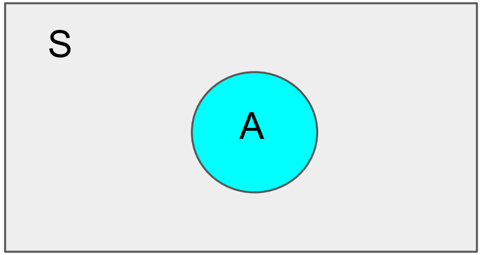
Section 2

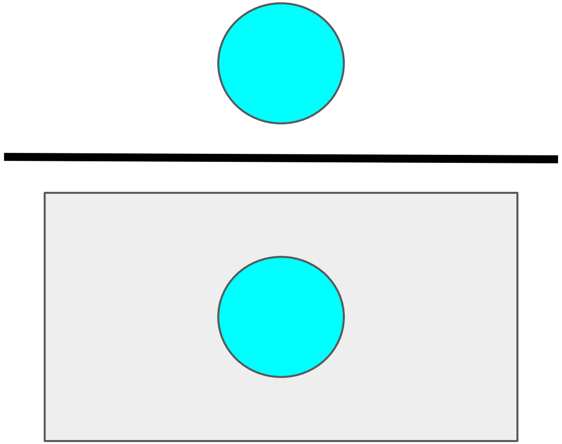
Probability, randomized algorithms, k-selection

Counting/probability spaces

- Probability in discrete domain is size(event)/size(sample space)



$P(A) = |A|/|S| =$



Independence

$P(E \text{ and } F) = P(E)P(F)$

e.g.:

$P(\text{coin1\_heads and coin2\_heads}) = P(\text{coin1\_heads})*P(\text{coin2\_heads})$

Equivalently, $P(E|F) = P(E)$

$P(\text{coin1\_heads given coin2\_heads}) = P(\text{coin1\_heads})$

Random Variables

A variable that probabilistically takes on different values.

- Can be a boolean: e.g., assign 1 if coin lands on heads, 0 if coin lands on tails
- Can be an integer: # of successive heads flipped/side of a die landing up
- Can be a real number: e.g., height of an individual sampled from a population

Expectation

$$E[X] = \sum xP(x)$$

Linear: E[aX + b] = aE[X] + b

Adding random variables: E[A + B] = E[A] + E[B] regardless of the relationship between A and B! (A and B do not have to be independent.)

However, E[AB] =/= E[A]E[B] when A and B are not independent. Be careful with this one.

Expectation II

$$E[X] = \sum xP(x)$$

Let A be a possible outcome of an experiment

Let X = # of times running the experiment before A occurs

Then E[X] = 1/P(A)

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Let A be a possible outcome of an experiment

Let X = # of times running the experiment before A occurs

Then E[X] = 1/P(A) because

E[X] = 1*P(A) + (1 + E[X])*(1-P(A))

E[X] - E[X]*1 + E[X]*P(A) = E[X]*P(A) = 1

Appendix

Conditional Probability

$$P(E \mid F) = \frac{P(EF)}{P(F)} = \frac{P(E \cap F)}{P(F)}$$

pronounced “probability of E ***given*** F”

Bayes’ Rule
(definition of conditional probability + chain rule to expand P(EF))

$$P(E \mid F) = \frac{P(F \mid E)P(E)}{P(F)}$$

can be expanded to ->
(to account for F
given all possibilities for E)

$$\frac{P(F \mid E)P(E)}{\sum_i P(F \mid E_i)P(E_i)}$$

Other potentially useful concepts

- DeMorgan's law
- Conditional independence $P(EF|G) = P(E|G)P(F|G)$
- Pigeonhole principle
- "Stars and bars"

For more practice/resources

- http://web.stanford.edu/class/cs109/