

Homework 9 of Introduction to Analysis(II)

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1. By observe, $\Delta(x) = \det(D^2f(x)) \geq 0$.

(a) Since $\Delta(x) \geq 0$ and $f_{xx}(x) > 0$, $D^2f(x)$ is semi-definite on $D(c, \delta)$. Thus, for any $x \in D(c, \delta)$,

$$\begin{aligned} Df(x) &= Df(x) - Df(c) \\ &= \int_0^1 D^2f(c + t(x - c))(x - c) dt \\ &\geq \int_0^1 0(x - c) dt \\ &\geq 0 \end{aligned}$$

Then,

$$\begin{aligned} f(x) - f(c) &= \int_0^1 Df(c + t(x - c))(x - c) dt \\ &\geq \int_0^1 0(x - c) dt \\ &\geq 0 \end{aligned}$$

Threrfore, c is a local minimun point of f .

(b) Since $f_{xx}(x) < 0$ for all x and $\Delta(x) \geq 0$, $f_{yy}(x) \leq 0$ for all x .

$$\text{For } u^T [D^2f(x)]u = \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} =$$

2. Since $f_x(x_0) = f_y(x_0) = 0$ and $f \in C^2(V, \mathbb{R})$, $Df(x_0) = 0$ and x_0 is a critical point.

If $f_{xy}(x_0) \neq 0$, $\det(D^2f(x_0)) = -(f_{xy}(x_0))^2 < 0$, then $D^2f(x_0)$ is not positive definite and negative definite. Thus, x_0 will be saddle point.

3. $R_{r-1}(x_0, h) = \frac{1}{r} D^r f(c)(h, \dots, h)$