Homework 8 of Introduction to Analysis(II)

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1. $f_x(0,0) = \lim\limits_{x\to 0} \dfrac{f(x,0)-f(0)}{x} = \lim\limits_{x\to 0} \dfrac{x\cdot 0(x^2-0^2)/(x^2+0^2)}{x} = 0$ exists. Also, we can esaily get that $f_y(0,0) = 0$. Then, $f_{xy}(0,0) = \dfrac{\partial f_x}{\partial y}(0,0) = \lim\limits_{y\to 0} \dfrac{y(0^4+4\cdot 0^2y^2-y^4)/(0^2+y^2)^2}{y} = \lim\limits_{y\to 0} \dfrac{-y^4}{y^4} = -1$. And, $f_{yx}(0,0) = 1$. Therefore, $f_{xy}(0,0) \neq f_y(0,0)$.

2. Since Df is continuous on S, for any $\varepsilon > 0$, there exists $\delta > 0$ s.t. $\|f(x)-f(y)\| < \varepsilon$ if $\|x-y\| < \delta$. And since S is a closed line in R^p, we can find a sequence $\{x_k\}_{k=1}^n$ s.t. $D(x_k, \dfrac{\delta}{2}) \supseteq S$ and $\|x_k+1-a\| > \|x_k-a\|$.

3. $\lim\limits_{u\to x} \|g(u)-g(x)-Dg(x)(u)\|$