aΛCDM

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2503.18924 with Nakagawa, Nakai, Yamada
2503.18120 & 2507.xxxxx with Luu, Tye

I. Introduction

ΛCDM and Cosmic anomalies

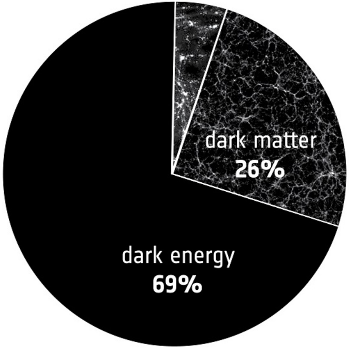


Figure: Copyright: ESA.

- Hubble Tension
- S_8/σ_8 Tension
- $w_{\rm DE} \neq -1$
- Cosmic Birefringence
- $\cdots$

The universe is expanding

$$v = HD$$

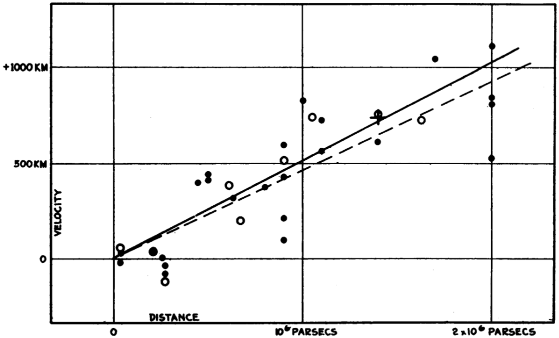


FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.

Figure: Hubble (1929)

The universe is acceleratingly expanding

The Nobel Prize in Physics 2011:
Saul Perlmutter (1/2),
Brian P. Schmidt (1/4)
and Adam G. Riess (1/4)
"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"

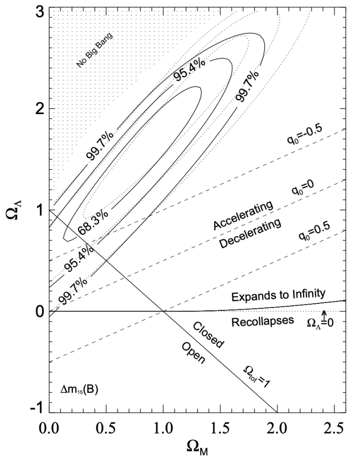


Figure: Riess et al. (1995)

Dark energy

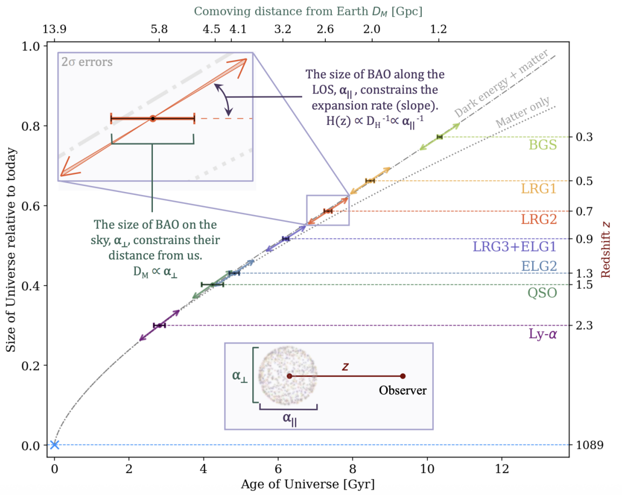
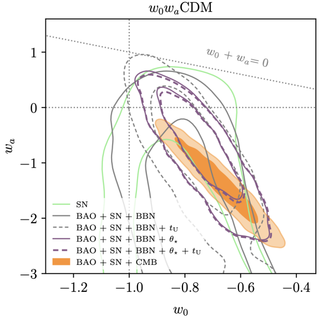
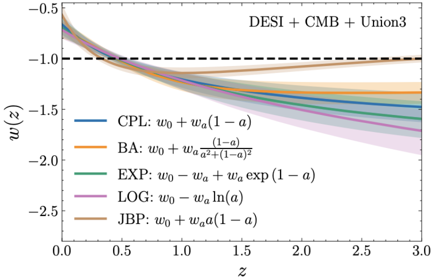


Figure: From DESI DR2.

