Homework 14 of Introduction to Analysis(II)

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1. Let $A_M = \{x \in E \mid f_M \text{ is discontinuous at } x\}$ and $A = \{x \in E \mid f \text{ is discontinuous at } x\}$. For any $M \in \mathbb{N}$, if x is a point that f_M is discontinuous at x, That means $f(x) \leq M$ and f is discontinuous at x. That is, $A_M \subseteq A$ for all M implies that $\cup A_m \subseteq A$.

   And for any $x \in A$, there exists a $N \in \mathbb{N}$ s.t. $f(x) < N$. Then, $x \in A_N$. Therefore, $A = \cup A_M$.

2. Let $\displaystyle\int_0^1 f(x)\, dx = \alpha$ and $\displaystyle\sup_{x\in[0,1]} \sup f(x) = M$.

   By 1,

   Since f is integrable, there exists a partition P s.t. $\int_0^1 f(x)\, dx - L(f,P) \leq \dfrac{\alpha}{4}$.

3. Let $A = \{x \in E \mid f(x) \neq 0\}$. If A is empty, then A is measure zero.

   Suppose A is non-empty. Since $\displaystyle\int_E f(x)\, dx = 0$, then for any $\varepsilon > 0$, there exists rectangles R_n cover E such that (U)$\displaystyle\int_E f(x)\, dx = \sum \sup_{x\in R_i} f(x) \cdot |R_i| < \varepsilon$.