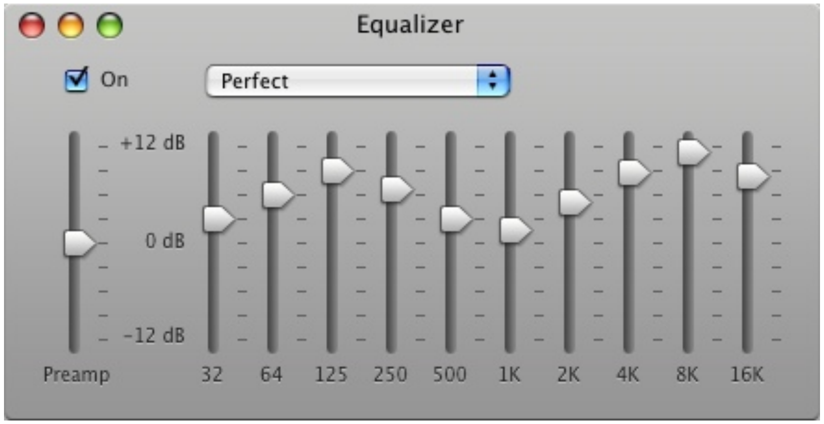
E40M

Inductors, RL Filters, Inductor Circuits & Applications

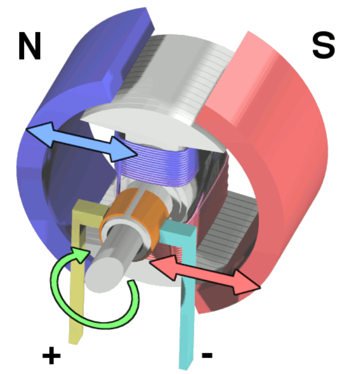
Reading

- Course Reader Chapter 9
- A&L: Chapter 9 – 9.1 to 9.3 (Inductors)
  Chapter 13 pages 732 – 741 (RL Circuits)
- Buck converter
  - http://en.wikipedia.org/wiki/Buck_converter

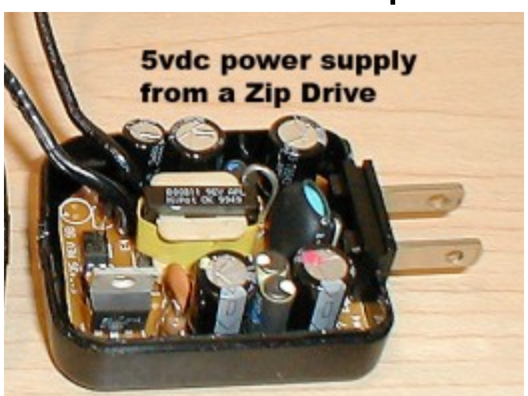
Why Are Inductors Useful/Important?



How do we design circuits that respond to certain frequencies?



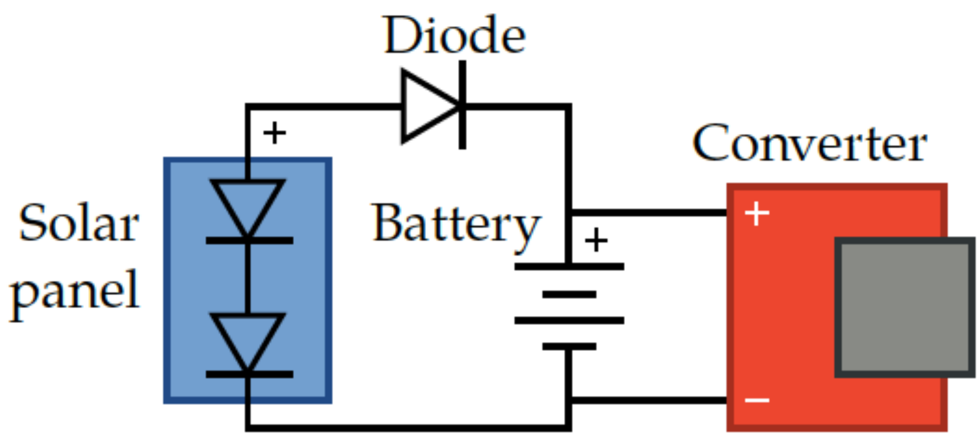
Motors convert electrical energy into mechanical energy. Generators do the reverse. Inductors are key to this.



http://hackedgadgets.com/2007/08/22/diy-wall-wart-usb-power-supply/

How do we convert 110V AC into voltages useful for electronic devices?

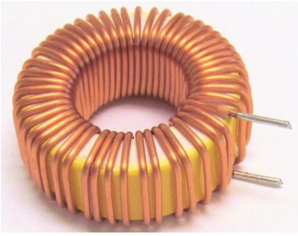
How Does This Work?

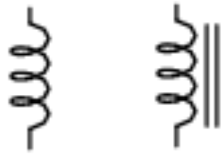


The voltage converter you used in the solar charger project is actually a “Boost” converter. It uses an inductor to boost the battery voltage to 5V.

Inductors

- An inductor is a new type of two terminal device
  - It is linear – double V and you will double i
  - Like a capacitor, it stores energy
    - Ideal inductors don't dissipate energy





- Defining equation: V = L di/dt L is inductance (in Henrys)
  - (see next page)

- If a sinewave current flows, $i = i_o \sin(2\pi F t)$ so $V = i_o 2\pi F L \cos(2\pi f t)$

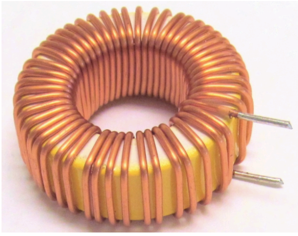
- Thus the impedance of an inductor is

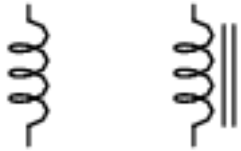
$$Z = \frac{V}{i} = \frac{2\pi F L i_o \cos(2\pi F t)}{i_o \sin(2\pi F t)} = j * 2\pi F L$$

So What Does It Do?

- It affects how fast a current can change
  - Voltage sets di/dt, and not i
  - Fast changes means lots of voltage! (We'll see examples later.)
- For very small Δt inductors look like current sources
  - They can supply very large voltages (+ or -)
  - And not change their current
- But for large Δt (after transients have died out, e.g. DC
  - Inductors look like short circuits (they are a wire)

Inductor Info, if You Know Physics E&M

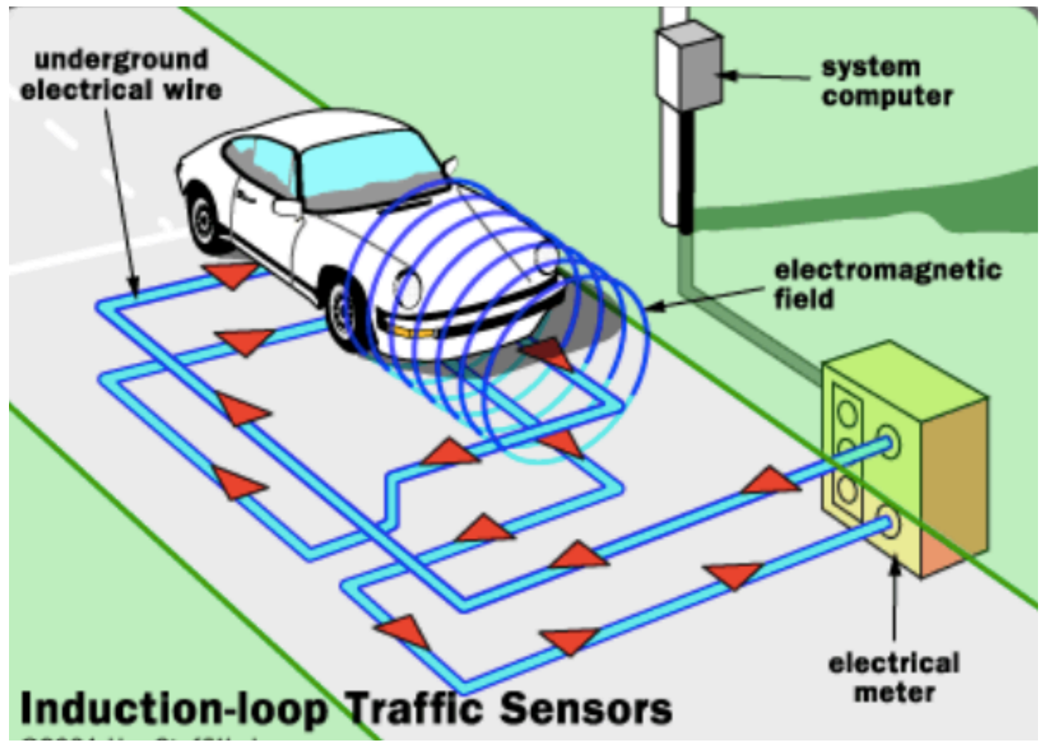




- Models the energy stored in magnetic fields
- An inductor is just a wire.
  - In the ideal case the wire has zero resistance
  - But current through a wire causes a magnetic field
  - $\phi = iL$ where ϕ is the magnetic flux created by the current
  - L is proportional to μ the magnetic permeability of the core material
- Changing magnetic flux induces a voltage
  - Faraday's Law: $V = \frac{d\phi}{dt}$ so that $V = \frac{d}{dt}\left(Li\right) = L\frac{di}{dt}$

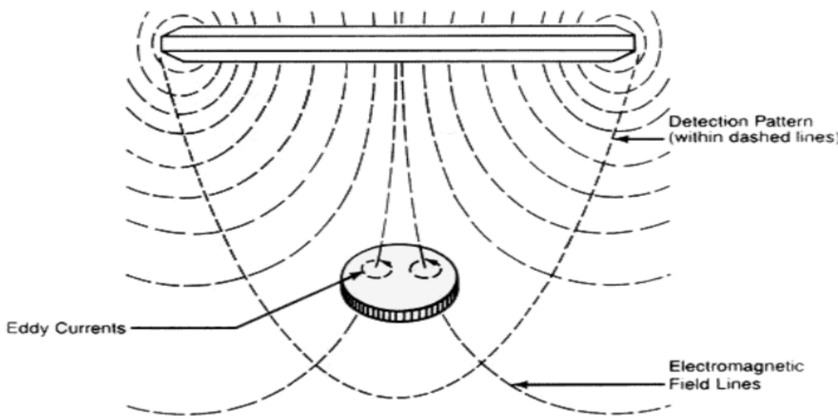
Interesting Application of Inductors

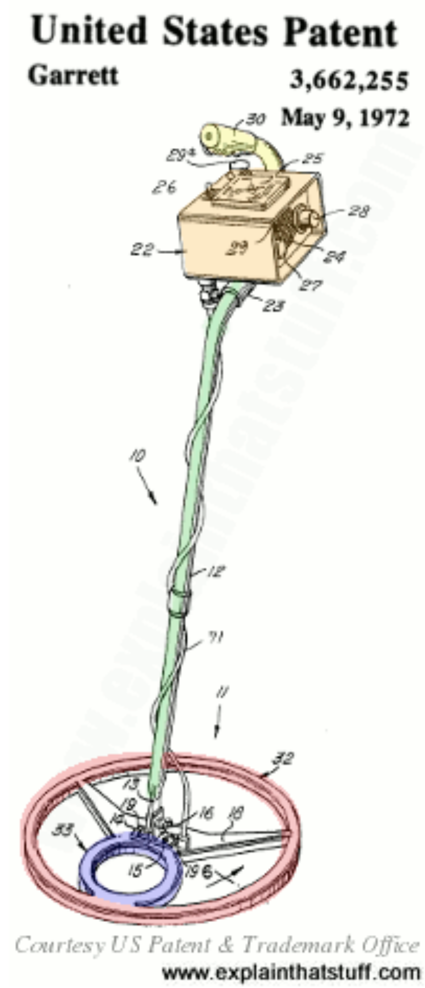
- Wire coil buried in road; constant current. $\phi = iL$
- Iron (steel) changes μ the magnetic permeability and hence changes L. System detects change in L, activates left turn light.



Another Interesting Application

- Metal detectors use a wire coil (inductor) as the “transmitter”.
- The EM field radiated by the inductor interacts with “buried” metals.
- Magnetic materials (iron) change the inductance. Conducting metals set up “eddy currents” which also produce an EM field.
- Usually a second “receiving” coil detects the EM field transmitted by the “buried” metal.





