OPTI536_Homework_2_Answer_pdf

Date	@February 25, 2025
Tags	Homework2
Person (Reporting Author)	Qi Wen Gan

This is the answer sheet for the homework 2 of OPTI365 Introduction to Imaging Science. To differentiate the required explanations from the problem, answers are provided in purple color background in all questions below.

Question 1

Problem 1

a) $\phi(x,y,z) = k[2\cos(\theta) - 2\sin(\theta)]$

since $\theta = 0$, $\cos(\theta) = 1$, $\sin(\theta) = 0$

$\phi(x,y,z) = k[2 \cdot 1 - 2 \cdot 0] = k2$

At $(x,y,z) = (0,0,1\,m)$

unwrapped $\rightarrow \phi(x,y,z) = k2$

$= \left(\frac{2\pi}{633\times10^{-9}\,m}\right) \times 1\,m$

$\simeq 9.928 \times 10^6$ rad

wrapped $\rightarrow \phi(x,y,z) = 9.928\times10^6$ rad mod 2π

~~1.579×10^6~~

$\simeq \frac{9.928\times10^6}{2\pi}$

$\simeq 1.579 \times 10^6$

take fractional part $0.579 \times 2\pi$

$\simeq 3.638$ rad

b) $\cos(1\,\text{mrad}) \simeq 1$

$\sin(1\,\text{mrad}) \simeq 0.001$

At $(x,y,z) = (0,0,1\,m)$

$\phi = k(1.1 - 0.001)$

$= k(1 - 0.001)$

$= k\,0.999$

$= \frac{2\pi}{633\times10^{-9}} \times 0.999$

$\simeq 9.948\times10^6$ rad unwrapped

unwrapped $\rightarrow \frac{9.948\times10^6}{2\pi} \simeq 1.578\times10^6$

take fractional part of 0.578

ϕ wrapped $\simeq 0.578 \times 2\pi$

$\simeq 3.632$ rad

(ii) $\Delta r(x,y,z) = (1m, 0, 1m)$

$\phi = k(1.1 - 1.0.001)$
$= k(1 - 0.001)$
$= k(0.999)$
$= 9.928 \times 10^6 (0.999)$
$\simeq 9.918 \times 10^6$ rad

$\phi_{wrapped} = \frac{9.918 \times 10^6}{2\pi}$
$\simeq 1.578 \times 10^6$

fraction part of 0.578,

$\phi_{wrapped} = 0.578 \times 2\pi$
$\simeq 3.632$ rad

(iii) $\Delta r(x,y,z) = (-1m, 0, 1m)$

$\phi = k(1.1 - (-1).0.001)$
$= k(1 + 0.001)$
$= k(1.001)$
$= 9.928 \times 10^6 (1.001)$
$\simeq 9.938 \times 10^6$ rad

$\phi_{wrapped} = \frac{9.938 \times 10^6}{2\pi}$
$\simeq 1.581 \times 10^6$

fraction part of 0.581

$\phi_{wrapped} \simeq 0.581 \times 2\pi$
$\simeq 3.650$ rad

(iv) $\Delta r(x,y,z) = (2m, 0, -1m)$

$\phi = k(-1.1 - 2.0.001)$
$= k(-1 - 0.002)$
$= k(-1.002)$
$= 9.928 \times 10^6 (-1.002)$
$\simeq -9.948 \times 10^6$ rad

$\phi_{wrapped} = -\frac{9.948 \times 10^6}{2\pi}$
$\simeq -1.583 \times 10^6$

fraction part of -0.583

$\phi_{wrapped} \simeq (1 - 0.583) \times 2\pi$
$\simeq 0.417 \times 2\pi$
$\simeq 2.62$ rad

c) i) $\cos(\theta) = \frac{k_z}{k}$

ii) $\sin(\theta) = \frac{k_x}{k}$

iii) $\phi(k_x, z) = (x_0, 0, z_0)$

$\phi(k_x, z) = k[z\cos(\theta) - x\sin(\theta)]$

$\phi = k\left(z\frac{k_z}{k} - x\frac{k_x}{k}\right)$

$= zk_z - xk_x$

At $(x_0, 0, z_0)$

$\phi = z_0 k_z - x_0 k_x$ ✱✱

d) $\tan\theta = \frac{o}{a}$

$= \frac{k_x}{k_z}$

$= \frac{24.815\ \text{rad}}{1\ \text{m}} \doteq \frac{9.926\times10^6\ \text{rad}}{1\ \text{m}}$

$= \frac{24.815\ \text{rad}}{1\ \text{m}^2} \times \frac{1\ \text{m}}{9.926\times10^6\ \text{rad}}$

$= \frac{24.815\ \text{rad}}{9.926\times10^6\ \text{rad}}$

$\simeq 2.5\times10^{-6}\ \text{rad}$

e) $\phi = k[z\cos(\theta) - x\sin\theta]$

At $(x=0, z=\Delta z)$

$\phi = k[\Delta z\cos\theta - 0\cdot\sin\theta]$

$= k\Delta z\cos\theta$

since $\cos\theta = \frac{k_z}{k}$, $k_z = \sqrt{k^2 - k_x^2}$

$\phi = k\Delta z\frac{k_z}{k}$

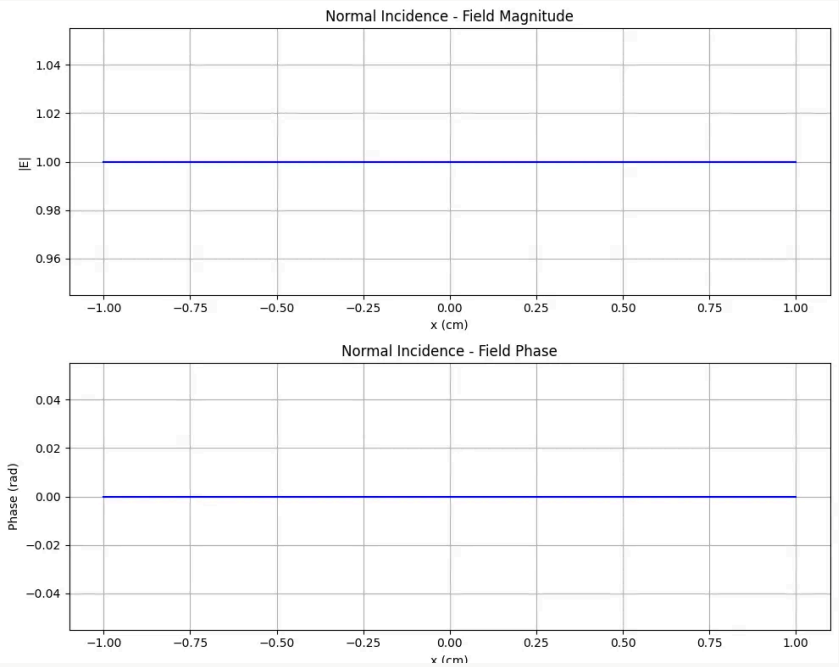
$= \Delta z\, k_z$

$= \Delta z\sqrt{k^2 - k_x^2}$ ✱

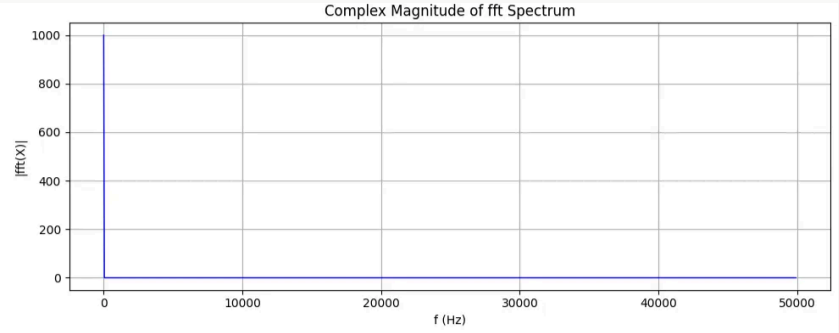
Question 2

A)

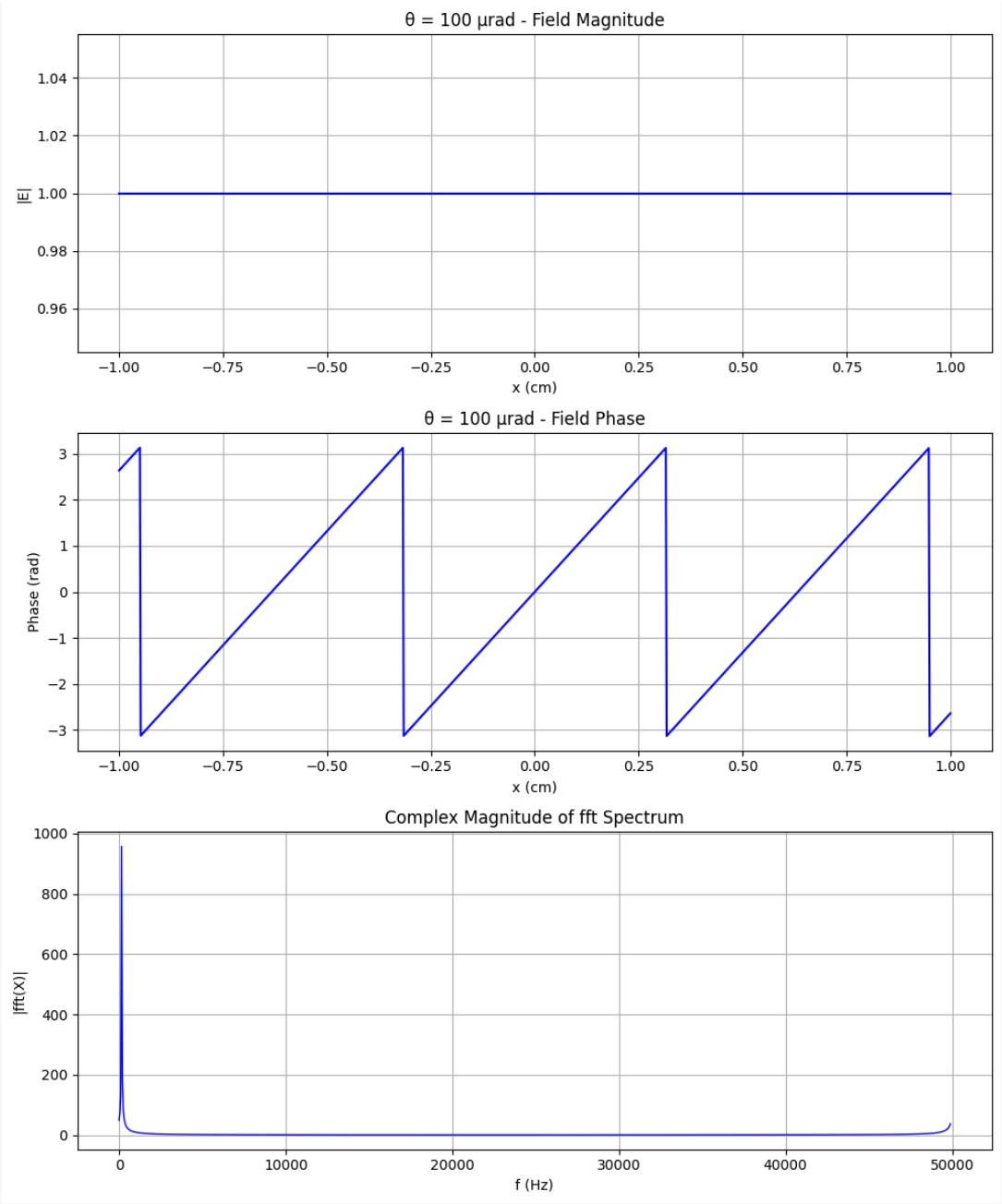
I)



