Axi-Higgs Cosmology

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May 25, 2023

2102.11257 & 2105.01631

I

Motivation

Tensions in ΛCDM

- Hubble tension: $H_{0,P18} = 67.36 \pm 0.54$ km/s/Mpc[Aghanim et al., 2022] vs. $H_{0,\text{late}} = 73.3 \pm 0.8$ km/s/Mpc [Verde et al., 2019] from $z \lesssim 2$.
  $H_{0,\text{late}} = 66.6^{+4.4}_{-3.3}$ km/s/Mpc from Refsdal's reappearance. (2305.06367)
- ^{7}Li Problem in BBN: the abundance ratio $^7\text{Li}/\text{H} \times 10^{10}$: 1.6 ± 0.3 (observed) vs. 5.6 ± 0.3 (theoretical)[Zyla et al. 2020, Pitrou et al., 2018, Iliadis and Coc, 2020].

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- The weak lensing measurement of S_8 together with the clustering parameter σ_8 [Troxel et al., 2018] yields a value smaller $S_{8,\text{DES}} = 0.773^{+0.026}_{-0.020}$ than given by the CMB-ΛCDM value, $S_{8,\text{CMB}} = 0.832 \pm 0.013$.
- Isotropic cosmic birefringence angle based on the cross-power (parity-violating) C_l^{EB} data in CMB[Minami and Komatsu, 2020], deviate from 0 by $\sim 2.4\sigma$.
  Later improved result: deviate from 0 by $\sim 3.6\sigma$[Eskilt and Komatsu, 2022].

Tensions in ΛCDM

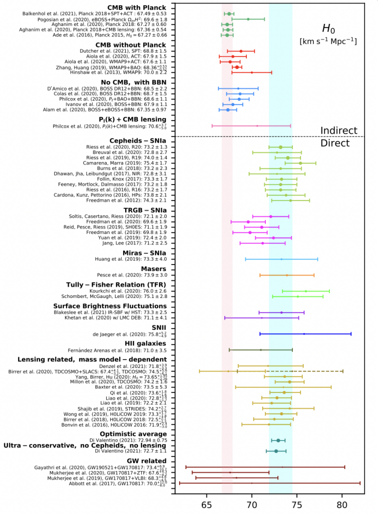

Figure: Valentino et al. 2021

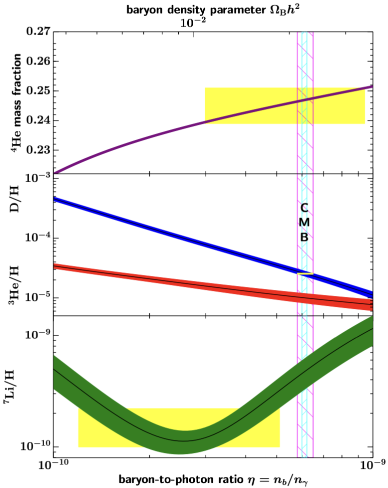

Figure: Zyla et al. 2020

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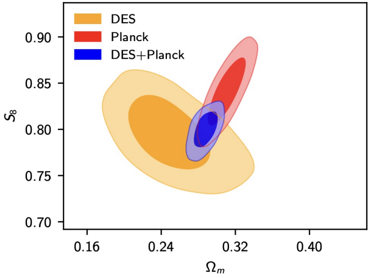

Figure: Handley and Lemos 2019

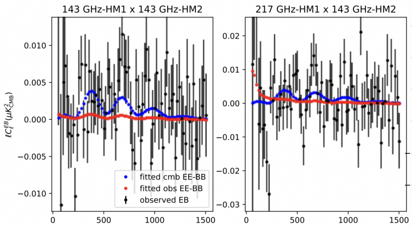

Figure: Minami and Komatsu 2020

II

Solutions

^{7}Li Puzzle

A Higher Higgs VEV during the BBN epoch.

- Fermi constant $G_F \propto v^{-2}$. A smaller G_F leads to an earlier freeze out of the $n \rightleftharpoons p$ and a longer n lifetime.
  A larger n density than that in the standard BBN.
- Electro mass $m_e \propto v$. A larger m_e will reduce the rate of $n \rightleftharpoons p$ and delay neutron decay.

