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Control Reconfigurability-Based Placement Strategy for FACTS Devices

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Abstract— **This paper considers the placement of Flexible AC transmission system devices into power systems with a criterion of maintaining a prescribed level of control reconfigurability. Control reconfigurability in this paper measures the small signal combined controllability and observability of a power system with an additional requirement on tolerance to loss of measurement data in any single measurement unit in the system, which have not been considered in the existing works. Control device placement is formulated as an optimization problem of finding a minimum number of new control devices to meet a prescribed control reconfigurability threshold. A binary search algorithm and a genetic algorithm are applied to Thyristor Controlled Series Compensators into a number of IEEE test systems. More specifically, Placement results for 14, 30, 57 and 118-bus IEEE test systems are presented.**

I. INTRODUCTION

The significance and necessity of maintaining an efficient, reliable, economic, and sustainable electric power system, as it is modernizing toward a smarter infrastructure, has been increasingly recognized. However, the aging transmission systems are increasingly working closer to the capacity and thermal limits, and thus making the system less reliable. A natural solution to this problem is to upgrade the transmission lines, substations and the associated equipment. Unfortunately, such upgrades have been proven to be expensive and time-consuming. Flexible AC transmission systems (FACTS) offered a cost-effective alternative [1]. FACTS are applied in the existing power system infrastructure to achieve higher transfer capability and enhance the controllability of the system.

FACTS control devices are power electronics based compensators to supply or absorb reactive power dynamically in power systems. There are three major types of FACTS devices: shunt device, such as Static Var Compensator (SVC), series device, such as Thyristor Controlled Series Compensator (TCSC), and the combination of shunt and series devices, such as Unified Power Flow Controller (UPFC), which is capable of controlling the bus voltage and power flow of a transmission line simultaneously. Since there is no moving or rotating components in FACTS devices, they react much faster than the synchronous condensers, and thus are more effective to improve the small signal stability.

The placement of FACTS devices has been investigated with different criteria. A primary objective of using FACTS device is to support a voltage. Rouco and Pagola used the eigenvalue sensitivities as the criterion for determining the location and controller design of series FACTS devices to damp out the power system oscillations [2]. With a similar objective, Kumar et al. calculated controllability index of inputs to modes for FACTS devices, and use them as the criteria for optimal placement [3]. However, the controllability and placement is concerned with only a few critical modes. From the voltage stability view point, Sharma et al. proposed a method called Extended Voltage Phasors Approach (EVPA) to identify the most critical segment or bus in a power network for placements of SVC or TCSC [4]. Other criteria include loading conditions [5] and voltage profiles [6].

The above papers do not consider fault-tolerance in power systems. In reality, different types of faults can interrupt the normal operation of power systems and significantly affect the control effectivenss of FACTS devices. This paper is focused on the fault-tolerant placement of FACTS devices. A new criterion for the placement of FACTS device is proposed based on the idea of control reconfigurability [7]. This criterion is most suitable when the goal of placement is to benefit the small signal control through feedback. In this case, the control effectiveness relies on the overall configuration of the sensing and control networks. The reconfigurability requirement incorporates consideration of tolerance to equipment or control/sensing device outages. It is assumed that synchronized phasor measurement units (PMU) [8], which have fast sample rate are used as the sensing devices. The consideration of using this criterion will be briefly presented in the next section. Before we introduce the criterion, the modeling of power systems and FACTS devices in power flow analysis will be discussed first. Section 3 proposes a formulation of the placement problem. In section 4, optimization algorithms to be applied in placement are described. Case studies and conclusions are presented in section 5 and 6, respectively.

II. BACKGROUND

This section reviews the background material necessary for the discussion of the new development in this paper.

A. Linear Models of Interconnected Power Networks

In small signal studies, an electric power system can be modeled as a linear time-invariant system based on Kirchhoff's circuit laws. Note that this is not a general model which looks into the details of generators, generators or condensers are considered as a voltage source. Suppose we use independent branch currents as state variables and voltage sources as input variables to model a power system, each

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set of independent KVL equations for the power network describes the dynamic behavior of the system by

$$\mathbf{L}\dot{\boldsymbol{x}} = \mathbf{R}\boldsymbol{x} + \mathbf{V}\boldsymbol{u} \tag{1}$$

where $\boldsymbol{x}, \boldsymbol{u}$ are arrays of state variables and input variables, respectively; $\mathbf{L}, \mathbf{R}$ are non-singular parameter matrices determined by the resistance and inductance of branches in the power network; matrix $\mathbf{V}$ represents the locations of the voltage sources.