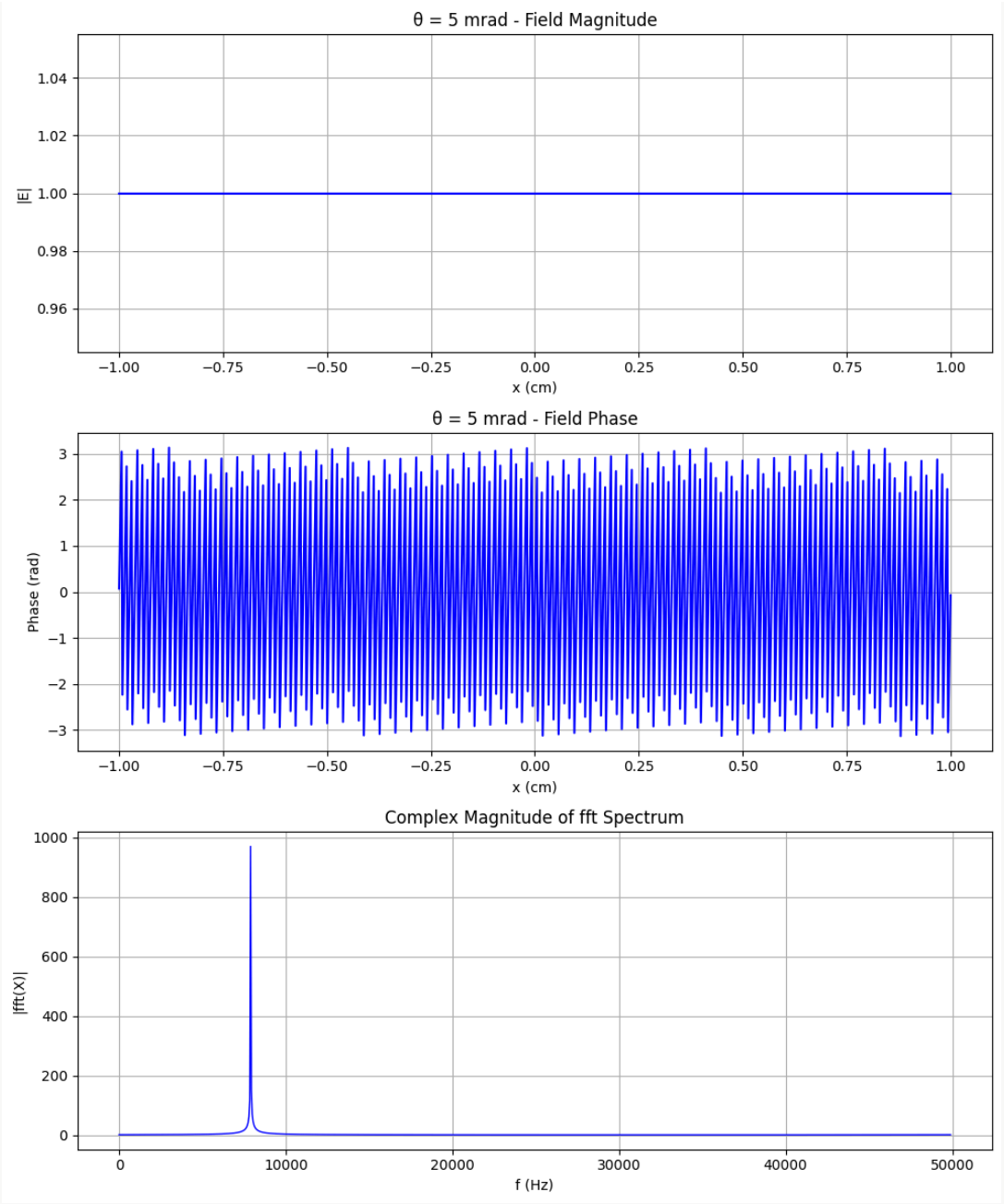
II)



III)


θ = 100 µrad - Field Magnitude
|E|
1.04
1.02
1.00
0.98
0.96
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x (cm)
θ = 100 µrad - Field Phase
Phase (rad)
3
2
1
0
−1
−2
−3
x (cm)
Complex Magnitude of fft Spectrum
|fft(X)|
1000
800
600
400
200
0
0
10000
20000
30000
40000
50000
f (Hz)

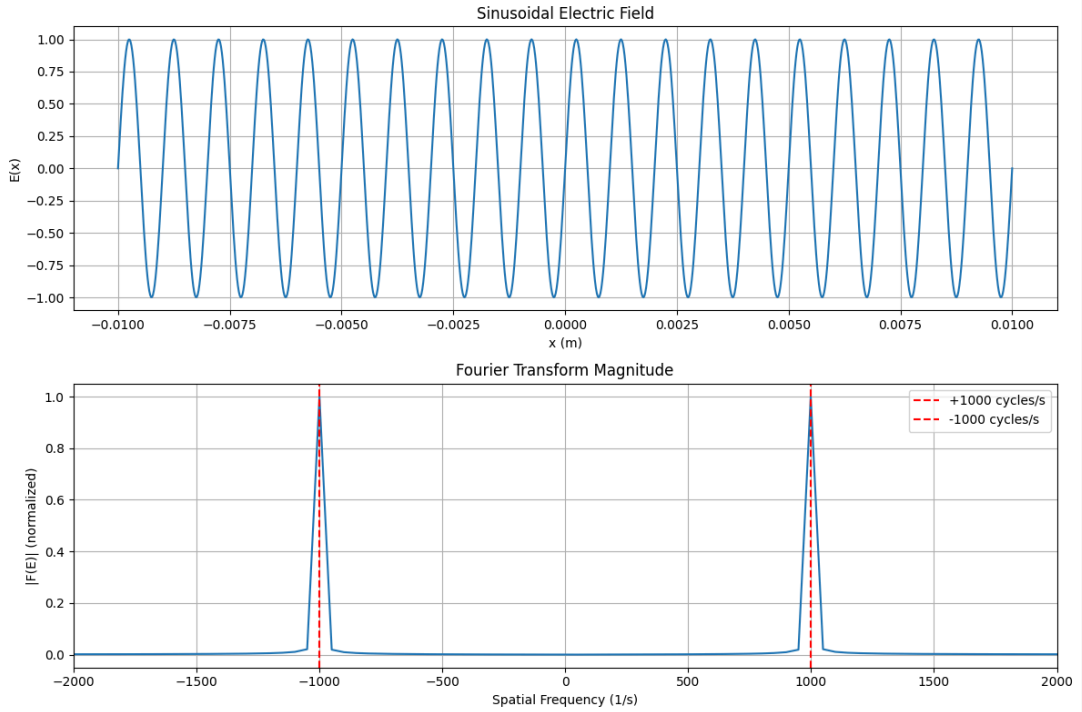
IV)



V) **ANS: As the plane wave angle increases, we see the peak of the signal start to deviate from the spatial frequency origin of 0. For the 100 urad phase field, we saw the peak of the signal is slightly larger than 0. For a larger plane wave angle like 5mrad, the signal peak deviate more toward the position of 10000 Hz in the spatial frequency domain.**

B)

I)



II) **ANS: After the Fourier transform operation, we observe two peaks of the signal in the spatial frequency domain. The peak of the signal deviates from 0 in both positive and negative directions with an equal magnitude of 1000 Hz.**

Question 3

A)

I)why do we take the Fourier transform in step 1?

ANS: By representing the wave field spatial information in a electrical field E format, the Fourier transform operation covert the wave from spatial domain (with a x, y coordinate) to frequency domain. Given the 2d Fourier transform operation of $FE(kx, ky, z = z0)$, the result will display the spatial frequency components of kx, ky, at initial propagate position z=0. Each frequency component is breaking down and represented by a plane wave number on its correspond direction (x and y), which make us easier to determine the evolution pattern wave field after it propagate in z-direction by adjusting the phrase of each wave number (i.e., kx, ky), rather than dealing it with a complex spatial pattern in a 3d coordinates system.

II) What is the kernel function and why do we multiply the Fourier transform by it, and where does the phase $\Delta z\sqrt{k^2 - kx^2 - ky^2}$of come from?

ANS: Given the Fourier transform of initial wave spatial frequency field, the kernel function $e^{j\Delta z\sqrt{k^2-kx^2-ky^2}}$ represent the wave propagation factor in a phrase shift format, specifically in z-direction. When we multiply $FE(kx, ky, z = z0)$ with this kernel, we are able to get the evolution of spatial frequency of he field from $z = z0$ to $z = z0 + \Delta z$. In frequency domain, the propagation is modeled with phrase shift for each wave plane component. Multiplying by the exponential kernel function adjusts the phrase of each component based on how far they travel (Δz) in the direction of (determine by kx and ky). This ensures the field at the new z-position reflects how the wave interfere after propagating. The term $\sqrt{k^2 - kx^2 - ky^2}$ is the z-component of the wave vector, kz, derived from a standard 3d wave vector $k^2 = kx^2 + ky^2 + kz^2$. When a wave propagates a distance in z direction Δz, the phrase shift is represented by Δzkz. Therefore, multiplying with the kernel $e^{j\Delta zkz}$ represent the phrase shift operation, helping us to evaluate evolution of the wave field at arbitrary Δz.

III) What are we doing qualitatively, and why does this method work

ANS: Qualitatively, we undergo three steps to get the field at the new position. First, we decompose the field by taking the field at $FE(kx, ky, z = z0)$ and breaking it into a sum of plane waves using the Fourier transform. Think of it as splitting a complex wave pattern into simple building blocks—each with its own direction and frequency. Secondly, we then shift each of these plane waves forward by adjusting their phases, based on how far they travel and their direction. That's what the kernel multiplication does—it simulates the wave

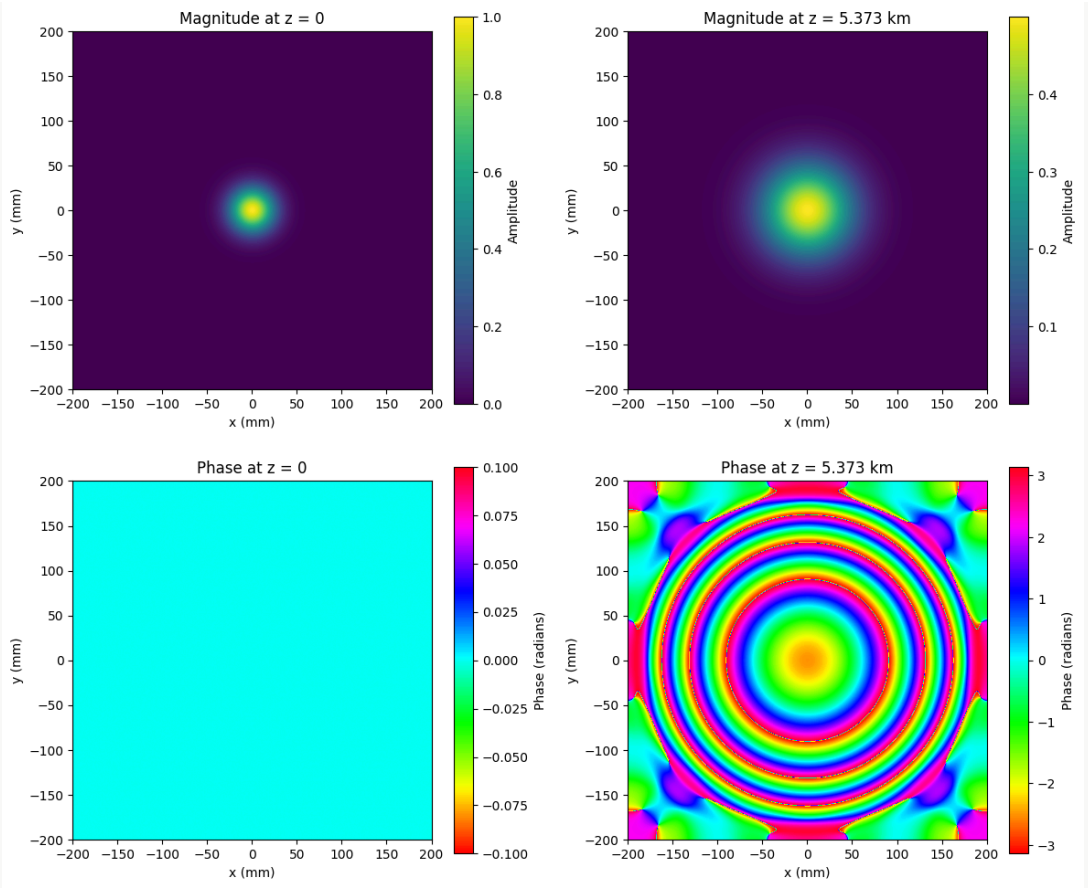
traveling through free space. Finally, we add all these shifted plane waves back together using the inverse Fourier transform to get the field at the new position $z = z0 + \Delta z$.