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- Mass difference $\Delta m_q = m_d - m_u \propto v$, which contributes to Δm_{np} and impact $n \rightleftharpoons p$ and neutron decay oppositely relative to G_F and m_e.
- Averaged light quark mass, which contributes to pion mass m_π. A larger pion mass makes nuclei less tightly bound. The nuclear-reaction rates thus may change substantially.

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The current experimental bounds on Y_p and D/H are still compatible for $\delta v_{\text{BBN}} \sim \mathcal{O}(1\%)$ and $\delta\eta \sim \mathcal{O}(1\%)$ (CMB baryon-to-photon ratio).

Following this, to addressing the ^{7}Li problem, one needs [Pitrou et al. 2018]

$$\delta v_{\text{BBN}} = (1.1 \pm 0.1)\% \ , \quad \delta\eta = (1.7 \pm 1.3)\% \ .$$

Hubble tension

Focus on the model ΛCDM$+m_e$. [Hart and Chluba, 2019]
$\delta m_e \approx \delta v_{\rm rec} \sim 1\%$.

- Thompson scattering cross-section, $\sigma_T \propto m_e^{-2}$.
- The atomic energy levels, $E_i \propto m_e$.
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$\Longrightarrow$ Shift up the redshift of the rec z_*, and the baryon drag redshift z_d.

$\Longrightarrow$ Sound horizon at rec decrease.

$\Longrightarrow$ To keep angular sound horizon at rec unchanged, H_0 increases.

Hubble tension

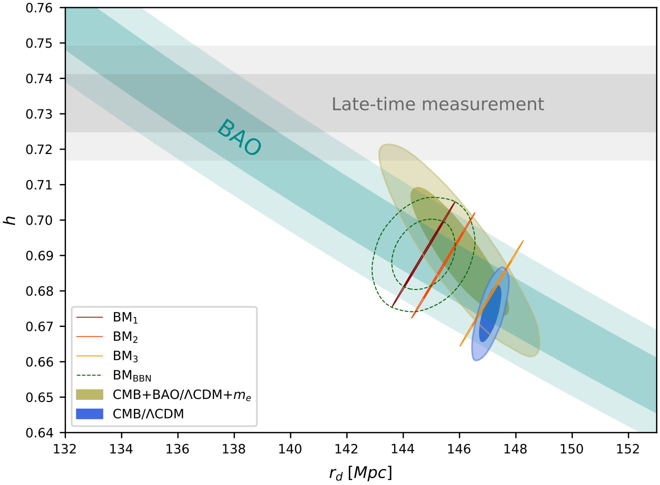
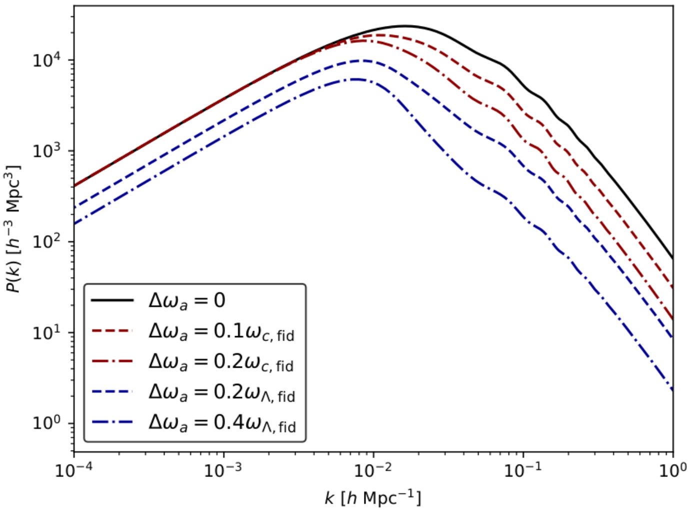


Figure: BM_1: $\delta v = 1.1\%$. BM_2: $\delta v = 1.0\%$. BM_1: $\delta v = 0\%$. $\mathrm{BM}_{\mathrm{BBN}}$: $\delta v = \delta v_{\mathrm{BBN}}$.