The number of independent state variable is the same as the number of independent KVL equations. Suppose there are n nodes (including the common reference node) and b branches in a power network, the number of independent KVL equations is $b - n + 1$ [9]. The independent KVL equations can be obtained by graph theory [10].

The measurement data in the system determine the output variables of the system

$$\boldsymbol{y} = \mathbf{C}\boldsymbol{x} \tag{2}$$

where $\boldsymbol{y}$ is an array of output variables; $\mathbf{C}$ is decided according to the sensor placement and their small signal gains.

Based on equations (1) and (2), a state space representation of the power system ensues:

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + \mathbf{B}\boldsymbol{u} \tag{3}$$

$$\boldsymbol{y} = \mathbf{C}\boldsymbol{x} \tag{4}$$

where $\mathbf{A} = \mathbf{L}^{-1}\mathbf{R}, \mathbf{B} = \mathbf{L}^{-1}\mathbf{V}$.

B. Modeling of FACTS Devices

FACTS Devices can be modeled as a combination of voltage source and inductance [11]. SVC is a shunt device with one end connected to the transmission system and the other end connected to the ground. TCSC is a series device with both ends connected to the transmission system. UPFC can be considered as the combination of SVC and TCSC. Figure 1 shows the models for SVC, TCSC and UPFC. FACTS devices are usually connected to the transmission system through transformers. The inductance in series with the voltage source is a portion of the transformer leakage inductance. The typical range of values for the series inductance is 0.01–0.05 per unit [12].

With the models shown in figure 1, when a FACTS device is placed into a power system, it can be considered as a branch connected to the system, from which the state-space model of the new system can be constructed. Suppose there are n states before placing a device, in the new state space model, those n states remain independent. For a shunt device, such as an SVC, there is an additional branch in the system. According to [9], the additional branch introduces an additional independent KVL equation, which implies that there is an additional state. Since there is a voltage source in the model of device, one more input is added to the system. For a series device, such a TCSC, its current is the same as the branch current into which the device is inserted. Thus, the state variables remain the same, and there is one more input variable due to the voltage source in the model. For hybrid devices such as UPFC, the system model can be modified accordingly.

C. Criterion for Placement

FACTS devices are expensive, which are usually installed in substations. In this paper, the concept of control reconfigurability [7] is used as the criterion for the placement of FACTS devices. The objective is to use a minimum number of devices to enhance the power system from a combined controllability and observability viewpoint with tolerance to some anticipated device outages. Since FACTS devices are generally highly reliable, while the data loss associated with measurement units, such as PMUs, are more likely to occur, our placement criterion focuses on achieving the fault tolerance of measurement units. This objective can also be achieved by placing additional measurement units into the network [13]. Also, the fault tolerance criterion can be extended to include other faults in the power systems.

Control reconfigurability measures the ability of a linear system to tolerate faults through feedback control [7]. Such ability is essentially measured by the remaining combined controllability and observability under the prescribed fault conditions. The combined controllability and observability is measured by the minimum Hankel singular value, i.e., the minimum value of the second-order modes, of the state space model of a linear stable system [14]. It is the square root of the minimum eigenvalue of the product of controllability Gramian and observability Gramian. Though in [14], it focused on model reduction, the meaning of the second-order modes remain to be the combined controllability and observability of the system. If the second order modes are all sufficiently large, the effort in terms of energy required to control and observe the states of the system should not be excessive. The worst case measure of the smallest Hankel singular value in the set of all anticipated system changes is defined as the reconfigurability [7].

