

Recloser Allocation and Placement for Rural Distribution Systems

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Abstract—Recloser allocation and placement for a rural distribution system is investigated using a hierarchical framework. This paper proposed a high level recloser allocation scheme in addition to the low level recloser placement. Allocating reclosers to certain feeders without knowing the exact circuit topology would be more efficient and desirable for the planning of utility companies. While the allocation scheme focused on minimizing SAIFI in this paper, other objectives and constraints can be included and formulated similarly. The allocation and placement is demonstrated on the Roy Billinton Test System (RBTS).

Index Terms—Switching Devices Placement, Distribution System Reliability, Reliability Evaluation

I. INTRODUCTION

Recloser is the major distribution system switching device used to expedite fault isolation and system reconfiguration. The placement of switching devices is generally formulated as an integer programming problem [1]–[5] or multi-objective programming [6], [7]. Reliability indices [8] such as System Average Interruption Frequency Index (SAIFI) and System Average Interruption Duration Index (SAIDI), served as either objective function or constraints in the optimization problem. SAIFI indicates the number of interruptions that a customer would experience each year, while SAIDI indicates the duration that a customer would experience for an interruption. The reliability indices are typically assessed by analytical or simulation methods [9]–[12], and the optimization problems are solved by various algorithms, including genetic algorithms [1], immune algorithm [2], simulated annealing [3], particle swarm algorithm [4], tabu search algorithm [5], and ant colony algorithm [7].

There are two general issues with the existing solutions. First, solutions proposed by the above paper typically used the average performance of the system to evaluate a particular solution. Since reliability indices are random variables, without considering the variance, a placement solution may violate a constraint with a chance of as high as 50%. This issue was addressed in our previous work [13], where recloser placement is investigated with more conservative constraints by using certain percentiles (e.g. 90%) instead of the mean values. Discrete event simulation is used to evaluate the percentiles and variance of the random variables. Therefore, an optimal solution

with a certain level of conservativeness can be obtained. The other issue of the existing solutions is that all the solutions above focused on finding the exact location of reclosers or switching devices. For a utility company with a large service area (which may consist of thousands of feeders), applying the existing solutions to find the exact placement locations is not efficient. In general, for a large scale distribution system, it would be more reasonable to solve the problem by a two stage “divide and conquer” process. In the first stage, the system is divided into small areas and the devices are allocated to each area based on statistical information without considering the detailed topology. In the second stage, exact locations are determined using specific reliability criteria.

This paper analyzed the impacts of installing reclosers in terms of SAIFI and CAIDI. Only sustained interruptions are considered in this paper, as monetary interruptions do not enter the calculation of SAIFI and CAIDI. The remainder of this paper is organized as follows. The next section presents the hierarchical framework for recloser allocation and placement. Section 3 briefly discusses the general improvement on reliability by using reclosers on a feeder. Section 4 proposes a device allocation scheme without considering the detailed topology. The allocation and placement on the Roy Billinton Test System (RBTS) [14], [15] are demonstrated in section 5. Conclusions are presented in section 6.

II. THE HIERARCHICAL FRAMEWORK

The two-stage “divide and conquer” process is especially suitable for rural distribution system because of the following: 1) Rural distribution system can be easily divided into smaller areas that are independent in terms of reliability indices; 2) Within a small area, the installation of the first recloser/sectionalizer can significantly improve the reliability, and the effect decreases as more devices are installed [16], [17]; 3) Due to factors such as weather, natural environment and power load, improvement on some small areas could be much more effective than the others.

In general, the deployment of reclosers by a utility can be described as a three-level hierarchical process as illustrated in Fig. 1. The first level allocates the reclosers to smaller areas (which can be as small as a few feeders) based on statistical information; the second level optimizes the placement of reclosers based on the detailed topology of the feeders in each

This work was supported by NYSERDA under contract #30733, managed by Michael Razanousky.

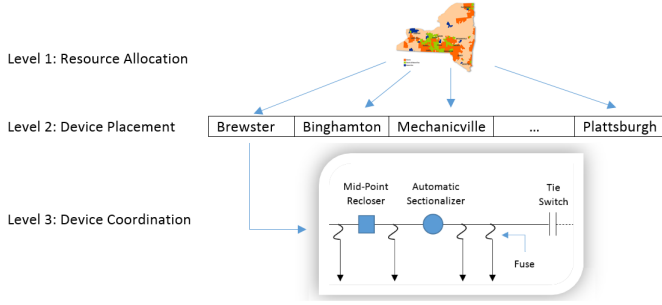


Fig. 1. Hierarchical Model for Recloser Placement Optimization

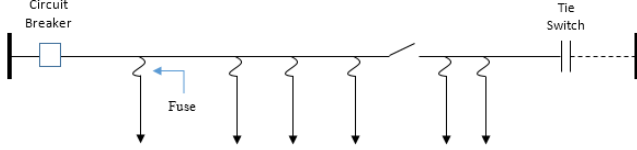


Fig. 2. A Typical Radial Feeder in Rural Area

area. The third level deals with the detailed coordinations of reclosers and other devices, such as fuses and tie switches.

