Homework 9 of Introduction to Analysis(II)

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1. By observe, $\Delta(x) = \det(D^2 f(x)) \geq 0$.

   (a) If $\Delta(c) > 0$, $\det(D^2 f(c)) > 0$ and $f_{xx}(c) > 0$ implies that $D^2 f(c)$ is positive definite. Then, c is local minimun.

   If $\Delta(c) = 0$, then $f_{yy}(c) > 0$ and $f_{xx}(c) f_{yy}(c) = (f_{xy}(c))^2$.

2. Since $f_x(x_0) = f_y(x_0) = 0$ and $f \in C^2(V, \mathbb{R})$, $Df(x_0) = 0$ and x_0 is a critical point.

   If $f_{xy}(x_0) \neq 0$, $\det(D^2 f(x_0)) = -(f_{xy}(x_0))^2 < 0$, then $D^2 f(x_0)$ is not positive definite and negative definite. Thus, x_0 will be saddle point.

3. $R_{r-1}(x_0, h) = \frac{1}{r} D^r f(c)(h, \cdots, h)$