B) Please refer q3.ipynb for more precise implementation

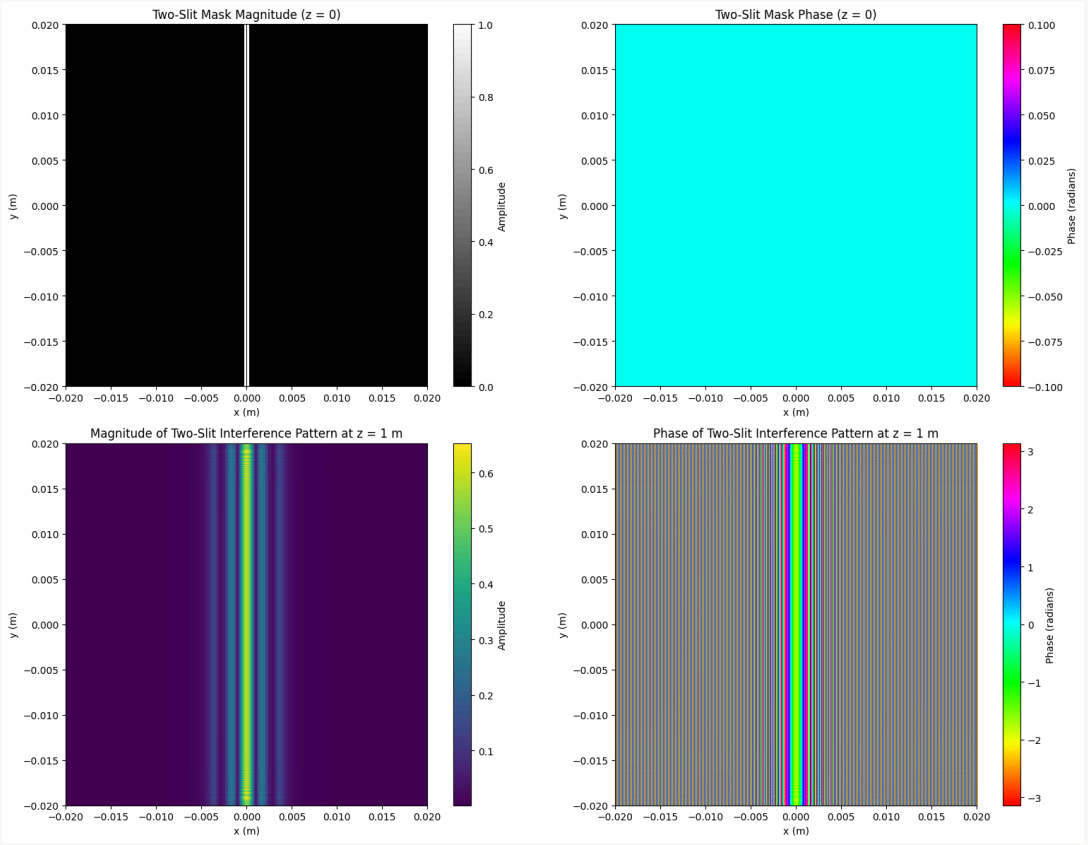
```
def angular_spectrum_propagation(E0, wavelength, z, xmax, N):
    """Calculate electric field propagation using angular spectrum method"""
    dx = 2 * xmax / N
    x = np.linspace(-xmax, xmax, N, endpoint=False)
    X, Y = np.meshgrid(x, x)

    k = 2 * np.pi / wavelength
    fx = np.fft.fftfreq(N, dx)
    FX, FY = np.meshgrid(fx, fx)

    kx = 2 * np.pi * FX
    ky = 2 * np.pi * FY
    kz = np.sqrt(k**2 - kx**2 - ky**2)
    H = np.exp(1j * kz * z)
# Handle evanescent waves (where kx^2 + ky^2 > k^2)
    H[np.isnan(kz) | (kx**2 + ky**2 > k**2)] = 0

    U0 = np.fft.fft2(E0)
    U = U0 * H
    E = np.fft.ifft2(U)

    mag = np.abs(E)
    phase = np.angle(E)
    return mag, phase, X, Y
```

C)

ANS: To validate the implementation of code, the width of the gaussian in right plot is approximately 2 times when we compared it to the left plot.


Magnitude at z = 0
Magnitude at z = 5.373 km
Phase at z = 0
Phase at z = 5.373 km
x (mm)
y (mm)
Amplitude
Phase (radians)

D)

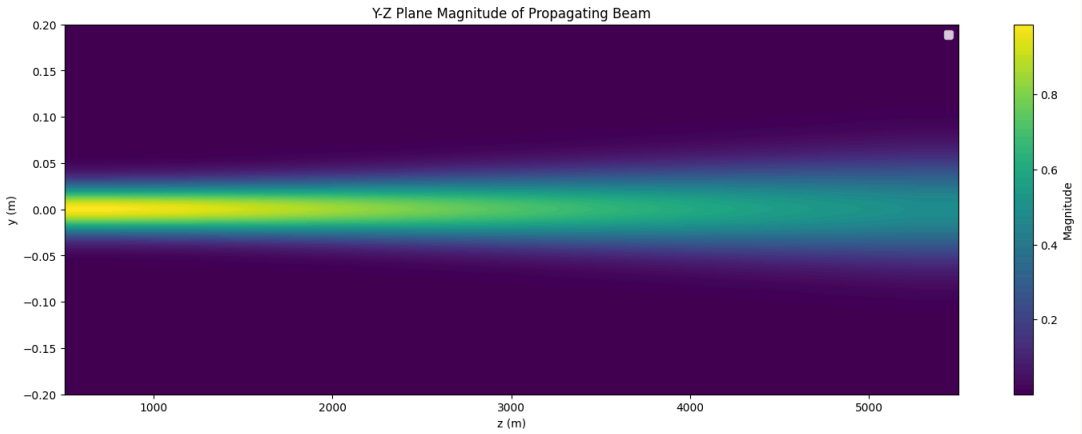

Two-Slit Mask Magnitude (z = 0)
Two-Slit Mask Phase (z = 0)
Magnitude of Two-Slit Interference Pattern at z = 1 m
Phase of Two-Slit Interference Pattern at z = 1 m
x (m)
y (m)
Amplitude
Phase (radians)

Explanation: The bottom-left plot shows the magnitude of the interference pattern observed at z = 1 m, displaying a series of bright and dark fringes along the x-axis, characteristic of two-slit interference, with the most intense central fringe at x = 0 m and fringe spacing of approximately 0.002532 m (2.532 mm). The bottom-right plot shows the phase of the interference pattern, revealing a complex, wavy pattern with alternating phase values corresponding to the interference fringes, primarily varying along the x-axis due to the vertical slits.

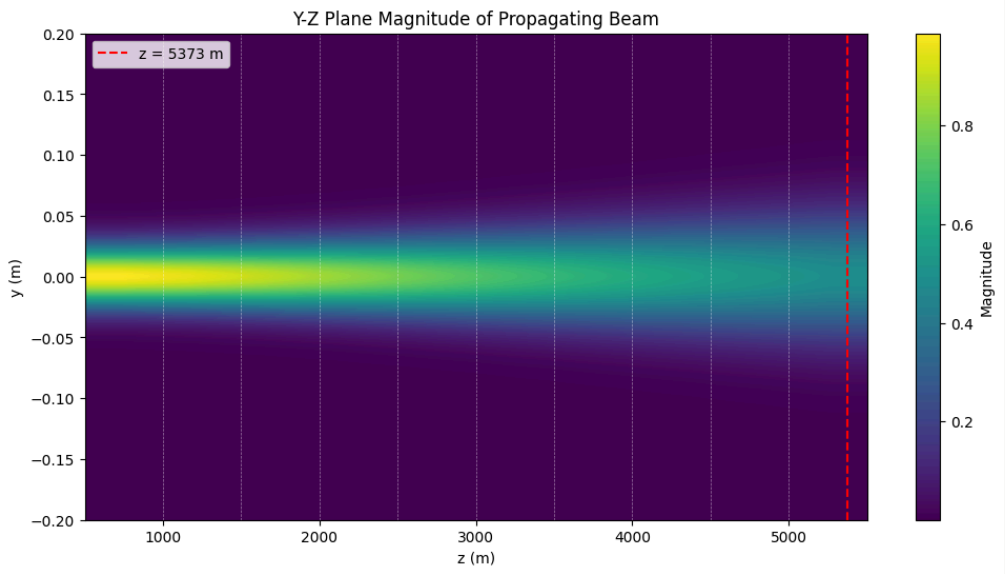
Question 4

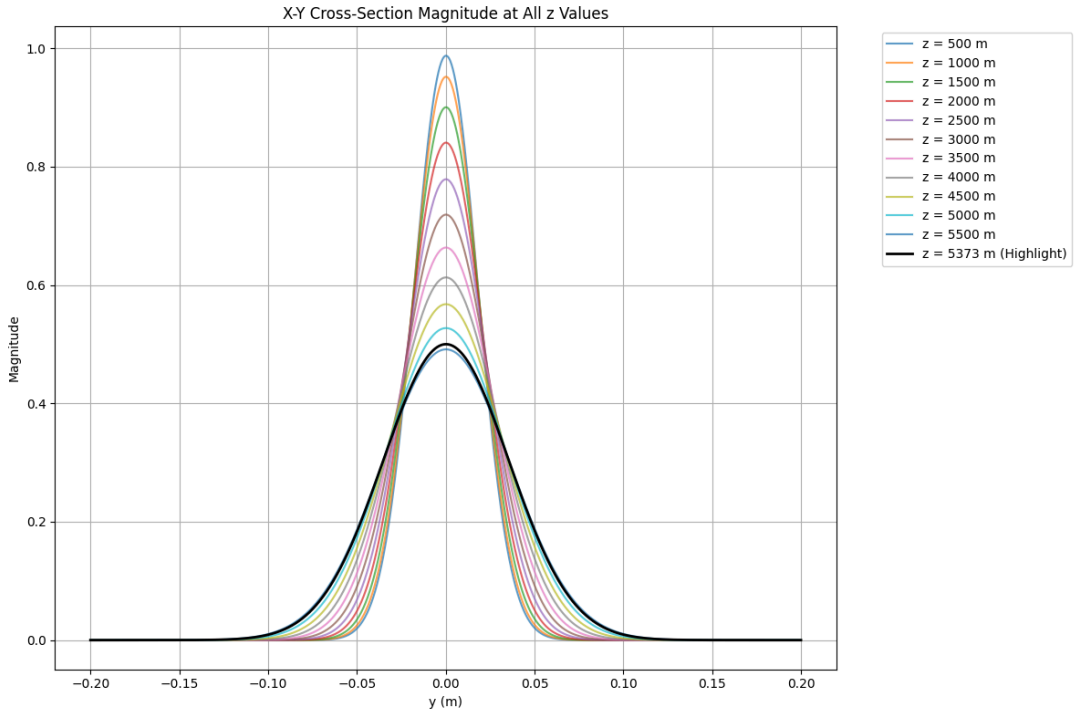
A)

