E40M

Review #2

Topics in Part 1 (Monday): - KCL, KVL, Power

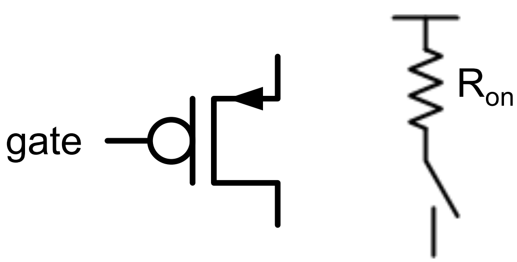
- Devices: V and I sources, R
- Nodal Analysis. Superposition
- Devices: Diodes, C, L
- Time Domain Diode, C, L Circuits

Topics in Part 2 (Today):

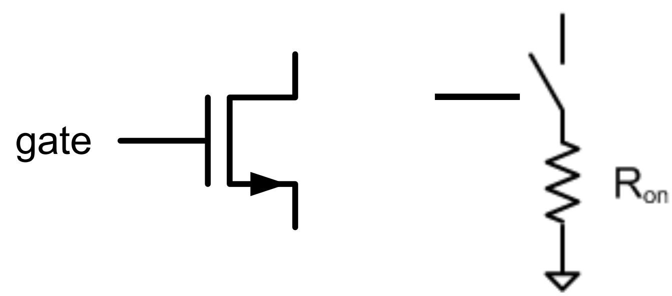
- MOSFETs, CMOS Circuits, Logic Gates
- Binary Numbers
- Time Division Multiplexing
- Frequency Domain Circuits, Impedance
- Filters, Bode Plots
- Op Amps

Electrical Device: MOSFETs

- Are very interesting devices
  - Come in two “flavors” – pMOS and nMOS
  - Symbols and equivalent circuits shown below
- Gate terminal takes no current (at least no DC current)
  - The gate voltage* controls whether the “switch” is ON or OFF



pMOS



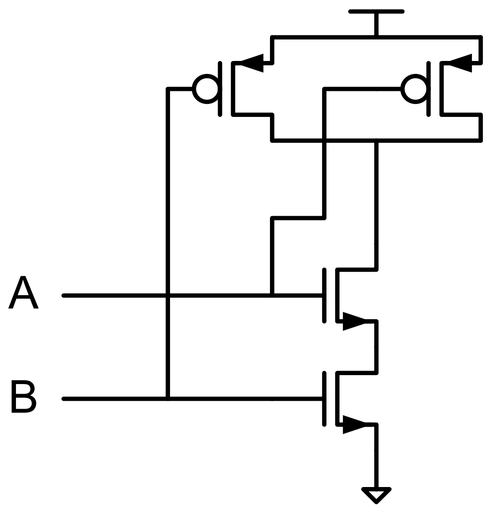
nMOS

* actually, the gate – to – source voltage, V_{GS}

Building Logic Gates from MOS Transistors

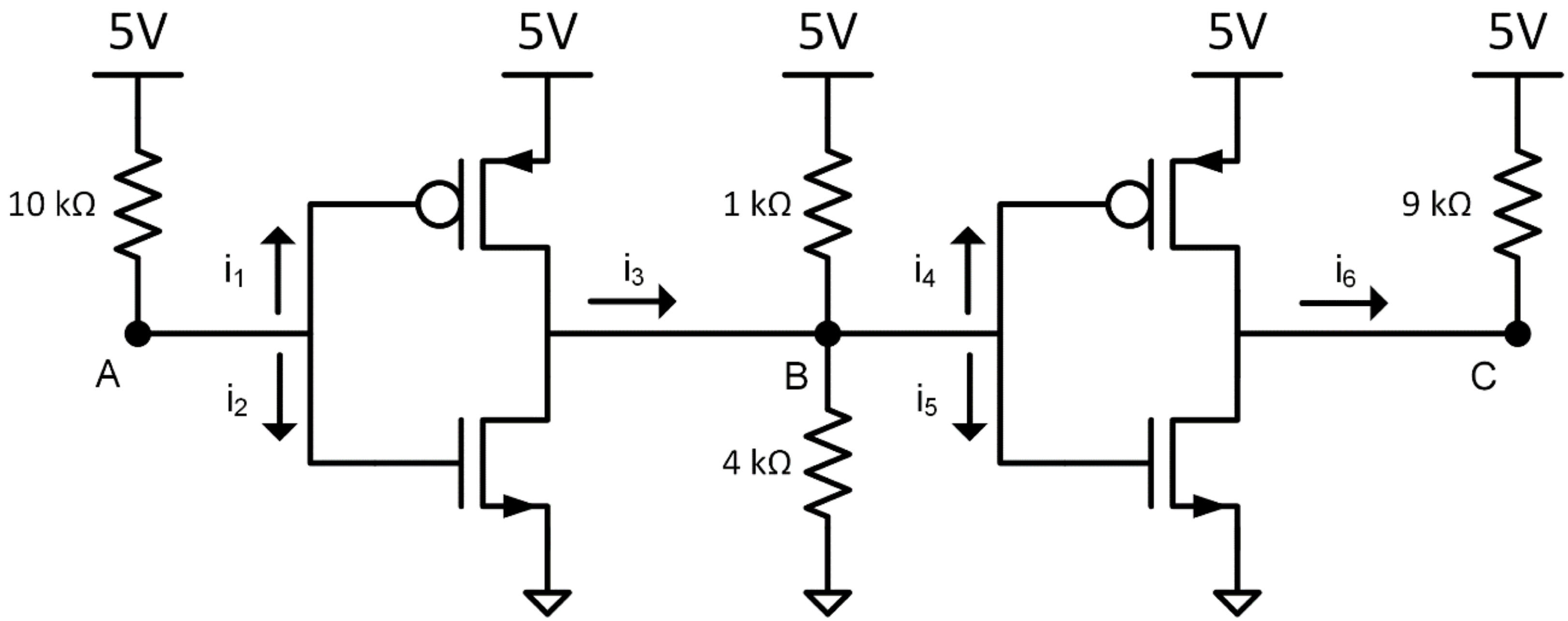
- pMOS connect to Vdd; nMOS connect to Gnd
  – Otherwise need voltage > Vdd, or < Gnd to turn them on

- Need to connect output to either Vdd, or Gnd
- For the gate shown below
  – The output is connect to Gnd when A and B are true
  – The output is connected to Vdd when A or B is false



Example: CMOS Circuits

- Find the node voltages and branch currents in the circuit below.
- Assume R_{on} for the transistors is 0Ω.



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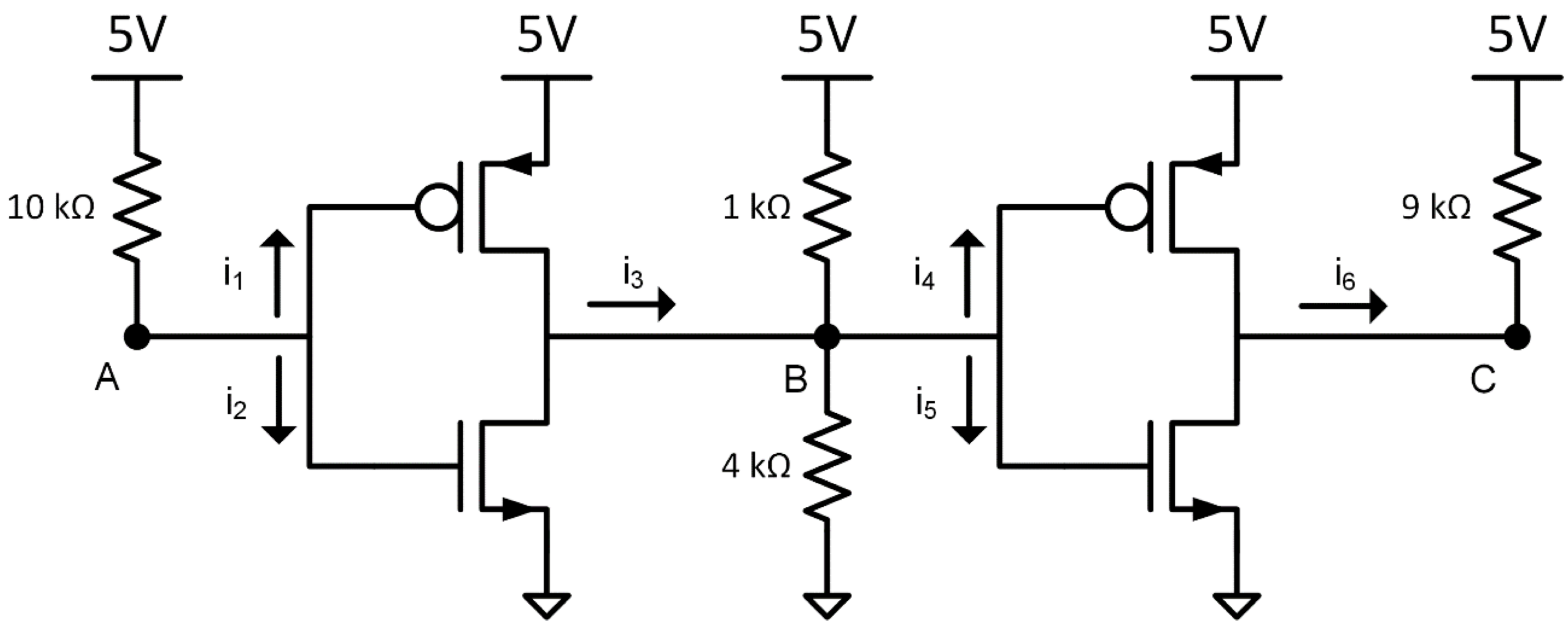


$$i_1 = i_2 = 0 \quad \therefore \quad V_A = 5V \qquad V_B = 0V \quad \therefore \quad i_3 = \frac{0-5V}{1k\Omega} = -5mA \qquad V_C = 5V \quad \therefore \quad i_6 = 0$$

