Solution 1

(a) Based on the lecture 2 slide, we have:

$$I \leftarrow gI^0 + g\sqrt{I^0}\epsilon_1 + \sqrt{(g^2\sigma_{2a}^2 + \sigma_{2b}^2)}\epsilon_2$$

and we know that:

$$I = gI^0 + \epsilon$$

so, we have:

$$\epsilon = g\sqrt{I^0}\epsilon_1 + \sqrt{(g^2\sigma_{2a}^2 + \sigma_{2b}^2)}\epsilon_2$$

then the variance of ϵ, which is δ^2 is:

$$\begin{aligned}\delta^2 &= (g\sqrt{I^0})^2 + (\sqrt{(g^2\sigma_{2a}^2 + \sigma_{2b}^2)})^2 \\ &= g^2 I^0 + g^2\sigma_{2a}^2 + \sigma_{2b}^2\end{aligned}$$

(b) Since we know that if we chose to have a shorter exposure time $\frac{T}{k}$, the corresponding ideal intensity would also be $\frac{I^0}{k}$, and also we would scale up the amplification factor to $g \times k$, so we can have:

$$\begin{aligned}I &= gk\frac{I^0}{k} + gk\sqrt{\frac{I^0}{k}}\epsilon_1 + \sqrt{(g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2)}\epsilon_2 \\ &= gI^0 + g\sqrt{I^0 k}\epsilon_1 + \sqrt{(g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2)}\epsilon_2\end{aligned}$$

then the expected variance δ^2 of noise ϵ is:

$$\begin{aligned}\delta^2 &= (g\sqrt{I^0 k})^2 + (\sqrt{(g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2)})^2 \\ &= g^2 I^0 k + g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2\end{aligned}$$

(c) We know that $I = \frac{\sum_{i=1}^{k} I_i}{k}$, and also from (b) we know that:

$$I_i = gI^0 + g\sqrt{I^0 k}\epsilon_1 + \sqrt{(g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2)}\epsilon_2$$

and we also know that δ_i^2 is:

$$\delta_i^2 = g^2 I^0 k + g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2$$

then, we have:

$$\epsilon = \frac{1}{k}(\epsilon_1 + \epsilon_2 + ... + \epsilon_k)$$

so, the noise variance δ^2 of this case should be:

$$\begin{aligned}\delta^2 &= \frac{k}{k^2}(g^2 I^0 k + g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2) \\ &= g^2 I^0 + g^2k\sigma_{2a}^2 + \frac{1}{k}\cdot\sigma_{2b}^2\end{aligned}$$

(d) I think it really depends. If we have a constant time budget, we need to compare two equations, which are:

$$\delta^2 = g^2 I^0 + g^2\sigma_{2a}^2 + \sigma_{2b}^2$$

and,

$$\delta^2 = g^2 I^0 + g^2k\sigma_{2a}^2 + \frac{1}{k}\cdot\sigma_{2b}^2$$

It is obviously that when $g^2k\sigma_{2a}^2$ scales up, then $\frac{1}{k}\cdot\sigma_{2b}^2$ scales down.So, in order to determine which strategy is better depends on k, σ_{2a} and σ_{2b}, if $k\sigma_{2a}^2 < \sigma_{2b}^2$, taking a single shot is preferable, if $k\sigma_{2a}^2 > \sigma_{2b}^2$, then taking k shots is preferable.

Solution 2

The input image is shown in Figure 1.

Figure 1: Input image for problem 2.

The output image is shown in Figure 2.

Figure 2: Output image for problem 2.

Solution 3

(a) The input image is shown in Figure 3 and the output image is shown in Figure 4.

Figure 3: Input image for problem 3a.

Figure 4: Output image for problem 3a.

(b) Figure 5 to Figure 7 show images before NMS corresponding to three different thresholds, and Figure 8 to Figure 10 show image after NMS corresponding to three different thresholds.