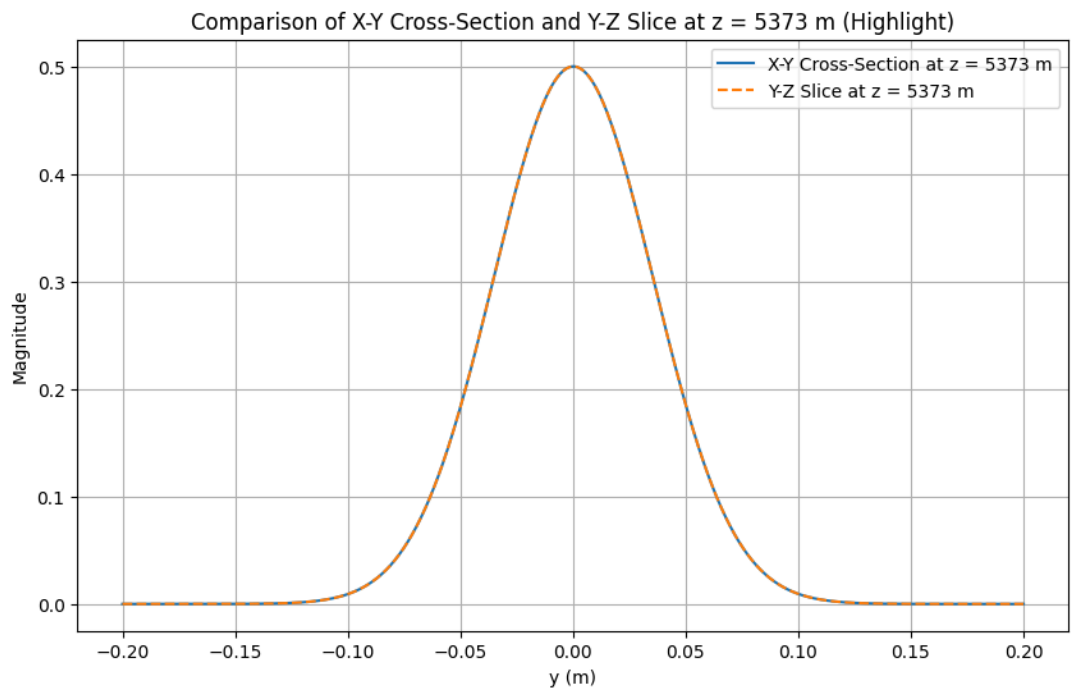
I)



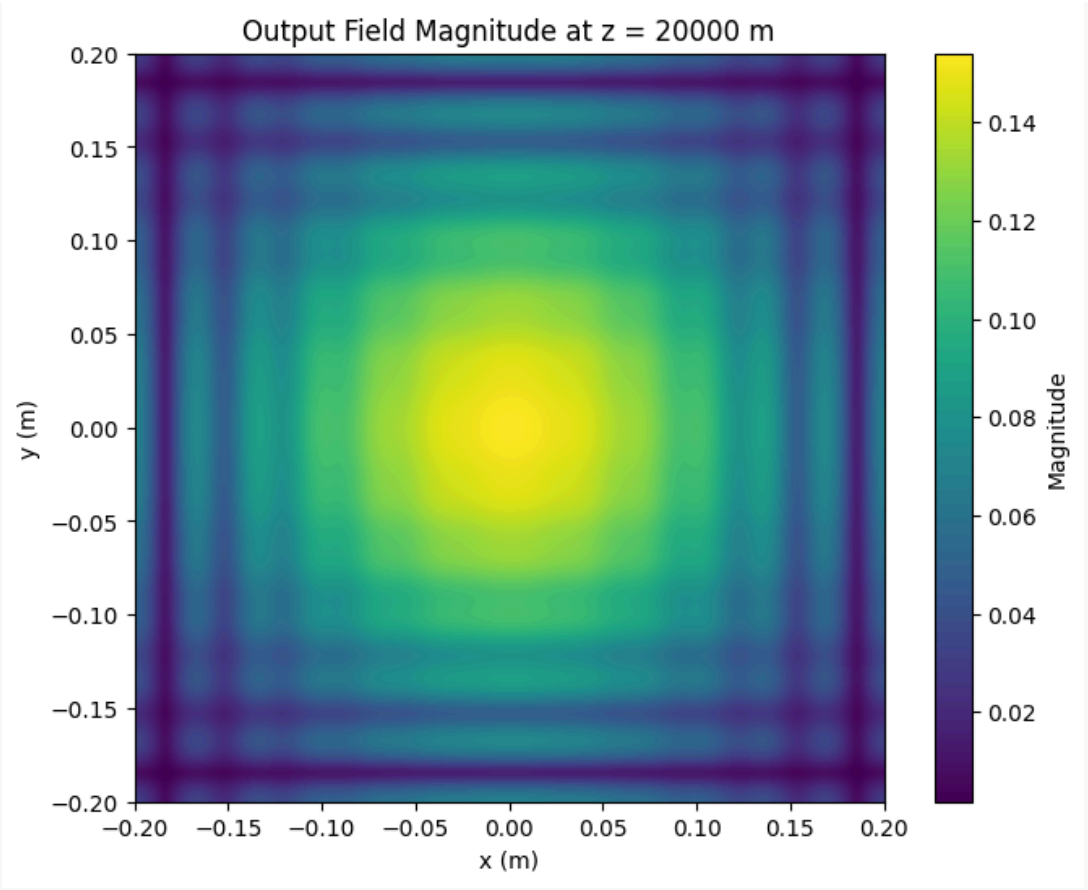
The white dotted line in the figure below depicts the Z cross-sections sampled at an interval of 500 m from 500 m to 5500 m.

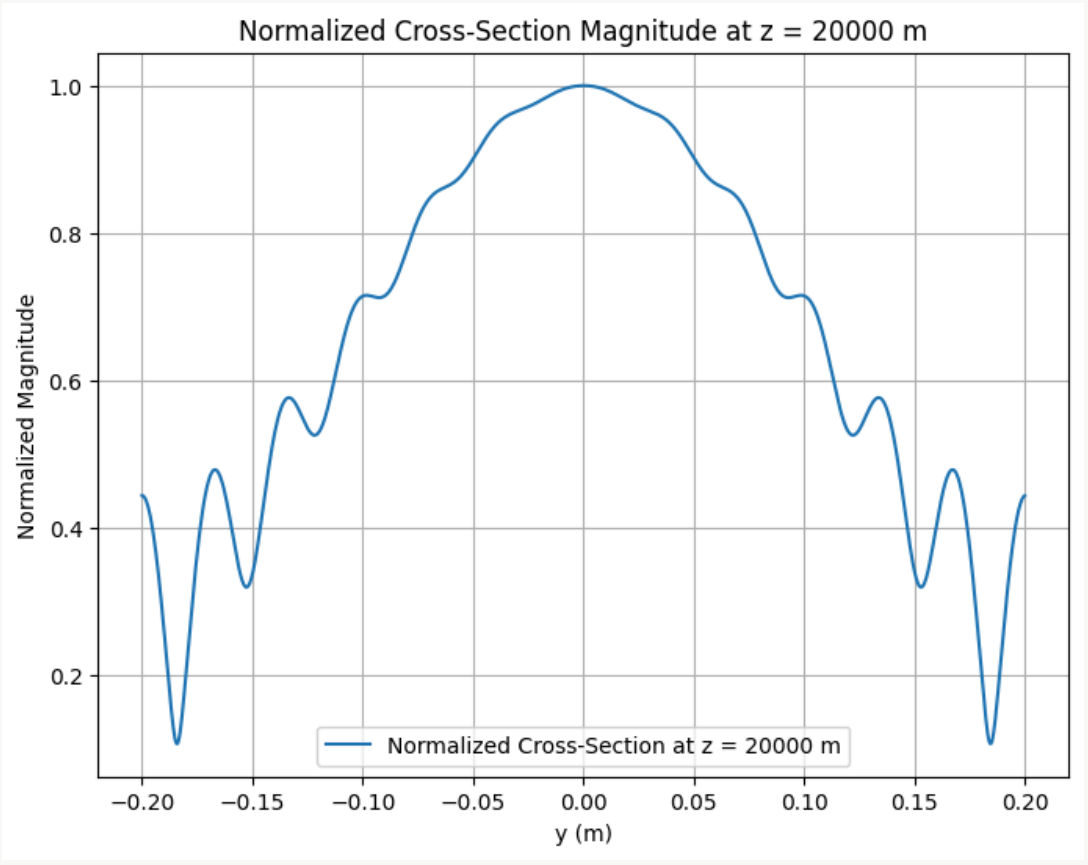






II) Propagate 20 km and Describe what you see.

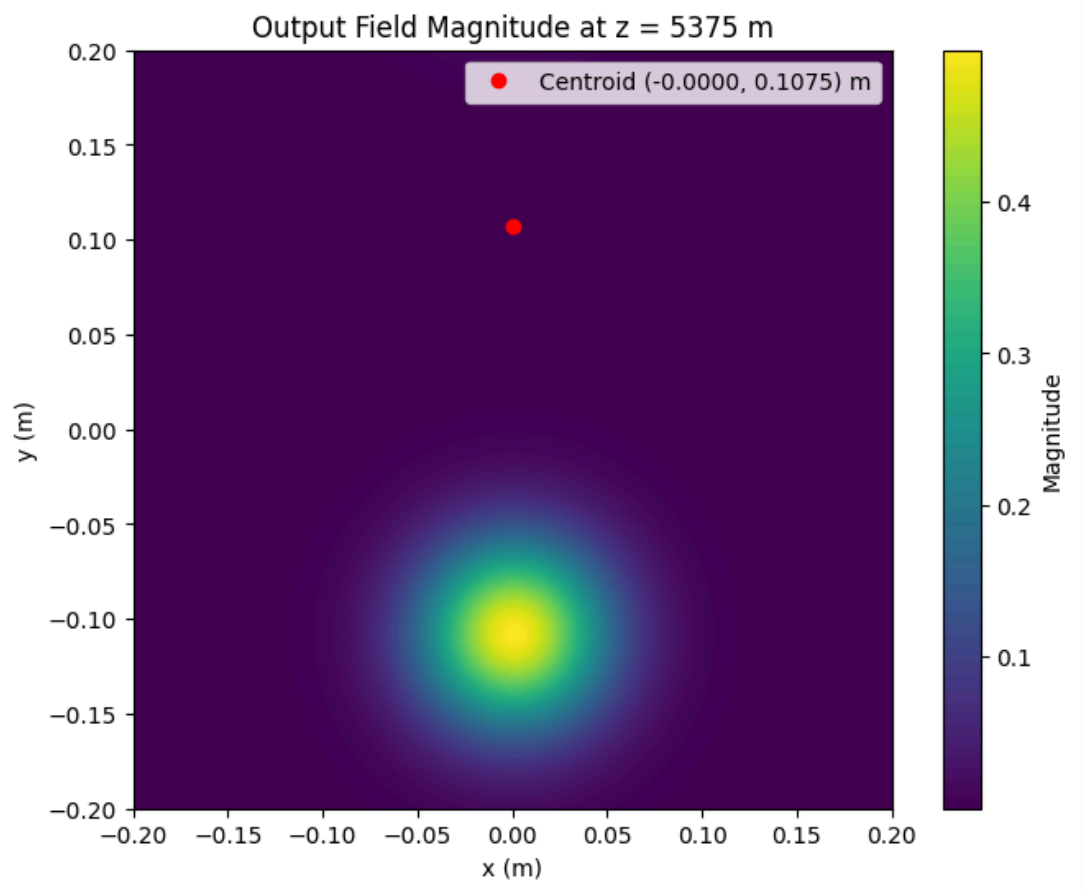

Output Field Magnitude at z = 20000 m
y (m)
x (m)
Magnitude
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
0.14
0.12
0.10
0.08
0.06
0.04
0.02

