Control Effectiveness of FACTS Devices in Power Systems

Qiu Qin[1] and N. Eva Wu[1]

Abstract— **This paper analyzes the fault tolerant control of a power grid with FACTS devices. Control effectiveness measures the small signal combined controllability and observability of a power system with tolerance to prescribed contingencies. This paper also focuses on the roles of FACTS devices in affecting the grids redundancy architecture. Under the small-signal modeling framework of the paper and the previously defined notion of control effectiveness, it is shown that introduction of FACTS devices always improves system control effectiveness. The device placement results on a 9-bus system are studied for their control effectiveness.**

I. Introduction

Automatic control of the power system has been recognized as an important aspect for maintaining and enhancing the grid's reliability and efficiency. From the perspective of control theory, controllability and observability are dual aspects of a power system's structural property. To improve the observability, measurement devices such as synchronized Phasor Measurement Unit (PMU) [1] are deployed into the system. To improve the controllability, Flexible AC Transmission Systems (FACTS) [2] are introduced to offer a cost-effective way for grid compensation and control.

FACTS devices are power electronics-based compensators for achieving higher transfer capability and enhancing the controllability of an existing system. In general, they control the power system through supplying or absorbing reactive power dynamically. Since there is no major mechanical component in FACTS controllers, they are able to take actions in a very short time (e.g., 25ms), which is much faster than other control methods (e.g. using synchronous condensers), and thus can be more effective in improving the small signal stability of the power system.

Voltage stability and rotor angle stability are the two major stability problems in power systems [3]. FACTS device placement has been investigated with a primary objective of improving voltage stability, i.e. providing voltage support and increasing the load carrying capacity on the transmission system. Voltage stability in essence involves only the steady-state behavior of a power system. Various criteria for FACTS device placement have been proposed to improve voltage stability [4]–[7], loading condition [8], [9] and voltage profiles [10]. Angle stability consists of transient stability and small signal stability. As a power system is a nonlinear system, transient stability studies focus on the nonlinear behavior of power systems. Such studies with FACTS include the analysis of trajectory sensitivity [11], stability margin [12], and energy functions [13]. Small signal stability studies focus on the behavior of power system under small disturbance. From the small signal stability viewpoint, the eigenvalue sensitivities as the criterion for FACTS placement were investigated in [14]. With a similar objective, optimal placement using controllability indices as the criteria was proposed in [15] to damp the system oscillations.

From a different perspective of small signal analysis, this paper analyzes the effects of FACTS devices in affecting the system's redundancy architecture through the concept of control effectiveness, which describes the combined controllability and observability with the tolerance of "$N-1$" contingencies. The framework developed here allows tolerance consideration of other faults that can be parameterized in a small signal state-space model. Control effectiveness originates from the idea of control reconfigurability [16]. Control effectiveness reveals the ability of FACTS devices for improving the small signal stability through feedback control with fault-tolerance.

The remainder of this paper is organized as follows: Section II describes the modeling of power system and FACTS devices. Section III defines the concept of control effectiveness and studies the effect of FACTS on it. Section IV presents a case study on a 9-bus power system.

II. Modeling

The control effectiveness of FACTS devices is studied on both electric (fast) and electromechanical (slow) dynamics of the power system. This section reviews the modeling of electric transmission network dynamics, generator dynamics and FACTS devices.

A. Models for Electric Transmission Network Dynamics

For studying the electric dynamics, generators are modeled as voltage sources behind their internal transient reactances and transmission lines are modeled as lumped equivalent π circuits. Loads in the system are modeled as constant impedances. Based on the Kirchhoff's circuit law, the electric transmission network is modeled as an LTI system with currents of the equivalent independent inductors and voltages of the equivalent independent capacitors as states. The inputs to the system are generator voltages and the outputs of the system are the measured voltages and currents.

