

Homework 8 of Introduction to Analysis(II)

AM15 黃琦翔 111652028

April 12, 2024

1. $f_x(0,0) = \lim_{x \rightarrow 0} \frac{f(x,0) - f(0)}{x} = \lim_{x \rightarrow 0} \frac{x \cdot 0(x^2 - 0^2)/(x^2 + 0^2)}{x} = 0$ exists. Also, we can easily get that $f_y(0,0) = 0$. Then, $f_{xy}(0,0) = \frac{\partial f_x}{\partial y}(0,0) = \lim_{y \rightarrow 0} \frac{y(0^4 + 4 \cdot 0^2 y^2 - y^4)/(0^2 + y^2)^2}{y} = \lim_{y \rightarrow 0} \frac{-y^4}{y^4} = -1$. And, $f_{yx}(0,0) = 1$. Therefore, $f_{xy}(0,0) \neq f_{yx}(0,0)$.
2. Since Df is continuous on S , for any $\varepsilon > 0$, there exists $\delta > 0$ s.t. $\|f(x) - f(y)\| < \varepsilon$ if $\|x - y\| < \delta$.
3. $\lim_{u \rightarrow x} \|g(u) - g(x) - Dg(x)(u)\|$