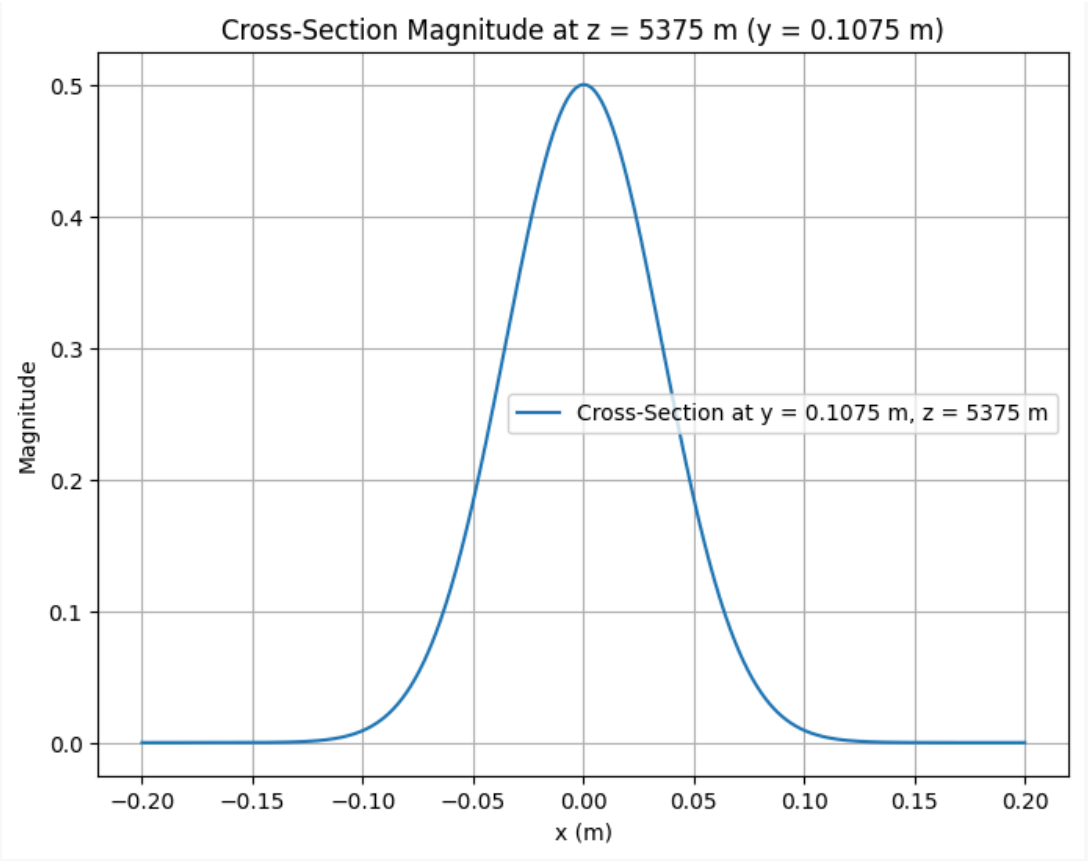

Normalized Cross-Section Magnitude at z = 20000 m
Normalized Magnitude
y (m)
Normalized Cross-Section at z = 20000 m
1.0
0.8
0.6
0.4
0.2
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20

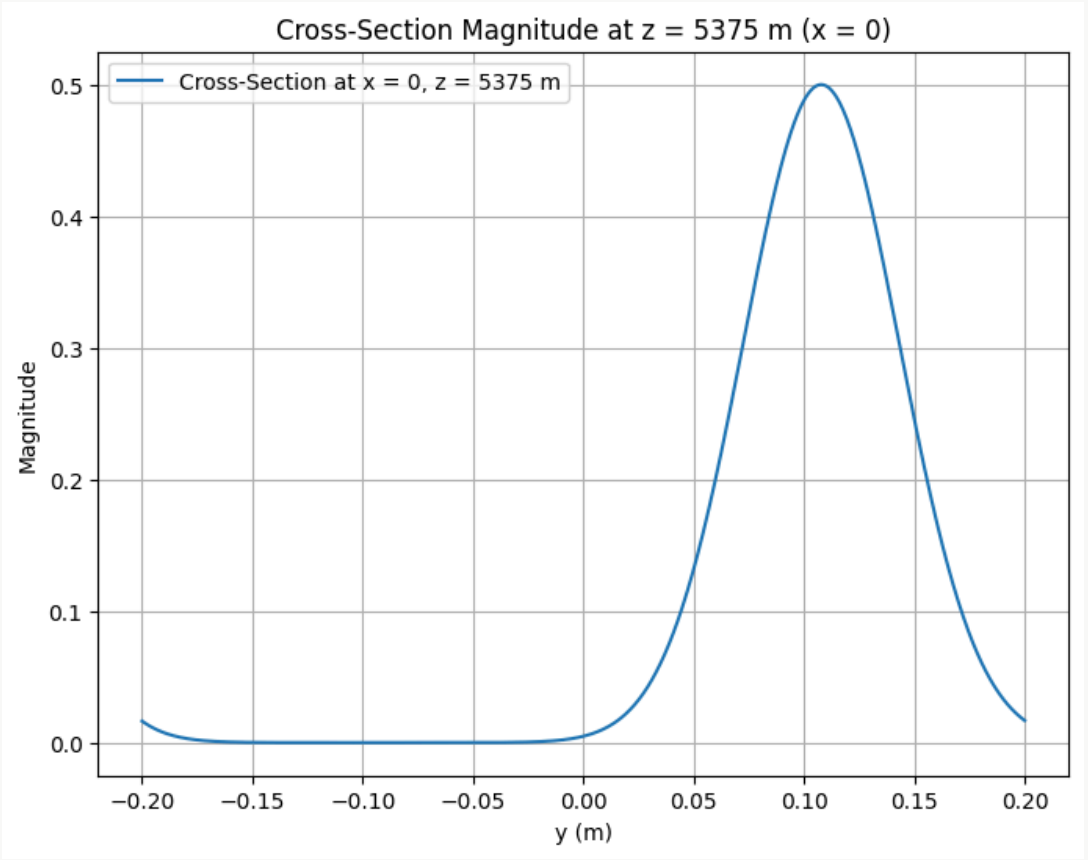
ANS: The peak value of the field at 20 km will be much lower than at z=0 (where it's 1.0) because the beam spreads significantly, reducing intensity. The beam has spread significantly due to diffraction over 20 km. The peak intensity is very low indicating the energy is distributed over a larger area. The simulation window (-0.2 m to 0.2 m) captures only the central portion of the beam, as its width far exceeds 0.4 m at this distance.
The pattern remains circularly symmetric, centered at (0, 0), with a smooth Gaussian-like distribution. The normalized plot shows a Gaussian profile with a maximum of 1 at y=0. At 5.373 km, the beam width doubled to ~0.05 m (50 mm), and the peak magnitude was higher (~0.02–0.03). At 20 km, the beam is over 3 times wider still, and the peak magnitude drops by another order of magnitude. The normalized cross-section confirms the Gaussian shape is preserved, just scaled wider.

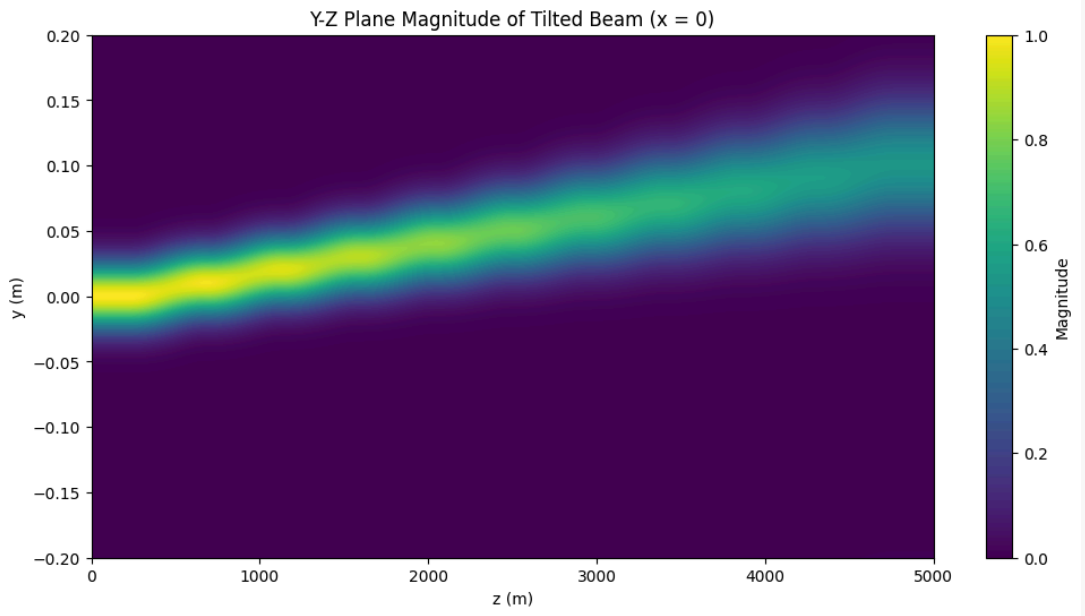
B)

I)

