Gemini dark matter

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I. Introduction

ΛCDM and Cosmic tensions

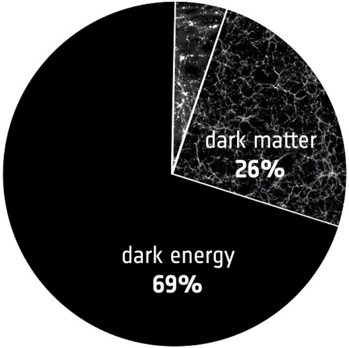


Figure: Copyright: ESA.

- Hubble Tension
- Cosmic Birefringence
- S_8/σ_8 Tension

(Less small structure observed)

  - less dark matter
  - decaying dark matter

Decaying dark matter and S_8/σ_8

- $CDM \to WDM + \cdots$
- The kinetic energy

$$\epsilon \equiv \frac{1}{2}\frac{m_{\text{CDM}}^2 - m_{\text{WDM}^2}}{m_{\text{CDM}}^2}$$

- The lifetime of CDM, τ.

Available parameter space:

$$\epsilon \sim \mathcal{O}(0.01)\text{–}\mathcal{O}(0.1)$$
$$\tau_8 \sim \mathcal{O}(10^{18})\,\text{s}$$

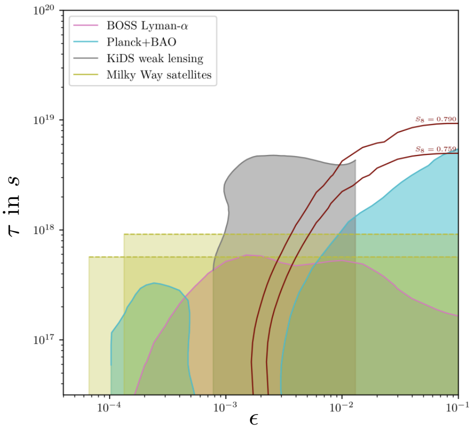


Figure: From 2403.15543 (Fuß, Garny & Ibarra)

Standard Model and mysteries

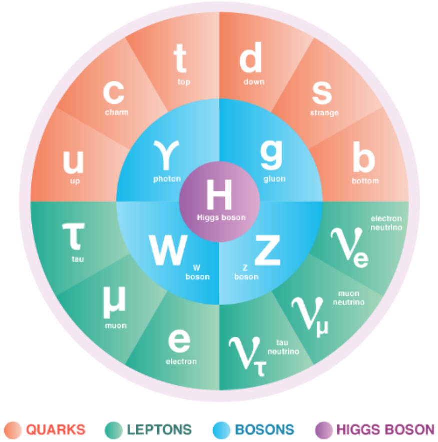


Figure: Artwork by Sandbox Studio, Chicago.

- Strong CP problem
- Naturalness problem
- Yukawa hierarchy
(Why Yukawa couplings are hierarchical?)
  - Landscape (statistical)
  - Clockwork
  - Flavon

Froggatt-Nielsen mechanism

Take charged lepton for example, $\frac{m_e}{m_\tau} \approx 10^{-4}$ and $\frac{m_\mu}{m_\tau} \approx 10^{-2}$.
FN mechanism considers a chiral $U(1)_{\mathrm{FN}}$. (Froggat and Nielsen (1979))
$U(1)_{\mathrm{F}}$ charge:

$$n_{\bar{\ell}_{\mathrm{L}}} = (1, 0.5, 0) \;, \quad n_{e_{\mathrm{R}}} = (4, 1, 0) \;, \quad n_\Phi = -1 \;, \quad n_{\mathcal{H}} = 0 \;.$$

Yukawa operator '$\bar{\ell}\mathcal{H}e$' is not allowed, only

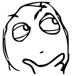
$$-\mathcal{L} \supset g_{ij} \left(\frac{\Phi}{\Lambda} \right)^{n_{ij}} \bar{\ell}^i_{\mathrm{L}} \mathcal{H} e^j_{\mathrm{R}} \;, \quad n_{ij} = n^i_{\bar{\ell}_{\mathrm{L}}} + n^j_{e_{\mathrm{R}}} \;, \quad g_{ij} \sim \mathcal{O}(1) \;.$$

