

逻辑回归

数据: $(X, Y), X \in R^n, Y \in \{0, 1\}$

模型: $P_1 = P(y_i = 1|x_i) = \frac{1}{1+e^{-Wx_i}}$

$$P_0 = P(y_i = 0|x_i) = \frac{e^{-Wx_i}}{1+e^{-Wx_i}}$$

$$P(y_i|x_i) = P_1^{y_i} P_0^{1-y_i}$$

MLE求解:

$$\begin{aligned}\hat{w} &= \arg \max_w \log P(Y|X) \\ &= \arg \max_w \log \prod P(y_i|x_i) \\ &= \arg \max_w \sum \log P(y_i|x_i) \\ &= \arg \max_w \sum \log P_1^{y_i} P_0^{1-y_i} \\ &= \arg \max_w \sum (y_i \log P_1 + (1 - y_i) \log P_0)\end{aligned}$$

得到交叉熵损失:

$$loss = - \sum (y_i \log P_1 + (1 - y_i) \log P_0)$$