Dynamical dark energy?

$$\frac{H^2}{H_0^2} = \begin{cases} \Omega_m a^{-3} + (1-\Omega_m) & \text{for } \Lambda\text{CDM} \\ \Omega_m a^{-3} + (1-\Omega_m) a^{-3(1+w)} & \text{for } w\text{CDM} \\ \Omega_m a^{-3} + (1-\Omega_m) a^{-3(1+w_0+w_a)} e^{-3w_a(1-a)} & \text{for} w_0 w_a\text{CDM} \\ \cdots & \end{cases}$$





Isotropic Cosmic Birefringence

$$C_l^{EB,obs} \neq 0 \implies \beta \neq 0$$

- $\beta = 0.35^\circ \pm 0.14^\circ$ (2.4σ) (Minami and Komatsu 2011.11254)
- $\beta = 0.34^\circ \pm 0.09^\circ$ (3.6σ) (Eskilt and Komatsu 2205.13962)
- $\beta = 0.20^\circ \pm 0.08^\circ$ (2.5σ) (ACT 2503.14452)

1. Isotropic
2. Frequency-blind

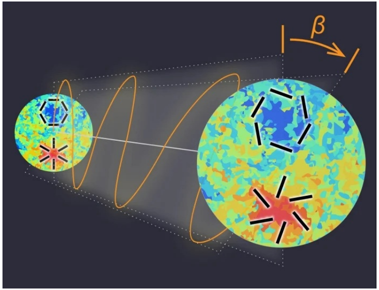


Figure: Credit: Yuto Minami

Misalignment

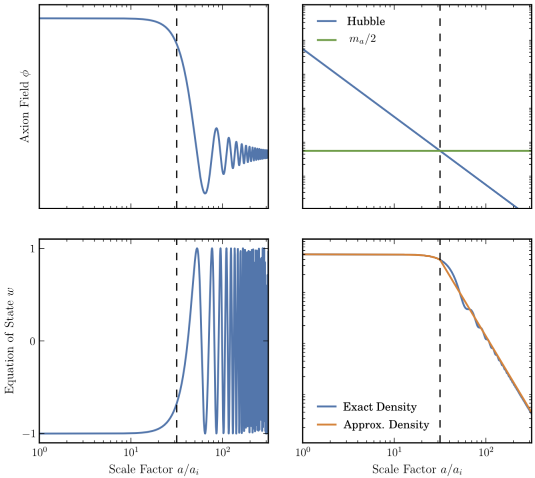


Figure: From Marsh (2016)

For a light (pseudo)scalar ϕ:

- When $H \gg m_\phi$,
  $\phi = \phi_i$, $\rho_\phi = \text{const.}$
- When $H \ll m_\phi$,
  $\phi \simeq e^{-Ht}\cos(m_\phi t)$
  $\langle \rho_\phi \rangle \propto a^{-3}$

Axion (ALP) Explanation

$$\mathcal{L} \supset c_\gamma \frac{\alpha}{4\pi}\frac{\phi}{f_\phi}F_{\mu\nu}\tilde{F}^{\mu\nu}$$ [Carroll, Field (1991)]

$$\implies \beta \simeq 0.067^\circ \times c_\gamma \frac{\Delta\phi}{f_\phi}$$

- Fuzzy dark matter axion, $m \sim 10^{-22}$ eV is too heavy to explain the β.
- axi-Higgs axion ($m \sim 10^{-29}$ eV) is just suitable. [Fung, Li, Liu, Luu, **Qiu**, Tye (2021)]
- For quintessence axion $m \sim H_0$, $\Delta\phi/f_\phi$ is small. So c_γ should be large. [Lin, Yanagida (2022)]

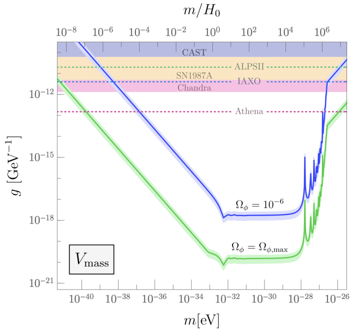


Figure: Fujita et al. 2011.11894

Can SMEFT explain β?