Φ SSB : $\langle \Phi \rangle / \Lambda \equiv \lambda < 1$ and

$$y_{ij} \equiv g_{ij} \lambda^{n_{ij}} \sim \begin{pmatrix} \lambda^5 & \lambda^2 & \lambda \\ \lambda^{4.5} & \lambda^{1.2} & \lambda^{0.5} \\ \lambda^4 & \lambda & 1 \end{pmatrix} \quad \Longrightarrow \quad m_\tau \gg m_\mu \gg m_e$$

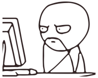
Flavon and Decaying DM

- DDM that resolves S_8 requires $\epsilon \sim \mathcal{O}(0.01)$ and $\tau_8 \sim \mathcal{O}(10^{18})$ s:



(i) DM visible decay is constrained by Indirect detection;
(ii) Almost degenerate spectrum is rare (without supersymmetry).

Gemini DM motivation

Interestingly: 

- FN mechanism predicts flavon, Φ.
  It couples with fermions: $g_\Psi \Phi \bar{\Psi}\Psi$, $g_\Psi \sim \mathcal{O}(m_\Psi/\Lambda)$.
  Φ couples to flavor-changing-currents.
  Suppressed decay channel!
- FN mass matrices are almost rank 1.
  Degenerate spectrum can be produced!

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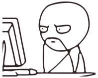
Interestingly: 

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A decaying dark matter model under the FN mechanism!

II. Gemini dark matter model

Gemini dark matter model

Dark sector lagrangian with chiral $U(1)_{\text{FN}}$:

$$\mathcal{L} = i\bar{\chi}_j\bar{\sigma}^\mu\partial_\mu\chi_j - \frac{\beta_{jk}}{2}\frac{\Phi^{n_j+n_k}}{\Lambda^{n_j+n_k-1}}\chi_j\chi_k + \text{h.c.}$$

- $U(1)_{\text{FN}}$ charge: $\chi_j(n_j)$ and $\Phi(-1)$.
- Φ SSB : $\Phi = (\langle\Phi\rangle + \phi)\, e^{ia/f_a}$ and $\{\phi, a\}$ are flavons.

$$\left(\frac{\Phi}{\Lambda}\right)^n = \lambda^n e^{in\frac{a}{f_a}}\left(1 + n\frac{\phi}{f_a} + \cdots\right) .$$

- Perform phase rotation $\chi_j \to e^{-in_j a/f_a}\chi_j$ to adjust the field space coordinates.
- The lagrangian becomes

$$\mathcal{L} = -\bar{\chi}_j\bar{\sigma}^\mu\partial_\mu\chi_j - \frac{1}{2}m_k\chi_k\chi_k - g^\phi_{jk}\phi\chi_j\chi_k - g^a_{jk}a\chi_j\chi_k + \text{h.c.} + \cdots$$

Gemini dark matter model

The mismatch between mass- and interaction-eigenstate gives the off-diagonal interactions between flavon and fermions.

$$\mathcal{L} = -\bar{\chi}_j \bar{\sigma}^\mu \partial_\mu \chi_j - \frac{1}{2} m_k \chi_k \chi_k - g^\phi_{jk} \phi \chi_j \chi_k - g^a_{jk} a \chi_j \chi_k + \text{h.c.}$$

$$D = \text{diag}(m_1, m_2, \cdots) = U^\top M U$$

$$g^\phi = \frac{1}{2} \text{sym} \left(U^\top \frac{1}{f_a} \frac{\partial(\lambda M)}{\partial \lambda} U \right)$$

$$g^a = \text{sym} \left(\frac{ND}{f_a} \right)$$

$$M_{jk} = \frac{1}{\sqrt{2}} f_a \beta_{jk} \lambda^{n_j + n_k - 1}$$

$$N_{jk} = (U^\dagger)_{ji} n_i U_{ik}$$

$$[\text{sym}(\cdots)]_{jk} = (\cdots)_{jk} + (\cdots)_{kj} - (\cdots)_{jj} \delta_{jk}$$

