Homework 10 of Introduction to Analysis(II)

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1. (a) $f'(0) = \lim\limits_{h \to 0} \dfrac{f(h) - f(0)}{h} = \lim\limits_{h \to 0} 1 + 2h\sin(\dfrac{1}{h}) = 1.$

   (b) $\lim\limits_{x \to 0} f(x) = \lim\limits_{h \to \infty} \dfrac{1}{h} + 2\dfrac{\sin(h)}{h^2} = 0.$ Thus, f is continuous on 0. And for any x close to 0, $|f(x +$

   $h) - f(x)| \le |h| + 2|(x+h)^2| + 2|x^2| \to 0$ as $h, x \to 0$, f is continuous on a small interval I_1.

   Since $f'(x) = 1 + 2(2x\sin(\dfrac{1}{x}) - \cos(\dfrac{1}{x}))$, there exsits x in any interval contains 0 s.t. $f'(x) < 0.$

   Therefore, f is neither increasing nor decreasing and not one to one in any interval contains 0.

   Thus, f is not invertible near 0.

   (c) This is not contradict to inverse function theorem.

2.

$$\|f(x_1) - f(x_2) - (x_1 - x_2)\| = \|x_1 + g(x_1) - (x_2 + g(x_2)) - (x_1 - x_2)\|$$

$$= \|g(x_1) - g(x_2)\|$$

$$\le a\|x_1 - x_2\|$$

And we can get g is a contraction mapping from $\mathbb{R}^n$ to $\mathbb{R}^n$, we can find an unique fixed point x^* s.t.

$g(x^*) = x^*$ and $f(x^*) = 2x^*.$

3.