In the context of small signal stability, the electric power

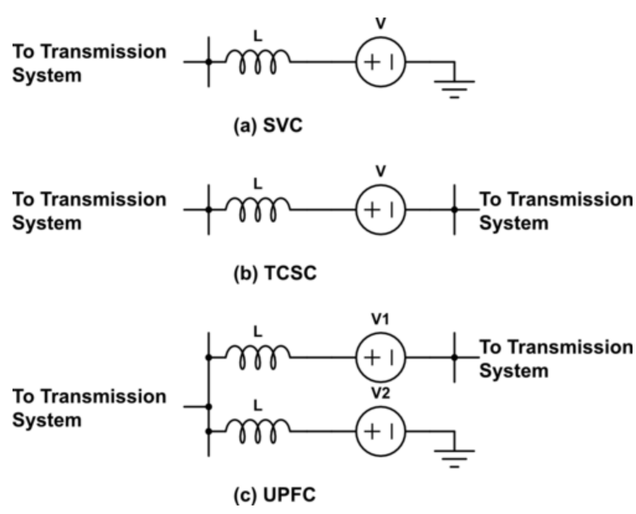

Fig. 1. Models of FACTS devices

system should be sufficiently controllable and observable in the face of anticipated loss of, say, measurement data. With the placement of FACTS devices, the reconfigurability should be enhanced.

Since the measurement data are represented by equation (4) in state space model, the loss of measurement data from a measurement unit can be represented by the loss of an output variable, i.e., the loss of a row in matrix $\mathbf{C}$. Suppose there are k measured output in the system and the availability of the measurements are represented by a vector $\boldsymbol{z}$, each element z_i in $\boldsymbol{z}$ indicates the status of one measurement unit. We have,

$$\boldsymbol{z} = [z_1, z_2, z_3, \ldots, z_k]^T \tag{5}$$

where $z_i = 1$ when the data are available at unit i, and $z_i = 0$ otherwise. Thus, matrix $\mathbf{C}$ of the system is a function of $\boldsymbol{z}$, so equation (4) becomes

$$\boldsymbol{y} = C(\boldsymbol{z})\boldsymbol{x} \tag{6}$$

With the sensor outage considerations, the control reconfigurability for the system described by (3) and (4) is the minimum Hankel singular value of the system with tolerance to loss of measurement data for any single measurement unit. Then the possible values of $\boldsymbol{z}$ are given by

$$\boldsymbol{S} = \{\boldsymbol{z} | \sum_{i=1}^{k} z_i = k - 1\} \tag{7}$$

where k is the total number of sensors involved in the control loops. If we use $\sigma_{min}(\boldsymbol{z})$ to denote the the minimum Hankel singular value, which is a function of $\boldsymbol{z}$, then the control reconfigurability can be defined as

$$\rho_S = \min_{\boldsymbol{z} \in S} \sigma_{min}(\boldsymbol{z}) \tag{8}$$

Once the placement of devices in the system is determined, the state space model is fixed and the control reconfigurability can be calculated by evaluating the minimum Hankel singular values for all the possible values of $\boldsymbol{z}$.

III. PROBLEM FORMULATION

Suppose the minimum Hankel singular value of the existing system is σ_0, the control reconfigurability of the system (with a tolerance to the loss of measurement data for any single measurement unit) is less than σ_0. Our objective is to enhance the control reconfigurability above σ_0 by placing a minimum number of FACTS devices into the system. With the models and the criterion presented above, the problem can be formulated as

$$\text{Minimize: } K \tag{9}$$

$$\text{Subject to: (3), (6), and} \rho_S = \min_{\boldsymbol{z} \in S} \sigma_{min}(\boldsymbol{z}) \geq \alpha\sigma_0 \tag{10}$$

where K is the number of additional FACTS devices to be placed into the system, ρ_S is the control reconfigurability of the system with additional FACTS devices, α is a constant to scale the minimum Hankel singular by a certain factor. For instance, one may select $\alpha = 2$ to double σ_0.

Even for a moderate electric power network, the search space for the above problem is enormous. Assuming there are N possible locations for new devices, the search space is $\sum_{K=1}^{N} \binom{N}{K}$, which could be a huge number.

