



Axi-Higgs
Cosmology

Yu-Cheng QIU

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Summary and
Outlook

Axi-Higgs Cosmology

Yu-Cheng QIU

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2102.11257 & 2105.01631

I

MOTIVATION

- ▶ Hubble tension: $H_{0,P18} = 67.36 \pm 0.54$ km/s/Mpc [Aghanim et al., 2022] vs. $H_{0,\text{late}} = 73.3 \pm 0.8$ km/s/Mpc [Verde et al., 2019] from $z \lesssim 2$.
- ▶ ${}^7\text{Li}$ Problem in BBN: the abundance ratio ${}^7\text{Li}/\text{H} \times 10^{10} : 1.6 \pm 0.3$ (observed) vs. 5.6 ± 0.3 (theoretical) [Zyla et al. 2020, Pitrou et al., 2018, Iliadis and Coc, 2020].

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- ▶ The weak lensing measurement of S_8 together with the clustering parameter σ_8 [Troxel et al., 2018] yields a value smaller $S_{8,\text{DES}} = 0.773^{+0.026}_{-0.020}$ than given by the CMB- Λ CDM value, $S_{8,\text{CMB}} = 0.832 \pm 0.013$.
- ▶ Isotropic cosmic birefringence angle based on the cross-power (parity-violating) C_l^{EB} data in CMB [Minami and Komatsu, 2020], deviate from 0 by $\sim 2.4\sigma$.
Later improved result: deviate from 0 by $\sim 3.6\sigma$ [Eskilt and Komatsu, 2022].

Tensions in Λ CDM

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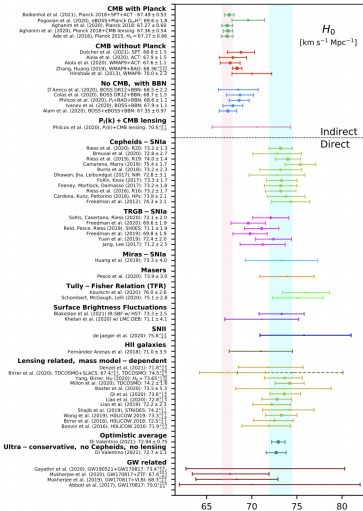


Figure: Valentino et al. 2021

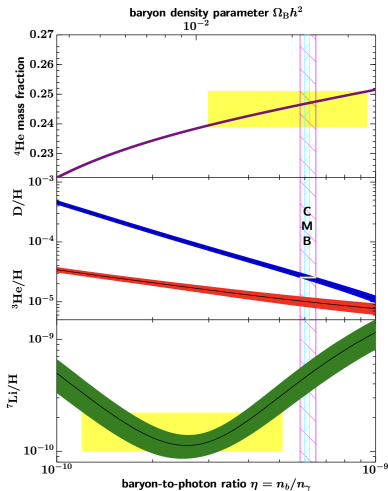


Figure: Zyla et al. 2020



II

SOLUTIONS

A Higher Higgs VEV during the BBN epoch.

- ▶ Fermi constant $G_F \propto v^{-2}$. A smaller G_F leads to an earlier freeze out of the $n \rightleftharpoons p$ and a longer n lifetime.

A larger n density than that in the standard BBN.

- ▶ Electro mass $m_e \propto v$. A larger m_e will reduce the rate of $n \rightleftharpoons p$ and delay neutron decay.

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- ▶ Electro mass $m_e \propto v$. A larger m_e will reduce the rate of $n \rightleftharpoons p$ and delay neutron decay.
- ▶ Mass difference $\Delta m_q = m_d - m_u \propto v$, which contributes to Δm_{np} and impact $n \rightleftharpoons p$ and neutron decay oppositely relative to G_F and m_e .
- ▶ Averaged light quark mass, which contributes to pion mass m_π . A larger pion mass makes nuclei less tightly bound. The nuclear-reaction rates thus may change substantially.

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- ▶ reduce the primordial mass fraction of Helium-4, Y_p ,
- ▶ raise the Deuterium primordial abundance D/H relative to Hydrogen.