In this paper, the recloser allocation problem in level 1 of Fig. 1 is formulated as an integer optimization problem. The allocation is designed for rural distribution systems. A limited number of reclosers will be allocated to the feeder sets with an objective of minimizing SAIFI. Other objective and constraints can be added as necessary. With the allocation solution, method proposed in [13] is used to obtain the exact optimal locations.

III. IMPROVEMENT ON RELIABILITY WITH RECLOSERS

Several papers discussed the impacts of reclosers on reliability [18]–[21]. All the results in these papers are based on the assumption that customers are evenly distributed along the main feeder. In this section, a more general example is provided in order to develop an device allocation scheme.

In general, rural distribution systems are radial networks, which is the most common topology. A typical circuit consists of one main feeder and several lateral feeders, as shown in Fig. 2. A fuse is installed at each of the lateral feeders. This typical model will be used to study the improvement on reliability by placing reclosers.

Typically, reclosers eliminate interruptions caused by a temporary fault. In this paper, we focus on the interruptions caused by sustained faults, and the optimized device allocation to minimize the impact of sustained faults.

A. Impacts of Faults on Lateral Feeders

Generally, reclosers should be installed with higher priority at the main feeder. When each lateral feeder is equipped with a fuse, for a sustained fault at lateral feeder, the fuse should isolate the fault without causing sustained interruption on the main feeder and other lateral feeders. Suppose that a recloser is installed at the substation or the substation breaker

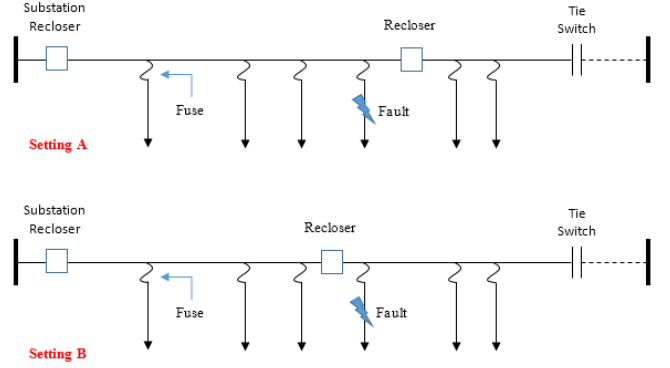


Fig. 3. Fault at Lateral Feeder with Different Recloser Settings

is capable of reclosing, a fault at a lateral feeder will always have the same impact on SAIFI and CAIDI regardless of where the reclosers are installed on the main feeder. For instance, in Fig. 3, when a sustained fault occurs at a lateral feeder, both setting A and setting B will have exactly the same sustained interruptions, i.e. only customers on the faulted lateral feeder will be interrupted, although the momentary interruptions would be different.

The overall SAIFI can be calculated by adding up the two SAIFIs caused by faults at main feeders ($SAIFI_M$) and lateral feeders ($SAIFI_L$), respectively.

$$SAIFI = SAIFI_M + SAIFI_L \quad (1)$$

$$SAIFI_M = \frac{\sum \lambda_{i,\text{main}} N_i}{\sum N_i}, \quad SAIFI_L = \frac{\sum \lambda_{i,\text{lateral}} N_i}{\sum N_i} \quad (2)$$

where, for the i th feeder, $\lambda_{i,\text{main}}$ is the interruption rate corresponding to failures at the main feeder; $\lambda_{i,\text{lateral}}$ is the interruption rate corresponding to failures at the lateral feeder; N_i is the number of customers. Note that failures at the i th lateral feeder only causes interruptions on the i th feeder.

In general, reclosers installed at the main feeder improve SAIFI by reducing $\lambda_{i,\text{main}}$, while $\lambda_{i,\text{lateral}}$ remains the same. In other words, only faults at main feeder will cause different impact on SAIFI and CAIDI for different recloser configurations on the main feeder. The expressions for CAIDI can be obtained similarly.

B. Impacts of Sectionalizing on a Main Feeder

The main purpose of using recloser along main feeder is to sectionalize the feeder and use alternative power supply to pick up the part of the loads when a sustained fault occurs.