In some applications, the number of devices to be placed is fixed to K_0, then the problem with the same criterion can be formulated as

$$\text{Maximize: } \rho_S = \min_{\boldsymbol{z} \in S} \sigma_{min}(\boldsymbol{z}) \tag{11}$$

$$\text{Subject to: (3), (6), and} K = K_0 \tag{12}$$

The search space for this formulation is $\binom{N}{K}$, which is smaller than the former one. However, for large number of N, the search space may still be sizable.

The above formulation is suitable for systems with fixed state variables. The placement of series devices does not affect the state variables, so it can be optimized with the above formulation. However, the placement of shunt devices introduces additional states, i.e., the current through the device. The minimum Hankel singular values of two systems with different states should not be compared directly. In order to optimize the placement of shunt devices with the criterion described in this paper, further investigation on a meaningful common measure of the system reconfigurability before and after the placement is required. This paper confines to the placement of series devices only. The optimization algorithms in this paper can be applied to more general types of devices once the suitable notion of control reconfigurability for systems with additional states is established.

IV. OPTIMIZATION ALGORITHMS

Since the optimization variable is a binary string, randomized algorithms are generally suitable for seeking the optimal solution. A genetic algorithm is used to solve the problem described in Equations (11) and (12). A binary search algorithm is used in combination with the genetic algorithm to optimize the problem described in Equations (9) and (10).

A. Genetic Algorithm

A genetic algorithm is used to maximize the value of control reconfigurability with a fixed number of devices to be placed. The main points of the genetic algorithm are as follows:

1) Encoding: The possible locations for placing devices are determined before the optimization and a binary string is used to encode the placement of devices. The number of bits in the string is the number of possible locations. Each bit in the string indicates the placement status of a device in a location. The presence of a device is represented by a 1, while the absence is represented by a 0.
2) Fitness Function: The value of control reconfigurability is used as the raw fitness. In addition, Goldberg's algorithm [15] will be used as a transformation to get a scaled fitness.
3) Initialization: The binary string is initialized randomly with a total number of 1s in the string fixed to the number of devices to be placed into the system.

4) Reproduction: The roulette-wheel selection [16] is used to select two binary strings for crossover. The crossover is achieved by performing bitwise 'AND' and 'OR' operations. Two off-springs are generated by the two operations, respectively. After that, some bits in the off-springs will be randomly set to 0 or 1 so that the total number of 1s in the string remains a fixed number (the number of devices to be placed into the system).
5) Mutation: There is a small mutation rate for each bit in the off-springs. The mutation always occurs in pairs in a string, where one mutation sets a bit to 1 and the other sets a bit to 0.
6) Termination: The algorithm terminates after a certain number of generations or when a result satisfies the requirements (e.g., a placement with a control reconfigurability higher than certain level is found).

B. Binary Search Algorithm

The binary search used in this paper assumes that the control reconfigurability does not decrease by placing new FACTS devices into the system. This is true for the series device. For other type of devices, such assumption is reasonable since we should expect a higher level of controllability and observability by using more FACTS devices.

Basically, the optimization problem in Equations (9) and (10) can be solved by iterating the genetic algorithm above with a decreasing K. To perform a more effective search, the following binary search algorithm is designed:

1) A lower bound L and upper bound H for the optimal value of K is used to specify the interval for the possible optimal values.
2) The genetic algorithm above is used repeatedly to determine if a feasible solution exists for a certain K. A single run of the genetic algorithm terminates if a feasible solution is found. If the algorithm cannot reach a feasible solution and the best known solution is not improved within certain number of generations, we consider that there is no feasible solution for a certain K.
3) Initially, the lower bound L_1 is set to 0 and the upper bound H_1 is set to the largest possible K_{max} (number of possible locations).
4) Start the first search for a feasible solution that satisfies equation (10) with the genetic algorithm and $K_1 = \beta(H_1 - L_1)$.
5) For the i^{th} search, if a feasible solution is found, set $H_{i+1} = K_i$, $L_{i+1} = L_i$ and
$$K_{i+1} = L_{i+1} + \beta(H_{i+1} - L_{i+1}) \qquad (13)$$
Else, set $L_{i+1} = K_i$, $H_{i+1} = H_i$ and
$$K_{i+1} = L_{i+1} + (1-\beta)(H_{i+1} - L_{i+1}) \qquad (14)$$
Then start the $(i+1)^{th}$ search with K_{i+1}. Here β is a constant factor between 0 and 1. K_{i+1} is an integer rounded to the ceiling in Equation (13) and round to the floor in Equation (14).