Parametrize couplings as

$$g^\phi_{jk} = \frac{1}{f_a} \left(m_j - m_k + m_j \delta_{jk} \right) \mathcal{A}_{jk}$$

$$g^a_{jk} = \frac{1}{f_a} \left(m_j - m_k + m_j \delta_{jk} \right) \mathcal{B}_{jk}$$

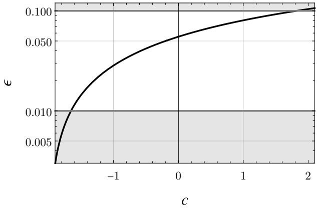
Gemini dark matter model

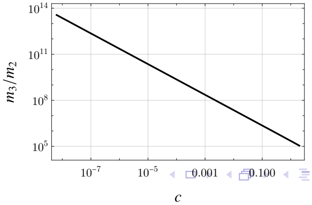
- In the limit of $\beta_{jk} \to 1$, $\text{rank}(M) = 1$.
  - if 2 generations, then $m_1 \approx 0$ and $m_2 \approx 1$. (scaled with the largest component)
  - if 3 generations, then $m_1 \approx m_2 \approx 0$ and $m_3 \approx 1$.

$\Longrightarrow$ To have a (nearly) degenerate spectrum, one needs at least 3 generations.

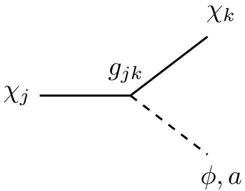
- Gemini dark matter: $\{\chi_1, \chi_2, \chi_3\}$, with $m_1 \lesssim m_2 \ll m_3$.

Take for example, $n_1 = 4.5 + n_3$, $n_2 = 2.5 + n_3$, and $\beta = \begin{pmatrix} 1 & 1 & 1+c \\ 1 & 1 & 1 \\ 1+c & 1 & 1 \end{pmatrix}$.





The benchmark model parameters



$$g_{jk} \sim \mathcal{O}\left(\frac{m_{j,k}}{f_a}\right)$$

$\chi_{1/2}$ are light enough to be stable. So they are DM.
m_a is from explicit breaking of $U(1)_{\mathrm{FN}}$.
We choose $m_a = 10^{-6}$ eV to avoid overproduction from misalignment.
ϕ is associated with SSB scale, which is heavy. Take $m_\phi = 10^9$ GeV.
So kinematically,

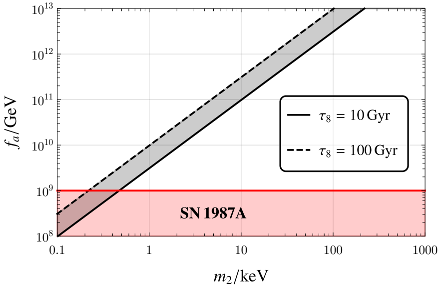
$$\chi_2 \to \chi_1 + a\ , \quad \chi_3 \to \chi_{1/2} + a\ , \quad \phi \to \chi_j + \chi_k$$

The benchmark model parameters

The decay $\chi_2 \to \chi_1 + a$ explains the S_8/σ_8 tension.

1. m_1 and m_2 are linked by $\epsilon \simeq 0.05$.
2. $m_{1/2}$ and f_a are linked by $\tau_8 \sim \mathcal{O}(10^{18})\,\mathrm{s}$:

$$m_{1/2} \approx 37 \times \left(\frac{f_a^2}{\tau_8}\right)^{1/3}$$



We call χ_1 and χ_2 the twins and χ_3 as the mother particle.
Free parameters are $\{f_a, m_3\}$.
They determine the production of Gemini DM.

