Homework 5 of Introduction to Analysis(II)

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1.   (a) If $x \in A$, $d(x,A) = \|x - x\| = 0$ and $d(x,B) = k > 0$, $\phi(x) = \dfrac{0}{0+k} = 0$. If $x \in B$, $d(x,A) = l > 0$

and $d(x,B) = 0$, $\phi(x) = \dfrac{l}{l+0} = 1$. If $x \notin A \wedge x \notin B$, $d(x,A) = l$ and $d(x,B) = k$, $\phi(x) = \dfrac{l}{l+k} < 1$

and is positive. Thus, $0 \le \phi(x) \le 1$ for all $x \in \mathbb{R}^n$.

(b) Let $\phi(x) = (b-a)\dfrac{d(x,A)}{d(x,A)+d(x,B)} + a$. From (a), we can get $\phi(x \in A) = (b-a) \cdot 0 + a = a$,

$\phi(x \in B) = (b-a) \cdot 1 + a = b$, and $a \le phi(x) \le b$ for all $x \in A$.

2.