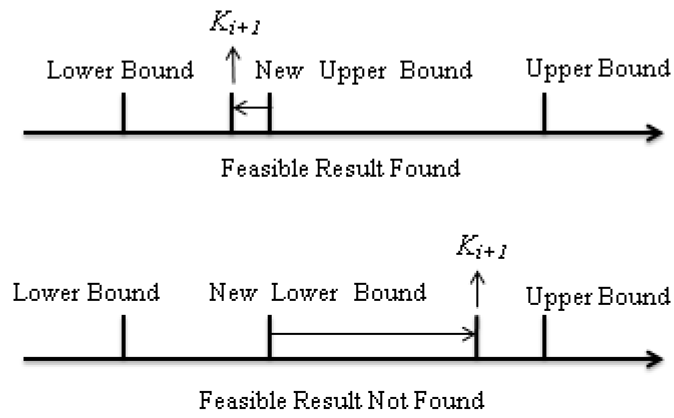

Fig. 2. Update Process of K

6) Once $K_i \geq H_i$ or $K_i < L_i$, the algorithm terminates. K_i is the optimal solution and the placement of the devices is indicated by the binary string returned by the genetic algorithm.

In our experiments, $\beta = 0.8$. Generally, for $\beta > 0.5$, once a feasible solution found, the algorithm decreases K_i by a small step, else the algorithm increases K_i by a big step. The reason for using an $\beta = 0.8$ rather than $\beta = 0.5$ or 0.618 is that finding a feasible solution when it exists for a certain K_i is much faster than waiting for the genetic algorithm to stop if there is no feasible solution. Figure 2 illustrates this algorithm.

V. CASE STUDIES

The placement of TCSC with the above formulation and algorithms are tested with the IEEE test systems for power flow analysis maintained by the University of Washington [17]. The 14, 30, 57 and 118 bus test cases are studied with MATLAB.

A. General Assumptions

In order to apply the formulation and algorithm to the test case, the following assumptions are used:

1) The generators and condensers in the systems are all considered as voltage sources.
2) The internal impedance of the generators and condensers are not given in the data. When considering the transient dynamics of the power system, a small impedance [18] for the generators should be included in the model. The system described by equation (3) may become decoupled if the internal impedance is ignored. The impedance assumed for all generators and condensers is $0.001 + j0.05$ per unit.
3) The impedance of loads are estimated by the voltage, active and reactive power, which is calculated by
$$R = \frac{V^2}{P^2 + Q^2} P \qquad (15)$$
$$X = \frac{V^2}{P^2 + Q^2} Q \qquad (16)$$

4) The shunt inductance is ignored.
5) The series inductance of the TCSC is assumed to be 0.02 per unit.
6) The TCSC will only be placed in the branches of the transmission system

B. Data Pre-Processing

In the data of the 14, 30 and 57 bus test cases, the resistance of transformer windings are specified as zero. There is a structure that two buses are connected by two transformers (one regular transformer and one three-winding transformer). Such a structure with zero resistant causes the magnitude of one of the eigenvalues of the system matrix A in equation (3) to be much smaller than the magnitude of the other eigenvalues and very close to zero (although the system is still stable), which may cause problems when we use MATLAB to calculate the Hankel singular values. To avoid this problem, a small resistance $R = 0.001$ per unit is added, which is less than 1% of the transformer inductance, to each transformer in the three test case data.

The data are processed by a MATLAB program to construct the state space models. The states are selected randomly. The program first constructs independent KVL equations based on graph theory and then construct the $\mathbf{L}$, $\mathbf{R}$, and $\mathbf{V}$ matrix in equation (1) by applying KCL to represent the branch currents by state variables.

The number of output variables are determined by the number of measurement devices, which are not specified in the test case data. To construct the matrix C in equation (4) and represent the availability of measurement data in the system, some of the state variables are selected directly as output variables, i.e., each measurement unit measures one state variable. The selection is based on the criterion described by Huang [13], which guarantees a certain level of control reconfigurability.