Fig. 1 shows the equivalent π circuit of a transmission line connecting two buses. There are 3 states for this circuit: the bus voltages v_A, v_B at the two ends and the branch current i_{AB} going through the equivalent inductor. The measured currents at the two ends are: $i_A = i_{AG} + i_{AB}$ and $i_B =$

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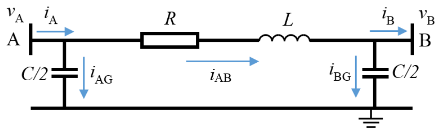

Fig. 1. Equivalent π Circuit of a Transmission Line

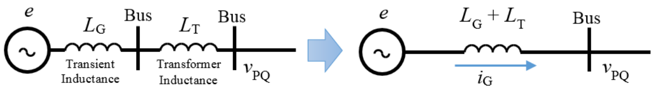

Fig. 2. Generator Modeling

$i_{AB} - i_{BG}$. The state equations are:

$$\dot{v}_A = 2(i_A - i_{AB})/C \tag{1}$$

$$\dot{v}_B = 2(i_{AB} - i_B)/C \tag{2}$$

$$\dot{i}_{AB} = (v_A - v_B - Ri_{AB})/L \tag{3}$$

Each transmission line in the network is modeled similarly.

A generator is modeled as an ideal voltage source connected to a terminal bus in series with the generator transient inductance. Each terminal bus is connected to a bus (with voltage v_{PQ}) in the transmission network through a transformer, which is simply modeled as a leakage inductor. As shown in Fig. 2, the transient inductance and transformer inductance are combined and the terminal bus is eliminated in the modeling process. The state equation for the current going through the inductance is:

$$\dot{i}_G = \frac{1}{L_G + L_T}(e - v_{PQ}) \tag{4}$$

where the generator voltage e is an input to the system.

For a system with M generators, N loads, L lines (including the transformers) and B buses. The LTI model will have M inputs and $B - M + N + L$ states.

B. Models for Generator Dynamics

The linearized classical generator model is used in which generator rotor angle and rotor speed are the states. The input to the system is mechanical power and the outputs of the system are bus voltage measurements in the transmission network.

$$\Delta\dot{\delta}_i = \Omega\Delta\omega_i \tag{5}$$

$$\Delta\dot{\omega}_i = \frac{1}{2H_i}\left(-\sum_{j=1}^{n}\frac{\partial P_i^e}{\partial \delta_j}\Delta\delta_j - D_i\Delta\omega_i + \Delta P_i^m\right) \tag{6}$$

There are two states for each generator, rotor angle $\Delta\delta_i$, relative to a synchronously rotating reference framework, and rotor speed $\Delta\omega_i$. Ignoring the electrical dynamics, the dynamic model for an n-generator system is the combination of dynamic models of all generators. The transmission network structure is reflected by the electrical power P^e in the above equations. Details for the system model and expression for P^e is presented in [17]. Therefore, there are a total of $2n$ states in a system with n generators. In this paper, the stable equilibrium point is chosen as the operating point, which can be obtained after solving for the power flow.

Here P_i^m is considered as the input variable. For an n-generator system, there are n input variables. It should be noted that since the machines are rotating, the n rotor angles are not independent. Assuming $\delta_{ij} = \delta_i - \delta_j$ as the angle difference between generators i and j and using δ_1 as the reference angle, the independent states are $[\boldsymbol{\delta}^T\ \boldsymbol{\omega}^T]^T$ consist of $\Delta\omega_i, i = 1, 2, 3, ..., n$ and $\Delta\delta_{i1}, i = 2, 3, 4, ..., n$ The corresponding state space representation of the system with $2n - 1$ states can be obtained by applying a similarity transformation on the $2n$ state system and eliminating the reference rotor angle. In short, the system is represented as

$$\begin{bmatrix} \dot{\boldsymbol{\delta}} \\ \dot{\boldsymbol{\omega}} \end{bmatrix} = \mathbf{A}\begin{bmatrix} \boldsymbol{\delta} \\ \boldsymbol{\omega} \end{bmatrix} + \mathbf{BP^m} \tag{7}$$

where $\mathbf{P^m}$ is a vector consist of ΔP_i^m.

The output equations of the system are determined by the measurement scheme. Here, the voltage phasor angle and magnitude of each bus in the system are used as the outputs of the system, assuming the system is near its stable operating point. For the system above, voltage angles and magnitudes are functions of rotor angles (δ_i). The detailed measurement models and output equations are discussed in [18].

