High-Quality Axions in a Class of Chiral $U(1)$ Gauge Theories

Yu-Cheng QIU

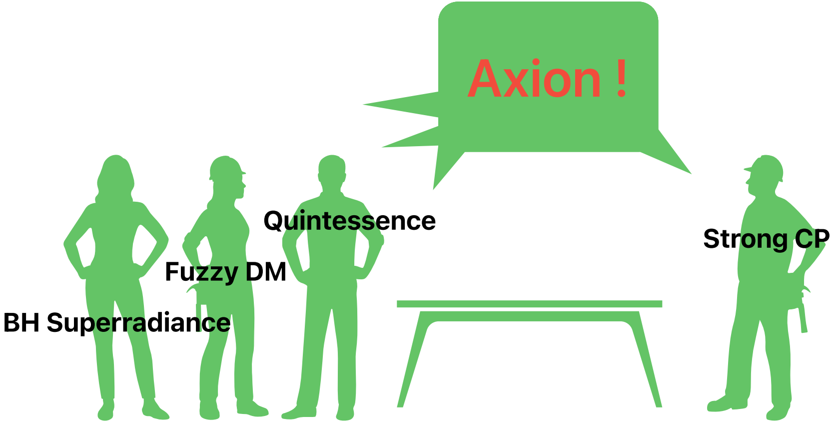
李政道研究所
Tsung-Dao Lee Institute

October 19, 2023

2301.02345 (PRL)

with Jin-Wei Wang, Tsutomu T. Yanagida

Who ordered the Axion?



Who ordered the Axion?

Strong CP problem

$$\mathcal{L} \supset \bar{\theta} G \tilde{G} \,, \quad \bar{\theta} < 10^{-10}$$

Why $\bar{\theta}$ is so small?

- Peccei–Quinn mechanism:

$$\text{anomalous global } U(1) \xrightarrow{SSB} \text{shift symmetry } \frac{a}{F_a} \to \frac{a}{F_a} + \delta_{\text{PQ}}$$

$$\xrightarrow{anomaly} \mathcal{L} \supset \left(\bar{\theta} + \frac{a}{F_a}\right) G \tilde{G} \xrightarrow{instanton} V(a) \sim \cos\left(\bar{\theta} + \frac{a}{F_a}\right)$$

Why axion?

- Observed Cosmological Constant

$$\Lambda_{\rm obs} \simeq 10^{-120} M_{\rm Pl}^4$$

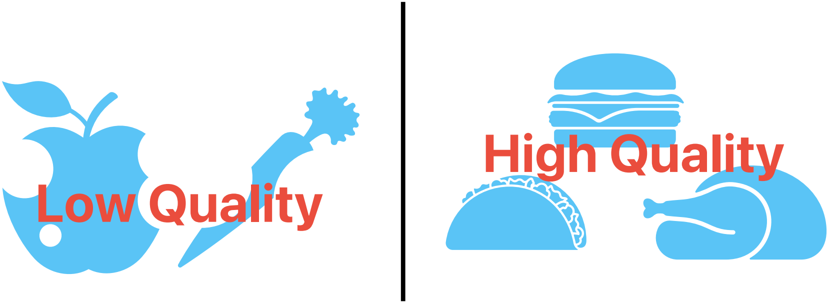
  - If it is the potential of a scalar, the mass must be extremely small, $\sim 10^{-33}\,\mathrm{eV}$.

- Fuzzy (Wave) Dark Matter

$$m \sim 10^{-22}\text{–}10^{-19}\,\mathrm{eV}$$

All of them could be Nambu-Goldstone boson! Axion!

How good is the axion served?



Axion Quality Problem

$U(1)_{\text{PQ}}$ is a global symmetry,
which could easily be explicitly broken at least by gravity (wormhole).

Suppose the axion belongs to $\Phi \sim e^{ia/F_a}$, then

$$g\frac{\Phi^n}{M_{\text{Pl}}^{n-4}} \xrightarrow{\langle\Phi\rangle\sim F_a} \Delta V(a) = M_{\text{Pl}}^4\left(\frac{F_a}{M_{\text{Pl}}}\right)^n \cos\left(\frac{a}{F_a} + \arg[g]\right)$$

$$V = V(a) + \Delta V(a) \xrightarrow{\bar{\theta}<10^{-10}} n > 10 \qquad \text{for } F_a \sim 10^{16}\,\text{GeV}$$

One shall suppress all terms at least up to order **10**!

Axion Quality Problem

"Gauged" PQ symmetry

The goal is to construct a (field theory) framework that can easily have high quality axions.

