Homework 2 of Introduction to Analysis(II)

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1. For all $\varepsilon > 0$, we can find a $N \in \mathbb{N}$ s.t. $|a_n - a| < \dfrac{\varepsilon}{2}$ for all $n > N$. And we can find a $N' > N$ s.t.
$\dfrac{\sum_{i=1}^{N} a_i - a}{N'} < \dfrac{\varepsilon}{2}$. Thus, for any $n > N'$,

$$
\begin{aligned}
|\frac{\sum_{i=1}^{n} a_i}{n} - a| &\leq |\frac{\sum_{i=1}^{N} a_i - a}{n}| + |\frac{\sum_{i=N+1}^{n} a_i - a}{n - N'}| \\
&< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} \\
&= \varepsilon
\end{aligned}
$$

Therefore, $\lim\limits_{n \to \infty} b_n = a$.

2. Since $\lim\limits_{n \to \infty} n a_n = 0$, $0 = \lim\limits_{n \to \infty} \dfrac{n a_n}{n} = \lim\limits_{n \to \infty} a_n$.

$$
\begin{aligned}
\sum_{n=0}^{N} a_n - A &= \sum_{n=0}^{N} a_n (1 - x^n) - \sum_{n=N+1}^{\infty} a_n x^n + (f(x) - A) \\
&=
\end{aligned}
$$