$$i_4 = i_5 = 0$$

Truth Tables & Logic Gates

A	B	AND
0	0	0
0	1	0
1	0	0
1	1	1

(A && B) AND

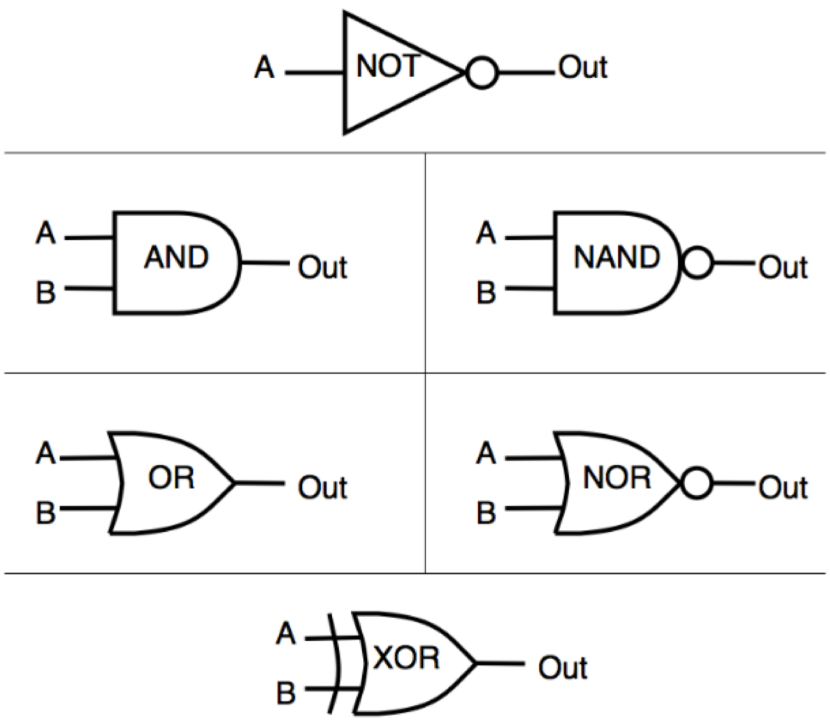
A	B	OR
0	0	0
0	1	1
1	0	1
1	1	1

(A || B) OR

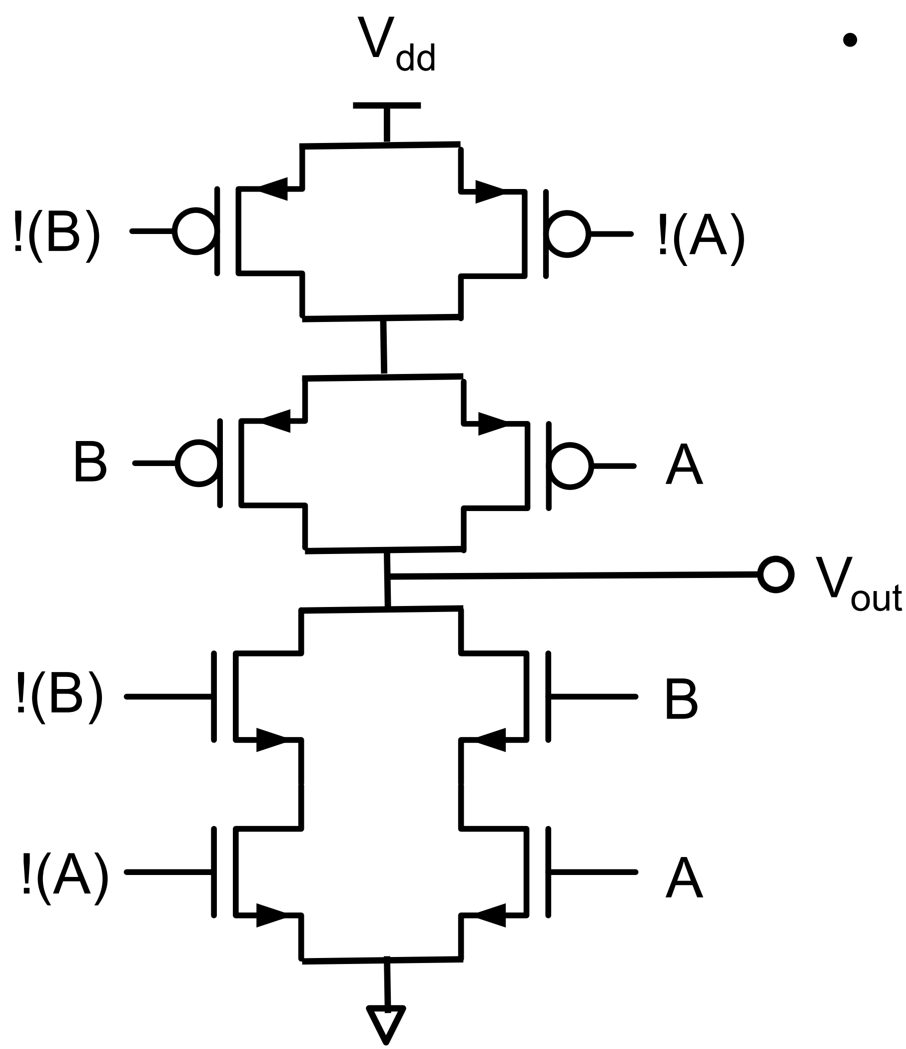
A	NOT
0	1
1	0

!(A) NOT

Logic Gate Symbols



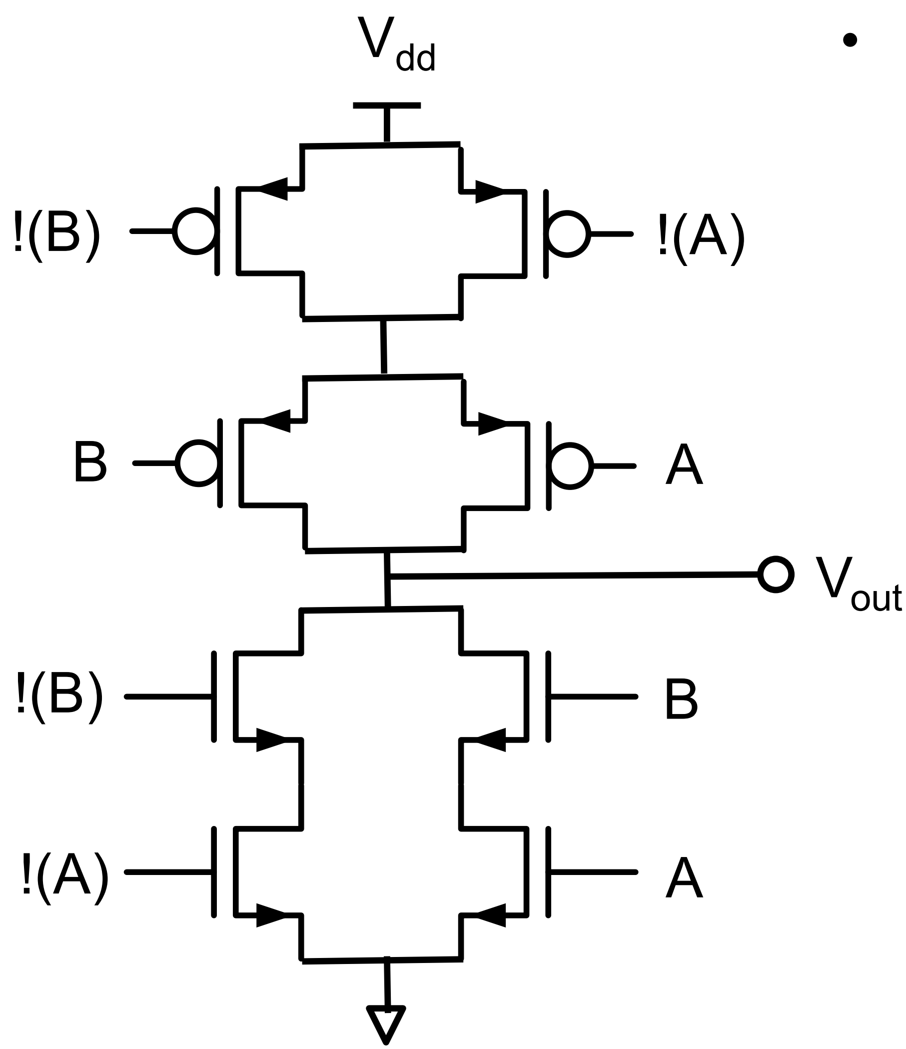
Example: CMOS Logic Gate



- Construct a truth table for the CMOS logic gate shown. What does it do?

A	**B**	V_{out}
0	0	
0	1	
1	0	
1	1	

Example: CMOS Logic Gate



- Construct a truth table for the CMOS logic gate shown. What does it do?

A	B	V_{out}
0	0	0
0	1	1
1	0	1
1	1	0

XOR Gate

Binary Numbers

Operation	Output	Remainder
$27/2^7$	0	27
$27/2^6$	0	27
$27/2^5$	0	27
$27/2^4$	1	11
$27/2^3$	1	3
$27/2^2$	0	3
$27/2^1$	1	1
$27/2^0$	1	1

27 in binary is 00011011

Carry		1	1	
11	1	0	1	1
3	0	0	1	1
Addition (14)	1	1	1	0

Add by carrying 1s

- Subtract by borrowing 2s.

Generating Two's Complement Numbers

- Take your number
  - Invert all the bits of the number
    - Make all "0" "1" and all "1" "0"
  - And then add 1 to the result

- Lets look at an example:

42	0	0	1	0	1	0	1	0
Flip 1 and 0	1	1	0	1	0	1	0	1
Add 1	1	1	0	1	0	1	1	0

- So -42 in two's complement is 11010110

Example:

Base 10: 11 – 23 = -12

Example:

Base 10: 11 – 23 = -12

6 bits + sign bit: 11 = 0001011

23 = 0010111

2's complement negative number:

-23 = 1101000 (first bit is sign bit)

+ 1 → 1101001

Sum:

0001011

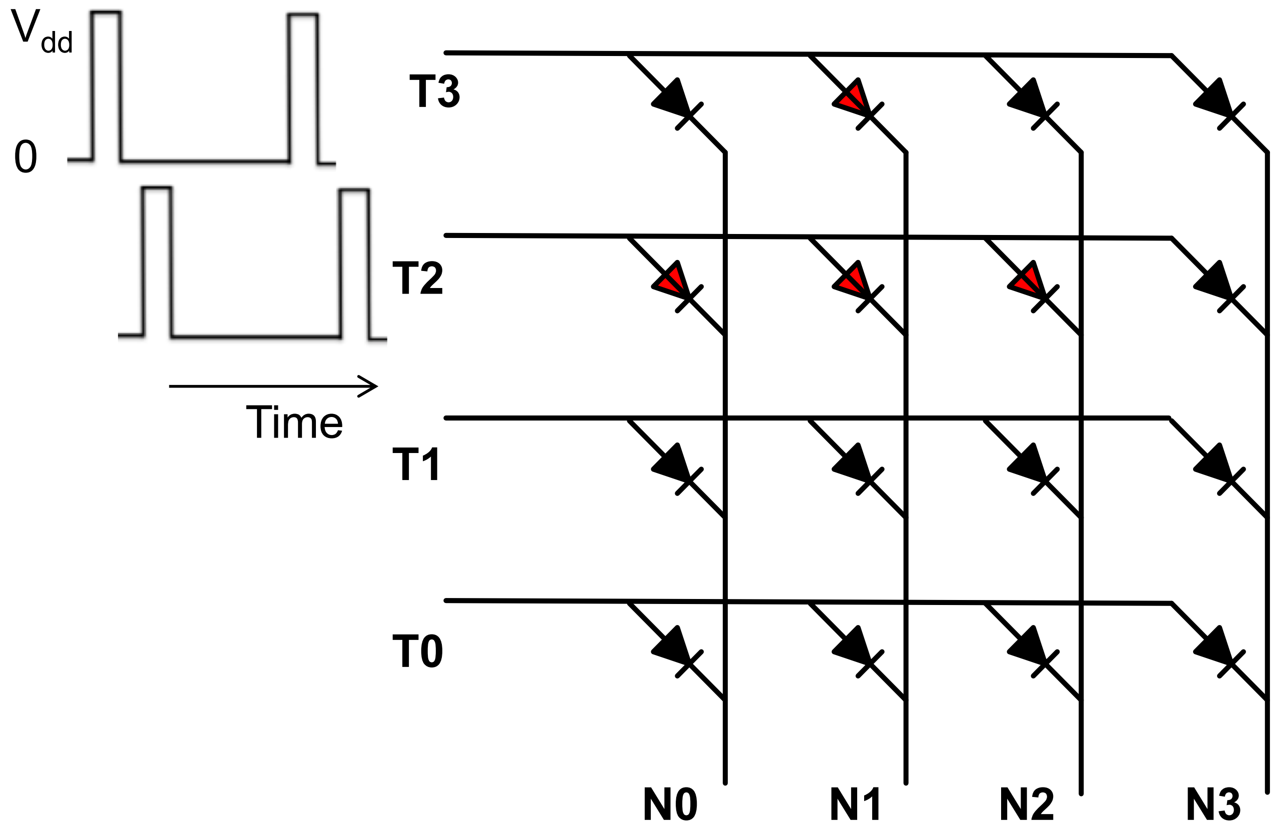
+1101001

= 1110100

Check: subtract 1 → 1110011;then flip bits: 0001100

Time Division Multiplexing

- If we use time division multiplexing to drive the LED array
  - How do you light up the red LEDs?