Gemini DM production: three stages

The Gemini DM production has three stages:

1. $T \gtrsim m_\phi$, ϕ stays in the thermal bath;
2. χ_3 freeze in from ϕ decay, $\phi \to \chi_3 + \chi_3$;

$$Y_3^{\text{f.i.}} \approx 0.3 \times \frac{T_0^3 M_{\text{Pl}}}{s_0 m_\phi} \frac{m_3^2}{f_a^2}$$

3. $\chi_{1/2}$ production from χ_3 decay, $\chi_3 \to \chi_{1/2} + a$. So $Y_1 + Y_2 \approx Y_3^{\text{f.i.}}$,

$$\Omega_{\text{DM}} h^2 = \frac{(\rho_1 + \rho_2) h^2}{3 M_{\text{Pl}}^2 H_0^2} \approx \frac{m_{1/2} Y_3^{\text{f.i.}} s_0 h^2}{3 M_{\text{Pl}}^2 H_0^2} \approx 4 \times \frac{h^2 T_0^3 m_3^2}{H_0^2 M_{\text{Pl}} m_\phi (\tau_8 f_a^4)^{1/3}}$$

Due to the (almost) degeneracy, χ_1 and χ_2 are produce together with almost same amount. Thus the name 'Gemini'.

Production

To have the correct DM relic abundance,

$$\Omega_{\mathrm{DM}} h^2 \approx 0.12 \times \left(\frac{m_3}{1.1 \times 10^4\,\mathrm{GeV}} \right)^2 \left(\frac{f_a}{2 \times 10^{10}\,\mathrm{GeV}} \right)^{-4/3}$$

Where we have take $\tau_8 = 10\,\mathrm{Gyr}$ and $H_0/h = 2.1 \times 10^{-42}\,\mathrm{GeV}$.

Successful DM production!

Warm or cold?

For $f_a = 10^{11}\,\mathrm{GeV}$, $m_{1/2} \simeq 10\,\mathrm{keV}$. (Is it warm?)

Warm or cold?

For $f_a = 10^{11}\,\text{GeV}$, $m_{1/2} \simeq 10\,\text{keV}$. (Is it warm?)
Suppose the instantaneous decay of $\chi_3 \to \chi_{1/2} + a$, happens at

$$T_3 \simeq 10^{-3} \times \frac{\sqrt{m_3^3 M_{\text{Pl}}}}{f_a} \, .$$

The twins $\chi_{1/2}$ obtain the average momentum $\langle p \rangle_3 \approx m_3/2$, which is redshifted to today. The free-streaming scale of the Gemini DM is

$$\lambda_{\text{fs}} \approx \frac{\langle v \rangle_0}{H_0} \approx \frac{\langle p \rangle_0}{H_0 m_{1/2}} \approx 4 \times 10^{-3}\,\text{Mpc}/h \, .$$

Clearly, $\lambda_{\text{fs}} \ll \mathcal{O}(1)\,\text{Mpc}/h$, the scale of Ly-$\alpha$ constraint.
It is cold.

Dark radiation

Any prediction from Gemini DM?

Dark radiation

Any prediction from Gemini DM?

a is light enough to be relativistic. So they contribute to $\Delta N_{\rm eff}$.

$$\Delta N_{\rm eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \left.\frac{\rho_a^{\rm th} + \Delta\rho_a}{\rho_\gamma}\right|_{\rm rec}$$

There are two possible production channels of such radiation:

1. thermal freeze out, $\rho_a^{\rm th} \implies \Delta N_{\rm eff} \geq 0.028$;
2. parturition process $\chi_3 \to \chi_{1/2} + a$, $\Delta\rho_a$.

SM prediction $(N_{\rm eff})_{\rm SM} \approx 3.044$.