C. Models of FACTS Devices

FACTS devices can be divided into three categories: shunt devices, such as Static Var Compensator (SVC), series devices, such as Thyristor Controlled Series Compensator (TCSC), and the combinations of shunt and series devices, such as Unified Power Flow Controller (UPFC). FACTS devices can be used in transmission, sub-transmission and distribution levels. In sub-transmission and distribution levels, FACTS devices are used for balancing three phase power supply and improving the power factor. These applications have a limited effect on the transmission system. This paper considers the application of FACTS device at the transmission level to help achieving effective control of the overall power system. FACTS devices introduce variable reactance in the grid. The dynamics of FACTS devices are ignored in this paper. By adjusting the reactance of a FACTS device, its output voltage and current can be controlled. Therefore, a FACTS device can also be considered as a variable voltage source or a variable current source. Simplified models of SVC and TCSC are used to represent the shunt and series FACTS devices, respectively. Different models for both SVC and TCSC are discussed here for studying fast and slow dynamics of the system.

For studying the electric transmission network dynamics, each shunt FACTS device is modeled as a controlled current source [19] connecting to a bus and each series FACTS device is modeled as a controlled voltage source [20] connecting in series with a transmission line. Fig. 3 shows the models of SVC and TCSC for this study. The currents of SVCs and voltages of each TCSCs are new inputs to the system.

For studying the generator dynamics, each shunt FACTS device is modeled as a controlled voltage source with a series

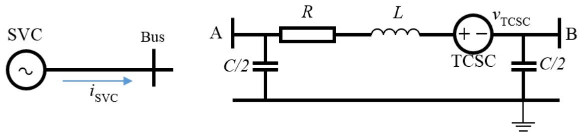

Fig. 3. Models of FACTS Device for Studying Electric Dynamics

Fig. 4. Models of FACTS Device for Studying Generator Dynamics

inductance [21], and each series devices is modeled as a controlled variable reactance [8]. Fig. 4 shows their models. According to [21], a typical value for the series inductance L in Fig. 4 is 0.01–0.05 pu. In this paper, $L = 0.01$ pu is used.

Without loss of generality, it is reasonable to assume that FACTS devices in transmission systems for control effectiveness purpose are configured to operate around zero reactive power/current injection point at steady state. For small signal analysis, this paper also assumes that the FACTS devices are always working in the linear control range.

With FACTS devices, equation (6) becomes:

$$\begin{aligned}\Delta\dot{\omega}_i = \frac{1}{2H_i}(&-\sum_{j=1}^{n}\frac{\partial P_i^e}{\partial \delta_j}\Delta\delta_j - D_i\Delta\omega_i \\ &+ \Delta P_i^m - \sum_{k=1}^{m}\frac{\partial P_i^e}{\partial V_k}\Delta V_k - \sum_{l=1}^{p}\frac{\partial P_i^e}{\partial X_l}\Delta X_l)\end{aligned} \tag{8}$$

where V_k is the voltage magnitude of the controlled voltage source in a shunt device model; X_l is the reactance of the controlled reactance in a series device model; m is the number of shunt devices and p is the number of series devices in the system.

III. Control Effectiveness

A. Control Reconfigurability

This paper defines the control effectiveness based on the idea of control reconfigurability [16]. Control Reconfigurability is defined as the Minimum Hankel Singular Value (MHSV) of a system under prescribed contingencies or faults. Hankel singular values are also called second order modes. They measure the energy of the states in a system and are used for balanced model reduction [22]. If the second order modes are all sufficiently large, the effort in terms of energy required to control and observe the states of the system should not be excessive [16]. Therefore, the MHSV can be used to measure the combined controllability and observability of a linear system. Control reconfigurability can also measure fault tolerance. For parameterized faults/contingencies, it is defined as:

$$\rho^S := \inf_{\theta\in S}[\sigma(\theta)] \tag{9}$$

where ρ^S is the control reconfigurability over set S, in which the prescribed faults/contingencies reside; θ is a point in set S; σ is the MHSV of the system at θ, defined by

$$\sigma(\theta) = \min(\sqrt{\lambda(\mathbf{PQ})}) \tag{10}$$

where $\lambda(\cdot)$ is the operation of calculating eigenvalues. $\mathbf{P}$ is the controllability Gramian and $\mathbf{Q}$ is the observability Gramian satisfying the following Lyapunov equations:

$$\mathbf{AP} + \mathbf{PA}^T + \mathbf{BB}^T = 0, \mathbf{A}^T\mathbf{Q} + \mathbf{QA} + \mathbf{C}^T\mathbf{C} \quad = 0 \tag{11}$$

where $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are system matrix, input matrix and output matrix at θ.