Time slot #1:
Time slot #2:

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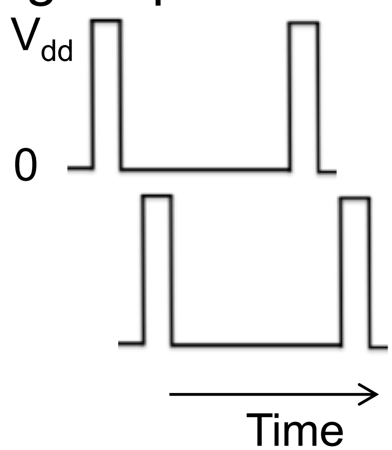
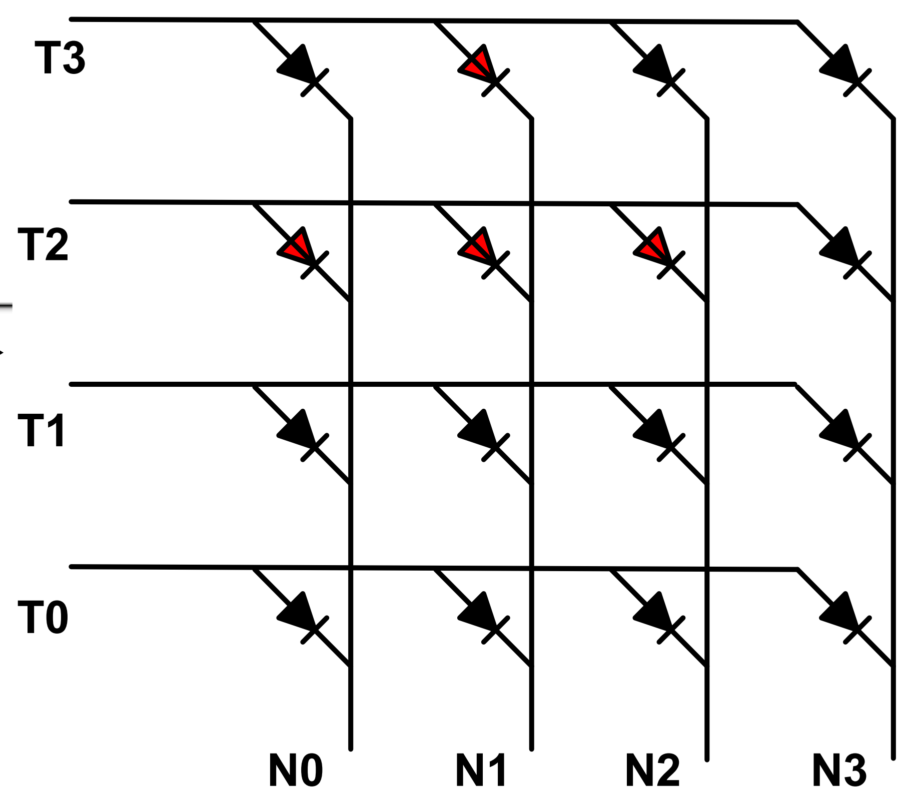




Time slot #1: N1 at Gnd.

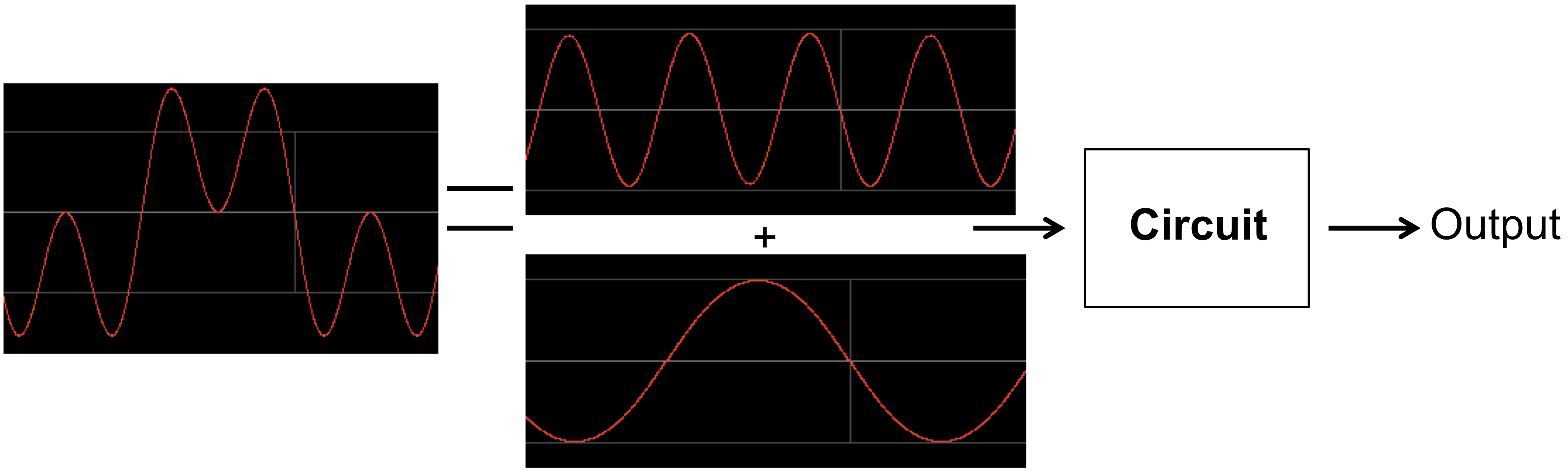
Time slot #2: N0, N1 and N2 at Gnd.

Duty Cycle Code

- For things that take real power
  - And where some elements can average
  - Can control output
    - By changing duty cycle of input

- Examples
  - Motor (inductance)
    - Switch voltage, current drives motor
  - Power supply converter (inductance)
    - Switch voltage, inductance/cap filter
  - LED (eyes)

Frequency Domain Analysis



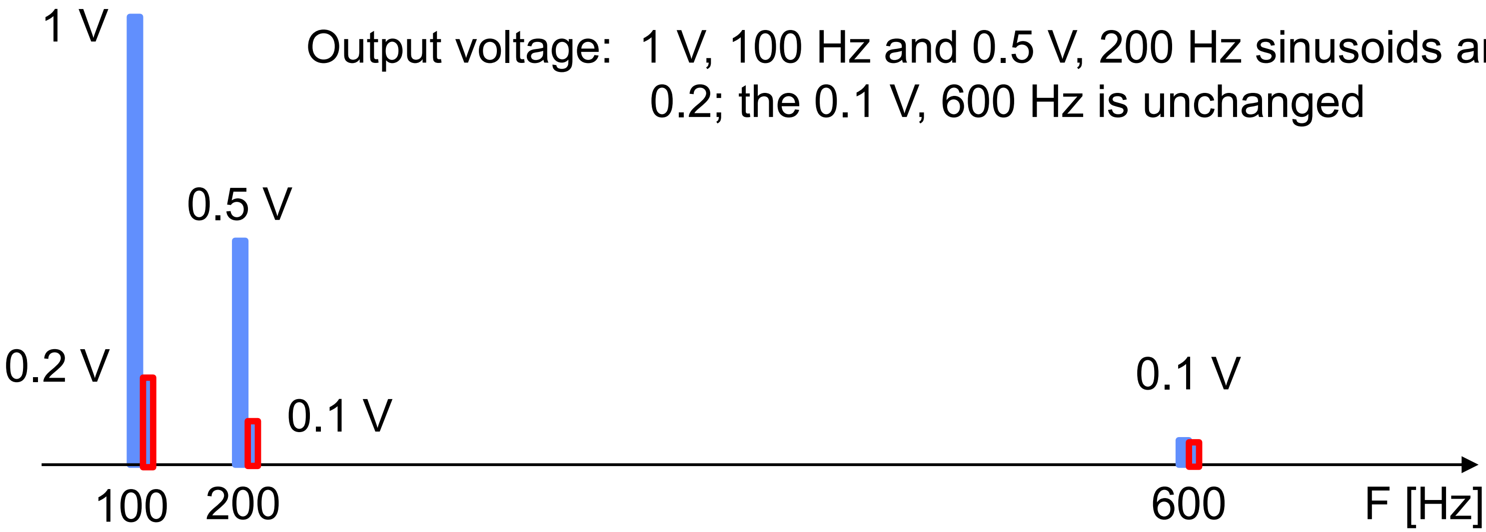
- If we have a circuit with an input voltage that varies with time, we can figure out what the output of that circuit will be by considering the individual frequency components of the input signal.

