

# Homework 1 of Computational Mathematics

AM15 黃琦翔 111652028

March 6, 2024

1.  $f(x) = x^3 + 2x + k$ , then  $f'(x) = 3x^2 + 2 > 0$  for all  $x$ . Thus, we assume there are two points  $a, b \in \mathbb{R}$  s.t.  $f(a) = f(b) = 0$ . By Rolle's Theorem, there exists a point  $c$  in  $[a, b]$ (or  $[b, a]$ ) s.t.  $f'(c) = 0$ (Contradiction).

And since  $f(x) \rightarrow \infty$  as  $x \rightarrow \infty$  and  $f(x) \rightarrow -\infty$  as  $x \rightarrow -\infty$ , by IVT, there exists at least one  $x$  s.t.  $f(x) = 0$ . Thus, the graph of  $f(x)$  crosses the  $x$ -axis exactly once whatever  $k$  is.

- 2.