$$\mathcal{L}_{\rm CS} = \frac{\alpha}{8\pi}\sum_a \frac{\tilde{\mathcal{O}}_a}{\Lambda_a^n} F_{\mu\nu}\tilde{F}^{\mu\nu} \ , \qquad n = \dim[\tilde{\mathcal{O}}_a]$$

Building Blocks:

$$H(\dim 1)\ , \quad D_\mu(\dim 1)\ , \quad \psi(\dim 3/2)\ , \quad X_{\mu\nu}(\dim 2)$$

- $n=2$: $H^\dagger H$
- $n=3$: (LEFT) $\mathcal{C}^{ij}\bar{e}^i P_{\rm L} e^j + \text{h.c.}, \quad (e \to \nu, d, u)$.
- $n=4$: $\sum_{X=F,Z,W,G} X_{\alpha\beta}X^{\alpha\beta} + X_{\alpha\beta}\tilde{X}^{\alpha\beta}$

No. All SM particles does not explain the observed β
[Nakai, Namba, obata, **Qiu**, Saito (2023)]

II. a(xion)ΛCDM

aΛCDM

$$\ddot{\phi} + 3H(t)\dot{\phi} + V'(\phi) = 0 \,, \quad V(\phi) = m^2 f_\phi^2 \left(1 - \cos\phi/f_\phi\right)$$

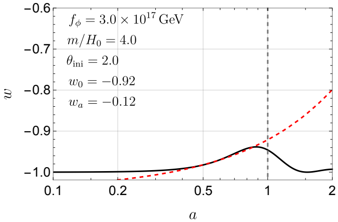
$$\frac{H^2}{H_0^2} = \frac{\Omega_m}{a^3} + \Omega_\Lambda + \frac{\rho_\phi(a)}{\rho_{\mathrm{crit},0}} \,, \quad \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

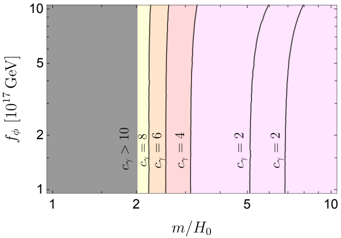
The dark energy is composed of Λ and ρ_ϕ. So the EoS

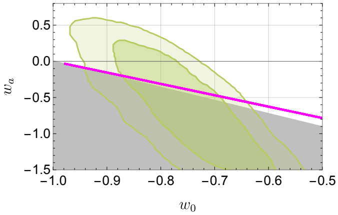
$$w_{\mathrm{DE}} = \frac{\dot{\phi}^2/2 - V(\phi) - \Lambda}{\dot{\phi}^2/2 + V(\phi) + \Lambda} = -1 + \frac{\dot{\phi}^2}{\dot{\phi}^2/2 + V(\phi) + \Lambda}$$

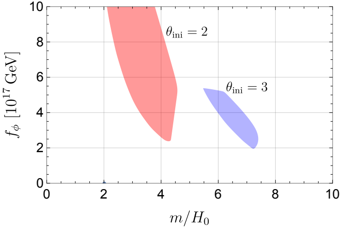
We propose the rolling of ϕ explains the β and $w_{\mathrm{DE}} \neq -1$.

aΛCDM









The contour region is best fit values from data set:
BAO + SN + BBN + $\theta_\star$ + $t_{\rm U}$.

We take appropriate $\Omega_\Lambda \in (0.3, 0.7)$ which gives correct relic. $\theta = \phi/f_\phi$

aΛCDM

To explain the $\beta \sim 0.3^\circ$ and $w_{\mathrm{DE}} \neq -1$, we have

- axion mass:

$$2H_0 \lesssim m \lesssim 7H_0 \ ,$$

- consistent with string motivated axion $f_\phi \sim \mathcal{O}(10^{17})$ GeV,
- small $c_\gamma \sim \mathcal{O}(1)$,
- and dS vacuum $\Lambda > 0$.

$\Lambda < 0$?