S_8/σ_8 'Tension'

Introduce the ultra-light axion-like particle, ω_a.



Isotropic Cosmic Birefringence

Consider the axion-photon coupling $aF\tilde{F}$. [Carroll and Field, 1990]

$$\mathcal{L} \supset \frac{1}{32\pi^2}\frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\Longrightarrow \quad \beta_{\rm obs} \sim \frac{1}{16\pi^2}\frac{a_{\rm ini}}{f_a}$$

$$\Longrightarrow \quad \frac{a_{\rm ini}}{f_a} \approx 1.0 \pm 0.3$$

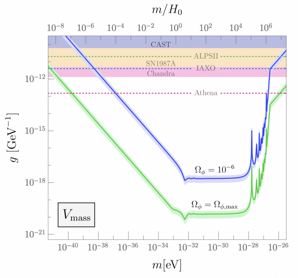


Figure: T. Fujita et. al., 2020

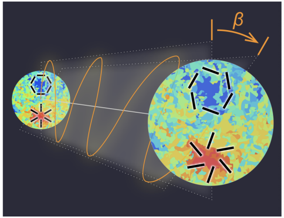


Figure: Credit: Y. Minami.

III

A(-xion) Solution

The candidate

- An up-shift of electron mass at the recombination era $\delta m_e \sim 1\%$ could resolve Hubble tension. [Hart & Chluba (2020)]
- ^{7}Li problem could be solved by lifting the Higgs VEV $\delta v \sim 1\%$. [2102.11257 and refs therein]

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A model with evolving Higgs VEV and the axion?

Axi-Higgs is constructed by introducing coupling between ultra-light axion(s) and the Higgs.

Axi-Higgs Potential

Scalar potential is $V = V_a + V_\phi$, where

$$V_a = m_a^2 f_a^2 \left(1 - \cos\frac{a}{f_a}\right) \approx \frac{1}{2} m_a^2 a^2 - \frac{1}{24}\frac{m_a^2}{f_a^2} a^4 + \cdots \ ,$$

$$V_\phi = \left| m_s^2 F(a) - \kappa \phi^\dagger \phi \right|^2 \ , \quad F(a) = 1 + \frac{Ca^2}{M_{\rm Pl}^2} \ .$$

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Neglect three Goldstone directions and let $\phi^\dagger\phi \to v^2/2$, then

$$V \approx \frac{1}{2} m_a^2 a^2 + |B(a,v)|^2 \,, \quad B = m_s^2 \left(1 + \frac{Ca^2}{M_{\mathrm{Pl}}^2}\right) - \frac{1}{2}\kappa v^2 \,.$$

Treating $a(t)$ as a background field, the Higgs VEV is given by minimize $\langle V_\phi \rangle$, which is

$$\langle \phi^\dagger \phi \rangle \equiv \frac{v^2}{2} = \frac{m_s^2}{\kappa} F(a)$$

Evolving Higgs VEV

- $m_a \sim 10^{-29}\,\text{eV} \quad \Longrightarrow \quad a(t)$ evolves in the cosmic time scale.
- Higgs VEV $v(a(t))$ also evolves in the cosmic time scale,

$$\delta v(t) = \frac{v(t) - v_0}{v_0} = [F(a(t))]^{1/2} - 1 \simeq \frac{Ca(t)^2}{2M_{\text{Pl}}^2}\ ,$$

where $v_0 = \sqrt{2}m_s/\sqrt{\kappa} = 246\,\text{GeV}$.

- $a(t)$ is determined by KG equation in the FLRW background,

$$\ddot{a} + 3H(t)\dot{a} + \frac{\partial V_a}{\partial a} = 0\ .$$

Evolving Higgs VEV is driven by misalignment mechanism of the ultra-light axion.

Coupled system

One may worry about the back reaction from the Higgs to the axion.