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The current experimental bounds on Y_p and D/H are still compatible for $\delta\nu_{\text{BBN}} \sim \mathcal{O}(1\%)$ and $\delta\eta \sim \mathcal{O}(1\%)$ (**CMB baryon-to-photon ratio**).

Following this, to addressing the ${}^7\text{Li}$ problem, one needs [[Pitrou et al. 2018](#)]

$$\delta\nu_{\text{BBN}} = (1.1 \pm 0.1)\% , \quad \delta\eta = (1.7 \pm 1.3)\% .$$

Focus on the model $\Lambda\text{CDM}+m_e$. [Hart and Chluba, 2019]

$$\delta m_e \approx \delta v_{\text{rec}} \sim 1\%.$$

- ▶ Thompson scattering cross-section, $\sigma_T \propto m_e^{-2}$.
- ▶ The atomic energy levels, $E_i \propto m_e$.
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\Rightarrow Sound horizon at rec decrease.

\Rightarrow To keep angular sound horizon at rec unchanged, H_0 increases.

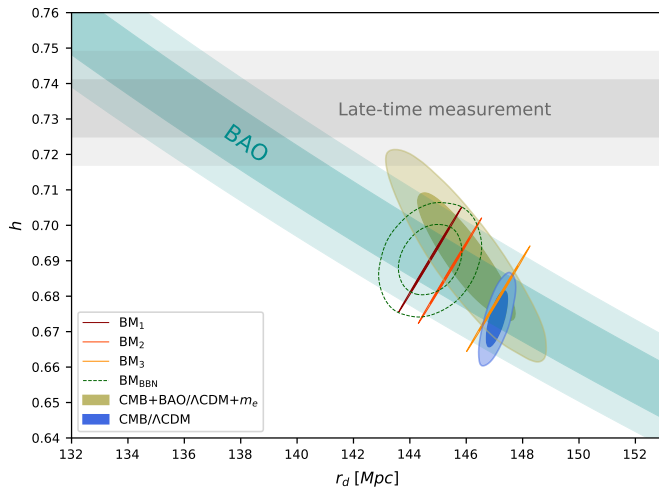
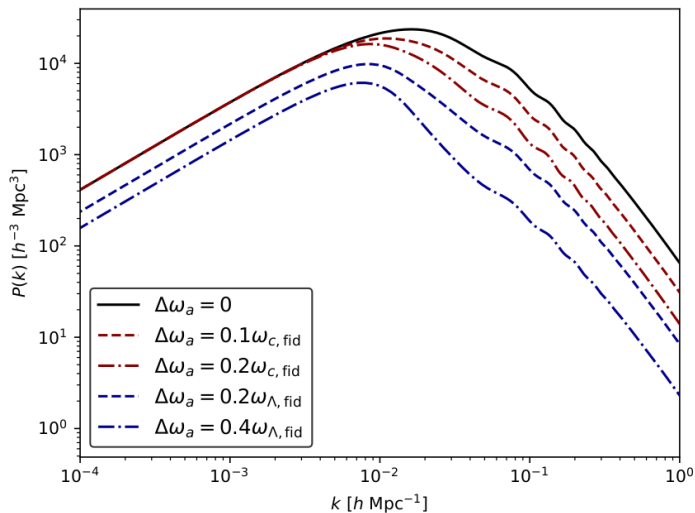


Figure: BM_1 : $\delta v = 1.1\%$. BM_2 : $\delta v = 1.0\%$. BM_3 : $\delta v = 0\%$. BM_{BBN} : $\delta v = \delta v_{BBN}$.

Introduce the ω_a .