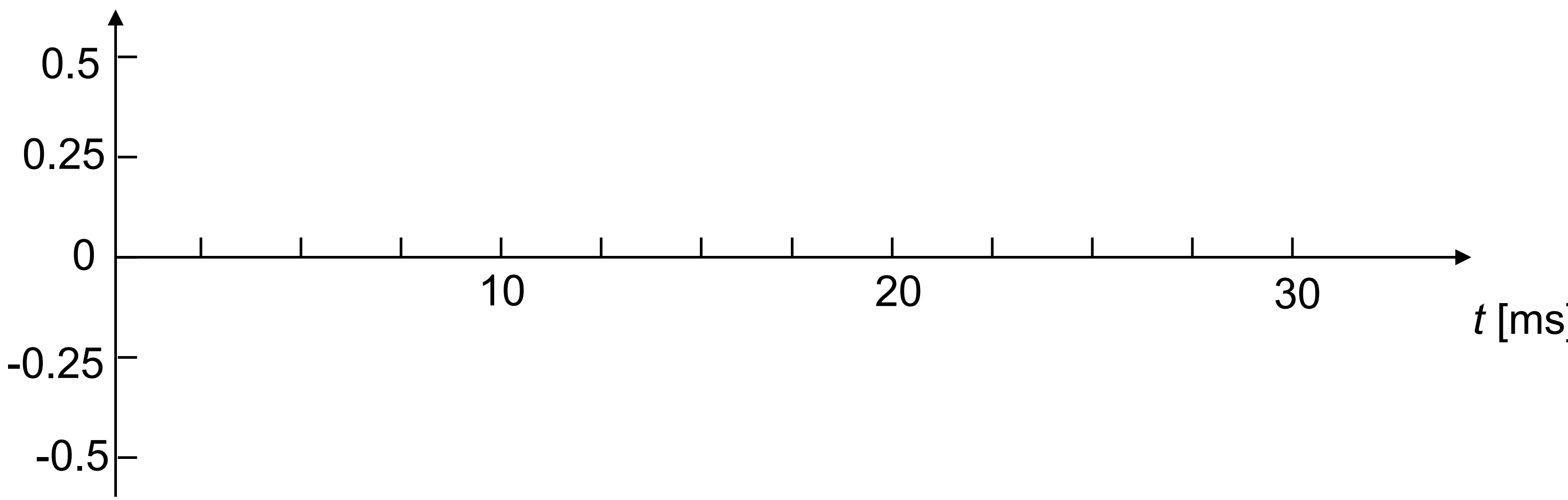
- Superposition will give us the resulting output.

Example: Frequency Decomposition

Example: Frequency Decomposition

Output voltage: 1 V, 100 Hz and 0.5 V, 200 Hz sinusoids are multiplied by 0.2; the 0.1 V, 600 Hz is unchanged





Impedance

- Impedance is a concept that is a generalization of resistance:

$$R = \frac{V}{i}$$

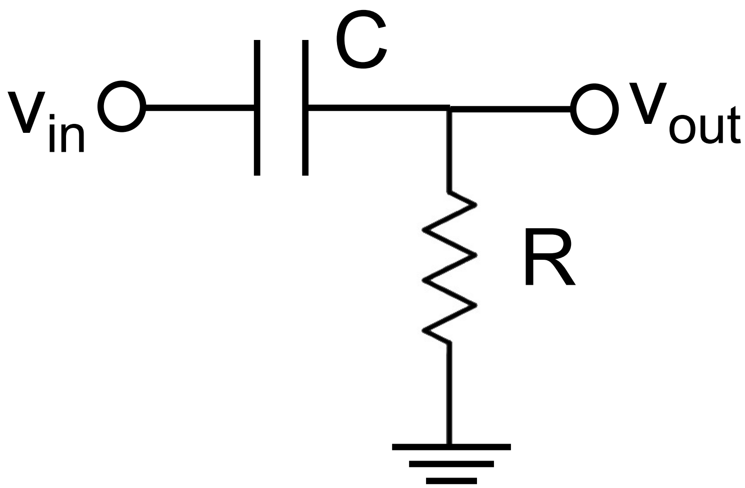
R is simply a number with the units of Ohms.

- What about capacitors and inductors? If V and i are sine waves,

$$Z_C = \frac{V}{i} = \frac{V}{CdV/dt} = \frac{V_0 \sin(2\pi Ft)}{2\pi FCV_0 \cos(2\pi Ft)} \qquad Z_C = \frac{V}{i} = \frac{1}{j * 2\pi FC}$$

$$Z_L = \frac{V}{i} = \frac{Ldi/dt}{i} = \frac{2\pi FLi_o \cos(2\pi Ft)}{i_o \sin(2\pi Ft)} \qquad Z_L = \frac{V}{i} = j * 2\pi FL$$

Analyzing RC, RL Circuits Using Impedance



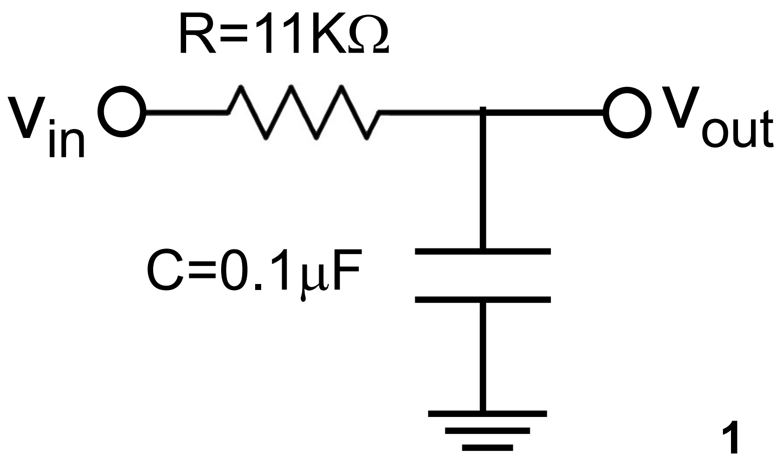
$$Z_C = \frac{1}{j * 2\pi FC} \qquad Z_R = R$$

- If the circuit had two resistors then we would know how to analyze it

$$\frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2} \text{ or more generally, } \frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$$

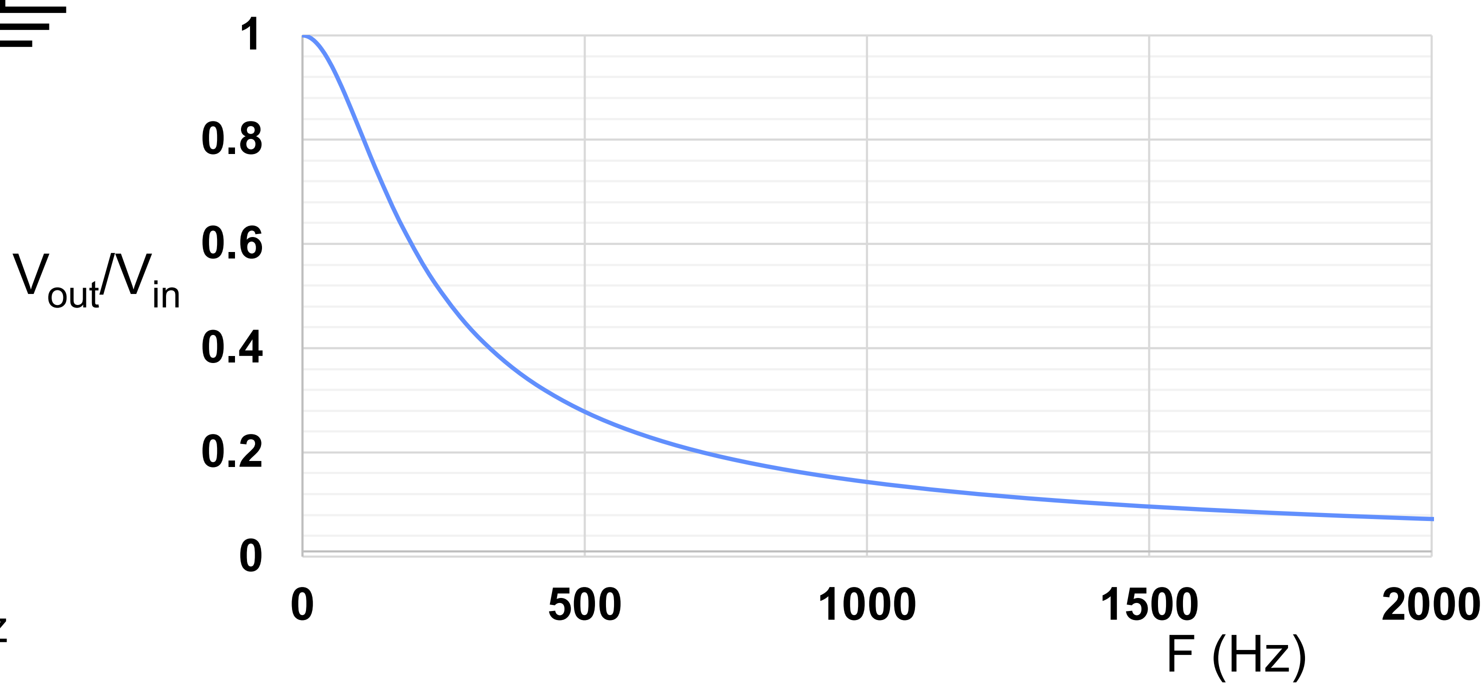
- So we can still use the voltage divider approach with impedances

RC Low Pass Filters



$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{j*2\pi FC}}{R+\frac{1}{j*2\pi FC}} = \frac{1}{1+j*2\pi FRC} = \frac{1}{1+jF/F_c}$$

$F_C = 1/[2\pi RC]$



RC = 1.1 ms
$F_c = 1/[2\pi RC]$ =145 Hz

Bode Plots

- Plot of log (gain = Vout/Vin) vs. log of frequency

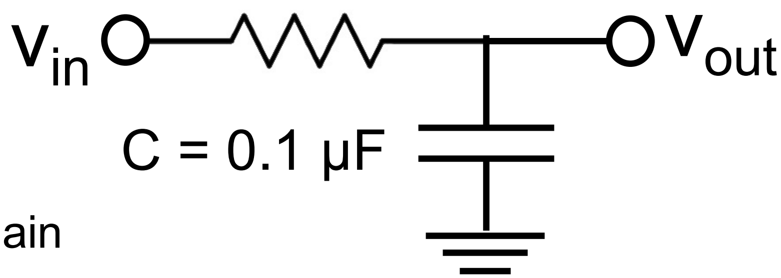
- Why plot log?
  - Log converts multiplication to addition

  - Makes the plots very simple
    - If gain proportional to F, the slope of the line is 1
    - If gain is constant, the slope of the line is 0
    - If gain is proportional to 1/F, the slope of the line is -1
    - If gain is proportional to $1/F^2$, the slope of the line is -2

