Physics Bowl D1 + D2 Cheat Sheets

Study Guide for Physics Bowl D1/D2

Version 1.7

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A Letter to the Reader

Dear Reader,

Thank you for opening this guide. It aspires to be a quiet companion on your learning journey. Physics may seem like a towering mountain from afar, but up close, it resembles a garden.

May this book encourage you to pause and admire the flowers by the roadside, to look up at the starry sky and moonlight.

D1 is like each new encounter; D2 invites you to take one more step: perform a small derivation, use an approximation, or see the same landscape with greater clarity. Don't fear D2—view it as an invitation to pause, ask "Why?", and then lift your gaze to take in the scenery.

If you get stuck, try these three things: sketch another diagram; study a simpler boundary case first; change only one variable at a time. Most confusions will eventually clear.

May this guide reward your Curiosity, Creativity, and Caution. And may the world hold just a little more wonder. Wishing you smooth learning and steady progress.

Ray Zhou

Reading Instructions

- **Purpose**: This guidebook is a study guide to Physics Bowl D1/D2. Use it to go over key concepts and trends, then practice.
- **Structure**: Material is divided into parts (Mechanics, Dynamics, Waves/EM, Optics). Inside each part, boxes contain formulas, heuristics, and solutions.
- **Practice pointers**: The list items are metadata only (Contest/Year/Division/Problem/Page). Problem statements are in official sources.
- **Navigation**: There are hyperlinks provided throughout the PDF to facilitate faster jumps.

Difficulty Legend

- **D1**: Baseline knowledge and methods; no calculus derivations; aimed for D1 questions.
- **D2**: More substantial topics and brief derivations (calculus/ODE/approximations); aimed for D2 or when principles are required.
- *D1 chapters may list some approximation results for memorization purposes alone; derivations will be at **D2**.*

Notation & Conventions

- **Units**: SI as default. Gravitation on Earth: $g = 9.8\,\mathrm{m/s^2}$ unless otherwise stated. Always $\mathrm{m/s^2}$ (or $\mathrm{m\,s^{-2}}$). Angles: radians except when a symbol for degrees is shown.
- **Vectors**: Vectors are denoted by arrows (e.g., $\vec{v}$); magnitudes $v = \|\vec{v}\|$. Use components v_x, v_y with $\hat{\imath}, \hat{\jmath}$.
- **emf vs field**: Use $\mathcal{E}$ for emf (scalar); $\vec{E}$ for electric field (vector). When magnitude is intended, write $|\vec{E}|$. *Example (Faraday–Lenz):* $\mathcal{E} = \oint_{\partial S} \vec{E} \cdot d\vec{l}$ (emf) is to be distinguished from the field $\vec{E}$.
- **Common symbols**: Circuits use V (voltage), I (current), R (resistance), C (capacitance). Induced emf uses $\mathcal{E}$ (not E). Electric field is $\vec{E}$. Waves use v (speed in medium), f (frequency), λ (wavelength).
- **Overloaded symbols**: P for pressure (fluids) or power (circuits) depending on context. Thermodynamics: $\Delta U = Q + W$ (convention: $W > 0$ means work done on the system).
- **Sign conventions**: Choose axes first and maintain consistency. For Doppler in a stationary medium, prefer the rule "approach increases frequency; recession decreases" and employ a sign diagram; see the Waves unit for the formula being given in detail. For KVL, direction of tracing loop; voltage rises are positive.

Disclaimer

This *"Physics Bowl D1 + D2 Cheat Sheets"* (Version 1.7 by Ray Zhou) is an independent, non-official study guide created for educational purposes only. It summarizes key concepts, formulas, heuristics, and original solutions based on standard physics principles and publicly available Physics Bowl metadata (e.g., Contest/Year/Division/Problem/Page).

Full problem statements are sourced from official AAPT materials—please visit the American Association of Physics Teachers (AAPT) website at aapt.org/programs/physicsbowl for complete exams and answers.

This guide is provided "as is" without warranties of accuracy or completeness. It is not endorsed by AAPT and should not be used as a substitute for official resources. Users should verify information independently.

For feedback or questions, contact zhouxinrui2025@163.com, or visit the project on GitHub: `https://github.com/REPLACE_WITH_LINK`.

Contents

Part I: Mechanics

1 Unit 1: Kinematics

Coverage checklist: Vectors vs scalars; constant-acceleration (SUVAT); projectile motion (same-level range/time/height); uniform circular motion ($a_c = v^2/r$, $T = 2\pi r/v$); relative motion; calculus forms $v = dx/dt$, $a = dv/dt$ **D2**; areas under v–t and a–t graphs **D2**

1.1 Vectors & Components D1

Formulas & Concepts

Concept explanation: Vectors have both magnitude and direction. The decomposition of a vector into orthogonal components translates geometric relations into algebraic relations such that calculations are simple component arithmetic.
Core formulas:

$$\begin{cases} \vec{v} = (v_x, v_y) \text{ (2D)}, \quad v = \|\vec{v}\| = \sqrt{v_x^2 + v_y^2}, \\ \text{Unit vectors: } \hat{\imath} = (1,0),\ \hat{\jmath} = (0,1),\ \vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}. \end{cases}$$

Variable definitions: v_x, v_y scalar components; v speed (magnitude of velocity); $\hat{\imath}, \hat{\jmath}$ orthonormal basis.
Prerequisites & scope: Axes must be orthogonal for Pythagorean magnitude; extend to 3D with $\hat{k}$ and $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$.

Heuristics & Pitfalls

- Align axes with directions of steepest slopes and launch angles to reduce components.
- Separate vectors before writing equations (Newton's laws, kinematics) to prevent late trigonometry.

- Mixing magnitude and component equations. Solution: write separate equations for x and y and add if applicable.

Problem #1. Resolve velocity vector $\vec{v} = (30\,\mathrm{m/s}, 40\,\mathrm{m/s})$ into magnitude and direction.

Solution

$v = \sqrt{30^2 + 40^2} = 50\,\mathrm{m/s}$. Direction $\theta = \arctan(40/30) \approx 53.1^\circ$ above the horizontal.

Problem #2. A force of $100\,\mathrm{N}$ acts at 30° to the horizontal. Find its horizontal and vertical components.

Solution

$F_x = 100\cos 30^\circ = 50\sqrt{3}\,\mathrm{N}$, $F_y = 100\sin 30^\circ = 50\,\mathrm{N}$.

1.2 Constant Acceleration (SUVAT) [D1]

Formulas & Concepts

Concept explanation: Position and velocity are quadratic/linear in time with constant acceleration, providing closed-form relations (SUVAT equations).
Core formulas:

$$\begin{cases} x = x_0 + v_0 t + \frac{1}{2}at^2, \\ v = v_0 + at, \\ v^2 = v_0^2 + 2a\,(x - x_0). \end{cases}$$

Variable definitions: x position; v velocity; a constant acceleration; subscript 0 initial value; t time; $\Delta x = x - x_0$.
Prerequisites & scope: Acceleration must be constant over the interval; otherwise use calculus forms.

Heuristics & Pitfalls

- Choose the equation that excludes the unknown you lack (e.g., use v^2-form when time is absent).
- Work symbolically until the end to avoid compounding rounding errors.
- Applying constant-a formulas when $a = a(t)$ or $a = a(v)$. Fix: switch to calculus forms or energy methods.

Problem #3. The velocity of a cart is given by the piecewise function $v(t) = \begin{cases} at, & 0 \le t \le T/2, \\ aT/2, & T/2 < t \le T, \end{cases}$ with constant $a > 0$. Find the total displacement in time T.

Solution

Area under v–t: first triangle area $\frac{1}{2}(aT/2)(T/2) = \frac{aT^2}{8}$; second rectangle area $(aT/2)(T/2) = \frac{aT^2}{4}$. Total $\Delta x = \frac{3aT^2}{8}$.

Problem #4. A car accelerates uniformly from rest to 20 m/s in 4 s. Find the acceleration and distance traveled.

Solution

$a = \dfrac{v - v_0}{t} = \dfrac{20 - 0}{4} = 5\,\mathrm{m/s^2}$. Distance $\Delta x = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2} \cdot 5 \cdot 16 = 40\,\mathrm{m}$.

1.3 Projectile (Equal Heights) D1

Formulas & Concepts

Concept explanation: Horizontal and vertical motions independent in uniform gravity. Decomposition into components provides closed-form solutions for flight time, range, and peak height at the same launch/landing height.

Core formulas:

$$\begin{cases} T = \dfrac{2v_0 \sin\theta}{g}, \\ R = \dfrac{v_0^2 \sin 2\theta}{g}, \\ H = \dfrac{v_0^2 \sin^2\theta}{2g}. \end{cases}$$

Variable definitions: v_0 launch speed; θ launch angle; g gravitational acceleration; T flight time; R range; H apex height.

Prerequisites & scope: Launch/landing at equal heights; neglect air resistance; for unequal heights, solve quadratic in t.

Heuristics & Pitfalls

- Separate x and y equations; eliminate t or use symmetry about the apex for time splits.
- For maximum range at equal heights, use $\theta = 45^\circ$.
- Using R formula when launch and landing heights differ. Fix: solve general quadratic and then compute $x(T)$.

Problem #5. A projectile is launched at speed v_0 and angle θ from flat ground and lands at the same height. Express the time of flight T, the maximum height H, and the range R in terms of v_0, θ, g.

Solution

$T = \dfrac{2v_0 \sin\theta}{g}$, $H = \dfrac{v_0^2 \sin^2\theta}{2g}$, $R = \dfrac{v_0^2 \sin 2\theta}{g}$, by separating vertical and horizontal components and using constant-g kinematics.

Problem #6. Two projectiles launched with the same v_0 at angles θ and $90° - \theta$ have equal ranges. What is the ratio of their maximum heights $H_\theta / H_{90°-\theta}$?

Solution

$H = \dfrac{v_0^2 \sin^2\theta}{2g}$. Hence $\dfrac{H_\theta}{H_{90°-\theta}} = \dfrac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$.

1.4 Uniform Circular Motion D1

Formulas & Concepts

Concept description: In uniform speed circular motion, the acceleration is centripetal (centerwards) with magnitude v^2/r; period and speed are related by circumference.

