# Computational Matiematics

## Homework 2

## Due by April 01, 2024

1.

Use a fixed-point iteration method to determine a solution accurate to within $10^{-2}$ for $x^3 - x - 1 = 0$ on [1, 2]. Use $p_0 = 1$.

2.

Use Theorem 2.1 to find a bound for the number of iterations needed to achieve an approximation with accuracy $10^{-3}$ to the solution of $x^3 + x - 4 = 0$ lying in the interval [1, 4]. Find an approximation to the root with this degree of accuracy.

3.

The following four methods are proposed to compute $21^{1/3}$. Rank them in order, based on their apparent speed of convergence, assuming $p_0 = 1$.

**a.** $p_n = \dfrac{20p_{n-1} + 21/p_{n-1}^2}{21}$

**b.** $p_n = p_{n-1} - \dfrac{p_{n-1}^3 - 21}{3p_{n-1}^2}$

**c.** $p_n = p_{n-1} - \dfrac{p_{n-1}^4 - 21p_{n-1}}{p_{n-1}^2 - 21}$

**d.** $p_n = \left(\dfrac{21}{p_{n-1}}\right)^{1/2}$

4.

Use Theorem 2.3 to show that $g(x) = 2^{-x}$ has a unique fixed point on $[\frac{1}{3}, 1]$. Use fixed-point iteration to find an approximation to the fixed point accurate to within $10^{-4}$. Use Corollary 2.5 to estimate the number of iterations required to achieve $10^{-4}$ accuracy, and compare this theoretical estimate to the number actually needed.

5.

Let $A$ be a given positive constant and $g(x) = 2x - Ax^2$.

**a.** Show that if fixed-point iteration converges to a nonzero limit, then the limit is $p = 1/A$, so the inverse of a number can be found using only multiplications and subtractions.

**b.** Find an interval about $1/A$ for which fixed-point iteration converges, provided $p_0$ is in that interval.

6.

Show that if $A$ is any positive number, then the sequence defined by

$$x_n = \frac{1}{2}x_{n-1} + \frac{A}{2x_{n-1}}, \quad \text{for } n \geq 1,$$

converges to $\sqrt{A}$ whenever $x_0 > 0$.

7.

Let $f(x) = -x^3 - \cos x$. With $p_0 = -1$ and $p_1 = 0$, find $p_3$.

**a.** Use the Secant method.

**b.** Use the method of False Position.

8.

Problems involving the amount of money required to pay off a mortgage over a fixed period of time involve the formula

$$A = \frac{P}{i}[1 - (1 + i)^{-n}],$$

known as an *ordinary annuity equation.* In this equation, $A$ is the amount of the mortgage, $P$ is the amount of each payment, and $i$ is the interest rate per period for the $n$ payment periods. Suppose that a 30-year home mortgage in the amount of \$135,000 is needed and that the borrower can afford house payments of at most \$1000 per month. What is the maximal interest rate the borrower can afford to pay?

9.

(a) Show that for any positive integer $k$, the sequence defined by $p_n = 1/n^k$ converges linearly to $p = 0$.

(b) Show that the sequence $p_n = 10^{-2^n}$ converges quadratically to 0.

10.

(a) The following sequences are linearly convergent. Generate the first five terms of the sequence $\{\hat{p}_n\}$ using Aitken's $\Delta^2$ method.

$$p_0 = 0.5, \quad p_n = \cos p_{n-1}, \quad n \geq 1$$

(b) Use Steffensen's method to find, to an accuracy of $10^{-4}$, the root of $x^3 - x - 1 = 0$ that lies in $[1, 2]$,

11. Given a polynomial $P(x) = x^3 - 5x^2 + 8x - 6$, do the following

(a) Evaluate $P(2), P'(2), P(4)$ and $P'(4)$ by Horner's Method

(b) Find the root of P(x) with error less than 0.00001 between [2,4] by using The Newton method with initial point $x_0 = 2$ and $x_0 = 4$. Determin which Initial point may lead to the root.

(c) Deflate P(x) into a quadartic ppolynomial by using the results in (b) and find the complex roots of P(x).

(d) Perform one step of Muller's Method starting from initial (0,P(0)), (1,P(1)) and (2,P(2)).

(e) Implement a matlab code of Muller's Method to find the complex root within error less than 0.00001 and compare with the answer you find in (c).