Using control reconfigurability as the placement criterion, the effects of series FACTS device on electrical dynamics of a power system has been studied in [23] with a simplified transmission network model without generator dynamics and shunt capacitors.

B. Effects of FACTS Devices on System Model

In general, the system model is

$$\dot{\mathbf{x}} = \mathbf{Ax} + [\mathbf{B}\ \mathbf{B_K}]\begin{bmatrix}\mathbf{u}\\ \mathbf{u_K}\end{bmatrix}, \mathbf{y} = \mathbf{Cx} \tag{12}$$

where $\mathbf{x}$ is the state vector and $\mathbf{y}$ is the output vector. $\mathbf{u}$ is the input to the system excluding FACTS devices. $\mathbf{u_K}$ is the input added by installing FACTS devices.

Adding K FACTS devices into the system has the same effect of adding K new input variables, i.e., K new columns into matrix $\mathbf{B}$.

C. Hankel Singular Values Enhancement

Installing a FACTS device adds an additional column to the system input matrix. The controllability Gramian is defined based on the system input matrix. In the following discussion, we define 3 systems[1]:

$$R_0 = \left(\begin{array}{c|c}\mathbf{A} & \mathbf{B}\\ \hline \mathbf{C} & 0\end{array}\right), R_K = \left(\begin{array}{c|c}\mathbf{A} & \mathbf{B_K}\\ \hline \mathbf{C} & 0\end{array}\right),$$
$$R = \left(\begin{array}{c|c}\mathbf{A} & [\mathbf{B}\ \mathbf{B_K}]\\ \hline \mathbf{C} & 0\end{array}\right).$$

Also, it is assumed that R_0 is controllable and observable.

Theorem 3.1: The MHSV of system R is bounded below by MHSV of system R_0.

Let $\mathbf{P_0}$, $\mathbf{P_K}$ and $\mathbf{P}$ be the controllability Gramians of the 3 systems, respectively.

Proof:

$$\mathbf{P} = \int_0^\infty e^{A\tau}[\mathbf{B}\ \mathbf{B_K}][\mathbf{B}\ \mathbf{B_K}]^T e^{A^T\tau}d\tau \tag{13}$$
$$= \int_0^\infty e^{A\tau}(\mathbf{BB^T} + \mathbf{B_K B_K^T})e^{A^T\tau}d\tau \tag{14}$$
$$= \mathbf{P_0} + \mathbf{P_K} \tag{15}$$

Gramians are semi-positive definite, thus, $\mathbf{P} \geq \mathbf{P_0}$.

Let $\lambda_j(\cdot)$ denote the jth largest eigenvalue. According to [25], for $n\times n$ symmetric matrices $\mathbf{M_1}$ and $\mathbf{M_2}$,

$$\lambda_j(\mathbf{M_1} + \mathbf{M_2}) \geq \lambda_j(\mathbf{M_1}) + \lambda_n(\mathbf{M_2}) \tag{16}$$

[1]The notation is inherited from [24]

Therefore,

$$\begin{aligned}\lambda_j(\mathbf{PQ}) &= \lambda_j(\mathbf{Q}^{-1/2}\mathbf{Q}^{1/2}\mathbf{P}\mathbf{Q}^{1/2}\mathbf{Q}^{1/2}) \\ &= \lambda_j(\mathbf{Q}^{1/2}\mathbf{P}\mathbf{Q}^{1/2}) \\ &= \lambda_j(\mathbf{Q}^{1/2}(\mathbf{P_0}+\mathbf{P_K})\mathbf{Q}^{1/2}) \\ &\geq \lambda_j(\mathbf{Q}^{1/2}(\mathbf{P_0})\mathbf{Q}^{1/2}) + \lambda_n(\mathbf{Q}^{1/2}(\mathbf{P_K})\mathbf{Q}^{1/2}) \\ &\geq \lambda_j(\mathbf{Q}^{1/2}(\mathbf{P_0})\mathbf{Q}^{1/2}) = \lambda_j(\mathbf{P_0}\mathbf{Q}) \end{aligned} \tag{17}$$

The MHSV ($j = n$) of the nth order system R,

$$\sigma = \min(\sqrt{\lambda(\mathbf{PQ})}) = \sqrt{\lambda_n(\mathbf{PQ})} \geq \sqrt{\lambda_n(\mathbf{P_0Q})} \tag{18}$$

■

Corollary 3.2: The control reconfigurability of system R is bounded below by the control reconfigurability of system R_0.