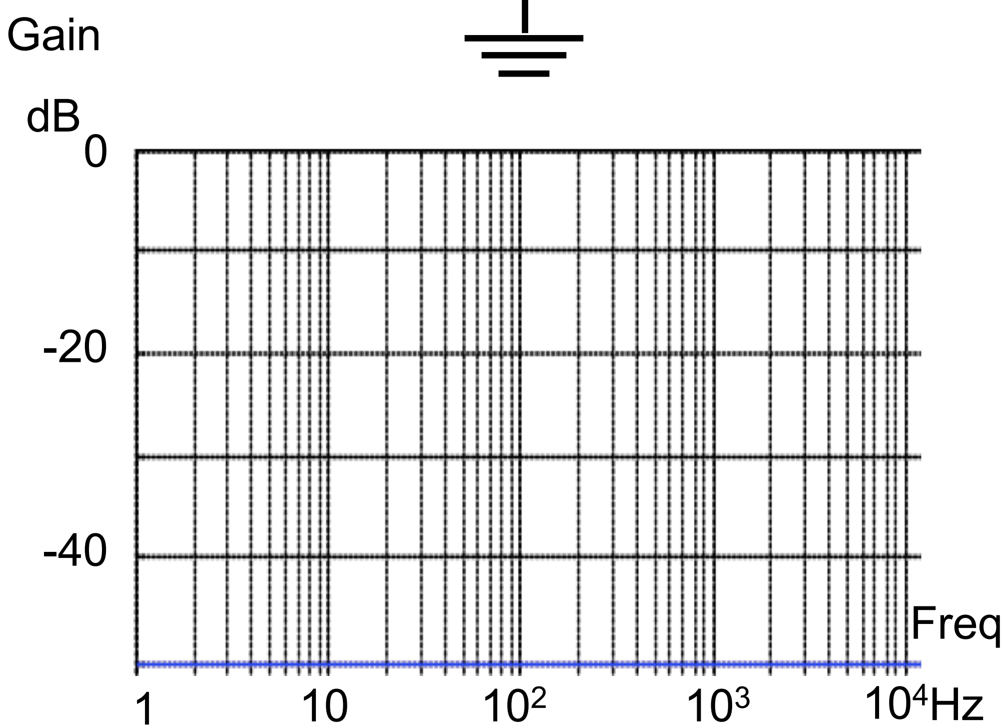
Plotting dB vs. Frequency

- Consider the simple low pass filter we looked at earlier

R = 11kΩ



$$\text{Gain} = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j * 2\pi FRC} = \frac{1}{1 + jF / F_c}$$



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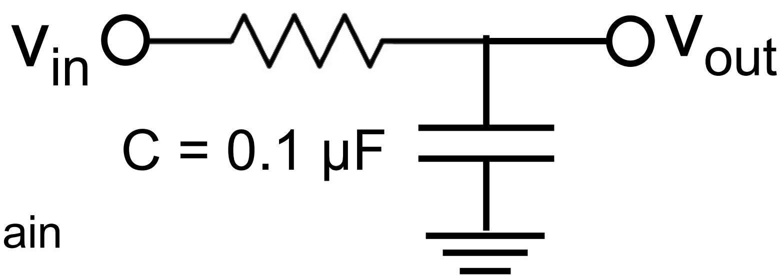


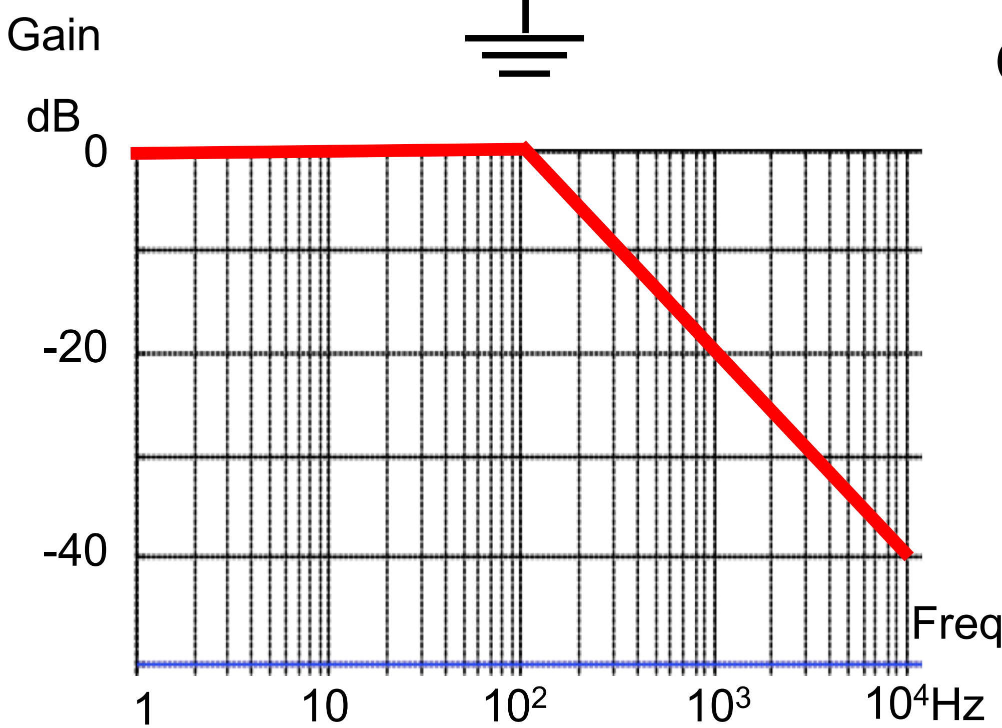
$$\text{Gain} = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j * 2\pi FRC} = \frac{1}{1 + jF/F_c}$$

$$\text{Gain}_{dB} = 20\log_{10}\left(\frac{1}{1 + jF/F_c}\right)$$

$$= 20\log_{10}(1) - 20\log_{10}(1 + jF/F_c)$$

$$\cong 0 - 20\log_{10}(F/F_c)$$

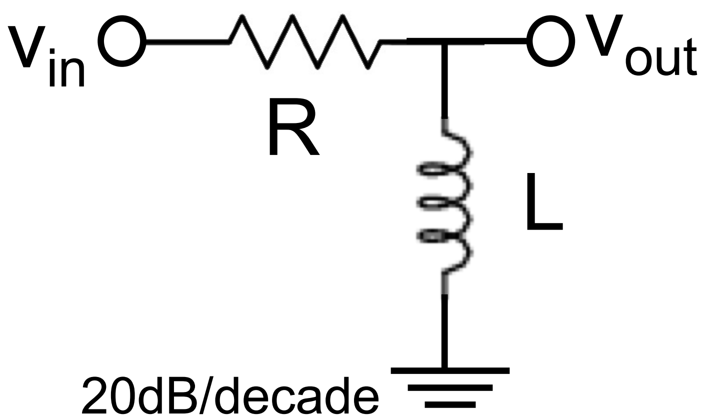
(assuming F is large and neglecting the phase)



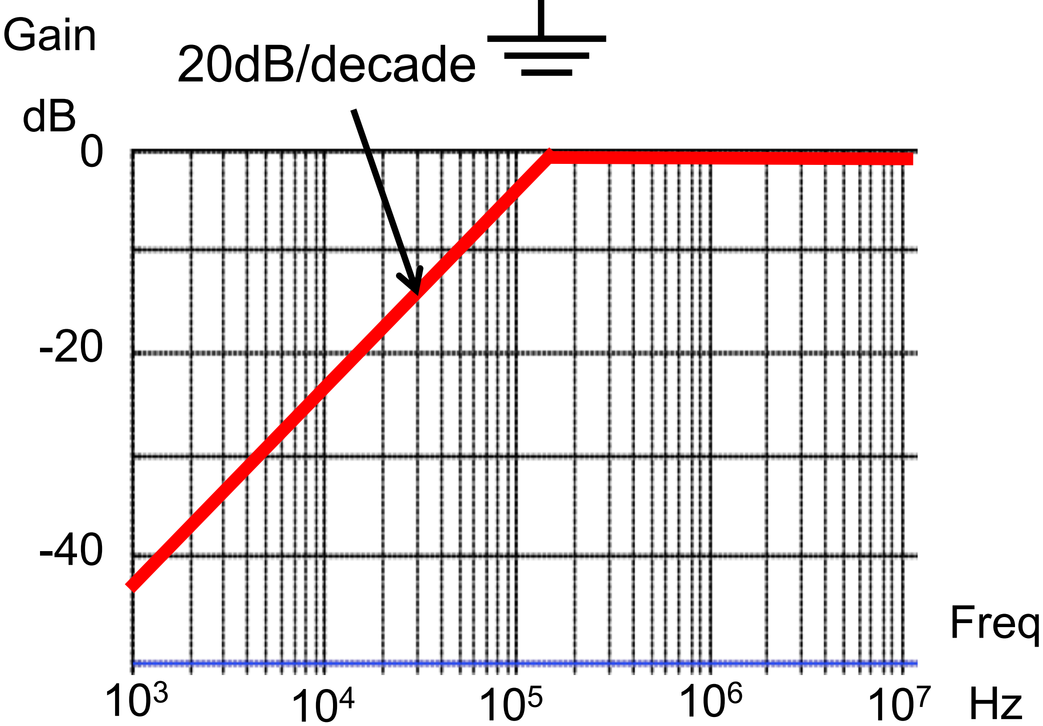
RC = 1.1 ms

F_c = 1/[2πRC] =145 Hz

RL High Pass



$$\frac{V_{out}}{V_{in}} = \frac{j * 2\pi FL}{R + j * 2\pi FL} = \frac{j * 2\pi F \frac{L}{R}}{1 + j * 2\pi F \frac{L}{R}}$$