C. Results

The pre-processed data are used for the optimization problems formulated in section 4. Table I shows the system parameters of the 4 test cases. The branch number in Table I considers only the branches within the transmission system, which does not include generators, condensers and loads. A TCSC may be placed on any one branch of the transmission system. Thus, the number of possible locations for placing TCSC is the same as the branch number.

TABLE I
SYSTEM PARAMETERS

System	Branch Number	State Variables	Input Variables
14 Bus	20	22	5
30 Bus	41	38	6
57 Bus	80	72	7
118 Bus	186	213	54

For the 14 and 30 bus systems, 5 measured outputs are assumed to observe 5 states directly in each system. For the 57 and 118 bus systems, 10 and 15 measured outputs are assumed, respectively. The minimum Hankel singular values (Min. HSV) and control reconfigurability (C. R.) of the system before optimization is showed in Table II.

TABLE II
ORIGINAL SYSTEM CHARACTERISTICS

System	Before Placement		
	Output Variables	Min HSV	C. R.
14	5	7.09E-07	6.37E-08
30	5	5.35E-13	3.60E-14
57	10	1.22E-15	9.31E-16
118	15	1.89E-14	1.65E-14

Based on the measurement data in the above table, for the problem formulated by equations (11) and (12), we assume that $K_0 = 2$ for the 14 and 30 bus systems, and $K_0 = 3$ for the 57 and 118 bus systems. Optimal placement results calculated by the genetic algorithm are shown in Table III. The number of possible placement combinations for each system is also shown in the table. Min HSV column indicates the combined controllability and observability achieved. C.R. (control reconfigurability is the combined controllability and observability under the worst measurement device outage.

TABLE III
RESULTS FOR FIXED NUMBER OF DEVICES

System	After Placement			
	TCSC No.	Possible Placements	Min HSV	C. R.
14 Bus	2	190	6.05E-06	1.25E-06
30 Bus	2	820	2.89E-11	3.80E-12
57 Bus	3	82160	2.75E-14	1.47E-14
118 Bus	3	1055240	3.21E-14	1.87E-14

For the problem formulated by equations (9) and (10), the binary search algorithm is used in combination with the genetic algorithm to search for the optimal solutions. Table IV shows the optimized results. Here we select the $\alpha = 2$ in equation (10) for the 14, 30 and 57 bus system. For the 118 bus system, we select $\alpha = 1.1$.

TABLE IV
RESUTLS FOR MINIMIZED NUMBER OF TCSC

System	After Placement		
	Min. TCSC No.	Min HSV	C. R.
14 Bus	2	1.12E-05	2.72E-06
30 Bus	2	1.62E-11	2.22E-12
57 Bus	4	1.40E-14	6.02E-15
118 Bus	5	4.02E-14	2.18E-14

The optimization results show that by using the criterion described in this paper to place the FACTS devices, we can enhance the combined controllability and observability of the electric power system while providing the ability to tolerate the loss of measurement data in a single sensor anywhere with a moderate cost.

The calculation of minimum Hankel singular values is computation intensive for large systems. For the 118 test case above, it may take hours to get a result. However, since placement is not a real-time optimization problem, the computation time is acceptable. Generally, the time to optimize the placement of TCSC is less than the time to optimize the placement of measurement data due to the fact that the number of TCSC to be placed is usually smaller. For a small number of devices, the binary search algorithml reduces the search space greatly after the first a few iterations on minimum Hankel singular values.

It should be mentioned that the optimal placement is highly related to the placement of measurement data. In practice, existing sensor placement in the system may not be optimized with the criteria of control reconfigurability. In such cases, the loss of a critical measurement unit may cause the minimum Hankel singular value to drop significantly . As a result, a small number of control device cannot effectively enhance the reconfigurability to a desired level. Thus, to achieve better results by using the criterion in this paper, the placement of measurement units should occur simultaneously whenever possible.