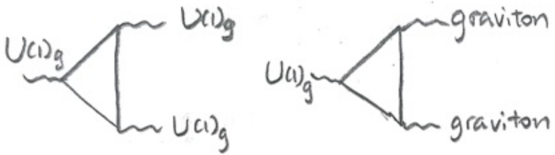
The recipe

- Suppose there are 2 complex scalar fields, then one has global $U(1) \times U(1)$. phase rotations
- One linear combination could be gauged, $U(1)_g$, and another one is the global $U(1)_{\mathrm{PQ}}$.
- To have an anomalous $U(1)_{\mathrm{PQ}}$, chiral fermions shall be introduced.

Chiral $U(1)$ Gauge Theory

Introduce N pairs of chiral fermions charged under $U(1)_g$, $\{\psi_i, \overline{\psi}_i\}$, where $\psi_i \in (\mathbf{3}, 1, 0)$ and $\overline{\psi}_i \in (\mathbf{3}^*, 1, 0)$ under $SU(3)_\mathrm{c} \times SU(2)_\mathrm{L} \times U(1)_\mathrm{Y}$.
Anomaly cancellation condition requires that

$$\sum_i^N Q_i^3 = 0 \ , \qquad \sum_i^N Q_i = 0$$



e.g. $N = 5$, $\{Q_i\} = \{-9, -5, -1, 7, 8\} \times 2$

Chiral $U(1)$ Gauge Theory

- Assume that 2 Higgs given all chiral fermion mass,
- and only through Yukawa interaction.

For $N = k + l$ pairs,

$$\mathcal{L} \supset -\phi_1 \sum_{i=1}^{k} c_i \psi_i \overline{\psi}_i - \phi_2 \sum_{j=1}^{l} c_j \psi_j \overline{\psi}_j$$

TABLE I. Fermion charge assignment.

i	1	2	3	4	$\cdots$				$k+1$	$k+2$	$\cdots$					
ψ_i	α_1	β_1	α_2	β_2	$\cdots$	γ_1	γ_2	$\cdots$	δ_1	η_1	δ_2	η_2	$\cdots$	σ_1	σ_2	$\cdots$
$\overline{\psi}_i$	β_1	α_1	β_2	α_2	$\cdots$	γ_1	γ_2	$\cdots$	η_1	δ_1	η_2	δ_2	$\cdots$	σ_1	σ_2	$\cdots$

Chiral $U(1)$ Gauge Theory

$$-q_1 = \alpha_1 + \beta_1 = \alpha_2 + \beta_2 = \cdots = 2\gamma_1 = 2\gamma_2 = \cdots$$

$$-q_2 = \delta_1 + \eta_1 = \delta_2 + \eta_2 = \cdots = 2\sigma_1 = 2\sigma_2 = \cdots$$

By $\sum Q_i = 0$, one could have

$$\sum_i (\alpha_i + \beta_i) + \sum_j \gamma_j + \sum_i (\delta_i + \eta_i) + \sum_j \sigma_j = 0$$

$$\Longrightarrow \quad -\frac{q_1}{q_2} = \frac{k}{l} = \frac{m}{n}$$

(n, m) are relatively prime numbers to each other.

The **lowest-order** non-renormalizable operator ($\phi_{1,2}$) that obey $U(1)_g$ is (?)

$$\mathcal{O} = \frac{1}{k!l!}\frac{\phi_1^k \phi_2^l}{M_{\rm Pl}^{N-4}}$$

Chiral $U(1)$ Gauge Theory

Focus on the axionic mode.

$$\phi_1 \sim e^{i\tilde{a}/f_1} , \quad \phi_2 \sim e^{i\tilde{b}/f_2}$$

Spontaneous symmetry breaking, $\langle\phi_1\rangle \sim f_1$ and $\langle\phi_2\rangle \sim f_2$.

$$|D_\mu\phi_1|^2 + |D_\mu\phi_2|^2 \quad \to \quad \frac{1}{2}(\partial_\mu a)^2 + m_A^2\left(A_\mu - \frac{1}{m_A}\partial_\mu b\right)^2$$

where

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{-1}{\sqrt{q_1^2 f_1^2 + q_2^2 f_2^2}} \begin{pmatrix} q_2 f_2 & -q_1 f_1 \\ q_1 f_1 & q_2 f_2 \end{pmatrix} \begin{pmatrix} \tilde{a} \\ \tilde{b} \end{pmatrix}$$

$$\frac{a}{F_a} \in [0, 2\pi) , \quad F_a = \frac{f_1 f_2}{\sqrt{m^2 f_1^2 + n^2 f_2^2}}$$