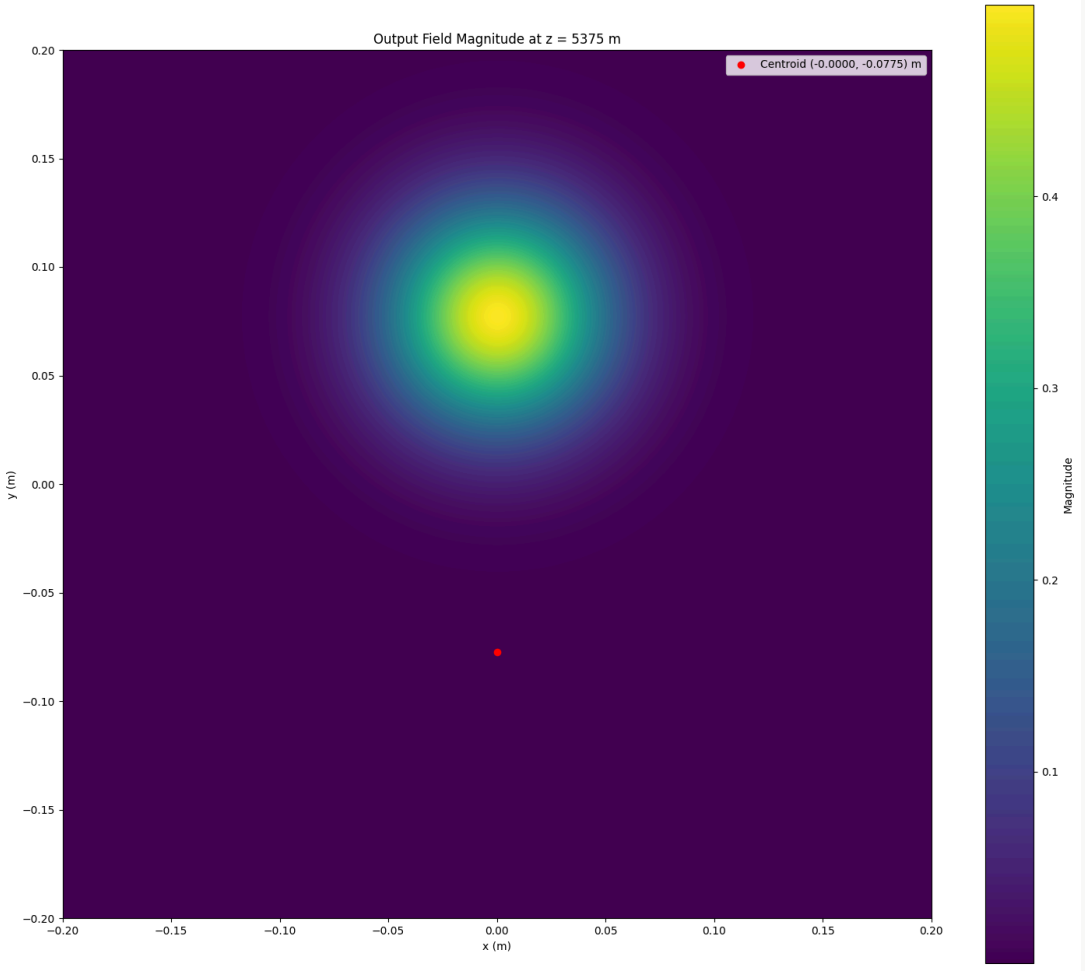


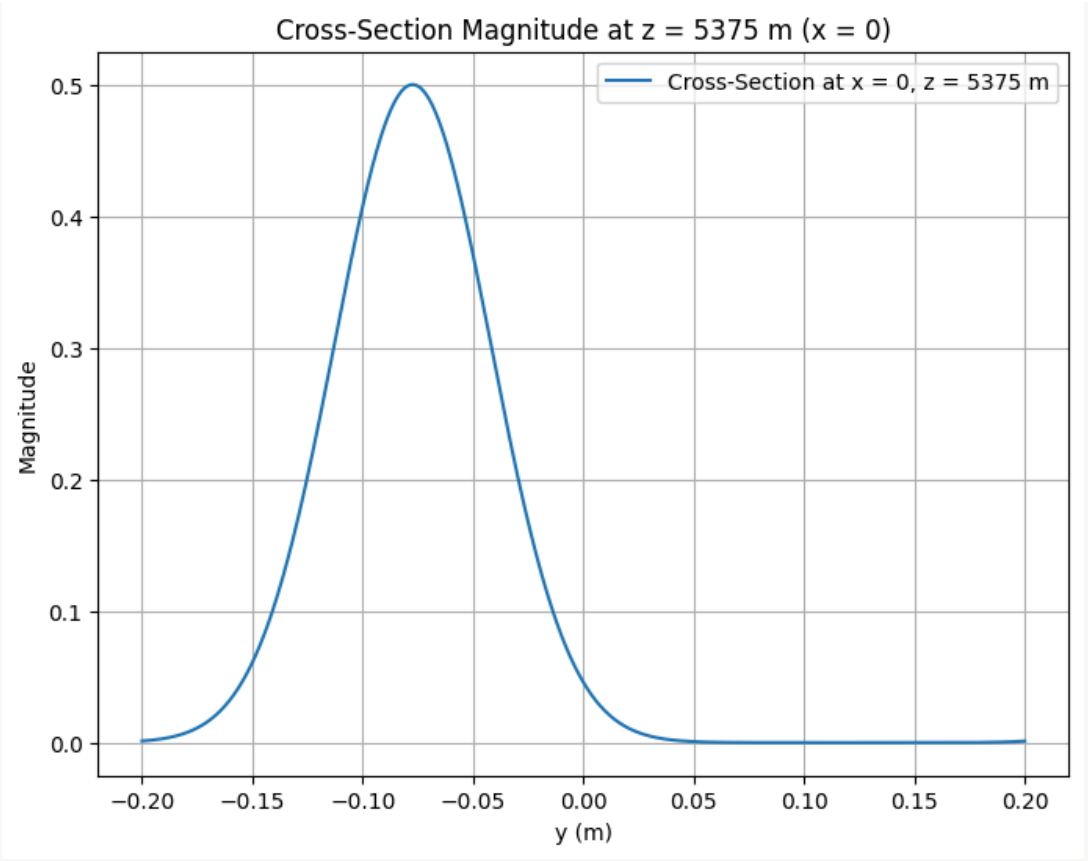

Cross-Section Magnitude at z = 5375 m (y = 0.1075 m)
Magnitude
x (m)
Cross-Section at y = 0.1075 m, z = 5375 m
0.0
0.1
0.2
0.3
0.4
0.5
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20

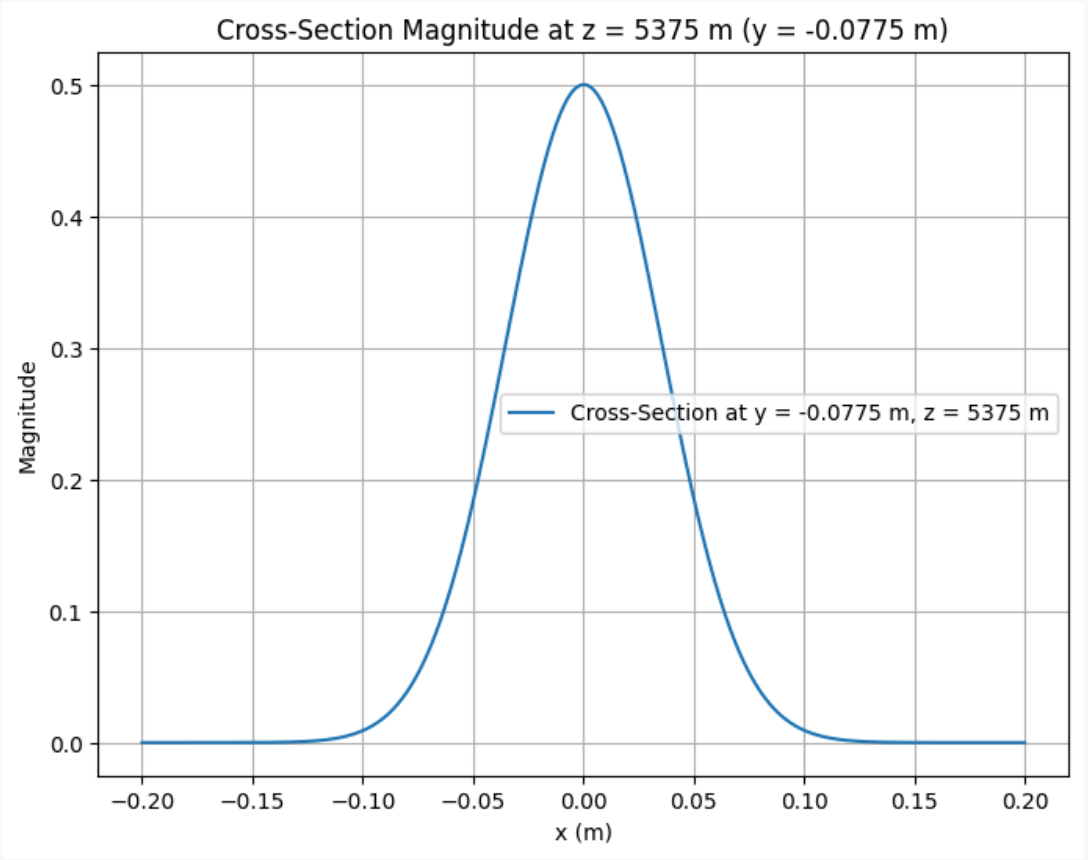

Cross-Section Magnitude at z = 5375 m (x = 0)
Magnitude
y (m)
Cross-Section at x = 0, z = 5375 m
0.0
0.1
0.2
0.3
0.4
0.5
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20

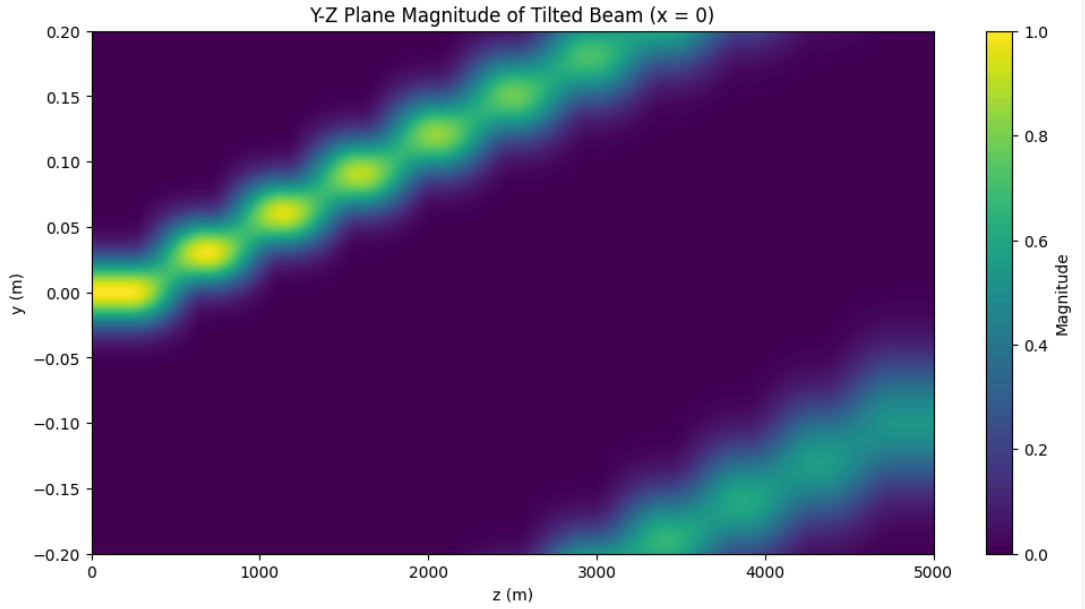


II) Repeat with 60 µrad. Describe what you see, and do your best to explain what might cause it.




Cross-Section Magnitude at z = 5375 m (x = 0)
Cross-Section at x = 0, z = 5375 m
Magnitude
0.0
0.1
0.2
0.3
0.4
0.5
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20
y (m)


Cross-Section Magnitude at z = 5375 m (y = -0.0775 m)
Cross-Section at y = -0.0775 m, z = 5375 m
Magnitude
0.0
0.1
0.2
0.3
0.4
0.5
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20
x (m)

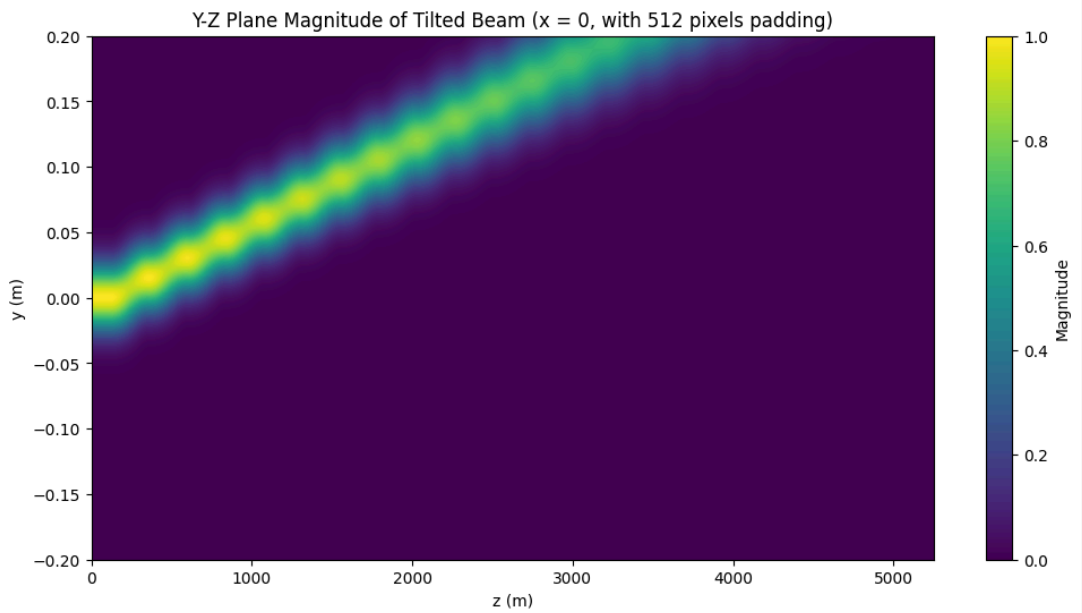


ANS: There is a repetitive beam pattern we are seeing in the Y-Z plane plot—where the beam's intensity appears to oscillate or repeat along the propagation direction (z-axis). This is a classic artifact caused by the lack of padding in the simulation.