Core formulas:

$$\begin{cases} a_c = \dfrac{v^2}{r} = \dfrac{4\pi^2 r}{T^2}, \\ T = \dfrac{2\pi r}{v}. \end{cases}$$

Variable definitions: r radius; v speed; T period; a_c centripetal acceleration.
Prerequisites & scope: Speed constant; acceleration direction changes; for non-uniform circular motion add tangential component.

Heuristics & Pitfalls

- Draw radial/tangential components explicitly; set $a_t = 0$ for UCM.
- Vertical circle normals: at the top, $T + mg = mv^2/r$; at the bottom, $T - mg = mv^2/r$.
- Minimum speed at the top for a taut string (no slack): $v_{\text{top}} \geq \sqrt{gr}$ (else $T = 0$ at the top).

- Treating centripetal force as an extra force. Fix: centripetal is the net radial component of existing forces.

Problem #7. A bead moves in a circle of radius r with period T. Compute its speed and centripetal acceleration.

Solution

$v = \dfrac{2\pi r}{T}$ and $a_c = \dfrac{v^2}{r} = \dfrac{4\pi^2 r}{T^2}$.

1.5 Relative Motion [D1]

Formulas & Concepts

Concept explanation: Relative velocity cancels observer motion: the velocity of A relative to B is $\vec{v}_A - \vec{v}_B$.
Core formula:

$$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B.$$

Variable definitions: $\vec{v}_{A/B}$ velocity of A relative to B; $\vec{v}_A, \vec{v}_B$ velocities in an inertial frame.
Prerequisites & scope: Within Galilean (non-relativistic) regime; in 2D/3D apply component-wise.

Heuristics & Pitfalls

- Move to the target's rest frame to align directions and simplify timing.
- For winds/rivers, remove drift by aiming to oppose the current component.
- Boundary condition: if $v \le u$, landing directly across is not possible; you will be swept downstream and must land downstream.

- Superposing speeds scalarly when directions are different. Remedy: take difference of vectors component-wise.

Problem #8. A river of width W flows east at speed u. A boat of speed v relative to water aims at angle α north of west to land directly across. Find α and the crossing time, assuming $v > u$.

Solution

Require zero east drift: west component equals current, so $v\cos\alpha = u$ and $\alpha = \arccos(u/v)$. North component is $v\sin\alpha = \sqrt{v^2 - u^2}$, so time $t = \dfrac{W}{\sqrt{v^2 - u^2}}$.

1.6 Calculus Forms & Graph Areas D2

Formulas & Concepts

Concept explanation: With acceleration varying, kinematics is found by integrating velocity and acceleration; graph areas are summaries of change.

Core formulas:

$$\begin{cases} v = \dfrac{dx}{dt}, \quad a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}. \\ v(t) = v(t_0) + \displaystyle\int_{t_0}^{t} a(\tau)\,d\tau, \quad x(t) = x(t_0) + \displaystyle\int_{t_0}^{t} v(\tau)\,d\tau. \\ \Delta x = \displaystyle\int v\,dt \text{ (area under } v\text{–}t), \quad \Delta v = \displaystyle\int a\,dt \text{ (area under } a\text{–}t). \\ \text{Chain rule: } a = \dfrac{dv}{dt} = \dfrac{dv}{dx}v \;\Rightarrow\; v\,dv = a(x)\,dx. \\ \text{If } a = \text{const: } v^2 = v_0^2 + 2a\,(x - x_0). \end{cases}$$

Variable definitions: x, v, a position/velocity/acceleration; t time; integrals are definite over the time interval.

Prerequisites & scope: Differentiability over interval; interpret signed areas for direction-sensitive quantities.

Heuristics & Pitfalls

- Read slopes of x–t as v and slopes of v–t as a; use areas for accumulated change.
- When $a = a(v)$ or $a = a(x)$, separate variables via $v\,dv = a\,dx$ to avoid time explicitly.
- If motion crosses turning points where $v = 0$, integrate piecewise and track signs to avoid taking an incorrect branch.
- Confusing displacement with distance on v–t when v changes sign. Fix: integrate absolute value for distance.

Problem #9. An object has acceleration $a(t) = a_0 + bt$ with constants a_0, b. If $x(0) = 0$ and $v(0) = v_0$, find $x(t)$.

Solution

Integrate: $v(t) = v_0 + \int_0^t (a_0 + b\tau)\,d\tau = v_0 + a_0 t + \frac{1}{2}bt^2$. Then $x(t) = \int_0^t v(\tau)\,d\tau = v_0 t + \frac{1}{2}a_0 t^2 + \frac{1}{6}bt^3$.

Problem #10. A particle obeys $\dfrac{dx}{dt} = k\sqrt{x}$ with $k > 0$ and $x(0) = 0$. Find $x(t)$.

Solution

Separate: $\dfrac{dx}{\sqrt{x}} = k\,dt \Rightarrow 2\sqrt{x} = kt + C$. With $x(0) = 0$, $C = 0$. Hence $x(t) = \dfrac{k^2t^2}{4}$ and $v(t) = \dfrac{k^2t}{2}$.

2 Unit 2: Newtonian Dynamics

Coverage checklist: Newton's laws; common forces (weight, spring, friction, normal); free-body diagrams; non-uniform circular motion; variable force and impulse

2.1 Newton's Laws and Free-Body Diagrams [D1]

Formulas & Concepts

$$\left\{ \sum \vec{F} = m\vec{a} \right. \text{; action–reaction pairs on separate bodies.}$$

Heuristics & Pitfalls

- Split each body; sketch clean FBDs and project along convenient directions.
- Don't place action–reaction pairs on the same drawing; they're on separate bodies and cancel only at the system level.
- Use radial/tangential axes for curves or inclined axes on ramps; use friction direction as unknown and solve for its sign.
- For connected bodies (strings/pulleys), apply kinematic constraints (e.g., equal string lengths imply proportional accelerations) and apply one tension per ideal massless string.
- In non-inertial frames (elevators, speeding cars), add $-m\,\vec{a}_{\text{frame}}$ only if you change frames explicitly.

Problem #1. A 5 kg block rests on a horizontal surface. A horizontal force $F = 20\,\mathrm{N}$ is applied. Find its acceleration if friction is negligible.

Solution

$\sum F = ma \Rightarrow 20 = 5a \Rightarrow a = 4\,\mathrm{m/s^2}$.

Problem #2. A 2 kg block hangs from a string. Find the tension in the string when the block is at rest.

Solution

$\sum F_y = 0 \Rightarrow T - mg = 0 \Rightarrow T = 2 \cdot 9.8 = 19.6\,\mathrm{N}$.

2.2 Friction and Springs [D1]

Formulas & Concepts

$$\begin{cases} \text{Friction: } f_s \le \mu_s N \text{ (variable up to max)}, \quad f_k = \mu_k N, \\ \text{Spring force: } F = -kx \text{ (Hooke) within elastic limit.} \end{cases}$$

Heuristics & Pitfalls

- Assume the direction of static friction to be unknown; solve and interpret its sign from the solution.
- Apply $f_s \le \mu_s N$; on the verge of sliding, take $f_{s,\max} = \mu_s N$. For steady sliding, use $f_k = \mu_k N$.
- For springs in series, use k_{eq}: in series $\frac{1}{k_{\mathrm{eq}}} = \sum \frac{1}{k_i}$, in parallel $k_{\mathrm{eq}} = \sum k_i$.

Problem #3. A 3 kg block on a horizontal surface has $\mu_k = 0.2$. Find the kinetic friction force when the block slides.

Solution

$N = mg = 3 \cdot 9.8 = 29.4\,\mathrm{N}$. $f_k = \mu_k N = 0.2 \cdot 29.4 = 5.88\,\mathrm{N}$.

Problem #4. A spring with constant $k = 200\,\text{N/m}$ is compressed by $x = 0.1\,\text{m}$. Find the restoring force.

Solution

$F = -kx = -200 \cdot 0.1 = -20\,\text{N}$ (opposite to compression).

2.3 Non-uniform Circular Motion D1

Formulas & Concepts

$$\Big\{\text{Radial: } \sum F_r = mv^2/r\ ;\ \text{tangential: } \sum F_t = ma_t.$$

Heuristics & Pitfalls

Split radial and tangential components explicitly; centripetal is net radial.

Problem #5. A car of mass m rounds a curve of radius $r = 50\,\text{m}$ at $v = 20\,\text{m/s}$. Find the required centripetal force.

Solution

$F_c = \dfrac{mv^2}{r} = m\dfrac{400}{50} = 8m\,\text{N}.$

Problem #6. A ball on a string swings in a vertical circle of radius r. At the top, the tension is T and speed is v. Write the centripetal force equation.

Solution

At the top, $T + mg = \dfrac{mv^2}{r}$ (both point inward).

2.4 Impulse and Variable Forces D2

Formulas & Concepts

$$\Big\{\text{Impulse: } \vec{J} = \int \vec{F}\,dt = \Delta\vec{p}.$$

Heuristics & Pitfalls

Extrapolate force-time profiles for varying forces; examine external impulses for system imparted momentum changes.

Problem #7. A constant force $F = 10\,\mathrm{N}$ acts on a mass for $\Delta t = 3\,\mathrm{s}$. Find the impulse delivered.

Solution

$J = F\Delta t = 10 \cdot 3 = 30\,\mathrm{N}\cdot\mathrm{s}$.

Problem #8. A particle's momentum changes from $\vec{p_i} = (5,0)\,\mathrm{kg}\cdot\mathrm{m/s}$ to $\vec{p_f} = (5,10)\,\mathrm{kg}\cdot\mathrm{m/s}$ in $2\,\mathrm{s}$. Find the average force.

Solution

$\vec{J} = \Delta\vec{p} = (0,10)\,\mathrm{kg}\cdot\mathrm{m/s}$. $\vec{F}_{\mathrm{avg}} = \dfrac{\vec{J}}{\Delta t} = (0,5)\,\mathrm{N}$.

