of the relay misoperations, which are caused by undetected defective settings, are false trips. Such defective settings are also referred as hidden failures (De La Ree, Liu, Mili, Phadke, & Dasilva, 2005).

Although backup relays are configured to operate when the primary relays failed to trip, the coordination of relay settings is increasingly complicated for modern power systems and it also suffers from hidden failures (Elizondo, De La Ree, Phadke, & Horowitz, 2001). Special Protection Scheme (SPS), or Remedial Action Scheme (RAS), uses wide area measurements to detect unusual system conditions and improve the stability of the system by various control actions such as tripping a line or generator (Adamiak et al., 2006; Anderson & LeReverend, 1996). While SPS has been widely used to increase the transfer capacity (McCalley & Fu, 1999), few studies have been done on correcting relay misoperations.

Various improved protection schemes have been proposed to deal with the relay misoperations. Adaptive relaying improves the reliability by integrating with the supervisory control and data acquisition (SCADA) system and central energy management system (EMS) (Horowitz, Phadke, & Thorp, 1988; Rockefeller, Wagner, Linders, Hicks, & Rizy, 1988). A few backup protection systems have been designed using wide area measurements. The backup expert decision system described by Tan, Crossley, Kirschen, Goody, and

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Downes (2000) processes the network topology and the relay operating response with an inference system. The agent-based backup protection system proposed by Wang et al. (2002) consists of rule-based controllers (agents) making decisions based on the local measurements and communications with other agents.

A self-healing protection system presented by Sheng, Li, Chan, Xiangjun, and Xianzhong (2006) combines the agent-based technique and expert system. With the idea of reducing communication burden, a hierarchical scheme was presented by He, Zhang, Chen, Malik, and Yin (2011). It first identifies the faulty area and then detects the fault by processing the voltages at both ends of each transmission line.

The deployment of Phasor measurement units (PMU) (Phadke & Thorp, 2008) enables the use of more accurate wide area measurements to detect faults and correct relay misoperations. Schweitzer, Whitehead, Zweigle, Ravikumar, and Rzepka (2010) discussed the benefit of using PMU data for real-time protection and control. Several methods are proposed for localized fault detection/location using synchronized PMU measurements (Brahma & Girgis, 2004; Dustegor, Poroseva, Hussaini, & Woodruff, 2010; Jiang, Yang, Lin, Liu, & Ma, 2000; Jiang, Lin, Yang, Too, & Liu, 2000). A relay-misoperation detection method using synchronized data from both ends of a transmission line is presented by Esmaeilian, Popovic, and Kezunovic (2015). Although these methods can obtain the exact location of a transmission line fault promptly, they are not practical as PMUs are required to be installed at both ends, or at least one end of every transmission line. However, according to the Department of Energy (2013), as of 2013, a total of 1126 PMUs were installed across the North America, which is far less than the number of transmission lines.

A few fault detection and identification schemes using sparse placement of PMU have been proposed for wide area backup protection. In Bo, Jiang, and Cao (2009), fault location is determined iteratively using genetic algorithm with the idea of superimposed impedance matrix during a fault. For large scale system the search space for this method could be very large. Eissa, Masoud, and Elanwar (2010) presented an application of PMU data in wide area back up protection. The proposed scheme divides the system into a few areas, each with PMU measurements. The voltage measurements and power flow directions are used to identify the fault area. However, to identify the fault on each transmission line, a large number of PMUs are still needed. Navalkar and Soman (2011) designed a remote fault detection scheme using the residual vector of the synchrophasor state estimator (Phadke, Thorp, & Karimi, 1986), and combined the fault detection result with backup relay to improve reliability. Based on the swing dynamics of generators, a maximum a posteriori (MAP) detector is developed by Valdez, Zhang, Torres, and Roy (2014) for fault location estimation. Since the time constants for swing dynamics are generally large, such dynamics are useful for applications with slower disturbances, such as load changes (Shames, Teixeira, Sandberg, & Johansson, 2011). They are, however, not suitable for diagnosing faults with critical clearing times of at most a few hundreds of milliseconds, such as transmission line faults.

Although the protection schemes mentioned above all addressed the relay's failure to trip, the false trip issue received little attention until recently (Wu & Qin, 2015). The authors have also performed hybrid simulations to assess the benefit of our diagnosis scheme (Qin & Wu, 2015). The efforts show that advancement of technology has made the implementation of real-time recovery from relay misoperations a possibility. In addition to detailing the improvement on multiple model filtering, and how multiple model method is applied to transmission fault diagnosis, this work also focuses on resolving the issue of computational complexity accompanying the inclusion of dynamic models of the transmission network. In this paper, faults are limited to transmission line three-phase short to ground faults, for

which the simple per phase model can be retained. Unsymmetrical faults, such as single phase short to ground, or phase to phase fault can be handle similarly by modeling the system with symmetrical components (Anderson & Fouad, 2002).

Time-stamped sampled measurements from the power system are used in the multiple model filtering process. Such measurements are also available as input data for PMUs.

An improved multiple model (MM) algorithm is designed for fault diagnosis in this paper. The MM algorithm was originally developed by Magill (1965). It is also referred as multiple model adaptive estimator (MMAE) (Maybeck & Hanlon, 1993). The measurement residuals of MM filtering can be used to calculate the likelihood for each model at which the system is operating. When each model represents a system operating mode, the associate likelihood is also referred as a mode probability. A few enhancements of the algorithm are discussed by Maybeck and Hanlon (1993). A numerically robust implementation of MM algorithm is presented by Li and Zhang (2000). For system with frequent mode jumps, the interacting MM algorithm (Blom, 1984) provides better performance by reinitializing the filter states based on a pre-defined transition matrix, which captures the prior knowledge about the system mode jumps. The variable structure MM algorithm introduced by Li and Bar-Shalom (1996) uses variable models instead of a fixed set of multiple models. The models are determined based on the knowledge of current system state. For the application of power system protection, since the system mode is generally not jumping frequently, the MM algorithm is used in this paper. Also, the state of each breaker is assumed to be known and used to construct the variable structure multiple model sets.

The remainder of this paper is organized as follows: Section 2 presented the multiple model algorithm for fault diagnosis. Section 3 discussed the communication and computational complexity. Section 4 proposed a reduced order multiple model algorithm. Section 5 presented a case study on the WSCC 9-bus system. Section 6 concludes this paper.

