Homework 9 of Introduction to Analysis(II)

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1. By observe, $\Delta(x) = \det(D^2 f(x)) \geq 0$.

   (a) Since $\Delta(x) \geq 0$ and $f_x x(x) > 0$, $D^2 f(x)$ is semi-definite on $D(c,\delta)$. Thus, for any $x \in D(c,\delta)$,

   $$\begin{aligned} Df(x) &= Df(x) - Df(c) \\ &= \int_0^1 D^2 f(c + t(x-c))(x-c)\, dt \\ &\geq \int_0^1 0(x-c)\, dt \\ &\geq 0 \end{aligned}$$

   Then,

   $$\begin{aligned} f(x) - f(c) &= \int_0^1 Df(c + t(x-c))(x-c)\, dt \\ &\geq \int_0^1 0(x-c)\, dt \\ &\geq 0 \end{aligned}$$

   (b)

2. Since $f_x(x_0) = f_y(x_0) = 0$ and $f \in C^2(V,\mathbb{R})$, $Df(x_0) = 0$ and x_0 is a critical point.

   If $f_{xy}(x_0) \neq 0$, $\det(D^2 f(x_0)) = -(f_{xy}(x_0))^2 < 0$, then $D^2 f(x_0)$ is not positive definite and negative definite. Thus, x_0 will be saddle point.

3. $R_{r-1}(x_0,h) = \dfrac{1}{r} D^r f(c)(h,\cdots,h)$