Isotropic Cosmic Birefringence

Consider the axion-photon coupling $aF\tilde{F}$. [Carroll and Field, 1990]

$$\begin{aligned}\mathcal{L} &\supset \frac{1}{32\pi^2} \frac{a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu} \\ \Rightarrow \beta_{\text{obs}} &\sim \frac{1}{16\pi^2} \frac{a_{\text{ini}}}{f_a} \\ \Rightarrow \frac{a_{\text{ini}}}{f_a} &\approx 1.0 \pm 0.3\end{aligned}$$

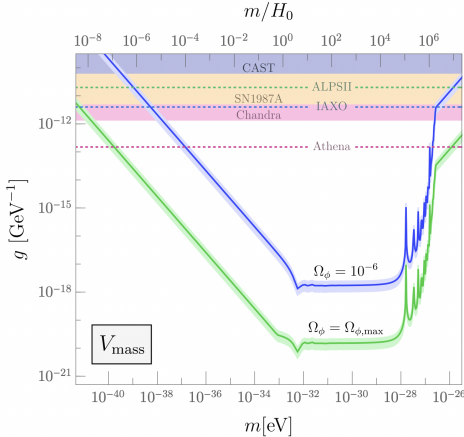


Figure: T. Fujita et. al., 2020



III

A(-XION) SOLUTION

- ▶ An up-shift of electron mass at the recombination era $\delta m_e \sim 1\%$ could resolve Hubble tension. [Hart & Chluba (2020)]
- ▶ ^7Li problem could be solved by lifting the Higgs VEV $\delta v \sim 1\%$. [2102.11257 and refs therein]

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- ▶ Ultra light axion could be used to explain ICB. [Minami & Komatsu (2020)]
- ▶ Introducing the ultra-light axion may be helpful suppressing the S_8/σ_8 . [KiDS-450 (2017), Handley & Lemos (2019)]

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A model with evolving Higgs VEV and the axion?

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A model with evolving Higgs VEV and the axion?

Axi-Higgs is constructed by introducing coupling between ultra-light axion(s) and the Higgs.

Spontaneous SUSY-breaking by $\overline{D3}$ -brane could be described by a nonlinear supergravity model. [Kallosh & Wrase (2014)]

$$K = X^\dagger X + \dots, \quad W = MX + \dots, \quad \underbrace{X^2 = 0}_{\text{nilpotent condition}}$$

The nilpotent condition projects out the scalar part of X ,

$$X = \frac{GG}{2F^X} + \sqrt{2}\theta G + \theta^2 F^X.$$

The X contributes to the scalar potential as

$$V_X = |M|^2$$



Projections

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Nilpotent superfield X could be used as projector to eliminate d.o.f.s in other superfields. [Lindstorm & Rocek (1979), Komargodski & Seiberg (2009), Dall'Agata & Farakos (2016)]

- ▶ $XQ = 0$ projects out scalar part of Q
- ▶ $XQ = \text{chiral}$ projects out fermionic d.o.f. of Q .
- ▶ ...

Higgs d.o.f are projected out to properly explain the cosmological constant problem. [Li, Qiu and Tye 2010.10089]

$$\begin{aligned} H_u H_d &\rightarrow \phi^\dagger \phi \\ \langle V_\mu \rangle + \langle V_D \rangle &\rightarrow 0 \end{aligned}$$

The axi-Higgs coupling

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Recall superpotential $W \supset X (\tilde{m}_s^2 + \tilde{\gamma} H_u H_d)$, where parameter \tilde{m}_s and $\tilde{\gamma}$ is in principle determined by geometric sector (U_i, S) , which intrinsically include axion-like fields. Thus, it is natural to introduce

$$V_X \rightarrow \left| m_s^2 G(a) - \kappa K(a) \phi^\dagger \phi \right|^2 = \left| K(a) \left[m_s^2 F(a) - \kappa \phi^\dagger \phi \right] \right|^2,$$

where $G(a=0) = K(a=0) = 1$,

$$G(a) = 1 + \frac{ga^2}{M_{\text{Pl}}^2}, \quad K(a) = 1 + \frac{ka^2}{M_{\text{Pl}}^2}, \quad F(a) = \frac{G(a)}{K(a)} \approx 1 + \frac{Ca^2}{M_{\text{Pl}}^2},$$

and $C = g - k$ is a constant whose positivity is undetermined.

$K(a)$ is not important. Let $K(a) = 1$.