## 2. Multiple model fault diagnosis

The fault diagnosis is achieved by using a variable structure multiple model algorithm. The algorithm processes the measurement data with a set of Kalman filters (Welch & Bishop, 1995). Fault diagnosis decision is made based on the measurement residuals of the Kalman filters. Here, variable structure represents the fact that the filter set is variable. At different time instances, different filters are included in the filter set according to the partial knowledge on the system configuration, such as the circuit breaker states, at that time.

### 2.1. System modes and models

Definitions of mode and model similar to that by Li, Zhao, and Li (2005) are used to describe the multiple model approach. A *mode* refers to a physical status of a system. For the diagnosis purpose, a system with  $N$  transmission lines has  $N+1$  modes, i.e. a fault-free mode, including both pre-fault and post-fault configurations, and  $N$  faulty modes. Each faulty mode corresponds to a particular transmission line shorted to ground, regardless of the exact fault location. In general, multiple configurations exist for each mode.

During the operation of power system, the system switches among different modes occasionally, depending on the relay operations or fault occurrence. The fault diagnosis aims to correctly identify the system mode.

A *model* refers to a mathematical representation of a system mode under a certain configuration. System configuration is fixed

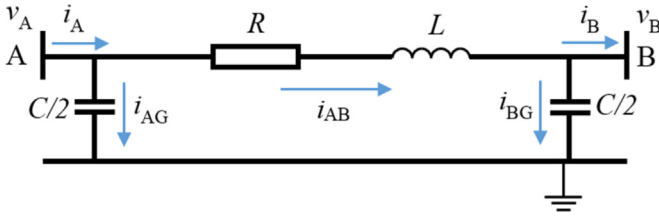


Fig. 1. Equivalent  $\pi$  circuit of a transmission line.

in a model. Generators are modeled as voltage sources behind their internal transient reactances and transmission lines are modeled as lumped equivalent  $\pi$  circuits. Loads in the system are modeled as constant impedances. Based on Kirchhoff's circuit law, the electric transmission network is modeled as an LTI system with currents of the equivalent independent inductors and voltages of the equivalent independent capacitors as states. The inputs to the system are generator voltages and the outputs of the system are the measured voltages and currents. In this paper, for each faulted model, the fault is modeled as a three-phase short to ground fault at a certain predetermined location. One or more models may be needed to represent a mode.

As an example, Fig. 1 shows the equivalent  $\pi$  circuit of a transmission line connecting two PQ buses.

There are 3 states for this circuit: the bus voltages  $v_A$ ,  $v_B$  at the two ends and the branch current  $i_{AB}$  going through the equivalent inductor. The measured currents at the two ends are  $i_A = i_{AG} + i_{AB}$  and  $i_B = i_{AB} - i_{BG}$ . The state equations are:

$$\dot{v}_A = \frac{2}{C}(i_A - i_{AB}) \quad (1)$$

$$\dot{v}_B = \frac{2}{C}(i_{AB} - i_B) \quad (2)$$

$$i_{AB} = \frac{1}{L}(v_A - v_B - Ri_{AB}) \quad (3)$$

Each transmission line in the network is modeled similarly. In an interconnected network,  $i_A$  and  $i_B$  in the state equations are replaced by linear combinations of some defined states based on Kirchhoff's current law.

The synchronized measurements sampled from voltage and current waveforms are used in this study instead of phasors. The output voltages of active elements, such as generators, are used as the input variables of the LTI system, whereas other measured current and voltage measurements are used as output variables of the LTI system.

In the multiple model filtering process, discrete-time models are used instead of continuous-time models. The continuous-time LTI system models are discretized using zero-order hold on the inputs at the selected sampling time  $kT$  of measurements, where  $T$  is a sufficiently small sample interval for protection purposes.  $T$  is to be suppressed in the subsequent model representation. The generic description of a discrete-time LTI system  $G_i$  can be written as:

$$\mathbf{x}_i(k+1) = \mathbf{A}_i\mathbf{x}_i(k) + \mathbf{B}_i\mathbf{u}(k) + \mathbf{w}(k) \quad (4)$$

$$\mathbf{y}_i(k+1) = \mathbf{C}_i\mathbf{x}_i(k) + \mathbf{v}(k) \quad (5)$$

where  $\mathbf{x}$  is the vector of state variables, i.e. the bus voltages and the currents of transmission lines.  $\mathbf{y}$  is the vector of measured currents and voltages.  $\mathbf{u}$  is the vector of active element output voltages, e.g. generator output voltage.  $\mathbf{A}_i$ ,  $\mathbf{B}_i$  and  $\mathbf{C}_i$  are constant matrices, where

subscript  $i$  identifies a particular configuration at a particular operating mode.  $\mathbf{w}$  and  $\mathbf{v}$  are process noise and measurement noise vectors, respectively. In the following discussion, it is assumed that  $\mathbf{w}$  and  $\mathbf{v}$  are independent white Gaussian noise with zero mean and diagonal constant covariance matrices  $\mathbf{Q}$  and  $\mathbf{R}$ , which are also assumed to be time-invariant. In Kalman filter applications,  $\mathbf{Q}$  and  $\mathbf{R}$  can be determined by tuning the filters off-line.

## 2.2. Modeling faults

In this paper, only three-phase short to ground faults on the transmission lines are studied to illustrate the proposed fault diagnosis principle for transmission short-circuit scenarios with the simplest models. In this case, each scenario of a three-phase solid short to ground at a predetermined location is modeled within the per phase setting. Note that a fault can be anywhere along a line for a particular faulty mode. Different faulty models may be used to represent a same faulty mode, and the purpose of diagnosis is to correctly identify the faulted line for removal.

The three phase shorted to ground fault considered in this paper is relatively rare. The majority of faults are unsymmetrical. A structural analysis method is proposed by Knüppel, Blanke, and Østergaard (2014) to detect common fault types. With the method presented in this paper, unsymmetrical faults, such as single phase short to ground or phase to phase short fault can be modeled using symmetrical components. Unsymmetrical faults, such as single phase short to ground or phase to phase short fault, can be modeled using symmetrical components. The application of symmetrical components in time domain to power system network calculation is described in Paap (2000). With symmetrical components, positive, negative and zero sequence networks are obtained for each unsymmetrical fault. Each of the sequence network models is handled similar to the per phase model studied in detail in this paper. While three phase to ground fault involves only the positive sequence network, a single phase short to ground fault is modeled by connecting the three sequence networks in series, a double phase short to ground fault is modeled by connecting the three sequence networks in parallel, and a phase to phase fault is modeled by connecting the positive sequence and negative sequence in parallel (Anderson & Fouad, 2002).