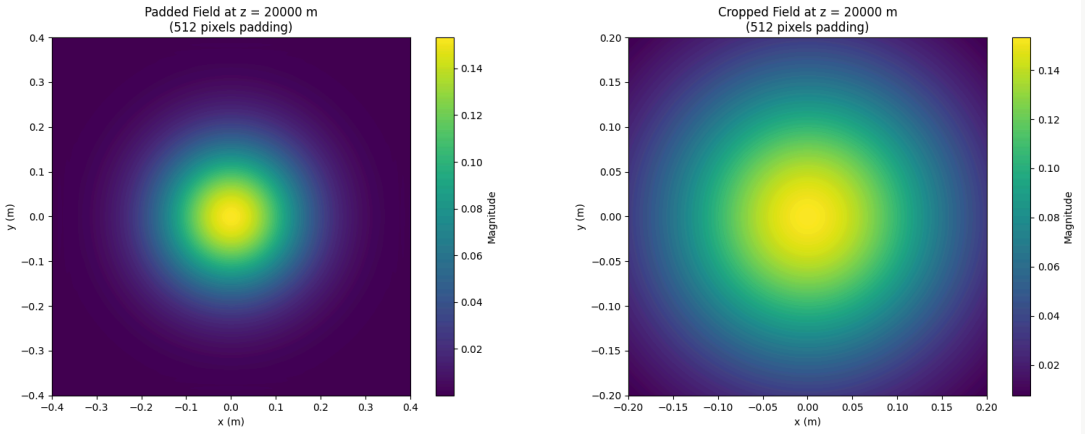
C) Please refer to the q4.ipynb code for a more precise calculation and results.

```
def angular_spectrum_propagation(E0, wavelength, z, xmax, N, padding_pixels=0):
    """Calculate electric field propagation using angular spectrum method with optional padding"""
    # If padding is requested, extend the field
    if padding_pixels > 0:
        N_padded = N + 2 * padding_pixels
        E0_padded = np.zeros((N_padded, N_padded), dtype=complex)
        start_idx = padding_pixels
        end_idx = padding_pixels + N
        E0_padded[start_idx:end_idx, start_idx:end_idx] = E0
        input_field = E0_padded
        total_N = N_padded
        total_xmax = xmax * (N_padded / N)  # Adjust domain size proportionally
    else:
        input_field = E0
        total_N = N
        total_xmax = xmax

    # Propagation
    dx = 2 * total_xmax / total_N
```

```
    x = np.linspace(-total_xmax, total_xmax, total_N, endpoint=False)
    X, Y = np.meshgrid(x, x)

    k = 2 * np.pi / wavelength
    fx = np.fft.fftfreq(total_N, dx)
    FX, FY = np.meshgrid(fx, fx)

    kx = 2 * np.pi * FX
    ky = 2 * np.pi * FY
    kz = np.sqrt(k**2 - kx**2 - ky**2)
    H = np.exp(1j * kz * z)
    H[np.isnan(kz) | (kx**2 + ky**2 > k**2)] = 0

    U0 = np.fft.fft2(input_field)
    U = U0 * H
    E = np.fft.ifft2(U)
    mag = np.abs(E)

    # Remove padding from the output if it was applied
    if padding_pixels > 0:
        mag = mag[start_idx:end_idx, start_idx:end_idx]
        X = X[start_idx:end_idx, start_idx:end_idx]
        Y = Y[start_idx:end_idx, start_idx:end_idx]

    return mag, X, Y
```



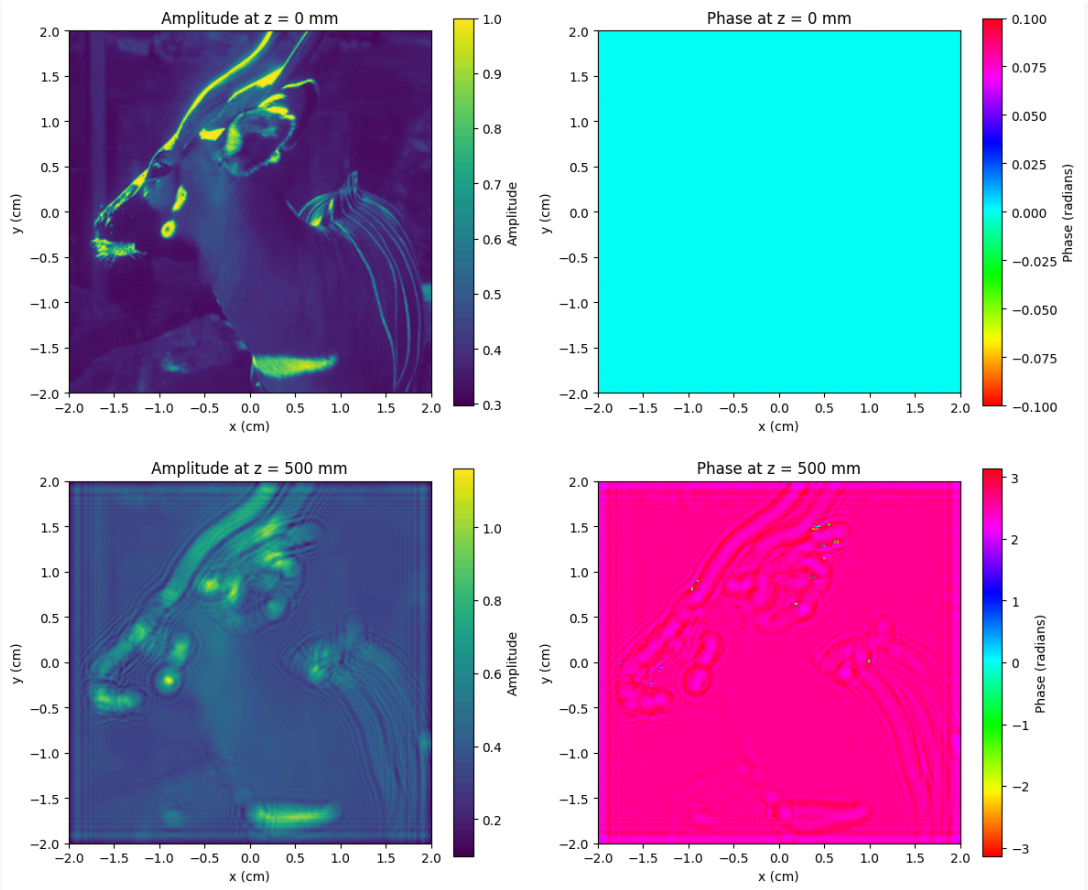
D) Please refer to the q4.ipynb code for a more precise calculation and results.


Padded Field at z = 20000 m
(512 pixels padding)
y (m)
x (m)
Magnitude
Cropped Field at z = 20000 m
(512 pixels padding)
y (m)
x (m)
Magnitude

Question 5

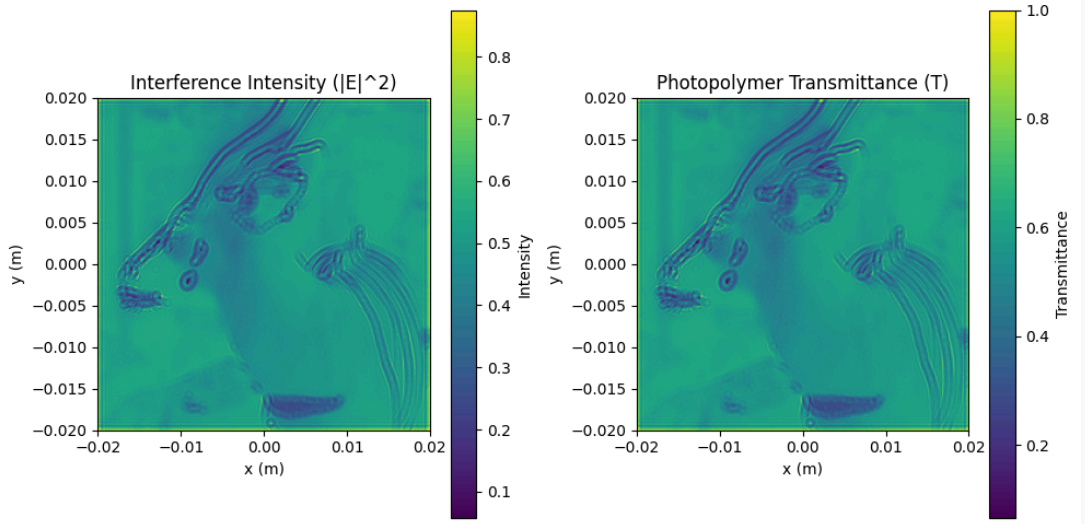
A)

B) Amplitude and Phase Plot after propagation

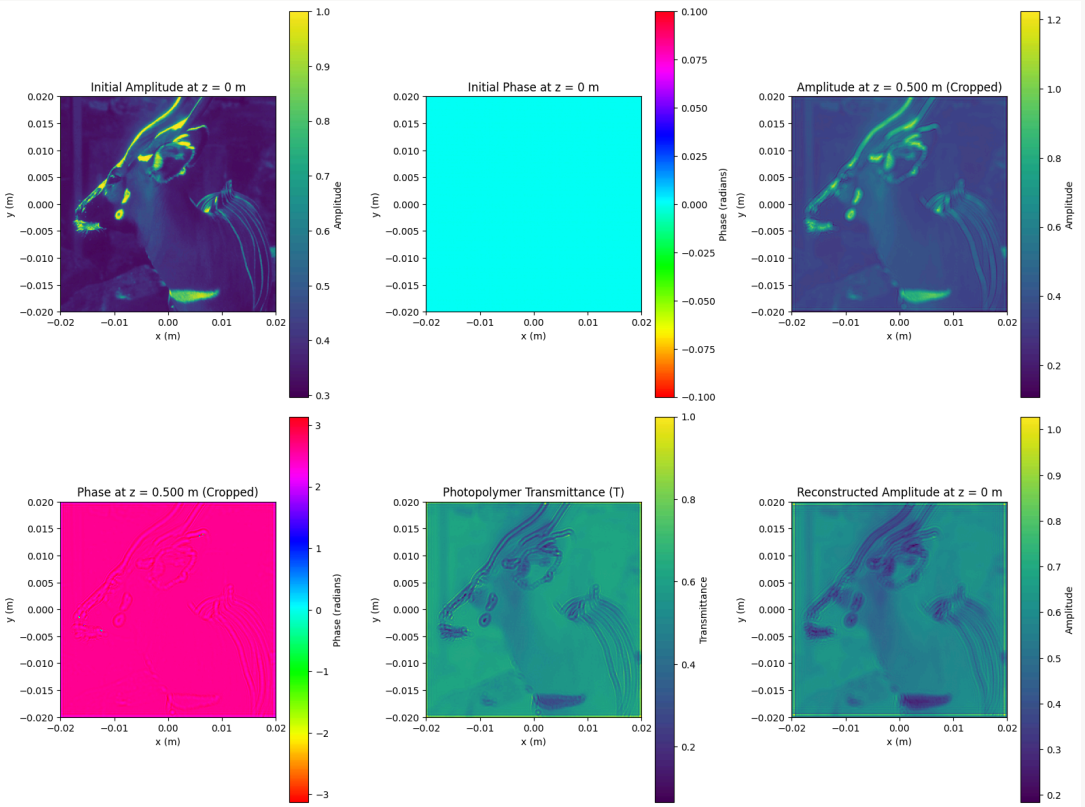


C) Here is the pseudo-code for the calculation. Please refer to the q5.ipynb code for a more precise calculation and results.

```
# Reconstruct the complex propagated field (E0) from amplitude and phase
E0 = mag_at_500mm * np.exp(1j * phase_cropped)

# Step 1: Interference with a unit-amplitude plane wave (reference beam)
E_interference = 1 + E0  # Complex field: 1 (reference) + E0 (object)

# Step 2: Calculate intensity (squared magnitude)
I_interference = np.abs(E_interference) ** 2

# Step 3: Calculate photopolymer transmittance (normalized intensity)
T = I_interference / np.max(I_interference)
```