Energy Flow in Inductors

- Power flow into a device is always iV
  - Total energy that flows into the inductor

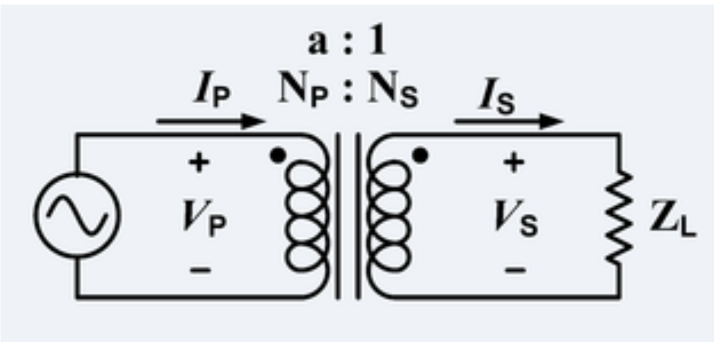
$$P = iV = iL\frac{di}{dt} \qquad \therefore \int_0^t P\,dt = \int_0^{t_{final}} Li\,di = \frac{1}{2}Li_{final}^2$$

  - Assumes current starts at 0A and ends at i_{final}

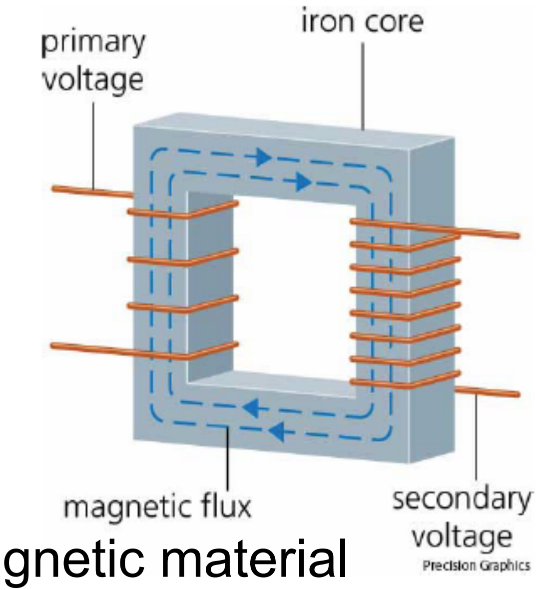
- The energy stored in the inductor is $\frac{1}{2}Li^2$
- Recall in a capacitor, energy stored is $\frac{1}{2}CV^2$
- In both cases, the energy is stored and later returned.

Transformers

- The energy is stored in the magnetic field
  - And the field is generated by the current



$$\phi = iL \text{ and } V = \frac{d\phi}{dt} = L\frac{di}{dt}$$

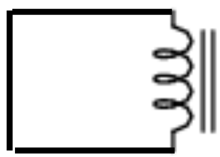


- If you have two coils of wire around the same magnetic material
  - Both wires will see the same magnetic field (ϕ)
  - You can transfer power between the two coils

- Drive one with a sinewave to create changing magnetic field
  - The other coil will develop a voltage across it
  - The voltage depends on the relative inductance

Ideal vs. Real Inductors

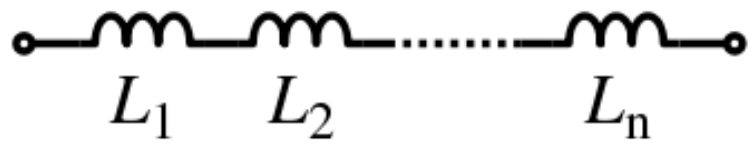
- Ideal inductors
  - Have no loss
  - Can store energy by letting the stored current circulate



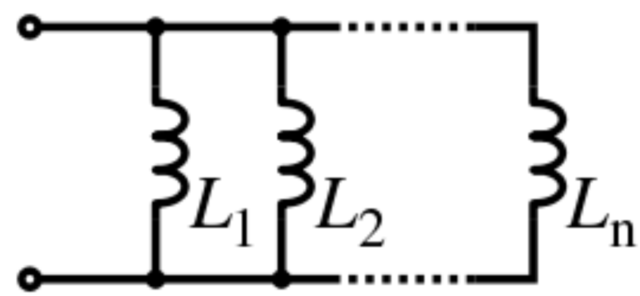
- Real Inductors
  - Are not that ideal (unless they are superconducting)
  - They have significant resistance from the wire
  - Can by modeled by an ideal inductor in series with a resistor
    - The resistance causes a voltage across the ideal inductor
    - Since the voltage across the ideal inductor is negative
      - The current decreases

Series and Parallel Inductors

- If you have inductors in series
  - Currents and di/dt are the same
  - $V = Ldi/dt = L_1 di/dt + L_2 di/dt + \ldots$
  - $L_{eqv} = L_1 + L_2 + \ldots.$



- If you have inductors in parallel
  - Voltages must be the same
    - $di_1/dt = V/L_1$
    - $di_2/dt = V/L_2$
  - Total current change
    - $di/dt = V\,(1/L_1 + 1/L_2 + \ldots) = V/L_{eqv}$
    - $1/L_{eqv} = 1/L_1 + 1/L_2 + \ldots$



- i.e. Inductors in series and parallel behave like resistors

Time Domain RL Circuits

- Suppose we have a simple RL circuit.

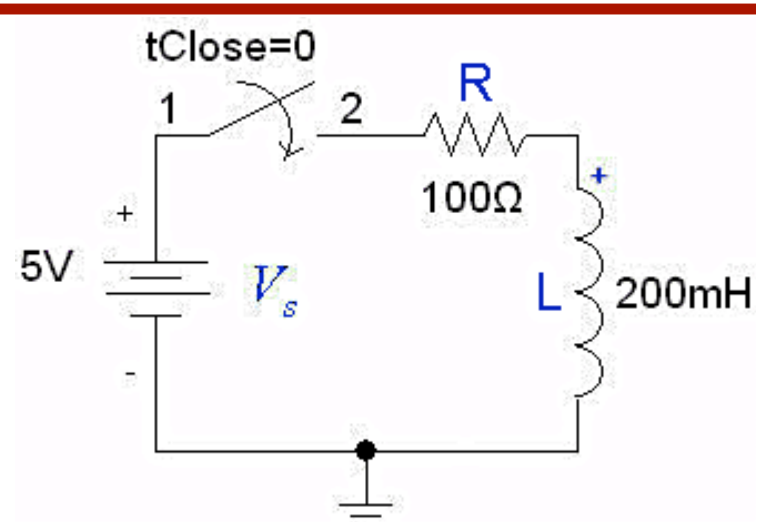
- In general, $V(t) = L\frac{di}{dt} + iR$

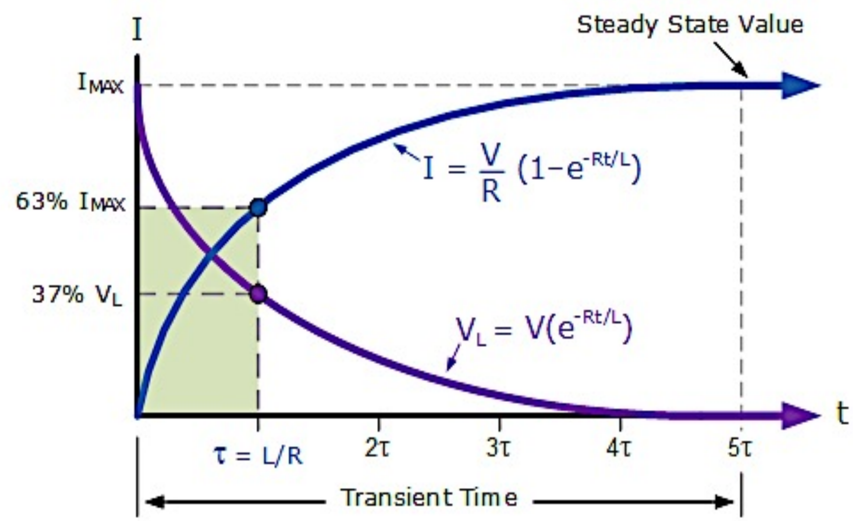
- The form of the solution is given by

$$L\frac{di}{dt} + iR = 0 \quad \text{so} \quad \frac{di}{i} = -\frac{R}{L}dt \quad \text{so} \quad i = Ae^{-t\frac{R}{L}}$$

- In inductor circuits the current through the inductor cannot change quickly (large di/dt would produce large V).But V can change quickly.

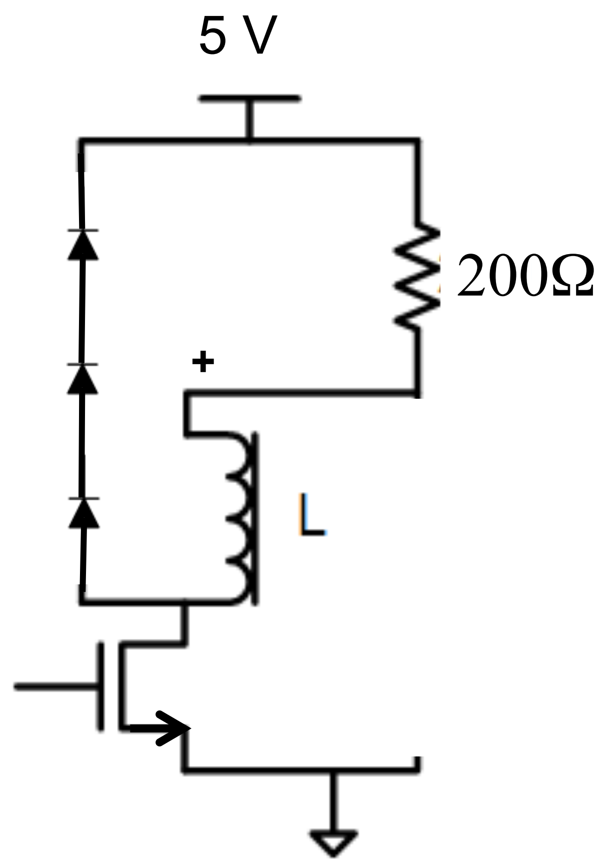
- These are opposite from capacitors.





An Interesting Inductor Circuit

- If the negative leads of the LEDs go to Vdd (5V)
  - How do the LEDs light up?
  - LEDs have 2V V_F, 20 mA I_{ON}



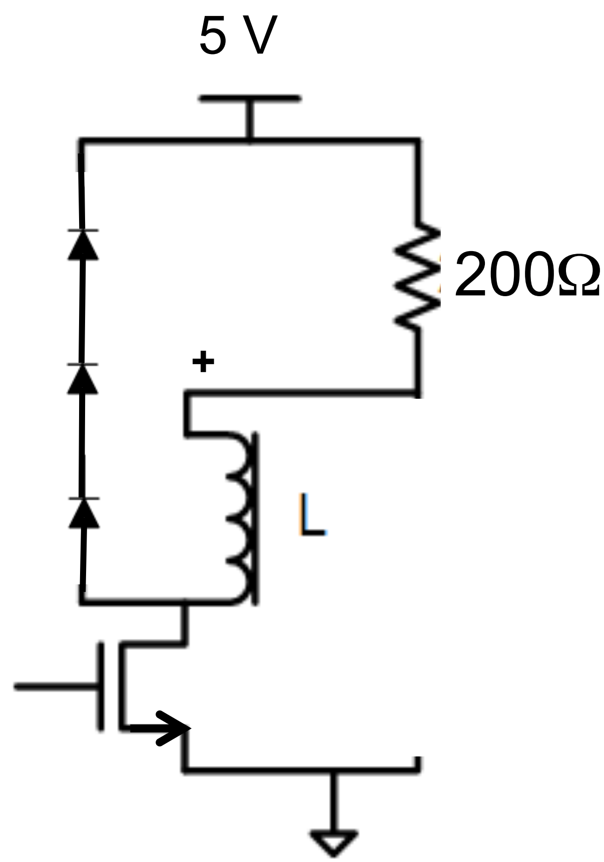
- What is the current through the inductor
  - Before the transistor turns off?
    - Assume transistor resistance is 1Ω
    - Assume inductor is around 30Ω
  - Right after the transistor turns off

- Where does this current go?

Working Through the Circuit

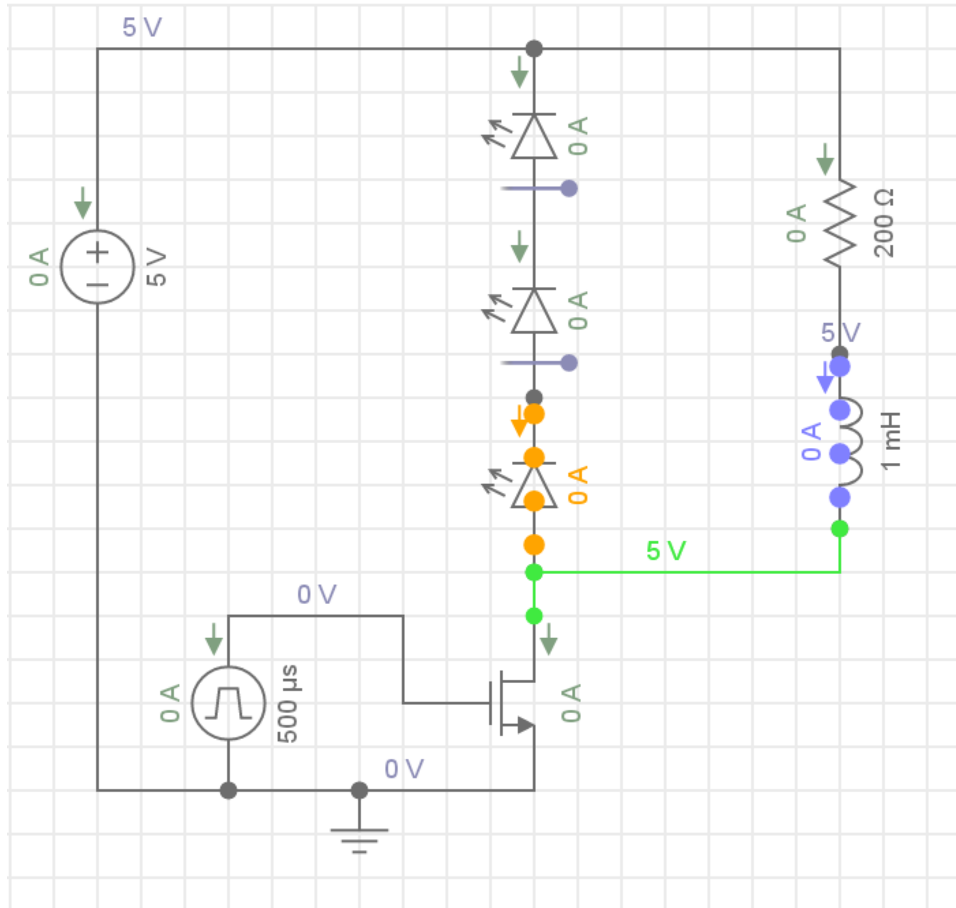
- What happens to current over time?