Figure 5: Problem 3_b_0 under T0 before NMS

Figure 6: Problem 3_b_1 under T1 before NMS

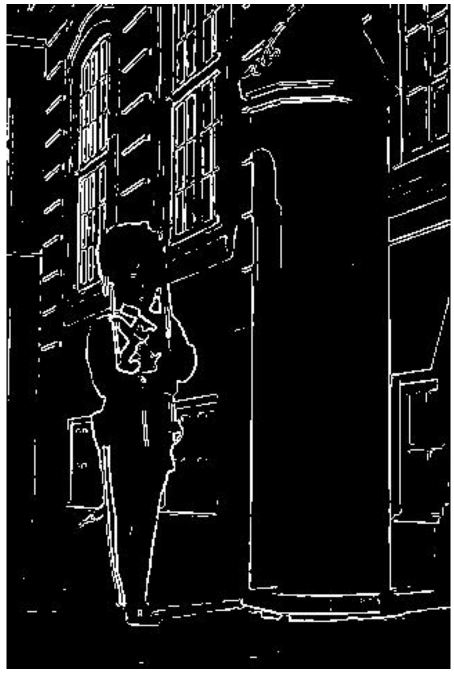

Figure 7: Problem 3_b_2 under T2 before NMS

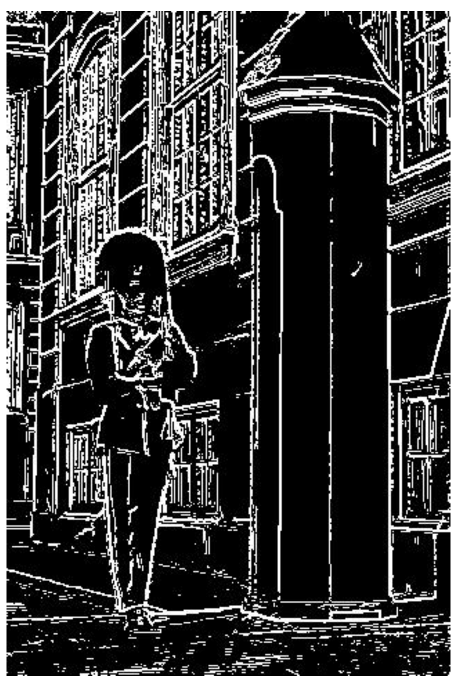

Figure 8: Problem 3_b_0 under T0 after NMS

Figure 9: Problem 3_b_1 under T1 after NMS

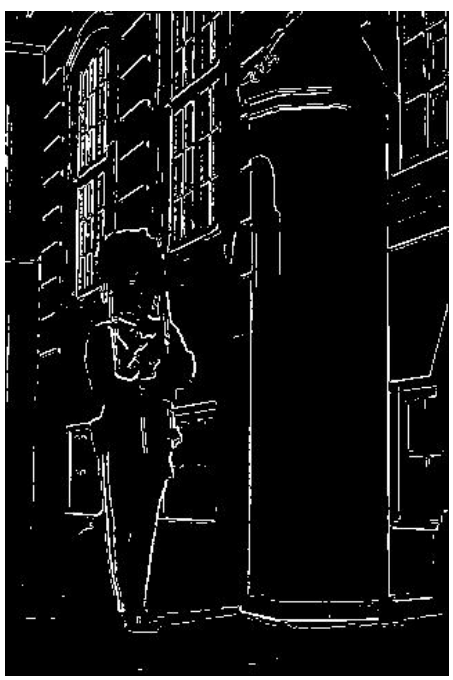

Figure 10: Problem 3_b_2 under T2 after NMS

Solution 4

Figures below show the result of Bilateral filtering, Figure 11 to Figure 13, they are affected by different parameters, and Figure 14 shows the result of repeated bilateral filtering, smooth but blur, and Figure 15 shows the result of more repeated time and more noise and make the image more smooth but more blur.

Figure 11: Problem 4_1_a result

Figure 12: Problem 4_1_b result

Figure 13: Problem 4_1_c result

Figure 14: Problem 4_1 repeated result

Figure 15: Problem 4_2 repeated result

Solution 5

(a) From the slide, we have the discrete 2D Fourier Transform as below:

$$F[\mu,\nu] = \frac{1}{WH}\sum_{n_x=0}^{W-1}\sum_{n_y=0}^{H-1} X[n_x,n_y]exp(-j2\pi(\frac{\mu n_x}{W}+\frac{\nu n_y}{H}))$$

we know that $F[\mu,\nu] = F[\mu+W,\nu] = F[\mu,\nu+H]$ because of periodicity, therefore, we typically store $F[\mu,\nu]$ for $\mu \in \{0,...,W-1\}$, $\nu \in \{0,...,H-1\}$. So, within one period, we have $F[-\mu,-\nu] = F[W-\mu,H-\nu]$, which indicates that $F[\mu,\nu]$ is central symmetry, so we can only store half of $F[\mu,\nu]$. The scalars can be stored discuss as follows:

1. When W_x is even and H_x is even, we can store $\mu \in \{0,..., \frac{W-2}{2}\}$ and $\nu \in \{0,...,H-1\}$, and the other half of the scalars can be recovered by $\mu' = W-\mu$, $\nu' = H-\nu$ for $\mu \in \{0,..., \frac{W-2}{2}\}$ and $\nu \in \{0,...,H-1\}$.
2. When W_x is odd and H_x is even, we can store $\mu \in \{0,...,W-1\}$ and $\nu \in \{0,..., \frac{H-2}{2}\}$, and the other half of the scalars can be recovered by $\mu' = W-\mu$, $\nu' = H-\nu$ for $\mu \in \{0,...,W-1\}$ and $\nu \in \{0,..., \frac{H-2}{2}\}$.
3. When W_x is even and H_x is odd, we can store $\mu \in \{0,..., \frac{W-2}{2}\}$ and $\nu \in \{0,...,H-1\}$, and the other half of the scalars can be recovered by $\mu' = W-\mu$, $\nu' = H-\nu$ for $\mu \in \{0,..., \frac{W-2}{2}\}$ and $\nu \in \{0,...,H-1\}$.
4. When W_x is odd and H_x is odd, we can store $\mu \in \{0,..., \frac{W-1}{2}\}$ and $\nu \in \{0,...,H-1\}$, and the other half of the scalars can be recovered by $\mu' = W-\mu$, $\nu' = H-\nu$ for $\mu \in \{0,..., \frac{W-3}{2}\}$ and $\nu \in \{0,...,H-1\}$.

(b) Figure 16 is the result of convolution in the Fourier domain.

Figure 16: Problem 5 convolution in the Fourier domain

Solution 6

(a) Figure 17 shows the result of Harr wavelet decomposition.

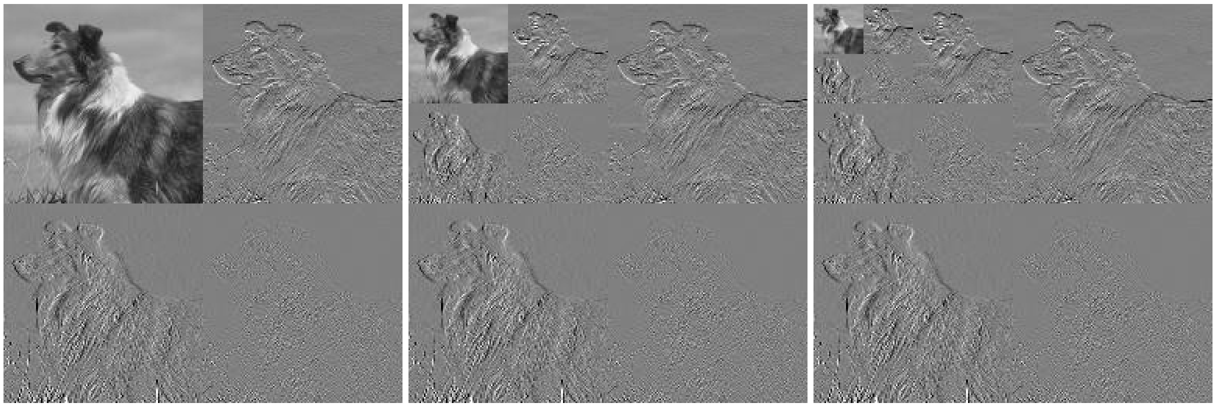

Figure 17: Problem 6a Harr wavelet decomposition

(b) Figure 18 shows the result of reconstruction from Harr wavelet decomposition, and it is fully reconstructed because it is just the same as the original image. Figure 19 shows the result of zeroed out the finest levels from 0 to 2.