Breaking Operator

$$\mathcal{O} = \frac{1}{k!\,l!}\frac{\phi_1^k \phi_2^l}{M_{\rm Pl}^{N-4}} \propto \exp\left(i\frac{aN_{\rm DW}}{F_a}\right), \quad N_{\rm DW} = {\rm GCD}[k,l]$$

Anomaly term is

$$\mathcal{L} \supset N_{\rm DW}\frac{a}{F_a} G\tilde{G}$$

- Choosing large k and l, one could have highly suppressed $\mathcal{O}$.
- Given k and l, one needs to solve for fermion charges Q_i under

$$\sum_i Q_i^3 = 0$$

A general mathematical problem. Tricks Required!

QCD Axion

Take $k = 3$ and $l = 2x$, and assume that $\mathrm{GCD}[3, 2x] = 1$.

$$\delta\bar{\theta} \sim \frac{2}{3!(2x)!}\left(\frac{F_a}{M_{\mathrm{Pl}}}\right)^N \left(\frac{9+4x^2}{2}\right)^{N/2} \frac{M_{\mathrm{Pl}}^4}{m_\pi^2 F_\pi^2}$$

- $F_a = 10^{12}$ GeV, $x = 7$, $\delta\bar{\theta} \sim 10^{-26}$

$$\{-19, -9, -14 | -17, 23, -4, 10, -2, 8, -2, 8, -2, 8, -2, 8, 1, 5\}$$

- $F_a = 10^9$ GeV, $x = 4$, $\delta\bar{\theta} \sim 10^{-23}$

$$\{-5, -3, -4 | -3, 6, 1, 2, 1, 2, 1, 2\}$$

Fuzzy DM

Assume that $\psi_i \in (1, \mathbf{2}, 0)$ and $\overline{\psi}_i \in (1, \mathbf{2}^*, 0)$ under SM gauge group.

$$\mathcal{L} \supset N_{\mathrm{DW}} \frac{a}{F_a} W \tilde{W}$$

• Electroweak instanton contribution negligible without SUSY.
Nomura, Watari, and Yanagida (2000)
• Potential mainly from $\mathcal{O}$.

$$V = \frac{\Lambda_a}{2} \left(1 - \cos \frac{a N_{\mathrm{DW}}}{F_a} \right) , \quad \Lambda_a = \frac{2^{2-N/2}}{k!l!} \frac{f_1^k f_2^l}{M_{\mathrm{Pl}}^{N-4}}$$

Fuzzy DM

The mass of axion is determined by $\partial_a^2 V|_{\min}$,

$$m_a = N_{\rm DW} M_{\rm Pl} \sqrt{\frac{2}{k!\,l!}} \left(\frac{m^2+n^2}{2}\right)^{N/4} \left(\frac{F_a}{M_{\rm Pl}}\right)^{-1+N/2}$$

- $k = 7, l = 42$ and $F_a = 10^{16}\,\text{GeV} \quad \longrightarrow \quad m_a = 2.5 \times 10^{-20}\,\text{eV}.$

$$\{-21|4,3,16,-9,20,-13\} \times 7\ , \quad N_{\rm DW} = 7$$

Quintessence Axion

Same as the Fuzzy DM scenario, the potential is

$$V = \frac{\Lambda_a}{2}\left(1 - \cos\frac{aN_{\rm DW}}{F_a}\right) \, , \quad \Lambda_a = \frac{2^{2-N/2}}{k!l!}\frac{f_1^k f_2^l}{M_{\rm Pl}^{N-4}}$$

- $k = 8$ and $l = 72$, $\Lambda_a = 1.88\Lambda_{\rm obs}$ and $F_a/N_{\rm DW} = 3.3 \times 10^{16}$ GeV.

$$\{-27|5, 1, 15, , -9, 19, -13, 29, -23, 3\} \times 8 \, , \quad N_{\rm DW} = 8$$

Lowest operator?

Higher order (non Yukawa) fermion mass term may generate

$$\mathcal{O} \overset{?}{\sim} \frac{1}{M_\psi^{N-4}} \phi_1^k \phi_2^l \, , \quad M_\psi < M_{\rm Pl}$$

Is there a general way to avoid this possible loop?

