Exercises(11)

April 30, 2024

1. (7 points) Show that the Inverse Function Theorem by the Implicit Function Theorem.

2. (7 points) Let $f : \mathbb{R}^2 \to \mathbb{R}$ belong to class $C^1(\mathbb{R}^2, \mathbb{R})$. Show that f is not injective (one-to-one).

3. (10 points) For each of the following, find all (local) extremum values of f subject to the given constraints

   (a) $f(x, y, z) = xy$, $x^2 + y^2 + z^2 = 1$ and $x + y + z = 0$;

   (b) $f(x, y, z, w) = 3x + y + w$, $3x^2 + y + 4z^3 = 1$ and $-x^3 + 3z^4 + w = 0$.

4. (6 points) Let $F : \mathbb{R}^2 \to \mathbb{R}$ be defined by $F(x, y) = y^2 - x$. Show that there does not exist a function ϕ defined on a neighborhood W of 0 such that $F(x, \phi(x)) = 0$ for all $x \in W$.