In supergravity, the supersymmetric vacuum is AdS.
AdS vacua are ubiquitious in String landscape.
If we forget about Cosmic Birefringence, $\Lambda < 0$ is allowed.

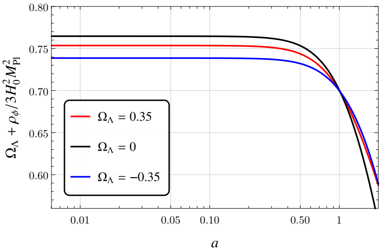


Figure: For $m = 2 \times 10^{-33}$ eV

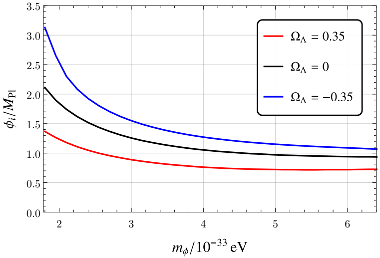
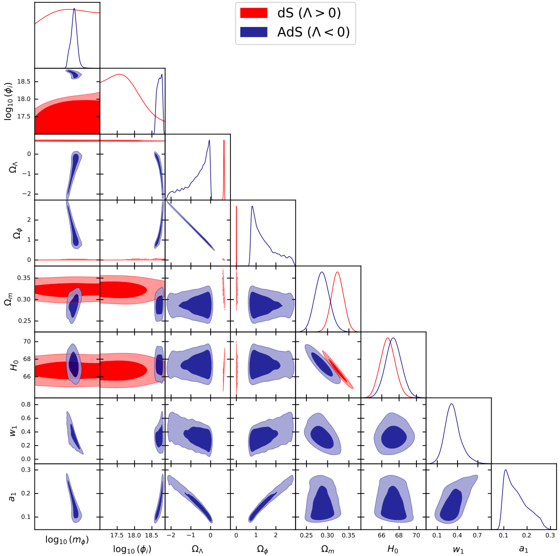


Figure: Parameter space that recreating the correct relic.

$\Lambda < 0$?

The units of m_ϕ, ϕ_i and H_0 are implicitly assumed as eV, GeV and $\mathrm{km}/s/\mathrm{Mpc}$.

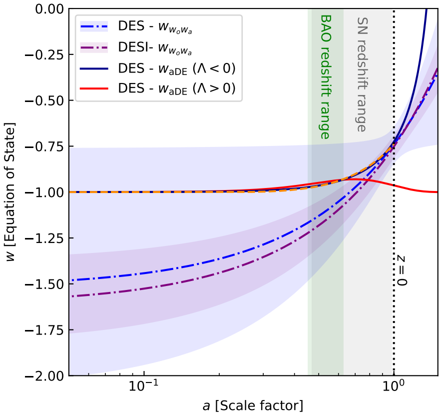


New fitting formula

$$w(a) = \begin{cases} -1 + w_1(a-a_1)^2 & 1 > a \geq a_1 \\ -1 & a < a_1 \end{cases}$$

Taking
$m \simeq 2.93 \times 10^{-33}$ eV
$\phi_i \simeq 6.28 \times 10^{18}$ GeV
$\Omega_\Lambda \simeq -1.61$
$\Omega_m \simeq 0.284$
$H_0 \simeq 67.46$ km/s/Mpc.

Best fit:
$w_1 \simeq 0.44$
$a_1 \simeq 0.23$



III. Towards the future

The future

In the unit of H_0,

$$\mathcal{H}^2 = \frac{\Omega_{\mathrm{m}}}{a^3} + \Omega_\Lambda + \Omega_\phi \ ,$$

$$\mathcal{H}' = -\frac{3}{2}\frac{\Omega_{\mathrm{m}}}{a^3} - \frac{3}{2}\left(\Omega_\Lambda + \Omega_\phi\right)\left(w_{\mathrm{DE}} + 1\right)$$