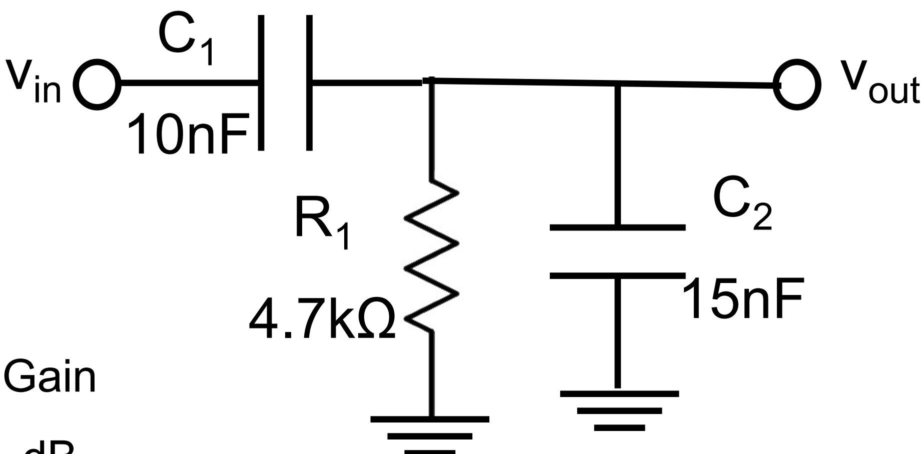


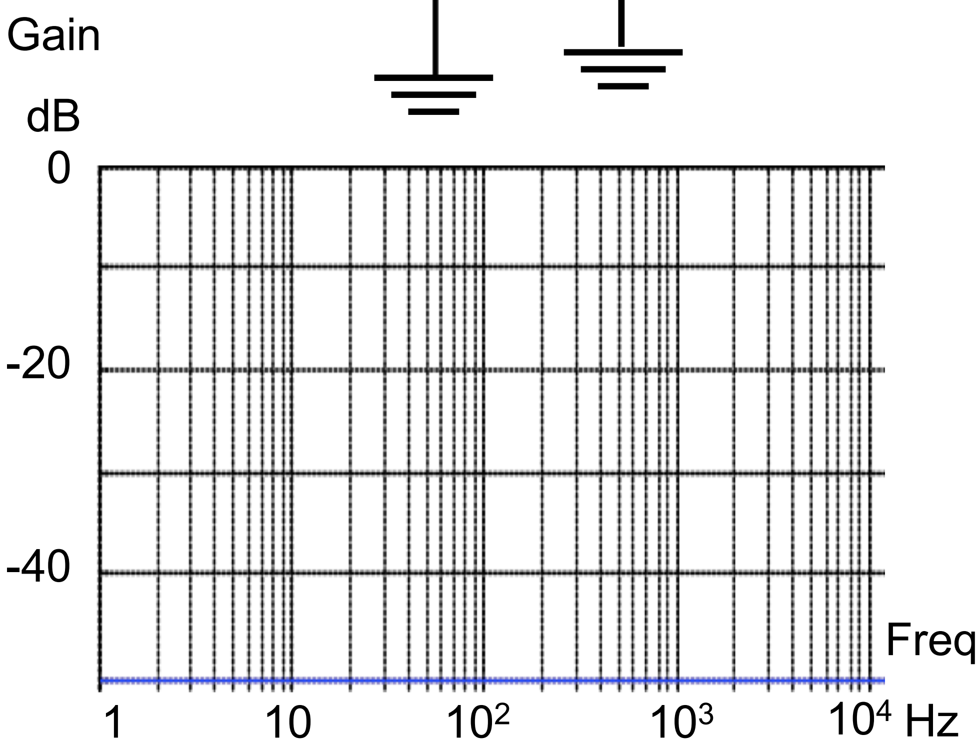
$$F_C = \frac{1}{2\pi \frac{L}{R}}$$

If R = 1 kΩ and L = 1 mH,

then $F_c \cong 159$ kHz

Example: Filter and Bode Plot





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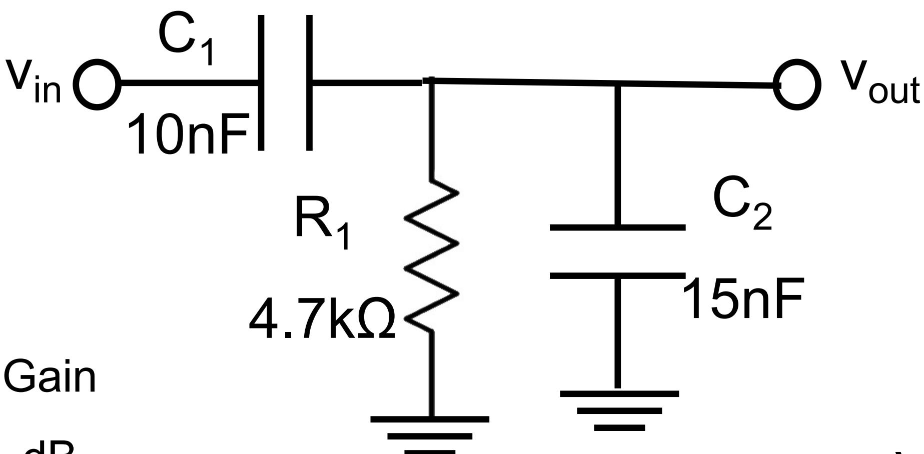
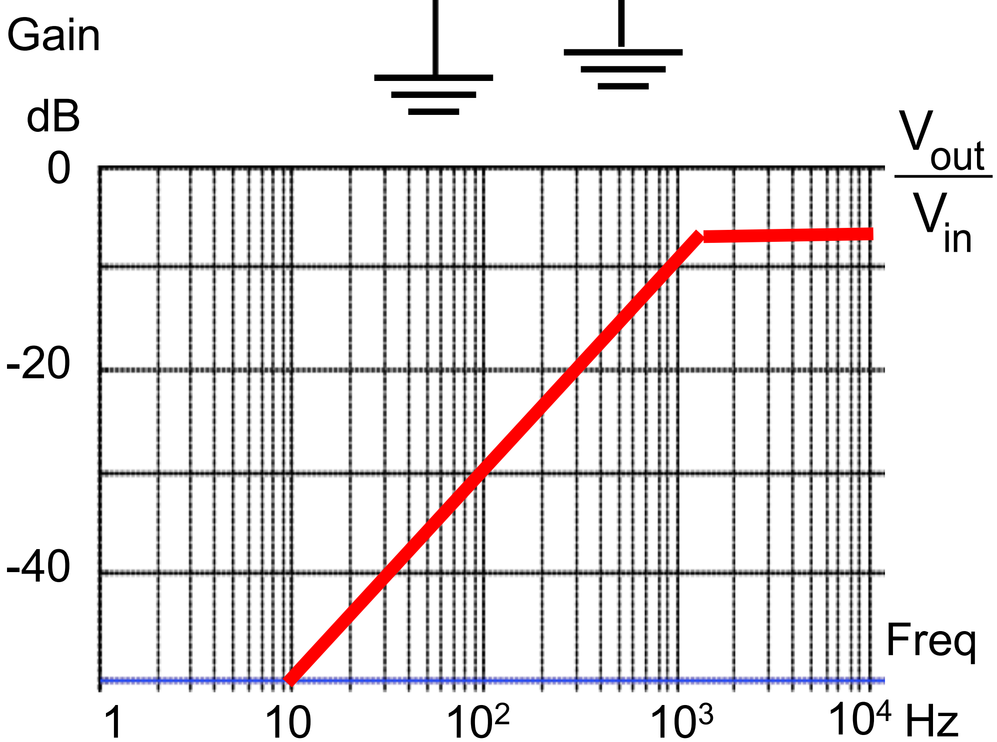




$$\frac{V_{out}}{V_{in}} = \frac{Z_{R1} \| Z_{C2}}{Z_{C1} + Z_{R1} \| Z_{C2}}$$

$$Z_{R1} \| Z_{C2} = \frac{Z_{R1} * Z_{C2}}{Z_{R1} + Z_{C2}} = \frac{R_1}{1 + 2\pi F R_1 C_2}$$

$$\frac{V_{out}}{V_{in}} = \frac{\dfrac{R_1}{1 + 2\pi F R_1 C_2}}{\dfrac{1}{2\pi F C_1} + \dfrac{R_1}{1 + 2\pi F R_1 C_2}} = \frac{2\pi F R_1 C_1}{1 + 2\pi F R_1 (C_1 + C_2)}$$

$$F_c = \frac{1}{2\pi R_1 (C_1 + C_2)} = 1.35\text{kHz}$$

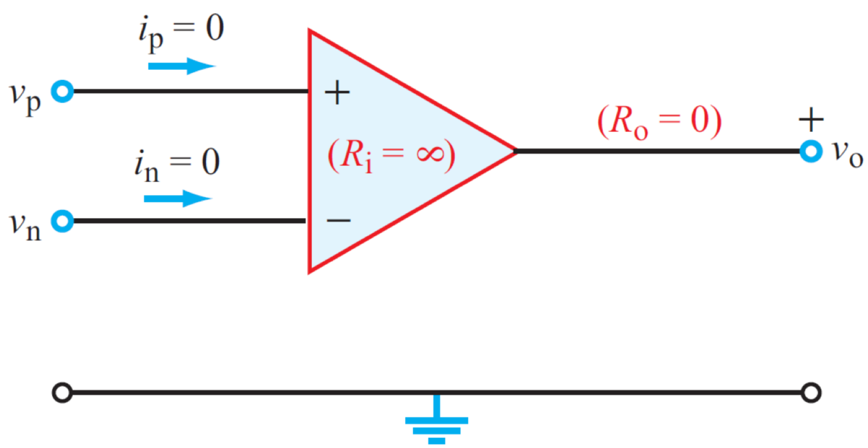
$$\text{Gain}_{dB} @ \text{HighF} = 20\log\left(\frac{C_1}{C_1 + C_2}\right) = -7.96\text{dB}$$

Op-Amps

- Are very high gain amplifiers
  - Have more gain then we need (1M)
  - Use feedback to get the gain we want

- Two ways to figure out output voltage
  - Write equations for V+ and V-
    - As a function of input and output voltages
    - Solve the equations
      - Vout = A(V+ - V-)
  - Use ideal Op-Amp equations
    - Assume A = infinite
    - Find the output voltage that makes V+ = V-

Ideal Op Amps



Ideal Op Amp

- Current constraint $i_p = i_n = 0$
- Voltage constraint $v_p = v_n$
- $A = \infty$ $R_i = \infty$ $R_o = 0$

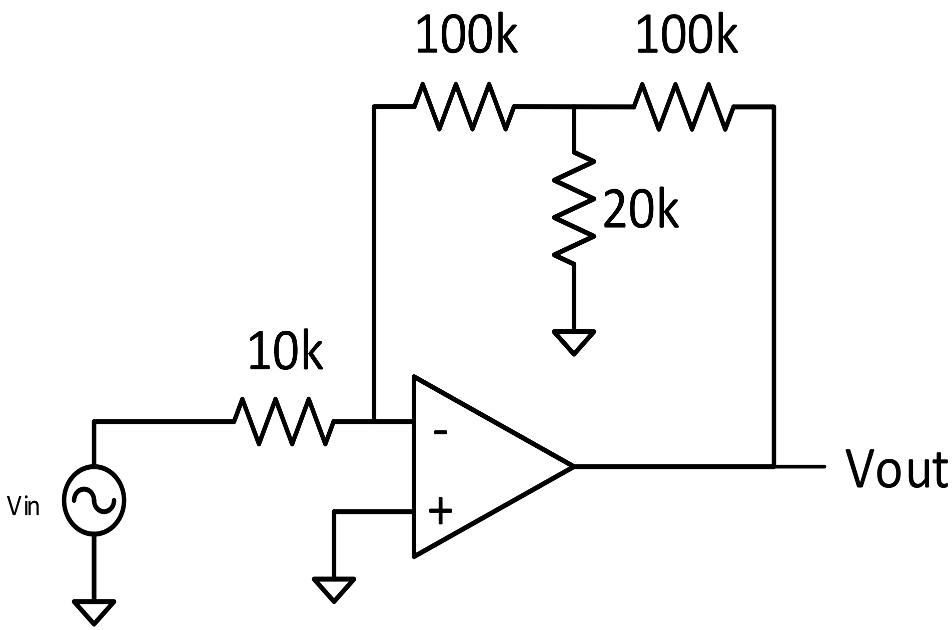
The Two Golden Rules for circuits with ideal op-amps*

1. $v_p = v_n$ (Ideal op-amp model). — **No voltage difference between op-amp input terminals**

2. $i_p = i_n = 0$ (Ideal op-amp model). — **No current into op-amp inputs**

*** when used in *negative feedback* amplifiers***

Example - Find The Gain
(see last Friday's Lecture Notes)

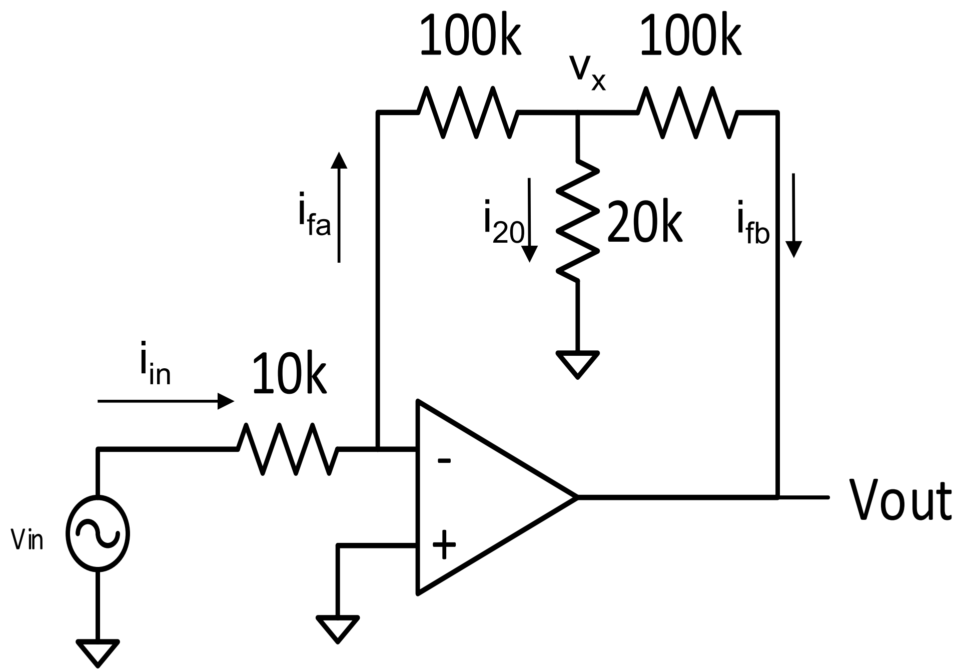


Change to divider at + input and explain RC time constant with DC gain of 10.