VI. CONCLUSIONS

In this paper, a criterion based on the idea of control reconfigurability is used for FACTS device placement. Control reconfigurability in this paper measures the combined controllability and observability with tolerance to loss of measurement data at any single measurement unit in the system. Placing additional devices into the system is formulated as an optimization problem and is solved with a binary search algorithm and a genetic algorithm. Comparing with the existing works, this paper considered fault-tolerance in the placement problem. The placement of TCSC is investigated with several IEEE test systems for power flow analysis. It shows that the placement strategy is feasible and is significant from the fault-tolerant control view point.

Only the placement of series devices is studied in this paper. For shunt devices, the problem needs further investigation since the minimum Hankel singular values of two systems with different states are not directly comparable. It is important to note that the concept of control reconfigurability can be extended to provide tolerance for any set of faults.

It is also important to note that when generator dynamics are included, more noticeable improvement through FACTS device placement is expected, as the generator inertia would results in much lower reconfigurability before placement. The investigation to quantify placement results in this more realistic setting will be reported in the near future.

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REFERENCES

[1] "The facts on resolving transmission gridlock," *Power and Energy Magazine, IEEE*, vol. 1, no. 5, pp. 41 – 46, sep-oct 2003.

[2] L. Rouco and F. Pagola, "An eigenvalue sensitivity approach to location and controller design of controllable series capacitors for damping power system oscillations," *Power Systems, IEEE Transactions on*, vol. 12, no. 4, pp. 1660 –1666, nov 1997.

[3] B. Kumar, S. Singh, and S. Srivastava, "Placement of facts controllers using modal controllability indices to damp out power system oscillations," *Generation, Transmission Distribution, IET*, vol. 1, no. 2, pp. 209 –217, march 2007.

[4] N. Sharma, A. Ghosh, and R. Varma, "A novel placement strategy for facts controllers," *Power Delivery, IEEE Transactions on*, vol. 18, no. 3, pp. 982 – 987, july 2003.

[5] R. Minguez, F. Milano, R. Zarate-Miano, and A. Conejo, "Optimal network placement of svc devices," *Power Systems, IEEE Transactions on*, vol. 22, no. 4, pp. 1851 –1860, nov. 2007.

[6] G. Yang, G. Hovland, R. Majumder, and Z. Dong, "Tcsc allocation based on line flow based equations via mixed-integer programming," in *Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century, 2008 IEEE*, July 2008, p. 1.

[7] N. Wu, K. Zhou, and G. Salomon, "Control reconfigurability of linear time-invariant systems," *Automatica*, vol. 36, no. 11, pp. 1767 – 1771, 2000.

[8] A. Phadke and J. Thorp, *Synchronized Phasor Measurements and Their Applications*. Springer, 2008.

[9] P. Feldmann and R. Rohrer, "Proof of the number of independent kirchhoff equations in an electrical circuit," *Circuits and Systems, IEEE Transactions on*, vol. 38, no. 7, pp. 681 –684, jul 1991.

[10] S. Banerjee, *Dynamics for Engineers*. Wiley, 2005.

[11] X.-P. Zhang, C. Rehtanz, and B. Pal, *Flexible AC Transmission Systems: Modelling and Control*. Springer, 2006.

[12] P. Kundur, *Power System Stability and Control*. McGraw-Hill, 1994.

[13] J. Huang and N. E. Wu, "Fault-tolerant sensor placement based on control reconfigurability," in *Proceedings of the 18th IFAC World Congress, 2011*, August 2011, pp. 14 841–14 819.

[14] B. Moore, "Principal component analysis in linear systems: Controllability, observability, and model reduction," *Automatic Control, IEEE Transactions on*, vol. 26, no. 1, pp. 17 –32, feb. 1981.

[15] D. E. Goldberg, *Genetic algorithms in search, optimization, and machine learning*. Addison-Wesley Pub. Co., 1989.

[16] M. Mitchell, *An introduction to genetic algorithms*. MIT Press, 1996.

[17] U. of Washington. (2012, Sep.) Power systems test case archive. [Online]. Available: http://www.ee.washington.edu/research/pstca/

[18] T. A. Loehlein, "White paper: Calculating generator reactances," Power Topic #6008 Technical information from Cummins Power Generation, Tech. Rep., 2006.