

Homework 2 of Introduction to Analysis(II)

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1. For all $\varepsilon > 0$, we can find a $N \in \mathbb{N}$ s.t. $|a_n - a| < \frac{\varepsilon}{2}$ for all $n > N$. And we can find a $N' > N$ s.t.

$$\frac{\sum_{i=1}^N a_i - a}{N'} < \frac{\varepsilon}{2}. \text{ Thus, for any } n > N',$$

$$\begin{aligned} \left| \frac{\sum_{i=1}^n a_i}{n} - a \right| &\leq \left| \frac{\sum_{i=1}^N a_i - a}{n} \right| + \left| \frac{\sum_{i=N}^n a_i - a}{n - N'} \right| \\ &< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} \\ &= \varepsilon \end{aligned}$$

Therefore, $\lim_{n \rightarrow \infty} b_n = a$.

2.