3 Unit 3: Work, Energy, Power

Coverage checklist: Work $W = \int \vec{F}\cdot d\vec{r}$; Work–Energy theorem; potential energies (gravity, spring); mechanical energy conservation; power (instantaneous vs average); conservative fields ($F_x = -dU/dx$)

3.1 Work and the Work–Energy Theorem [D1]

Formulas & Concepts

$$\begin{cases} \text{For constant } F \text{ and displacement } d \text{ at angle } \theta: \quad W = Fd\cos\theta, \\ \text{Work–Energy: } \Delta K = W_{\mathrm{net}}, \\ \text{Power: instantaneous } P = \vec{F}\cdot\vec{v}, \quad \text{average over } \Delta t: \ \bar{P} = \dfrac{\Delta W}{\Delta t}. \end{cases}$$

Heuristics & Pitfalls

- Choose the system to eliminate internal forces; only external work should be considered.
- Include nonconservative work explicitly (friction, thrust). For rolling without slipping, static friction often does no work on the rolling body.
- Check for indications by comparing the endpoints: $K_i + U_i$ vs. $K_f + U_f$.

Problem #1. A 10 kg block is pushed 5 m horizontally by a constant force $F = 30\,\text{N}$. Find the work done.

Solution

$W = Fd\cos\theta = 30 \cdot 5 \cdot \cos 0 = 150\,\text{J}$.

Problem #2. A net force of 50 N accelerates a 5 kg mass from rest over 10 m. Find the final kinetic energy and speed.

Solution

$W_{\text{net}} = 50 \cdot 10 = 500\,\text{J}$. By work-energy theorem, $\Delta K = W_{\text{net}} = 500\,\text{J}$. Then $\frac{1}{2}mv^2 = 500 \Rightarrow v = \sqrt{200} = 10\sqrt{2}\,\text{m/s}$.

3.2 Potential Energy and Conservation D1

Formulas & Concepts

$$\begin{cases} U_g = mgh \text{ (near-Earth)}, \quad U_s = \frac{1}{2}kx^2, \\ \text{Conservation (when nonconservative work zero): } K_i + U_i = K_f + U_f. \end{cases}$$

Heuristics & Pitfalls

Use reference $U = 0$ conveniently; only differences matter.

Problem #3. A 2 kg block falls $h = 5\,\text{m}$ from rest. Find its speed at the bottom using energy conservation.

Solution

$K_i + U_i = K_f + U_f \Rightarrow 0 + mgh = \frac{1}{2}mv^2 + 0 \Rightarrow v = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \cdot 5} = \sqrt{98} \approx 9.9\,\text{m/s}$.

Problem #4. A spring ($k = 100\,\text{N/m}$) is compressed by $x = 0.2\,\text{m}$. Compute the stored elastic potential energy.

Solution

$U_s = \frac{1}{2}kx^2 = \frac{1}{2} \cdot 100 \cdot 0.04 = 2\,\text{J}$.

3.3 Conservative Fields D2

Formulas & Concepts

$$F_x = -, dfracdUdx$$

Path independence in conservative fields; use energy methods when applicable.

Heuristics & Pitfalls

Use energy methods if forces are conservative; verify curl-free areas.

Problem #5. A mass moves in a potential $U(x) = \frac{1}{2}kx^2$. Find the force acting on it.

Solution

$\vec{F} = -\dfrac{dU}{dx}\,\hat{\imath} = -kx\,\hat{\imath}$ (Hooke's law in 1D).

Problem #6. Show that gravitational potential $U = -\dfrac{GMm}{r}$ gives force $F = -\dfrac{GMm}{r^2}$ (radial).

Solution

First compute $\dfrac{dU}{dr} = \dfrac{d}{dr}\Big(-GMm\,r^{-1}\Big) = +\dfrac{GMm}{r^2}$. Therefore the radial force is $\vec{F} = -\dfrac{dU}{dr}\,\hat{\mathbf{r}} = -\dfrac{GMm}{r^2}\,\hat{\mathbf{r}}$ (attractive, inward).

4 Unit 4: Momentum & Collisions

Coverage checklist: Momentum $\vec{p} = m\vec{v}$; impulse; momentum conservation; elastic/inelastic/fully inelastic collisions; center of mass; rocket equation

4.1 Momentum and Impulse D1

Formulas & Concepts

Concept explanation: Momentum measures motion; impulse is the total effect of force over time and is equal to the change in momentum.
Core formulas:

$$\begin{cases} \vec{p} = m\,\vec{v}, \\ \vec{J} = \displaystyle\int_{t_1}^{t_2} \vec{F}\,dt = \Delta\vec{p}, \\ \text{System:} \quad \vec{P} = \displaystyle\sum_i m_i\vec{v}_i, \quad \Delta\vec{P} = \vec{J}_{\text{ext}}.\ (\text{If } \vec{J}_{\text{ext}} = 0,\ \vec{P} \text{ conserved}) \end{cases}$$

Variable definitions: $\vec{p}$ momentum; $\vec{P}$ total momentum; $\vec{J}$ impulse; $\vec{F}$ external force; m mass; $\vec{v}$ velocity.
Prerequisites & scope: Valid for Newtonian mechanics; for varying mass systems, take care with momentum flux (see rockets).

Heuristics & Pitfalls

- Identify a closed system (no external impulse) to use momentum conservation immediately.
- For short-duration large forces (collisions), use impulse–momentum rather than force–time details.

- Treating internal forces as external. Fix: define the system to include interacting bodies so internal forces cancel.

Problem #1. A 4 kg object moves with velocity $\vec{v} = (6, 8)\,\text{m/s}$. Find its momentum vector and magnitude.

Solution

$\vec{p} = m\vec{v} = 4(6, 8) = (24, 32)\,\text{kg}\cdot\text{m/s}$, so $|\vec{p}| = \sqrt{24^2 + 32^2} = 40\,\text{kg}\cdot\text{m/s}$ and the direction matches $\vec{v}$.

Problem #2. A force $F(t) = 10t\,\text{N}$ acts on a particle from $t = 0$ to $t = 2\,\text{s}$. Find the impulse delivered.

Solution

$J = \int_0^2 10t\,dt = [5t^2]_0^2 = 20\,\text{N}\cdot\text{s}$.

4.2 Elastic and Inelastic Collisions D1

Formulas & Concepts

Concept explanation: Collisions conserve total momentum; elastic ones also conserve kinetic energy. The center-of-mass (COM) frame makes algebra simpler.
Core formulas (1D):

$$\begin{cases} \text{Momentum: } m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2, \\ \text{Elastic energy: } \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2, \\ \text{Result (elastic): } v_1 = \dfrac{(m_1 - m_2)u_1 + 2m_2u_2}{m_1 + m_2}, \quad v_2 = \dfrac{2m_1u_1 + (m_2 - m_1)u_2}{m_1 + m_2}, \\ \text{Completely inelastic: stick } (v_1 = v_2 = v) = \dfrac{m_1u_1 + m_2u_2}{m_1 + m_2}. \end{cases}$$

Variable definitions: u_i initial, v_i final velocities; m_i masses; all along one line (1D).
Prerequisites & scope: For oblique/2D, conserve vector momentum and use geometry; kinetic energy changes via deformation/heat in inelastic cases.

Heuristics & Pitfalls

- Switch to the COM frame where total momentum is zero; velocities reverse in elastic 1D collisions.
- For 2D glancing collisions, conserve components along orthogonal axes; use restitution or geometry for angles.

- Enforcing kinetic energy conservation in inelastic impact. Fix: only momentum is guaranteed; account for energy loss.

Problem #3. Two carts collide elastically in 1D. Mass $m_1 = 2\,\text{kg}$ at $u_1 = 5\,\text{m/s}$ hits mass $m_2 = 3\,\text{kg}$ at rest. Find v_1 and v_2.

Solution

$v_1 = \dfrac{(2-3)\cdot 5 + 2\cdot 3\cdot 0}{5} = \dfrac{-5}{5} = -1\,\text{m/s}$. $v_2 = \dfrac{2\cdot 2\cdot 5 + (3-2)\cdot 0}{5} = \dfrac{20}{5} = 4\,\text{m/s}$.

Problem #4. In a perfectly inelastic collision, $m_1 = 4\,\text{kg}$ at $u_1 = 6\,\text{m/s}$ collides with $m_2 = 2\,\text{kg}$ at rest. Find the final velocity.

Solution

$$v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{4\cdot 6 + 0}{6} = 4\,\text{m/s}.$$

4.3 Center of Mass and System Dynamics D1

Formulas & Concepts

Conceptual explanation: The center of mass (COM) is an average of the mass distribution of a system; external forces accelerate the COM as if all the mass were centered at the COM.
Core formulas:

$$\begin{cases} \vec{R} = \dfrac{\sum m_i \vec{r}_i}{\sum m_i}, \quad \vec{V} = \dot{\vec{R}} = \dfrac{\sum m_i \vec{v}_i}{M}, \\ M\,\ddot{\vec{R}} = \sum \vec{F}_{\text{ext}}. \text{ (Internal forces cancel in pairs)} \end{cases}$$

Variable definitions: $\vec{r}_i, \vec{v}_i$ positions/velocities; $M = \sum m_i$ total mass; $\vec{R}$ COM position.
Prerequisites & scope: Requires Newton's third law in internal pairs; for variable mass, include momentum flux.

Heuristics & Pitfalls

- Compute COM motion to track overall translation, then analyze internal relative motion separately.
- In explosions/fragmentation, the COM continues with pre-event velocity if external forces are negligible.

- Treating internal impulses as changing COM momentum. Fix: only external impulse changes total momentum.

Problem #5. Two masses $m_1 = 3\,\text{kg}$ at $x_1 = 0$ and $m_2 = 2\,\text{kg}$ at $x_2 = 5\,\text{m}$ lie on a line. Find the COM position.

Solution

$$X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{0 + 10}{5} = 2\,\text{m}.$$

Problem #6. A system has two equal masses m moving at velocities $\vec{v}_1 = (2, 0)$ and $\vec{v}_2 = (-1, 3)$ m/s. Find the velocity of the COM.