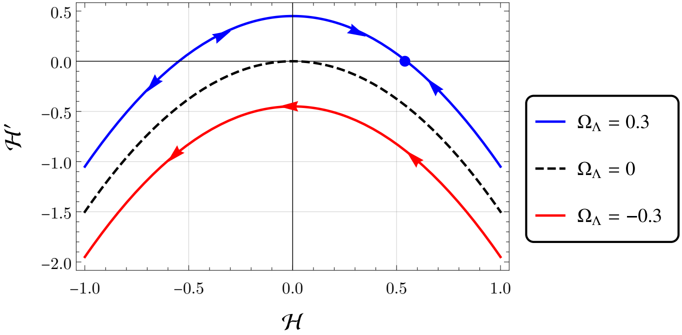
For $\Omega_\phi \to 0$, one has

$$\mathcal{H}' = -\frac{3}{2}\left(\mathcal{H}^2 - \Omega_\Lambda\right)$$

A phase space relation!

The future

$$\mathcal{H}' = -\frac{3}{2}\left(\mathcal{H}^2 - \Omega_\Lambda\right)$$



The future

Axion can be approximated as

$$\Omega_\phi(a) \simeq \begin{cases} \Omega_{\phi,0} & a \leq 1 \\ \Omega_{\phi,0} a^{-3} & a > 1 \end{cases} ,$$

So it is an effective ΛCDM, with $\Omega_\mathrm{m} + \Omega_{\phi,0} \to \Omega_\mathrm{m}$.

$$\frac{a'}{a} = \sqrt{\mathcal{H}^2} = \sqrt{\frac{1-\Omega_\Lambda}{a^3} + \Omega_\Lambda}$$

$$a(t)^3 = \frac{\Omega_\Lambda - 1}{\Omega_\Lambda} \sin^2\left[\frac{3}{2}\sqrt{-\Omega_\Lambda} H_0 \left(t - t_0\right) + \arctan(\sqrt{-\Omega_\Lambda})\right] ,$$

The future

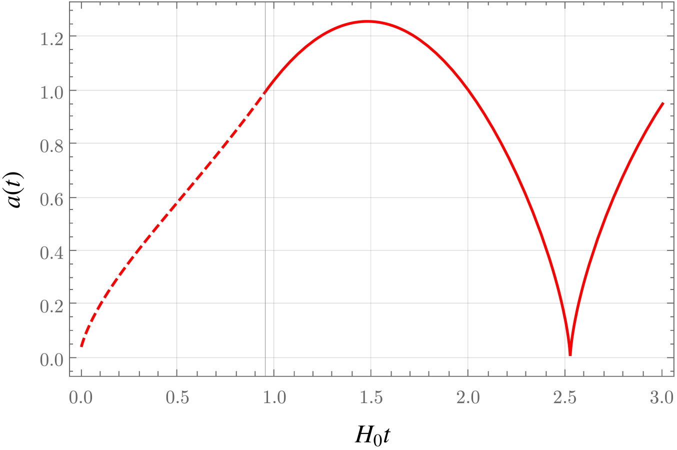


Figure: Effective ΛCDM with $\Omega_\Lambda = -1$ for $t > t_0$ and $\Omega_\Lambda = 0.7$ for $t < t_0$. The age of universe is $t_0 = 13.8$ Gyr, and $H_0 t_0 = 0.952$.

The future

Numerical evolution from $H_0 t_0 = 0.952$:

$$\theta'' + 3\mathcal{H}\theta' + \frac{m_\phi^2}{H_0^2}\sin\theta = 0$$

$$a' = \mathcal{H}a$$

$$\mathcal{H}' = -\frac{3}{2}\frac{\Omega_m}{a^3} - \frac{1}{2}\theta'^2$$

Initial conditions can be obtained from observation on EoS today w_0:

$$\overline{V}(\theta_0) = \frac{1}{2}(1-\Omega_\mathrm{m})(1-w_0) - \Omega_\Lambda \,,$$

