Homework 2 of Introduction to Analysis(II)

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1. Suppose $f_k(x) = \sum_{n=1}^k \frac{x}{n^{\alpha}(1+nx^2)}$ and $E_l = [-l,l]$ for $l \in \mathbb{N}$. Then, we want to proof that for all $\varepsilon > 0$, there exists $N \in \mathbb{N}$ s.t. $|f_k(x) - f_l(x)| < \varepsilon$ for all k, l > N and all $x \in E_L$.

First, suppose that
$$l > k > N$$
, $|f_k(x) - f_l(x)| = \sum_{n=k}^{l} \frac{x}{n^{\alpha}(1 + nx^2)}$