Solution

$$\vec{V} = \frac{m\vec{v}_1 + m\vec{v}_2}{2m} = \frac{(1,3)}{2} = (0.5, 1.5)\,\text{m/s}.$$

4.4 Variable Mass and Rockets D2

Formulas & Concepts

Concept explanation: For mass-exchange systems (rockets), momentum conservation for rocket+exhaust yields logarithmic change in velocity.
Core formulas:

$$\begin{cases} \text{Continuous: } m\,d\vec{v} = -\,\vec{v}_e\,dm, \\ \text{with } dm < 0 \text{ (mass loss), } \vec{v}_e \text{ exhaust speed relative to rocket,} \\ \text{Ideal Tsiolkovsky (1D): } \Delta v = v_e \ln \dfrac{m_i}{m_f}. \end{cases}$$

Variable definitions: $\vec{v}_e$ exhaust velocity relative to rocket (magnitude v_e); m_i, m_f initial/final mass; m instantaneous mass.
Prerequisites & scope: Neglect external forces (or include gravity drag separately); v_e constant; exhaust ejected at steady relative speed.

Heuristics & Pitfalls

- Include gravity losses by subtracting $g\,\Delta t$ from Δv when appropriate (vertical ascent approximation).
- Use staging by summing $v_e \ln(m_i/m_f)$ per stage.

- Using exhaust speed relative to Earth instead of rocket. Fix: v_e is defined relative to the rocket.

Problem #7. A rocket has initial mass $m_i = 1000\,\mathrm{kg}$, final mass $m_f = 400\,\mathrm{kg}$, and exhaust speed $v_e = 2000\,\mathrm{m/s}$. Find Δv in space (ignoring gravity).

Solution

$$\Delta v = v_e \ln \frac{m_i}{m_f} = 2000 \ln \frac{1000}{400} = 2000 \ln 2.5 \approx 1833\,\mathrm{m/s}.$$

Problem #8. A rocket ejects mass at rate $\dot{m} = 10\,\mathrm{kg/s}$ with effective exhaust speed $v_e = 1500\,\mathrm{m/s}$. Find the instantaneous thrust force.

Solution

$$\text{Thrust} = \dot{m}\, v_e = 10 \cdot 1500 = 15000\,\mathrm{N}.$$

5 Unit 5: Rotational Motion

Coverage checklist: Angular kinematics; torque ($|\boldsymbol{\tau}| = rF\sin\theta$); moment of inertia; $\tau_{\text{net}} = I\alpha$; rotational kinetic energy; angular momentum and conservation; rolling without slipping; inertia integrals

5.1 Angular Kinematics and Dynamics [D1]

Formulas & Concepts

Concept description: Rotational motion is analog to linear motion: torque is the rotational analog to force, and moment of inertia of mass.
Core formulas:

$$\begin{cases} \omega = \dfrac{d\theta}{dt}, \quad \alpha = \dfrac{d\omega}{dt}, \text{ with constant-}\alpha\text{ kinematics.} \\ \boldsymbol{\tau} = \vec{r} \times \vec{F}, \quad |\boldsymbol{\tau}| = r_\perp F = rF\sin\theta, \quad \sum \tau = I\alpha, \\ K_r = \frac{1}{2} I\omega^2, \quad P = \tau\,\omega. \end{cases}$$

Variable definitions: θ, ω, α angular position/velocity/acceleration; τ torque; I moment of inertia.
Prerequisites & scope: Rigid body about a fixed axis; I constant in time.

Heuristics & Pitfalls

- Use the perpendicular lever arm $r_\perp$ for torques; sum about convenient pivots to kill unknown forces.
- Prefer energy when forces are complicated but conservative; otherwise use $\sum \tau = I\alpha$ about the COM or a fixed axis.
- Mixing signs of torques from different reference senses. Fix: choose a positive rotation sense and stick with it.

Problem #1. A disk of radius $r = 0.5\,\mathrm{m}$ and moment of inertia $I = 2\,\mathrm{kg}\cdot\mathrm{m}^2$ experiences a net torque $\tau = 10\,\mathrm{N}\cdot\mathrm{m}$. Find its angular acceleration.

Solution

$$\alpha = \frac{\tau}{I} = \frac{10}{2} = 5\,\mathrm{rad/s}^2.$$

Problem #2. A flywheel rotating at $\omega = 20\,\mathrm{rad/s}$ has $I = 5\,\mathrm{kg}\cdot\mathrm{m}^2$. Find its rotational kinetic energy.

Solution

$K_r = \frac{1}{2}I\omega^2 = \frac{1}{2}\cdot 5 \cdot 400 = 1000\,\mathrm{J}$.

5.2 Angular Momentum and Conservation D1

Formulas & Concepts

Explanation of concept: Angular momentum is conserved when no external torque is applied.
Core formulas:

$$\begin{cases} \vec{L} = I\,\vec{\omega}\ \text{(about fixed axis)}, \quad \sum \tau_{\text{ext}} = \dfrac{d\vec{L}}{dt}. \\ \text{If } \sum \tau_{\text{ext}} = 0,\ \vec{L} \text{ conserved.} \end{cases}$$

Variable definitions: $\vec{L}$ angular momentum; I moment of inertia.
Prerequisites & scope: Axis and point of reference must be specified; rolling applies at instantaneous point of contact.

Heuristics & Pitfalls

- For isolated systems with negligible external torques, apply $\vec{L}$ conservation about a fixed axis.
- Choose the reference point wisely to eliminate unknown torques.

- Forgetting that static friction can act either way in rolling. Fix: determine its direction from torque/acceleration requirements.

Problem #3. A disk of radius R rolls without slipping at $v_{cm} = 5\,\text{m/s}$. Find ω.

Solution

$$v_{cm} = \omega R \Rightarrow \omega = \frac{v_{cm}}{R} = \frac{5}{R}\,\text{rad/s}.$$

Problem #4. A solid sphere ($I = \frac{2}{5}mR^2$) rolls at $\omega = 10\,\text{rad/s}$ and $v_{cm} = 2\,\text{m/s}$. Find its total kinetic energy if $m = 3\,\text{kg}$.

Solution

Using no-slip $v_{cm} = \omega R$, we have $I\omega^2 = \frac{2}{5}mR^2\omega^2 = \frac{2}{5}mv_{cm}^2$. Thus $K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2 = \left(\frac{1}{2} + \frac{1}{5}\right)mv_{cm}^2 = \frac{7}{10}\,mv_{cm}^2 = \frac{7}{10} \cdot 3 \cdot 4 = 8.4\,\text{J}$.

5.3 Moments of Inertia D2

Formulas & Concepts

Concept description: Moment of inertia is a measure of resistance to angular acceleration; composition rules and standard shapes allow for rapid calculation.
Core formulas:

$$\begin{cases} I = \displaystyle\int r^2\,dm \text{ (axis distance } r), \\ \text{Parallel axis: } I = I_{\text{cm}} + Md^2, \\ \text{Perpendicular axis (planar lamina): } I_z = I_x + I_y. \end{cases}$$

Variable definitions: I_{cm} about COM axis; d offset; M total mass.
Prerequisites & scope: Perpendicular-axis requires lamina in the plane; parallel-axis requires fixed, parallel axes.

Heuristics & Pitfalls

- Decompose into standard shapes and sum moments about the same axis.
- Use symmetry to eliminate products of inertia; choose axes through COM when possible.

- Applying perpendicular-axis to 3D bodies. Fix: valid only for planar laminae.

Problem #5. A thin rod of mass m and length L rotates about one end. Find its moment of inertia.

Solution

$I = \int_0^L x^2\,dm = \int_0^L x^2 \dfrac{m}{L}\,dx = \dfrac{m}{L}\dfrac{L^3}{3} = \dfrac{mL^2}{3}.$

Problem #6. A disk of mass M and radius R has $I_{cm} = \frac{1}{2}MR^2$. Using the parallel-axis theorem, find I about a point on its rim.

Solution

$I = I_{cm} + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2.$

5.4 Rolling Without Slipping [D1]

Formulas & Concepts

Concept description: Rolling is a mixture of translation of the center of mass and rotation; the no-slip condition ties them together.
Core formulas:

$$\begin{cases} v_{\text{cm}} = \omega R, \\ K = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2. \end{cases}$$

Variable definitions: R radius; M mass; I moment of inertia; v_{cm} center-of-mass speed.

Heuristics & Pitfalls

- Static friction can accelerate or decelerate rolling bodies but does no work on the body in pure rolling.
- Use energy for ramp problems; use $\sum\tau = I\alpha$ when forces/accelerations are requested.

Problem #7. A solid cylinder of mass m and radius R is released from rest to roll without slipping down an incline of height h. Find its speed at the bottom.

Solution

Energy: $mgh = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}(\frac{1}{2}mR^2)(v_{cm}^2/R^2) = \frac{1}{2}mv_{cm}^2 + \frac{1}{4}mv_{cm}^2 = \frac{3}{4}mv_{cm}^2$, so $v_{cm} = \sqrt{\frac{4}{3}gh}$.

Problem #8. A rolling sphere moves to the right and speeds up. Determine the direction of static friction on the sphere.

Solution

For a solid sphere with $a > 0$, no slip implies friction acts up the incline or in the direction that provides a torque to increase ω: here, friction acts forward (to the right) at the contact point to produce a counterclockwise torque consistent with $\dot{\omega} > 0$.

Part I: Mechanics Practice Pointers

Part II: Oscillations, Waves, Thermodynamics & Fluids

6 Unit 6: Oscillations & Waves

Coverage checklist: SHM; pendulum (small-angle); wave speed $v = f\lambda$; superposition/standing waves; sound and Doppler; SHM ODE

6.1 Simple Harmonic Motion [D1]

Formulas & Concepts

Concept explanation: SHM occurs when acceleration is proportional to and opposite to displacement; solutions are sinusoidal with constant amplitude (no damping).
Core formulas:

$$\begin{cases} x = A\cos(\omega t + \phi), \quad v = -A\omega\sin(\omega t + \phi), \quad a = -\omega^2 x, \\ T = \dfrac{2\pi}{\omega}, \quad \omega = \sqrt{\dfrac{k}{m}} \text{ (mass–spring)}, \quad T_{\text{pend}} \approx 2\pi\sqrt{\dfrac{\ell}{g}} \text{ (small angle)}. \end{cases}$$

Variable definitions: A amplitude; ω angular frequency; ϕ phase; k spring constant; ℓ pendulum length.
Prerequisites & scope: No damping/driving; small-angle approximation for pendulum.