## 2.3. Multiple model filtering

Fig. 2 shows the structure of the multiple model algorithm with  $L$  models to identify  $N+1$  modes. The multiple model filtering algorithm is presented by Maybeck and Hanlon (1993) and Li and Zhang (2000). Given the inputs and outputs of the system, the goal is to determine which model, in a set of predefined models, best approximates the actual system configuration. Then the mode represented by the model is identified as the actual system mode. A Kalman filter is built for each model. The inputs and measurements of the system are applied to each Kalman filter built on its assumed model and the differences between the estimated outputs and actual outputs, i.e. the residual is processed to obtain the likelihood of a model. With the system described by (4) and (5), each iteration of the Kalman filter operates in two steps: time update and measurement update (Welch & Bishop, 1995). In time update, the system states and state error covariance matrix are predicted based on the system inputs and the assumed design model:

$$\mathbf{x}_i^-(k) = \mathbf{A}_i\mathbf{x}_i(k-1) + \mathbf{B}_i\mathbf{u}(k-1) \quad (6)$$

$$\mathbf{P}_i^-(k) = \mathbf{A}_i\mathbf{P}_i(k-1)\mathbf{A}_i^T + \mathbf{Q} \quad (7)$$

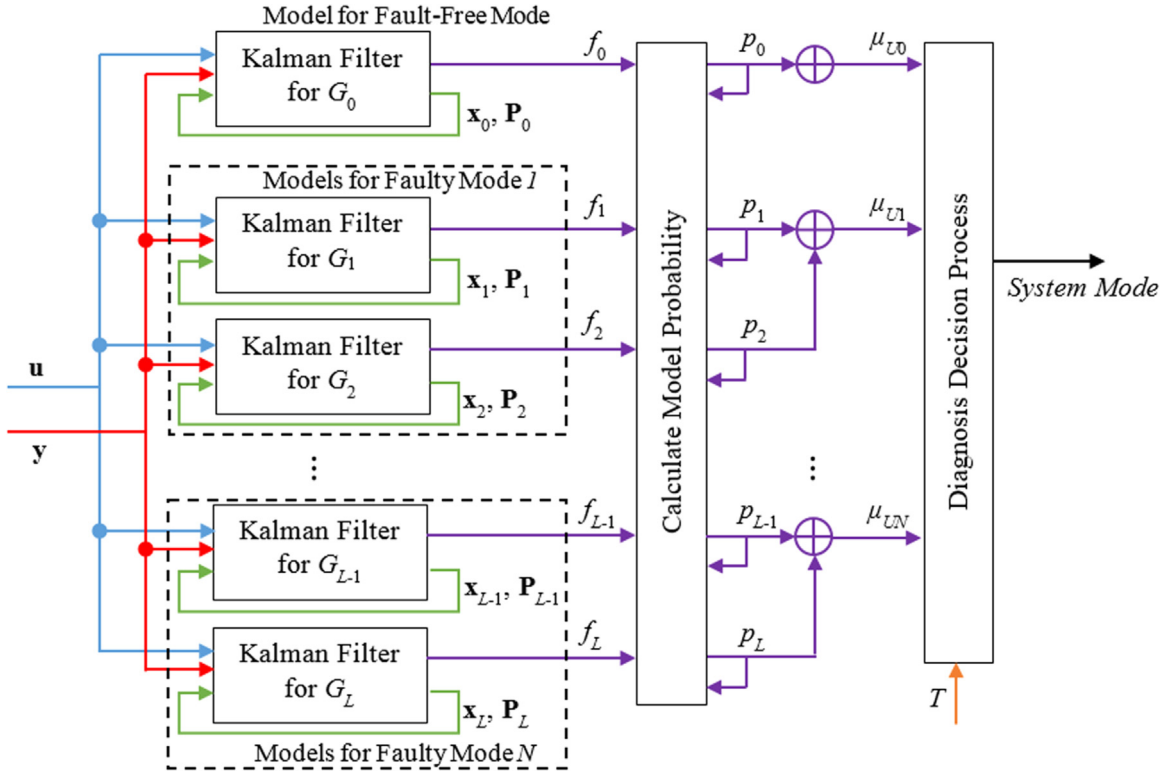


Fig. 2. Structure of multiple model algorithm.

In the measurement update, system states and state error covariance matrix are estimated by the system outputs:

$$\mathbf{x}_i(k) = \mathbf{x}_i^-(k) + \mathbf{K}_i(k)\mathbf{r}_i(k) \quad (8)$$

$$\mathbf{P}_i(k) = (\mathbf{I} - \mathbf{K}_i(k)\mathbf{C}_i)\mathbf{P}_i^-(k) \quad (9)$$

where  $\mathbf{K}$  is the Kalman gain,  $\mathbf{r}$  is the measurement residual, and  $\mathbf{S}$  is the residual covariance matrix:

$$\mathbf{K}_i(k) = \mathbf{P}_i^-(k)\mathbf{C}_i^T\mathbf{S}_i^{-1} \quad (10)$$

$$\mathbf{S}_i(k) = \mathbf{C}_i\mathbf{P}_i^-(k)\mathbf{C}_i^T + \mathbf{R} \quad (11)$$

$$\mathbf{r}_i(k) = \mathbf{y}(k) - \mathbf{C}_i\mathbf{x}_i^-(k) \quad (12)$$

When  $\mathbf{Q}$  and  $\mathbf{R}$  are time-invariant, both  $\mathbf{P}$  and  $\mathbf{K}$  will converge to constants quickly and can be pre-computed off-line (Grewal & Andrews, 1993; Maybeck & Hanlon, 1993; Welch & Bishop, 1995).

