Homework 6 of Introduction to Analysis(II)

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1. Let $\phi(x) = \arctan(x)$, then $\phi \circ f(x) = \phi(f(x))$ is bdd by $(-\frac{\pi}{2}, \frac{\pi}{2})$. Then, by Tietze's Extension Theorem, there exists $g \in C(\mathbb{R}^n, \mathbb{R})$ s.t. $g(x) = \phi(f(x))$ for all $x \in D$ and $\sup|g(x)| = \sup|\phi(f(x))| \leq \frac{\pi}{2}$.

   Thus, there exists $h(x) = \tan(g(x))$ in $C(\mathbb{R}^n, \mathbb{R})$ and $h(x) = f(x)$ for all $x \in D$ by $\phi(x)$ is invertible.

2. We want to show that $\lim_{h \to 0} \frac{\|f(x+h) - f(x) - h \cdot Df(x)\|}{h}$ exists for all x.