Homework 1 (deadline: Oct. 10)

Demonstrate barotropic instability numerically based on the shallow water quasi-geostrophic equation in the presence of a background zonal wind shear $U(y)$. You can build your own code based on the python code shown in the class (the python code can be downloaded from the course website `https://qiuyang50.github.io/_pages/modeling_2024fall/`). The shallow water quasi-geostrophic equation reads as follows,

$$\frac{D}{Dt}\left(\nabla^2\psi - f^2F\psi\right) + \beta\frac{\partial\psi}{\partial x} = 0 \tag{1}$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} - \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}$, f, F, β are all dimensionless constant, $U(y)$ has the following profile,

$$U(y) = \begin{cases} 1 & y > a \\ y/a & -a \leq y \leq a \\ -1 & y < -a \end{cases}$$

Step 1: initialize the variable ψ. Hint: you can set the initial value of ψ as a combination of background state and a perturbation, $\psi(x, y, 0) = \Psi(y) + \psi'$, where $\Psi(y)$ satisfies $U(y) = -\frac{\partial\Psi(y)}{\partial y}$.

Step 2: add a perturbation ψ', which has to be small in amplitude and periodic at the domain boundaries. For example,

$$\psi' = 10^{-4}\sin\left(\frac{2\pi k}{L}x\right)e^{-\frac{y2}{a^2}}$$

You may try other forms of perturbations.

Step 3: consider multiple scenarios with different values of a and k and compare your results with the theoretical prediction mentioned in the class. Hint: $a * k$ should be small enough for the emergence of instability. See chapter 6.2 (page 256) in Vallis's book.

Homework requirement: please summarize all you findings in a report, particularly including several snapshots of the vorticity field to demonstrate the barotropic instability. A thorough discussion about the choice of a and k in terms of barotropic instability is appreciated.