This follows Theorem 3.1 as it applies to each prescribed contingency.

The essence of the above is to affirm the meaningfulness of control effectiveness as a device placement criterion, which guarantees that new device placement always improves control effectiveness. The same argument applies to sensor placement.

D. Criteria for Comparing Different Systems

It should be noted that $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{P}$, and $\mathbf{Q}$ may all depend on θ. However, only MHSVs of systems where a state-space is preserved before and after a FACTS placement are considered. A more restrictive but simplifying condition is to consider those systems with the same $\mathbf{A}$ matrix. Control reconfigurability is suitable for measuring the tolerance to faults that do not affect the system matrix, such as actuator or sensor faults. The term control effectiveness is coined to reflect our intention to analyze the effect of placement of FACTS devices into a power system. Control effectiveness extends control reconfigurability to include faults that change the state space, for example, the "$N-1$" contingencies in power system.

As described in the last section, each FACTS device is modeled as an additional input to the system without changing the $\mathbf{A}$ matrix. Let $\sigma_0(\theta)$ denote the MHSV of the system at θ before installing the devices and $\sigma_i(\theta)$ denote the MHSV of the system at θ after installing the device at location i. The control effectiveness of installing the device at location i is defined as:

$$\rho_i^S := \inf_{\theta \in S} [\frac{\sigma_i(\theta)}{\sigma_0(\theta)}] \tag{19}$$

Therefore, the MHSVs before and after installing a set of FACTS devices can be compared for each contingency. Control effectiveness of installing a new set of FACTS devices is defined as the minimum improvement on the MHSV for the prescribed contingencies. This definition allows contingencies that change the system matrix to be evaluated.

Since the control effectiveness is calculated based on the Hankel singular values of a multiple-input multiple-output (MIMO) system, a proper scaling of input/output [26] in the system model is necessary. In this paper, the inputs and outputs consist of variables with different units and magnitudes. Scaling is achieved by dividing each input/output by a factor representing the range of the variable. After scaling, the input/output becomes a percentage of the factor. With scaling, the input matrix $\mathbf{B}'$ and output matrix $\mathbf{C}'$ can be written as:

$$\mathbf{B}' = \mathbf{B}\mathbf{Q_u}, \mathbf{Q_u} = \text{diag}(u_1^N, u_2^N, ...) \tag{20}$$

$$\mathbf{C}' = \mathbf{Q_y}\mathbf{C}, \mathbf{Q_y} = \text{diag}(1/y_1^N, 1/y_2^N, ...) \tag{21}$$

where $\mathbf{Q_u}$ and $\mathbf{Q_y}$ are diagonal matrices. $u_1^N, u_2^N, ...$ and $y_1^N, y_2^N, ...$ are the scaling factors for the inputs and outputs. The selection of scaling factors depends on the practical applications.

IV. A Case Study

Power systems are generally required to satisfy the "$N-1$" security criterion [27], which ensures the system to operate without causing an overload failure when removing any one component from the system. In this section, we restrict the "$N-1$" contingencies to the removal of any single transmission line. The control effectiveness measure incorporates fault tolerance into the placement of FACTS devices. It is calculated as the worst case improvement on combined controllability and observability within "$N-1$" contingencies.

The Western System Coordinating Council (WSCC) 3-machine, 9-bus power system [17], shown in Fig. 5, is to be used for this study. There are 6 transmission lines. Control effectiveness of FACTS devices is studied with 6 contingencies in addition to the normal operating situation (7 cases altogether for each control effectiveness evaluation).

A. Control Effectiveness on Electric Network Dynamics

For studying electric network dynamics, output measurements are selected as the bus voltages at bus 4, 7 and 9, plus the currents of all branches connected to these 3 buses. Without FACTS devices, there are 3 inputs and 12 outputs in the network's LTI model. The inputs and outputs are quasi sinusoidal waveforms. Generator voltages, bus voltages and branch currents are scaled with their magnitudes at the operating point as scaling factors, where the operating point is determined by the power flow solution [17]. The current of SVC and the voltage of TCSC are scaled with their maximum

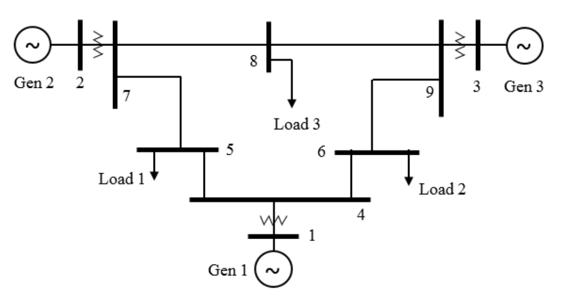

Fig. 5. WSCC 3-Machine 9-Bus System

operating limits as scaling factors. The limits depend on the practical applications. Here, for illustration purpose, the limit for SVC current is set to 1 per unit and the limit for TCSC voltage is set to 0.5 per unit.

