Exercises(6)

March 26, 2024

1. (10 points) Use Tietze's extension Theorem to show that if $D \subseteq \mathbb{R}^n$ is closed and if f is an unbounded continuous function on $D \to \mathbb{R}$, then there exists a continuous extension of f to all of $\mathbb{R}^n$. (Hint: consider the composition $\phi \circ f$, where $\phi(x) = \tan^{-1} x$.)

2. (10 points) Let $f : A(\text{open}) \subseteq \mathbb{R} \to \mathbb{R}^m$ and assume $\frac{df_i}{dx}$ exists for $i = 1, \ldots, m$. Show that Df exists.

3. (10 points) Suppose that f is a real-value function defined in an open set $E \subseteq \mathbb{R}^n$, and that the partial derivatives $\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n}$ are bounded in E. Prove that f is continuous in E.