

Homework 12 of Introduction to Analysis(II)

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1. (a) Since E is Jordan region, $\text{Vol}(\partial E) = 0$.

(b) Since $\text{cl}(E) = \text{int}(E) \cup \partial E$ and $\text{int}(E) \subseteq E \subseteq \text{cl}(E)$,

$$\text{Vol}(\text{cl}(E)) = \text{Vol}(\text{int}(E)) + \text{Vol}(\partial E) = \text{Vol}(\text{int}(E)) \leq \text{Vol}(E) \leq \text{Vol}(\text{cl}(E)).$$

Therefore, $\text{Vol}(\text{cl}(E)) = \text{Vol}(\text{int}(E)) = \text{Vol}(E)$.

(c)

(\implies) From (b), we know $\text{Vol}(\text{int}(E)) = \text{Vol}(E) > 0$, then we can find a set of rectangles R_n s.t.

$$\sum |R_n| > 0 \text{ and } \cup R_n \subseteq \text{int}(E). \text{ Therefore, } \text{int}(E) \neq \emptyset.$$