$$\theta_0' = \pm\sqrt{\frac{3}{\beta^2}(1-\Omega_\mathrm{m})(1+w_0)}$$

The future

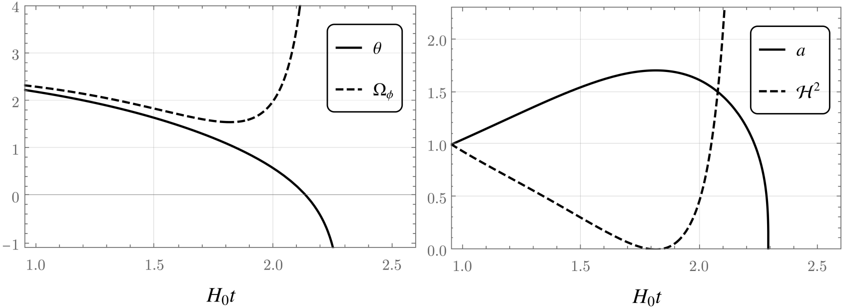


Figure: The cosmic evolution of best-fit parameters: $m_\phi = 2.93 \times 10^{-33}$ eV, $\Omega_\Lambda = -1.61$, $\theta_i = 0.82\pi$, $\Omega_m = 0.284$, $f = M_{Pl}$.

The lifespan is $T = 33.3$ Gyr!

The future

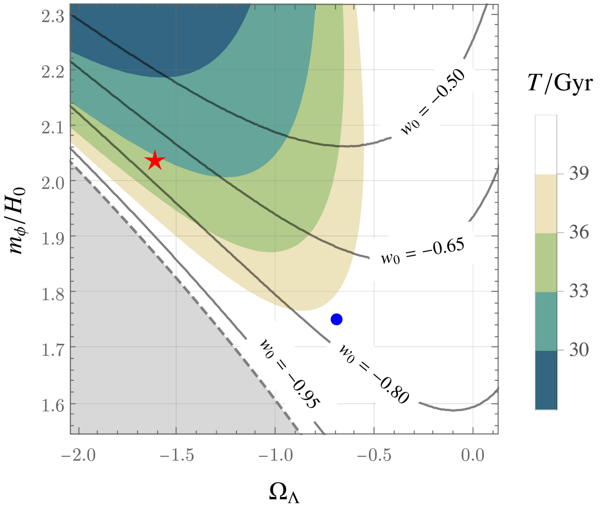


Figure: The red star is the best-fit. The blue dot is the average.

Summary

1. We propose a simple extension to ΛCDM to explain cosmic birefringence and dynamical dark energy.
2. The axion mass is pinned down to $2H_0 \lesssim m \lesssim 7H_0$, with $f_\phi \sim \mathcal{O}(10^{17})$ GeV and small c_γ.
3. Neglecting the cosmic birefringence, the data allow AdS vacuum.
4. New fitting formula for the dynamical dark energy is proposed, pending for future analysis.
5. Best-fit lifespan is 33 billion years.

Thank you

Equation of motion

The equation of motion expressed in scale factor a is

$$\theta'' + \left(\frac{4}{a} + \frac{H'}{H}\right)\theta' + \frac{m^2}{a^2 H^2}\sin\theta = 0 \ . \tag{1}$$

Two Friedmann equations are

$$\frac{H^2}{H_0^2} = \frac{\Omega_m}{a^3} + \Omega_\Lambda + \frac{1}{6}\frac{H^2}{H_0^2}\frac{f_\phi^2}{M_{\mathrm{Pl}}^2}\theta'^2 + \frac{V(\theta)}{3H_0^2 M_{\mathrm{Pl}}^2} \tag{2}$$

$$a\frac{HH'}{H_0^2} = -\frac{3}{2}\frac{\Omega_m}{a^3} - \frac{1}{4}\frac{f_\phi^2}{M_{\mathrm{Pl}}^2}\frac{a^2 H^2 \theta'^2}{H_0^2} \ . \tag{3}$$

Combining the above two gives

$$\frac{H'}{H} = -\frac{3}{2}\frac{\Omega_m}{a^4}\frac{H_0^2}{H^2} - \frac{1}{4}a\frac{f_\phi^2}{M_{\mathrm{Pl}}^2}\theta'^2 \tag{4}$$

$$\frac{H^2}{H_0^2} = \left(\frac{\Omega_m}{a^3} + \Omega_\Lambda + \frac{V(\theta)}{3H_0^2 M_{\mathrm{Pl}}^2}\right)\left(1 - \frac{1}{6}\frac{f_\phi^2}{M_{\mathrm{Pl}}^2}\theta'^2\right)^{-1} \tag{5}$$