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Consider the coupled EoMs for $a(t)$ and $v(t)$,

$$\ddot{a} + 3H\dot{a} + \left[m_a^2 + \frac{4Cm_s^2}{M_{\rm Pl}^2} B(a,v) \right] a \approx 0$$
$$\ddot{v} + (3H + \Gamma_\phi)\,\dot{v} - 2\kappa B(a,v)v = 0 \ ,$$

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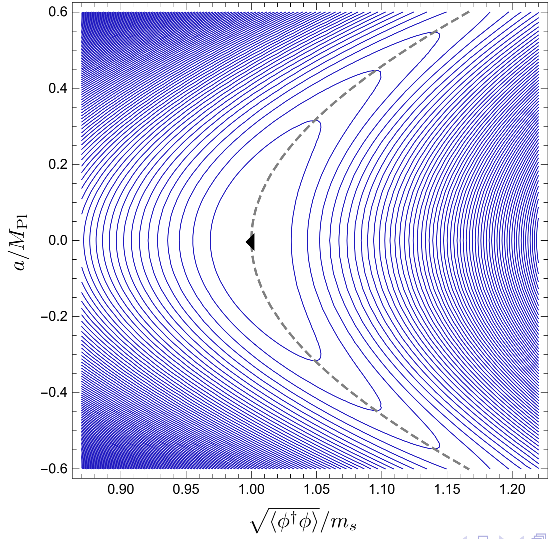
- At first sight, $m_s^2 B/M_{\rm Pl}^2 \sim 10^{50} m_a^2$ and second term in potential driven force dominated over m_a. Fortunately, this is not the case.
- Thanks to the presence of Γ_ϕ, Higgs field profile got damped quickly to the value where $B(a,v) = 0$.

The coupled system will evolve along the valley in a-ϕ configuration space.

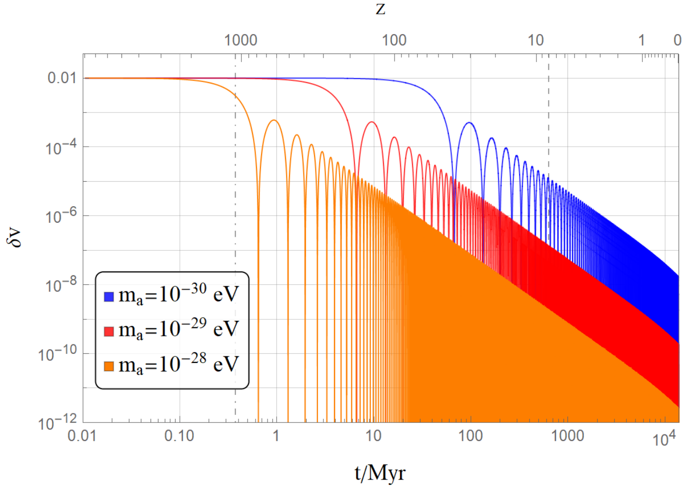
Coupled system



Evolving Higgs VEV

$$\delta v(t) = \frac{Ca(t)^2}{2M_{\rm Pl}^2} = \delta v_{\rm ini}\frac{a(t)^2}{a_{\rm ini}^2}$$

Set $\delta v_{\rm ini} = 1\%$.



Axi-Higgs Parameters

- One has two bounds on the axion mass.
  1. Keep $\delta v \geq 1\%$ until $t \gtrsim t_{\text{rec}}$.
  2. $d(\delta v)/dt \lesssim 10^{-16}\,\text{yr}^{-1}$ required by experimental bound.

  This leads to

$$1.0 \lesssim \frac{m_a}{10^{-29}\,\text{eV}} \lesssim 3.3\,, \quad 68\%\ \text{C.L.}$$

- ICB determines $a_{\text{ini}}/f_a \approx 1$.
- Suppose a takes up x fraction of today's matter energy density and one could constrain a_{ini}.

