CS 161 W19: Recitation 5 Problems

February 2019

Exercise 0

Solve the following recurrences — that is, get a tight bound of the form $T(n) = O(f(n))$ for the appropriate function f. You may use the master theorem if it applies. Ignore floors and ceilings, and assume $T(0) = T(1) = 1$.

(a) $T(n) = T(n/5) + 64n$

(b) $T(n) = 4T(n/4) + n$

(c) $T(n) = T(n-2) + 9n$

Exercise 1

Random sort (also known as bogosort) attempts to sort a list by first checking if it sorted, and if not it randomly permutes all the elements and then checks again. It repeats this process until it successfully generates the sorted list. In this problem, assume that the operation of randomly permuting n elements takes $O(n)$ time, and assume that the list we are attempting to sort consists of distinct integers. What is the expected runtime of the algorithm for worst-case input? Is the runtime of the algorithm bounded?

Exercise 2

A review on hashing: Assuming you are using a uniformly random hash function,

1. if you were to hash n items into $b = n$ buckets, how many items would you expect to be hashed to the 3rd bucket? (Assume $n > 3$.)
2. in expectation, what is the number of items per bucket?
3. what is the probability that the 3rd bucket has exactly 3 items?
4. what is the big-O runtime of INSERT, DELETE, SEARCH, and SELECT where, for example, SELECT(5) finds the 5th smallest element in the hash table?

Exercise 3

True or false:

1. If (u, v) is an edge in an undirected graph and during DFS, $finish(v) < finish(u)$, then u is an ancestor of v in the DFS tree.
2. In a directed graph, if there is a path from u to v and $start(u) < start(v)$ then u is an ancestor of v in the DFS tree.

Exercise 4

You have n boxes. The i-th box has dimensions $w_i \times h_i$. Box i can fit inside box j if and only if $w_i < w_j$ and $h_i < h_j$. A sequence of boxes $b_1, b_2, ..., b_k$ form a chain if box b_i fits inside box b_{i+1} for each $1 \leq i < k$. Design an algorithm which takes as input a list of dimensions $w_i \times h_i$ and returns a longest possible chain of boxes. You must construct a directed graph as part of your solution.

Exercise 5

Suppose that G is a graph with $2n$ nodes and no triangles (cycles of length 3). G is a proper graph, *i.e.* it has no self-loops or multiple edges between the same pair of nodes. Use induction to prove that G has at most n^2 edges.