Heuristics & Pitfalls

- Use energy partition $K + U = \frac{1}{2}kA^2$ to find speeds at positions; use phase to compute time fractions.
- For compound oscillators, reduce to effective k_{eff} or ℓ_{eff} before applying SHM formulas.
- Small-angle pendulum: check $\theta_{\max} \lesssim 10°$ for $T \approx 2\pi\sqrt{\ell/g}$ to be within a percent; otherwise expect longer T. First-order correction (radians; [D2]result, memorize only): $T \approx 2\pi\sqrt{\ell/g}\left(1 + \frac{\theta_0^2}{16}\right)$.

- Using pendulum period formula at large angles. Fix: restrict to small angles or use elliptic corrections.

Problem #1. A mass on a spring follows SHM with amplitude A and period T. What fraction of the period is spent with $|x| > \frac{A}{2}$?

Solution

Let $x(t) = A\cos(\omega t)$. The condition $|x(t)| > A/2$ is equivalent to $|\cos(\omega t)| > 1/2$. It is simpler to calculate the fraction of time for the complementary condition, $|x(t)| \le A/2$, which corresponds to $|\cos(\omega t)| \le 1/2$. In one full cycle $\theta = \omega t \in [0, 2\pi)$, this holds for $\theta \in [\pi/3, 2\pi/3]$ and $\theta \in [4\pi/3, 5\pi/3]$. The total angular duration is $(\frac{2\pi}{3} - \frac{\pi}{3}) + (\frac{5\pi}{3} - \frac{4\pi}{3}) = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$. The fraction of the period for this condition is $\frac{2\pi/3}{2\pi} = \frac{1}{3}$. Therefore, the fraction of the period spent with $|x| > A/2$ is $1 - 1/3 = 2/3$.

6.2 Waves (Traveling) D1

Formulas & Concepts

Concept explanation: Traveling waves follow $v = f\lambda$; boundary conditions set standing-wave modes; source/observer motion shifts frequency (Doppler).
Core formulas:

$$\left\{ v = f\lambda, \quad y(x,t) = A\cos(kx - \omega t + \phi), \; k = \frac{2\pi}{\lambda}, \; \omega = 2\pi f. \right.$$

Variable definitions: v wave speed in medium; L length; v_o observer speed; v_s source speed.
Prerequisites & scope: Linear superposition; small amplitudes; Doppler formula assumes $v_o, v_s \ll v$ (nonrelativistic).

Heuristics & Pitfalls

- Draw mode shapes to match node/antinode boundary conditions before writing f_n.
- Use a sign diagram for Doppler to avoid sign errors; approaching increases frequency. For the formula given, this means observer towards source ($+v_o$, numerator) and source towards observer ($-v_s$, denominator). For receding, reverse these signs.
- Using v of sound/light incorrectly across media. Fix: use the correct medium speed for $v = f\lambda$.

Problem #2. A wave has wavelength $\lambda = 2\,\mathrm{m}$ and frequency $f = 50\,\mathrm{Hz}$. Find its speed.

Solution

$v = f\lambda = 50 \cdot 2 = 100\,\mathrm{m/s}$.

6.3 Standing Waves (Strings/Pipes) D1

Formulas & Concepts

Concept explanation: Boundaries reflect waves; interference of forward and backward waves creates nodes and antinodes with discrete mode frequencies set by geometry and boundary conditions. **Core formulas:**

$$\begin{cases} \text{String fixed ends: } f_n = \dfrac{nv}{2L},\ n = 1, 2, \dots \\ \text{Open/closed pipe: } \begin{cases} f_n = \dfrac{nv}{2L}, & n = 1, 2, 3, \dots \text{ (both ends open)}, \\ f_n = \dfrac{(2n-1)v}{4L}, & n = 1, 2, 3, \dots \text{ (one end closed)}. \end{cases} \end{cases}$$

Problem #3. A string of length $L = 1.2\,\mathrm{m}$ fixed at both ends has fundamental frequency $f_1 = 200\,\mathrm{Hz}$. Find the wave speed.

Solution

$f_1 = \dfrac{v}{2L} \Rightarrow v = 2Lf_1 = 2 \cdot 1.2 \cdot 200 = 480\,\mathrm{m/s}$.

6.4 Doppler Effect (Fixed Medium) D1

Formulas & Concepts

Concept explanation: Relative motion between source and observer shifts the detected frequency: approaching raises f' and receding lowers it; the medium is stationary. **Core formula:**

$$f' = \frac{v \pm v_o}{v \mp v_s} f \text{ (approach uses top signs).}$$

Variable definitions: v wave speed; v_o observer speed; v_s source speed.

Problem #4. An acoustic source emits $f = 440\,\text{Hz}$ in air with $v = 340\,\text{m/s}$. The observer moves toward the source at $v_o = 10\,\text{m/s}$ while the source moves toward the observer at $v_s = 20\,\text{m/s}$. Find the observed frequency.

Solution

$$f' = \frac{v + v_o}{v - v_s} f = \frac{350}{320} \cdot 440 \approx 481\,\text{Hz}.$$

6.5 SHM ODE and Driven Forms D2

Formulas & Concepts

Concept explanation: The homogeneous SHM ODE has sinusoidal solutions; with driving and damping, the steady-state response depends on drive frequency and damping ratio.
Core formulas:

$$\begin{cases} x'' + \omega_0^2 x = 0 \Rightarrow x(t) = C\cos\omega_0 t + D\sin\omega_0 t, \\ \text{Damped: } x'' + 2\zeta\omega_0 x' + \omega_0^2 x = 0, \ \text{under/critical/over-damped by } \zeta, \\ \text{Driven: } x'' + 2\zeta\omega_0 x' + \omega_0^2 x = \dfrac{F_0}{m}\cos\omega t, \\ \ \Rightarrow \ \text{amplitude peaks near } \omega \approx \omega_0, \\ \text{for small damping } \omega_{\text{peak}} \approx \omega_0\sqrt{1 - 2\zeta^2} \ (\zeta \ll 1). \end{cases}$$

Variable definitions: ω_0 natural frequency; ζ damping ratio; F_0 drive amplitude.
Prerequisites & scope: Linear oscillator model; small oscillations; steady-state assumes transients have decayed.

Heuristics & Pitfalls

- Identify regime via ζ; near resonance, estimate amplification and phase shift.

- Confusing natural and driving frequencies. Fix: keep ω_0 (system) distinct from ω (drive).

Problem #5. For $x'' + \omega^2 x = 0$ with $x(0) = 0$ and $\dot{x}(0) = v_0$, find $x(t)$ and the maximum speed.

Solution

The general solution is $x(t) = C\cos(\omega t) + D\sin(\omega t)$. $x(0) = 0 \Rightarrow C = 0$. $\dot{x}(t) = D\omega\cos(\omega t)$, so $\dot{x}(0) = v_0 \Rightarrow D\omega = v_0 \Rightarrow D = v_0/\omega$. Thus, $x(t) = \dfrac{v_0}{\omega}\sin(\omega t)$. The velocity is $v(t) = \dot{x}(t) = v_0\cos(\omega t)$. The maximum speed is the amplitude of $v(t)$, which is $|v_0|$.

7 Unit 7: Fluids & Thermodynamics

Coverage checklist: Hydrostatics (pressure/buoyancy); continuity; Bernoulli; ideal gas; First Law and engines; entropy

7.1 Hydrostatics and Buoyancy D1

Formulas & Concepts

Concept explanation: Static fluids exert pressure that increases with depth; the buoyant force equals the weight of displaced fluid (Archimedes).
Core formulas:

$$\begin{cases} P = P_0 + \rho g h \text{ (hydrostatic pressure)}, \\ F_b = \rho g V_{\text{disp}} \text{ (buoyancy)}. \end{cases}$$

Variable definitions: P_0 reference pressure (often atmospheric at $h = 0$); ρ fluid density; h depth; V_{disp} displaced volume.
Prerequisites & scope: Fluid at rest (no flow), constant ρ with depth (or integrate if varying); neglect surface tension unless specified.

Heuristics & Pitfalls

- Draw free-body diagrams (FBD) of floating/sinking bodies: set F_b vs weight vs any tension to solve equilibrium.
- Choose a definite reference level for h and keep P_0 the same when comparing points.
- Submerged fraction for floating: $\dfrac{V_{sub}}{V} = \dfrac{\rho_b}{\rho}$ (with $0 < \rho_b \le \rho$).
- If $\rho_b > \rho$ and the object is released freely, the initial net force is downward: $mg - F_b > 0$.
- If later supported (bottom contact or tension), static equilibrium requires $T + F_b = mg$.
- Gauge vs absolute pressure: $\Delta P = \rho g h$ is a *gauge* difference; absolute pressure is $P = P_{\text{atm}} + \rho g h$ when the surface is open to atmosphere.

- Using object's volume instead of displaced volume for F_b. Fix: use actual displaced fluid volume (submerged part only).

Problem #1. A block of volume V and density ρ_b floats in a liquid of density ρ. What fraction of its volume is submerged?

Solution

At equilibrium $\rho g V_{sub} = \rho_b g V$, so $V_{sub}/V = \rho_b/\rho$.

7.2 Continuity and Bernoulli [D1]

Formulas & Concepts

Concept explanation: In steady incompressible flow, mass conservation gives $Av =$ const; along a streamline with negligible viscosity, mechanical energy per volume is constant (Bernoulli).
Core formulas:

$$\begin{cases} \text{Continuity: } A_1 v_1 = A_2 v_2 \ (\rho \text{ constant}). \\ \text{Bernoulli: } P + \frac{1}{2}\rho v^2 + \rho g y = \text{const (along a streamline)}. \end{cases}$$

Variable definitions: A cross-sectional area; v speed; P pressure; y elevation; ρ density.
Prerequisites & scope: Steady, incompressible, non-viscous flow; apply Bernoulli along a streamline, not across shocks or with pumps/turbines unaccounted.

Heuristics & Pitfalls

- Check assumptions (steady/incompressible/irrotational) before using Bernoulli; otherwise use energy loss terms.
- Combine continuity with Bernoulli to eliminate speeds or pressures efficiently.
- Use stagnation points: where $v = 0$, the total (stagnation) pressure is $P_0 = P + \frac{1}{2}\rho v^2$ upstream along a streamline.

