# Homework 12 of Introduction to Analysis(II)

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1. (a) Since $E$ is Jordan region, $\text{Vol}(\partial E) = 0$.

   (b) Since $\text{cl}(E) = \text{int}(E) \cup \partial E$ and $\text{int}(E) \subseteq E \subseteq \text{cl}(E)$,

   $\text{Vol}(\text{cl}(E)) = \text{Vol}(\text{int}(E)) + \text{Vol}(\partial E) = \text{Vol}(\text{int}(E)) \leq \text{Vol}(E) \leq \text{Vol}(\text{cl}(E))$.

   Therefore, $\text{Vol}(\text{cl}(E)) = \text{Vol}(\text{int}(E)) = E$.

   (c)

   ($\Longrightarrow$) From $(b)$, we know $\text{Vol}(\text{int}(E)) = \text{Vol}(E) > 0$, then we can find a set of rectangles $R_n$ s.t. $\sum |R_n| > 0$ and $\cup R_n \subseteq \text{int}(E)$. Therefore, $\text{int}(E) \neq \emptyset$.

   ($\Longleftarrow$) Since $\text{int}(E)$ is non-empty, for any $x_0 \in \text{int}(E)$, there exists $\varepsilon > 0$ s.t. $D(x_0, \varepsilon) \subseteq \text{int}(E)$. Then, we can find a small rectangle $R$ with each length is $\dfrac{\varepsilon}{2}$ and $R$ is contained in $D(x_0, \varepsilon)$. Thus, $\text{Vol}(\text{int}(E)) > \left(\dfrac{\varepsilon}{2}\right)^2 > 0$.

   (d)