- So inductors can generate quite large voltages if we use them in circuits where they have to do so to keep i_L constant!



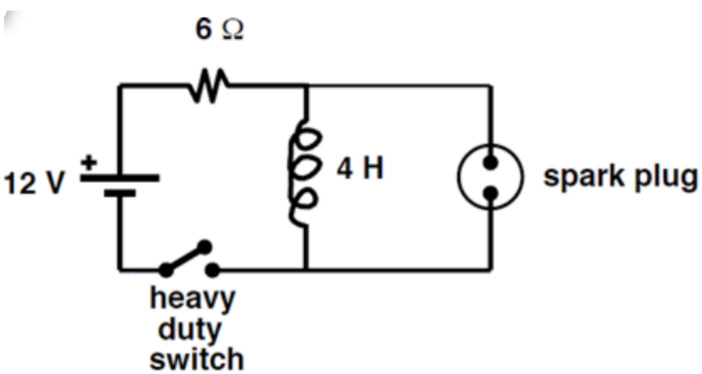
EveryCircuit Simulation

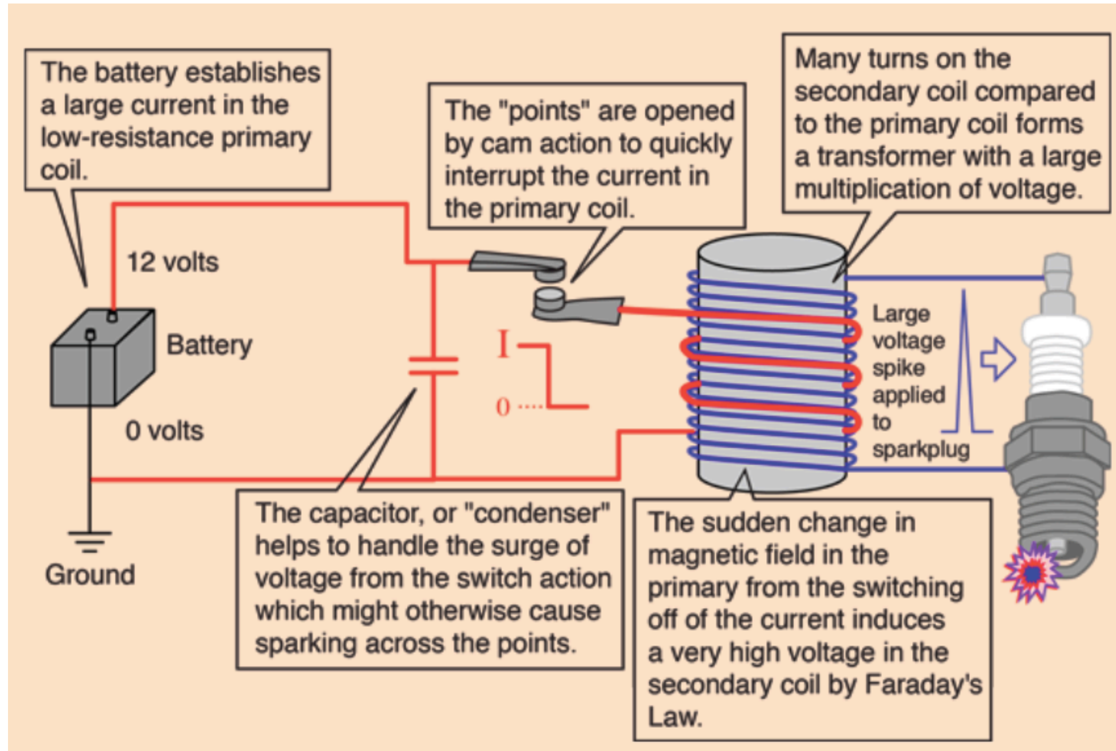
- In the simulation the NMOS transistor is turned ON and OFF.
- When the NMOS is ON, current flows through the 100 ohm resistor and through the inductor to ground.
- Where does i go when the NMOS device is OFF?



Practical Application

- Ignition circuit



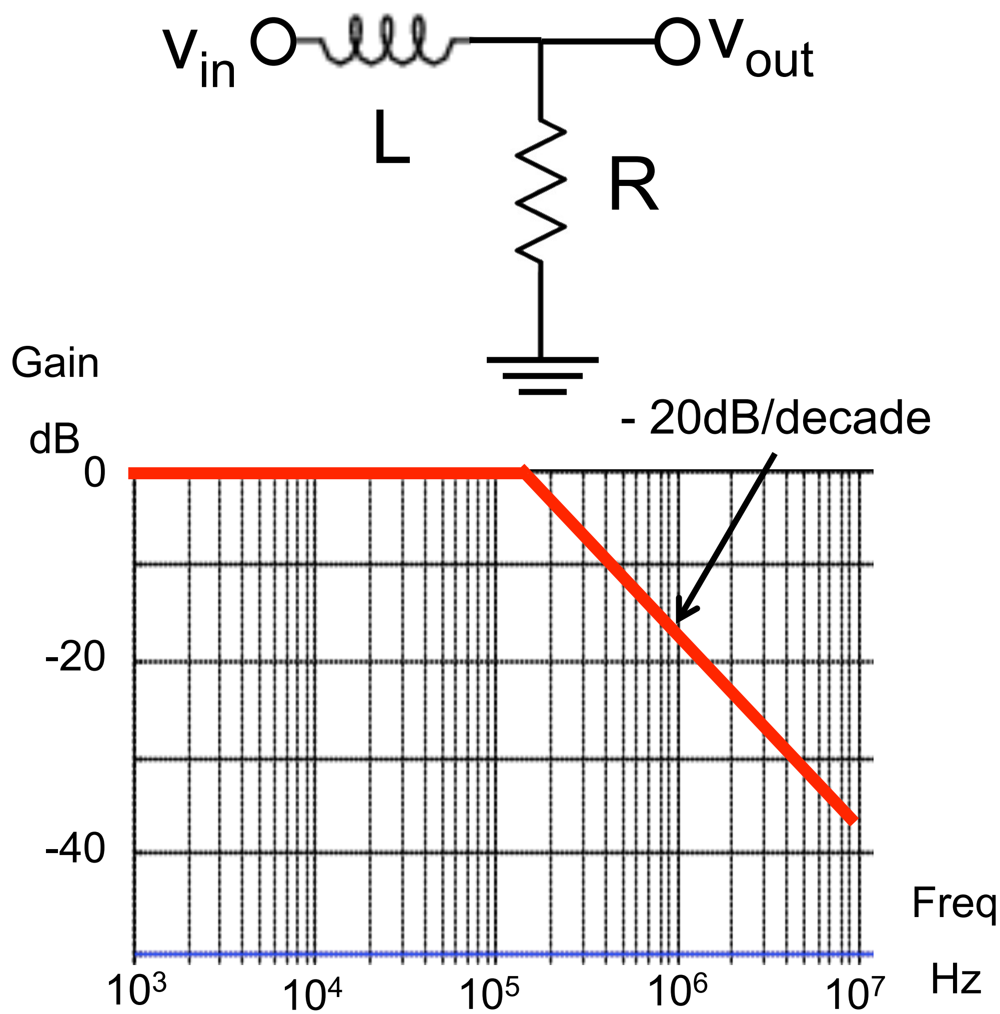


- 12V battery establishes current through inductor. Switch opens. Inductor responds with V = Ldi/dt. Transformer steps up V. 20,000 – 40,000 V fires spark plug igniting gas/fuel mixture.

