

# Homework 13 of Introduction to Analysis(II)

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1. (a) For any  $\varepsilon > 0$ , we can find an open cube  $D(x_0, \delta)$  that  $R_j \subseteq D(x_0, \delta)$  and  
(b) Since for any  $E_i$  is measure zero, we can find  $\{R_{i,j}\}_j$  s.t.  $\sum_j |R_{i,j}| < \frac{\varepsilon}{2^i}$ . Then, let  $R_{1,1} = R_1$ ,  
 $R_{1,2} = R_2, R_{2,1} = R_3, R_{1,3} = R_4, R_{2,2} = R_5$  etc. And we can get  $\{R_k\}_k$  contains  $\bigcup_i E_i$  and  $\sum_k |R_k| < \varepsilon(\frac{1}{2} + \frac{1}{4} + \cdots) = \varepsilon$ . Therefore, union of countable measure zero sets is measure zero.  
(c) Since  $A$  is compact,  $A$  is closed and bounded.
2. For any grid  $\mathfrak{O}$ ,  $U(1, g) = L(1, g)$