The residual  $\mathbf{r}$  is a multivariate random variable that follows Gaussian distribution (Maybeck, 1979). Therefore, given a residual and the corresponding covariance matrix, a probability  $f_i(k)$  can be obtained from the probability density function of normal distribution  $\mathbf{r}(k) \sim N(0, \mathbf{S}(k))$ . This probability indicates how likely the system inputs and outputs match those resulting from a particular model. The conditional probability that a model infers the actual system mode (conditioned on the measurement) is calculated by Maybeck and Hanlon (1993) and Li and Zhang (2000):

$$p_i(k) = P[G(k) = G_i | \mathbf{y} = \mathbf{y}_k] \quad (13)$$

$$p_i(k) = \frac{p_i(k-1)f_i(k)}{\sum_{j \in \Omega} p_j(k-1)f_j(k)} \quad (14)$$

where  $G(k)$  denote the actual system model at step  $k$ , and  $\Omega$  is the set of all models in the filter set. The initial values for  $\mathbf{x}$ ,  $\mathbf{P}$  and  $p_i$  are set based on some prior knowledge.

#### 2.4. Designing multiple model set

At least one model should be designed to represent the each system mode. Since a fault may occur anywhere along a transmission line, each mode space, except the normal mode space, is a continuum of models parameterized by the fault location. Robust diagnosis filters (Blanke, Kinnaert, Lunze, & Staroswiecki, 2006; Chen & Patton, 2012; Ding, 2008) can be designed to address the fault location uncertainty. In this paper, an alternative approach with multiple representative models is used as it is suitable to addressing our diagnostic objective that is to identify which piece of equipment should be removed in order for the system to enter a planned  $N-1$  secure mode commonly required of a power system. A representative model is a deterministic mathematical representation of a single model of the continuum. The problem of designing a finite model set based on prior knowledge through discretizing the spacial parameter is discussed by Li et al. (2005).

In the case of power system faults, if possible fault locations are uniformly distributed along the transmission lines, according to either minimum probability mismatch design or minimum distance design criterion described by Li et al. (2005), a reasonable model set is to use midpoint faulty models. Such a representative model set is used by Qin et al. (2015). With this model set, it is observed through simulations that the incorrect fault identification is likely to occur when the fault is near one end of the transmission line.

In this paper, an improved model set is designed. In addition to the fault-free model, for each faulty mode, two models with faults near the two ends of the line are used. Additional models could be used for long transmission lines. Let  $U_x \subset \Omega$  denote the model set used for mode  $x$ ,  $x = 0, 1, \dots, N$ , the mode probability is calculated as



$$\mu_{U_k}(k) = \sum_{i \in U_k} p_i(k) \quad (15)$$

### 2.5. Diagnosis decision process

The mode with highest mode probability is considered as the current system mode. To be conservative, a fault diagnosis decision can be made only if the faulty mode probability is higher than a predefined threshold  $\theta$  for 6 consecutive samples. In addition, to avoid fluctuations, mode probabilities are averaged over a number of samples (10 samples in the case study of this paper).

### 2.6. Variable structure multiple models

In this paper, it is assumed that the state of each circuit breaker is known and this information is incorporated in the models. Once a breaker operates, the models in the MM algorithm are also updated to reflect the changes. The possible combinations of breaker state are predetermined and all possible models are computed off-line. In the on-line process, active models are selected and included in the set  $\Omega$  based on the actual breaker state. This multiple model procedure is also referred to as variable structure multiple model algorithm (Li & Bar-Shalom, 1996).

## 3. Communication and computation issues

Using the wide area measurement data for fault diagnosis requires all measurement data to be transmitted and processed at a data center. According to Naduvathuparambil, Valenti, and Feliachi (2002), by using fiber-optic cables or digital microwave links, the associated delay would be 100–150 ms. Since the capacity of fiber-optic cables or digital microwave links are sufficiently large for practical application, this delay is generally not affected by the amount of measurement data.

In order for the fault diagnosis scheme to provide backup protection, the critical clearing time for the particular fault should be greater than the sum of communication and computation delay of the fault diagnosis process. A 20 cycle clearing time corresponds to about 300 ms. The computation delay depends on the computer performance and the scale of the power system. Computational complexity provides a measurement for the performance of the algorithm on large scale system.

Computational complexity in terms of the rough number of floating point operations (FLOPS) for each iteration of a standard Kalman filter implementation is  $O(n^3)$ , where  $n$  is the order of the system model. The  $n^3$  term is associated with the matrix inverse operation. However, since (7) and (9)–(11) can be pre-computed offline, the real-time computation for Kalman filter involves only (6), (8) and (12). Let  $n_s$ ,  $n_l$ , and  $n_o$  denote the number of states, inputs and outputs, respectively. Each scalar addition or multiplication counts as an operation. The total FLOPS count for (6), (8) and (12) are

$$\text{FLOPS}_6 = (2n_s - 1)n_s + (2n_l - 1)n_s + n_s = O(n_s^2 + n_l n_s)$$

$$\text{FLOPS}_8 = (2n_o - 1)n_s + n_s = O(n_s n_o)$$

$$\text{FLOPS}_{12} = (2n_s - 1)n_o + n_o = O(n_s n_o)$$

For  $L$  filters running concurrently, the FLOPS count for (14) is  $3L - 1$ . The time complexity for each iteration of the MM algorithm is

$$L \cdot O(n_s^2 + n_s n_l + n_s n_o) + 3L - 1 = O(L n_s (n_s + n_l + n_o)) \quad (16)$$

When the system is large and involves many transmission lines,  $L$  can grow as fast as  $n_s$ . Theoretically, the performance of MM

algorithm tends to be poor when  $L$  is too large or too small (Li & Bar-Shalom, 1996). To improve the performance of the MM algorithm,  $L$  should be limited to a relative small number. In addition, to improve the computational efficiency, the system model order ( $n_s$ ) should be also limited.

An intuitive approach is to replace the single set of  $n_s$ th order LTI system models in the MM algorithm by  $K$  sets of lower order subsystem models. Suppose  $L$  and  $n_s$  are limited to relatively small constants, the time complexity becomes

$$O(K L n_s (n_s + n_l + n_o)) = O(K (n_l + n_o)) \quad (17)$$

In the next section, the reduced-order MM algorithm is presented along with a heuristic to construct  $K$  sets of subsystem.