With one recloser installed on the feeder and the alternative power supply, as shown in Fig. 4, the probabilities for a fault to occur at the each of the two sections are

$$p_1 = L_1/L, p_2 = L_2/L \quad (3)$$

where $L = L_1 + L_2$ is the total length of the feeder.

The total number of customer interruptions I and total interruption duration T are:

$$I = \lambda L(p_1 N_1 + p_2 N_2) \quad (4)$$

$$T = \mu(p_1 N_1 + p_2 N_2) \quad (5)$$

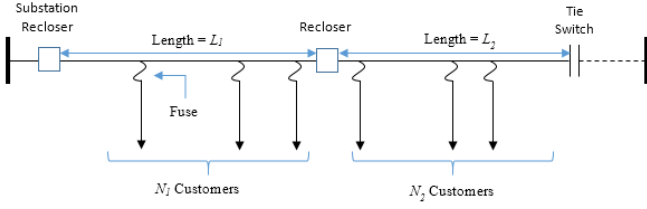


Fig. 4. Feeder with One Midpoint Recloser

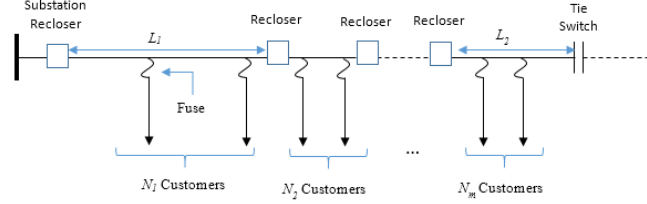


Fig. 5. Feeder with m Sections

where λ is the failure rate per unit line length for the main feeder. μ is the repair time for a failure on the main feeder. $N = N_1 + N_2$ is the total number of customers. Therefore,

$$\text{SAIFI}_M = \lambda L \frac{p_1 N_1 + p_2 N_2}{N_1 + N_2} = \lambda \frac{L_1 N_1 + L_2 N_2}{N} \quad (6)$$

$$\text{CAIDI}_M = \mu \frac{p_1 N_1 + p_2 N_2}{p_1 N_1 + p_2 N_2} = \mu \quad (7)$$

Note that the above results do not assume that customers are distributed evenly along the feeder. Based on the above results, if only sustained faults are considered, sectionalizing has no effect on CAIDI.

For a more general result, if the feeder is sectionalized into m sections as shown in Fig. 5, SAIFI is calculated as:

$$\text{SAIFI}_M = \lambda \frac{\sum_{j=1}^m L_j N_j}{N} \quad (8)$$

where N_j is the number of customers at the j th section.

C. Impacts of Distribution of Customers

When customers are evenly distributed along the main feeder, if the feeder is sectionalized into m sections:

$$N_j = N L_j / L \quad (9)$$

$$\text{SAIFI}_M = \lambda \frac{\sum_{j=1}^m L_j^2}{L} \quad (10)$$

the minimum SAIFI is achieved when

$$L_1 = L_2 = \dots = L_m = L/m \quad (11)$$

In this case,

$$\text{SAIFI}_M = \lambda \frac{\sum_{j=1}^m (L/m)^2}{L} = \lambda L/m \quad (12)$$

Therefore, if customers are evenly distributed along the main feeder, the optimal placement of reclosers is to install them with equal distance in between. However, it is usually the case that customers are not evenly distributed.

In fact, the value of (12) can always be achieved by setting $L_j = L/m$ in (8), regardless of the distribution of customers. In this sense, (12) can be considered as an upper bound.

When the customers are not evenly distributed, a smaller SAIFI can be achieved. For instance, in (6), if $L_1/L = 0.2$, $L_2/L = 0.8$, $N_1/N = 0.8$, and $N_2/N = 0.2$, then $\text{SAIFI}_M = 0.32 < 0.5$.

Based on the above analysis, it can be noticed that even though the assumption of evenly distributed customers are not usually the case, the SAIFI obtained by using this assumption is generally conservative. It would be safe to use this assumption in the planning stage.

D. Impacts on CAIDI by using Recloser

According to (7), CAIDI is not affected by the placement of recloser. Using reclosers generally limits the number of customers being interrupted, but the interruption duration is not affected. CAIDI mainly depends the repair time of a fault.

In some cases, when upgrading devices on feeders by replacing manual switches by reclosers, CAIDI can increase after installing reclosers. This is because the reclosers eliminate the interruptions caused by manually operating the switches. This phenomenon has been reported in [13], [21]. In general, distribution automation increases the system reliability by eliminating small interruptions, which results in longer average interruption durations. The most effective way to improve CAIDI is to improve the repair or recovery process for sustained faults.