Scalar potential is $V = V_a + V_\phi$, where

$$V_a = m_a^2 f_a^2 \left(1 - \cos \frac{a}{f_a} \right) \approx \frac{1}{2} m_a^2 a^2 - \frac{1}{24} \frac{m_a^2}{f_a^2} a^4 + \dots ,$$

$$V_\phi = \left| m_s^2 F(a) - \kappa \phi^\dagger \phi \right|^2 , \quad F(a) = 1 + \frac{C a^2}{M_{\text{Pl}}^2} .$$

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$$V_\phi = \left| m_s^2 F(a) - \kappa \phi^\dagger \phi \right|^2 , \quad F(a) = 1 + \frac{C a^2}{M_{\text{Pl}}^2} .$$

Neglect three Goldstone directions and let $\phi^\dagger \phi \rightarrow v^2/2$, then

$$V \approx \frac{1}{2} m_a^2 a^2 + |B(a, v)|^2 , \quad B = m_s^2 \left(1 + \frac{C a^2}{M_{\text{Pl}}^2} \right) - \frac{1}{2} \kappa v^2 .$$

Treating $a(t)$ as a background field, the Higgs VEV is given by minimize $\langle V_\phi \rangle$, which is

$$\langle \phi^\dagger \phi \rangle \equiv \frac{v^2}{2} = \frac{m_s^2}{\kappa} F(a)$$



Evolving Higgs VEV

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- ▶ $m_a \sim 10^{-29} \text{ eV} \implies a(t)$ evolves in the cosmic time scale.
- ▶ Higgs VEV $v(a(t))$ also evolves in the cosmic time scale,

$$\delta v(t) = \frac{v(t) - v_0}{v_0} = [F(a(t))]^{1/2} - 1 \simeq \frac{Ca(t)^2}{2M_{\text{Pl}}^2},$$

where $v_0 = \sqrt{2}m_s/\sqrt{\kappa} = 246 \text{ GeV}$.

- ▶ $a(t)$ is determined by KG equation in the FLRW background,

$$\ddot{a} + 3H(t)\dot{a} + \frac{\partial V_a}{\partial a} = 0.$$

Evolving Higgs VEV is driven by misalignment mechanism of the ultra-light axion.

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One may worry about the back reaction from the Higgs to the axion.

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Consider the coupled EoMs for $a(t)$ and $v(t)$,

$$\ddot{a} + 3H\dot{a} + \left[m_a^2 + \frac{4Cm_s^2}{M_{\text{Pl}}^2} B(a, v) \right] a \approx 0$$
$$\ddot{v} + (3H + \Gamma_\phi) \dot{v} - 2\kappa B(a, v)v = 0 ,$$

where $\Gamma_\phi \sim 4 \text{ MeV}$ is effective Higgs field dissipation.

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where $\Gamma_\phi \sim 4 \text{ MeV}$ is effective Higgs field dissipation.

- ▶ At first sight, $m_s^2 B / M_{\text{Pl}}^2 \sim 10^{50} m_a^2$ and second term in potential driven force dominated over m_a . Fortunately, this is not the case.
- ▶ Thanks to the presence of Γ_ϕ , Higgs field profile got damped quickly to the value where $B(a, v) = 0$.

The coupled system will evolve along the valley in a - ϕ configuration space.