## 4. Reduced order multiple model algorithm

The structure of the reduced order approach with  $K$  sets of subsystems is shown in Fig. 3. The MM algorithm implemented on each set of subsystems has the same structure as the one shown in Fig. 2. A system level fault diagnosis is devised to combine all the mode identification results from MM algorithm implementations for each set of subsystems.

Partitioning the system physically requires detailed investigation of the system topology. In this paper, a novel two-step heuristic is proposed to obtain subsystem sets. This approach is based on the state space model of the system without using any topological information. In general, the first step creates several sets of subsystem models with different sets of observable states through sensor set selection. The second step removes the unobservable states and some weakly observable states so that each set of subsystem models is only used to diagnose a subset of faults.

It should be noted that this two-step procedure needs to be repeated for each case when a line is tripped (i.e., removed from the system), as well as the normal operating condition. Therefore, once a line is tripped, the  $K$  sets of subsystems are also updated to reflect the change.

### 4.1. Selection of measurements

In the first step,  $K$  subsets of system measurements are selected to create  $K$  sets of subsystems. The subsystems have the same inputs as the original system. The measurement selection is based on the sensors in the system. Each sensor may provide multiple channels of measurements. A set of subsystems is defined as a set of  $L$  models with the same measurements. Each set contains measurements from a small number of  $M$  sensors. Each subsystem has the same model order as the original system.

The concept of mode identifiability is now defined in order to describe the criteria for the selection of measurements. Let  $G_i$  denote a transfer function model representing mode  $i$ , as defined in Section 2.1. The mode is identifiable within a set of models  $\Omega$ , if  $G_i$  is significantly different from other models.

$$\forall j \in \Omega, j \neq i, \quad \exists \|G_i - G_j\| > \rho \quad (18)$$

where  $\rho$  is a predefined threshold.

In each set of models, only the models corresponding to the identifiable modes are retained. Let  $S$  denote the set of all modes that are identifiable by the set of system models with all measurements, and  $S_i$  denote the subset of modes that are identifiable in the  $i$ th set of subsystem models, the criteria for measurement selection are to satisfy

$$S_1 \cup S_2 \cup S_3 \cup \dots \cup S_K = S \quad (19)$$

Different criteria can be used to measure  $\|G_i - G_j\|$ , such as

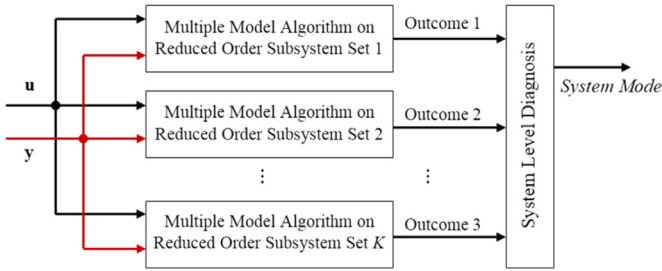


Fig. 3. Structure of reduced order multiple model algorithm.

$\mathcal{H}_2/\mathcal{H}_\infty$  norm (Zhou & Doyle, 1998), gap metric (Vinnicombe, 1992; Zames & El-Sakkary, 1980). Those criteria measure the distance between two systems over the entire frequency range. In the paper, each input signal is a generator output voltage, which has a frequency  $\omega$  close to the power system nominal frequency  $\omega_n$  (e.g. 60 Hz in the United States). Once a fault occurs,  $\omega$  will deviate from  $\omega_n$ . However, the deviation is generally insignificant. That is to say the nominal frequency components contain most energy of the input signals. Therefore, the frequency response of  $G_i$  at nominal frequency  $G(\omega_n)$  is used to quantify  $\|G_i - G_j\|$ . Specializing the norm in (18) to Frobenius norm yields,

$$\forall j \in \Omega, j \neq i, \quad \exists \|G_i(\omega_n) - G_j(\omega_n)\|_F > \rho \quad (20)$$

When all inputs are retained for the creation of each subsystem  $G_i$ , as long as a sufficient number of measurements are selected, (20) captures both magnitude and input and output signal direction information of nominal frequency response.

Fig. 4 shows an example of creating two sets of subsystems ( $G^{S1}$  and  $G^{S2}$ ) from a set of systems with 8 outputs and 3 models.  $G_1(\omega_n)$ ,  $G_2(\omega_n)$  and  $G_3(\omega_n)$  are the transfer functions of the system models with all 8 measurements.  $G_i^{S1}$  and  $G_i^{S2}$  ( $i = 1, 2, 3$ ) are the transfer functions of the subsystems with 4 measurements. Each  $g_j$  represents a row in the transfer function.

For a system with a total of  $H$  sensors, the procedure for selection is as follows:

1.  $M=0$ .
2.  $M = M + 1$ .
3. List all possible  $K = \binom{H}{M}$  combinations of sensors. Use each combination as a sensor set  $S_j$  to construct  $K$  sets of subsystems.
4. For each set, check the identifiability based on (20). Remove faulty models corresponding to modes that are not identifiable. (Note that a fault-free model is always kept in a set.)
5. If (19) is satisfied, stop the procedure and use the measurements from the selected sensors for each set. Otherwise, go to step 2.

With this procedure, some modes may be identifiable in more than one set of subsystems. Such redundancy is used to improve the reliability of fault diagnosis.

#### 4.2. Balanced model reduction

In the second step, balanced model reduction is applied on each subsystem so that the computational complexity is reduced. The balanced truncation model reduction method proposed by Moore (1981) is used here to obtain stable reduced order systems with a guaranteed error bound in the  $\mathcal{L}_\infty$  norm (Zhou & Doyle, 1998). Hankel singular values of a system are calculated as the square roots of the eigenvalues for the product of the controllability Gramian and the observability Gramian. Suppose that the Hankel singular values  $\sigma_k^H$  are ordered by  $\sigma_1^H > \sigma_2^H > \dots > \sigma_r^H > \dots > \sigma_n^H$ , if the system is reduced to  $r$ th order, the error bound is given as

$$\|G - G^r\|_\infty \leq 2 \sum_{k>r} \sigma_k^H \quad (21)$$

where  $G^r$  is the reduced order system and  $\sigma_k$  are the Hankel singular values corresponding to the truncated states.