Planck collaboration gives $(N_{\rm eff})_{\rm P18} = 2.88^{+0.44}_{-0.42}$.

$$\Delta N_{\rm eff} = (N_{\rm eff})_{\rm P18} - (N_{\rm eff})_{\rm SM} \leq 0.276$$

Dark radiation prediction

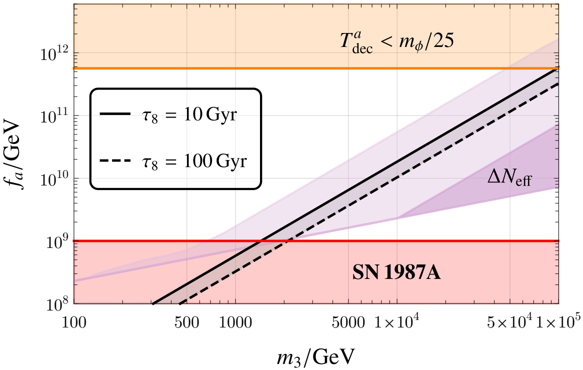


Figure: Benchmark model: $m_a = 10^{-6}\,$eV, $m_\phi = 10^9\,$GeV and $\epsilon = 0.05$. ▢ is the range where $\Delta N_{\rm eff} > 0.276$ and ▢ is where $\Delta N_{\rm eff} > 0.04$. The sensitivity of future CMB-S4 could reach $\Delta N_{\rm eff} \sim 0.02$. ▢ indicates where radiation from $\phi \to a + a$ cannot be thermalized.

III. Summary and discussion

Summary and discussion

- We propose the Gemini dark matter model to explain the S_8/σ_8 tension.
  - The dark matter couple to the flavon that explains the Yukawa hierarchy.
  - Flavon provides the decay channel for S_8/σ_8 and DM production channel.
- The cry of the twins during parturition (dark radiation) is predicted and can be probed in the future CMB-S4.

Discussion

- Gemini DM could be the sterile neutrino.
- Light flavon indicates fifth force.
- Extremely light flavon(a) can be used to explain the cosmic birefringence.

Thank you!

$\mathcal{A}$ and $\mathcal{B}$

Consider $n_1 = 4.5 + n_3$, $n_2 = 2.5 + n_3$ and $\beta = \begin{pmatrix} 1 & 1 & 1+c \\ 1 & 1 & 1 \\ 1+c & 1 & 1 \end{pmatrix}$.

Only c will affect the mixing patterns. Parametrize the coupling as

$$g^{\phi}_{jk} = \frac{1}{f_a}\left(m_j - m_k + m_j\delta_{jk}\right)\mathcal{A}_{jk}$$
$$g^{a}_{jk} = \frac{1}{f_a}\left(m_j - m_k + m_j\delta_{jk}\right)\mathcal{B}_{jk} \ .$$

(ij)	$\lvert\mathcal{A}_{ij}\rvert^2$	$\lvert\mathcal{B}_{ij}\rvert^2$
(11)	$6.64^{+0.49}_{-0.47} \times 10^3$	$48.6^{+0.4}_{-0.4}$
(22)	$6.76^{+0.48}_{-0.48} \times 10^3$	$49.4^{+0.3}_{-0.4}$
(33)	$1.57 \times 10^3 \pm 10^{-3}$	12.3 ± 10^{-5}
(21)	$1.37 \times 10^2 \pm 10^{-1}$	$0.999^{+0.001}_{-0.002}$
(31)	$5.77^{+0.95}_{-0.88} \times 10^{-2}$	$4.21^{+0.70}_{-0.65} \times 10^{-4}$
(32)	$6.79^{+1.11}_{-0.99} \times 10^{-2}$	$4.98^{+0.79}_{-0.75} \times 10^{-4}$

Statistics of matrix elements $|\mathcal{A}_{ij}|^2$ and $|\mathcal{B}_{ij}|^2$ with $\epsilon \in (0.01, 0.1)$, and randomly sampled c under uniform distribution. Central values are averages. The upper and the lower uncertainties indicate the maximum and the minimum.

