Homework 2 of Introduction to Analysis(II)

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1. Suppose $f_k(x) = \sum_{n=1}^{k} \frac{x}{n^\alpha(1+nx^2)}$ and $E_L = [-L, L]$ for $L \in \mathbb{N}$. Then, we want to proof that for all $\varepsilon > 0$, there exists $N \in \mathbb{N}$ s.t. $|f_k(x) - f_l(x)| < \varepsilon$ for all $k, l > N$ and all $x \in E_L$.

   First, suppose that $l > k > N$, then

$$\begin{aligned} |f_k(x) - f_l(x)| &= \sum_{n=k}^{l} \frac{x}{n^\alpha(1+nx^2)} \\ &\leq \sum_{n=k}^{l} \frac{L}{n^\alpha(1+nL^2)} \end{aligned}$$