We study the control effectiveness by installing a single SVC at one of the 6 buses of the transmission network (i.e. bus 4–9). Fig. 6 shows the MHSVs for the 7 cases. It should be noted that when a line is removed, the corresponding output is also removed from the model. The percentage improvements are calculated before and after installing SVC for each bus. Control effectiveness is calculated based on (19), which essentially reflects the minimum improvement on MHSV among the 7 cases. The results show that the best control effectiveness is achieved when installing an SVC at bus 7. From the results, we also observed that the improvements of installing SVC at bus 4, 5, and 6 are quite limited. The MHSVs for different contingencies vary greatly.

Similarly, Fig. 7 shows the MHSVs for the 7 cases when installing a TCSC at each of the 6 transmission lines. When TCSC is installed at a particular line, the contingency of removing that line is ignored, as removing the line will also remove the TCSC. The results show that the best control effectiveness is achieved when installing the TCSC at line 6-9. Comparing with the results in Fig. 6, in this 9 bus system, a TCSC is seen to provide wider range of impacts. As the inputs of SVCs and TCSCs to the system are normalized with their respective operational limits, the control effectiveness of SVC and TCSC can be compared in the sense that they require equivalent effort to be operated at their limits. In this case, SVC provides better control effectiveness.

B. Control Effectiveness on Generator Dynamics

For studying the generator dynamics, output measurements are selected as the bus voltage magnitudes of all the 9 buses and inputs are the generator output voltages. Without FACTS devices, there are 3 inputs and 9 outputs in the small signal model. Since the voltages in this model are phasors, they are generally around 0.95 to 1.05 per unit. The bus voltages are scaled with a factor of 0.05, i.e. one half of the range. As the mechanical power cannot change rapidly, the mechanical power inputs are scaled with a factor of 0.1 per unit. The SVC voltage and TCSC reactance are scaled according to their operational ranges. In this study, we assume that the SVC voltage ranges from 0.95 to 1.05 per unit and the TCSC reactance ranges from -0.2 per unit to 0.2 per unit, i.e. the scaling factors are set to 0.05 for SVC and 0.2 for TCSC. Fig. 8 and 9 shows the MHSVs for the 7 cases when installing a single SVC/TCSC into the system at a time.

The results show that, with an SVC, the improvements of MHSV on generator dynamics (slow modes) is more uniform than those on electric dynamics (fast modes). For a TCSC, it generally improves the MHSV significantly for the contingency of removing a neighborhood line. For instance, when TCSC is installed at line 4-5, the MHSV for the case of removing line 5-7 improves greatly. In the sense of control effectiveness, the best location for SVC is at bus 5 and the best location for TCSC is in line 4-5. With the specified operational ranges, a TCSC achieves a slightly better control effectiveness than an SVC.

V. Conclusions

The paper studied the control effectiveness of FACTS devices, when one is introduced into a particular location in power systems. Control effectiveness quantifies the minimum combined controllability and observability of the system under prescribed contingencies. The LTI model for electric network dynamics and the small signal model for generator dynamics are used in combination with simplified FACTS devices models. When studying electric dynamics, shunt device is modeled as a controlled current source and series device is modeled as a controlled voltage source. When studying the generator dynamics, shunt device is modeled as a controlled voltage source and series device is modeled as a controlled reactance. The use of different models for FACTS devices avoids the variation of system matrix when a new FACTS devices is installed, thus enabling the comparison of Hankel singular values before and after. The inputs and outputs of the system are scaled, so that the control effectiveness of different FACTS devices can be compared. It should be noted that the operational limit of a device is also reflected in the control effectiveness through scaling. Therefore, devices with different sizes are also comparable for control effectiveness.

Though the placement approach seems to be exhaustive enumerating, it in fact is reasonable because there are usually very limited number of candidate locations and number in each placement opportunity.