- Using Bernoulli across different streamlines where viscous losses or pumps exist. Fix: apply along a single streamline and include head gains/losses when needed.

Problem #2. Water flows through a horizontal pipe from diameter $D_1 = 0.1\,\text{m}$ to $D_2 = 0.05\,\text{m}$. If $v_1 = 2\,\text{m/s}$, find v_2 using continuity.

Solution

$$A_1 v_1 = A_2 v_2 \Rightarrow \pi(D_1/2)^2 v_1 = \pi(D_2/2)^2 v_2 \Rightarrow v_2 = v_1 \frac{D_1^2}{D_2^2} = 2\frac{0.01}{0.0025} = 8\,\text{m/s}.$$

Problem #3. At point 1 in a pipe, $P_1 = 1.0 \times 10^5\,\text{Pa}$, $v_1 = 2\,\text{m/s}$, $y_1 = 0$. At point 2, $v_2 = 5\,\text{m/s}$, $y_2 = 3\,\text{m}$. Find P_2 for water ($\rho = 1000\,\text{kg/m}^3$).

Solution

Bernoulli: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$. Compute in SI with scientific notation: $\frac{1}{2}\rho(v_1^2 - v_2^2) = 0.5 \times 10^3\,(4 - 25) = -1.05 \times 10^4\,\text{Pa}$ and $\rho g(y_2 - y_1) = 10^3 \times 9.8 \times 3 = 2.94 \times 10^4\,\text{Pa}$. Hence $P_2 = 1.00 \times 10^5 - 1.05 \times 10^4 - 2.94 \times 10^4 = 6.01 \times 10^4\,\text{Pa}$.

7.3 Ideal Gas and First Law D1

Formulas & Concepts

Concept explanation: $PV = nRT$ holds for ideal gases; the First Law links changes in internal energy to heat and work with clear sign conventions.
Core formulas:

$$\begin{cases} PV = nRT, \quad U = \frac{f}{2}nRT, \\ \text{where } f \text{ is dof (e.g., } f = 3 \text{ monatomic, } 5 \text{ diatomic at room T),} \\ \Delta U = Q + W_{\text{on}} \quad (W_{\text{on}} = \text{work done on the system}), \\ W_{\text{on}} = -\displaystyle\int P\,dV \quad (\text{so } W_{\text{by}} \equiv -W_{\text{on}} = \displaystyle\int P\,dV). \end{cases}$$

Variable definitions: P, V, T pressure/volume/temperature; n moles; R gas constant; Q heat into system; W_{on} work on system; W_{by} work done by the gas; U internal energy.
Prerequisites & scope: Ideal gas approximation; U depends only on T for ideal gases; sign convention must be consistent.

Heuristics & Pitfalls

- Identify process (isochoric/isobaric/isothermal/adiabatic) to pick W, Q, ΔU quickly.
- Draw P–V diagrams: areas give work; direction indicates sign.

- Mixing sign conventions for work. Fix: adopt $\Delta U = Q + W_{\text{on}}$ (work on system positive) consistently; then $W_{\text{by}} = -W_{\text{on}}$.

Problem #4. An ideal gas undergoes an isothermal expansion from volume V_1 to V_2 at temperature T. Find the work done by the gas.

Solution

$W = \int_{V_1}^{V_2} \frac{nRT}{V}\, dV = nRT \ln \frac{V_2}{V_1}$.

Problem #5. An ideal gas at $P = 2 \times 10^5\,\mathrm{Pa}$ and $V = 0.01\,\mathrm{m}^3$ has $n = 1\,\mathrm{mol}$. Find the temperature T using $PV = nRT$ with $R = 8.314\,\mathrm{J/(mol \cdot K)}$.

Solution

$$T = \frac{PV}{nR} = \frac{2 \times 10^5 \cdot 0.01}{1 \cdot 8.314} \approx 240\,\mathrm{K}.$$

7.4 Entropy and Carnot [D1]

Formulas & Concepts

Concept explanation: Entropy quantifies thermal disorder and increases in irreversible processes; Carnot gives the maximum efficiency of heat engines between two reservoirs.
Core formulas:

$$\begin{cases} \Delta S = \displaystyle\int_{\text{rev}} \frac{\delta Q}{T}, \\ \text{Carnot bound: } \eta_{\max} = 1 - \dfrac{T_c}{T_h}. \end{cases}$$

Variable definitions: S entropy; T_h, T_c hot/cold absolute temperatures; δQ infinitesimal heat (reversible path).
Prerequisites & scope: Absolute temperatures (Kelvin); reversible paths for definition; real engines achieve less than Carnot due to irreversibilities.

Heuristics & Pitfalls

- Compute ΔS along a convenient reversible path (e.g., isothermal + isochoric steps).
- For engine limits, compare cycle temperatures to T_h, T_c to bound η quickly.
- Using Celsius in $\eta = 1 - T_c/T_h$. Fix: convert to Kelvin.

Problem #6. A Carnot engine operates between $T_h = 500\,\text{K}$ and $T_c = 300\,\text{K}$. Find its maximum efficiency.

Solution

$$\eta_{\max} = 1 - \frac{T_c}{T_h} = 1 - \frac{300}{500} = 0.4 = 40\%.$$

Problem #7. An ideal gas expands reversibly at constant $T = 400\,\text{K}$ from $V_1 = 1\,\text{m}^3$ to $V_2 = 2\,\text{m}^3$. Find the entropy change if $n = 1\,\text{mol}$.

Solution

$$\Delta S = \frac{Q_{rev}}{T} = \frac{nRT\ln(V_2/V_1)}{T} = nR\ln 2 = 8.314\ln 2 \approx 5.76\,\text{J/K}.$$

7.5 Heat Engines and Efficiency [D1]

Formulas & Concepts

Core formulas:

$$\begin{cases} W_{\text{by}} = \int P\,dV, \quad W_{\text{on}} = -W_{\text{by}}, \quad \eta = \dfrac{W_{\text{by}}}{Q_h}, \\ \eta_{\max,\,\text{Carnot}} = 1 - \dfrac{T_c}{T_h}. \end{cases}$$

Variable definitions: Q_h heat absorbed from hot reservoir; T_h, T_c absolute temperatures of hot/cold reservoirs; W_{by} work done by the gas.

Part II: Oscillations, Waves, Thermodynamics and Fluids Practice Pointers

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- Physics Bowl Thermodynamics & Phase Change Problem 4 Page: 5
- Physics Bowl Thermodynamics & Phase Equilibrium Problem 6 Page: 7
- Physics Bowl Fluid Mechanics Problem 11 Page: 12
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Part III: Electricity & Magnetism

8 Unit 8: Electrostatics

Coverage checklist: Coulomb force; electric field/potential; capacitors and energy; Gauss's law

8.1 Coulomb, Field and Potential D1

Formulas & Concepts

Concept explanation: Point charges interact by an inverse-square law; electric field and potential describe force per unit charge and energy per unit charge.
Core formulas:

$$\begin{cases} |\vec{F}| = k\,\dfrac{|q_1 q_2|}{r^2} \text{ (along the line of centers)}, \\ \vec{E} = \dfrac{\vec{F}}{q}, \quad V = \dfrac{U}{q}, \quad \Delta U = -q \displaystyle\int \vec{E} \cdot d\vec{r}, \\ \text{Point charge: } E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2},\ V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r}. \end{cases}$$

Variable definitions: q, Q charges; r separation; $k = 1/(4\pi\varepsilon_0)$; U potential energy.
Prerequisites & scope: Electrostatics (charges at rest); superposition holds; signs determine directions.

Heuristics & Pitfalls

- Apply symmetry (dipoles, rings, infinite sheets) to cancel parts before integrating.
- Apply potential for conservative additions first, then differentiate to get fields.
- Not remembering vector directions for $\vec{E}$ and $\vec{F}$. Fix: graph direction first, calculate magnitude second.

Problem #1. Two point charges $+Q$ are at $(\pm a, 0)$. Find the electric field on the y-axis at $(0, y)$.

Solution

Horizontal components cancel; vertical add: $E_y = 2\,kQ\,\dfrac{y}{(a^2 + y^2)^{3/2}}$.

Key Insight

Constants/units: $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $c = 3.00 \times 10^8$ m/s, $e = 1.60 \times 10^{-19}$ C. Use SI unless specified.

8.2 Capacitors and Energy [D1]

Formulas & Concepts

Concept explanation: A capacitor is a device that holds equal and opposite charge on two conductors with a gap or dielectric between them. The stored charge at a given potential difference will be proportional to the geometry and material. Networks simplify by simple series/parallel formulas and energy can be traced by $U = \frac{1}{2}CV^2$.
Core formulas:

$$\begin{cases} C = \varepsilon_0 \dfrac{A}{d} \text{ (parallel plates in vacuum)}, \quad C = \varepsilon_r \varepsilon_0 \dfrac{A}{d} \text{ (uniform dielectric)}, \\ \text{Series: } \dfrac{1}{C_s} = \sum \dfrac{1}{C_i}, \quad \text{Parallel: } C_p = \sum C_i, \\ U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \dfrac{Q^2}{2C}. \end{cases}$$

Variable definitions: A plate area; d separation; ε_0 vacuum permittivity; ε_r relative permittivity; Q charge; V voltage.
Prerequisites & scope: Edge effects neglected; linear dielectrics; use equivalent capacitance to reduce networks.

Heuristics & Pitfalls

- Parallel by series and symmetry simplify before writing node/loop equations.
- Use $U = \frac{1}{2}CV^2$ to compare energy storage or redistribution after reconfiguration.
- Assuming charge conservation on each plate when switches change connectivity. Fix: conserve charge on isolated conductors only.

Problem #2. Two capacitors $C_1 = 2\,\mu\text{F}$ and $C_2 = 3\,\mu\text{F}$ are in series. Find the equivalent capacitance.

Solution

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6} \Rightarrow C_s = \frac{6}{5} = 1.2\,\mu\text{F}.$$

Problem #3. A capacitor with $C = 10\,\mu\text{F}$ is charged to $V = 50\,\text{V}$. Find the energy stored.