Current amp or trans-R?

Former was on Sp2017 final

Example - Find The Gain



$v_n = v_p = 0 \rightarrow\ i_{fa} = (0 - v_x)/100k$

$i_n = i_p = 0 \rightarrow\ i_{in} = i_{fa} = (v_{in} - 0)/10k$

KCL at v_x node: $i_{fa} = i_{20} + i_{fb}$

$-v_x/100k = v_x/20k + (v_x - v_{out})/100k$

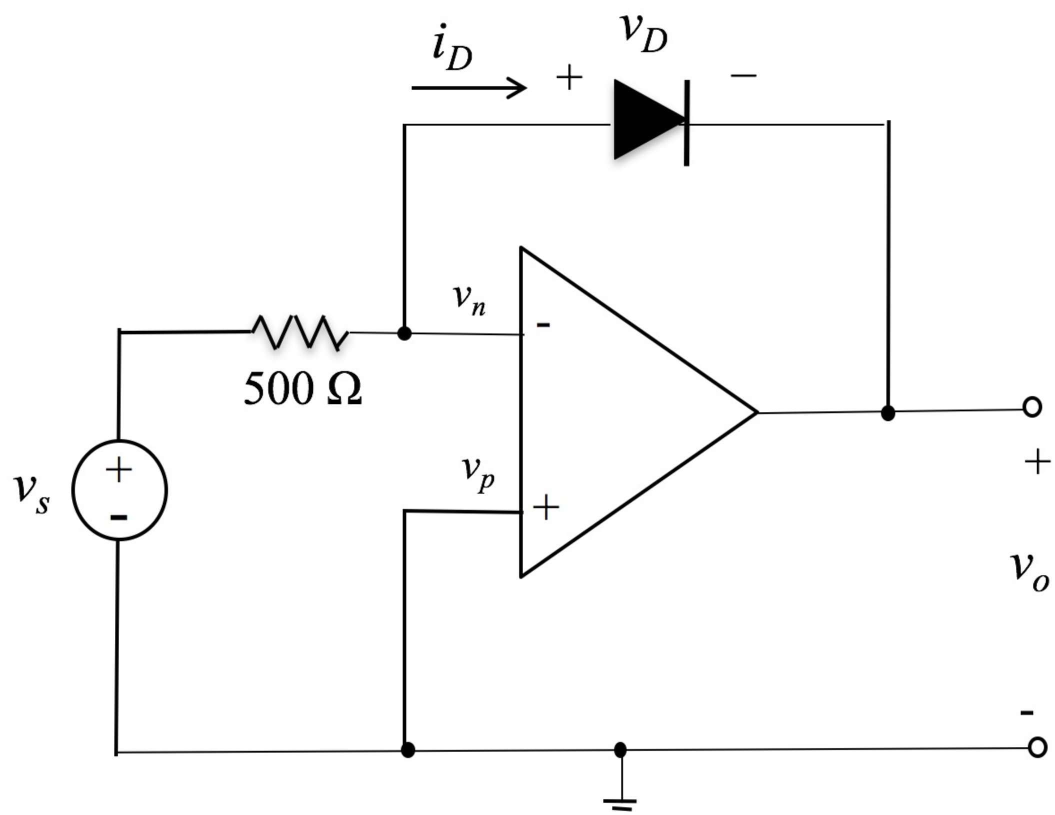
$v_{out} = 100k[1/100k + 1/20k + 1/100k]v_x$

$= 7v_x$

$v_{in}/10k = -v_x/100k \rightarrow\ v_x = -10v_{in}$

$v_{out} = 7(-10)v_{in} = -70v_{in}$

Example: Diode in the Feedback Loop

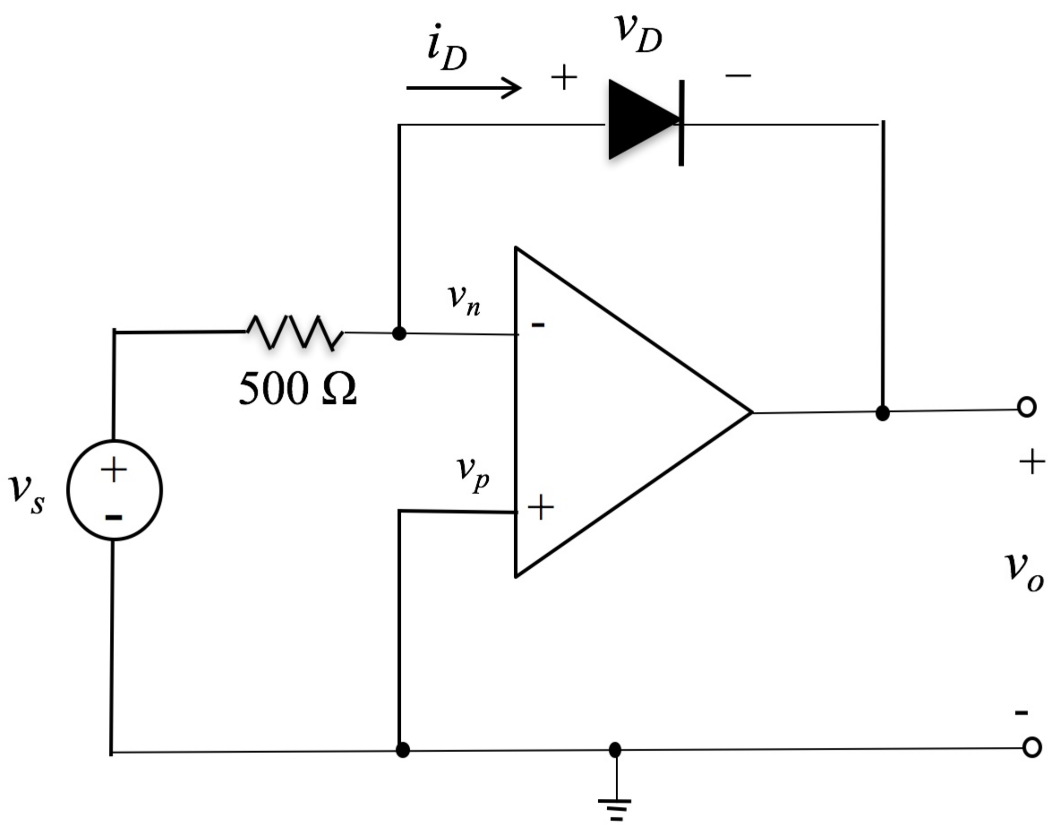


Given – the diode current is:

i_D = (100 fA)$e^{-v_D/(0.025\text{V})}$

Find v_o as a function of v_s

Example: Diode in the Feedback Loop



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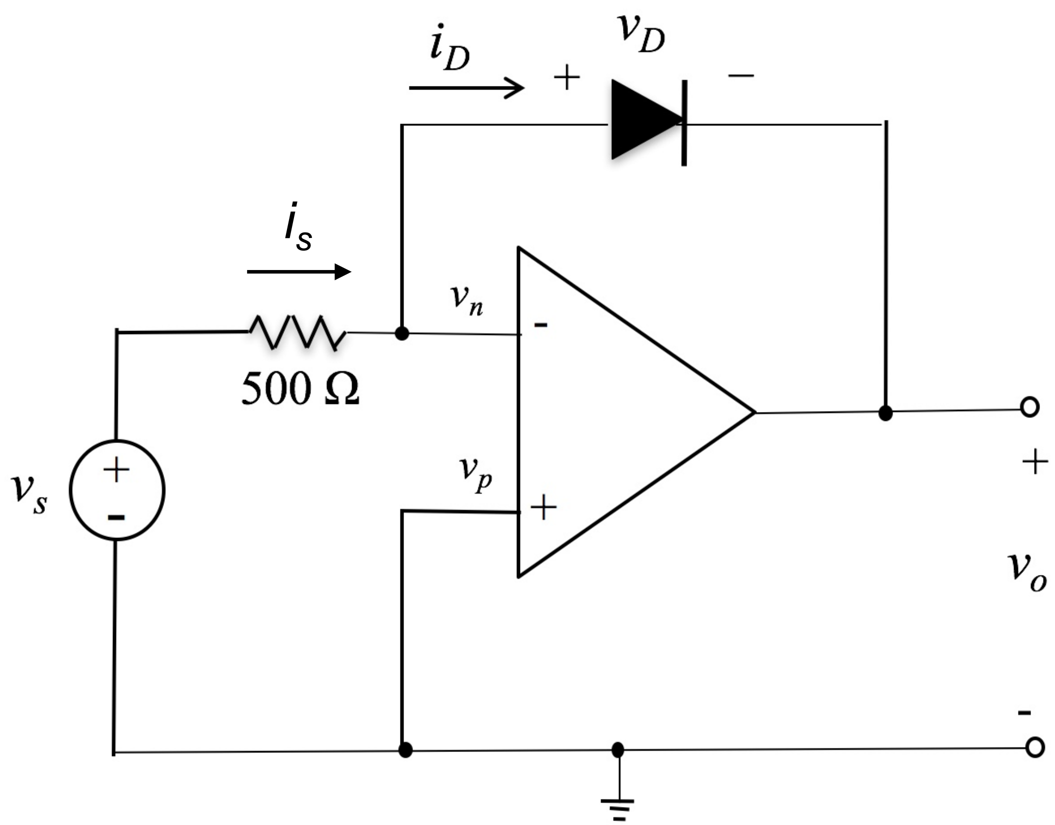