Studying the control effectiveness of FACTS devices is significant from the view point of fault-tolerant control. It exploits the potentialities of applying FACTS devices in wide area damping and multiple band stabilization. This paper in fact solved an optimally robust placement problem for the 9-bus system. Future works include the study of a larger system and the placement of multiple FACTS devices.

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	No SVC	SVC at Bus 4		SVC at Bus 5		SVC at Bus 6		SVC at Bus 7		SVC at Bus 8		SVC at Bus 9	
	min HSV	min HSV	Improvement	min HSV	Improvement	min HSV	Improvement	min HSV	Improvement	min HSV	Improvement	min HSV	Improvement
Normal	5.672E-02	5.781E-02	1.91%	6.136E-02	8.17%	6.790E-02	19.70%	1.438E-01	153.45%	1.193E-01	110.28%	6.538E-02	15.27%
Line45 Removed	9.356E-02	1.006E-01	7.55%	1.476E-01	57.80%	1.064E-01	13.68%	2.996E-01	220.26%	1.086E-01	16.06%	3.557E-01	280.22%
Line57 Removed	2.942E-02	3.020E-02	2.66%	3.043E-02	3.43%	3.287E-02	11.71%	1.113E-01	278.39%	1.934E-01	557.19%	4.412E-02	49.96%
Line46 Removed	1.167E-01	1.270E-01	8.89%	1.548E-01	32.70%	1.196E-01	2.50%	1.699E-01	45.67%	1.706E-01	46.28%	3.031E-01	159.83%
Line69 Removed	4.030E-02	4.101E-02	1.75%	4.716E-02	17.02%	4.422E-02	9.71%	3.168E-01	686.06%	2.901E-01	619.71%	8.581E-02	112.91%
Line78 Removed	5.397E-02	5.727E-02	6.12%	5.397E-02	0.00%	5.495E-02	1.81%	1.293E-01	139.55%	2.678E-01	396.30%	7.739E-02	43.39%
Line89 Removed	7.076E-02	7.249E-02	2.44%	8.748E-02	23.63%	8.043E-02	13.67%	2.863E-01	304.58%	1.473E-01	108.15%	1.694E-01	139.35%
min. % Improvement			1.75%		0.00%		1.81%		45.67%		16.06%		15.27%
Control Effectiveness			1.0175		1.0000		1.0181		1.4567		1.1606		1.1527

Fig. 6. Minimum Hankel Singular Values of the Electrical Dynamics with SVC

	No TCSC	TCSC at Line 45		TCSC at Line 57		TCSC at Line 46		TCSC at Line 69		TCSC at Line 78		TCSC at Line 89	
	min HSV	min HSV	Improvement	min HSV	Improvement	min HSV	Improvement	min HSV	Improvement	min HSV	Improvement	min HSV	Improvement
Normal	5.672E-02	1.450E-01	155.66%	9.659E-02	70.28%	1.345E-01	137.07%	7.996E-02	40.96%	8.158E-02	43.83%	6.519E-02	14.93%
Line45 Removed	9.356E-02	N/A		2.488E-01	165.94%	9.441E-02	0.91%	1.083E-01	15.80%	2.922E-01	212.35%	1.449E-01	54.89%
Line57 Removed	2.942E-02	9.409E-02	219.78%	N/A		1.108E-01	276.56%	5.751E-02	95.48%	2.997E-02	1.86%	4.215E-02	43.27%
Line46 Removed	1.167E-01	1.202E-01	3.05%	1.501E-01	28.65%	N/A		1.651E-01	41.51%	1.957E-01	67.80%	1.914E-01	64.09%
Line69 Removed	4.030E-02	2.093E-01	419.35%	9.577E-02	137.62%	2.031E-01	403.85%	N/A		7.948E-02	97.22%	4.324E-02	7.30%
Line78 Removed	5.397E-02	1.091E-01	102.24%	5.894E-02	9.21%	1.331E-01	146.67%	8.621E-02	59.73%	N/A		7.232E-02	34.00%
Line89 Removed	7.076E-02	1.568E-01	121.53%	1.464E-01	106.88%	1.371E-01	93.77%	8.090E-02	14.33%	1.379E-01	94.94%	N/A	
min. % Improvement			3.05%		9.21%		0.91%		14.33%		1.86%		7.30%
Control Effectiveness			1.0305		1.0921		1.0091		1.1433		1.0186		1.0730