Axi-Higgs Parameters

Numerically, parameters are

$$a_{\rm ini} \approx 3.7 \times 10^{17}\,\mathrm{GeV} \left(\frac{x}{0.01}\right)^{1/2} \left(\frac{\xi}{1.5}\right)^{-1} ,$$

$$f_a \approx 3.8 \times 10^{17}\,\mathrm{GeV} \left(\frac{x}{0.01}\right)^{1/2} \left(\frac{\xi}{1.5}\right)^{-1} ,$$

$$C \approx 0.84 \left(\frac{\delta v_{\rm ini}}{0.01}\right) \left(\frac{x}{0.01}\right)^{-1} \left(\frac{\xi}{1.5}\right)^{2} ,$$

where $x = \omega_a/(\omega_a + \omega_b + \omega_c)$,
and ξ is the numerical factor from equation $\xi H(z_a) = m_a$.
Be aware that m_a does not come in above parameters.

Axi-Higgs Parameters

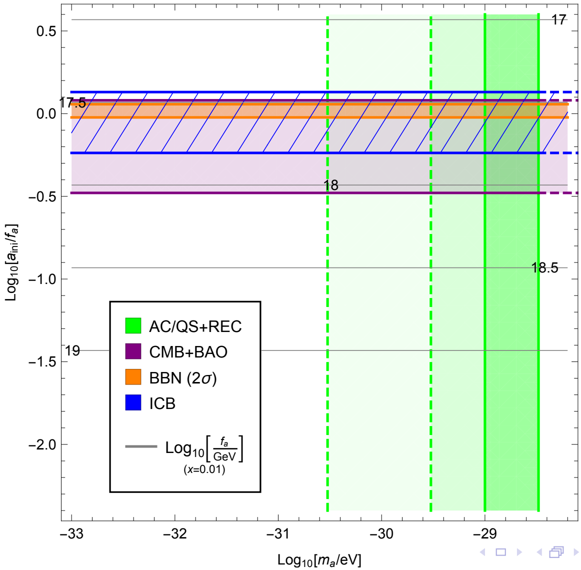
There are four parameters in the single axion axi-Higgs model,

$$m_a,\ \delta v_{\rm ini},\ a_{\rm ini},\ f_a$$

which are all relatively well-constrained.

- $\delta v_{\rm BBN} \approx \delta v_{\rm rec} > \delta v_0 = 0 \implies m_a \approx 10^{-30}\,\text{eV}\text{–}10^{-29}\,\text{eV}$
- ^{7}Li and H_0 puzzle $\implies \delta v_{\rm ini} \approx 1\%$
- H_0 tension & S_8/σ_8 tension $\implies a_{\rm ini} \approx 10^{17}\,\text{GeV}$
- CMB Birefringence $\implies f_a \approx a_{\rm ini} \approx 10^{17}\,\text{GeV}$

Axi-Higgs Parameters



Two-axion Axi-Higgs

- As the evolution of single-axion, one would expect that $\delta v_{\mathrm{BBN}} > \delta v_{\mathrm{rec}}$.
- To better resolve Hubble tension, one would require $\delta v_{\mathrm{rec}} > 1\%$. Meanwhile, BBN analysis prefers $\delta v_{\mathrm{BBN}} < 1.1\%$.

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- In fuzzy dark matter scenario, an axion with mass $\sim 10^{-22}\,\text{eV}$ as the cold dark matter can resolve a number of problems in the weakly interacting massive particle (WIMP) model.
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Consider two axions with mass $m_1 = 10^{-29}\,$eV and $m_2 = 10^{-22}\,$eV. Neglecting interaction between a_1 and a_2, function F is given by

$$F\left(a_1, a_2\right) = 1 + \frac{C_1 a_1^2}{M_{\rm Pl}^2} + \frac{C_2 a_2^2}{M_{\rm Pl}^2}\,.$$

We expect small deviation of Higgs VEV, which could be approximated by

$$\delta v(t) = F^{1/2} - 1 \approx \frac{C_1 a_1^2}{2M_{\rm Pl}^2} + \frac{C_2 a_2^2}{2M_{\rm Pl}^2}\,.$$