While the measurement selection step obtains subsystems of the full system yet still captures the entire set of faults, the model reduction step requires the reduced order model to be sufficiently close to the corresponding full order model. In this paper, the order of the model  $r$  is selected such that the error bound in (21) is

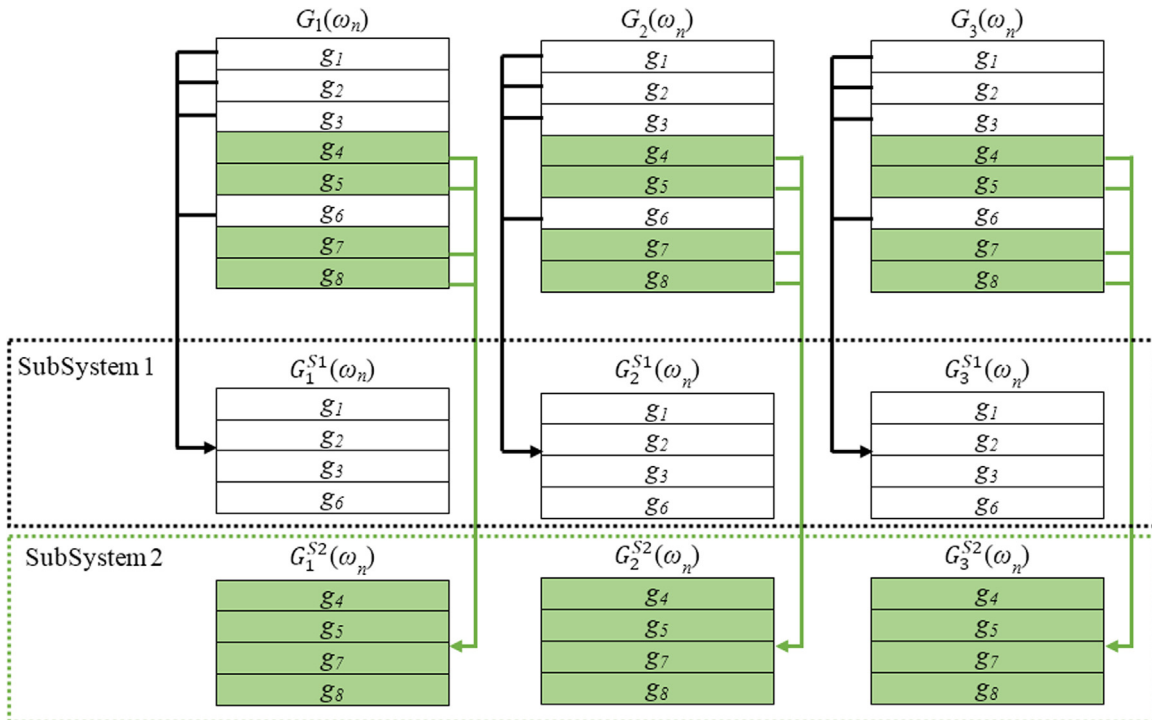


Fig. 4. Creating subsystems using rows from frequency response matrices.

less than a pre-defined percentage  $\beta$  of the maximum Hankel singular value  $\sigma_1^H$ :

$$2 \sum_{k>r} \sigma_k^H < \beta \sigma_1^H \quad (22)$$

#### 4.3. System level diagnosis

Among many choices, the following criterion for system level diagnosis is used in this paper. When a fault is identifiable in only one set of subsystems, the fault decision is made when its mode probability is higher than the threshold  $T$ . When a fault is identifiable in more than one set of subsystems, to be conservative, a fault decision is made only if the mode probabilities of the fault in all subsystems, where the fault is identifiable, are all higher than the threshold  $T$ .

### 5. Case study

The Western System Coordinating Council (WSCC) 3-machine, 9-bus power system (Anderson & Fouad, 2002), which includes 3 generators, 3 loads, 3 transformers and 6 transmission lines, is used for case study. Topology of the system is shown in Fig. 5. Details on the parameters of the system can be found in Anderson and Fouad (2002).

Three sensors are assumed to be available at Buses 4, 7 and 9. Each sensor provides 4 measurement outputs, i.e. the bus voltage and 3 currents of the 3 lines connected to the bus. Therefore, there are a total of 12 measurements. Generator output voltages are also assumed to be available. An estimation method is presented in Wu and Qin (2015) and Qin et al. (2015) to estimate the generator output voltages using the nearby measurements. All measurements are synchronized time domain samples. The sampling rate is 12 samples per AC cycle. The nominal frequency of the system is 60 Hz.

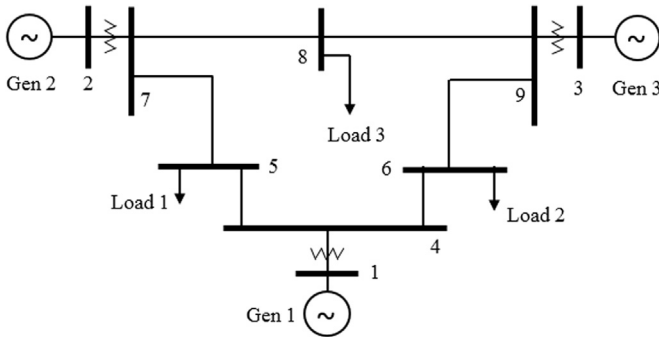


Fig. 5. The WSCC 9-Bus system.

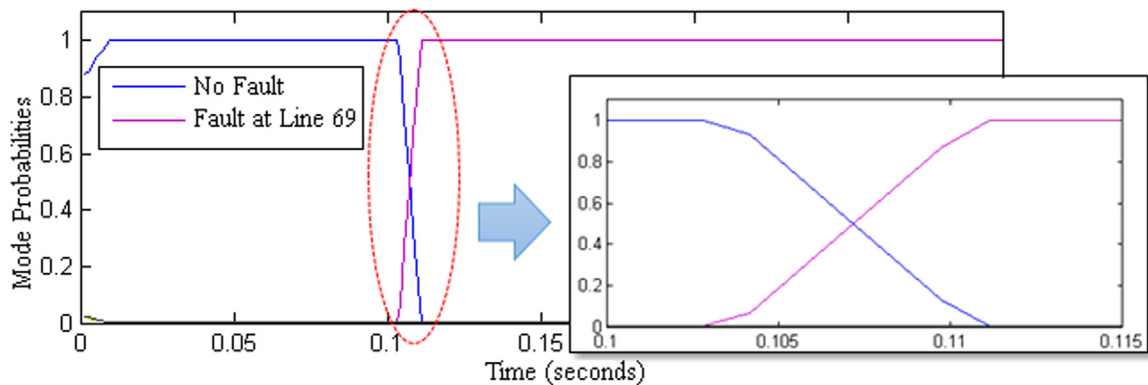


Fig. 6. Mode probabilities for full order MM algorithm.