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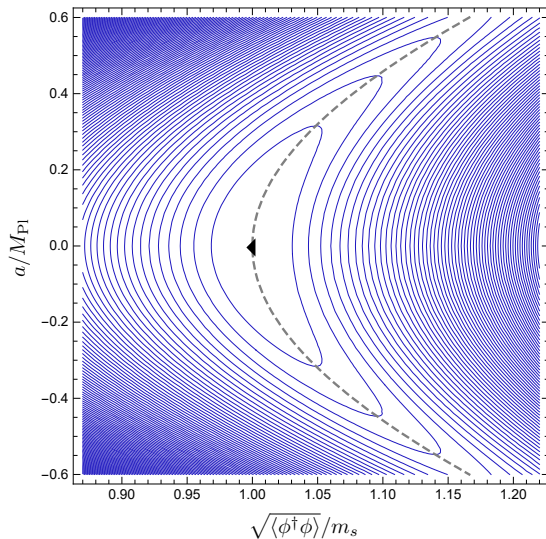
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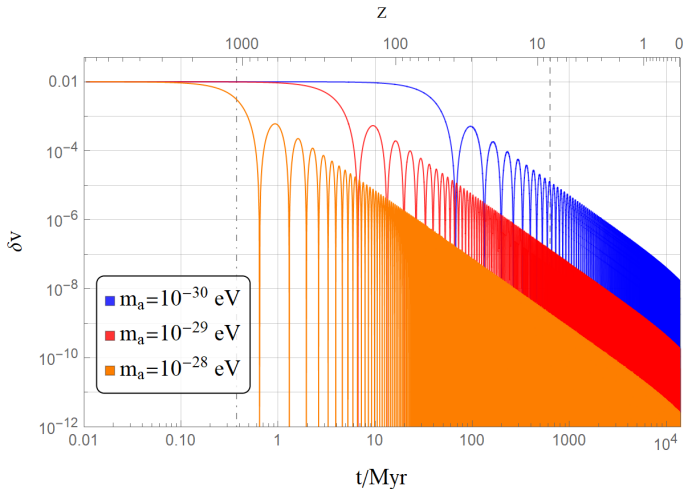
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$$\begin{aligned}\delta v(t) &= \frac{Ca(t)^2}{2M_{\text{Pl}}^2} \\ &= \delta v_{\text{ini}} \frac{a(t)^2}{a_{\text{ini}}^2}\end{aligned}$$

Set $\delta v_{\text{ini}} = 1\%$.



- ▶ One has two bounds on the axion mass.
 1. Keep $\delta v \geq 1\%$ until $t \gtrsim t_{\text{rec}}$.
 2. $d(\delta v)/dt \lesssim 10^{-16} \text{ yr}^{-1}$ required by experimental bound.

This leads to

$$1.0 \lesssim \frac{m_a}{10^{-29} \text{ eV}} \lesssim 3.3, \quad 68\% \text{ C.L.}$$

- ▶ ICB determines $a_{\text{ini}}/f_a \approx 1$.
- ▶ Suppose a takes up x fraction of today's matter energy density and one could constrain a_{ini} .

Numerically, parameters are

$$a_{\text{ini}} \approx 3.7 \times 10^{17} \text{ GeV} \left(\frac{x}{0.01} \right)^{1/2} \left(\frac{\xi}{1.5} \right)^{-1},$$

$$f_a \approx 3.8 \times 10^{17} \text{ GeV} \left(\frac{x}{0.01} \right)^{1/2} \left(\frac{\xi}{1.5} \right)^{-1},$$

$$C \approx 0.84 \left(\frac{\delta v_{\text{ini}}}{0.01} \right) \left(\frac{x}{0.01} \right)^{-1} \left(\frac{\xi}{1.5} \right)^2,$$

where $x = \omega_a / (\omega_a + \omega_b + \omega_c)$,

and ξ is the numerical factor from equation $\xi H(z_a) = m_a$.

Be aware that m_a does not come in above parameters.

There are four parameters in the single axion axi-Higgs model,

$$m_a, \delta v_{\text{ini}}, a_{\text{ini}}, f_a$$

which are all relatively well-constrained.

- ▶ $\delta v_{\text{BBN}} \approx \delta v_{\text{rec}} > \delta v_0 = 0 \implies m_a \approx 10^{-30} \text{ eV} - 10^{-29} \text{ eV}$
- ▶ ${}^7\text{Li}$ and H_0 puzzle $\implies \delta v_{\text{ini}} \approx 1\%$
- ▶ H_0 tension & S_8/σ_8 tension $\implies a_{\text{ini}} \approx 10^{17} \text{ GeV}$
- ▶ CMB Birefringence $\implies f_a \approx a_{\text{ini}} \approx 10^{17} \text{ GeV}$

