Homework 13 of Introduction to Analysis(II)

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1. (a) For any $\varepsilon > 0$, we can find an open cube $D(x_0, \delta)$ that $R_j \subseteq D(x_0, \delta)$ and

   (b) Since for any E_i is measure zero, we can find $\{R_{i,j}\}_j$ s.t. $\sum_j |R_{i,j}| < \frac{\varepsilon}{2^i}$. Then, let $R_{1,1} = R_1$, $R_{1,2} = R_2, R_{2,1} = R_3, R_{1,3} = R_4, R_{2,2} = R_5$ etc. And we can get $\{R_k\}_k$ contains $\bigcup_i E_i$ and $\sum_k |R_k| < \varepsilon(\frac{1}{2} + \frac{1}{4} + \cdots) = \varepsilon$. Therefore, union of countable measure zero sets is measure zero.

   (c)