IV. RECLOSER ALLOCATION SCHEME

The Recloser allocation scheme proposed in this paper is based on the lumped representation of feeders. By representing a set of feeders as lumped elements, detailed topology is not considered during the allocation process, which leads to an efficient planning process. Once the devices are allocated to particular sets of feeders, the exact placement location can be efficiently determined using the existing placement solutions. As showed in the last section, installing new reclosers will not improve CAIDI. For the recloser allocation scheme, the goal is set to be minimizing SAIFI. Since reclosers improve SAIFI through reducing SAIFI_M in (1), the objective for allocation becomes minimizing SAIFI_M .

A. Lumped Representation of Feeders

In the last section, it is shown that (12) is a conservative estimation for SAIFI_M . The failure rate for a particular feeder can be obtained statistically from the historical data. Through using the historical data, weather impacts are also included in the failure rate.

To account for the distribution of customers, a customer distribution factor ρ may be included:

$$\text{SAIFI}_M = \lambda \rho L/m \quad (13)$$

Since a lower SAIFI could be archived when customers are not evenly distributed. ρ represents how close the customers are to being evenly distributed. When the customers are almost

evenly distributed, $\rho = 1$. Otherwise, $\rho < 1$. ρ could be a small number when most customers are clustered at a short section of the feeder. Further investigation is needed to define ρ as a rigorous function of customer distribution.

B. General Formulation for Recloser Allocation

With the lumped representation of feeders, once the number of devices allocated to the feeders is determined, an estimate of SAIFI can be determined. Since only feeders with substation reclosers and alternative power supplies can benefit from sectionalization using reclosers, in the following recloser allocation problem, it is assumed that a substation recloser is already installed and alternative power supply is available for each feeder. The feeders that are providing alternative supplies to one another should be grouped together and be considered as a lumped element in device allocation.

Suppose the feeders are divided into K sets, for each set j , let n_j denote the number of feeders in the set and x_j denote the number of reclosers (excluding substation recloser). There are a total of $x_j + n_j$ sections in the set. N_j , SAIFI $_j$, λ_j , ρ_j and L_j denote corresponding number of customers, SAIFI, failure rate, customer distribution factor, total feeder length, respectively. The overall SAIFI $_M$ can be estimated as:

$$\begin{aligned} \text{SAIFI}_M &= \frac{\sum_{j=1}^K (\text{SAIFI}_j N_j)}{\sum_{j=1}^K N_j} \\ &= \frac{\sum_{j=1}^K (\lambda_j \rho_j L_j N_j / (x_j + n_j))}{\sum_{j=1}^K N_j} \end{aligned} \quad (14)$$

A device allocation scheme can be formulated based on the budget and constraints of a utilities company. As an example, to minimize the estimated overall SAIFI with a fixed total number of reclosers, the problem can be formulated as

$$\text{Minimize: SAIFI}_M \quad (15)$$

$$\text{Subject to: } \sum_{j=1}^K x_j = M, 0 \leq x_j \leq M_j \quad (16)$$

where, M is the total number of reclosers, and M_j is the maximum number of reclosers allowed to be installed at the j th set of feeders. In general, no more than 3 reclosers should be installed on one feeder.

In general, device allocation formulations yield integer programming problems, which could be intractable. However, various algorithms have been developed for such problems, the detail algorithms are beyond the scope of this paper.

V. CASE STUDY: THE RBTS SYSTEM

The Roy Billinton Test System (RBTS) has been widely used for studying the reliability of distribution systems. The RBTS is briefly described here. Detailed data of the system are available in [14], [15].

The overall RBTS consists of 6 buses, 5 of which are load buses, i.e., bus 2 through bus 6. Bus 2 and bus 4 are mixed commercial and residential distribution networks. Bus 3 represents a network for typical and large users. Bus 5 is a

TABLE I
STATISTICAL DATA OF FEEDER SETS

j	Bus No.	Feeder No.	n_j	λ_j	L_j	N_j	M_j
1	2	1,2	2	0.065	4.20	654	4
2	2	3,4	2	0.065	5.80	1254	6
3	3	1,2	2	0.065	6.40	1458	6
4	3	3,4	2	0.065	7.60	2406	6
5	3	5,6	2	0.065	7.20	1936	6
6	3	7,8	2	0.065	4.80	6	4
7	4	1,7	2	0.065	7.30	2390	6
8	4	2,5,6	3	0.065	6.70	9	9
9	4	3,4	2	0.065	7.35	2380	6
10	5	1,2	2	0.065	14.30	1699	6
11	5	3,4	2	0.065	13.80	1159	6
12	6	1,2	2	0.065	18.10	1733	6
13	6	3,4	2	0.065	67.35	1205	0

typical urban type network and bus 6 is a typical rural area network. All of the 5 distribution systems are radial network.