Two-axion Axi-Higgs

- $a_{1,\mathrm{ini}}$ the same as single-axion case. Consider $a_{2,\mathrm{ini}}$ consists of most of the dark matter and it starts to roll down at $z_2 \sim 2 \times 10^6$, one has

$$a_{1,\mathrm{ini}} \approx 3.7 \times 10^{17}\ \mathrm{GeV}\ , \quad a_{2,\mathrm{ini}} \approx 1.5 \times 10^{17}\ \mathrm{GeV}\ .$$

- $C_{1,2}$ are determined by

$$\frac{C_1 a_{1,\mathrm{ini}}^2}{2M_{\mathrm{Pl}}^2} + \frac{C_2 a_{2,\mathrm{ini}}^2}{2M_{\mathrm{Pl}}^2} = \delta v_{\mathrm{BBN}}\ , \quad \frac{C_1 a_1^2(t_{\mathrm{rec}})}{2M_{\mathrm{Pl}}^2} = \delta v_{\mathrm{rec}}\ .$$

For $\delta v_{\mathrm{BBN}} = 1\%$, $\delta v_{\mathrm{rec}} = 2\%$ and relation $a_1(t_{\mathrm{rec}}) \approx 0.99 a_{1,\mathrm{ini}}$,

$$C_1 \approx 1.7\ , \quad C_2 \approx -5.1\ .$$

Test of Axi-Higgs

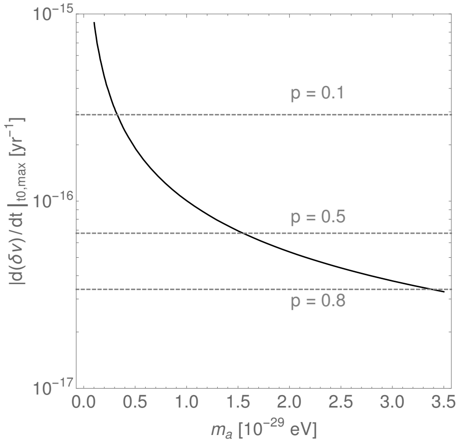

Figure: Drift rate of v.

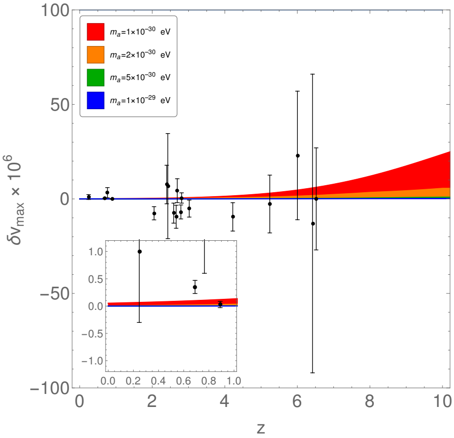

Figure: δv from Quasar Spectrum.

IV

Summary

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- A slightly higher v in the early universe helps resolve Hubble tension and ^{7}Li problem.
- The introduction of axion helps explain the ICB and suppress S_8/σ_8.
- The parameters are tightly constrained and could be tested in the near future.

Thank you for your attention.

Projections

- Consider $X^2 = 0$ with $X = x + \sqrt{2}\theta G + \theta^2 F^X$.

$$x^2 \,, \quad xG_\alpha = 0 \,, \quad 2xF^X - GG = 0 \quad \Longrightarrow \quad X = \frac{GG}{2F^X} + \sqrt{2}\theta G + \theta^2 F^X \,.$$

- For $X^2 = XQ = 0$ with $Q = q + \sqrt{2}\theta\phi + \theta^2 F^Q$,

$$x = \frac{GG}{2F^X} \,, \quad xq = 0 \,, \quad qG_\alpha + x\psi_\alpha = 0 \,, \quad qF^X + xF^Q - G\psi = 0$$

$$\Longrightarrow \quad q = \frac{1}{F^X}\left(\psi - \frac{F^Q G}{2F^X}\right) G$$

