# Homework 11 of Introduction to Analysis(II)

AM15 黃琦翔 111652028

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1.

2. Suppose $f \in C^1(\mathbb{R}^2, \mathbb{R})$, $Df(x_0, y_0) \neq 0$ for some $x_0, y_0$ (or $f$ is constant function and not one-to-one). Then, suppose $\frac{\partial f}{\partial x} \neq 0$ for neighborhood of $(x_0, y_0)$, and let $h(x, y) = f(x, y) - f(x_0, y_0)$ with $\frac{\partial h}{\partial x} \neq 0$. by Implicit Function Theorem, there is a neighborhood $U \subseteq \mathbb{R}^2$ and $W \subseteq \mathbb{R}$ s.t. $(x_0, y_0) \in U$ and $y_0 \in W$ and a function $g : W \to \mathbb{R}^2$ s.t. $h(g(y), y) = f(g(y), y) - f(x_0, y_0) = 0$. Then, $f(g(y), y) = f(x_0, y_0)$ for $y \in W$ and $f$ is not one-to-one. If $\frac{\partial f}{\partial x} = 0$, then $\frac{\partial f}{\partial y} \neq 0$ and use the same argument can get the same result.

3.