Chiral $U(1)$ Gauge Theory

$$\mathcal{L} \supset -\phi_1 \sum_i c_i \psi_i \overline{\psi}_i - \phi_2 \sum_j c_j \psi_j \overline{\psi}_j$$

Interestingly, there is an anomaly-free $\mathbf{Z}_{2N}$ symmetry, where

$$\psi_i(+1)\,, \quad \overline{\psi}_i(+1)\,, \quad \phi_{1,2}(-2)\,.$$

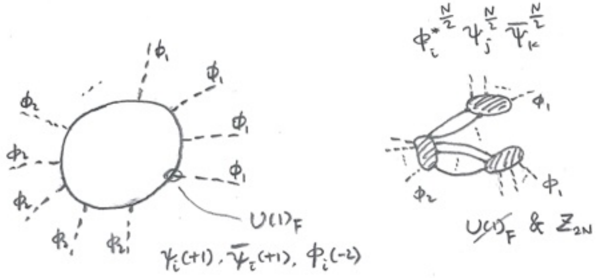
We gauge it.

Now we have gauged $U(1)_g$ and $\mathbf{Z}_{2N}$.

The $\mathbf{Z}_{2N}$ and fermion loop.

$$\mathcal{L}_0 = -\phi_1 \sum_i c_i \psi_i \overline{\psi}_i - \phi_2 \sum_j c_j \psi_j \overline{\psi}_j$$

Accidentally, it contains a global $U(1)_{\mathrm{F}}$.



- Converging diagram does not contribute to $\mathcal{O}$.
- Diverging diagrams are more suppressed due to the loop factors.

Instability Problem

Cosmic evolution

$$\ddot{a} + 3H(t)\dot{a} + \partial_a V = 0$$

Suppose $a_{\rm ini}$ locates around the bottom, $\partial_a V \sim m_a^2 a$,

$$m_a \sim \sqrt{\frac{\Lambda_a N_{\rm DW}}{F_a^2}} > \sqrt{N_{\rm DW}} \times 10^{-33}\,\text{eV} \quad \text{for} \quad F_a \lesssim M_{\rm Pl}$$

One has to put $a_{\rm ini}$ around the hilltop.
One needs large enough $\frac{F_a}{N_{\rm DW}}$ to stabilize it.

Previously,

- $k = 8$ and $l = 72$, $\Lambda_a = 1.88\Lambda_{\rm obs}$ and $\frac{F_a}{N_{\rm DW}} = 3.3 \times 10^{16}\,\text{GeV}$.

Too complicated!

Gauged $\mathbf{Z}_{2N}$ and $N = 80$ pairs of fermions are needed!
Instability!

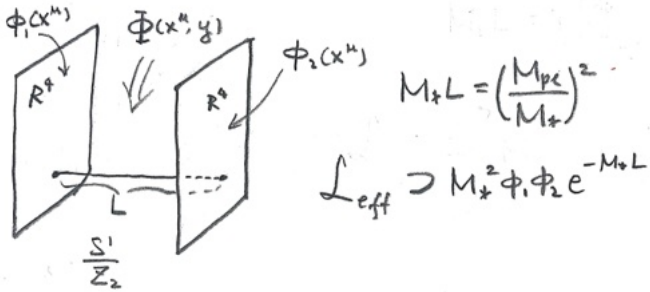
ugly!

How about something simple and beautiful?

Extra Dimension!

New Scenario on Quintessence Axion

Flat extra dimension. Fundamental scale is M_*.



2303.02852 (PRD)

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New Scenario on Quintessence Axion

$$V = \frac{\Lambda_a}{2}\left(1 - \cos\frac{a}{F_a}\right)$$

$$\Lambda_a \approx \Lambda_{\rm obs}\left(\frac{M_*}{1.47\times 10^{17}\,\text{GeV}}\right)^{515} \quad , \quad F_a = \frac{M_*}{\sqrt{2}} \approx 1.04\times 10^{17}\,\text{GeV}$$

- Gauged $\mathbf{Z}_{2N}$ is not needed.
- Only 2 pairs of fermions are needed.
- No instability problem. $F_a > 10^{17}$ GeV.
- Simple $U(1)_g$ charge assignment.

Summary

- High quality axion(s) could be easily constructed in this framework.
- The quality is protected by gauged $U(1)_g$ and $\mathbf{Z}_{2N}$.
- The QCD axion constructed under this framework has $N_{\rm DW} = 1$.
- $N_{\rm DW} \frac{a}{F_a} W\tilde{W} \longrightarrow c_\gamma \frac{a}{F_a} F\tilde{F}$, quintessence axion could explain the isotropic cosmic birefringence. Lin & Yanagida (2022)
- If extra dimension is introduced, model become simpler and quintessence axion becomes more attractive.
- Lots of massive fermions may leave traces in the Cosmological Collider.
- ...

Thank you!