ϕ, a in thermal

ϕ coupled to SM in the form $\lambda^n \frac{\phi}{f_a} \bar{Q}\mathcal{H}q$, which leads to

$$\phi + \mathcal{H} \to \bar{Q} + q\,, \quad \phi + Q \to \mathcal{H} + q\,, \quad \phi + q \to \mathcal{H} + Q\,.$$

The amplitudes are $|\mathcal{M}|^2 = \frac{\lambda^2}{f_a^2}(2p_Q p_q)$. (take $n = 1$ for leading contribution)
The Boltzmann equation for ϕ is

$$\dot{n}_\phi + 3Hn_\phi = -\tilde{\Gamma}_\phi \left(n_\phi - n_\phi^{\text{eq}}\right)$$
$$\tilde{\Gamma}_\phi = \langle\sigma_{\mathcal{H}} v\rangle n_{\mathcal{H}}^{\text{eq}} + \langle\sigma_Q v\rangle n_Q^{\text{eq}} + \langle\sigma_q v\rangle n_q^{\text{eq}}$$

Decoupling temperatures are

$$3H(T_{\text{dec}}^a) = \tilde{\Gamma}_a(T_{\text{dec}}^a)\,, \quad 3H(T_{\text{dec}}^\phi) = \tilde{\Gamma}_\phi(T_{\text{dec}}^\phi)$$

ϕ, a in thermal

Take $\{\mathcal{H}, Q, q\}$ massless.

$$
\begin{aligned}
\tilde{\Gamma}_\phi &= \langle \sigma_\mathcal{H} v \rangle n_\mathcal{H}^{\rm eq} + \langle \sigma_Q v \rangle n_Q^{\rm eq} + \langle \sigma_q v \rangle n_q^{\rm eq} \\
&\approx \frac{1}{n_\phi^{\rm eq}} \int \prod_j \frac{g_j d^3 p_j}{(2\pi)^3 (2E_j)} \frac{\lambda^2}{f_a^2} \left(2p_Q p_q + 2p_\mathcal{H} p_q + 2p_Q p_\mathcal{H}\right) \\
&\qquad \times (2\pi)^4 \delta^4(p_\phi + p_\mathcal{H} - p_Q - p_q) e^{-(E_\phi + E_\mathcal{H})/T} \\
&= \frac{g_\mathcal{H} g_Q g_q \lambda^2}{16(2\pi)^3} \frac{T^5}{f_a^2 m_\phi^2} \frac{\mathcal{I}(m_\phi/T)}{K_2(m_\phi/T)}
\end{aligned}
$$

$$
\mathcal{I}(\zeta) = \int_\zeta^\infty d\xi (\xi^2 - \zeta^2)(2\xi^2 - \zeta^2) K_1(\xi)
$$

The interaction rate for a is obtained by taking massless limit of $\tilde{\Gamma}_\phi$,

$$
\tilde{\Gamma}_a \approx \frac{g_\mathcal{H} g_Q g_q \lambda^2}{(2\pi)^3} \frac{T^3}{f_a^2} \,.
$$

ϕ, a in thermal

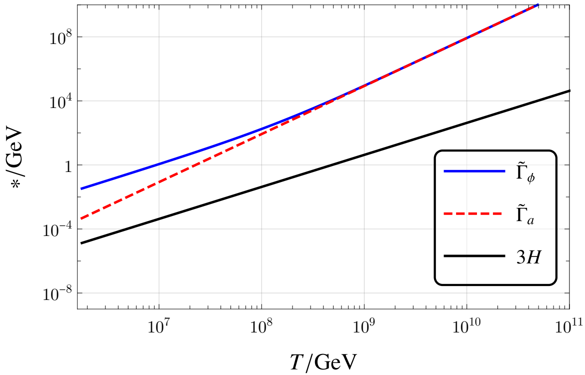


Figure: Here $f_a = 2 \times 10^{11}$ GeV and $m_\phi = 10^9$ GeV. So $T^a_{\mathrm{dec}} \approx 5 \times 10^4$ GeV.