Homework 2 of Computational Mathematics

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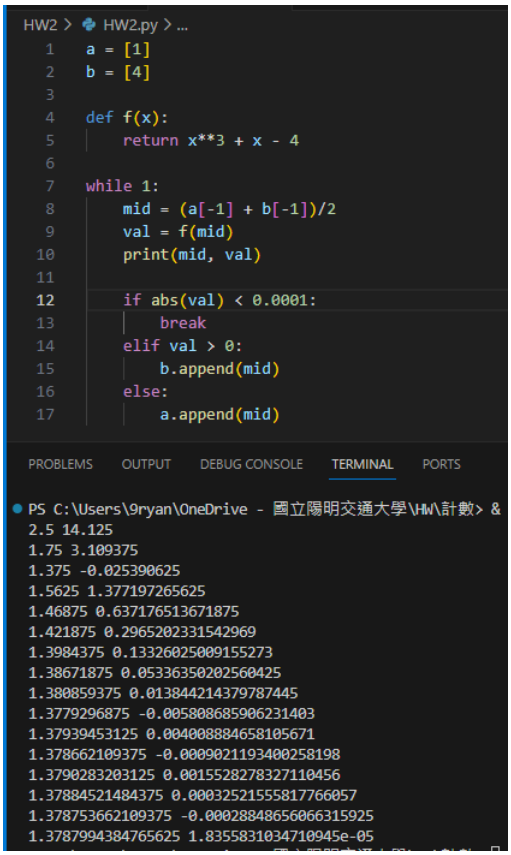
1. $x^3 = x+1 \implies x^2 = 1+\frac{1}{x} \implies x = \sqrt{1+\frac{1}{x}} = g(x)$. $p_1 = g(p_0) = \sqrt{1+1} = \sqrt{2} \approx 1.414$. $p_2 = g(p_1) = \sqrt{1+\frac{1}{\sqrt{2}}} \approx 1.3065$. $p_3 = g(p_2) \approx 1.3172$. $p_4 = g(p_3) \approx 1.326$. $p_5 = g(p_4) \approx 1.324$ Then, p_4 is the answer that we want to find.

2. Let $f(x) = x^3 + x - 4$, $f'(x) = 3x^2 + 1 < 49$ for all $x \in [1,4]$. Thus, for $|x-y| < \frac{10^{-3}}{49} \approx 2.0409e-5$, $|f(x) - f(y)| < 10^3$. Find n s.t. $3 \cdot 2^{-n} < 2.0409e-5$, $n > -\log_2(\frac{2.0409e-5}{3}) \approx 17.1653$. Thus, the bound of the number of iteration is 18. Then, by python code below, the root is about 1.3787.



3. (a) $\lim_{n\to\infty} \frac{|20p_n + 21/p_{n-1}^2 - \sqrt[3]{21}|}{|20p_{n-1} + 21/p_{n-2}^2 - \sqrt[3]{21}|^q}$