- $\overline{D}_{\dot\alpha}\left(X\overline{Z}\right) = X^2 = 0$ with $Z = z + \sqrt{2}\theta\chi + \theta^2 F^Z$ gives

$$\chi = i\partial_\mu z \sigma^\mu \frac{\overline{G}}{\overline{F^X}}$$

$$F^Z = -\partial_\mu \left(\frac{\overline{G}}{\overline{F^X}}\right) \bar\sigma^\nu \sigma^\mu \frac{\overline{G}}{\overline{F^X}} \partial_\nu z + \frac{\overline{GG}}{2(\overline{F^X})^2} \partial^2 z$$

a-ϕ Mixing

The axi-Higgs mass matrix is given by

$$\mathbf{M} = \begin{pmatrix} m_a^2 + \dfrac{8m_s^4 a^2}{M_{\rm Pl}^4} & -\dfrac{2\sqrt{2}m_s^2 av}{M_{\rm Pl}^2} \\ -\dfrac{2\sqrt{2}m_s^2 av}{M_{\rm Pl}^2} & v^2 \end{pmatrix} , \qquad \lim_{m_a \to 0} \det \mathbf{M} = 0$$

Diagonalize $\mathbf{M}$, one has

$$\left(m_\phi^{\rm phys}\right)^2 \approx 4m_s^2 \left(1 + \frac{a^2}{M_{\rm Pl}^2}\right) + \mathcal{O}\left(m_a^2\right)$$

$$\left(m_a^{\rm phys}\right)^2 \approx m_a^2 + \mathcal{O}\left(m_a^4\right)$$

- Above the scale $\sqrt{m_a f_a}$, the shift symmetry of axion is restored.

$$\Delta m_a^2 \sim \frac{1}{\pi^2}\left(\frac{m_s^2}{M_{\rm Pl}^2}\right)\left(\frac{m_a^2 f_a^2}{m_\phi^2}\right) \lesssim m_a^2$$

Nonlinear SUGRA

Spontaneous SUSY-breaking by $\overline{\text{D3}}$-brane could be described by a nonlinear supergravity model. [Kallosh & Wrase (2014)]

$$K = X^\dagger X + \cdots \,, \quad W = MX + \cdots \,, \quad \underbrace{X^2 = 0}_{\text{nilpotent condition}}$$

The nilpotent condition projects out the scalar part of X,

$$X = \frac{GG}{2F^X} + \sqrt{2}\theta G + \theta^2 F^X \,.$$

The X contributes to the scalar potential as

$$V_X = |M|^2$$

Projections

Nilpotent superfield X could be used as projector to eliminate d.o.f.s in other superfields. [Lindstorm & Rocek (1979), Komargodski & Seiberg (2009), Dall'Agata & Farakos (2016)]

- $XQ = 0$ projects out scalar part of Q
- $XQ =$ chiral projects out fermionic d.o.f. of Q.
- $\dots$

Higgs d.o.f are projected out to properly explain the cosmological constant problem. [Li, Qiu and Tye 2010.10089]

$$\begin{aligned} H_u H_d &\rightarrow \phi^\dagger \phi \\ \langle V_\mu \rangle + \langle V_D \rangle &\rightarrow 0 \end{aligned}$$

The axi-Higgs coupling

Recall superpotential $W \supset X\left(\tilde{m}_s^2 + \tilde{\gamma} H_u H_d\right)$, where parameter $\tilde{m}_s$ and $\tilde{\gamma}$ is in principle determined by geometric sector (U_i, S), which intrinsically include axion-like fields. Thus, it is natural to introduce

$$V_X \quad \to \quad \left| m_s^2 G(a) - \kappa K(a) \phi^\dagger \phi \right|^2 = \left| K(a) \left[m_s^2 F(a) - \kappa \phi^\dagger \phi \right] \right|^2 ,$$

where $G(a=0) = K(a=0) = 1$,

$$G(a) = 1 + \frac{g a^2}{M_{\rm Pl}^2} , \quad K(a) = 1 + \frac{k a^2}{M_{\rm Pl}^2} , \quad F(a) = \frac{G(a)}{K(a)} \approx 1 + \frac{C a^2}{M_{\rm Pl}^2} ,$$

and $C = g - k$ is a constant whose positivity is undetermined.
$K(a)$ is not important. Let $K(a) = 1$.