https://www.youtube.com/watch?v=W94iksaQwUo

RL and RLC FILTERS

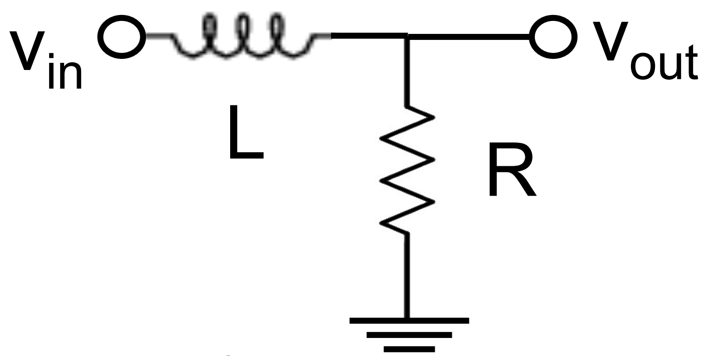
RL Filters – Low Pass

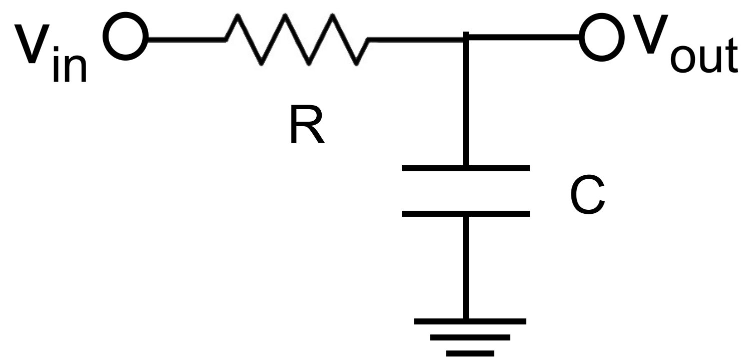


If R = 1 kΩ and L = 1 mH,

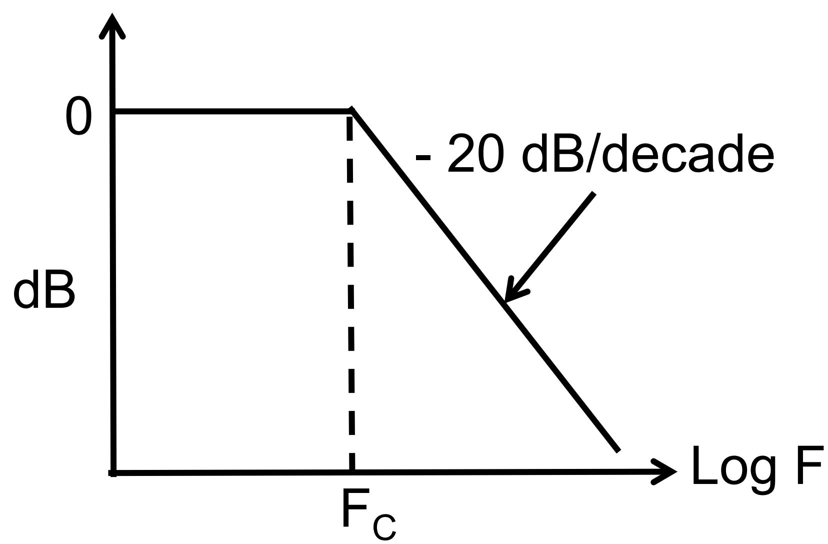
then $F_c \cong 159$ kHz

RL and RC Low Pass Filters



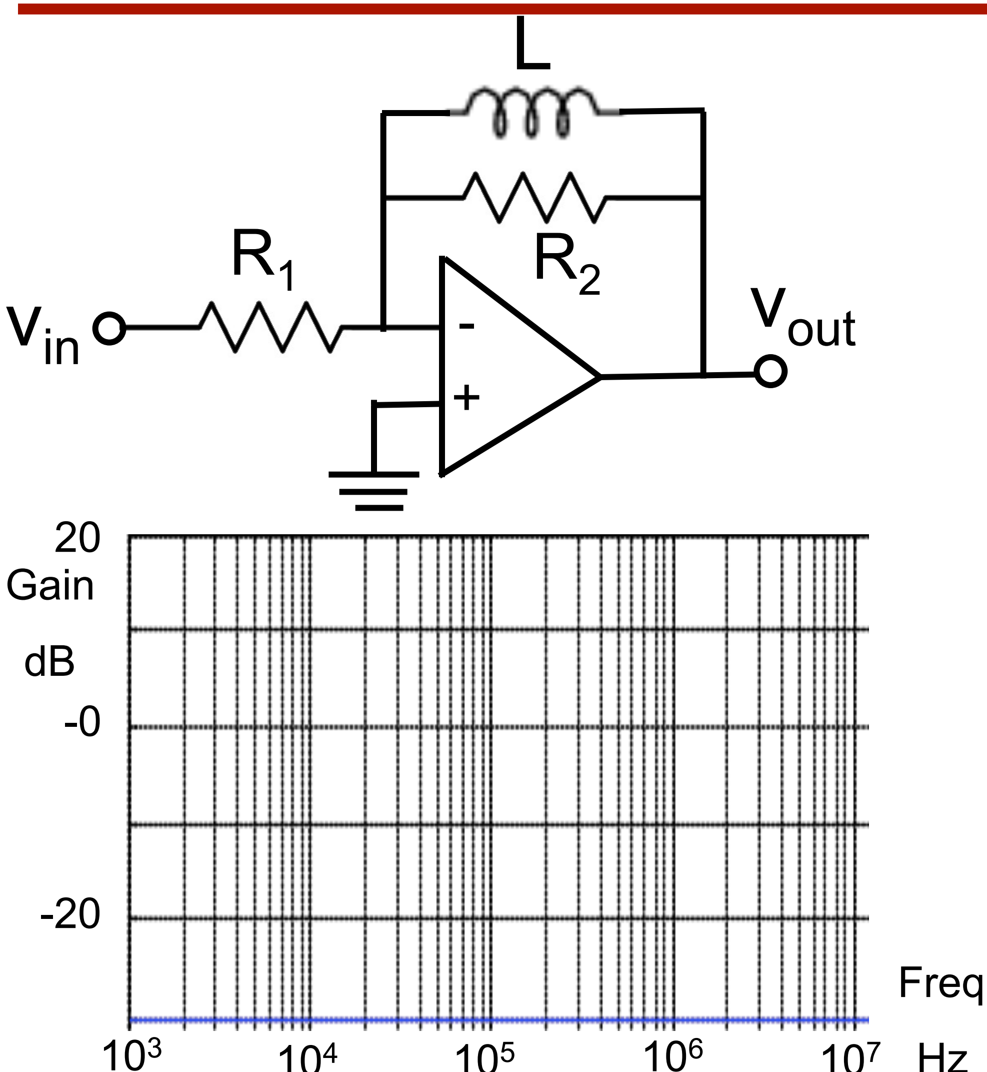


$$F_C = \frac{1}{2\pi \frac{L}{R}}$$

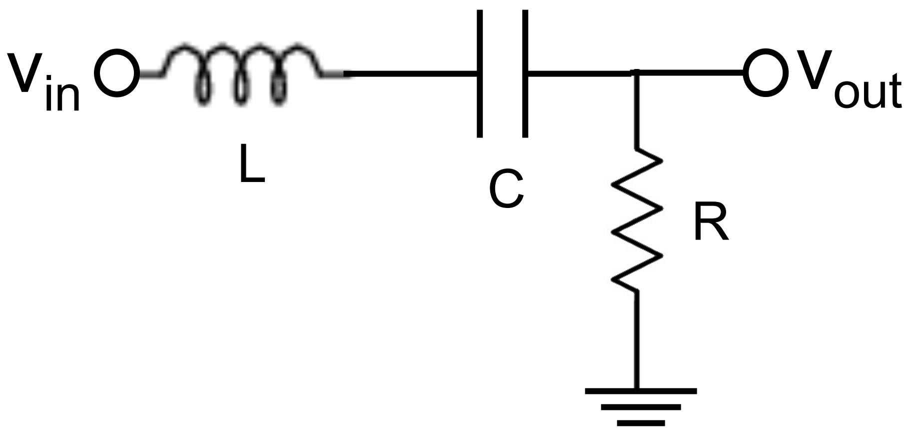


$$F_C = \frac{1}{2\pi RC}$$

RL High Pass

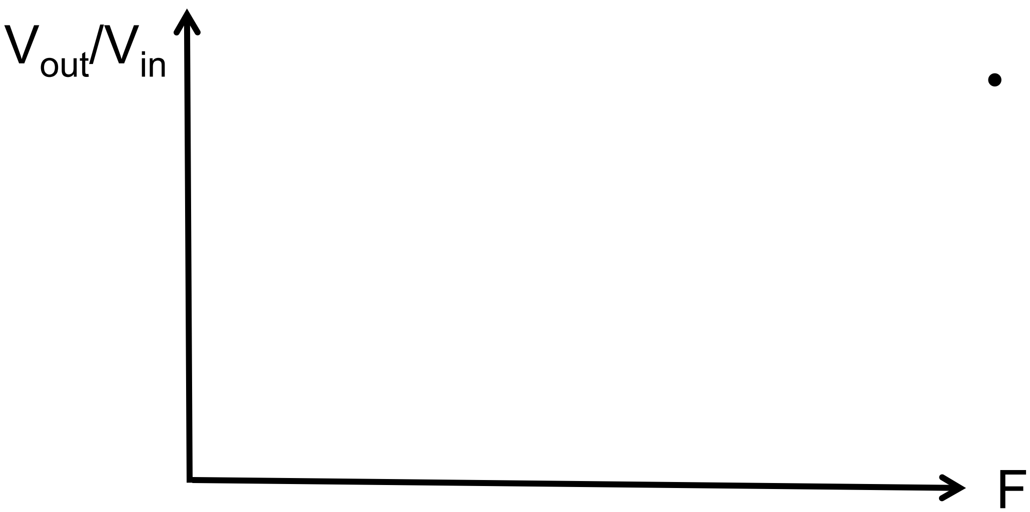


RLC Filter



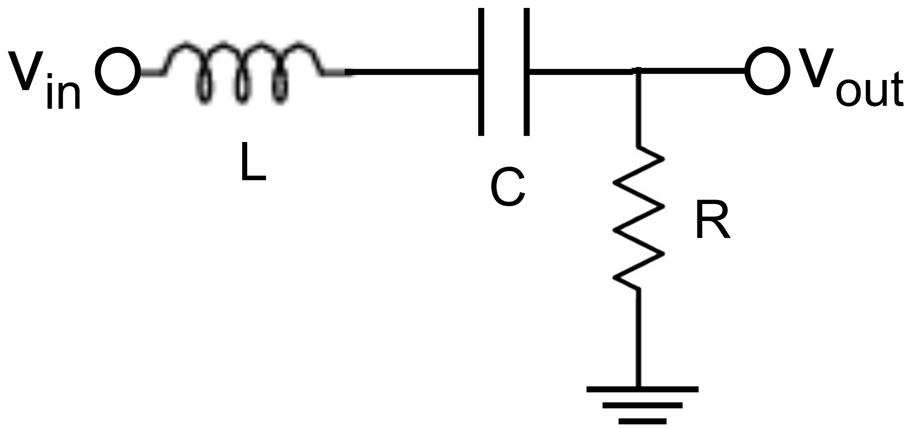
- Let's think about this before we do any math

- Very low frequencies:



- Very high frequencies:

RLC Filter



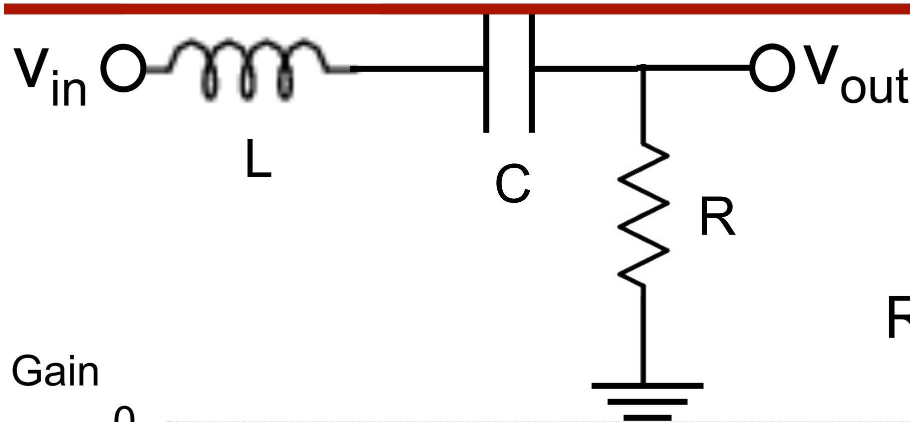
Let $R = 1\ k\Omega$, $L = 10\ mH$, and $C = 10\ \mu F$

We can estimate the corner frequencies as follows:

Neglecting the F^2 term, $F_c = \dfrac{1}{2\pi RC} = 15.9\ Hz$

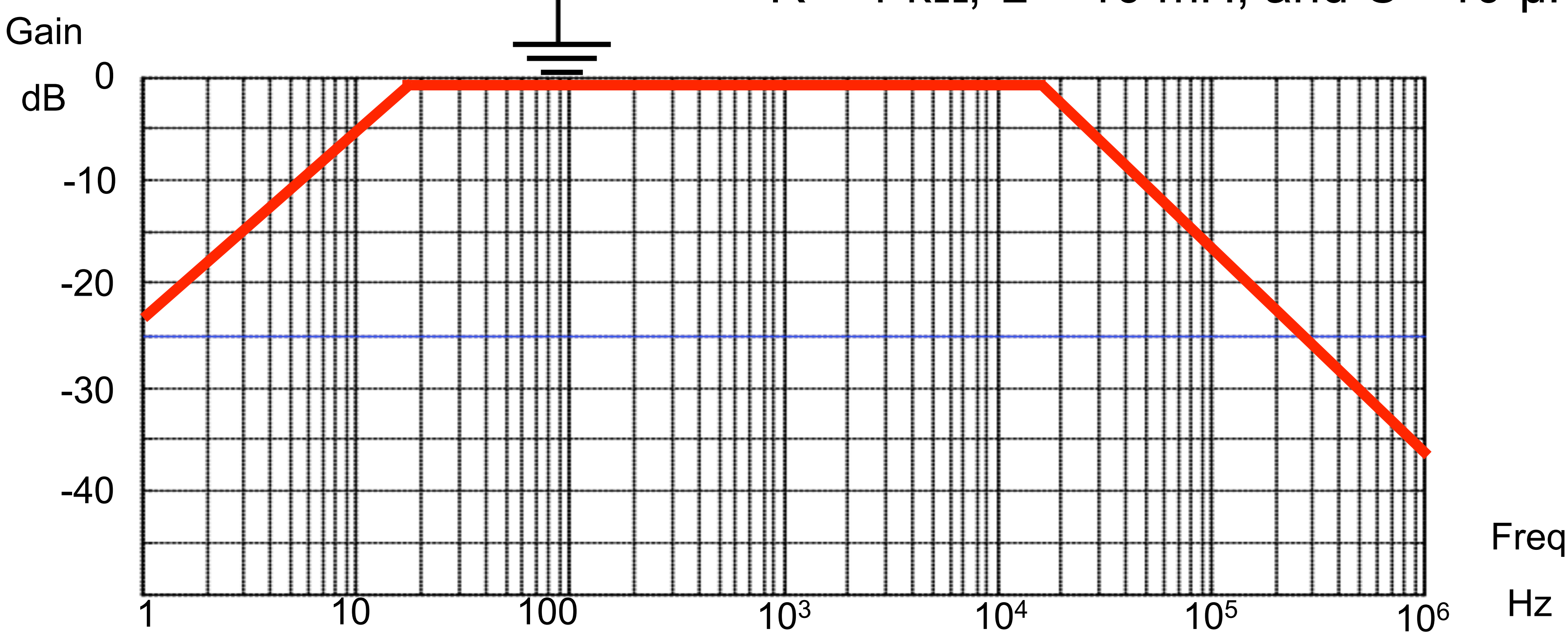
The F^2 term dominates when $(2\pi F)^2 LC = 2\pi FRC$ or $F = \dfrac{1}{2\pi \frac{L}{R}} = 15.9\ kHz$

RLC Filter

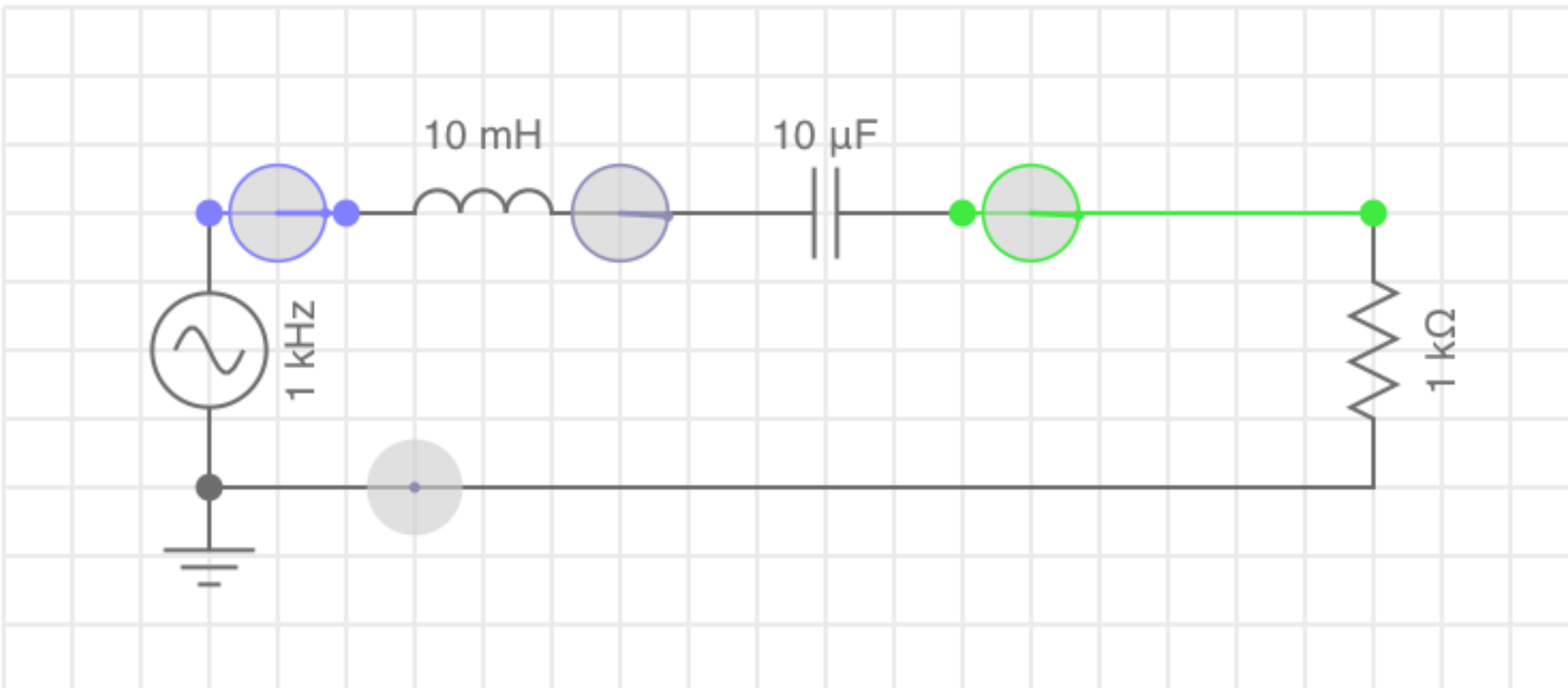


$$\frac{V_{out}}{V_{in}} = \frac{j*2\pi FRC}{1+j*2\pi FRC+\left(j*2\pi F\right)^2 LC}$$