Fig. 7. Minimum Hankel Singular Values of the Electrical Dynamics with TCSC

	No SVC	SVC at Bus 4		SVC at Bus 5		SVC at Bus 6		SVC at Bus 7		SVC at Bus 8		SVC at Bus 9	
	min HSV	min HSV	Improvement	min HSV	Improvement	min HSV	Improvement	min HSV	Improvement	min HSV	Improvement	min HSV	Improvement
Normal	1.881E-03	2.150E-03	14.30%	2.072E-03	10.15%	2.005E-03	6.63%	2.130E-03	13.25%	2.075E-03	10.34%	2.066E-03	9.87%
Line45 Removed	3.312E-04	3.478E-04	5.01%	3.595E-04	8.54%	3.433E-04	3.65%	3.872E-04	16.90%	3.793E-04	14.50%	3.701E-04	11.73%
Line57 Removed	5.950E-03	6.702E-03	12.64%	6.380E-03	7.23%	6.433E-03	8.11%	6.380E-03	7.23%	6.409E-03	7.71%	6.556E-03	10.18%
Line46 Removed	4.923E-04	5.294E-04	7.53%	5.277E-04	7.18%	5.121E-04	4.02%	5.651E-04	14.78%	5.546E-04	12.66%	5.485E-04	11.41%
Line69 Removed	5.378E-03	6.117E-03	13.75%	5.987E-03	11.32%	5.638E-03	4.84%	6.083E-03	13.11%	5.827E-03	8.35%	5.683E-03	5.67%
Line78 Removed	7.853E-04	9.067E-04	15.47%	8.561E-04	9.02%	8.501E-04	8.26%	8.270E-04	5.32%	8.256E-04	5.13%	8.433E-04	7.39%
Line89 Removed	2.411E-03	2.749E-03	14.04%	2.684E-03	11.34%	2.523E-03	4.64%	2.685E-03	11.38%	2.587E-03	7.31%	2.467E-03	2.31%
min. % Improvement			5.01%		7.18%		3.65%		5.32%		5.13%		2.31%
Control Effectiveness			1.0501		1.0718		1.0365		1.0532		1.0513		1.0231

Fig. 8. Minimum Hankel Singular Values of the Generator Dynamics with SVC

	No TCSC	TCSC at Line 45		TCSC at Line 57		TCSC at Line 46		TCSC at Line 69		TCSC at Line 78		TCSC at Line 89	
	min HSV	min HSV	Improvement	min HSV	Improvement	min HSV	Improvement	min HSV	Improvement	min HSV	Improvement	min HSV	Improvement
Normal	1.881E-03	2.125E-03	12.97%	1.955E-03	3.94%	1.896E-03	0.79%	1.896E-03	0.80%	1.945E-03	3.41%	1.895E-03	0.74%
Line45 Removed	3.312E-04	N/A		8.816E-04	166.15%	3.427E-04	3.46%	3.313E-04	0.01%	3.315E-04	0.07%	3.836E-04	15.81%
Line57 Removed	5.950E-03	1.121E-02	88.37%	N/A		6.401E-03	7.57%	7.429E-03	24.85%	8.064E-03	35.53%	6.141E-03	3.21%
Line46 Removed	4.923E-04	5.942E-04	20.70%	4.958E-04	0.70%	N/A		6.346E-04	28.91%	5.813E-04	18.08%	4.925E-04	0.04%
Line69 Removed	5.378E-03	6.359E-03	18.24%	7.118E-03	32.37%	6.584E-03	22.43%	N/A		5.404E-03	0.49%	5.649E-03	5.05%
Line78 Removed	7.853E-04	9.505E-04	21.04%	1.032E-03	31.39%	8.485E-04	8.05%	7.855E-04	0.04%	N/A		1.079E-03	37.35%
Line89 Removed	2.411E-03	2.966E-03	23.01%	2.445E-03	1.43%	2.424E-03	0.53%	2.445E-03	1.41%	3.435E-03	42.49%	N/A	
min. % Improvement			12.97%		0.70%		0.53%		0.01%		0.07%		0.04%
Control Effectiveness			1.1297		1.0070		1.0053		1.0001		1.0007		1.0004

Fig. 9. Minimum Hankel Singular Values of the Generator Dynamics with TCSC

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