Homework 8 of Introduction to Analysis(II)

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1. $f_x(0,0) = \lim_{x\to 0} \frac{f(x,0)-f(0)}{x} = \lim_{x\to 0} \frac{x\cdot 0(x^2-0^2)/(x^2+0^2)}{x} = 0$ exists. Also, we can esaily get that $f_y(0,0) = 0$. Then, $f_{xy}(0,0) = \frac{\partial f_x}{\partial y}(0,0) = \lim_{y\to 0} \frac{y(0^4+4\cdot 0^2y^2-y^4)/(0^2+y^2)^2}{y} = \lim_{y\to 0} \frac{-y^4}{y^4} = -1$. And, $f_{yx}(0,0) = 1$. Therefore, $f_{xy}(0,0) \neq f_y(0,0)$.

2. Since Df is continuous on S(close interval in R^p), Df is Riemann intergrable on S. Then, let $g(x) = \int_0^x Df(a+t(b-a))(b-a)dt$. For any $\varepsilon > 0$, we can find $\delta > 0$ s.t. $\|Df(x)-Df(y)\| < \frac{\varepsilon}{\sup_{s\in S}|Df(s)|}$ for $\|x-y\| < \delta$.