Algorithm

YIXIANG QIU

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1   Asymptotic Analysis

Asymptotic Analysis is a method used to quantify the **time** and **space** cost in an algorithm.

First we can define the order of growth of Θ. Instead of saying a function has order of growth W, we say that the function belongs to $\Theta(W)$. In other words it belongs to the family of functions that have the same order of growth.

Theorem 1

For some function $R(N)$ with order of growth $f(N)$ we write that $R(N) \in \Theta(f(N))$.

Example 2

Suppose a function is defined to be $R(N^3 + 3N^4)$ then the order of growth is N^4. Then we can write as $R(N^3 + 3N^4) \in \Theta(N^4)$.

The Difference between $O(N)$ and $\Theta(N)$:

- The Θ means that the same order of growth, which also means the tight-bound (both upper-bound and lower-bound).

- The O can be thought as less then or equal to some order of growth. Which is equivalent to the upper bound.

Example 3

Suppose $f(N) = 2N$, all of these statements are true.

$$f(N) \in \Theta(N) \quad \text{and} \quad f(N) \in O(N) \quad \text{and} \quad f(N) \in O(N^2)$$

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2   Searching

3 Sorting

Sorting Algorithm is an important strategy in Computer. In this section will introduce some useful sorting algorithm. And analysis their Time and Space Complexity.

3.1 Selection Sort

Selection sort is perhaps the easiest sorting algorithm. A formal selection sort consists of three steps. Suppose an array has N elements.

1. Find the smallest element.

2. Move it to the front.

3. Selection sort the remaining $N - 1$ elements.