R = 1 kΩ, L = 10 mH, and C = 10 μF



EveryCircuit Simulation



Another Important Application of Indictors
Problem: Convert 12V to 1V

- Your laptop / tablet adapter generates around 15V
  - Its internal battery is probably around 10-12V
- But the processor requires a Vdd of around 1V
  - The transistors are so small they cannot support more voltage
  - Just as important, $P \propto V^2$ so reducing P means reducing V
- Processors dissipate 10W - 30W in a laptop which means 10 - 30A!
- So we need to convert the energy from battery/wall
  - To 1V, and we don't want to waste energy …

Apple MacBook Charger

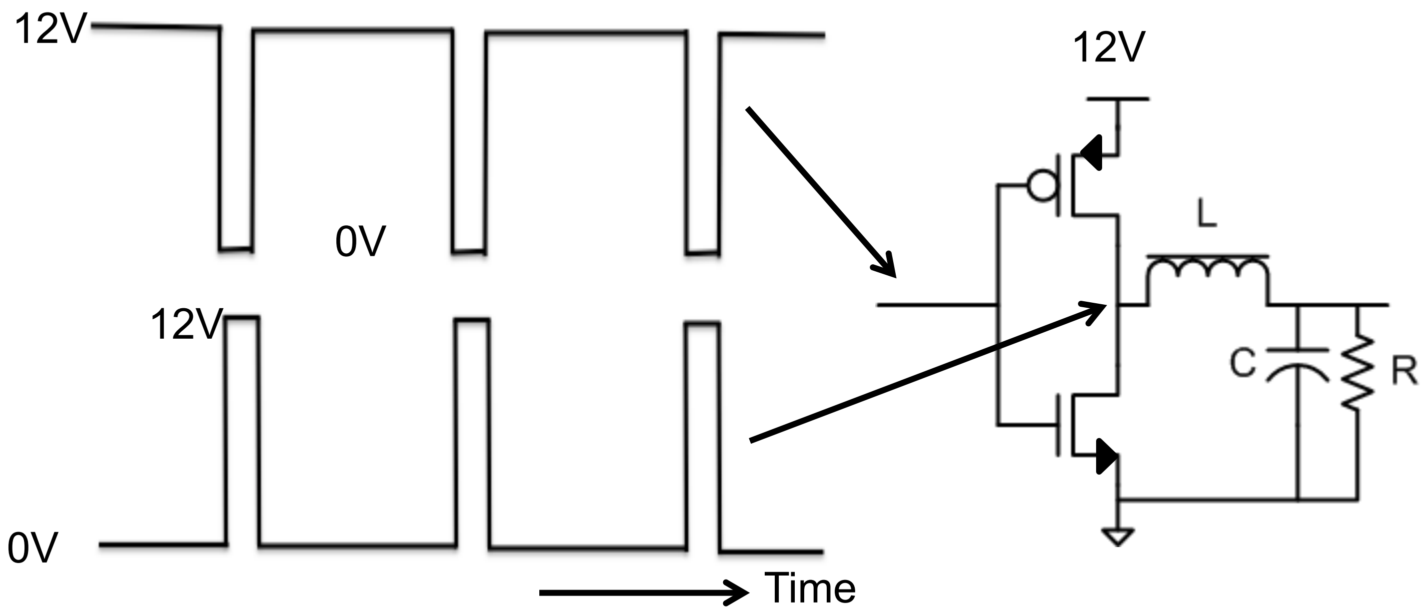
18.5 V, 4.6 A

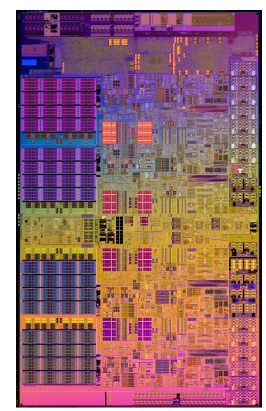
Apple MacBook Battery

11.3 V, 95 Whr

Basic Buck Converter

What happens if we drive a power inverter from a 12V supply (your laptop battery)?





Your laptop CPU

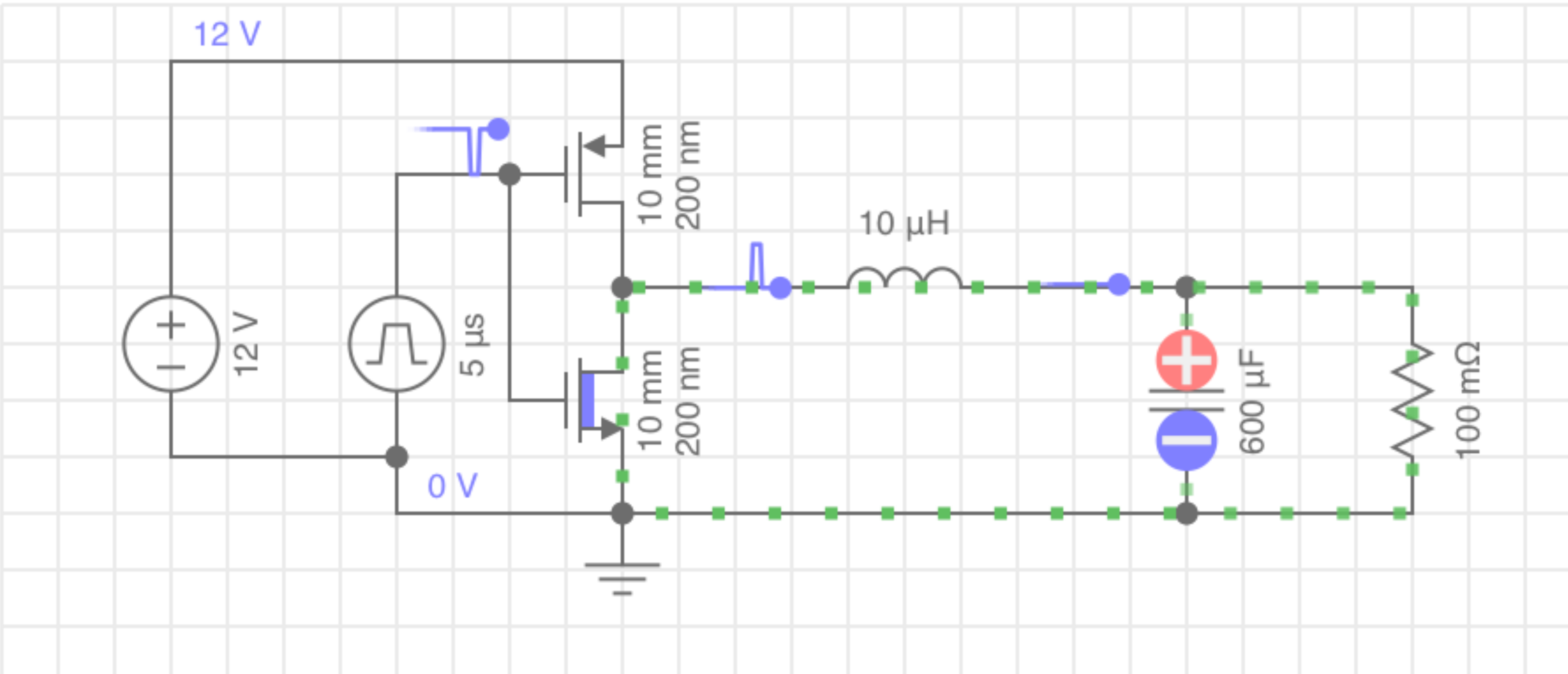
Needs 1V, 10A

How do we analyze this?

Assume R = 0.1Ω (10A @ 1V, your laptop CPU)

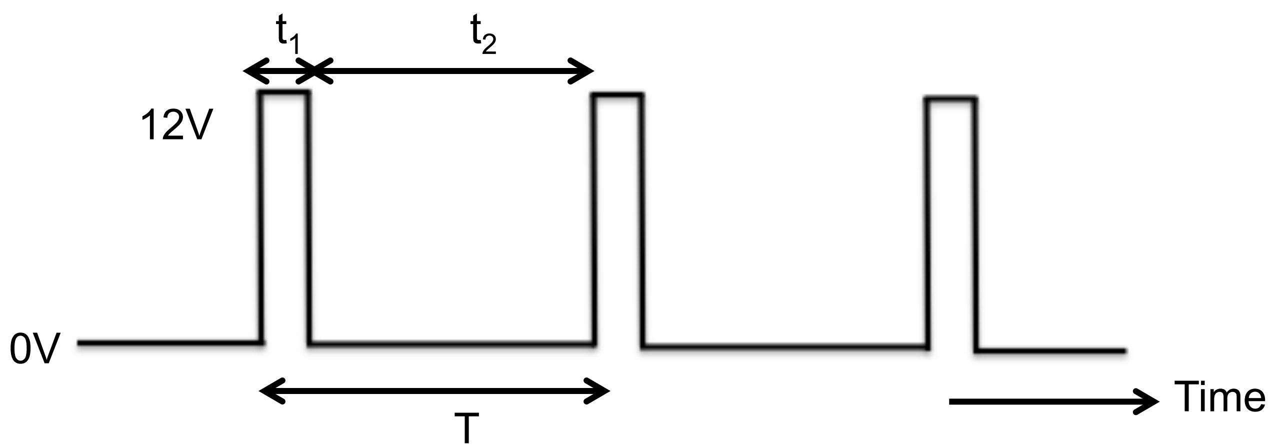
EveryCircuit To The Rescue

- If we have no idea what this circuit does, let's try EveryCircuit.



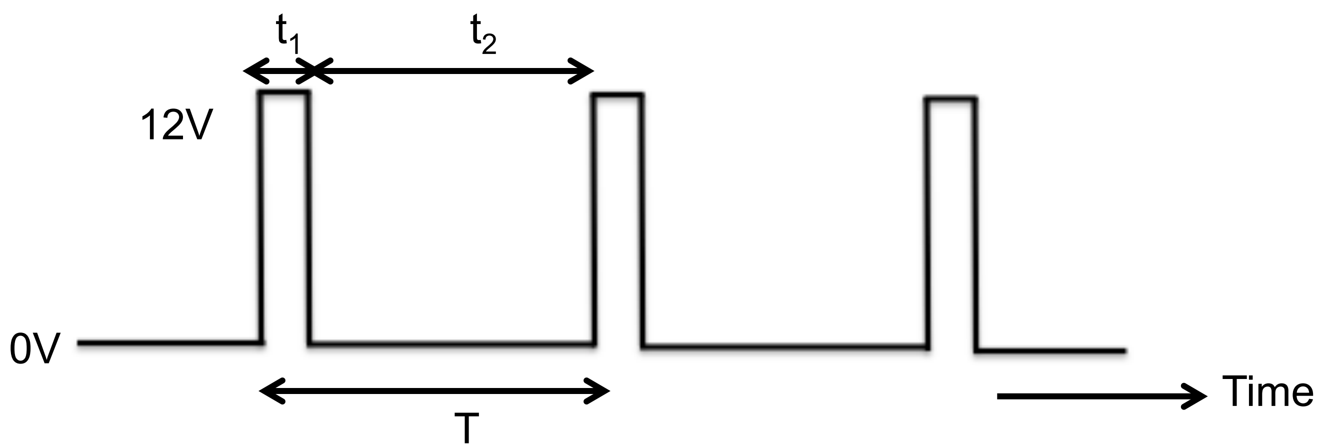
- So the output is 1 V! But how does this work?