Axi-Higgs Parameters

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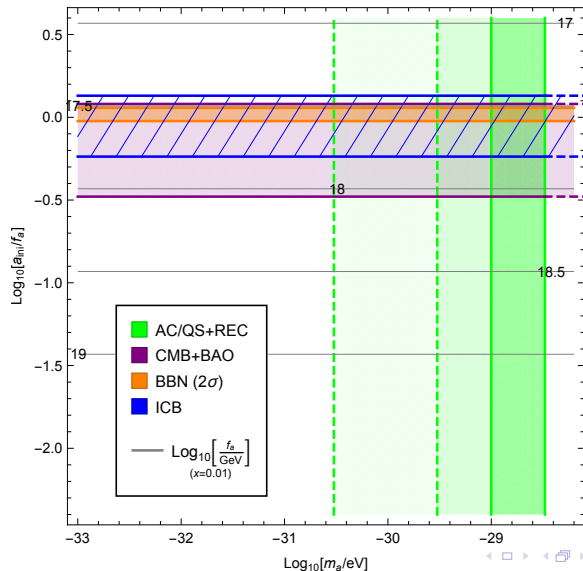
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- ▶ As the evolution of single-axion, one would expect that $\delta v_{\text{BBN}} > \delta v_{\text{rec}}$.
- ▶ To better resolve Hubble tension, one would require $\delta v_{\text{rec}} > 1\%$.
Meanwhile, BBN analysis prefers $\delta v_{\text{BBN}} < 1.1\%$.

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- ▶ In fuzzy dark matter scenario, an axion with mass $\sim 10^{-22}$ eV as the cold dark matter can resolve a number of problems in the weakly interacting massive particle (WIMP) model.
- ▶ A second axion could naturally appear in function F that actually responsible for Higgs VEV.

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Consider two axions with mass $m_1 = 10^{-29}$ eV and $m_2 = 10^{-22}$ eV. Neglecting interaction between a_1 and a_2 , function F is given by

$$F(a_1, a_2) = 1 + \frac{C_1 a_1^2}{M_{\text{Pl}}^2} + \frac{C_2 a_2^2}{M_{\text{Pl}}^2}.$$

We expect small deviation of Higgs VEV, which could be approximated by

$$\delta v(t) = F^{1/2} - 1 \approx \frac{C_1 a_1^2}{2M_{\text{Pl}}^2} + \frac{C_2 a_2^2}{2M_{\text{Pl}}^2}.$$

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- ▶ $a_{1,\text{ini}}$ the same as single-axion case. Consider $a_{2,\text{ini}}$ consists of most of the dark matter and it starts to roll down at $z_2 \sim 2 \times 10^6$, one has

$$a_{1,\text{ini}} \approx 3.7 \times 10^{17} \text{ GeV} , \quad a_{2,\text{ini}} \approx 1.5 \times 10^{17} \text{ GeV} .$$

- ▶ $C_{1,2}$ are determined by

$$\frac{C_1 a_{1,\text{ini}}^2}{2M_{\text{Pl}}^2} + \frac{C_2 a_{2,\text{ini}}^2}{2M_{\text{Pl}}^2} = \delta v_{\text{BBN}} , \quad \frac{C_1 a_1^2(t_{\text{rec}})}{2M_{\text{Pl}}^2} = \delta v_{\text{rec}} .$$

For $\delta v_{\text{BBN}} = 1\%$, $\delta v_{\text{rec}} = 2\%$ and relation $a_1(t_{\text{rec}}) \approx 0.99 a_{1,\text{ini}}$,

$$C_1 \approx 1.7 , \quad C_2 \approx -5.1 .$$

Test of Axi-Higgs

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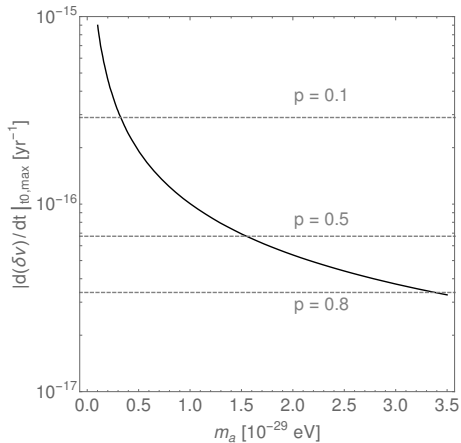


Figure: Drift rate of ν .