D) Here is the pseudo-code for the calculation. Please refer to the q5.ipynb code for a more precise calculation and results.



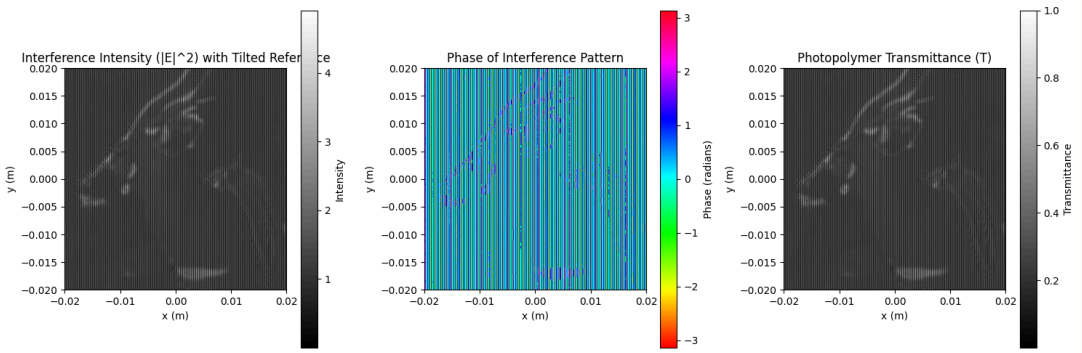

```
    # Step 1: Interference with reference beam (unit-amplitude plane wave)
    # Calculate interference pattern intensity (using E0 for photopolymer recor
ding)
    E_interference = 1 + E0  # Unit-amplitude reference beam (amplitude = 1), c
omplex E0
    I_interference = np.abs(E_interference) ** 2
```

```
    T = I_interference / np.max(I_interference)  # Photopolymer transmittance
    # Step 2: Reconstruction beam - Unit-amplitude plane wave traveling in op
posite direction
    # The photopolymer modulates the reconstruction beam: E(x, y) = T(x, y)
(amplitude only, flat phase)
    E_reconstruction = T * np.exp(1j * 0)  # Unit amplitude modulated by T, flat
phase

    # Step 3: Propagate this field back 500 mm (z = -500 mm) to the starting pl
ane (z = 0)
   (mag_padded_back, phase_padded_back,= \
        angular_spectrum_propagation(E_reconstruction, lambda_, -z, xmax, N,
padding_pixels)

    # The reconstructed field amplitude is the magnitude of the cropped field a
t z = 0
    amplitude_reconstructed = mag_cropped_back
```

E) Why and How ?

ANS: Why Issues Occur in the Simulation:
In the simulation, a finite grid (256 × 256 pixels, padded to 512 pixels, covering 10 cm × 10 cm) is used with the angular spectrum propagation method at z = 500 mm and λ = 633 nm. The diffraction pattern may exceed this grid, causing aliasing or loss of high-spatial-frequency details due to the FFT's assumption of periodicity. This can result in blurring, ringing, or missing fine details in the reconstructed image. While the code correctly incorporates amplitude and phase (E_o = mag_cropped * exp(1j * phase_cropped)), errors in phase computation—due to propagation artifacts, insufficient padding, or numerical precision—can distort the wavefront, leading to an imperfect reconstruction of the original image (e.g., the deer). Additionally, using a flat, unit-amplitude plane wave as the reference beam (perpendicular to the z-axis) may cause overlap between the reconstructed object beam and the undiffracted reference beam, introducing noise and distortion. A tilted reference beam could mitigate this by reducing interference with zero-order diffraction.

How to Improve:
Two solutions are proposed. First, a tilted reference beam introduces a linear phase gradient, separating the reconstructed object beam from the undiffracted and conjugate beams, thus reducing noise and enhancing clarity. Second, the "Amplitude+Phase Holography" approach focuses on accurately recording and reconstructing both amplitude and phase, ensuring a more faithful wavefront reconstruction and improved image quality. Please the the result below.



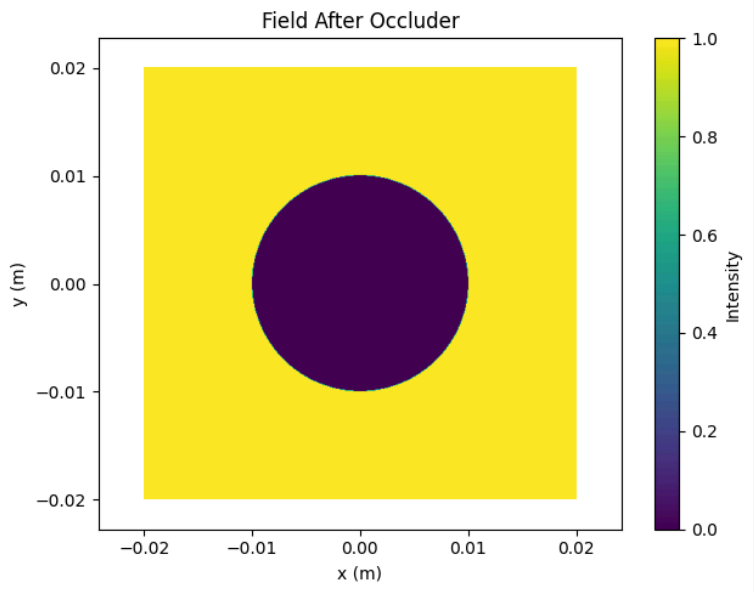
⚠️ Please refer to the q4.ipynb code for the implementation of two improvements.

Question 6 ✅

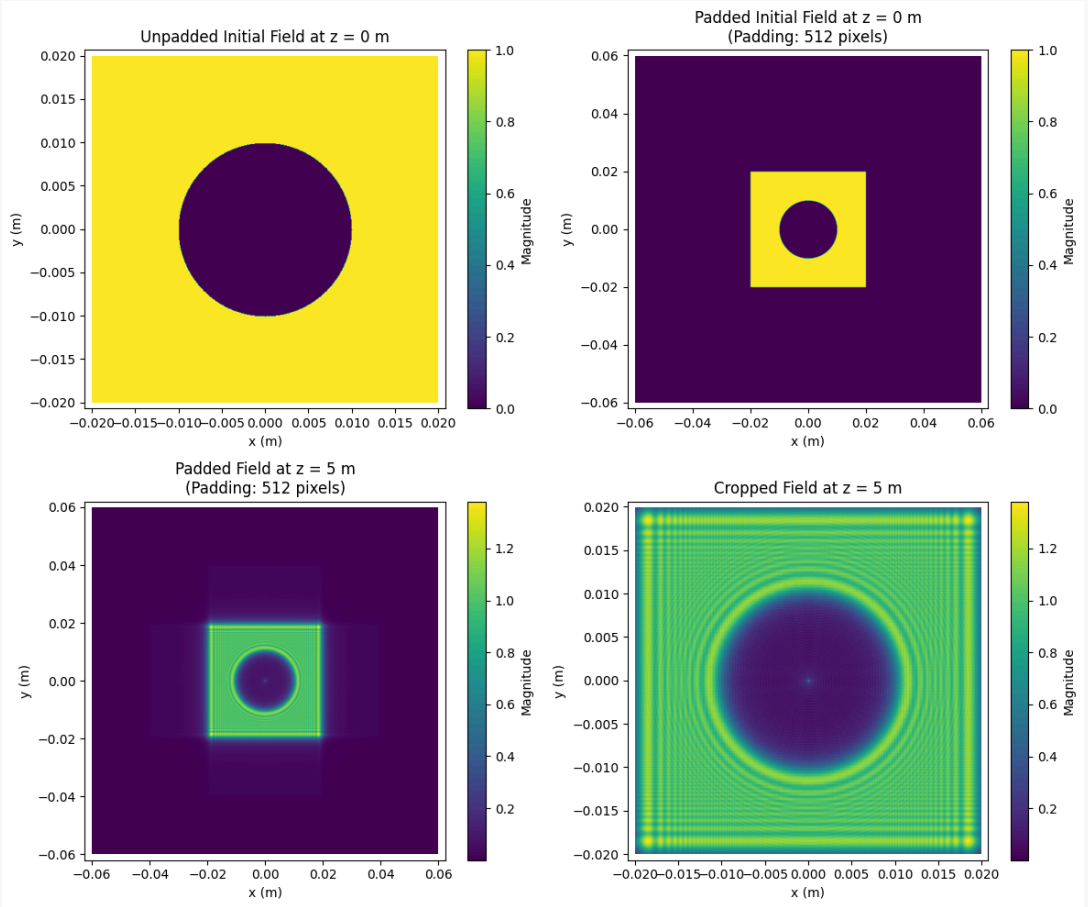
A)

```
def field_calculation():
    wavelength = 633e-9  # meters (633 nm)
    D = 0.02  # meters (2 cm)
    xmax = 0.02  # meters (2 cm)
    N = 512

    dx = 2 * xmax / N
    x = np.linspace(-xmax, xmax, N)
    y = np.linspace(-xmax, xmax, N)
    X, Y = np.meshgrid(x, y)
    R = np.sqrt(X**2 + Y**2)

    mask = np.ones((N, N))
    mask[R < D/2] = 0
    incident_field = np.ones((N, N))
    field_after = incident_field * mask


    return x, field_after
```

Field Magnitude



B) Focus point plot



C)

ANS: The Poisson spot is a diffraction phenomenon that occurs when light waves pass around the edges of a circular obstacle. The waves diffract (bend) around the edges of the occluder and interfere with each other in the region behind it (Goodman, 2005).
In the shadow region (behind the occluder), these diffracted waves can interfere constructively at the center (directly behind the occluder), creating a bright spot. This spot is unexpected from a purely geometric optics perspective (which predicts a dark shadow).

In our simulation, by z = 5 m, the light has propagated far enough for diffraction effects to become significant, allowing edge waves to interfere constructively at the center, forming the Poisson spot.

Reference

Goodman, J. W. (2005). *Introduction to Fourier Optics* (3rd ed.). Roberts & Company Publishers.