Let's Start by Thinking About The Voltage Driving The LCR Circuit



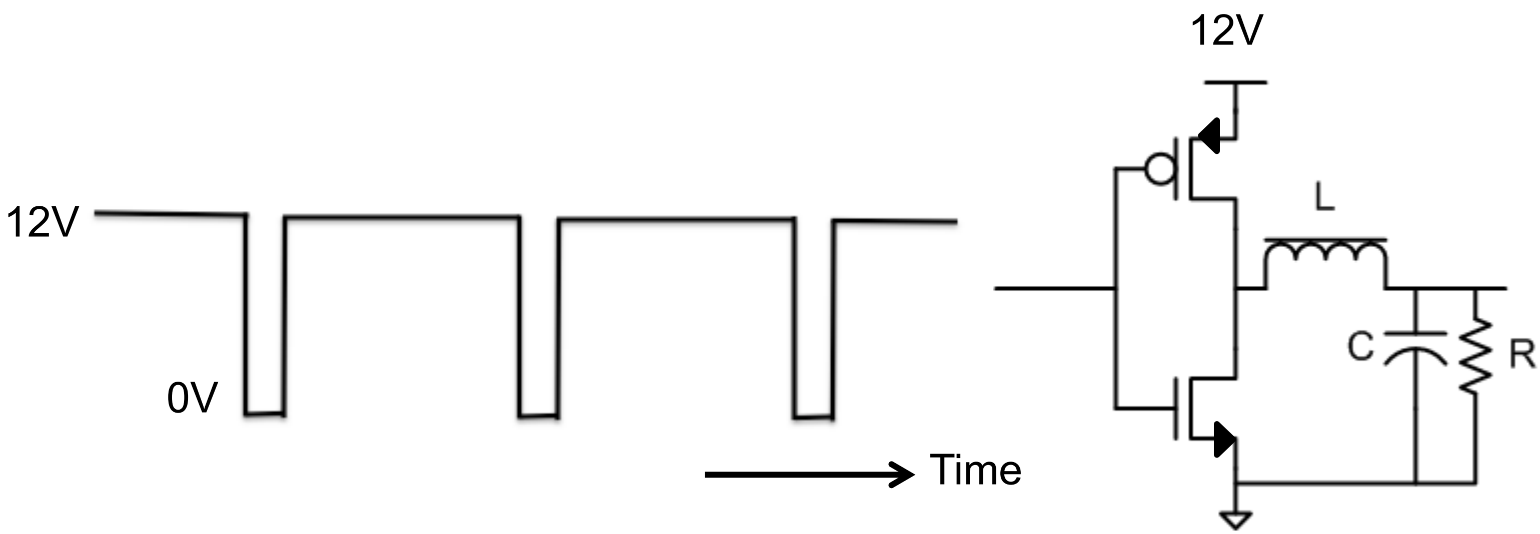
- We know we can represent a time varying voltage by adding sine wave components.
- If the input has a period T, then the fundamental frequency is F=1/T. There will also be higher frequency components.
- But there is also a DC component: DC (average V) =(12V) t_1/T

Let's Start by Thinking About The Voltage Driving The LCR Circuit

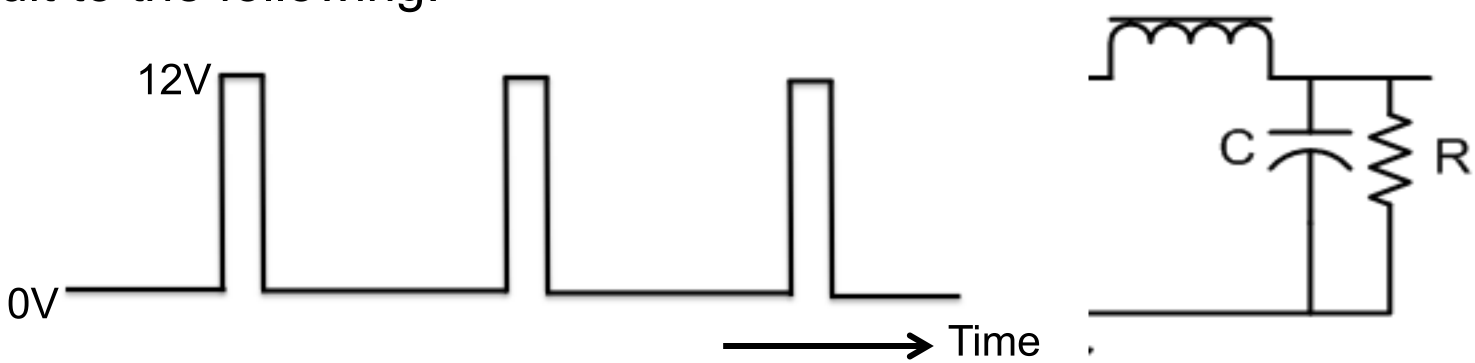


- Example: Suppose T = 5 µsec. Then F = 200 KHz.
- Suppose t_1 = (5/12) µsec, then DC component = 12(1/12) = 1.0 V
- So if we had a circuit that passed the DC component through to the output, but blocked the high frequencies, then . . . . . .

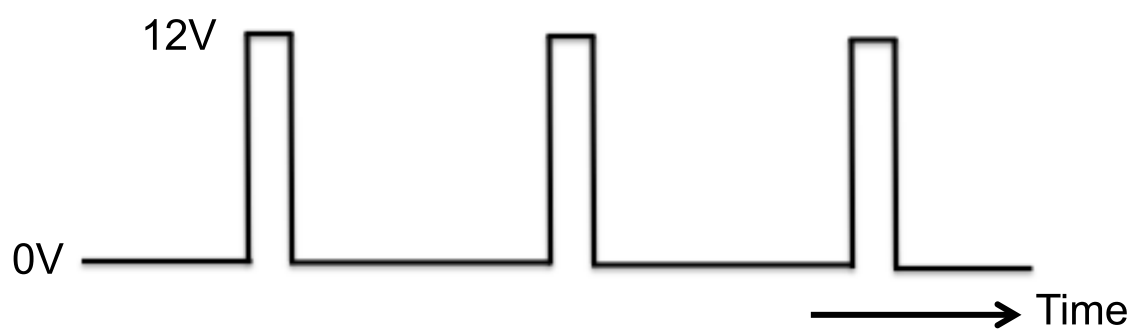
Basic Buck Converter

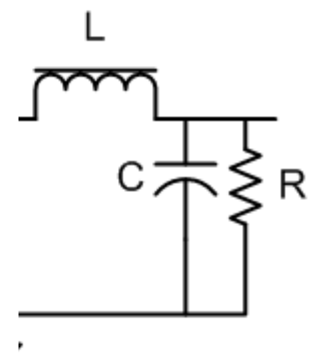


- The CMOS inverter will invert the digital signal, so we can reduce this circuit to the following:



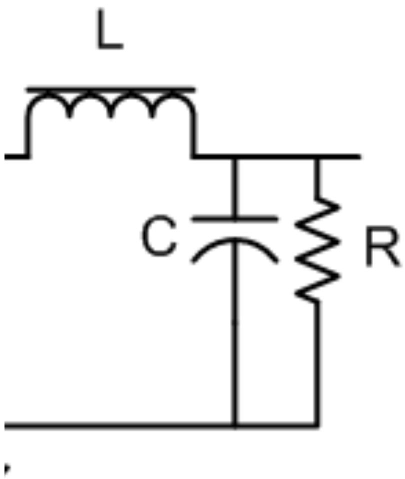
Basic Buck Converter





- L C circuit is a low pass filter
  - Z_L = j2πfL which = 0 at DC, so DC voltages will go directly to V_{OUT}
  - Z_L = 2πFL which at high F $\rightarrow \infty$ so high frequencies will be blocked

Buck Converter Circuit – Impedance Analysis



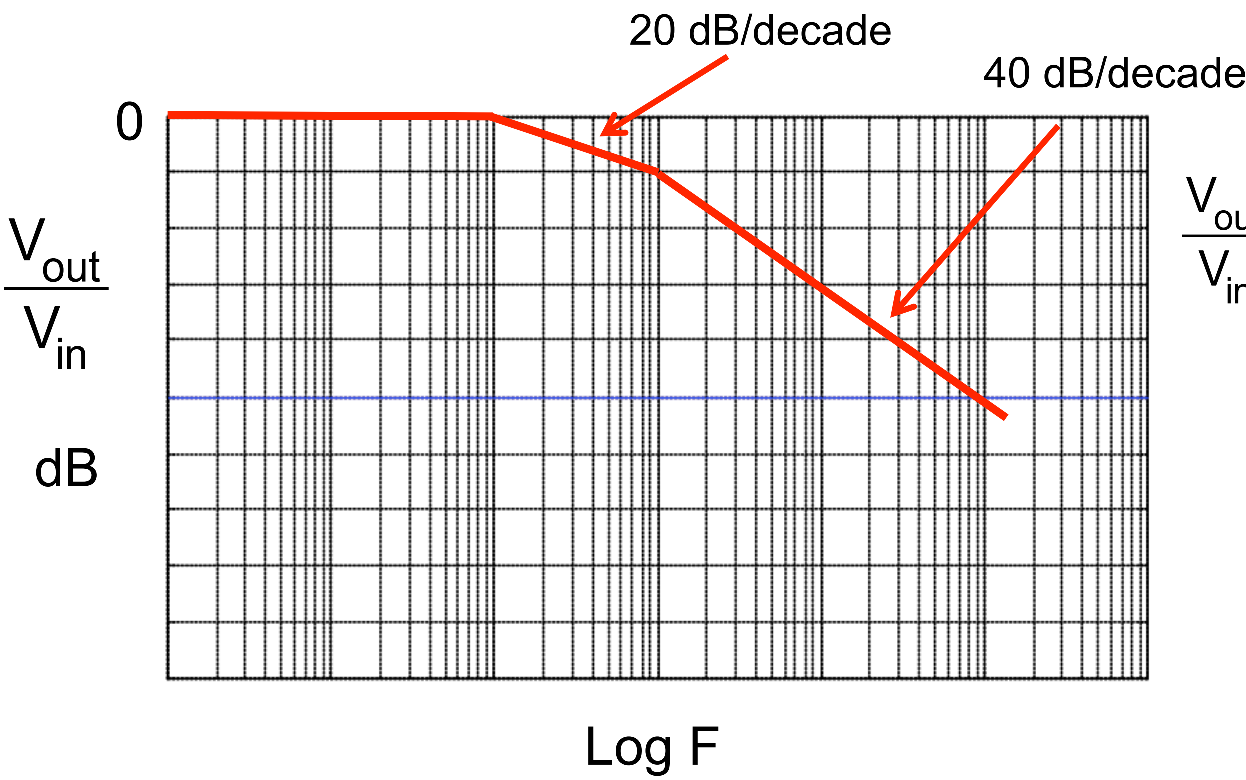
$$Z_2 = \frac{1}{\frac{1}{R} + j*2\pi FC} = \frac{R}{1+j*2\pi FRC}$$

$$Z_1 = j*2\pi FL$$

$$\frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1+Z_2} = \frac{\frac{R}{1+j*2\pi FRC}}{\frac{R}{1+j*2\pi FRC}+j*2\pi FL} = \frac{\frac{R}{1+j*2\pi FRC}}{\frac{R+j*2\pi FL(1+j*2\pi FRC)}{1+j*2\pi FRC}} = \frac{1}{1+j*2\pi F\frac{L}{R}+LC\left(j*2\pi F\right)^2}$$

- So as we expected, DC is passed through to the output and high frequencies are attenuated as $1/f$ or $1/f^2$. So it really is a low pass filter but it attenuates high frequencies as $1/f^2$.

Bode Plot



$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + j*2\pi F\frac{L}{R} + LC\left(j*2\pi F\right)^2}$$

Let L = 20 μH

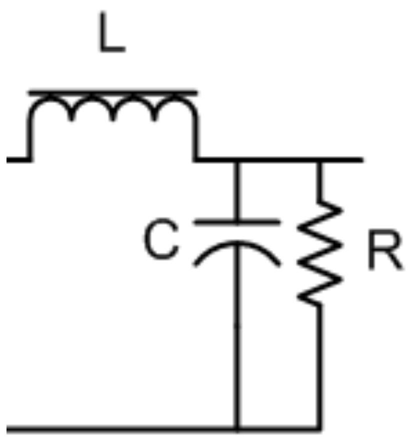
C = 300 μF

R = 0.1 ohms

- DC is passed to the output, high frequencies are attenuated as $1/F^2$ at high frequencies (-40 dB/decade).