Figure 18: Problem 6b Reconstruction from Harr wavelet decomposition

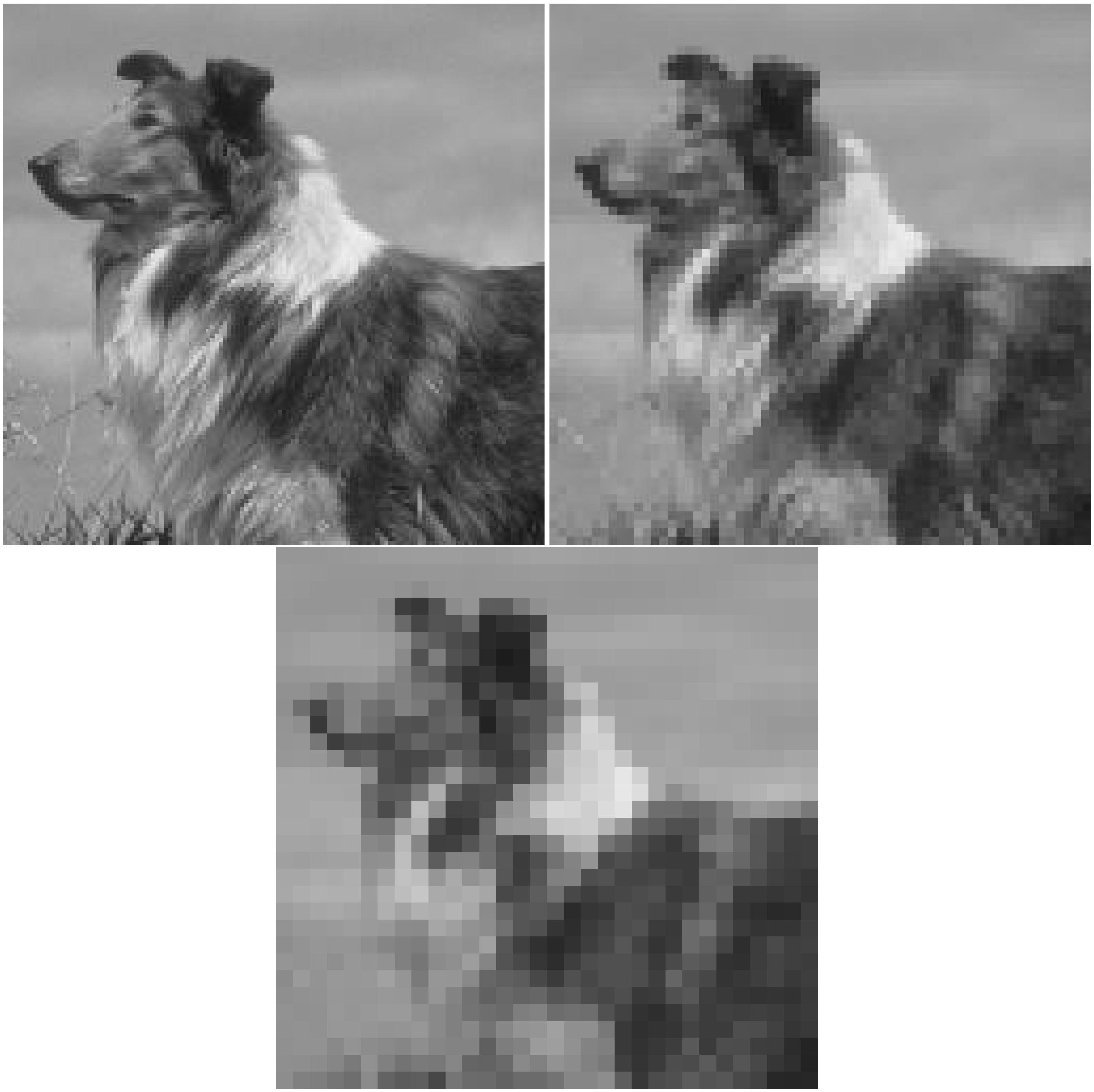

Figure 19: Problem 6b Reconstruction from Harr wavelet decomposition

Information

This problem set took approximately 42 hours of effort.

I discussed this problem set with:

- Jiayao Cheng
- Yukun Li

I also got hints from the following sources:

- Wikipedia article on matrix calculus at https://en.wikipedia.org/wiki/Matrix_calculus
- Read numpy tutorial from http://cs231n.github.io/python-numpy-tutorial/
- Read about histogram equalization from https://www.math.uci.edu/icamp/courses/math77c/demos/hist_eq.pdf
- Read about grayscale information from https://en.wikipedia.org/wiki/Grayscale
- Some hints for problem 4 are from TAs and piazza.
- Read about bilateral filtering from http://www.numerical-tours.com/matlab/denoisingadv_8_bilateral/