Homework 1 of Computational Mathematics

AM15 黃琦翔 111652028

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1. $f(x) = x^3 + 2x + k$, then $f'(x) = 3x^2 + 2 > 0$ for all x. Thus, we assume there are two points $a, b \in \mathbb{R}$ s.t. $f(a) = f(b) = 0$. By Rolle's Theorem, there exists a point c in $[a,b]$(or $[b,a]$) s.t. $f'(c) = 0$(Contradiction).

   And since $f(x) \to \infty$ as $x \to \infty$ and $f(x) \to -\infty$ as $x \to -\infty$, by IVT, there eists at least one x s.t. $f(x) = 0$. Thus, the graph of $f(x)$ crosses the x-axis exactly once whatever k is.

2. By EVT, we know that the maximun occurs either $f'(x) = 0$ or a, b.

   (a) $f'(x) = \frac{1}{3}(2 - e^x) = 0$ when $x = \ln(2)$. And since $f'(x) > 0$ when $x \in (0, \ln(2))$ and $f'(x) < 0$ when $x \in (\ln(2), 1)$, $f(\ln(2)) = \frac{1}{3}(2 - 2 + 2\ln 2) = \frac{2\ln(2)}{3}$ is the maximun.

   (b) $f'(x) = \dfrac{4x^2 - 8x - (4x-3)(2x-2)}{x^4 - 4x^3 + 4x^2} = \dfrac{-4x^2 + 6x - 8}{x^4 - 4x^3 + 4x^2} < 0$ for $x \in [0.5, 1]$. Thus, $f(0.5) = \dfrac{2-3}{0.25-2} = \dfrac{4}{7}$.

   (c)