Solution

$U = \frac{1}{2}CV^2 = \frac{1}{2} \cdot 10 \times 10^{-6} \cdot 2500 = 12.5 \times 10^{-3} = 12.5\,\text{mJ}.$

8.3 Gauss's Law D1

Formulas & Concepts

Concept explanation: The flux of $\vec{E}$ through a closed surface equals enclosed charge over ε_0; symmetry lets you get fields without integration.
Core formula:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}.$$

Variable definitions: Q_{enc} charge enclosed; $d\vec{A}$ outward area element; choose Gaussian surface aligned to symmetry.
Prerequisites & scope: Use for infinite planes/cylinders/spheres; for conductors, $E = 0$ inside and charges reside on surfaces. Within uniform dielectrics/insulators with embedded charge, fields may exist inside the material (i.e., E need not vanish).
*Under **D1**: memorize the integral statement; derivations are not required.*

Heuristics & Pitfalls

- Pick surfaces where E is constant and parallel to $d\vec{A}$ over large patches (sphere/-cylinder/plane).
- For conductors in electrostatics, set $E_{\text{inside}} = 0$ and use boundary conditions for surface charge; for dielectrics, prefer symmetry and superposition without advanced $\vec{D}$ formalism.
- Choosing a Gaussian surface that doesn't match symmetry, forcing difficult integrals. Fix: reselect surface to exploit symmetry.

Problem #4. Using Gauss's law, find $E(r)$ outside a uniformly charged sphere of radius R and total charge Q.

Solution

Gaussian sphere: $E \cdot 4\pi r^2 = Q/\varepsilon_0$, so $E(r) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$ for $r \geq R$.

9 Unit 9: DC Circuits

Coverage checklist: Ohm's law and power; series/parallel reductions; Kirchhoff (KCL/KVL); RC qualitative; RC exact

9.1 Ohm's Law and Reductions [D1]

Formulas & Concepts

Concept explanation: Ohm's law relates voltage, current, and resistance; power forms help rank dissipation; series/parallel laws simplify networks.
Core formulas:

$$\begin{cases} V = IR, \quad P = IV = I^2R = \dfrac{V^2}{R}, \\ R_s = \sum R_i, \quad \dfrac{1}{R_p} = \sum \dfrac{1}{R_i}. \end{cases}$$

Variable definitions: V voltage; I current; R resistance; P power.
Prerequisites & scope: Ohmic elements only; temperature dependence ignored unless specified.

Heuristics & Pitfalls

- Reduce networks with series/parallel and symmetry; then solve KCL/KVL for the rest of unknowns alone.
- Equate power by using I^2R or V^2/R based on conditions of fixed current/voltage.
- In bridge-type circuits, look for equal potentials in a branch (through symmetry or KCL) to remove it.

- Mixing fixed-voltage and fixed-current contexts when comparing brightness. Fix: pick the power form consistent with constraints.

Problem #1. Three resistors of R are in parallel; find the equivalent resistance.

Solution

$\dfrac{1}{R_{eq}} = \dfrac{1}{R} + \dfrac{1}{R} + \dfrac{1}{R}$ so $R_{eq} = R/3$.

9.2 Kirchhoff Laws (KCL/KVL) [D1]

Formulas & Concepts

Concept explanation: Kirchhoff's current and voltage laws enforce charge and energy conservation; first-order RC circuits charge and discharge exponentially with time constant τ.

Core formulas:

$$\begin{cases} \text{KCL: } \sum I_{\text{in}} = \sum I_{\text{out}} \text{ (at a node)}, \\ \text{KVL: } \sum \Delta V = 0 \text{ (around a loop)}, \\ \text{Charging (step from } V_0 \to V) : \\ \quad V_C(t) = V + \big(V_0 - V\big) e^{-t/RC}, \\ \quad I(t) = \dfrac{V - V_0}{R} \, e^{-t/RC}, \\ \text{Discharging (to 0):} \\ \quad V_C(t) = V_0 \, e^{-t/RC}, \\ \quad I(t) = -\dfrac{V_0}{R} e^{-t/RC}, \\ \text{General: } \; V_C(t) = V_\infty + \big(V_0 - V_\infty\big) e^{-t/\tau}. \end{cases}$$

Variable definitions: $\tau = RC$ time constant; V source voltage; V_C capacitor voltage; I branch current.
Prerequisites & scope: Linear time-invariant components; piecewise-constant sources for standard transients.

Heuristics & Pitfalls

- Use series/parallel reductions or source transformations conceptually when helpful, but solve RC timing with baseline KCL/KVL forms.
- Check for limiting values at $t = 0^+$ and $t = \infty$ to test expressions; impose continuity of V_C at switching times.
- Time constant: $\tau = RC$ for the basic first-order RC considered here.

- Letting capacitor voltage jump at $t = 0$. Fix: enforce continuity of V_C and initial condition from prior steady state.

Problem #2. An RC circuit with V applied at $t = 0$ has $R = 2\,\Omega$, $C = 1\,\text{F}$. Find $V_C(t)$.

Solution

$V_C(t) = V(1 - e^{-t/RC}) = V(1 - e^{-t/2})$.

Problem #3. Quick check for limits: For any first-order RC with step to V_∞, verify $V_C(0^+) = V_C(0^-)$ (no jump) and $V_C(\infty) = V_\infty$. With $V_C(0^-) = V_0$, the standard form $V_C(t) = V_\infty + (V_0 - V_\infty)e^{-t/\tau}$ satisfies both.

Solution

Evaluate: $V_C(0^+) = V_\infty + (V_0 - V_\infty) = V_0$ (continuous). As $t \to \infty$, $e^{-t/\tau} \to 0$, so $V_C \to V_\infty$.

9.3 RC Transients (First Order) D1

Formulas & Concepts

Core results:

$$\begin{cases} \tau = RC, \\ \text{Charge to } V: \; V_C(t) = V + \big(V_0 - V\big)e^{-t/RC}, \\ \text{Discharge to 0: } V_C(t) = V_0 e^{-t/RC}. \end{cases}$$

Problem #4. An RC circuit with V applied at $t = 0$ has $R = 2\,\Omega$, $C = 1\,\text{F}$. Find $V_C(t)$.

Solution

$V_C(t) = V(1 - e^{-t/RC}) = V(1 - e^{-t/2})$.

10 Unit 10: Magnetism & Induction

Coverage checklist: Lorentz force (charges/wires); Faraday-Lenz induction; EM spectrum; Ampere law

10.1 Lorentz Force D1

Formulas & Concepts

Concept explanation: A moving charge feels $q\vec{E}$ and $q\,\vec{v} \times \vec{B}$; the magnetic force stays perpendicular to velocity, so it deflects direction without changing speed.
Core formulas:

$$\left\{ \vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B}) \right.$$

Variable definitions: q charge; $\vec{v}$ particle velocity; $\vec{E}, \vec{B}$ fields.
Prerequisites & scope: Nonrelativistic; right-hand rule for cross products.

Heuristics & Pitfalls

- Use right-hand rule consistently; reverse direction for negative charges.

- Magnetic force does no work (always perpendicular to $\vec{v}$), so magnetic fields alone cannot change particle speed.

- Using q's sign incorrectly in $q\,\vec{v} \times \vec{B}$. Fix: compute direction for positive charge, then flip if $q < 0$.

Problem #1. A particle of charge q enters a uniform magnetic field $\vec{B}$ perpendicular to its velocity with speed v. Find the radius and period of its circular motion (neglect $\vec{E}$).

Solution

Magnetic force provides centripetal: $qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$. The period is $T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$.

10.2 Magnetic Force on Wires D1

Formulas & Concepts

Core formulas:

$$\vec{F} = I\,\vec{L} \times \vec{B}, \quad |\vec{F}| = ILB\sin\theta.$$

Variable definitions: I current; $\vec{L}$ directed along the current segment with magnitude L; $\vec{B}$ magnetic field; θ angle between $\vec{L}$ and $\vec{B}$.

Heuristics & Pitfalls

- For loops, integrate $d\vec{F} = I\,d\vec{l} \times \vec{B}$ and exploit symmetry.

Problem #2. A wire of length L carries current I in a uniform field $\vec{B}$ perpendicular to the wire. Find the magnitude of magnetic force.

Solution

$F = ILB$.

10.3 Faraday-Lenz D1

Formulas & Concepts

Concept explanation: Changing magnetic flux induces an emf that opposes the change (Lenz); steady currents set magnetic fields constrained by Ampère's law.

Core formulas:

$$\begin{cases} \mathcal{E} = \oint_{\partial S} \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}, \quad \Phi_B = \iint_S \vec{B} \cdot d\vec{A}, \quad \Phi_E = \iint_S \vec{E} \cdot d\vec{A}, \\ \text{(see Ampère/Maxwell–Ampère for magnetic circulation)} \end{cases}$$

Variable definitions: $\mathcal{E}$ induced emf (scalar, $\mathcal{E} = \oint_{\partial S} \vec{E} \cdot d\vec{l}$); $\Phi_B = \iint_S \vec{B} \cdot d\vec{A}$ magnetic flux; $\Phi_E = \iint_S \vec{E} \cdot d\vec{A}$ electric flux; I_{enc} enclosed current.
Prerequisites & scope: Under **D1**memorize the integral form (no derivation). Generally assume quasi-static fields; for time-varying fields use Maxwell–Ampère with displacement current.

Key Insight

Terminology note: *Electric circulation* refers to the line integral of the electric field that defines emf, $\mathcal{E} = \oint_{\partial S} \vec{E} \cdot d\vec{l}$. *Magnetic circulation* refers to $\oint \vec{B} \cdot d\vec{l}$ as used in Ampère/Maxwell–Ampère. These are distinct: $\mathcal{E}$ is a scalar (emf), while $\vec{E}$ and $\vec{B}$ are fields.

Heuristics & Pitfalls

- Sketch the loop and determine the positive normal; apply Lenz's rule to deduce the direction of the induced current.
- Apply circular/rectangular Amperian loops along symmetry for infinite wires/-solenoids.
- Missing displacement current for charging capacitors. Fix: include $\varepsilon_0 \, d\Phi_E/dt$ in Maxwell–Ampère when fields vary.