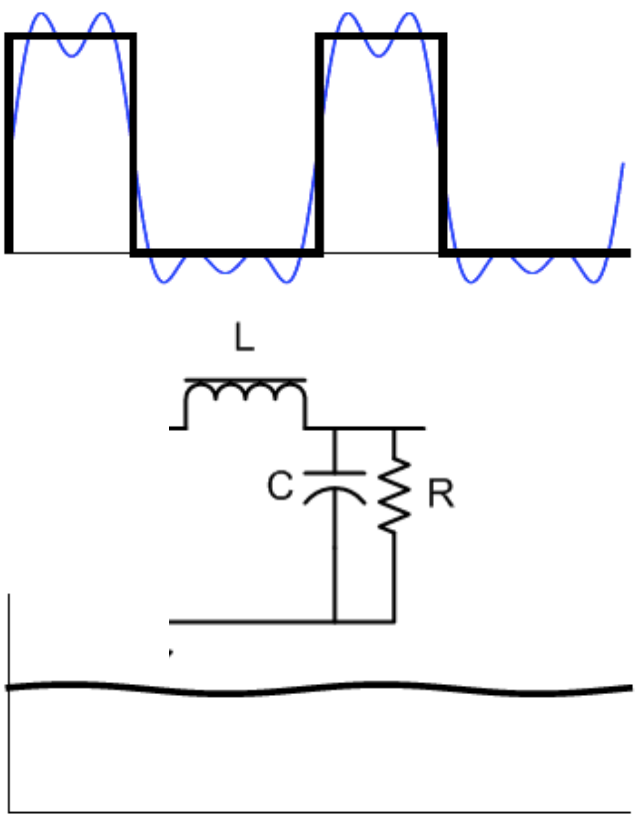
Some Reasonable Values

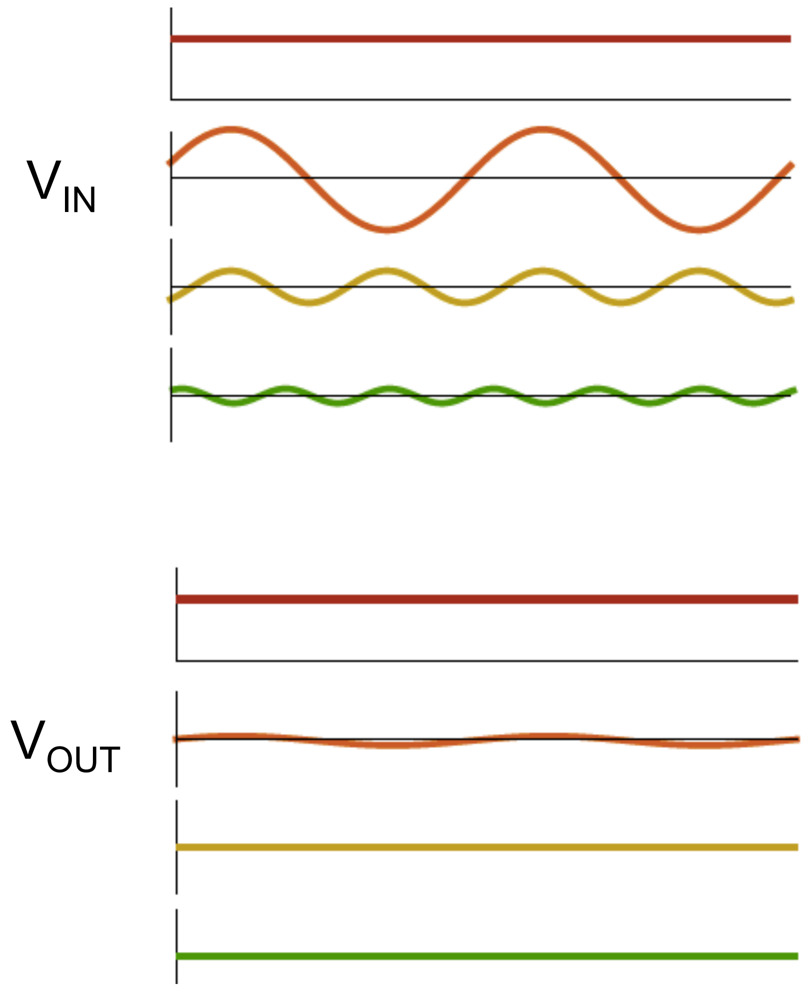
- L = 20μH   C = 300 μF
- If the input is 0 to 12 V with a basic frequency of 200 KHz
  - $LC(2\pi f)^2 \approx 9500$
  - So the output at 200 KHz is ≈ mV
- But if the input is at 12 V 1/12 of the time and 0 V the rest of the time,
  - The DC output voltage = 1 V
- So we have a 12 V to 1 V converter!



$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + j * 2\pi F \frac{L}{R} + LC\left(j * 2\pi F\right)^2}$$

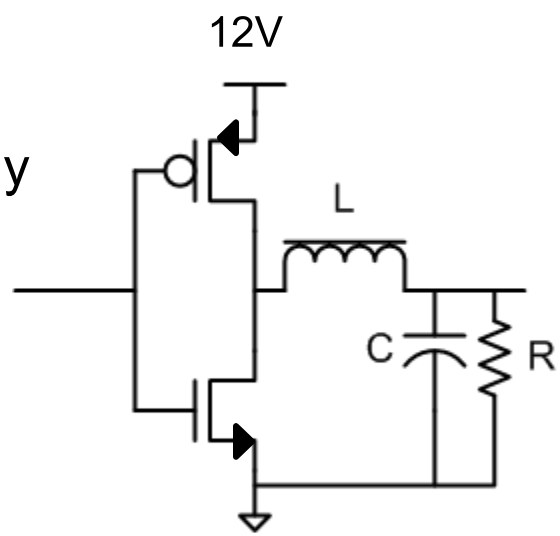
Buck Converter As A Filter





Is This Energy Efficient?

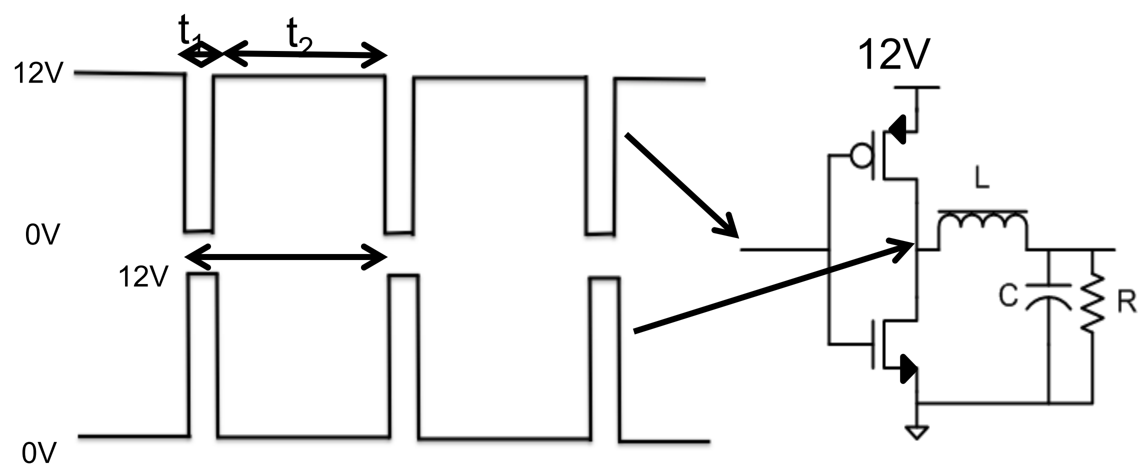
- Ideal C, L don't dissipate power
- Can make the R of the transistors small
- So it doesn't seem to dissipate much energy



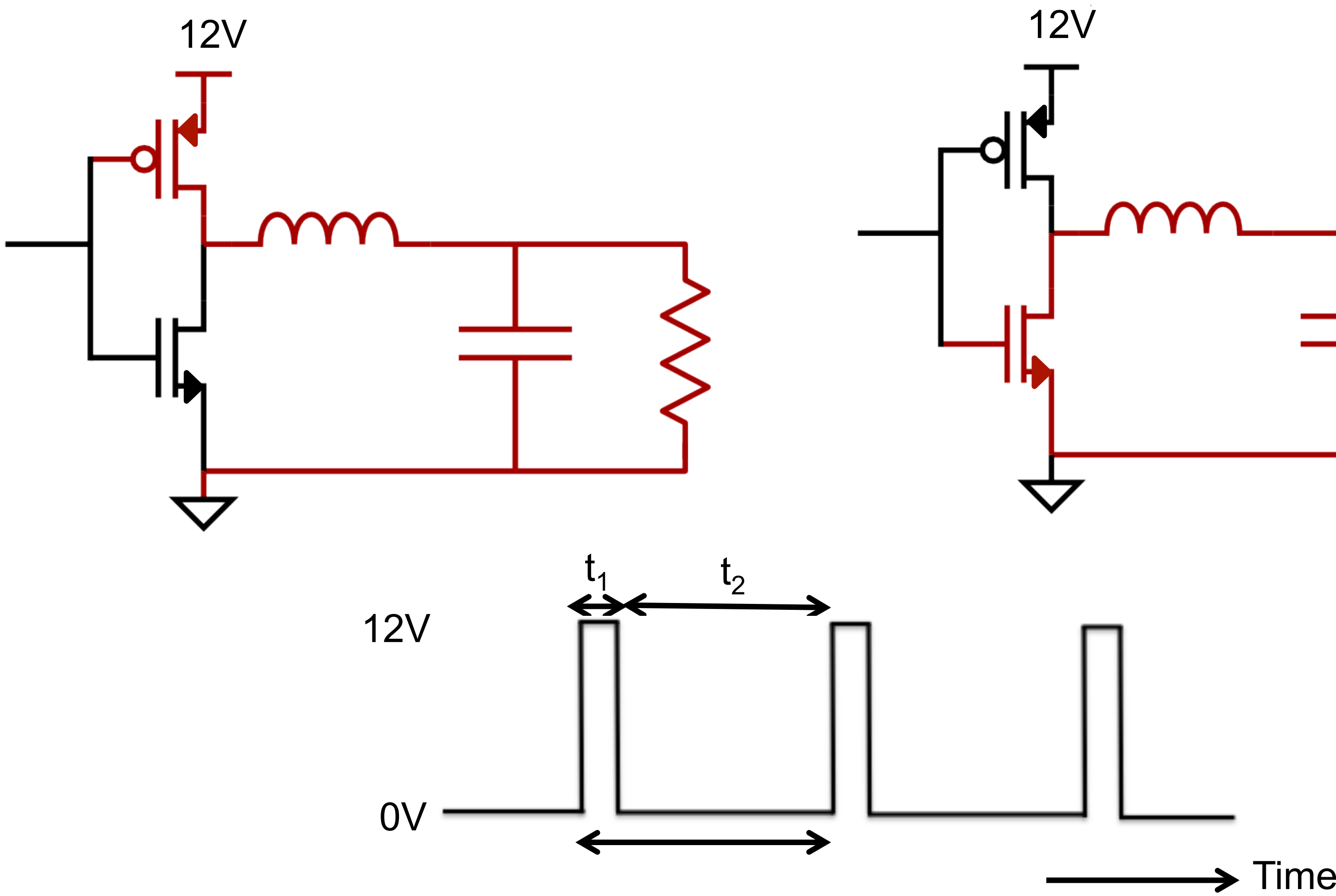
- If it is energy efficient
  - Power @ 12V supply ≈ Power @1V
    - But this means i@1V = 12 i@12V
  - Is this possible?

How Can This Work?

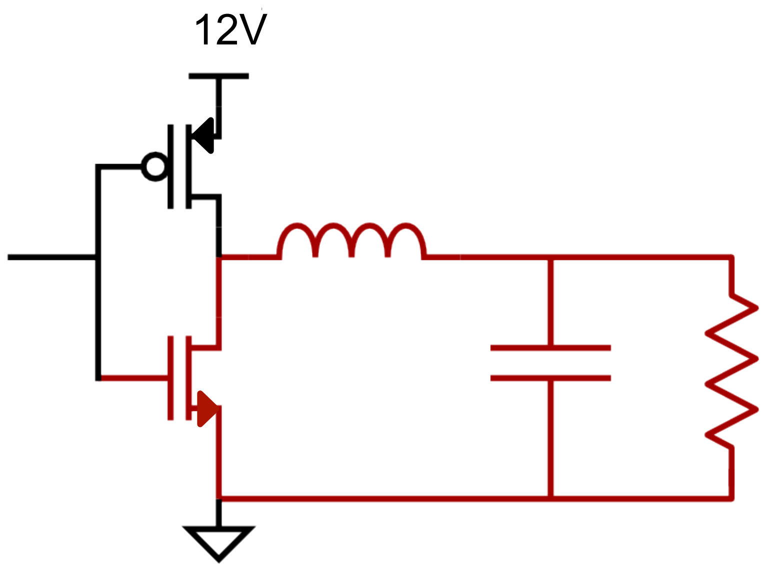
- Suppose V_{OUT} is 1V and the load R draws 10A (i.e. R = 0.1 ohm).
  - Remember the inductor current only changes slowly
    - di/dt=V/L =12V/20μH = .6A/μs; t_1= .4μs
  - So $i_{INDUCTOR} \approx$ 10A.
- But this current comes from the 12V supply
  - Only when the pMOS transistor is on
  - The rest of the time the current comes from Gnd!



Current Flow In Buck Converter



Inductor Current



$$V = L\frac{di_L}{dt}$$

$$\text{so} \quad \frac{di_L}{dt} = -\frac{V_{OUT}}{L}$$

$$i_L = C\frac{dV_{OUT}}{dt} + \frac{V_{OUT}}{R}$$

- We could solve the circuit using KCL and KVL
- But that will give
  - A 2nd order differential equation
    - Would prefer a simpler method if possible

Make Two Approximations

(both are good approximations)

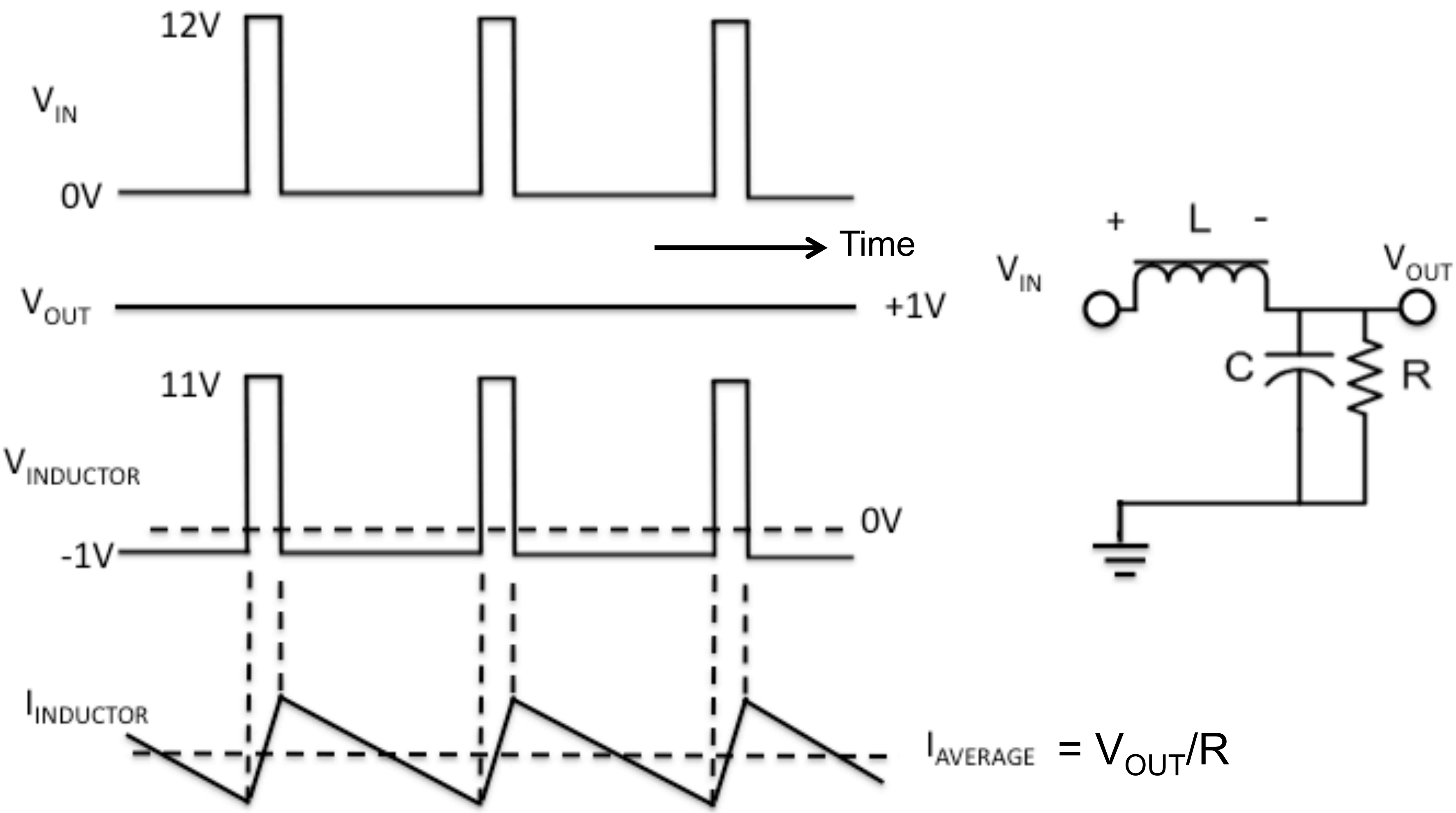
- The circuit will settle into some repeating cycle