$i_D = (100\ \text{fA})e^{-v_D/(0.026\text{V})}$

$v_n = v_p = 0$

$i_s = v_s / R_s$

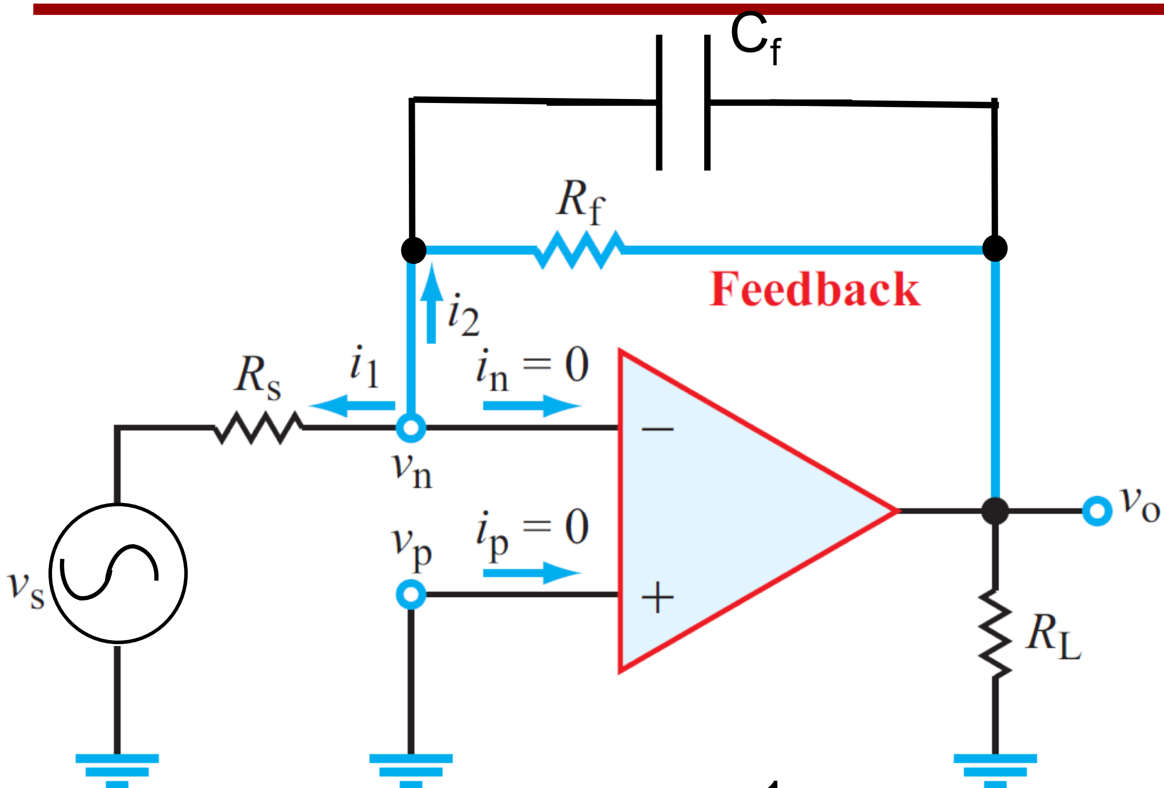
$i_n = 0 \quad \rightarrow \quad i_s = i_D$

$v_s / R_s = (100\ \text{fA})e^{-v_D/(0.026\text{V})}$

$v_D = (26\ \text{mV}) \ln[\, v_s / (50\text{pV})]$

$v_o = -\, v_D = -\,(60\ \text{mV}) \log[\, v_s / (50\text{pV})]$

Adding Capacitors



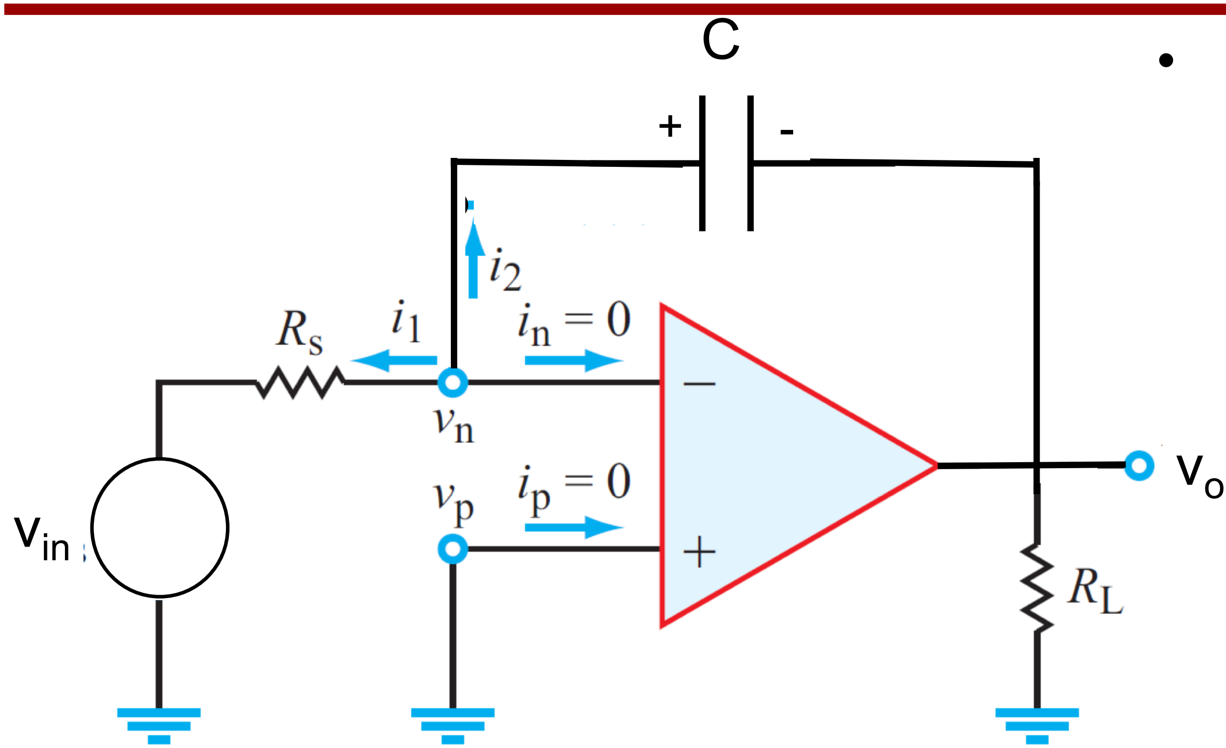
- Suppose we add a capacitor in the feedback
- We can treat this exactly as we did the earlier circuits by using impedances.
- Our earlier analysis showed

$$v_o = -v_s \frac{R_f}{R_s}$$

$$Z_s = R_s \qquad Z_f = \frac{1}{\frac{1}{R_f} + j * 2\pi F C_f}$$

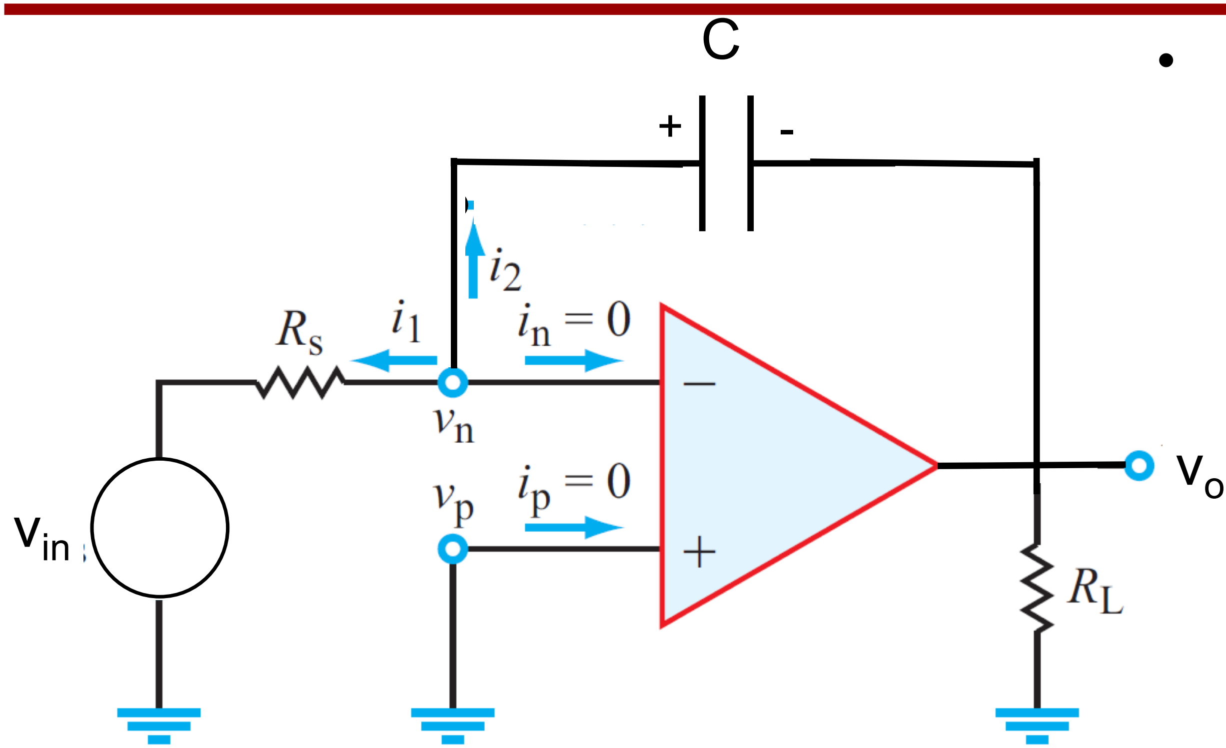
$$\therefore v_o = -v_s \frac{Z_f}{Z_s} = -\frac{\dfrac{1}{\frac{1}{R_f} + j * 2\pi F C_f}}{R_s} = -v_s \frac{R_f}{R_s}\left(\frac{1}{1 + j * 2\pi F R_f C_f}\right)$$

Example: Time Domain Analysis



- What is the transfer function in the time domain?

Example: Time Domain Analysis



- What is the transfer function in the time domain?

$$i_1 + i_2 = 0$$

$$\therefore -\frac{v_{in}}{R_s} + C\frac{dV_c}{dt} = 0$$

$$\therefore -\frac{v_{in}}{R_s} = C\frac{dV_o}{dt}$$

This is an op amp integrator!

$$\therefore \frac{dV_o}{dt} = -\frac{v_{in}}{R_s C} \quad \text{or} \quad v_o = -\frac{1}{R_s C}\int_0^t v_{in}\, dt$$

Finite State Machines – Useless Box

- **SwitchOn** is either true or false; **Limit** is either true of false;
- Can represent **Motor** using two Boolean variables
  - **Forward** is either true or false; **Reverse** is either true or false
    - It is an error is both are true

- What is the Boolean expression for this FSM (Finite State Machine)

  - Forward: on

  - Reverse !on && !limit