Using the method described in Section 2, the system is modeled as a 18th order system for the normal operating condition. The states includes 6 voltages (buses 4–7), 3 load currents, 3 generator output currents and 6 transmission line currents. Voltages at buses 1–3 are generator output voltages. When a short to ground fault occurs, a branch breaks into two and an additional state is added to the system model. When a line is removed from the system, the corresponding state is also removed.

A MATLAB Simulink model, including both the generator dynamics and LTI transmission network, is built to simulate the 9-bus system. White Gaussian process noise and measurement noise are introduced into the system in Simulink. For each channel the noise power is 3% of the signal power when the system is operating normally. Two types of situations are simulated for fault diagnosis in the presence of relay misoperations: failure to trip (i.e. a fault occurs but breakers fails to trip), and false trip fault-on (i.e. a fault occurs at one line but another line is tripped, which is likely when the system is under stress). Each type of misoperation is simulated for 1000 runs with faults induced at random locations (uniformly distributed along 6 transmission lines) and random protection misoperation (line tripping, if applicable, uniformly distributed among 6 transmission lines). The full order approach and reduced order approach are applied for fault diagnosis, independently.

For the full order approach described in Section 2, 13 models are built in the MM algorithm, i.e. 1 fault-free model and 2 models for each of the 6 transmission line fault. In the 2 models for each transmission line fault, the fault locations are selected as 10% line length from each end of the line. Depending on whether there is a fault or line removal, the order of the models can be 17, 18, or 19. The threshold on mode probability for diagnosis decision is set as  $\theta=0.5$ .

Fig. 6 shows the mode probability  $\mu$  for a diagnosis process using full order approach. A fault occurs at  $t=0.1$  s at line 69. The fault location is around 0.75% of the line length from bus 6. The mode probability of fault on line 69 reached the threshold in less than 0.015 s (about 1 AC cycle).

For the reduced order approach described in Section 4. When no line is tripped, following the measurement selection procedure with a threshold of  $\rho=5$ ,  $M=1$  and measurements from each sensor forms a measurement set. A total of 3 sets of subsystems are created. Each subsystem has 4 measurements, including 1 bus voltage and 3 branch currents. With this design, each faulty mode is identifiable in two sets of subsystems, as shown in Table 1. Similar to the full order approach, for each set, 9 models are built, i.e. one fault-free model and two models for each of the 4 identifiable transmission line faults. In the model reduction process, the threshold  $\beta$  is set to 15%. As shown in Table 1, the model orders are reduced to 6, 7 and 4. When a line is tripped, the measurement selection and model reduction procedure need to be repeated to

**Table 1**  
Sets of subsystems in reduced order approach (No line tripped).

Set	Measurements	Identifiable line faults	Order
1	$v_4, i_{41}, i_{45}, i_{46}$	Line45, Line57, Line46, Line69	6
2	$v_7, i_{72}, i_{75}, i_{78}$	Line45, Line57, Line78, Line89	7
3	$v_9, i_{93}, i_{96}, i_{98}$	Line46, Line69, Line78, Line89	4

**Table 2**  
Sets of subsystems in reduced order approach (Line 57 tripped).

Set	Measurements	Identifiable line faults	Order
1	$v_4, i_{41}, i_{45}, i_{46}$	Line45, Line46, Line69	3
2	$v_7, i_{72}, i_{75}, i_{78}$	Line78, Line89	2
3	$v_9, i_{93}, i_{96}, i_{98}$	Line46, Line69, Line78, Line89	2

obtain different sets of subsystems. As a result, when any single line in the system is tripped, the measurement selection results are the same, however the identifiable faulty modes are different. As an example, the measurement selection and model reduction result for tripping line 57 is shown in Table 2.

The correct identification rate and average identification time for both fault diagnosis approaches in 1000 simulation runs are shown in Table 3.

According to the results in Chiang (2011), the critical clearing time for this 9-bus system is generally around 0.3 s. Considering that the communication and computational delay is around 0.1 s, most of the faults can be correctly diagnosed and relay misoperations can be mitigated using the approach presented in this paper. It is also observed that the decision time is reduced as anticipated when using the reduced order approach.

Further breakdowns of the diagnosis outcomes are shown in Table 4. It should be noted that most faults should be correctly identified by the existing primary protection system, and the rates in the table indicate how the approaches in this paper would improve the fault diagnosis. Here “Correct Action” is defined as identifying the faulty mode correctly despite the conventional protection misoperations, which is the same as it is in Table 3. Such correct actions contribute to correcting the relay misoperations. Assuming no correlation between the fault diagnosed by our approach and the primary protection system, over 90% of the

**Table 3**  
Fault diagnosis results in 1000 simulations.

Type	Correct decision rate (%)	Average decision time (s)
Full order approach		
Failure to trip	98.5	0.047
False trip fault-on	99.2	0.048
Reduced order approach		
Failure to trip	95.1	0.023
False trip fault-on	90.3	0.019

**Table 4**  
Fault decision breakdowns in 1000 simulations.

Type	Correct action (%)	No action (%)	Incorrect action (%)
Full order approach			
Failure to Trip	98.5	0.0	1.5
False trip fault-on	99.2	0.0	0.8
Reduced order approach			
Failure to trip	95.1	4.4	0.5
False trip fault-on	90.3	8.9	0.8

misoperations can be corrected. “No Action” refers to the cases that the fault diagnosis algorithm fails to diagnose a fault, i.e. identified a faulty system as fault-free. In such cases, the diagnosis does not help the primary protection system. Since most of these faulty cases are picked up by the primary protection system, no harm is done by the secondary protection. “Incorrect Action” refers to the case where our fault diagnosis algorithm draws a wrong diagnosis conclusions, such as misidentifying an intact line as the faulty line. Such outcomes could be harmful to the power system. Both “No Action” and “Incorrect Action” diagnosis outcomes can be attributed to the design methods used, number of models involved, and the specified thresholds for measurement set selection, for reduced model order, and for diagnosis decisions. The results show that, although the “Correct Action” rate is a bit lower in the reduced order approach, the “Incorrect Action” rate decreased for the failure to trip cases and increased only slightly (0.1%) for the false trip Fault-on cases. This is due to the conservative diagnosis decision rules in the reduced order approach.