  V_{OUT} and i_L must be the same at the beginning and end of each cycle

- The output voltage is approximately constant
  - Want stable output voltage

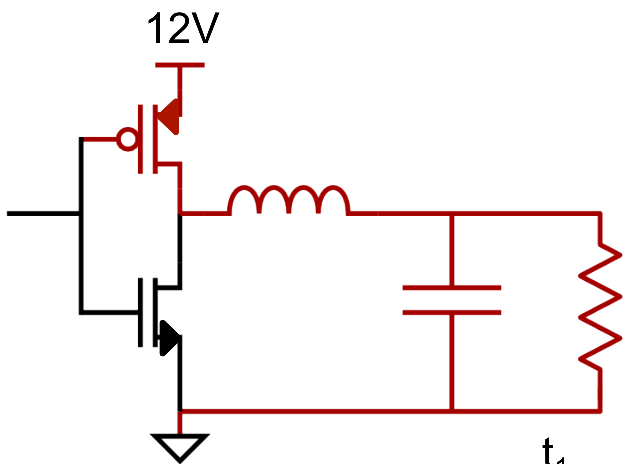
  This means the voltage across the inductor is roughly constant in each phase of the converter. Thus the inductor current will increase or decrease linearly during each phase

Operation with Our Assumptions

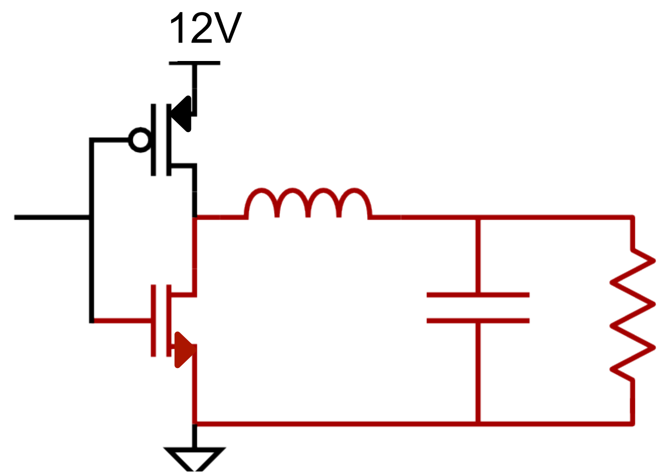


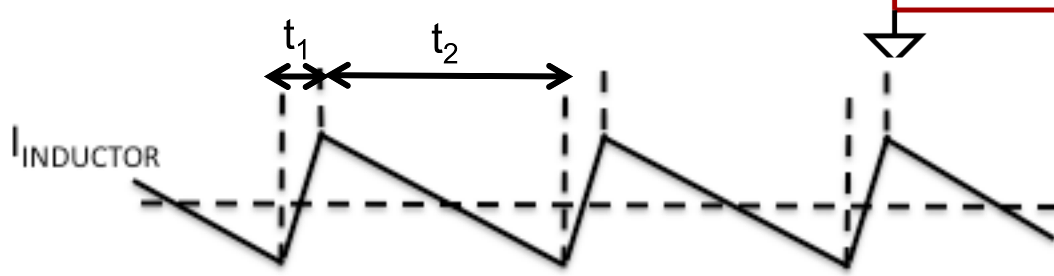
$I_{AVERAGE} = V_{OUT}/R$

Inductor Current



$$V = L\frac{di}{dt}$$





$$\frac{di}{dt} = \frac{12 - V_{OUT}}{L}$$

$$\frac{di}{dt} = -\frac{V_{OUT}}{L}$$

$$\Delta i_{charge} = t_1 \cdot \frac{12 - V_{OUT}}{L}$$

$$\Delta i_{discharge} = -t_2 \cdot \frac{V_{OUT}}{L}$$

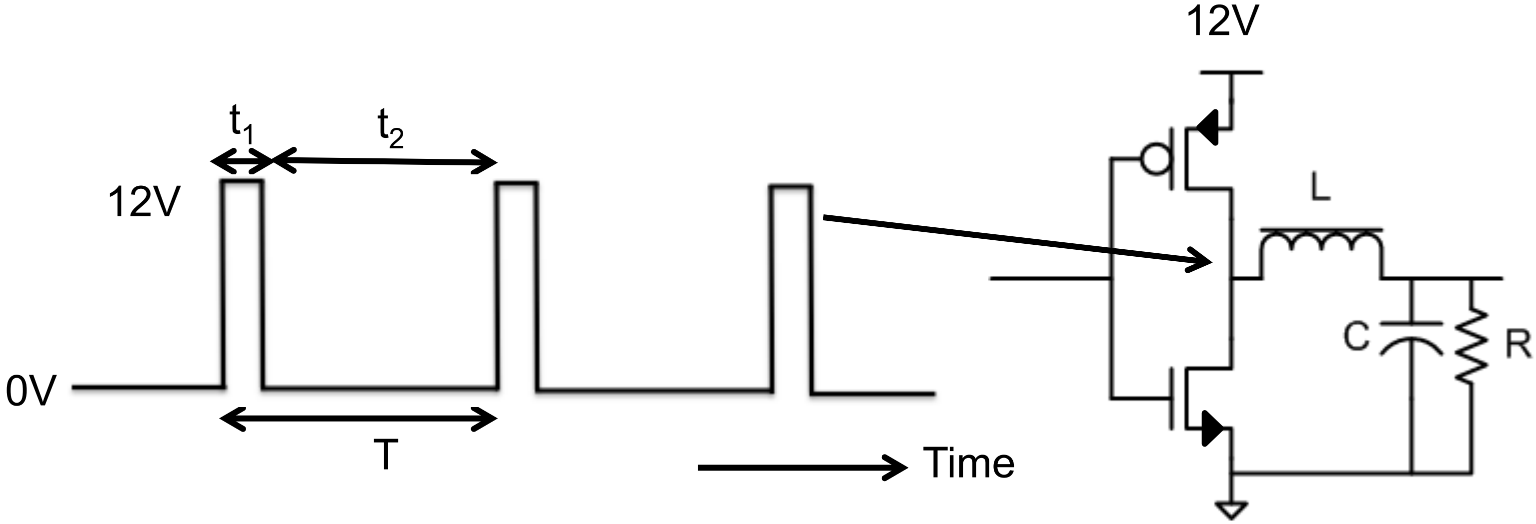
Balancing the Equations

Since the current at the start and end of the cycle are the same,

$$\Delta i_{charge} + \Delta i_{discharge} = 0$$

So we substitute the equations in:

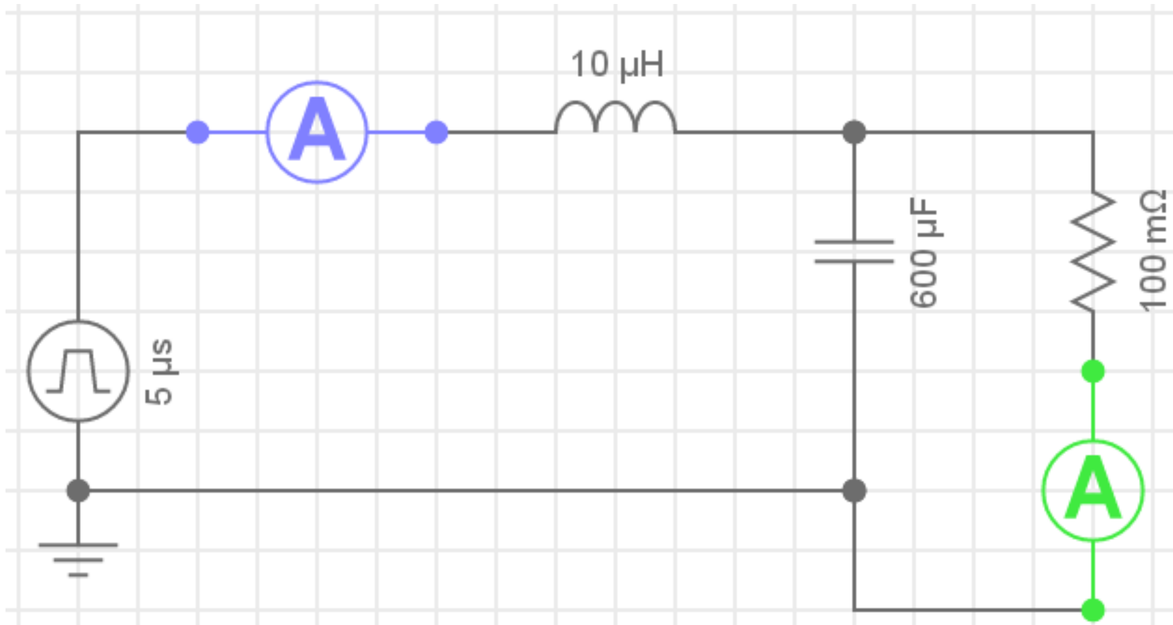
$$t_1 \cdot \frac{12 - V_{OUT}}{L} = t_2 \cdot \frac{V_{OUT}}{L} \qquad V_{OUT} = 12 \cdot \frac{t_1}{t_1 + t_2}$$

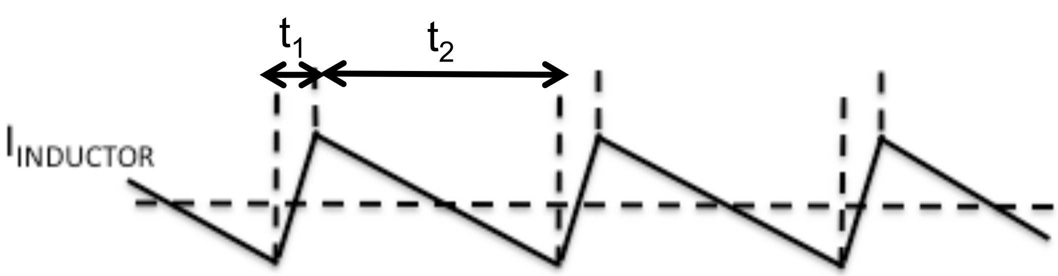


Output Voltage Only Depends on Duty Cycle

- The output voltage doesn't depend on load current (R)!
- The output is the "average" voltage created by inverter
  - It is V_{dd} times percentage of time the output is high
  - The LC circuit is "filtering" the square wave voltage
    - Smoothing out the signal.
- The "ripple" in the current is $\propto 1/L$ $\quad \Delta i_{discharge} = -t_2 \cdot \frac{V_{OUT}}{L}$
- Our model did not include resistance of transistor, inductor
  - If it did, the output voltage would depend on the current
    - But this would be like having an ideal "average" voltage source plus a series resistor

EveryCircuit Simulation – Buck Converter





$I_{AVERAGE} = V_{OUT}/R$

Learning Objectives

- Understand what an inductor is
  - $V = L\, di/dt$
  - It is a device that tries to keep current constant
    - Generates voltage (in either direction) to resist current changes
- Understand that ideal inductors and capacitors are lossless
  - They store energy, and don't dissipate it
  - Energy that goes into an LC circuit, must come out
  - We can use this to convert energy from one voltage to another
  - Size of the components is related to the energy they can store
- Be able to use impedance to
  - Solve for the output voltage of RL filters

Learning Objectives

- Understand the basic operations of a buck converter
  - Key point to remember is that the output is pretty stable
  - Find the output voltage as a function of input duty cycle
    - Vin * T_{high}/T_{cycle}, (the average value of the input waveform)
  - Output voltage doesn't depend on output current
    - Average input current does depend on output current
- For the final: not responsible for E&M applications (traffic sensors, metal detectors) and not responsible for details of Buck Converter (should understand 12V to 1V principles – see above).