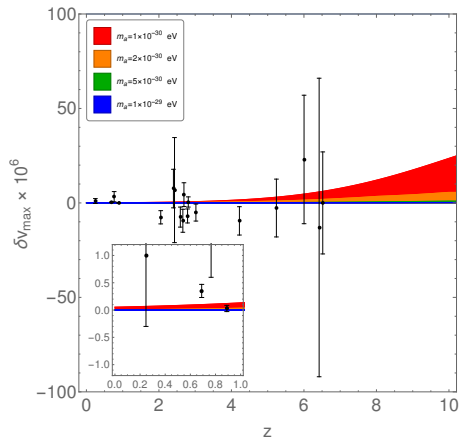


Figure: $\delta\nu$ from Quasar Spectrum.



IV

SUMMARY

- ▶ Axi-Higgs model introduces an evolving Higgs VEV using $|\cdots|^2$.
- ▶ This form $|\cdots|^2$ ensures the cosmic evolution of v and protects the axion evolution from the Higgs back-reaction.

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- ▶ A slightly higher v in the early universe helps resolve Hubble tension and ${}^7\text{Li}$ problem.
- ▶ The introduction of axion helps explain the ICB and suppress S_8/σ_8 .
- ▶ The parameters are tightly constrained and could be tested in the near future.

Thank you for your attention.

- Consider $X^2 = 0$ with $X = x + \sqrt{2}\theta G + \theta^2 F^X$.

$$x^2, \quad xG_\alpha = 0, \quad 2xF^X - GG = 0 \quad \Rightarrow \quad X = \frac{GG}{2F^X} + \sqrt{2}\theta G + \theta^2 F^X.$$

- For $X^2 = XQ = 0$ with $Q = q + \sqrt{2}\theta\phi + \theta^2 F^Q$,

$$x = \frac{GG}{2F^X}, \quad xq = 0, \quad qG_\alpha + x\psi_\alpha = 0, \quad qF^X + xF^Q - G\psi = 0$$

$$\Rightarrow \quad q = \frac{1}{F^X} \left(\psi - \frac{F^Q G}{2F^X} \right) G$$

- $\bar{D}_{\dot{\alpha}}(X\bar{Z}) = X^2 = 0$ with $Z = z + \sqrt{2}\theta\chi + \theta^2 F^Z$ gives

$$\chi = i\partial_\mu z \sigma^\mu \frac{\bar{G}}{\bar{F}^X}$$

$$F^Z = -\partial_\mu \left(\frac{\bar{G}}{\bar{F}^X} \right) \bar{\sigma}^\nu \sigma^\mu \frac{\bar{G}}{\bar{F}^X} \partial_\nu z + \frac{\bar{G}\bar{G}}{2(\bar{F}^X)^2} \partial^2 z$$

The axi-Higgs mass matrix is given by

$$\mathbf{M} = \begin{pmatrix} m_a^2 + \frac{8m_s^4 a^2}{M_{\text{Pl}}^4} & -\frac{2\sqrt{2}m_s^2 a v}{M_{\text{Pl}}^2} \\ -\frac{2\sqrt{2}m_s^2 a v}{M_{\text{Pl}}^2} & v^2 \end{pmatrix}, \quad \lim_{m_a \rightarrow 0} \det \mathbf{M} = 0$$

Diagonalize \mathbf{M} , one has

$$\left(m_\phi^{\text{phys}}\right)^2 \approx 4m_s^2 \left(1 + \frac{a^2}{M_{\text{Pl}}^2}\right) + \mathcal{O}(m_a^2)$$

$$\left(m_a^{\text{phys}}\right)^2 \approx m_a^2 + \mathcal{O}(m_a^4)$$

- Above the scale $\sqrt{m_a f_a}$, the shift symmetry of axion is restored.

$$\Delta m_a^2 \sim \frac{1}{\pi^2} \left(\frac{m_s^2}{M_{\text{Pl}}^2}\right) \left(\frac{m_a^2 f_a^2}{m_\phi^2}\right) \lesssim m_a^2$$