

Problem 1(20 points)

Consider the initial-value problem $y' = y$, $y(0) = 1$

- (a) Write the iterative expressions for the following three methods: (1) the Euler method, (2) the trapezoidal method, and (3) the predictor-corrector using the Euler and trapezoidal as the predictor and corrector, respectively.
 (b) For $h=0.1$, list in a table the calculated results using these three methods for the first four steps, as well as the error at each point.

(a)

(1) $y(i+1)=y(i)+hy(i)$

(2) $y(i+1)=y(i)+\frac{h}{2}(y(i)+y(i+1)) \rightarrow y(i+1)=\frac{2+h}{2-h}y(i)$

(3) predictor: $y(i+1)=y(i)+hy(i)$; corrector: $y(i+1)=y(i)+\frac{h}{2}(y(i)+y(i+1))$.

(b)

| Method | $y = e^x$ | Euler method | | Trapezoidal method | | predictor-corrector | |
|--------|-----------|--------------|---------|--------------------|------------------|---------------------|------------------|
| | | result | error | result | error 1.0e-03 | result | error 1.0e-03 |
| Step 1 | 1.1052 | 1.1 | -0.0052 | 1.1053 | 0.0922 | 1.1050 | -0.1709 |
| Step 2 | 1.2214 | 1.21 | -0.0114 | 1.2216 | 0.2039 | 1.2210 | -0.3778 |
| Step 3 | 1.3499 | 1.331 | -0.0189 | 1.3502 | 0.3380 | 1.3492 | -0.6262 |
| Step 4 | 1.4918 | 1.4641 | -0.0277 | 1.4923 | 0.4981 | 1.4909 | -0.9226 |

Problem 2 (80 points)

Consider the example problem of a driven pendulum under damping in the textbook. Change the driving force to a square wave with $f_d(t) = f_0$ for $0 < T < T_0/2$ and $f_d(t) = -f_0$ for $T_0/2 < T < T_0$, where T_0 is the period of the driving force that repeats periodically.

(a) Write a computer program to solve the equation of motion by using the fourth-order Runge-Kutta method.

(2) Plot two figures (similar to Fig. 4.4 and Fig. 4.5 in the book), one for regular motion and one for chaotic motion, with different choices of the parameters.

Note:

For problem 2, please hand in a printed copy of your source code and output file along with two figures, and also send a copy of your source code to mailbox ruc_phys_guo@163.com

(a)

```
%computational-physics homework-1 (b)
%function:theta' '=fd-q*theltha'-sin(theta)
%fd=f0    when  0<t<T0/2
%fd=-f0   when  T0/2<t<T0
%q=k/m*sqrt(1/g);w0=dimensionless parameter
%-----
%depart it to two one-oder ODE
%y(1)=theta
%y(2)=theta'
%
%dy(1)/dt=y(2)
%dy(2)/dt=fd-qy(2)-sin(theta)
%-----

clc,clear
t0=0; t1=500; step=0.5;
n=(t1-t0)/step;
t=linspace(t0,t1,n);
y1=zeros(1,length(t));
y2=zeros(1,length(t));
y1(1)=0;    %theta=0
y2(1)=1.4;   %theta'=2
%²ÎËý%q,T0,f0
q=0.5;
T0=100;
f0=2.55 ;
for i=1:length(t)-1
    %-----
```

```

k11=y2(i);
k12=y2(i)+step/2*k11;
k13=y2(i)+step/2*k12;
k14=y2(i)+step*k13;

y1(i+1)=y1(i)+step/6*(k11+2*k12+2*k13+k14);
%-----
if mod(i*step,T0)<T0/2
    fd=f0;
else
    fd=-f0;
end

k21=fd-q*y2(i)-sin(y1(i));
if mod(i*step,T0)+step/2<T0/2
    fd=f0;
else
    fd=-f0;
end
k22=fd-q*(y2(i)+0.5*step*k21)-sin(y1(i)+0.5*step*k11);
k23=fd-q*(y2(i)+0.5*step*k22)-sin(y1(i)+0.5*step*k12);
if mod(i*step,T0)+step<T0/2
    fd=f0;
else
    fd=-f0;
end
k24=fd-q*(y2(i)+step*k23)-sin(y1(i)+step*k13);

y2(i+1)=y2(i)+step/6*(k21+2*k22+2*k23+k24);

if y1(i+1)>0

    if mod(y1(i+1),2*pi)<pi
        y1(i+1)=mod(y1(i+1),2*pi);
    else
        y1(i+1)=mod(y1(i+1),2*pi)-2*pi;
    end

else
    if (-mod(-y1(i+1),2*pi))>-pi
        y1(i+1)=-mod(-y1(i+1),2*pi);
    else
        y1(i+1)=-mod(-y1(i+1),2*pi)+2*pi;
    end
end

```

```

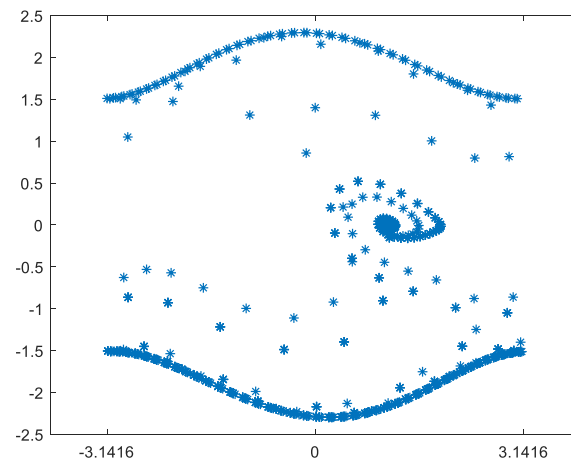
end
end
%-----
end
plot(y1,y2,'*')
set(gca,'XTick',[-2*pi:pi:2*pi])

```

(b)

One for regular motion

选取参数 $q=0.5, t_1=500, T_0=100, f_0=0.9$, step=0.5



One for chaotic motion,

选取参数 $q=0.5, t_1=500, T_0=100, f_0=2.55$, step=0.5