Problem #3. In a loop of area A, the magnetic field increases as $B(t) = B_0 + kt$. Find the induced emf.

Solution

$$\mathcal{E} = \left|\frac{d\Phi}{dt}\right| = \left|\frac{d(BA)}{dt}\right| = kA.$$

10.4 Ampere and Maxwell–Ampere D2

Formulas & Concepts

Core formulas:

$$\begin{cases} \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \text{ (steady currents)}, \\ \text{Maxwell–Ampere (general): } \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \dfrac{d\Phi_E}{dt}. \end{cases}$$

10.5 EM Spectrum and Maxwell (Concept) [D1]

Formulas & Concepts

Concept explanation: Electromagnetic waves range from radio to gamma; Maxwell's equations couple $\vec{E}$ and $\vec{B}$ and give wave speed c in vacuum.
Formulas & Concepts:

$$\begin{cases} c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}, \\ \text{Spectrum ordering by frequency: radio } \to \text{ microwave } \to \text{ IR } \to \text{ visible,} \\ \text{then UV } \to \text{ X } \to \text{ gamma.} \end{cases}$$

Prerequisites & scope: Vacuum relations shown; material dispersion alters speed and wavelength.

Heuristics & Pitfalls

- Recall typical sources: antennas (radio), thermal (IR), electronic transitions (visible/UV), inner-shell transitions (X/gamma).
- Use $c = f\lambda$ with medium refractive index n via $v = c/n$.

Problem #4. Light in vacuum has wavelength $\lambda = 600\,\text{nm}$ and speed $c = 3 \times 10^8\,\text{m/s}$. Find its frequency.

Solution

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{600 \times 10^{-9}} = 5 \times 10^{14}\,\text{Hz}.$$

Problem #5. Rank the following by increasing photon energy: radio, visible, X-ray.

Solution

Higher frequency means higher photon energy $E = hf$. Ordering: radio $<$ visible $<$ X-ray.

Part III: Electricity & Magnetism Practice Pointers

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- Physics Bowl DC Circuits Problem 22 Page: 23

Part IV: Optics & Modern Physics

11 Unit 11: Optics

Coverage checklist: Reflection; refraction (Snell); thin lens and magnification; interference/diffraction

11.1 Reflection and Refraction [D1]

Formulas & Concepts

Concept explanation: Light reflects with equal incident and reflected angles; refraction across media obeys Snell's law.
Core formulas:

$$\begin{cases} \theta_i = \theta_r, \\ n_1 \sin\theta_1 = n_2 \sin\theta_2, \quad \text{TIR when } \theta_1 > \theta_c = \arcsin(n_2/n_1)\ (n_1 > n_2). \end{cases}$$

Variable definitions: n refractive index; θ angles measured to the normal.
Prerequisites & scope: Geometric optics regime; isotropic media; polarization effects ignored here.

Heuristics & Pitfalls

- Draw the normal and principal rays first; search for total internal reflection when going to a lower-n medium.
- Use reversibility of light to validate constructions.

Problem #1. Light travels from air ($n_1 = 1$) into water ($n_2 = 1.33$) at incidence angle $\theta_1 = 40°$. Find the refraction angle θ_2.

Solution

Snell: $n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow \sin\theta_2 = \dfrac{\sin 40°}{1.33} \Rightarrow \theta_2 \approx 28.9°$.

Problem #2. Light moves from glass ($n = 1.5$) to air ($n = 1$). Find the critical angle for total internal reflection.

Solution

$$\theta_c = \arcsin\frac{n_2}{n_1} = \arcsin\frac{1}{1.5} \approx 41.8°.$$

11.2 Thin Lenses and Sign Conventions [D1]

Formulas & Concepts

Concept explanation: Thin lens imaging follows the lens equation with sign conventions; magnification uses image and object sizes/orientations.

Core formulas:

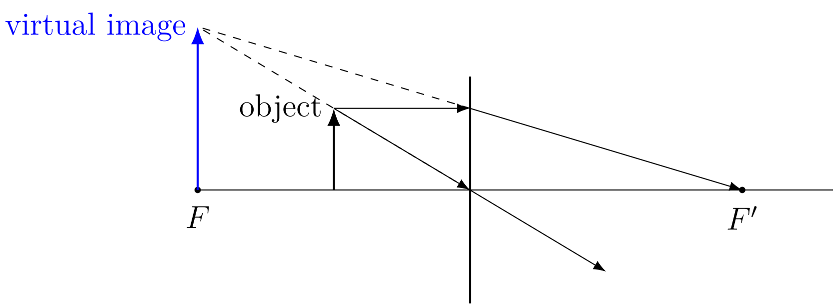
$$\begin{cases} \frac{1}{f} = \frac{1}{s} + \frac{1}{s'}, \quad m = -\frac{s'}{s} = \frac{h'}{h}. \end{cases}$$

Variable definitions: f focal length; s object distance; s' image distance; m magnification; h', h image/object heights.

Prerequisites & scope: Use consistent sign convention (e.g., real is positive); paraxial approximation.

Heuristics & Pitfalls

- Combine equation + ray diagram: draw two principal rays to confirm the algebraic image location.
- Remember that negative m indicates inversion; $|m| > 1$ indicates magnification.
- Sign convention (real-is-positive): take $s > 0$ for real objects and $s' > 0$ for real images on the opposite side of the lens from the object; $s' < 0$ indicates a virtual image on the object side (then $m > 0$ and the image is upright).



Problem #3. An object at $s = 30\,\text{cm}$ forms an image at $s' = -60\,\text{cm}$ using a thin lens. Find the focal length f and magnification m.

Solution

Lens equation: $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{30} + \frac{1}{-60} = \frac{1}{60}$, so $f = 60\,\text{cm}$. Magnification: $m = -s'/s = -(-60)/30 = 2$. Since $f > 0$, it is a converging lens. Since $s' < 0$, the image is virtual. Since $m > 0$, the image is upright.

11.3 Interference and Diffraction [D1]

Formulas & Concepts

Concept explanation: Coherent sources produce interference patterns; finite apertures make the light diffract, establishing angular scales by wavelength/aperture.
Core formulas:

$$\begin{cases} \text{Double-slit maxima: } d\sin\theta = m\lambda, \\ \text{Single-slit minima: } a\sin\theta = m\lambda, \ \ m = \pm1, \pm2, \ldots \end{cases}$$

Variable definitions: d slit separation; a slit width; λ wavelength; θ diffraction angle.
Prerequisites & scope: Small-angle approximations $\sin\theta \approx \theta$ valid near axis; coherence required for stable fringes.

Heuristics & Pitfalls

- Map angles to screen positions with $y \approx L\tan\theta \approx L\theta$ for small θ.
- To resolve features, compare λ to a and d to predict fringe spacing/envelope width.

Problem #4. For double-slit with spacing d and wavelength λ, what is the angle of the m-th bright fringe?

Solution

$d\sin\theta = m\lambda \Rightarrow \theta = \arcsin(m\lambda/d)$ (small-angle: $\theta \approx m\lambda/d$).

12 Unit 12: Modern Physics

Coverage checklist: Special relativity (γ, time dilation, length contraction, $E = mc^2$); photoelectric effect; atomic spectra; nuclear decay/half-life

12.1 Special Relativity [D1]

Formulas & Concepts

Concept explanation: At high speeds, time dilates and lengths contract; energy–mass equivalence relates rest mass to rest energy.

Core formulas (with proper vs observed):

$$\begin{cases} \gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}, \\ \text{Time dilation: } \Delta t = \gamma\,\Delta\tau \;\; (\Delta\tau \text{ proper time in moving clock's frame}), \\ \text{Length contraction: } L = \dfrac{L_0}{\gamma} \;\; (L_0 \text{ proper length measured at rest with the rod}), \\ \text{Relativistic energy: } E = \gamma mc^2 \; (E_0 = mc^2), \quad p = \gamma mv, \quad E^2 = (pc)^2 + (mc^2)^2. \end{cases}$$

Variable definitions: γ Lorentz factor; v relative speed; c speed of light; $\Delta\tau$ proper time (clock's rest frame); L_0 proper length (object's rest frame); E total energy; E_0 rest energy; p relativistic momentum.
Prerequisites & scope: Inertial frames; v along one axis for simple forms; proper quantities measured in an object's rest frame.

Heuristics & Pitfalls

- Label frames (S, S') and identify proper time/length before applying formulas.

D2 Approximation for $v \ll c$: $\gamma \approx 1 + \frac{1}{2}(v/c)^2$ (derivation and series methods belong to **D2**).

Problem #1. A spaceship moves at $0.8c$ relative to Earth. What factor relates proper time to dilated time?

Solution

$\gamma = 1/\sqrt{1-0.8^2} = \frac{5}{3}$.

12.2 Photoelectric Effect D1

Formulas & Concepts

Concept explanation: Electrons emit when photon energy exceeds the work function; the threshold frequency is $f_{th} = \phi/h$.
Core formulas:

$$\begin{cases} K_{\max} = hf - \phi, \quad f_{\text{th}} = \phi/h, \end{cases}$$

Variable definitions: h Planck constant; ϕ work function.
Prerequisites & scope: Idealized models; surface effects and detector thresholds may alter observed $K_{\max}$.

Heuristics & Pitfalls

- Stopping potential depends on frequency (threshold via $f_{\text{th}} = \phi/h$), not intensity.
- In a $K_{\max}$–f plot, slope $= h$, vertical intercept $= -\phi$.

- Increasing intensity raises saturation current but does not change stopping potential.

Problem #2. Light of frequency f hits a metal with work function ϕ. Write the maximum kinetic energy of ejected electrons.

Solution

$K_{\max} = hf - \phi$.

12.3 Nuclear Decay Basics D1

Formulas & Concepts

Core formulas:

$$N(t) = N_0\, 2^{-t/T_{1/2}} = N_0 e^{-\lambda t}, \ \lambda = \frac{\ln 2}{T_{1/2}}.$$

Heuristics & Pitfalls

- For decay chains, use activity $A = \lambda N$; independent branches superpose exponentials.
- Plot $\ln N$ vs t to extract λ from the slope.

Part IV: Optics & Modern Physics Practice Pointers

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