## 6. Conclusions and future works

In this paper, multiple model approaches are proposed for fault diagnosis in the presence of relay misoperations. In the multiple model filtering, two or more models are used to represent a faulty mode in order to improve the accuracy. Simulation results show that the multiple model approaches can promptly diagnose the faults and mitigate relay misoperations. The fault diagnosis is robust in the sense that it can identify the faulty transmission line regardless of the fault location along the line. The reduced order approach makes this diagnosis method more scalable in terms of computational complexity by using several sets of lower order models instead of one set of higher order models of the entire electric network as the filters’ design models. Although the measurement selection and the model reduction in designing the reduced order models are based on the model characteristics at nominal frequency, the mode identification is based on the dynamics of the electric network. The identifiability criterion of reduced order approach is intended to guarantee that, in the worst case, the mode can be identified once the system enters steady-state. A more rigorous theoretical support is needed to justify the two-step design procedure. Further investigations should be made with regards to the determination of thresholds in the algorithms. This paper only investigated three phase short to ground faults on the transmission line. Other types of faults, such as single phase short to ground, can be diagnosed similar to appropriate models using symmetrical components.

The diagnosis approaches in this paper operate independent of the existing primary protection system. The coordination for the faults identified by our diagnosis approach and the existing protection system would be an interesting topic for future study. Such a study would also provide a better interpretation for the rates in Table 4.

In general, these approaches do not require dense sensor placement in the system. The basic requirement for all modes to be identifiable is that the electric network states should be observable with all the available sensors. Its relation to the requirements on mode identifiability would be an interesting topic for the future works. Even if the system is not observable, these approaches can still identify a subset of the faulty modes.

Communication and computational issues for these approaches are briefly discussed. In the case study of 9-bus system, noticed that for each set of subsystems, only local measurements (in one sensor) are needed to perform the mode identification. The computation can be done locally and only the mode identification results from different locations need to be combined to obtain a



fault decision. This indicates a potential for applying distributed computing for the fault diagnosis in the future.

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## Appendix A

### A.1. Model of the electric network for the 3-generator 9-bus system

The following is the electric network dynamics in state-space form for the 9-bus system at the pre-fault state. It is used as a design model for one of the multiple model filters used for fault diagnosis. During the normal operation, the system is of 18th order. The reader is referred to Fig. 2 in Wu and Qin (2015) for the definition of the circuit parameters and variables.

Let  $C_4 = 0.5(C_{45} + C_{46})$ ,  $C_5 = 0.5(C_{45} + C_{57})$ ,  $C_6 = 0.5(C_{69} + C_{46})$ ,  $C_7 = 0.5(C_{57} + C_{78})$ ,  $C_8 = 0.5(C_{78} + C_{89})$ , and  $C_9 = 0.5(C_{69} + C_{89})$ . The following are the 18 state equations of the linear electric network:

$$\begin{aligned} C_4 \dot{V}_4 &= -i_{41} - i_{45} - i_{46}, \quad C_5 \dot{V}_5 = -i_{5A} + i_{75} + i_{45}, \\ C_6 \dot{V}_6 &= -i_{6B} + i_{96} + i_{46}, \quad C_7 \dot{V}_7 = -i_{72} - i_{78} - i_{75}, \\ C_8 \dot{V}_8 &= -i_{8C} + i_{78} + i_{98}, \quad C_9 \dot{V}_9 = -i_{93} - i_{98} - i_{96}, \\ L_A \dot{i}_{5A} &= v_5 - R_A i_{5A}, \quad L_B \dot{i}_{6B} = v_6 - R_B i_{6B}, \\ L_C \dot{i}_{8C} &= v_8 - R_C i_{8C}, \quad L_1 \dot{i}_{41} = v_4 - e_1, \\ L_{45} \dot{i}_{45} &= v_4 - v_5 - R_{45} i_{45}, \quad L_{46} \dot{i}_{46} = v_4 - v_6 - R_{46} i_{46}, \\ L_2 \dot{i}_{72} &= v_7 - e_2, \quad L_{57} \dot{i}_{75} = v_7 - v_5 - R_{57} i_{75}, \\ L_{78} \dot{i}_{78} &= v_7 - v_8 - R_{78} i_{78}, \quad L_3 \dot{i}_{93} = v_9 - e_3, \\ L_{69} \dot{i}_{69} &= v_9 - v_6 - R_{69} i_{69}, \quad L_{89} \dot{i}_{98} = v_9 - v_8 - R_{89} i_{98}. \end{aligned}$$

$e_1(t)$ ,  $e_2(t)$ , and  $e_3(t)$  are the inputs to the linear network, which are considered known as they are quasi-steady-state sinusoidal signals.

The next set is the 11 output equations of the network, representing all measurements made by the 3 PMUs.  $y_1 = v_4$ ,  $y_2 = i_{41}$ ,

$$y_3 = i_{45} + \frac{C_{45}}{2C_4}(-i_{41} - i_{45} - i_{46}), \quad y_4 = i_{46} + \frac{C_{46}}{2C_4}(-i_{41} - i_{45} - i_{46}),$$

$$y_5 = v_7, \quad y_6 = i_{75} + \frac{C_{57}}{2C_7}(-i_{72} - i_{75} - i_{78}), \quad y_7 = i_{72},$$

$$y_8 = i_{78} + \frac{C_{78}}{2C_7}(-i_{72} - i_{75} - i_{78}), \quad y_9 = v_9,$$

$$y_{10} = i_{96} + \frac{C_{69}}{2C_9}(-i_{93} - i_{96} - i_{98}), \quad \text{and}$$

$$y_{11} = i_{98} + \frac{C_{89}}{2C_9}(-i_{93} - i_{96} - i_{98}).$$

The model order varies as the power system configuration changes. At the occurrence of a short to ground fault, the model order becomes 19 because a transmission line is broken into two. At the removal of a transmission line, the order of the model is reduced to 17.

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