Homework 11 of Introduction to Analysis(II)

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1. For $f \in \mathscr{C}^1(E, \mathbb{R}^n)$ with E is open subset of $\mathbb{R}^n$ and $Jf(x_0) \neq 0$ for some $x_0 \in E$ and $y_0 = f(x_0)$. Then, let $h(x) = f(x) - f(x_0)$ and $h(x_0) = 0$.

2. Suppose $f \in \mathscr{C}^1(\mathbb{R}^2, \mathbb{R})$, $Df(x_0, y_0) \neq 0$ for some x_0, y_0 (or f is constant function and not one-to-one). Then, suppose $\dfrac{\partial f}{\partial x} \neq 0$ for neighborhood of (x_0, y_0), and let $h(x, y) = f(x, y) - f(x_0, y_0)$ with $\dfrac{\partial h}{\partial x} \neq 0$. by Implicit Function Theorem, there is a neighborhood $U \subseteq \mathbb{R}^2$ and $W \subseteq \mathbb{R}$ s.t. $(x_0, y_0) \in U$ and $y_0 \in W$ and a function $g : W \to \mathbb{R}^2$ s.t. $h(g(y), y) = f(g(y), y) - f(x_0, y_0) = 0$. Then, $f(g(y), y) = f(x_0, y_0)$ for $y \in W$ and f is not one-to-one. If $\dfrac{\partial f}{\partial x} = 0$, then $\dfrac{\partial f}{\partial y} \neq 0$ and use the same argument can get the same result.

3.