A. Device Allocation

Feeders providing alternative supply to each other are grouped into a set. The system is divided into 13 sets. Statistical data for each set are calculated in Table I. Noted that there are two feeders in bus 6 without any alternative power supply routes. These two feeders are excluded in the optimization by setting up a set of No.13 with $M_{13} = 0$. To be conservative, $\rho_j = 1.0$ is assumed for all sets.

If $M = 20$ reclosers are to be installed into the system, with the formulation of (15)-(16), the optimization problem can be solved by using CVX, a package for specifying and solving convex programs [22], [23], and Gurobi [24], a solver which enables CVX to solve integer programming problems. The optimal allocation is as follows:

$$\begin{aligned} x_1 = 0, x_2 = 1, x_3 = 1, x_4 = 3, x_5 = 2, x_6 = 0, x_7 = 2 \\ x_8 = 0, x_9 = 2, x_{10} = 3, x_{11} = 2, x_{12} = 4, x_{13} = 0 \end{aligned}$$

B. Device Placement

The exact locations for recloser placement are obtained by using the simulation-based optimization proposed in [13]. With an objective of minimizing SAIFI, for the j th set of feeders, the optimization problem is formulated as:

$$\text{Minimize: } S_j \quad (17)$$

$$\text{Subject to: } \mathbf{P}(\text{SAIFI}_j < S_j) > 0.9 \quad (18)$$

$$\text{Number of Reclosers} = m_j \quad (19)$$

where, $\mathbf{P}(\text{SAIFI} < S)$ is the probability of SAIFI less than a specific number S . In this case, $S = 0.3$ is used.

The optimal locations for installing reclosers are as follows, Bus 2: 21; Bus 3: 8,21,35,38,45,57; Bus 4: 8,23,36,63; Bus 5: 4,7,18,30,42; Bus 6: 9,14,18,22. The numbers are the line section numbers specified in [14], [15]. Simulation is performed for the overall system with the above recloser placement and the histograms of 10,000 samples of SAIFI and CAIDI are shown in Fig. 6. Detailed interpretations of the histogram is provided in [13].

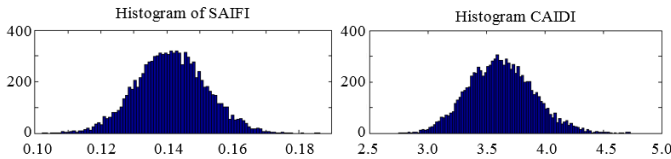


Fig. 6. Distribution of SAIFI and CAIDI for RBTS with reclosers

VI. CONCLUSIONS

This paper proposed a recloser allocation and placement framework for rural distribution systems. The traditional recloser placement problem is formulated as a two-level optimization problem. At the first level, reclosers are allocated to smaller areas, i.e., a set of feeders, based on a conservative estimation of SAIFI. At the second level, the exact locations are obtained using a simulation-based method, which provides certain level of conservativeness.

For the allocation problem, this paper analyzed the impacts of installing reclosers on main feeders on SAIFI and CAIDI. The following assumptions are made, which are typical features of distribution systems in rural area: 1) The distribution system is a radial system. Each feeder generally consists of one main feeder and several lateral feeders. 2) Each lateral feeder is equipped with a fuse, which can isolate sustained faults from affecting the main feeder and other lateral feeder. 3) Reliability index for each feeder is independent. In addition, the allocation problem only considers feeders with substation reclosers and alternative supplies that can be coordinated.

Based on the above assumptions and the analysis in this paper, we have the following conclusions: 1) Faults on lateral feeder have the same impact regardless of the reclosers on the main feeder. 2) The impacts of reclosers on SAIFI can be estimated as a function of the number of sections on a feeder. 3) The SAIFI value estimated by assuming evenly distributed customers can serve as an upper bound. A lower SAIFI can be achieved when customers are not evenly distributed. 4) Using reclosers to replace manual swatches could increase CAIDI since it eliminates shorter interruptions. A lumped representation for feeders is developed and a recloser allocation scheme is proposed without considering the detailed system topology. Such device allocation is more desirable and more efficient in the planning stage. The optimization for resource allocation can be solved by using existing software packages.

A customer distribution factor is suggested to describe how evenly the customers are distributed along a main feeder. A rigorous formulation of this factor should be investigated in the future.

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